Digital Signal Processing

DSP

Digital Signal Processing.

3rd edition.

Digital processing

natural processing

↓

and we also have to bridge the two.

1, 2, 3, 4, 5, 6, 7, 8

Chap 1 : **bridging** → how to go from nature to computer.

> **Transistor**
> introduced a very different way of calculations.

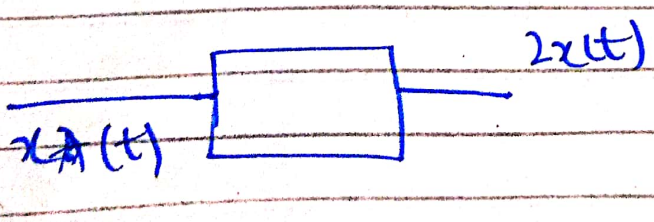
units, tens, hundreds
Boolean algebra

Computer

↓ Analogue Computational device

↓ Digital Computational Device.

[Mathematical systems]



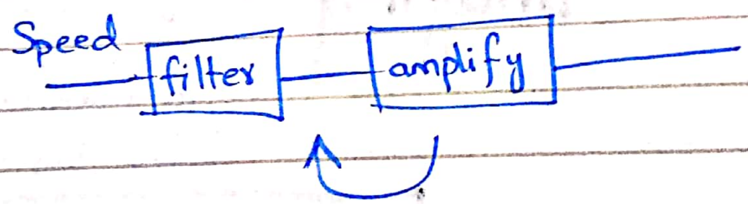
if we want

$3x(t) \rightarrow$ change circuit (property of physical devices)

In general purpose
⇓
can multiply $2x(t)$ or transistors & also $3x(t)$

In digital, it is not ability of circuit but of programming
→ coz its calculations based.

In Analogue:



To inverse we have to switch devices
or the other hand, if we use computer.
Some work is done logically.

main()
input x;
$x_1 = $ filter(x)
Amplify(x_1)

$x_1 = $ amplify(x)
filter(x)

Reorder

Analogue devices

(3)

→ high power consumption.
→ not portable
→ Great size.

DSP → More accurate as compared to analogue signals.

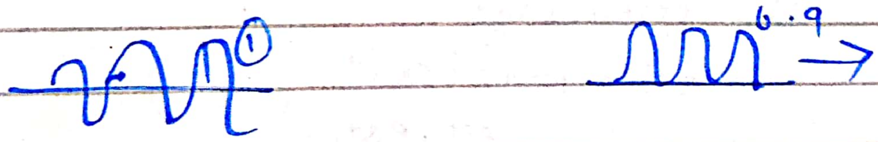
In digital we can detect error.

In analogue decimal are included

↳ we cannot detect error.

as 1 & 0.9 both are acceptable

1011 → 10(0.9) – error.



Now correction:

In digital → easy corrected as compared to analogue.

0.9 → near to 1 → so we take it as 1.

in 100 out of 1 → 1 is converted to 0.

other 99 → 1 not less than 0.5

U

0 " greater " "

Means correction is correct.

That is why things are rapidy changing from A to D.

1. Signal is physical

→ cannot see but can be counted or measured (mathematically expressed)

→ phyical or local device described by relations b/w input & output.

Converted or logical.

System: In simpler language called amplifier.

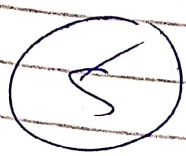
Signal Processing: Relation b/w signal & sys to study signal & system.

Lecture # 2.

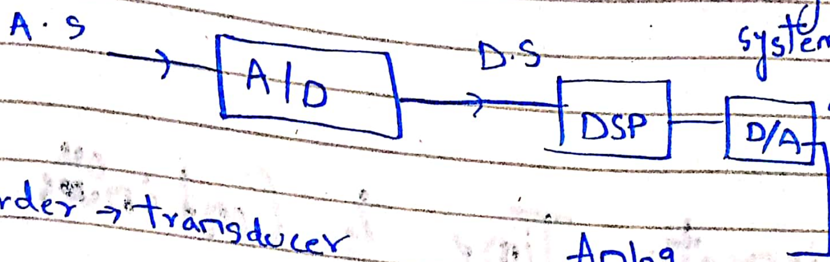
1.1.1

Basic elements of DSP System.

Natural signals
are analouge.



eg
hearing
system
analogue.



Recorder → transducer

Anlog Signal.

Output & Input of DSP system
is signal.

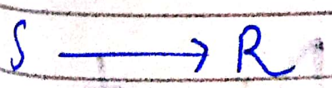
· Filtering
· Transmission

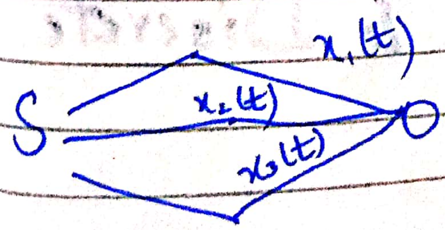
Classification (1.2) OF Signals:

1.2.1

① Multichannel and Multidimensional

one person talking
↓
single
source





∴ signals can combine when they are of same Nature.

$x(t) = x_1(t) + x_2(t) + x_3(t)$.

1.2.1

Multi dimensional

ek signal kitne independant variable par depend.

$$f(x) = 2x, x^2+1, \sqrt{x}.$$

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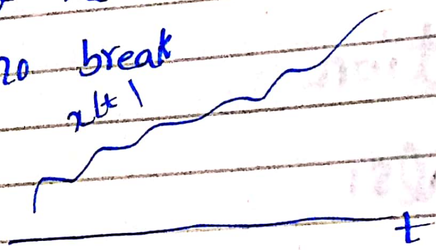
$$f\,x = x^2+y^2$$

1.2.2

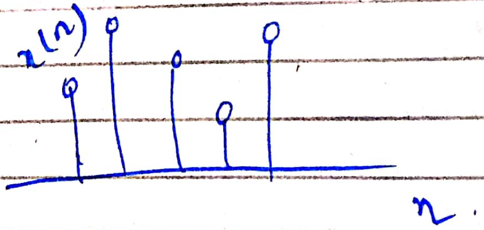
Discrete Time & Continuous time signals:

→ All real Values are defined

no break



→ D.Time: Only on specific time it is defined.



1.2.3

Continuous Value & Discrete Valued Signal:

⇒ if all the values are real then it can be any Real Value (Continuous)

1

⇒ Values are fixed in discrete valued Signal.

No. of decimal point fix
↓
Discrete

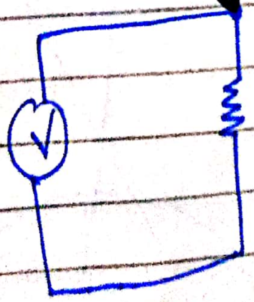
No. of decimal point not fix
↓
Continuous.

→ Jab kise Signal me dono bath ajaye tho inke time & value ko discrete kardety known as Discrete time & Discrete value.

No. of levels fix hotay

1.2.4.

Deterministic vs Random Signals:



$R = 2\Omega$

$i = \frac{V}{R^2}$

if current

⇒ indepent i depend on dependant v.

8

Time and temperature relation not deterministic.

Random Signals:

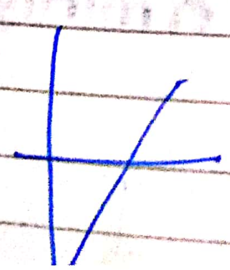
Here we take some range of values.

1.3. Concept of Frequeny.

→ The concept of f in CT & DTS transmission.

the more the variation the more it can carry information.

How to measure →



→ the third . more will have high priority.

Discrete.

Max — 0-2 pi
frequency.
conti ← infinity

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Signals → Sinusoidal → best.

$x(t) = A\cos(\Omega + \theta)$

$x(n) = A\cos(\omega n + \phi)$

↓

if A=1

$x(n) = \cos(\omega n + \phi)$

↓ standard cos.

shift right & left. $x(t) = \cancel{A}\cos(\Omega t + \theta)$.

