Hypothesis Testing

Basics & Fundamentals

Hypothesis Testing:

Statistical technique to test some hypothesis about the parent population from which the sample is actually drawn.

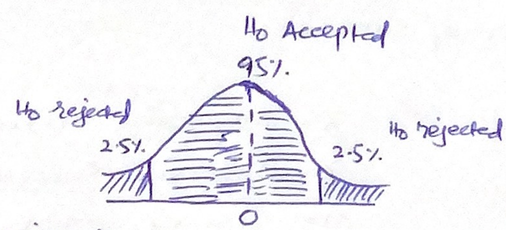
Estimation: It uses the statistics obtained from the sample as estimate of the unknown parameters of the population from which the sample is drawn.

- "A hypothesis in statistics is simply a quantitative statement about population."

Procedure:

1. Setup the Hypothesis.
   H_0 (Null hypothesis) ; H_a: (Alternative hypothesis)

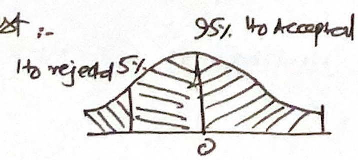
2. Set up a suitable significant level.
   - Test of validity of H_0 against H_a at certain level of significant. Such as 5%, 1% etc. Sometimes it is denoted by "α"



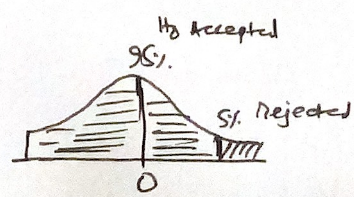
* This is called two-tailed test
2.5% left side & 2.5% right side.

- 5% chances of having error.
- 5% level of signifi; means 5% wrong and 95% right decision

→ One-tailed test :-



Left sided one tailed test.



Right sided one tailed test.

Level of significance	10%	5%	1%	0.1%
one tailed	1.65	1.96	2.58	3.29
two tailed	1.28	1.64	2.33	3.10

Z-Value: It is a standardized score that describes how many standard deviation (δ) an element is from the mean (μ).

③ Setting up a test Criterion: Selection of an appropriate probability distribution for the test such as T-test, Chi^2 test, F test etc.

④ Perform computation, &

⑤ Decision taking & conclusion.

• TEST OF SIGNIFICANCE FOR ATTRIBUTES.

∴ Test for number of successes:

$$Z = \frac{x - np}{\sqrt{npq}}$$

∴ Z = calculated critical value.
x = observed success.
n = size of sample.
P = probability of success.
q = (1-P); probability of failure.

Sol: Consider the coin is unbiased.

H_0: No of Heads = No. of tails

H_a: No; of Heads ≠ No of tails

Given data: $n = 484$

$x = 265$

$P = 1/2 = 0.5$

$q = 0.5$

$$Z = \frac{265 - 242}{\sqrt{484 \times 1/4}}$$

$$\Rightarrow 23/\sqrt{121} = \frac{23}{11}$$

$$\boxed{Z = 2.09}$$

$Z_{tab} = 1.96$ ∴ For two tailed test

-1.96 to 1.96

Ho is rejected and The coin is biased.

Q#02: Sol: Let's consider The die is fair

H_0: No of odd points = No of even points.

H_a: " " " " ≠ " " " ".

Given data:

$n = 256$

$x = 122$

$\alpha = 0.05$

$P = 0.5$

$q = 0.5$

$$Z = \frac{x - np}{\sqrt{npq}}$$

$$\Rightarrow \frac{122-128}{\sqrt{256 \times 1/4}} = \frac{-6}{\sqrt{64}} = -6/8$$

$$\boxed{Z = -0.75}$$

$Z_{tab} = 1.96$

H_0 is passed and accepted.

Decision = The given die is fair.

Test for proportion of successes:

$$\underset{\text{(porportion)}}{S.E} = \sqrt{\frac{pq}{n}}$$

where: p = probability of success.
$q = (1-p)$; Probability of failure
n = size of The sample.

