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Section: A
Semester: 5th
Subject: DSP (Assignment 2)
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Question # 1:

Find z-Transform of following signals using properties of z-transform.

a) $x_1[n] = n\left(\frac{1}{3}\right)^{n-2} u[n-1]$

$u[n] \xleftrightarrow{z} \frac{1}{1-z^{-1}}$

$u[n-1] \xleftrightarrow{z} \frac{1}{z-1}$ ⇒ Shifting property

$a^n u[n-1] \xleftrightarrow{z} \frac{1}{z/a-1}$ ⇒ Scaling Property

$n a^n u[n-1] \longleftrightarrow -z \frac{d}{dz}\left(\frac{1}{z/a-1}\right)$ ⇒ Differentiation Property

Given: $x_1[n] = n\left(\frac{1}{3}\right)^{n-2} u[n-1]$

This can be written as:

$x_1[n] = n\left(\frac{1}{3}\right)^{-2}\left(\frac{1}{3}\right)^{n} u[n-1]$

Applying all properties,

$X_1[z] = -z(3)^2 \frac{d}{dz}\left(\frac{1}{\frac{z}{a}-1}\right)$

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$$X_2[z] = -9z \frac{d}{dz}\left(\frac{9}{z-a}\right)$$

$$= -9z \frac{d}{dz}\left(\frac{1/3}{z-1/3}\right)$$

$$= -9z \frac{d}{dz}\left(\frac{1/3}{(3z-1)/3}\right)$$

$$= -9z \frac{d}{dz}\left(\frac{1}{3z-1}\right)$$

$$= -9z \frac{(3)}{(3z-1)^2}$$

$$X_1[z] = = \frac{-27z}{(3z-1)^2}$$

So $n\left(\frac{1}{3}\right)^{n-2} u(n-1) \xleftrightarrow{z} \frac{-27z}{(3z-1)^2}$

b) $x_2[n] = u[n-1] - \left(\frac{1}{3}\right)^n u[-n-1]$

This signal can be written as;

$x_2[n] = x_a[n] + x_b[n]$

where $x_a[n] = u[n-1]$

and $x_b[n] = -(1/3)^n u[-n-1]$

Linearity Property:

$x_a[n] + x_b[n] \xleftrightarrow{z} X_a[z] + X_b[z]$

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Solving $x_a[n]$:

$$x_a[n] = u[n-1]$$

Use shifting property:

$$X_a[z] = z^{-1}\, u[z].$$

$$= z^{-1}\, \frac{1}{1-z^{-1}}$$

$$= \frac{1/z}{\frac{z-1}{z}}$$

$$X_a[z] = \frac{1}{z-1}$$

Solving $x_b[n]$:

$$x_b[n] = -(1/3)^{-n}\, u[-n-1]$$

can be written as;

$$x_b[n] = -\left[\left(\frac{1}{3}\right)^{-1}\right]^n u[-n-1]$$

$$\therefore -a^n u(-n-1) \xleftrightarrow{z} \frac{1}{1-az^{-1}}$$

$$x_b[n] = -(3)^n u[-n-1]$$

$$X_b[z] = \frac{1}{1-(3)z^{-1}}$$

$$X_b[z] = \frac{1}{1-\frac{3}{z}}$$

$$X_b[z] = \frac{z}{z-3}$$

$$X_2[z] = X_a[z] + X_b[z]$$

$$X_2[z] = \frac{1}{z-1} + \frac{z}{z-3}$$

$$\Rightarrow X_2[z] = \frac{z-3+z}{(z-1)(z-3)}$$

$$X_2[z] = \frac{2z-3}{(z-1)(z-3)}$$

c) $x_3[n] = x_1[n] * x_2[n-1]$

$x_1(n) * x_2(n) \xleftrightarrow{z} X_1(z) \cdot X_2(z)$

∴ Convolution in time domain $\xleftrightarrow{z}$ Multiplication in z-domain

$X_3[z] = X_1[z] \cdot X_2[z]$ ∴ Convolution property is used

$x_2[n-1] \xleftrightarrow{} z^{-1} \cdot \frac{2z-3}{(z-1)(z-3)}$ ∴ shifting property is used

