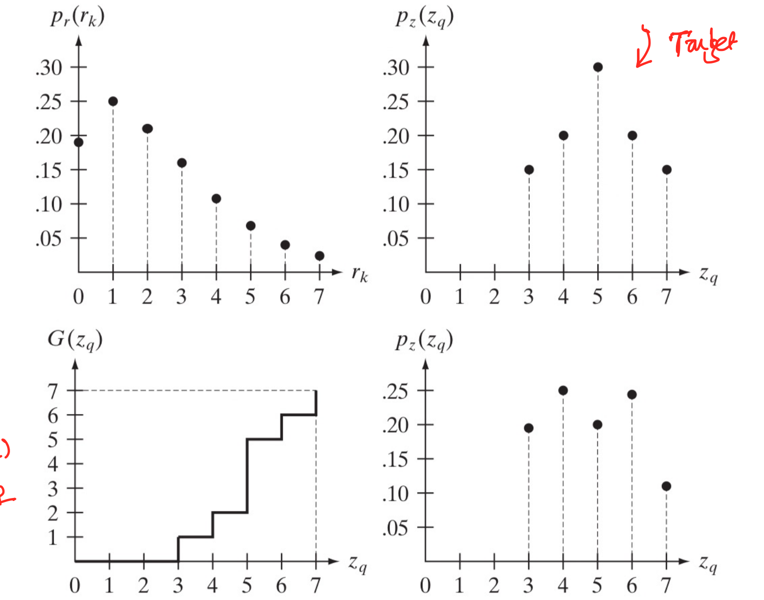
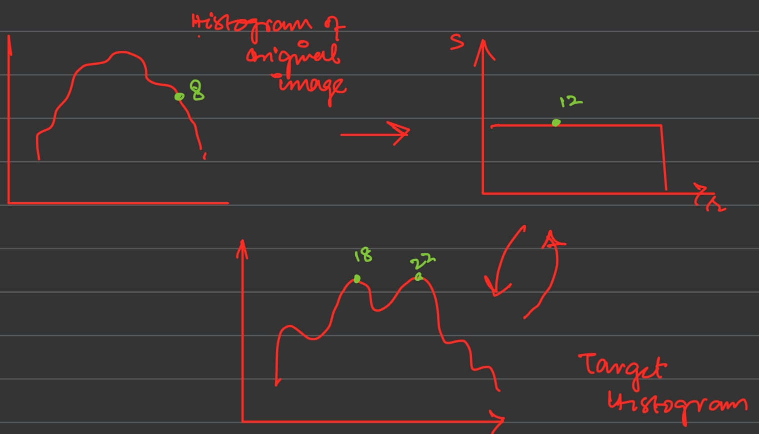
Histogram Matching

a b
c d

FIGURE 3.22
(a) Histogram of a 3-bit image. (b) Specified histogram.
(c) Transformation function obtained from the specified histogram.
(d) Result of performing histogram specification. Compare (b) and (d).

T(z) map

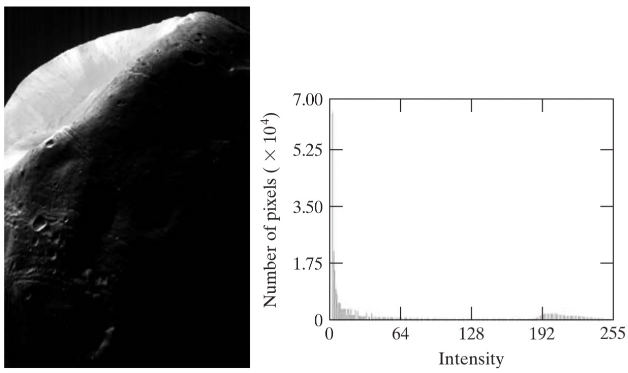


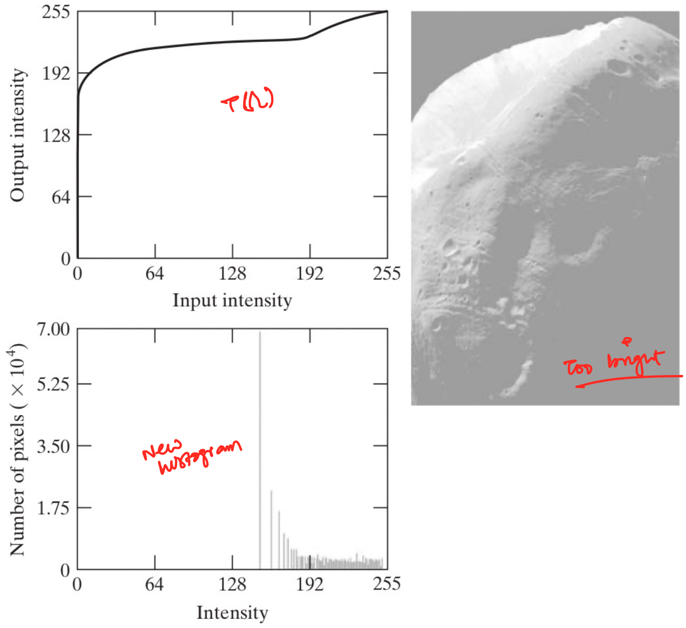


There may not be one to one mapping in the inverse. The most common decision is to choose the pixel value close to the original

a b

FIGURE 3.23
(a) Image of the Mars moon Phobos taken by NASA's *Mars Global Surveyor.*
(b) Histogram.
(Original image courtesy of NASA.)