For determination of limit in the porportion of successes.

$$= P \pm Z \times S.E \text{ (Porportion)}$$

Q#01: Solution:

Givendata:

$n = 1000$, $P = 100/1000 = 0.1$, $q = 0.9$

$$\Rightarrow P \pm Z\sqrt{\frac{pq}{n}} = 0.1 \pm 1.96 \times \sqrt{\frac{0.1 \times 0.9}{1000}}$$

$$= 0.1 \pm 1.96\sqrt{0.00009}$$

$$= 0.1 \pm 1.96(0.0095)$$

$$\Rightarrow 0.1 \pm 0.0186 =$$

Limit:

0.0814 to 0.1186

$= \boxed{8.14\% \text{ to } 11.86\%}$

Q#02:

Solution: let's consider the hypothesis that the claim of the wholeseller is correct i.e., 5% defective item only.

H_0: Defective item is only 5%

H_a: Defective item is more than 5%.
Less

Given data:

$n = 500$, $P = 0.05$ (5%), $q = 0.95$ (95%)

$Z_{(tab)}$ at 5% (level of Significance) = 1.96

$$= P \pm Z\left(\sqrt{\frac{pq}{n}}\right)$$

$$= 0.05 \pm 1.96\sqrt{\frac{0.05 \times 0.95}{500}}$$

$$= 0.05 \pm 1.96 \times 0.0097 = 0.05 \pm 0.019$$

Limits:

$0.05 - 0.019 = 0.031$ to $0.05 + 0.019 = 0.069$

0.031×500 $\Rightarrow 15.5$ to 34.5
0.069×500

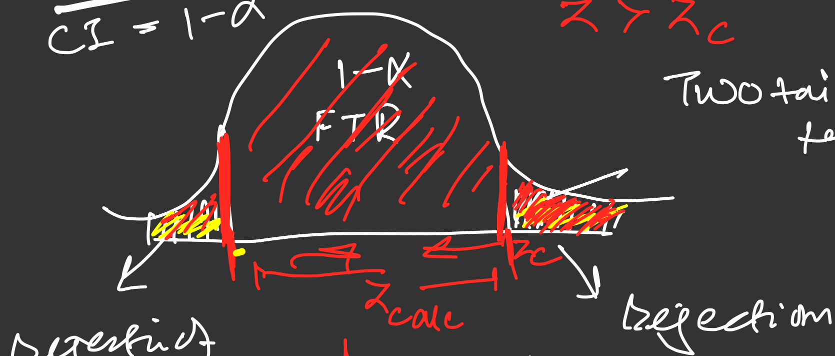
H_0 is accepted & passed i.e.,

28 defective item is supplied by the wholeseller.

Hypothesis testing → Z test

$\alpha = 5\%$ } → chance of error
$\alpha = 1\%$ }

$CI = 1 - \alpha$



$Z > Z_c$

Two tailed test

α

$Z_{calc} < Z_c$ → Ho accept

- one tailed test
- two " "

CI → confidence of interval

α : Level of significance

$\alpha = 0.05$

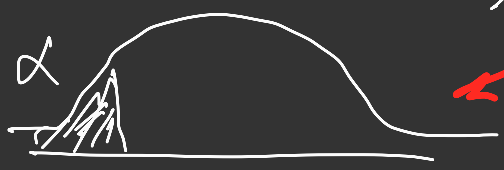
Ho : Null –

Ha : Alternative –

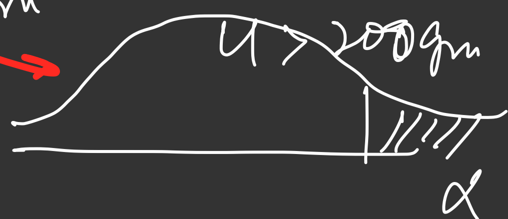
Ho : μ = value or Ha : $\mu \neq$ value. (2-tailed)

$\mu >$ or $\mu <$ value (1-Tailed test.)