$$\Rightarrow X_2[z-1] = \frac{2z-3}{z(z-1)(z-3)}$$

$$\Rightarrow X_3[z] = \frac{-27z}{(3z-1)^2} \times \frac{2z-3}{z(z-1)(z-3)}$$

$$X_3[z] = \frac{-54z+81}{(9z^2-6z+1).(z^2-3z-z+3)}$$

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$$X_3[z] = \frac{-54z + 81}{(9z^2 - 6z + 1)(z^2 - 4z + 3)}$$

$$X_3[z] = \frac{-54z + 81}{9z^4 - 36z^3 + 27z^2 - 6z^3 + 24z^2 - 18z + z^2 - 4z + 3}$$

$$X_3[z] = \frac{-54z + 81}{9z^4 - 42z^3 + 52z^2 - 22z + 3}$$

Question # 2:

Find the systems function for casual LTI system :-

a) $y(n) = \frac{3}{4} y(n-1) - \frac{1}{8} y(n-2) + x(n)$

Z-transform notation is :-

$$Y(z) = \frac{3}{4} z^{-1} Y(z) - \frac{1}{8} z^{-2} Y(z) + X(z)$$

$$Y(z) - \frac{3}{4} z^{-1} Y(z) + \frac{1}{8} z^{-2} Y(z) = X(z)$$

$$\left(1 - \frac{3}{4} z^{-1} + \frac{1}{8} z^{-2}\right) Y(z) = X(z)$$

$$\Rightarrow \frac{Y(z)}{X(z)} = \frac{1}{1 - 3/4 z^{-1} + 1/8 z^{-2}}$$

System function is $H(z) = \frac{Y(z)}{X(z)}$

$$H(z) = \frac{1}{(1 - 1/2 z^{-1})(1 - 1/4 z^{-1})}$$

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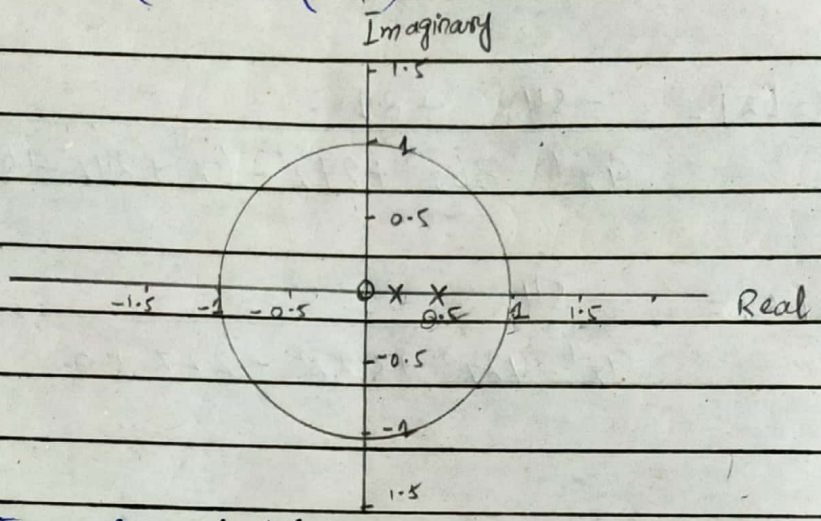
- Zeros : at origin z=(0)

Poles : at (1/2) and (1/4)