↑

$x(t) = A+1\cos(\Omega + \theta)$

when $\theta = 0$

cos peak par Chala jata ha.

standard cos

0—2π

Lecture #3

3/10/19

DSP

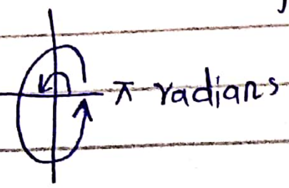
1.3.1 Continuous-time Sinusoidal Signals

radian form:

$$x(t) = A\cos(\underbrace{\Omega}_{\text{frequency} \to \text{angular frequency}} t + \theta)$$

1C

angle covered per unit time = Angular frequency.



$1\ \text{rad} = 57.29$ degrees

3.142 rad.

radians	cycle
π radian	half cycle
2π radian	1 cycle

cycle form:

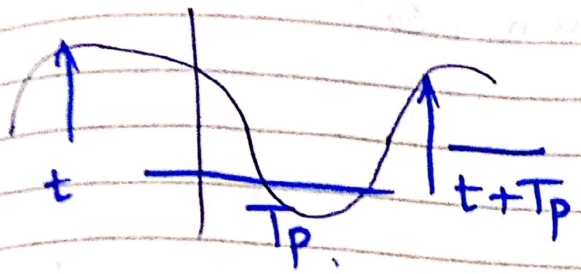
$$x(t) = A\cos(2\pi f t + \theta)$$

Properties of Ω

$\boxed{A1}$ CTS are always periodic Signals.

⇒ Periodicity : The signal will repeat itself after specific intervals.

$$x(t+T_p) = x(t)$$



(11)

For all values of Ω, the signal will be periodic.

$\boxed{A2}$

Sinusoidal signals are distant whose frequencies are distant.

⇒ Each frequency has its distant signal. $(-\infty < f < \infty)$.

all the cosine signals are periodic

e.g

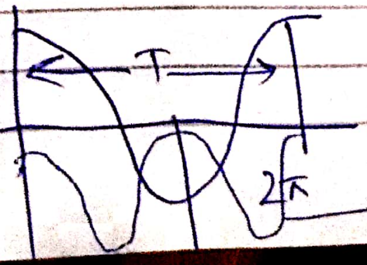
$$x_1(t) = A\cos(\Omega_1 t + \theta)$$
$$x_2(t) = A\cos(\Omega_2 t + \theta)$$

if their frequencies are diff then these signals are different.

$\boxed{A3}$

if f of CTS is increased then T will be decreased

$$T = \frac{2\pi}{\omega}$$



→ half time me repeat hoga.

⇒ As f ↑, the rate of oscillation increases

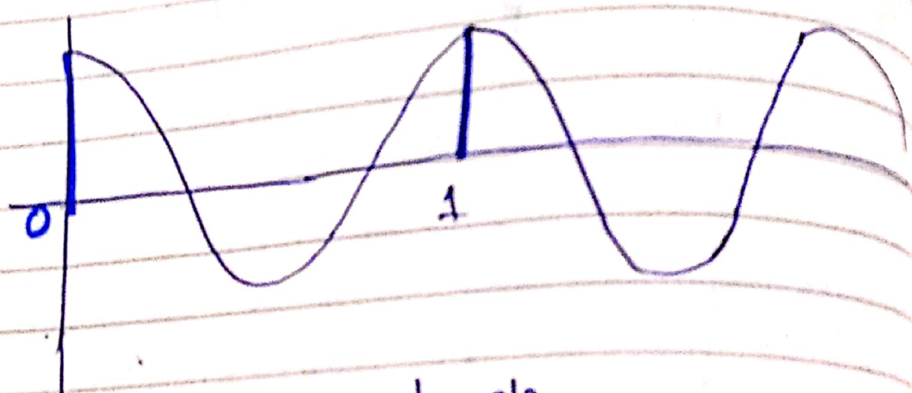
Discrete-Time Sinusoidal Signal.

$x[n] = A \cos(\omega n + \theta)$

↳ angle covered per unit time (continuous)

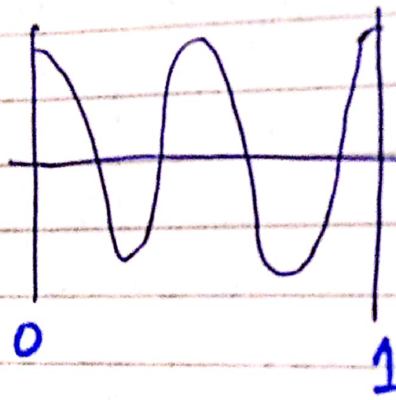
CONTINUOUS

2π

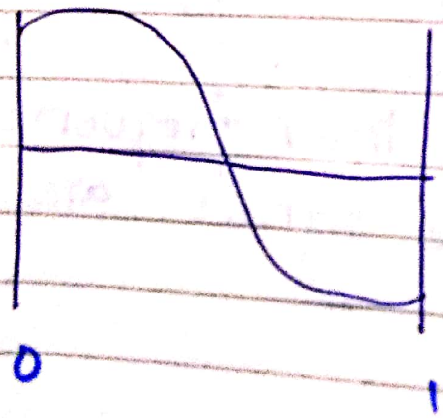


1 sec → 2π or 1 cycle.

π



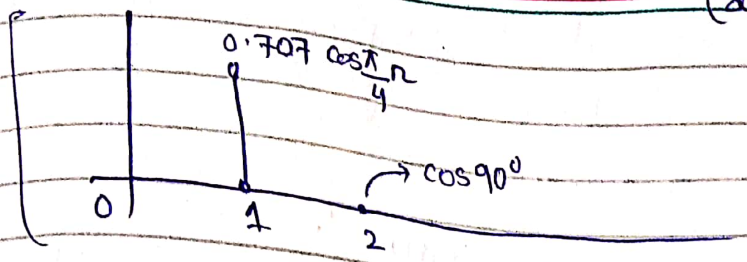
$\frac{1}{2}\pi$



B/B1

$\omega \rightarrow$ angle covered per unit sample (discrete).

Discrete.



Angle check.

B/B2

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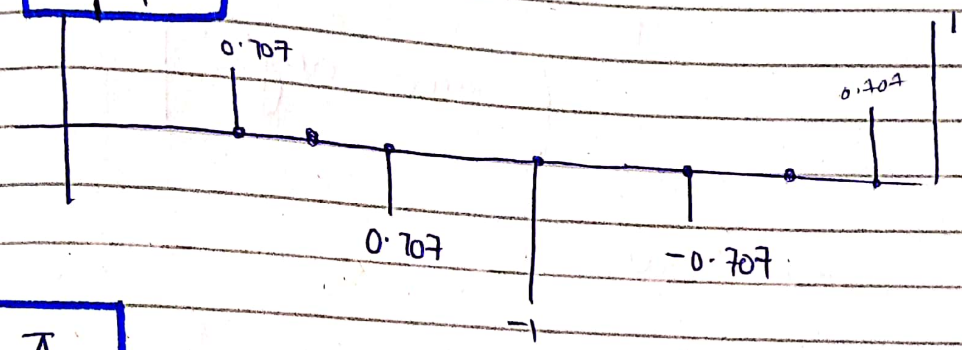
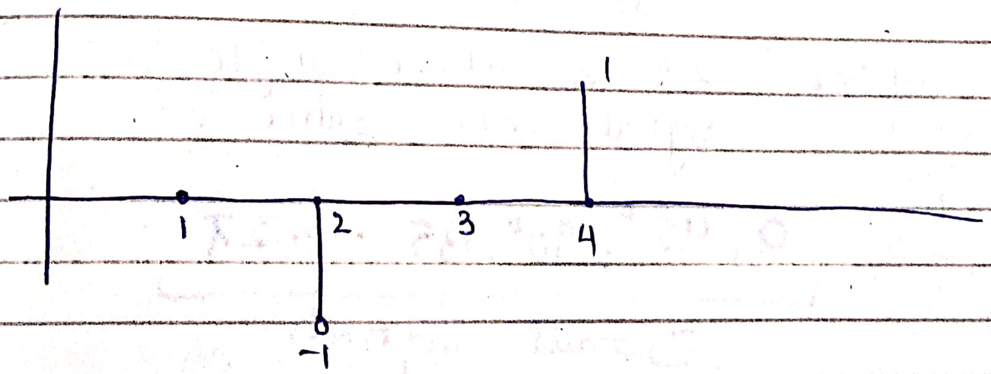
$\boxed{\cos\frac{\pi}{4}n}$