$\mu < 200$gm



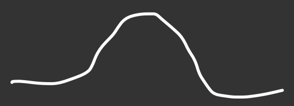
$\mu > 200$gm



Z - test :

Sample size < 30

t - test

- Sample size ≥ 30
- Random Samples.
- independent
- 

σ^2 = known

Z, f, t } test

Parametric test

Define hypothesis

1. H_0 : Null
   H_a : Alternate.

2. Suitable H. Test.

3. → Level of Significance → identify

4: → Setting up a test criteria

5: → Computation

6. → Decision.

Level of Significance	10%	5%	1%	0.1%
1- Tailed test	1.65	1.96	2.58	3.29
2- " "	1.28	1.64	2.33	3.10

Test Of Significance for Attributes

$$Z = \frac{x - np}{\sqrt{npq}}$$

Z : calc critical value

x : observed success

n = size of sample

p = proba; of succ;

$(1-p) = q$ = " of failure

H_0 : No of H = No of Tail

$\boxed{H_a : \text{" " " } \neq \text{ " " "}}$

Given Data :

$n = 484/2$

$$Z = \frac{265 - 242}{\sqrt{484 \times \frac{1}{2} \times \frac{1}{2}}}$$

$$= \frac{23}{\sqrt{121}} = 23/11$$

$$\boxed{Z = 2.09}$$

$\alpha = 5\%$

$Z_{calc} > Z_{Tab}$

H_0 is Rejected

H_a is accepted

Test for proportion of Successes :

$S.E = \sqrt{\dfrac{pq}{n}}$

$p =$ prob; of success

$q = 1 - p$

$n =$ Sample size

limit

Tab; c value

α 1%

$p \pm z * S.E$

$p \pm z \sqrt{\dfrac{pq}{n}}$.

$0.1 \pm 1.96 \sqrt{\dfrac{0.1 \times 0.9}{1000}}$

$= 0.1 \pm 0.0186$

$= 0.0814 \longleftarrow 0.1186$

$\boxed{8.14\% \text{ to } 11.86\%}$

H0: city A = city B
H1: ≠

Test for diff; between proportion

$$Z = \frac{p_1 - p_2}{\sqrt{pq\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$\because$ p = prob; of S
q = 1 - p
p_1 = sample$_1$ size
p_2 = sample$_2$ size
n_1 = size of pop$_1$
n_2 = size of pop$_2$

$$\therefore p = \frac{x_1 + x_2}{n_1 + n_2} \quad \text{or} \quad \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2}$$

x_1 = no of occurence in sample$_1$
x_2 = " " " sample$_2$

$$Z = \frac{p_1 - p_2}{\sqrt{pq\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$p = \frac{x_1 + x_2}{n_1 + n_2} = \frac{500 + 650}{2200} = 0.52$$

$$q = 1 - p = 0.48$$

$$Z = \frac{0.54 - 0.50}{\sqrt{0.52 \times 0.48 \left(\frac{1}{\underset{n_1}{1000}} + \frac{1}{\underset{n_2}{1000}} \right)}}$$

$P_1 = \frac{500}{1000} = 0.5$

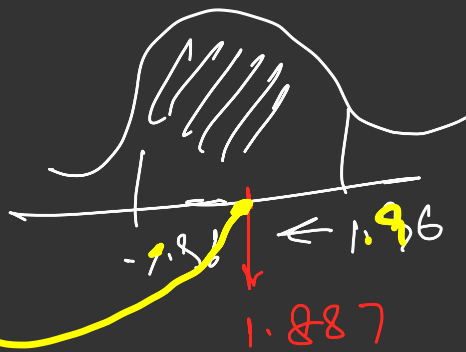
$P_2 = \frac{550}{1000} = 0.54$

$P_1 = 0.54$

$P_2 = 0.50$

$Z = 1.887$

5%



H_0 is accepted

T-test

Student test

Size of sample < 30

δ = unknown

o) Helps in testing μ

o) Mathematically derived under the assumption of a normally distributed population.