System is **stable.**

b) $y(n) = y(n-1) - 0.5y(n-2) + x(n) + x(n-1)$

Z- Transform notation is :

$$Y(z) = z^{-1}Y(z-1) - 0.5z^{-2}Y(z) + X(z) + X(z)\cdot z^{-1}$$

$$Y(z) - z^{-1}Y(z-1) + 0.5z^{-2}Y(z) = X(z) + z^{-1}X(z)$$

$$(1 - z^{-1} + 0.5z^{-2})\,Y(z) = (1 + z^{-1})\,X(z)$$

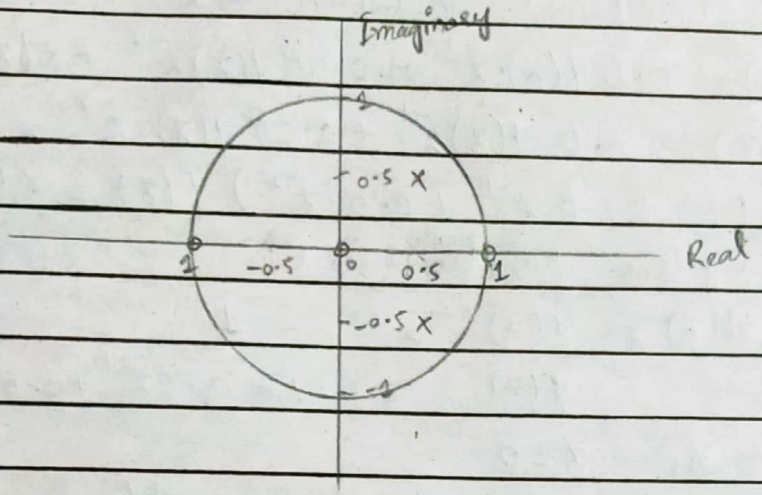
$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 + z^{-1}}{(1 - z^{-1} + 0.5z^{-2})}$$

$$H(z) = \frac{1 + 1/z}{\frac{z^2 - z + 0.5}{z^2}}$$

$$H(z) = \frac{z(z+1)}{z^2 - z + 0.5}$$

Zeros: at $z=0$ and $z=1$

Poles: $(1/2 \pm 1/2\,j)$ or $\left(\frac{1 \pm j}{2}\right)$



System is **stable.**

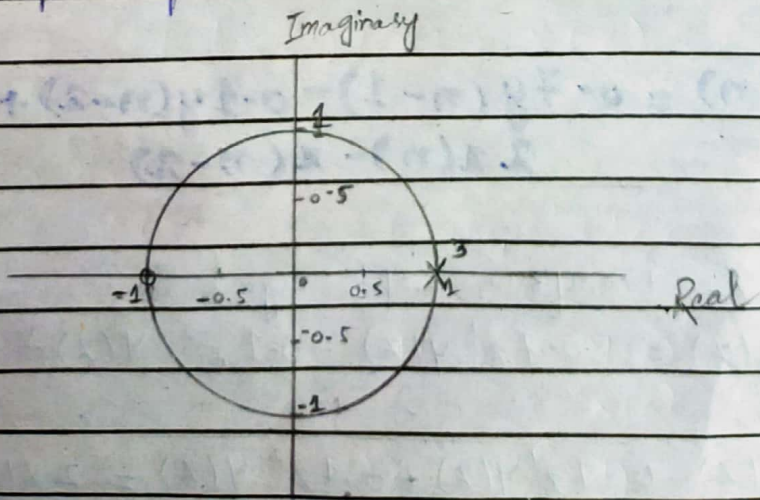
c) $$H(z) = \frac{z^{-1}(1+z^{-1})}{(1-z^{-1})^3}$$

System function is already given.

Zeros: at $z = -1$

Poles:

Triple pole at $z = +1$



System is **unstable.**

d) $y(n) = 0.6y(n-1) - 0.08y(n-2) + x(n)$

Z- Transform notation is:

$Y(z) = 0.6\,Y(z)\cdot z^{-1} - 0.08\,Y(z)z^{-2} + X(z)$.