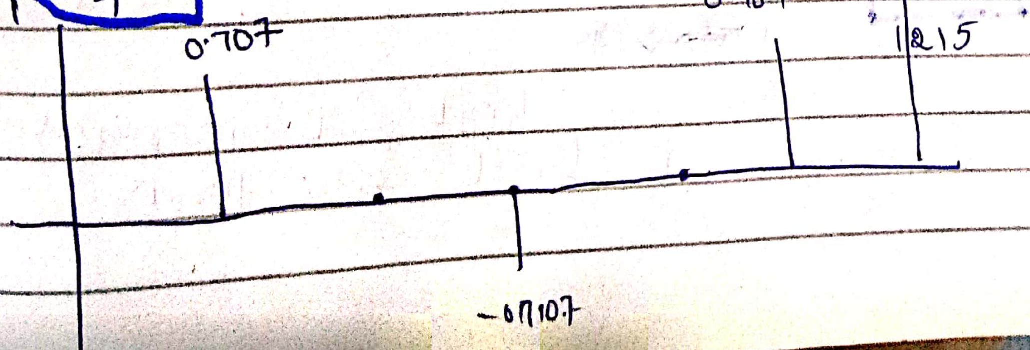


$\boxed{\cos\frac{\pi}{2}}$



$\boxed{\cos\frac{9\pi}{4}n}$ $\Rightarrow 45^\circ \times 9 = 405^\circ$

405×2
$\Rightarrow 810$



→ if two signal's frequencies are diff but their signals can be same.

⇒ Why signals are same?

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$\cos\frac{\pi}{4}$ & $\cos\frac{9\pi}{4}$

45° & $\underbrace{360^\circ + 45^\circ}_{\boxed{\text{same angle}}}$

w_1, w_2

$w_1 - w_2 = 2\pi$ then they will be same signal.

After 2π & above angle .. signals are same.

Range $\boxed{0 - 2\pi}$

$\boxed{-\pi \text{ to } +\pi}$

$\underbrace{0, 45^\circ, 90^\circ, 135 \dots 2\pi}$

Distant Signals
Different frequencies.

$\boxed{B3}$

There's limit, it increases upto some limit and then decreases.

$\cos\frac{\pi}{4}n$ — 8 samples

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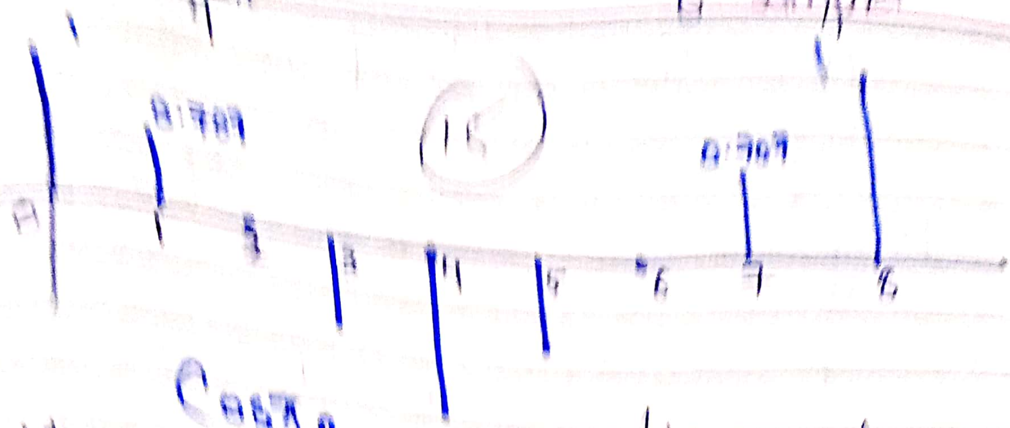
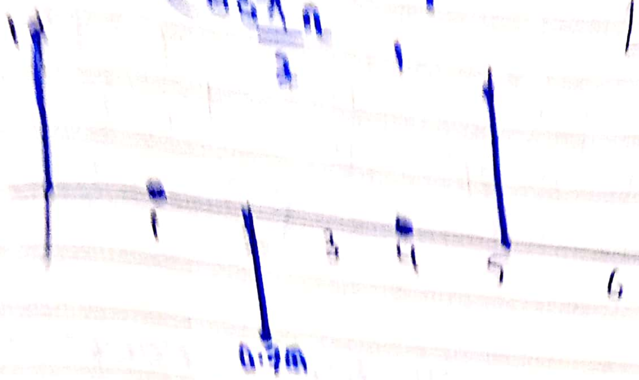
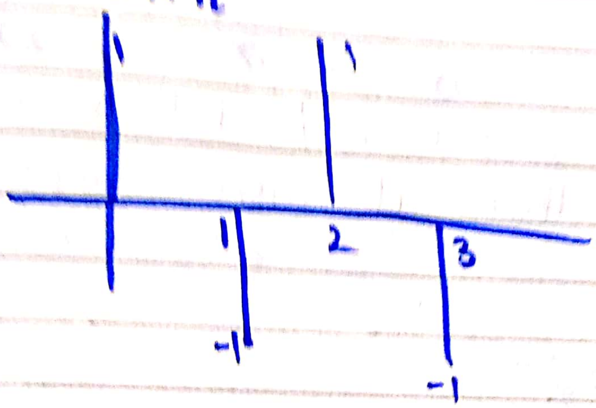


$\cos\frac{\pi}{2}n$ — 4 samples.



