University of Engineering and Technology, Peshawar

Engineering Economics

CSE-305

(Chapter 03b)

Principles of Money-Time Relationship

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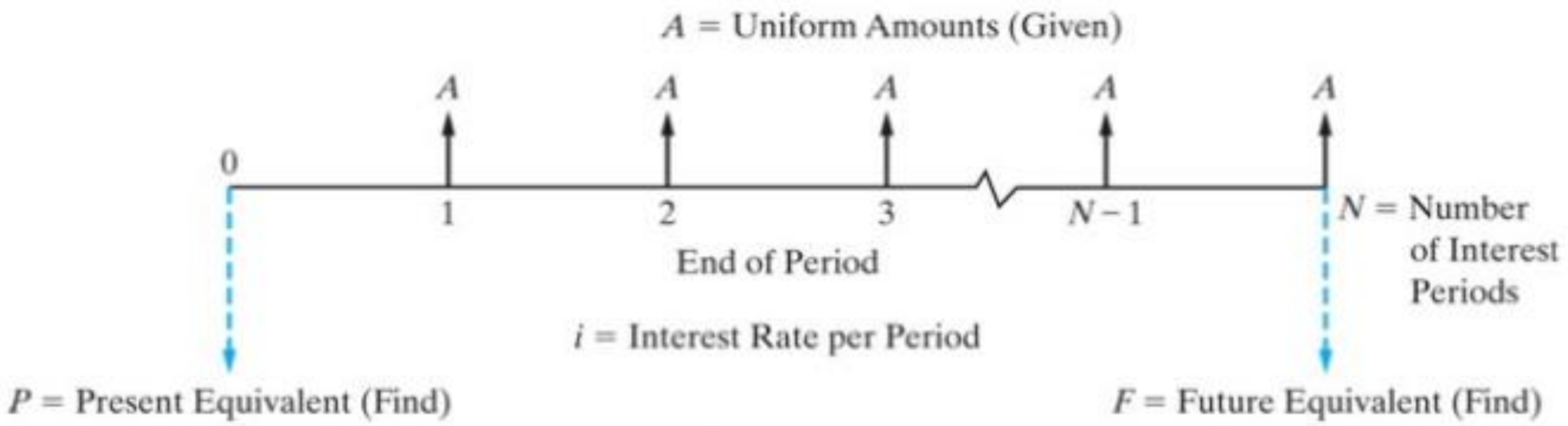
Agenda

- Annuity- Uniform Payment Series
- Uniform Series: Compound Amount Factor
- Uniform Series: Capital Recovery Factor
- Uniform Series: Sinking Fund Factor
- Deferred Loan Repayment Plans
- Early Savings Plan
- Deferred Savings Plan

Uniform Series: Annuity

➢ **A series of uniform receipts A, at the end of each period for N periods with interest at i% per period.**



General cash flow diagram relating uniform series(ordinary annuity) to present worth and future worth

PW, FW and AW

- Present worth occurs one period before A (uniform Payment)
- Future worth occurs at the same time as the last A and N periods after P
- Annual worth occurs at the end of periods 1 through N, Inclusive

Finding F given A:

$$F = A[\frac{(1+i)^N - 1}{i}] \qquad F = A(F/A, i\%, N)$$

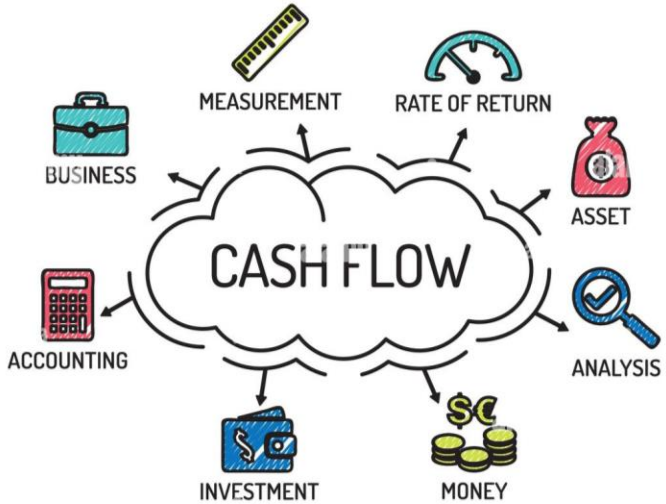
Uniform Series Compound Amount Factor.

Uniform Series: Cash Flows

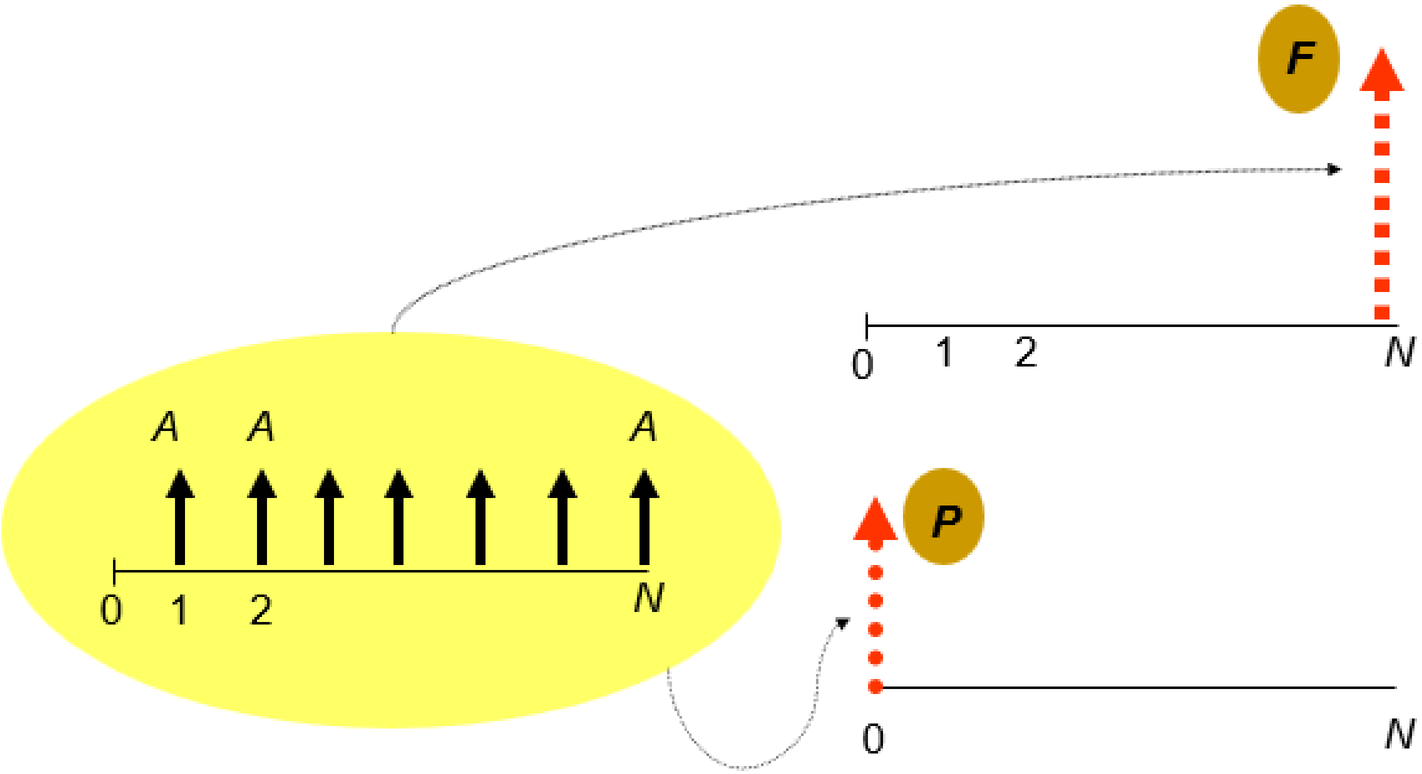
- Time Scale A woman wishes to have \$100,000 in her retirement savings plan after working for 25 years. She will accomplish this by depositing A dollars each year in each year in a savings account that earns 6% per year. How much must she save each year?