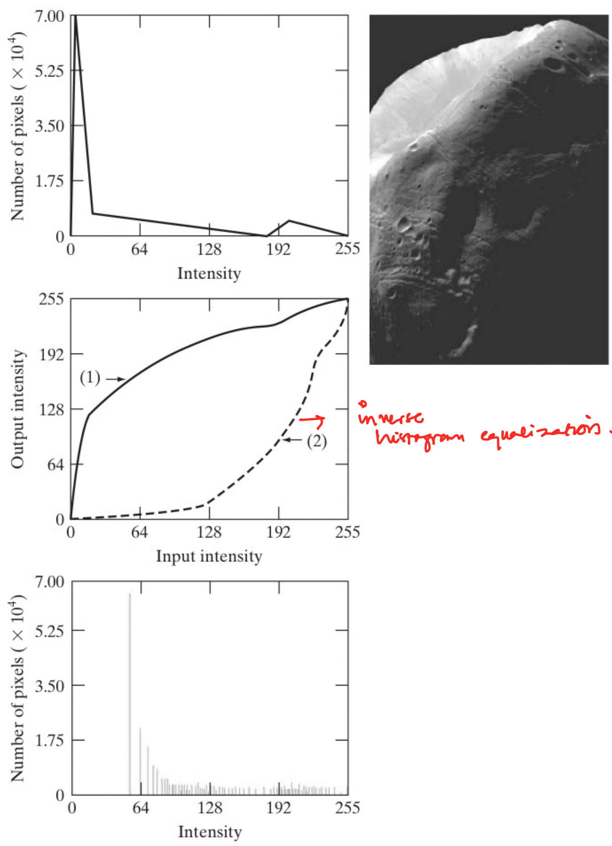
a b
c

FIGURE 3.24
(a) Transformation function for histogram equalization.
(b) Histogram-equalized image (note the washed-out appearance).
(c) Histogram of (b).

After Histogram matching

a c
b
d

FIGURE 3.25
(a) Specified histogram.
(b) Transformations.
(c) Enhanced image using mappings from curve (2).
(d) Histogram of (c).



inverse histogram equalization.

Q. Assuming continuous intensity values, let say an image has the intensity PDF such as :

$$P_r(r) = \begin{cases} \dfrac{2r}{(L-1)^2} & 0 \le r \le L-1 \\ 0 & \text{otherwise} \end{cases}$$

Find the transformation function that will produce an image whose intensity PDF is :

$$P_z(z) = \begin{cases} \dfrac{3z^2}{(L-1)^3} & 0 \le r \le L-1 \\ 0 & \text{otherwise} \end{cases}$$

Solution: The transformation function is given as:

$$S = T(r) = (L-1)\int_0^r P(w)\,dw \rightarrow (1)$$

$$T(r) = (L-1)\int_0^r \frac{2w}{(L-1)^2}\,dw$$

$$= \frac{2\cancel{(L-1)}}{(L-1)^{\cancel{2}}}\int_0^r w\,dw$$

$$= \frac{\cancel{2}}{(L-1)} \cdot \frac{r^2}{\cancel{2}}$$

$$\boxed{S = T(r) = \frac{r^2}{L-1}} \rightarrow (2)$$

$$G(z) = (L-1)\int_0^z P_z(t)\,dt \rightarrow (3)$$

$$= (L-1)\int_0^z \frac{3t^2}{(L-1)^3}\,dt$$

$$= \frac{3\cancel{(L-1)}}{(L-1)^{\cancel{3}}}\int_0^z t^2\,dt$$

$$= \frac{\cancel{3}}{(L-1)^2} \cdot \frac{z^3}{\cancel{3}} = \frac{z^3}{(L-1)^2}$$

$$\boxed{G(z) = \frac{z^3}{(L-1)^2}} \rightarrow (4)$$

We know that,

$$G(z) = s$$

$$\Rightarrow \frac{z^3}{(L-1)^2} = s$$

$$\Rightarrow z = \left[(L-1)^2 \cdot s\right]^{1/3}$$

From eq (1) we have $s = \frac{r^2}{L-1}$

$$z = \left[(L-1)^{\cancel{2}} \cdot \frac{r^2}{\cancel{L-1}}\right]^{1/3}$$

$$z = \left[(L-1) \cdot r^2\right]^{1/3} \rightarrow (5)$$

$$\begin{tikzcd} r \arrow[r] \arrow[rr, bend right, "map"'] & s \arrow[r] & z \end{tikzcd}$$