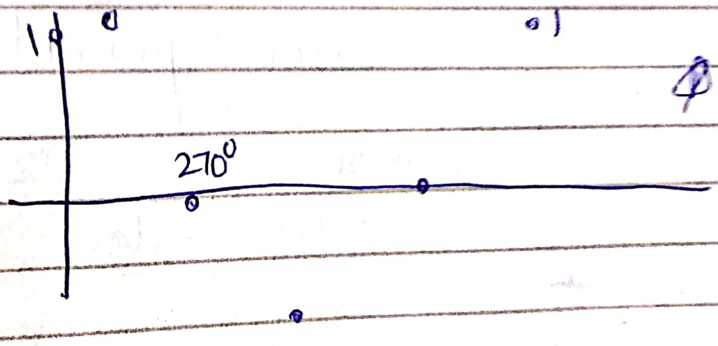
$\cos \pi n$

2 samples



oscillation ↑

$\cos\frac{3\pi}{2}n$



4 samples

Oscillation ↑

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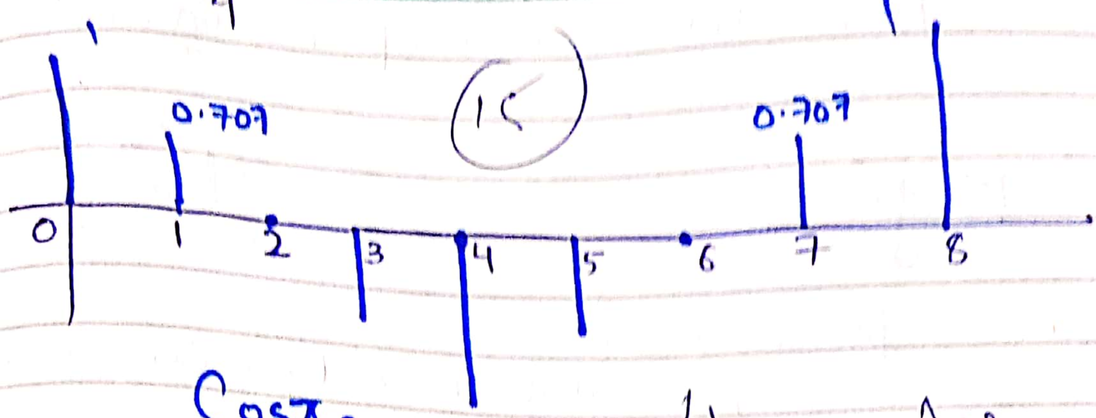
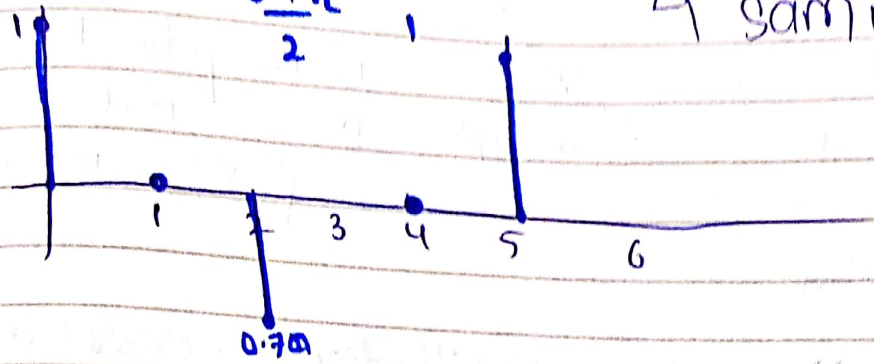
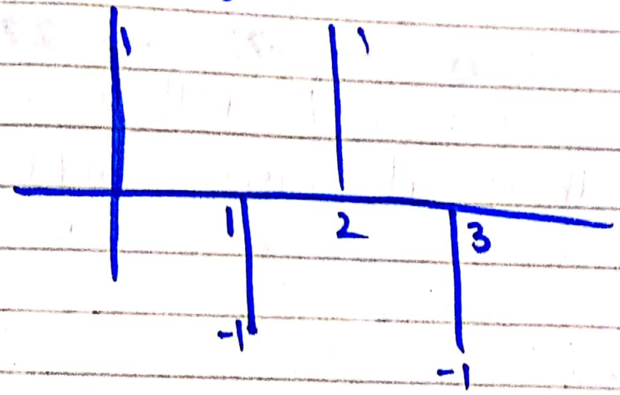
$\cos \frac{\pi}{4} n$ 8 samples



$\cos \frac{\pi}{2} n$ 4 samples.



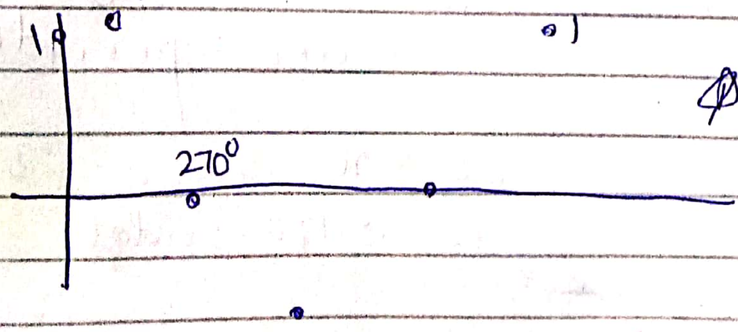
$\cos \pi n$



2 samples

oscillation ↑.

$\cos \frac{3\pi}{2} n$



4 samples
Oscillation ↑.

Above 180°, rate of oscillation decreases

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Why:

for oscillation, signal need to cover 360°

* when 180° se barh jata ha tho 360° multiple me nahi atta aur pir similar signals nahi atay.

Maximum oscillation occurs in distrete at $\omega = \pi$

0 —— π —— 2π

The ocillation ↑ till $0-\pi$ & then decreases. after that

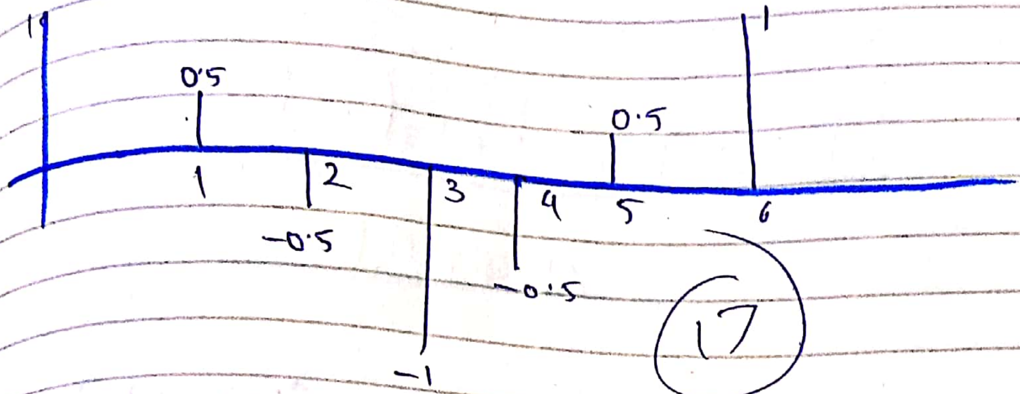
⇒ In DTS,

$\cos \frac{\pi}{4} n$, $\cos \frac{\pi}{2} n$ are periodic

$\cos \pi n$ — 70° is sinusoidal but not periodic.

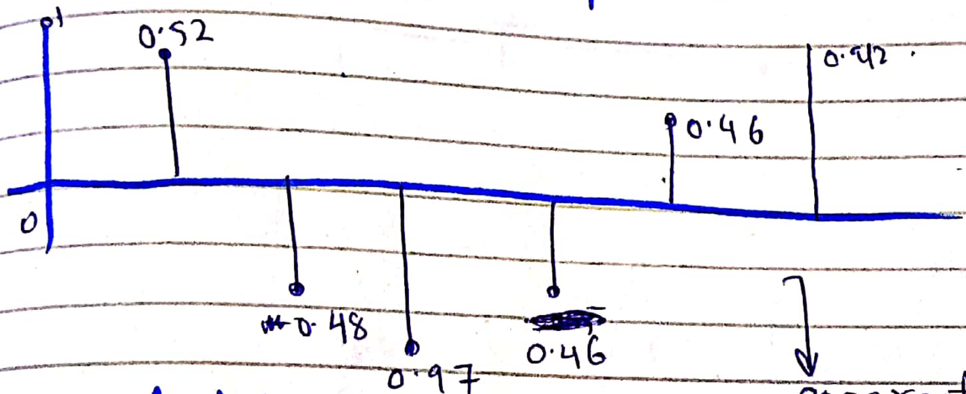
$\cos \frac{\pi}{3} n$ — 60°

$\cos \frac{\pi}{3}$



$\cos 1n$

↳ 1 radian per sample.



apparently it seems to be periodic but it is not coz it skips 1.

[Sinusoidal to be periodic needs 2π or its multiple]

[changing integer ($\cos 1n$) never gives π.]

$\frac{\omega}{2\pi}$ → Rational → the signal will be periodic

↓

omega k andar π hoga thab periodic hoge.

repeat thab ayega if 1 ajaye kahi

lekin wo kabhi nahi ayega

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$\Omega \rightarrow (2\pi)k \rightarrow X$

↓

discrete (1, 2, 3, 4, 5)

Lecture 4

★ (next page)

1.3.3 Harmonic Related Complex Exponentials

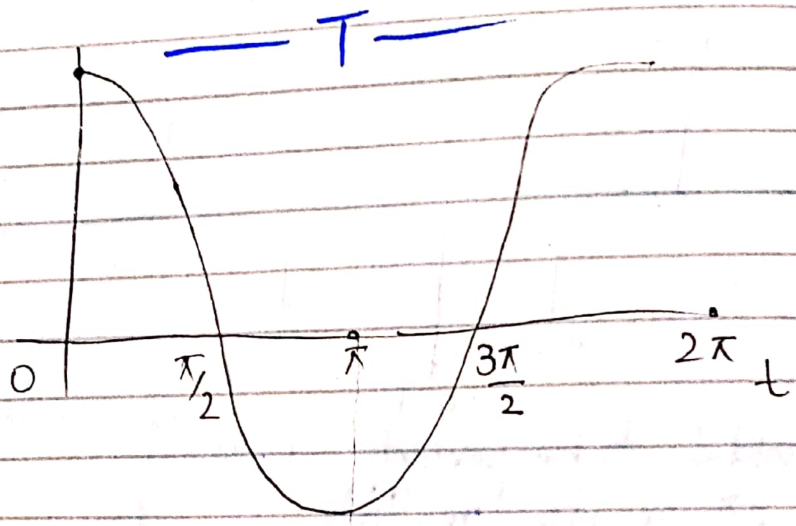
(18)