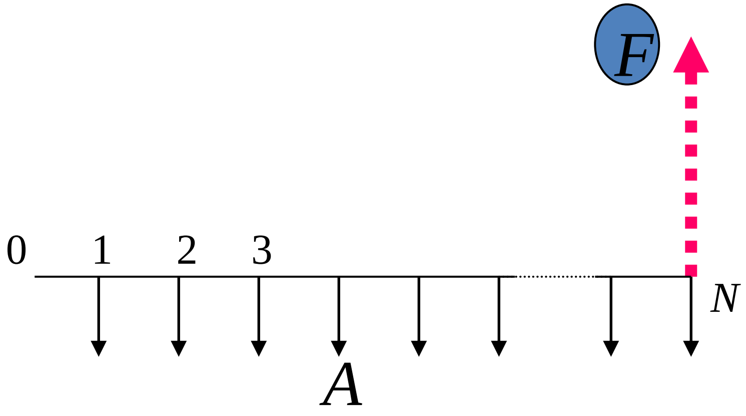
The annual deposit required to accumulate \$100,00 at 6% annual interest is:

$$A = \$100{,}000\,(A/F, 6\%, 25)$$
$$= \$100{,}000\,(0.0182)$$
$$= \$1{,}820{,}000$$



Uniform Series: Equal Cash Flows



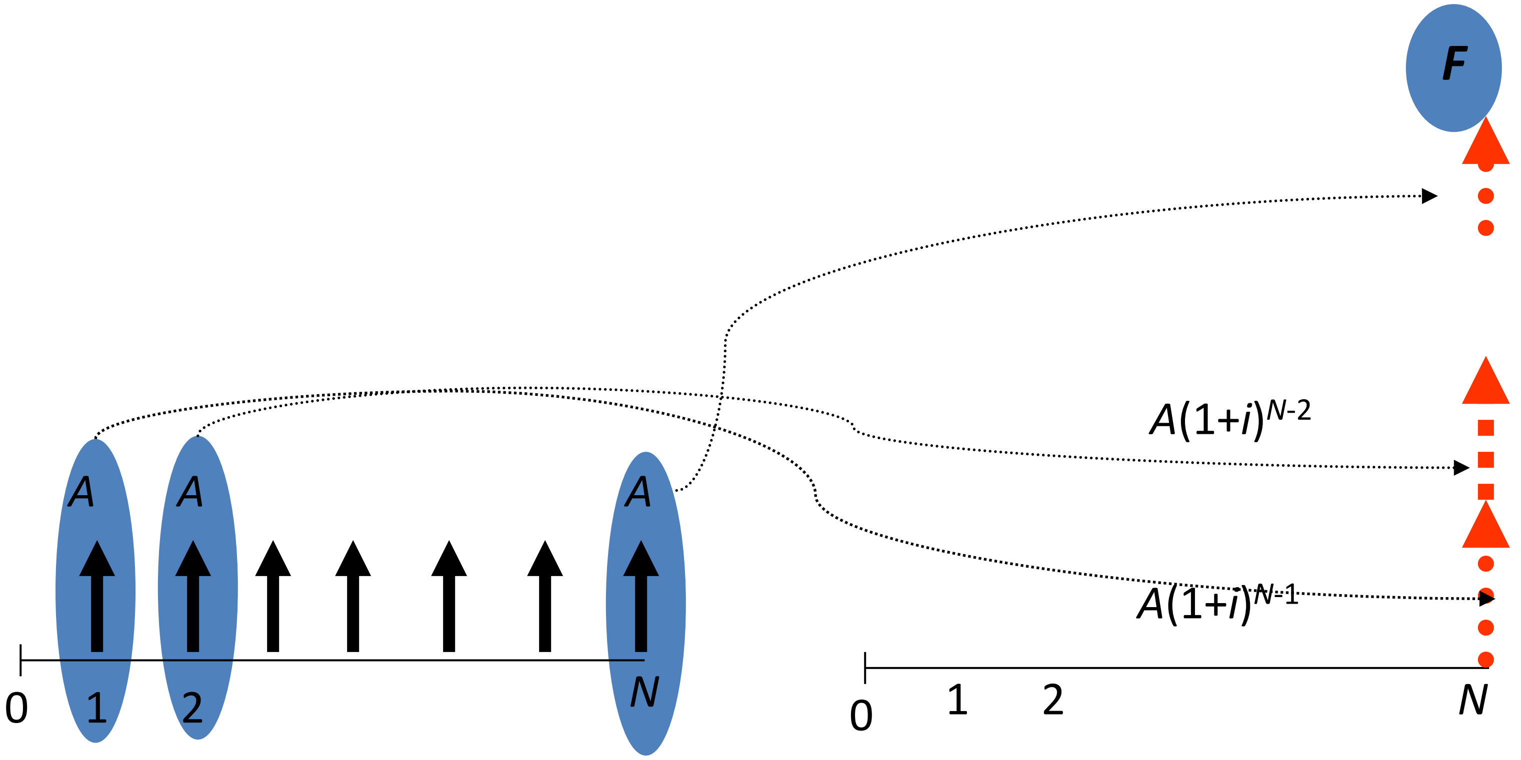


$$F = A\boxed{\frac{(1+i)^N - 1}{i}}$$

$$= A(F/A, i, N)$$

$$F = A(1+i)^{N-1} + A(1+i)^{N-2} + \cdots + A$$

Equal Payment Series: Compound Amount Factor



Example:

Given: A = \$5,000,

N = 5 years,

and i = 6%

Finding: F

Solution:

F = \$5,000(F/A,6%,5)

= \$28,185.46

Equal Payment Series: Compound Amount Factor

$$\$5,000(1+0.06)^4 = \$6,312.38$$

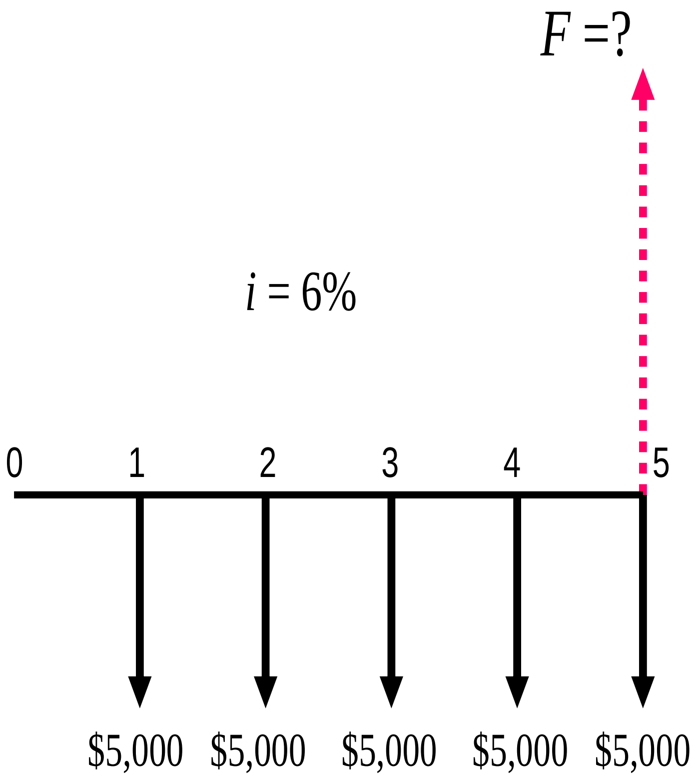
$$\$5,000(1+0.06)^3 = \$5,955.08$$

$$\$5,000(1+0.06)^2 = \$5,618.00$$

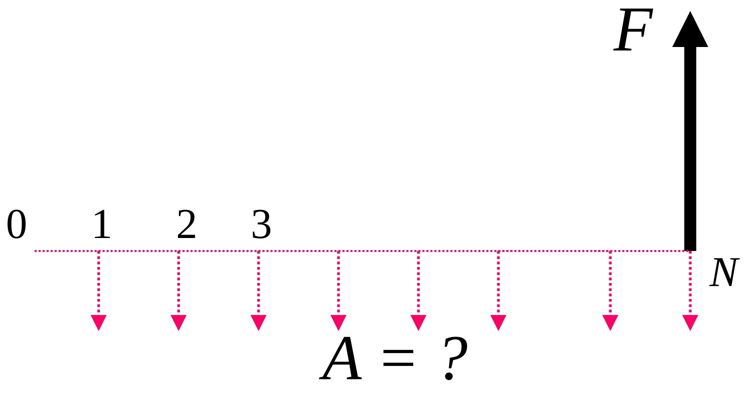
$$\$5,000(1+0.06)^1 = \$5,300.00$$

$$\$5,000(1+0.06)^0 = \$5,000.00$$

$$\$28.185.46$$



Equal Payment Series: Compound Amount Factor

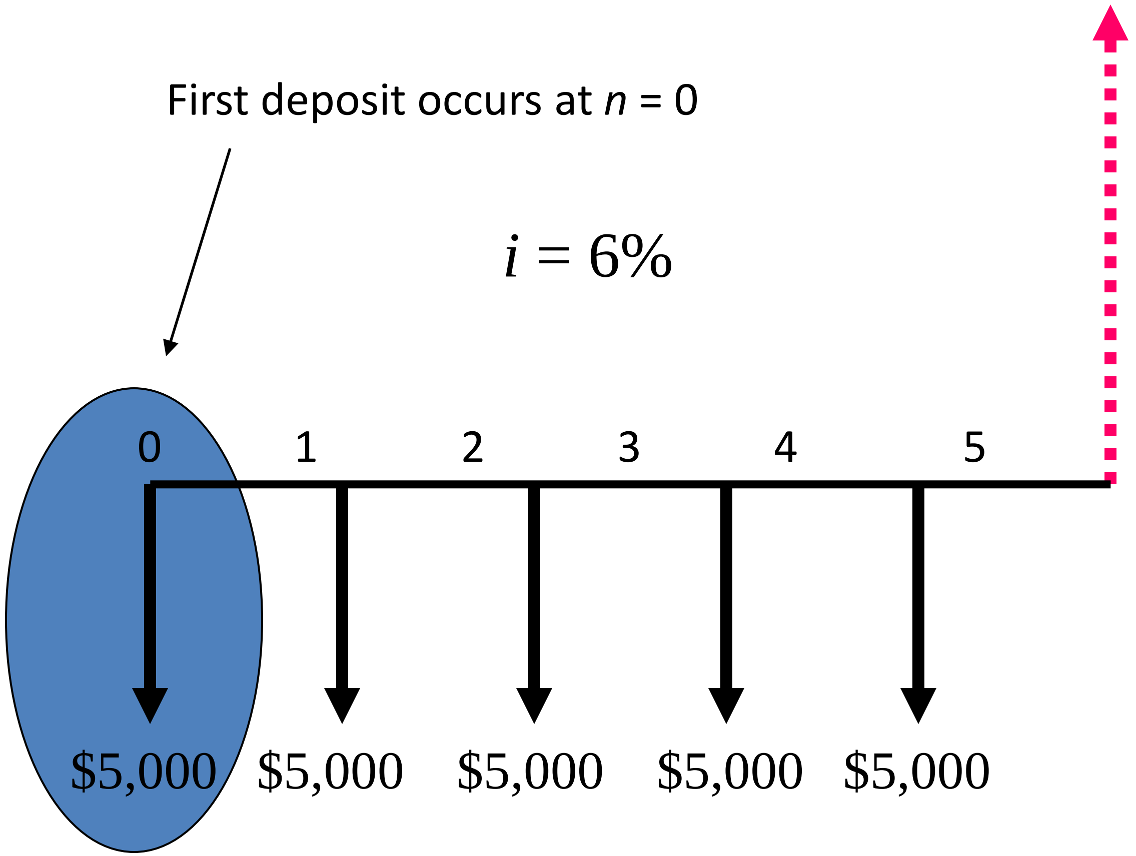


$$A = F\left[\frac{i}{(1+i)^N - 1}\right] = F(A/F, i, N)$$

Example:

- Given: F = \$5,000, N = 5 years, and i = 7%
- Find: A
- Solution: A = \$5,000(A/F,7%,5) = \$869.50

Equal Payment Series: Compound Amount Factor

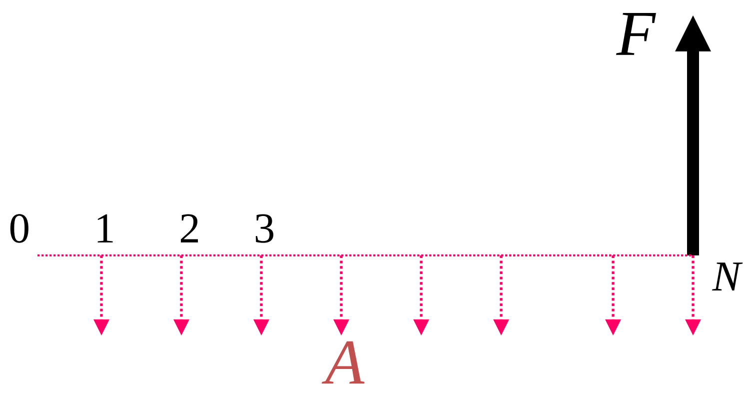


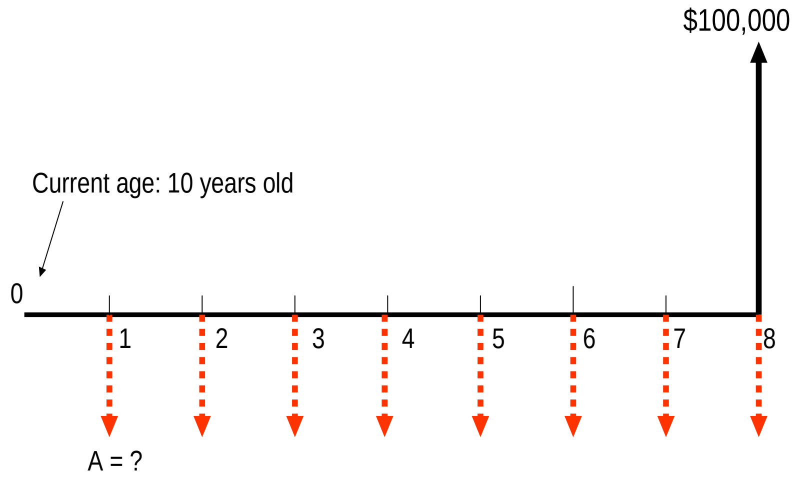
$F = ?$

Annuity Due

$$F_5 = \$5{,}000(F/A, 6\%, 5)(1.06)$$
$$= \$29{,}876.59$$

Equal Payment Series: Sinking Fund Factor





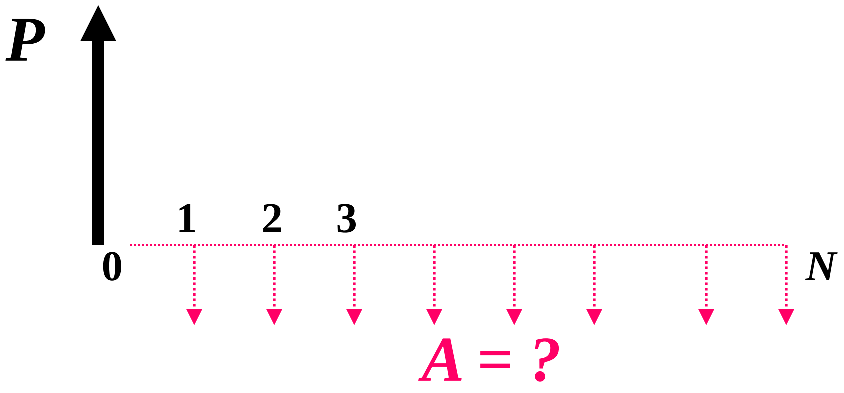
Example: College Savings Plan:

- Given: F = \$100,000, N = 8 years, and i = 7%
- Solution:

A = \$100,000(A/F,7%,8) = \$9,746.78

$$A = F\frac{i}{(1+i)^N - 1}$$

Uniform Series: Capital Recovery Factor



$$A = P\boxed{\frac{i(1+i)^N}{(1+i)^N - 1}}$$

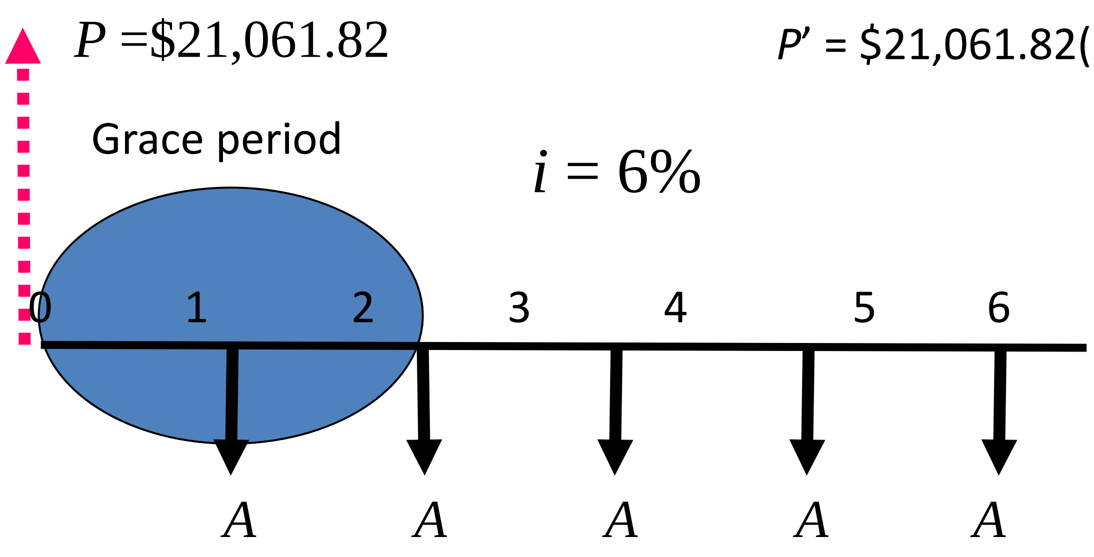
$$= P(A/P, i, N)$$

Example 2.12: Paying Off Education Loan

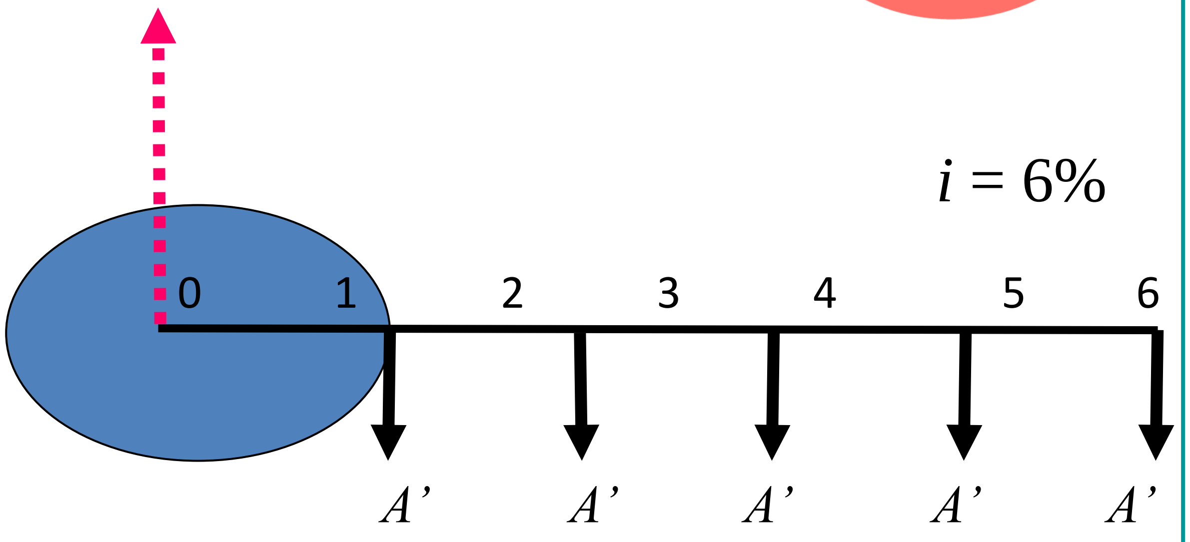
- Given: P = \$21,061.82, N = 5 years, and i = 6%
- Find: A
- Solution: A = \$21,061.82(A/P,6%,5) = \$5,000

Deferred Loan Repayment Plan

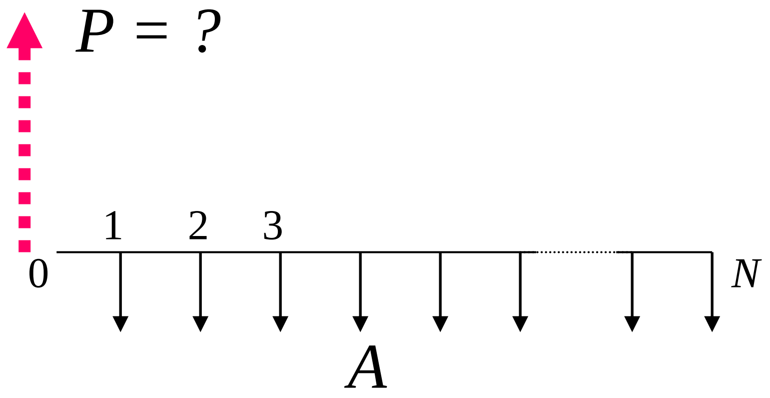
$P' = \$21{,}061.82(F/P, 6\%, 1)$



$$
\begin{aligned}
P' &= \$21{,}061.82(F/P, 6\%, 1) \\
&= \$22{,}325.53 \\
A &= \$22{,}325.53(A/P, 6\%, 5) \\
&= \$5{,}300
\end{aligned}
$$