- For the discrete case:

The transformation function for discrete intensity values is formulated as:

$$s_k = T(r_k) \rightarrow (1)$$

$$s_k = (L-1) \sum_{j=0}^{k} p_r(r_j) \rightarrow (2)$$ Normalized histogram

$$p_r(r_j) = \frac{n_k}{MN} \qquad k = 0, 1, 2 \ldots L-1$$

$$s_k = (L-1) \sum_{j=0}^{k} \frac{n_j}{MN}$$

$$= \frac{(L-1)}{MN} \sum_{j=0}^{k} n_j \rightarrow (3)$$

The transformation function for specified pdf is formulated as:

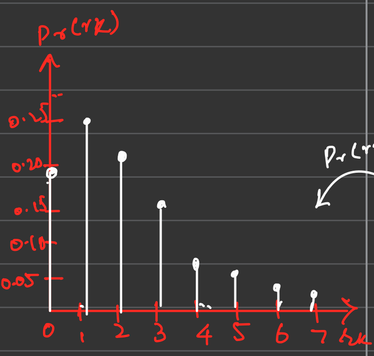
$$G(z_q) = (L-1) \sum_{j=0}^{q} p_z(z_j) \rightarrow (4)$$

$G(z_q) = T(r_k)$

$G(z_q) = S$

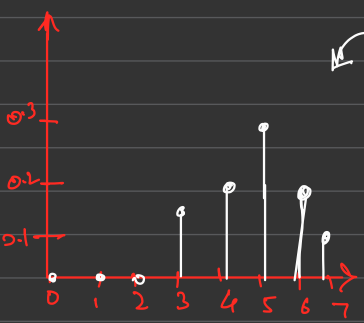
$\boxed{z_q = G^{-1}(S)} \rightarrow 5$

Problem :



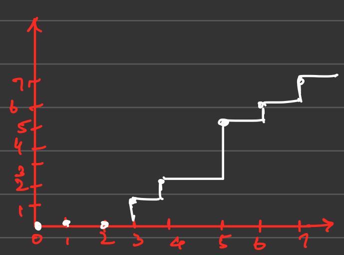
Pr(rk)

r_k	n_k	$P_r(r_k) = \frac{n_k}{MN}$	s_k	Round off	$P_s(s_k)$
0	790	0.19	1.33	1	0.19
1	1023	0.25	3.08	3	0.25
2	850	0.21	4.55	5	0.21
3	656	0.16	5.67	6	0.24
4	329	0.08	6.23	6	
5	245	0.06	6.55	7	0.10
6	122	0.03	6.86	7	
7	81	0.02	7.00	7	



z_q	$P_z(z_q)$	Actual value $P_z(z_k)$	z_q	$G(z_q)$
$z_0 = 0$	0.00	0	0	0
$z_1 = 1$	0.00	0	1	0
$z_2 = 2$	0.00	0	2	0
$= 3$	0.15	0.19	3	1
$= 4$	0.20	0.25	4	2
$= 5$	0.30	0.21	5	5
$= 6$	0.20	0.24	6	6
$= 7$	0.15	0.109	7	7

$$G(z) = (L-1)\sum_{j=0}^{q} P_z(z_j)$$



$q=0 \quad G(z_0) = 7 \times P(z_0) = 7 \times 0 = 0$

$q=1 \quad G(z_1) = 7 \times P(z_0) + 7 \times P(z_1) = 0$

$q=2 \quad G(z_2) = 0 + 7P(z_2) = 0 + 0 = 0$

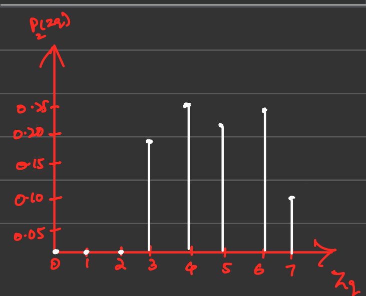
$q_3 = G(z_3) = 0 + 7P(z_3) = 0 + 7 \times 0.15 = 1.05 \simeq 1$

$q_4 = G(z_4) = 1.05 + 7 \times P(z_4) = 1.05 + 7 \times 0.20 = 2.45 \simeq 2$

$q_5 = G(z_5) = 2.45 + 7 \times P(z_5) = 2.45 + 7 \times 0.30 = 4.55 \simeq 5$

$q_6 = G(z_6) = 4.55 + 7 \times P(z_6) = 4.55 + 7 \times 0.20 = 5.95 \simeq 6$

$q_7 = G(z_7) = 5.95 + 7 \times P(z_7) = 5.95 + 7 \times 0.15 = 7.00$



s_k	z_q
1	3
3	4
5	5
6	6
7	7