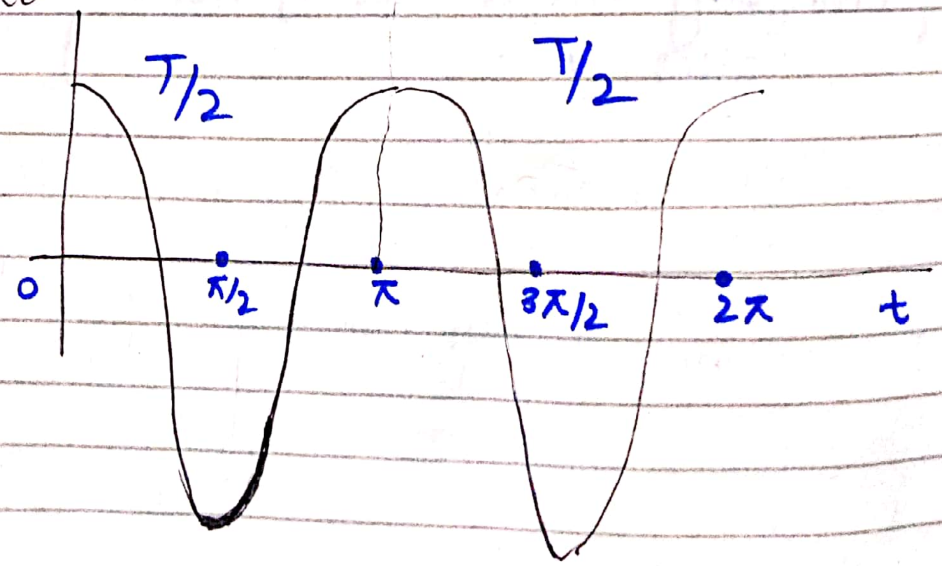
Exponential

$x(t) = Ae^{j(\Omega t+\Theta)}$

Sinusoidal.

$A\cos(\Omega t+\Theta)$





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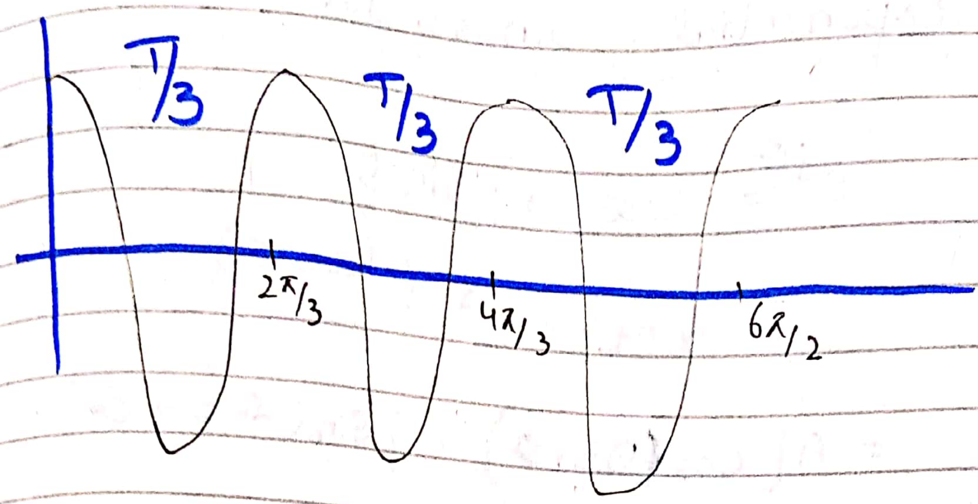
(19)

Cos 3t



⇒ if time period ek dusre k multiple ho then phase repeat hoga ⇒ ?

This is known as Harmonic Related Sinusoidal.

Continuous:

$$x(t) = \sum_{k=-\infty}^{\infty} C_k e^{jk\omega t}$$

$k = \infty \rightarrow +\infty$

↓

harmonics.

(harmonically related

k = 1

k = 2

k = 4

⋮)

Sum of Complex Exponential.

Discrete:

$$x[n] = \sum C_k e^{jk\omega_r t}$$

$k = 0 \rightarrow 2\pi$.

$$\sum C_k e^{jk\left(\frac{2\pi}{N}\right)n}$$

0 & 2π freq is same.

Lecture #4.

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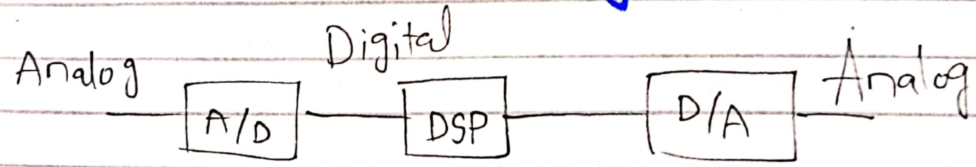
Exponential Sinusoidal:

$$e^{j\theta} = \cos\theta + j\sin\theta .$$

$$x(t) = Ae^{j(\Omega t + \theta)}$$

$$= A[\cos(\Omega t + \theta) + j\sin(\Omega t + \theta)]$$

1.4 Analog to Digital & Digital to Analog Conversions:



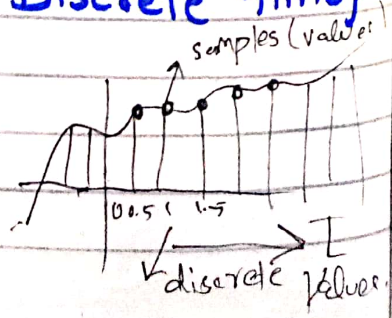
Cont time & Cont signal is call Continuous Signal

values

↳ Digital time & Digital signal " Digital "

Step 1 (cont time to Discrete time):

⇒ Sampling.



Sampling time period = T_s

F_s : $1/T_s$

(2)

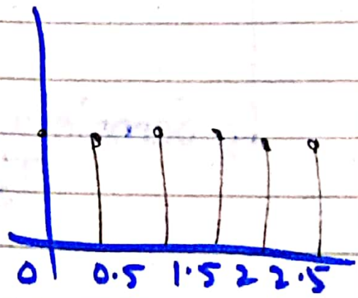
(The time after which sample are taken. (F_s)

(Samples taken per unit time T_s)

↳ At first its time was continuous now through sample humne discrete kardia.

Step 2:

Quantization.



To assign specific levels to each sample.

nT_s → sample no, T.P

1.4.1

Sampling of Analog Signals:

increase no. of samples → good Quality

↓

how much increase?

↓

ziada increase nahi k transmit na hosake.. We need to find optimized solution.

freq $\propto$ variation in signal.

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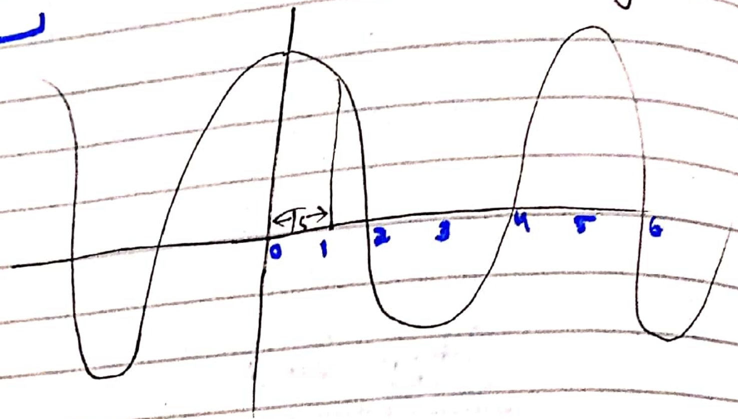
Formula :

$$x(t) = A\cos(\cancel{\omega t}\,\Omega t + \theta)$$

$$= A\cos(2\pi F t + \theta)$$ ↳ sinusoidal signal.

Signal having freq F.