p density fn → $f(t) = c\left(1+\frac{t^2}{\nu}\right)^{-\frac{\nu+1}{2}}$

ν = dof

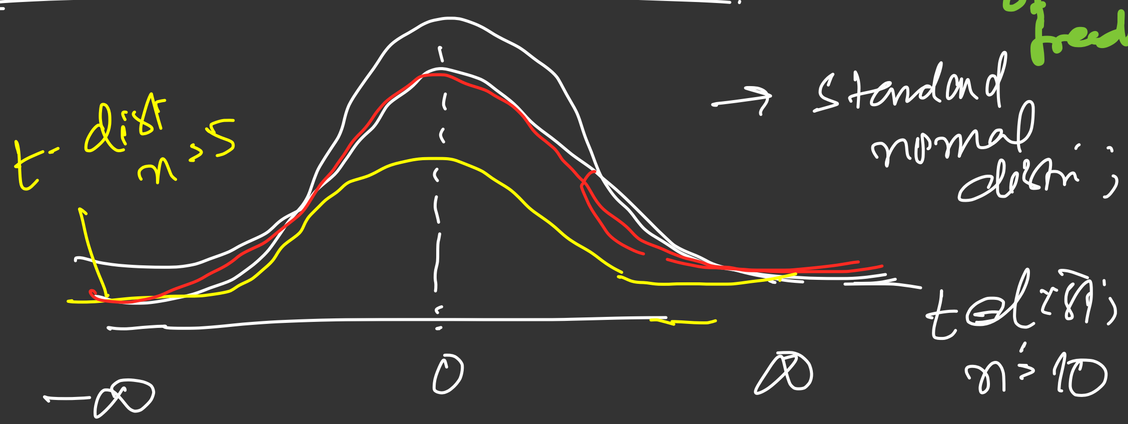
degree of freedom

degree of freedom

$\nu = n-1$

$n = ?$

$t \rightarrow$



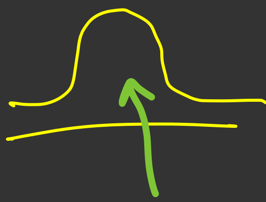
$\boxed{\text{Power of } t} \rightarrow$ even $f(t)$

Symmetrical

like ND

$\nu = n-1$

S. size (n)	ν	t-value
5	4	2.776
10	9	2.262
30	29	2.045
∞	—	$\boxed{1.96}$



$z = 1.96$

Table

i) Calc t-value $\geq$ Tab; t-value

→ Significant Difference b/w Pop. mean & sample mean.

→ Calc t-value < tab; t-value

→ No significant diff;

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1. To test the significance of the mean of a random sample

$$t = \frac{|(\bar{x} - \mu)| * \sqrt{n}}{s}$$

$$s = \sqrt{\frac{\sum (x - \bar{x})^2}{n-1}}$$

$\bar{x}$: mean of sample.

μ = " " pop

n = sample size

s = std of sample

S = sample

$\boxed{H_0 = \mu = \bar{x}}$; $\boxed{H_a = \mu \neq \bar{x}}$

life of bulbs

x

$(x - \bar{x})$

$\bar{x} = 19$

$\bar{x} = \frac{\sum x}{n}$

$\frac{133}{7} = 19$

19
21
25
16
17
14
21

Life of bulbs

x

$\bar{x} = 19$

$\bar{x} = \frac{\sum x}{n}$

x	$(x-\bar{x})$	$(x-\bar{x})^2$
19	0	0
21	2	4
25	6	36
16	-3	9
17	-2	4
14	-5	25
21	2	4
$\sum x = 133$		82

$\bar{x} = 133/7 = 19$

$$s = \sqrt{82/6} = \sqrt{13.67} \cong 3.7$$

$t_{0.05}$

$t_{0.01}$

$$t = \frac{|\bar{x} - \mu| * \sqrt{n}}{s}$$

tab t-value

3.707

$$= \frac{|19-20| * \sqrt{7}}{3.7} = 0.716$$

$H_0 = \mu = \bar{x} \to$ sample mean

$\downarrow$

pop mean

1: To Test the difference b/w means of the two samples (independent sample) Unpaired

$$t = \frac{(\bar{x}_1 - \bar{x}_2)}{S} \times \sqrt{\frac{n_1 \times n_2}{n_1 + n_2}}$$

$$S = \sqrt{\frac{\Sigma(x - \bar{x}_1)^2 + \Sigma(x_2 - \bar{x}_2)^2}{n_1 + n_2 - 2}}$$

v

$H_0 =$ Efficacy of Drug A = Eff. of Drug B.