Uniform Series: Present Worth Factor

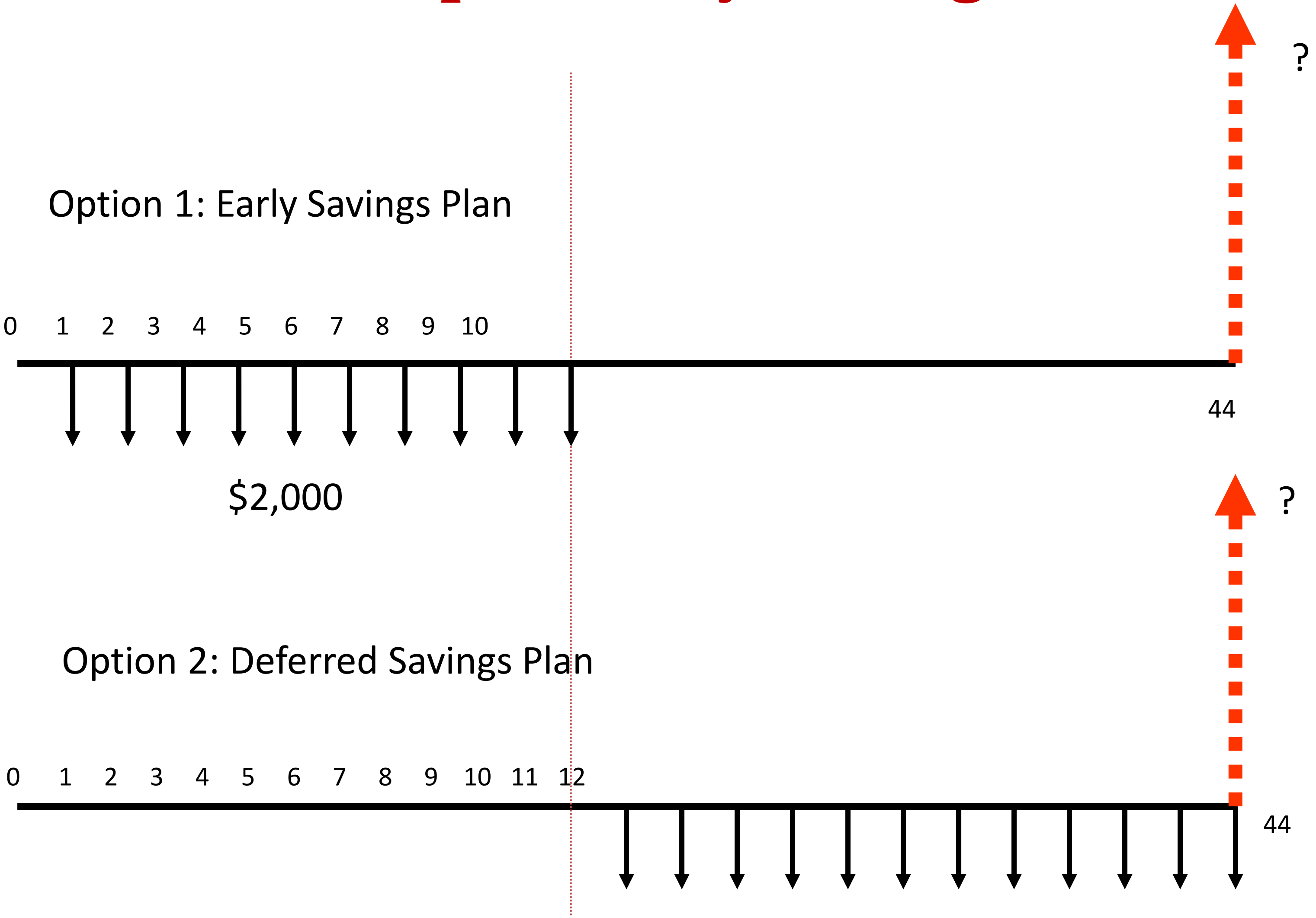


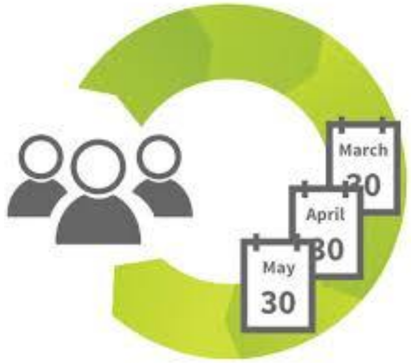
$$P = A\boxed{\frac{(1+i)^N - 1}{i(1+i)^N}}$$
$$= A(P/A, i, N)$$

Example 2.14: Powerball Lottery

- Given: A = \$7.92M, N = 25 years, and i = 8%
- Find: P
- Solution: P = \$7.92M(P/A,8%,25) = \$84.54M

Example: Early Savings Plan

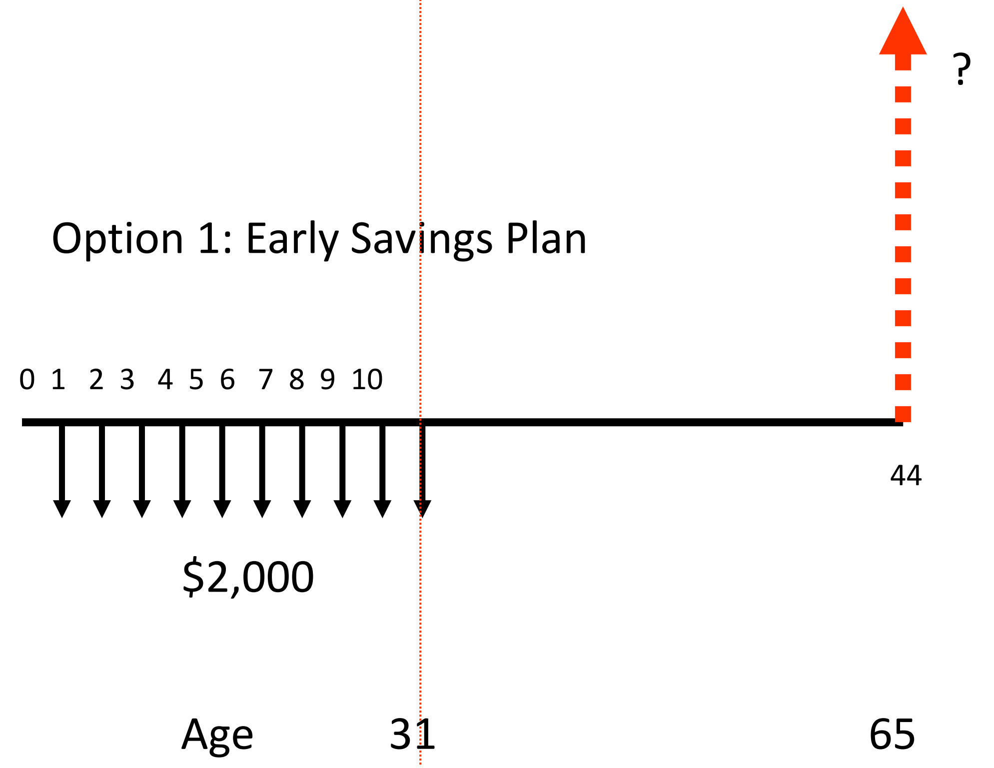




Option 1: Early Savings Plan

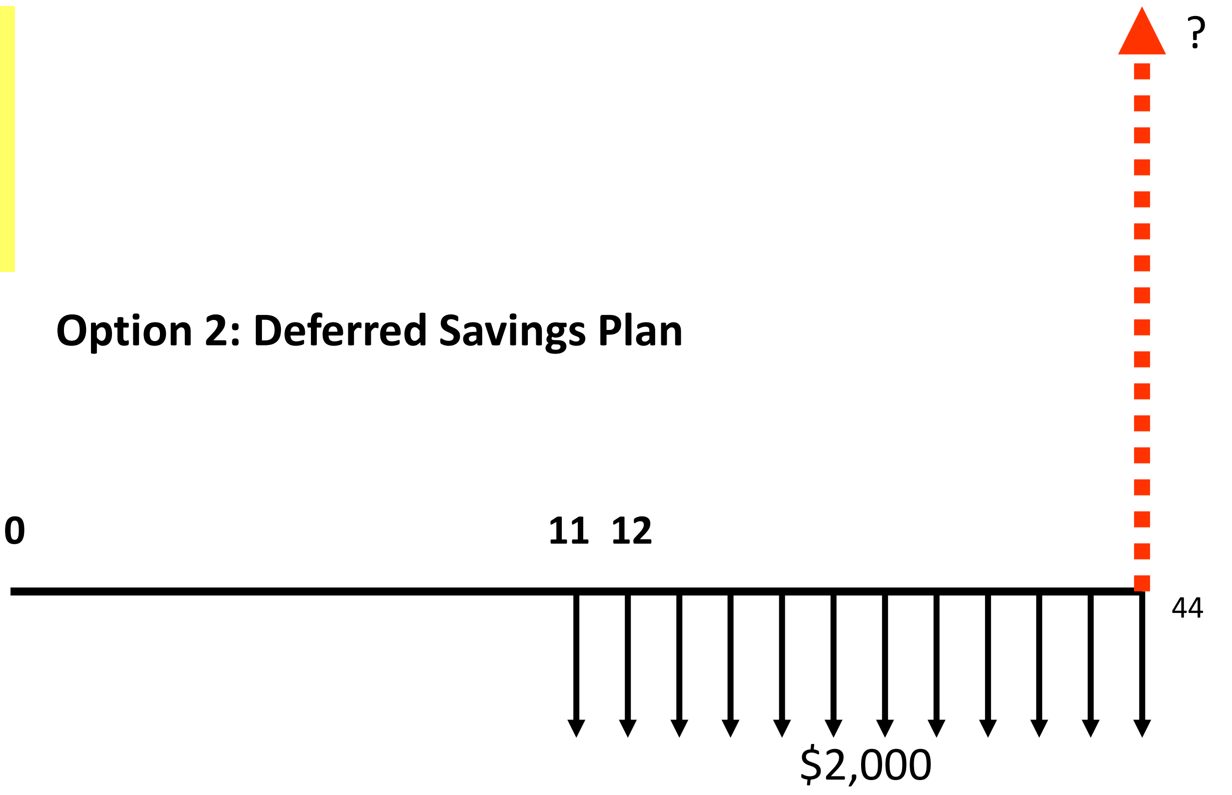
$$F_{10} = \$2,000(F/A, 8\%, 10)$$
$$= \$28,973$$