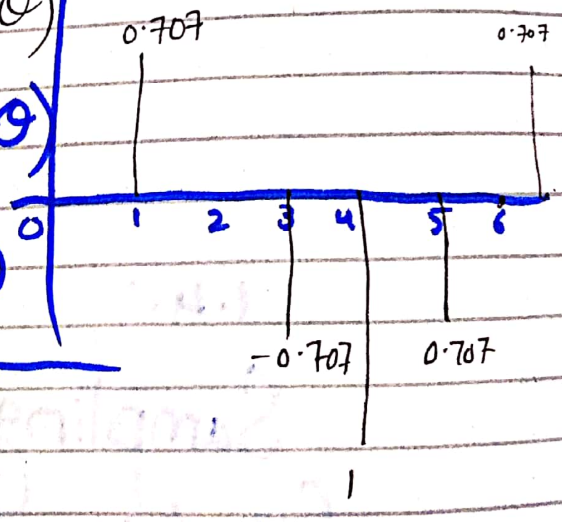


↳ Sample with T_s or F_s.

$$x[nT_s] = A\cos(2\pi F \cdot nT_s + \theta)$$

$$= A\cos\left(2\pi F \frac{1}{F_s} n + \theta\right)$$

$$= A\cos\left(2\pi\left(\frac{F}{F_s}\right)n + \theta\right) \quad \text{—(1)}$$



$$x[n] = A\cos(2\pi f n + \theta)$$

$$\cancel{2\pi}\frac{F}{F_s}\cancel{n} = \cancel{2\pi} f n$$ ⇒ Discrete time sinusoidal

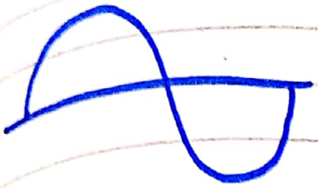
$$\boxed{f = \frac{F}{F_s}}$$

natural F

freq of DTS.

sampling frequency.

Lab (Dsp)



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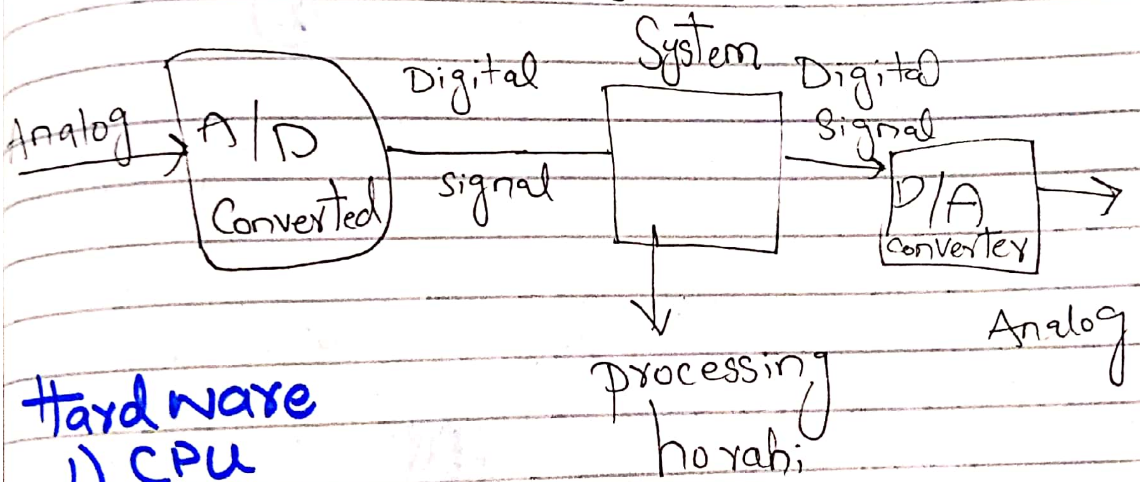
1. Continuous value Continuous Time
↓ Sampling.
2. Continuous " Discrete Time
↓ Quantization
3. Discrete " C "
4. D " D "

↳ Digital Signal

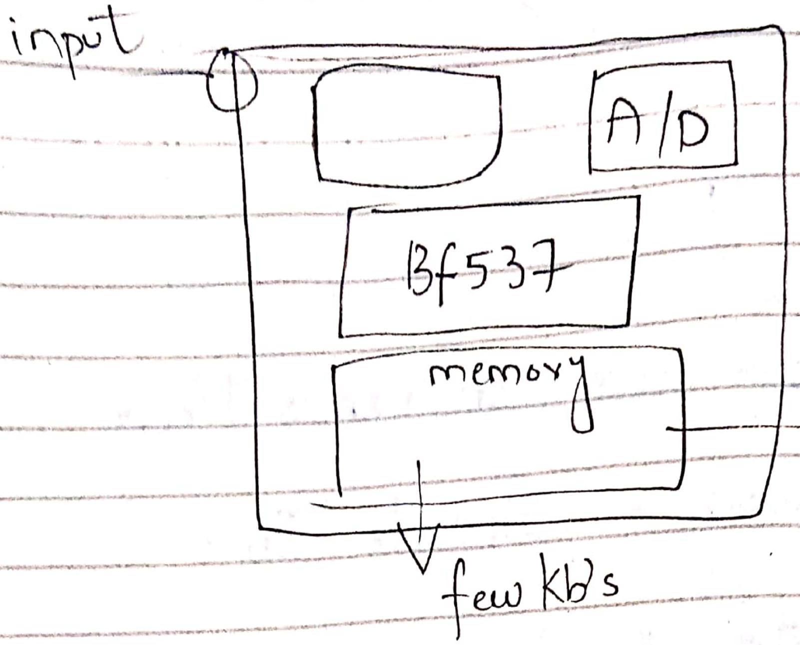
All signals ara Analog



Hardware
1) CPU
2) FPGA's
3) Digital signal Processor.
↓ (specifically designed)
↳ blackfin.

RISC
↓
limited Instruction Set.

Simple Instruc format



→ onchip Memory

↓

IDE → Visualdsp++.

IDE

↓

1 editor

↓

type prog

↓

2 Compiler.

↓

Linker

↓

(Code) Loader

↓ (RAM)

onchip Memory. ← exe. code

Lecture #5

Natural F is not changed.

→ By making samples we make digital signals.

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→ Control DF through Samples.

$f = \omega = -\pi < \omega < \pi$

$f = -\frac{1}{2} < f < \frac{1}{2}$

adjust F_s, so that f remains

→ $-\frac{1}{2}$ to $\frac{1}{2}$

double of numerator $= \frac{1}{2}$

$= < \frac{1}{2}$

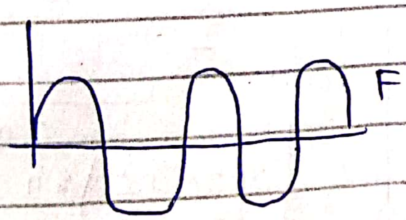
more than double

$F_s = 2F$

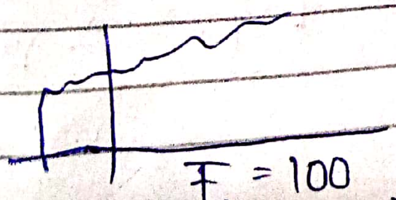
Or

$F_s \geq 2F$

F_s should be greater or double of F.



→ pure sin wave



$F_1 = 100$

$F_1' = 200$ → Take this as base $\frac{200}{400} = \frac{1}{2}$

Greater f + base → will fit smaller frequency.