$H_a =$ " $\neq$ "

D_A = (avg) : 8, 10, 12, 9, 14, 13 $\quad n_1 = 6$

D_B = " : 7, 9, 14, 12, 8 $\quad n_2 = 5$

x_1	$x_1 - \bar{x}_1$	$(x_1 - \bar{x}_1)^2$
8	-3	9
10	-1	1
12	1	1
9	-2	4
14	3	9
13	2	4

$\Sigma x = 66$ $\quad \Sigma (x_1 - \bar{x}_1)^2 = 28$

$\bar{x}_1 = \frac{\Sigma x}{n_1} = 11$

$\bar{x}_1$

x_2	$x_2 - \bar{x}_2$	$(x_2 - \bar{x}_2)^2$
7	-3	9
9	-1	1
14	4	16
12	2	4
8	-2	4

$\Sigma x_2 = 50$ $\quad \Sigma = 34$

$\bar{x}_2 = \frac{\Sigma \bar{x}_2}{n_2} = 50/5 \to 10$

$$S = \sqrt{\frac{28+34}{9}} \Rightarrow \boxed{S = 2.62}$$

$$t = \frac{|11-10|}{2.62} * \sqrt{3\cdot}$$

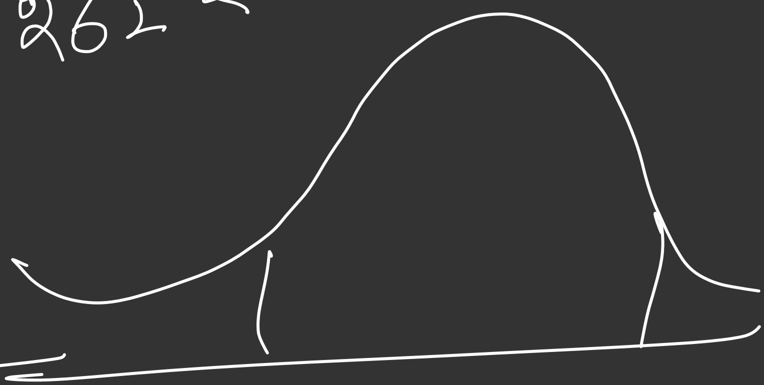
$$= \frac{1}{2.62} * \sqrt{30/11}$$

$$t = 0.63$$

$v = 9$

tab t- value.

$t_{0.05} = 2.262$



H_0 is accepted

Date: 11/01/23

1) 1803, 1848, 1784, 1782, 1834
1740, 1810, 1762, 1815, 1847, 1852, 1854
1856, 1778, 1773, 1768, 1766,
1806, 1804, 1755, 1788, 1807, 1781
1844, 1838, 1835, 1843, 1796

Date: 12/09/23

1795, 1789

Ashfaq Ahmad

A. Rahim

Test for difference between proportions.

$$Z = \frac{P_1 - P_2}{\sqrt{pq(1/n_1 + 1/n_2)}}$$

Where: P = Probability of Success
q = " " Failure (1-P)
P_1 = Size of sample 1
P_2 = Size of sample 2
n_1 = Size of population 1
n_2 = Size of population 2.

• $P = \frac{x_1 + x_2}{n_1 + n_2}$ or $\frac{n_1 P_1 + n_2 P_2}{n_1 + n_2}$

x_1 : No of occurence in Sample 1
x_2 : No of occurence in Sample 2.