$$F_{44} = \$28,973(F/P, 8\%, 34)$$
$$= \$396,645$$

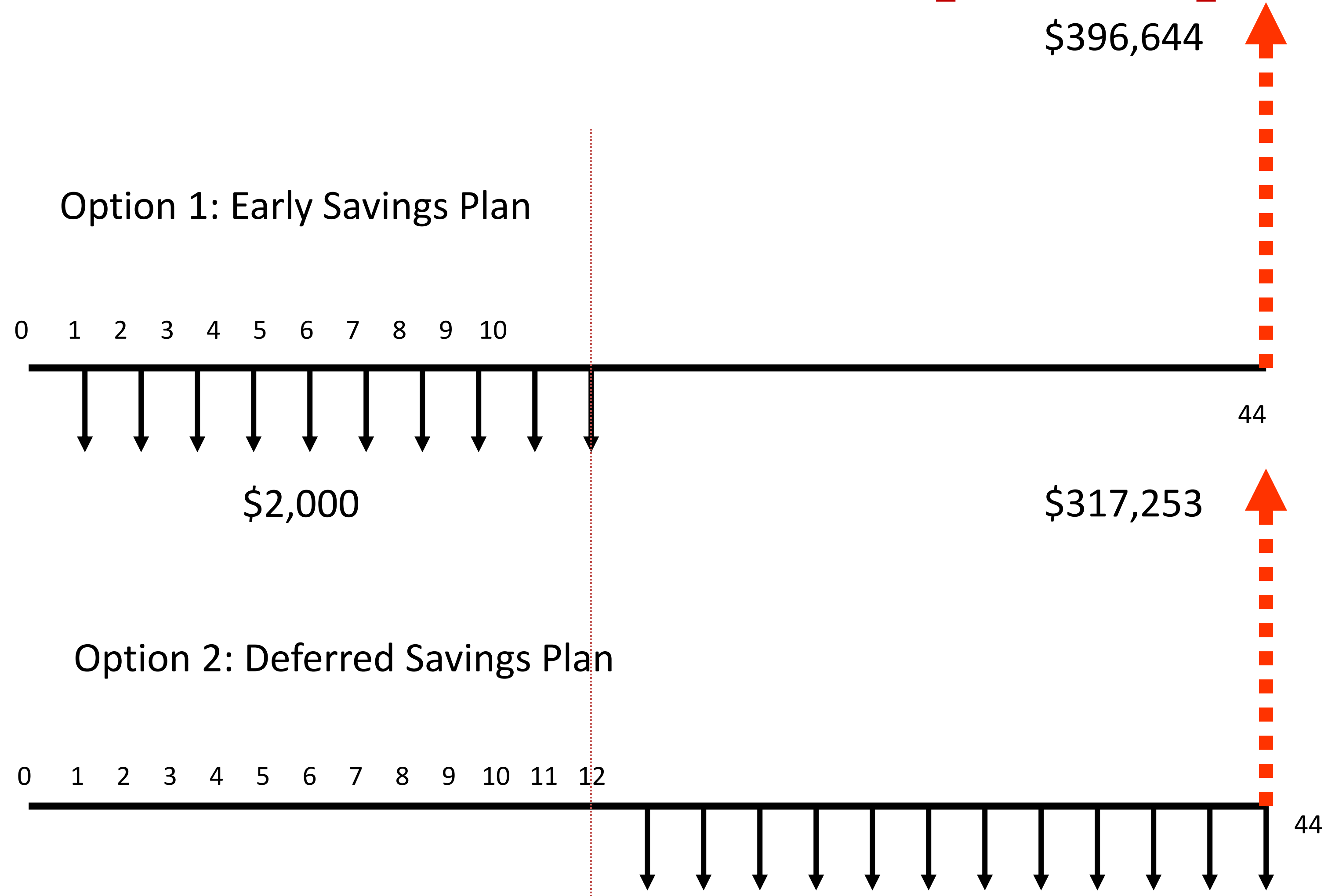


Option 2: Deferred Savings Plan

$$F_{44} = \$2,000(F/A, 8\%, 34)$$
$$= \$317,233$$



What interest would make these plans Equivalent?



Finding Equivalence

Option 1:

$$F_{44} = \$2000\ (F/A,\ i\%,\ 10)\ (F/P,\ i\%,\ 34)$$

Option 2:

$$F_{44} = \$2000\ (F/A,\ i\%,\ 34)$$

Option 1= Option 2

$$\$2000\ (F/A,\ i\%,\ 10)\ (F/P,\ 8\%,\ 34) = F_{44} = \$2000\ (F/A,\ i\%,\ 34)$$

Solve for i.

Finding Present Worth

For Present Worth in uniform Series can be evaluated from Compound Amount Factor:

$$P(1+i)^N = \frac{A[(1+i)^N - 1)]}{i}$$

Hence:

$$P = A[\frac{[(1+i)^N - 1)]}{(1+i)^N i}]$$

The quantity in the square brackets is known as **Uniform Series Present Worth Factor (P/A, i%, N)**

Present Worth: Example

If a certain machine undergoes a major overhaul, its output can be increased by 20% which translates into an extra cash flow of $20,000 at the end of each year for 5 years. If i=15% per year, how much can we afford to invest to overhaul this machine?

Solution:

The increase in cash flow is $20,000 per year and it continues for 5 years at 15% annual interest. The upper limit on what we can Afford to spend is:

$$P = \$20{,}000\ (P/A, 15\%, 5)$$

$$= \$20{,}000\ (3.3522) = \$\ 67{,}044$$

Present Worth: Example

Suppose your rich uncle has $1,000,000 that he wishes to distribute among his heirs at the rate of $100,000 per year. If the amount is deposited in a bank account that earns 6% effective interest each year, how many years will it take to completely deplete the account? How long will it take if the interest is 8%?