" That's why F_1 should be double of highest frequency "

If F_s is not selected as double of Frequency ?? ↳ In communication (signals are not PURE)

$$\boxed{\begin{aligned} x_1(t) &= \cos 2\pi (10)t \\ x_2(t) &= \cos 2\pi (50)t \end{aligned}} \qquad \boxed{F_s = 40}$$

$f_1 = \frac{10}{40} = \frac{1}{4}$

$x_1[n] = \cos 2\pi \frac{1}{4} n$

$= \cos \frac{\pi}{2} n$

$f_2 = \frac{50}{40} = \frac{5}{4}$

$x_2[n] = \cos 2\pi \left(\frac{5}{4}\right) n$

$= \cancel{5\cos}\ \cos \frac{5\pi n}{2}$

$= \frac{5}{2}$ greater than $1/2$

In invalid range

$= \frac{1}{2} = 90^\circ, \frac{5}{2} = 450^\circ$ same sinusoid

$= \cos\left(\frac{4\pi}{2} + \frac{\pi}{2}\right) n$

$= \cos\left(2\pi + \frac{\pi}{2}\right) n$

$= \cos \frac{\pi}{2} n$

$x(t) = x_1(t) + x_2(t)$

$= \cos 2\pi(10)t + \cos 2\pi (50)t$

$x[n] = \cos 2\pi \left(\frac{10}{40}\right) n + \cos 2\pi \left(\frac{50}{40}\right) n$

$= \cos \frac{\pi}{2} n + \cos \frac{5\pi}{2} n$

$= \underbrace{\cos \frac{\pi}{2} n + \cos \frac{\pi}{2} n}_{\text{similar}}$

can't differentiate

• The signal allies with each other

$F_s = 40$ -> It's half is holding frequency.

$20 \Leftrightarrow 20$ -> how much analogue it can

folding frequency , $F = \frac{1}{2} \longleftrightarrow \frac{1}{2}$ dicretite in $-\frac{1}{2}$ to $\frac{1}{2}$ range.

$F = 40$, $f = 1$ (invalid)

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$-\frac{1}{2} < f < \frac{1}{2}$ (0, 0.25) | $\frac{1}{2} < f < \frac{3}{2}$ (1, 1.25)

f of $-\frac{1}{2}$ & f of $\frac{1}{2}$

allising on one another.

8 KHz (minimum) 10 KHz, 12 KHz

-> greater sampling

speech 0 - 4 KHz -> better speech

sampling f = 8 kHz

Clock cycle = upper than that we can sample frequency

Processor : MHz => ~~Theoritical~~ Theoratically possible but not practically

-> can't take infinite sample.

2 million → in comp not possible
→ 2 GHz

1.4.2

Quantization of Continuous amplitude signals.

$x(t) = (0.9)^t$

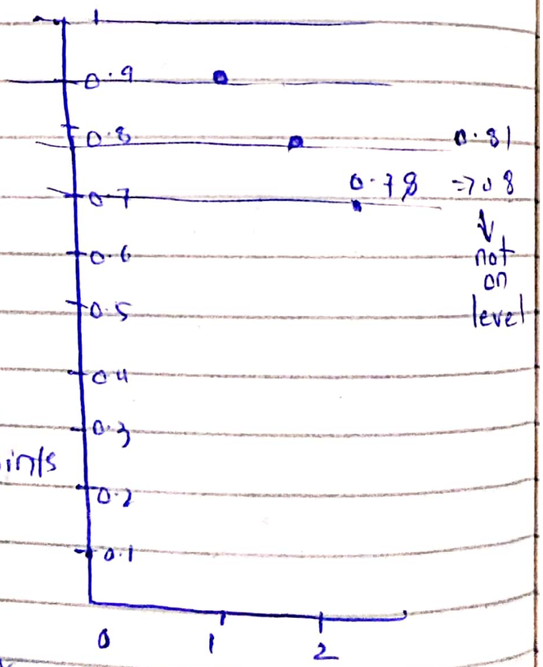
$F_s = 1$

$x[n] = [0.9]^n$

discrete : 0.1, 0.2

Cont : 0.1, 0.19

n	$x[n]$
0	1
1	0.9
2	0.81
3	0.7~~29~~
4	0.6 ~~561~~
5	0.5 ~~9049~~
6	0.4 ~~782~~



greater decimal place : greater level.

point on level → quantized.

In quantized → we move all points on level.

0.5 <
Truncation & rounding same

remove decimal place

cut decimal places → Truncation

round → Rounding

0.5 >
Truncation & rounding different

In rounding
↓
0.81
↓
check 1st if
greater than
↓
time &
resources
used.

0.59045
↓
need 4,5
operations in
rounding

29

→ In Truncation to do
not need operations.
Speed of Is speed = 8KHz → rounding
→ 3 operations
in each
computer.

0.8 include 0.81, 0.82, 0.83
but in decoding don't known 0.8
is made of 0.81, 0.81...
etc.

→ Loss → try to reduce Loss

0.71 → 0.7 → error → 0.01 (less
error
0.72 → 0.02 if near
to
maximum error 0.79 (near to 0.8) 0.7)

Less error → no automatically fall on 0.7

minimum error → 0
maximum " → siz of level ↕ Δ

0.7
Trunction : 0 ≤ e < Δ → Δ added → next level.
↳ not error

Rounding 0.75 →
error 0 → $\frac{\Delta}{2}$ (0.5) half of quantization level.
less than $\Delta/2$.

0·69 rounded to 0·7

0·7 → 0·65 to 0·75 to 0·7

$$-\frac{\Delta}{2} < e < \frac{\Delta}{2}$$

$e = x[n] - x_q[n]$, $x_q[n] - x[n]$

x_q → x quantized x

= difference gives error.

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0·65 → 0·7 + ive error

0·75 → 0·7 − ive error

$e \propto \Delta$

step size / Δ → Less → Less quantization error → No. of levels will increase

$\Delta = 0·1$, $L = 11$ } code → 16 bits

$\Delta = 0·01$, $L = 101$ → need 64 bits to create level.

For every sample → no. of bits will increase → size will increase (data size).

Lecture # 7

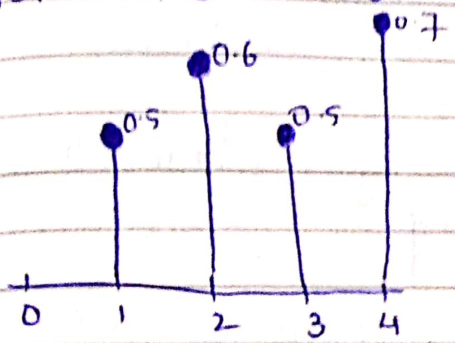
Digital to Analogue

⇒ Decode.

(31)

Group into 4, 4 bits.

0101 0110 0101 0111

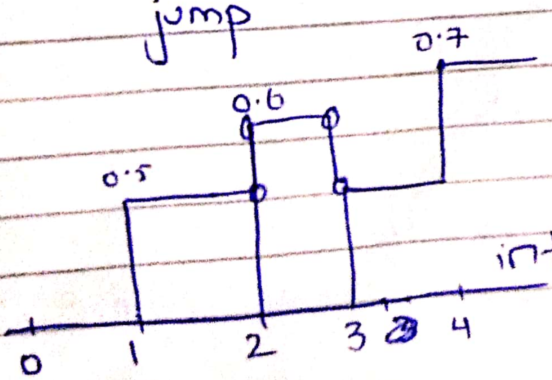


⇒ No Reverse process (Quantization)
Its lose cannot be recovered.

Quantization
Many to 1.

⇒ Samples → continuous (interpolation):

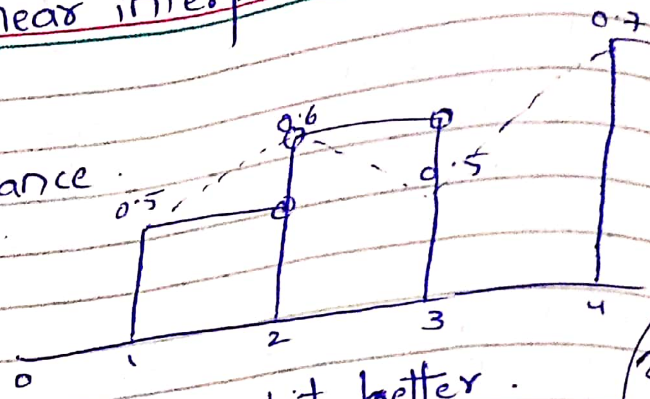
we retain the old value & then jump



(1) zero or hold interpolation.

2 Linear interpolation

=> it is better in performance.



32

Now, this signal is a bit better.