Sol:

$H_0 : P_1 = P_2$

$H_a : P_1 \neq P_2$

Given data:

$n_1 = 1000,\ x_1 = 500,\ P_1 = 500/1000 = 0.50$

$n_2 = 1200,\ x_2 = 650,\ P_2 = 650/1200 \approx 0.54$

$$Z = \frac{P_1 - P_2}{\sqrt{pq(1/n_1 + 1/n_2)}}$$; Here q & p are unknowns

$$P = \frac{x_1 + x_2}{n_1 + n_2} = \frac{500 + 650}{2200} = 0.52$$

$$q = 1 - 0.52 = 0.48$$

Now:

$$Z = \frac{0.54 - 0.50}{\sqrt{0.52 \times 0.48 (1/1000 + 1/1200)}}$$

$$= \frac{\pm 0.04}{\sqrt{0.249 \times 0.0018}} = \frac{\pm 0.04}{0.0212} = \pm 1.887$$

$Z_{tab} = -1.96$ to 1.96 for 5% level of significance.

* H_0 is passed & accepted: There is no difference in the eating habit of rice b/w the two cities

QUESTION NO: 02

$H_0 : P_1 = P_2$; $H_a : P_1 \neq P_2$

Solution :

$P_1 = 800$, $x_1 = 600$, $p_1 = 600/800 = 0.75$

$P_2 = 1200$, $x_2 = 700$, $p_2 = 700/1200 = 0.58$

$$P = \frac{600+700}{1200+800} = \frac{1300}{2000} = 0.65 \; ; \; q = 1-P = 0.35$$

$$Z = \frac{0.75-0.58}{\sqrt{0.65 \times 0.35\left(\frac{1}{800}+\frac{1}{1200}\right)}} = \frac{0.17}{\sqrt{0.228(0.0020)}}$$

$$= \frac{0.17}{\sqrt{0.00048}} = 7.763$$

$Z_{tab} = -2.58$ to 2.58

H_0 is rejected. There is a significant difference in smoking habit of city A & city B.

One tailed test OR Two tailed test

$H_0: \mu = 100g$

$\mu \neq 100g$

$\mu \neq$ a value of assumptions.

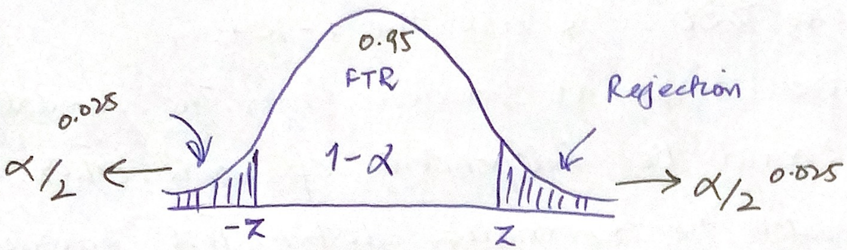
Then we need to perform two tailed test

95% confidence level.

$C = 0.95$

$\alpha = 1 - 0.95$

$\alpha = 0.05$



critical values.

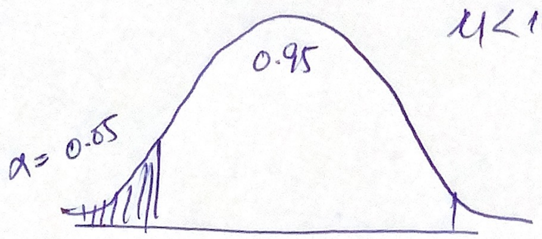
Z_c : Calculated.

$Z_c > Z \rightarrow$ Rejected

Z_c with the level $\rightarrow$ accepted.

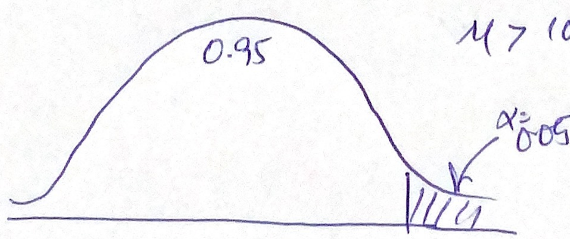
Ha:

$\mu < 100g$



Ha:

$\mu > 100g.$



Z-Test

- It is a statistical method to determine whether the distribution of the test statistics can be approximated by a normal distribution.

When to use Z-test :

- Sample size ≥ 30, else use the t-test.
- Samples are drawn at random from the population.
- Samples should be independent of each other.
- Data should be normally distributed, however for large sample size, it is assumed to have a normal distribution.