Solution:

<u>For i= 6%:</u>

$$P = A\,(P/A, i\%, N)$$

$$\$1{,}000{,}000 = \$100{,}000\,(P/A, 6\%, N))$$

$$N = 15.7$$

<u>For i= 8%:</u>

$$P = A\,(P/A, i\%, N)$$

$$\$1{,}000{,}000 = \$100{,}000\,(P/A, 8\%, N))$$

$$N = ?$$

Capital Recovery Factor

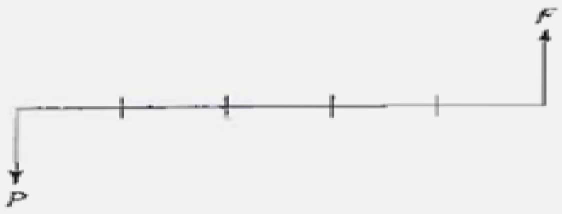
Finding A given P.

$$A = P\left[\frac{i(1+i)^N}{(1+i)^N - 1}\right]$$

The quantity in square brackets is known as Uniform Series Capital Recovery Factor (A/P, i%, N)

Discrete Cash Flows: Equivalence Chart

Single Payment



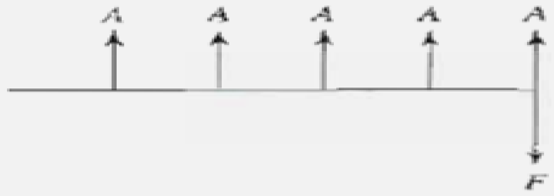
Compound Amount:

To Find F Given P — $(F/P,i,n)$ — $F = P(1+i)^n$

Present Worth:

To Find P Given F — $(P/F,i,n)$ — $P = F(1+i)^{-n}$

Uniform Series

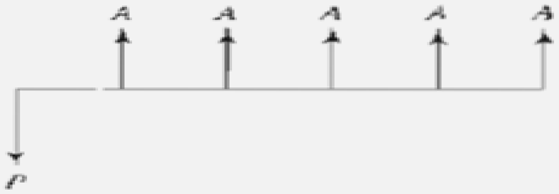


Series Compound Amount:

To Find F Given A — $(F/A,i,n)$ — $F = A\left[\dfrac{(1+i)^n - 1}{i}\right]$

Sinking Fund:

To Find A Given F — $(A/F,i,n)$ — $A = F\left[\dfrac{i}{(1+i)^n - 1}\right]$



Capital Recovery:

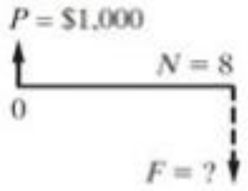
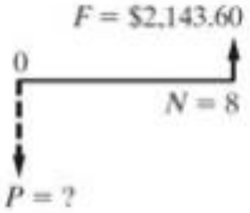
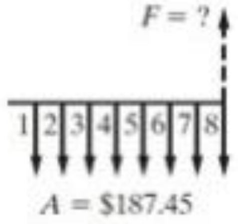
To Find A Given P — $(A/P,i,n)$ — $A = P\left[\dfrac{i(1+i)^n}{(1+i)^n - 1}\right]$

Series Present Worth:

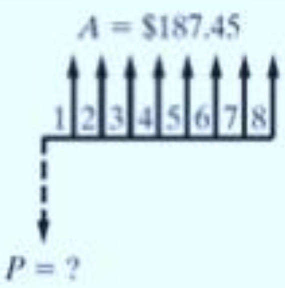
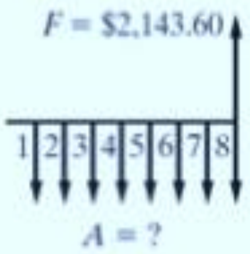
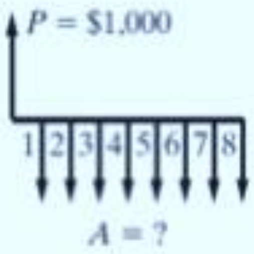
To Find P Given A — $(P/A,i,n)$ — $P = A\left[\dfrac{(1+i)^n - 1}{i(1+i)^n}\right]$

Discrete Cash Flows: Examples

Example Problems (All Using an Interest Rate of $i = 10\%$ per Year—See Table C-13 of Appendix C)

To Find:	Given:	(a) In Borrowing–Lending Terminology:	(b) In Equivalence Terminology:	Cash-Flow Diagram[a]	Solution
For single cash flows:					
F	P	A firm borrows $1,000 for eight years. How much must it repay in a lump sum at the end of the eighth year?	What is the future equivalent at the end of eight years of $1,000 at the beginning of those eight years?		$F = P(F/P, 10\%, 8)$ $= \$1{,}000(2.1436)$ $= \$2{,}143.60$
P	F	A firm wishes to have $2,143.60 eight years from now. What amount should be deposited now to provide for it?	What is the present equivalent of $2,143.60 received eight years from now?		$P = F(P/F, 10\%, 8)$ $= \$2{,}143.60(0.4665)$ $= \$1{,}000.00$
For uniform series:					
F	A	If eight annual deposits of $187.45 each are placed in an account, how much money has accumulated immediately after the last deposit?	What amount at the end of the eighth year is equivalent to eight EOY payments of $187.45 each?		$F = A(F/A, 10\%, 8)$ $= \$187.45(11.4359)$ $= \$2{,}143.60$

Discrete Cash Flows: Examples

P	A	How much should be deposited in a fund now to provide for eight EOY withdrawals of \$187.45 each?	What is the present equivalent of eight EOY payments of \$187.45 each?		$P = A(P/A, 10\%, 8)$ $= \$187.45(5.3349)$ $= \$1{,}000.00$
A	F	What uniform annual amount should be deposited each year in order to accumulate \$2,143.60 at the time of the eighth annual deposit?	What uniform payment at the end of eight successive years is equivalent to \$2,143.60 at the end of the eighth year?		$A = F(A/F, 10\%, 8)$ $= \$2{,}143.60(0.0874)$ $= \$187.45$
A	P	What is the size of eight equal annual payments to repay a loan of \$1,000? The first payment is due one year after receiving the loan.	What uniform payment at the end of eight successive years is equivalent to \$1,000 at the beginning of the first year?		$A = P(A/P, 10\%, 8)$ $= \$1{,}000(0.18745)$ $= \$187.45$

Summary

- Annuity- Uniform Payment Series
- Uniform Series: Compound Amount Factor
- Uniform Series: Capital Recovery Factor
- Uniform Series: Sinking Fund Factor
- Deferred Loan Repayment Plans
- Early Savings Plan
- Deferred Savings Plan