$Y(z) - 0.6\,Y(z)z^{-1} + 0.08\,Y(z)\,z^{-2} = X(z)$

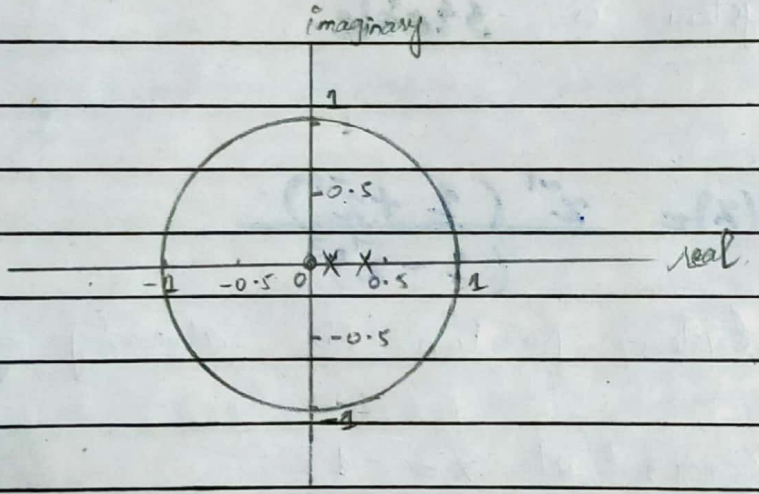
$(1 - 0.6z^{-1} + 0.08z^{-2})\,Y(z) = X(z)$

System function is:

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1 - 0.6z^{-1} + 0.08z^{-2}} = \frac{1}{(1-\frac{1}{5}z^{-1})(1-\frac{2}{5}z^{-1})}$$

Zeros: $z = 0$

Poles: at $z = 2/5$ and $z = 1/5$.



System is **stable**.

e) $y(n) = 0.7y(n-1) - 0.1y(n-2) + 2x(n) - x(n-2)$

Z- transform of signal is;

$Y(z) = 0.7z^{-1}\,Y(z) - 0.1z^{-2}.\,Y(z) + 2X(z) - X(z).z^{-2}$

$Y(z) - 0.7z^{-1}Y(z) + 0.1z^{-2}\,Y(z) = 2X(z) - X(z)\cdot z^{-2}$

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$$(1 - 0.7z^{-1} + 0.1z^{-2})\, Y(z) = 2X(z) - X(z)\cdot z^{-2}$$

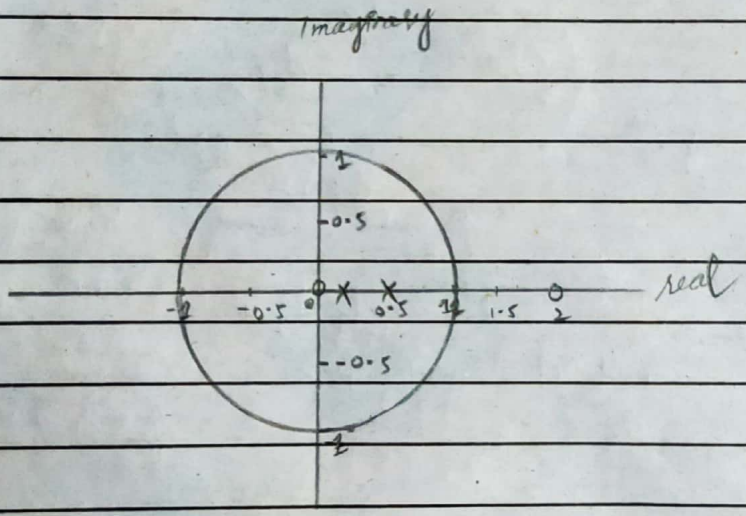
$$\Rightarrow (1 - 0.7z^{-1} + 0.1z^{-2})\, Y(z) = (2 - z^{-2})\, X(z)$$

System function is;

$$H(z) = \frac{Y(z)}{X(z)} = \frac{2 - z^{-2}}{1 - 0.7z^{-1} + 0.1z^{-2}}$$

Zeros: at $z=0$, at $z=2$.

Poles: at $z = 1/2$ and $z = 1/5$.



System is **Stable**.