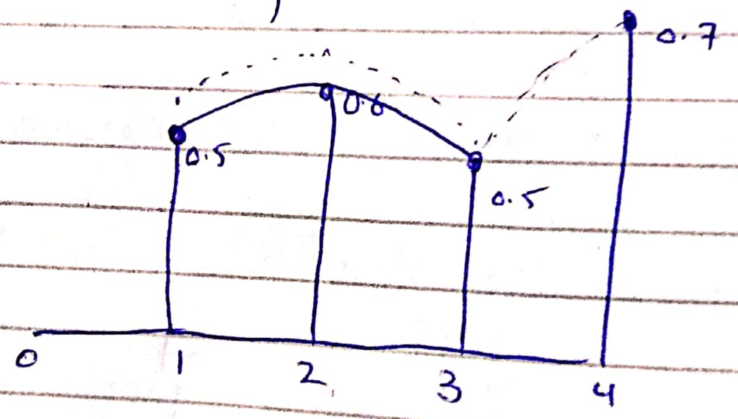
then why zero or hold interpolation?

=> No calculation, just retain the old point & jump.

In Linear, we need to calculate each point.

3 Quardratic Interpolation

Combine 3 points.



$y = ax + b$

$y = ax^2 + bx + c$

↙ 2nd order eqn

=> Need more time to Calculate

Chap #2.

Discrete-time Signal & System.

Here we will discuss discrete signals.

Most natural signals are Analogue ⇒ Converted to Discrete.

Signals can be inherently discrete.

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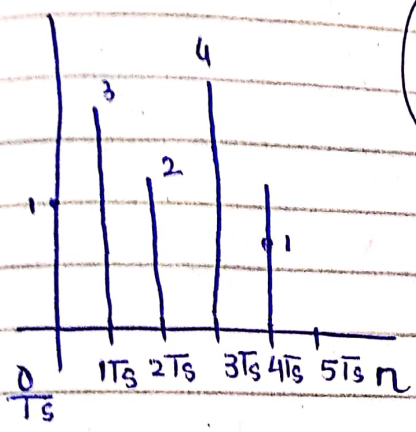
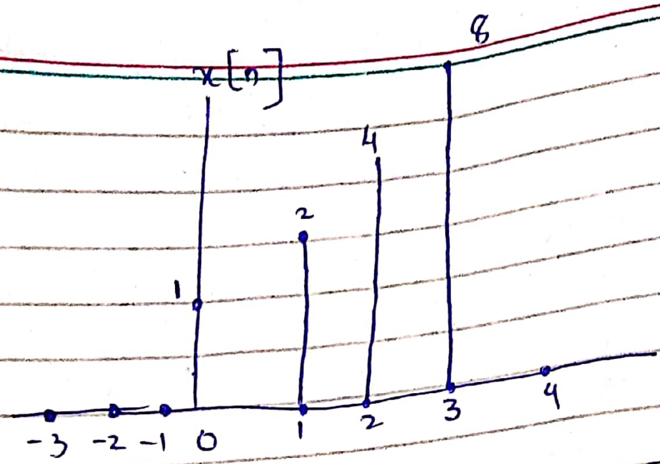


⇒ indexes of Discrete signals will be integers.

Functional form:

$$x[n] = \begin{cases} 0 & n<0 \\ 1 & n=0 \\ 3 & n=1 \\ 2 & n=2 \\ 1 & n=4 \\ 0 & n>4 \\ 4 & n=3 \end{cases}$$



$$x[n] = \begin{cases} 0 & n < 0 \\ 2^n & \end{cases}$$

Tabular Form:

n	-2	-1	0	1	2	3	4	5
x[n]	0	0	1	3	2	1	0	0

Sequence Form:

Improved form of Tabular form

zero location

x[n]

0	0	1	3	2	1	0	0

1 location must be identified.

Some Elementary DT Signals:

(1) ⇒ Unit Sample Signal / Impulse Signal:

Special signal

$$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \\ & \text{(otherwise)} \end{cases}$$

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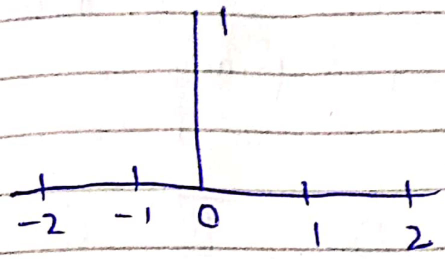


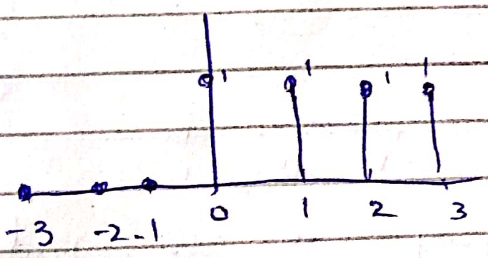
(2) Unit Step Signal:

Special signal.

↓

1 Step & that's it.

$$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$

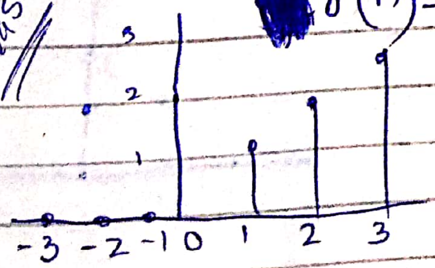


→ to extract portions from signal.

(3) Unit Ramp:

$$r(n) = \begin{cases} 0 & n < 0 \\ n & n \geq 0 \end{cases}$$

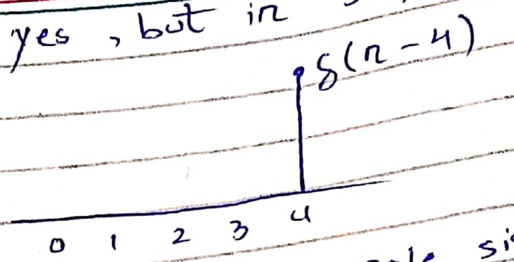
linear increase



As n increases signal increases.

can it be opposite?

yes, but in shifted form



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This is unit sample signal but in shifted form.

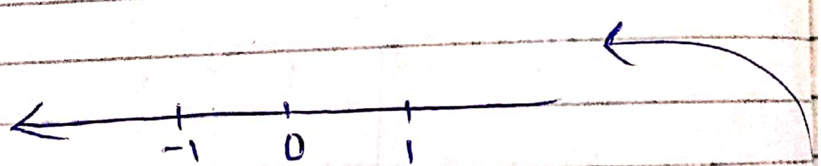
Lecture # 8

Exponential Signal : → fast growing signal

$$x[n] = a^n$$

a_{real} → Different behaviour

a_{comp} → Different behaviour.



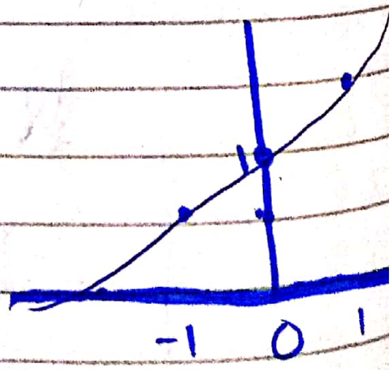
when we say a is real so a is somewhere from this line

Real if

$\boxed{a<1}$

$a = 2$

$$x[n] = 2^n$$



base > 1

power ↑

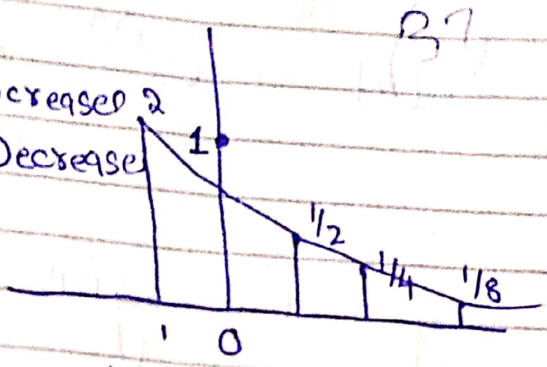
signal ↑

Case 2:

$$\boxed{0 < a < 1}$$

$$x[n] = \left(\tfrac{1}{2}\right)^n$$

As n increases, signal Decrease

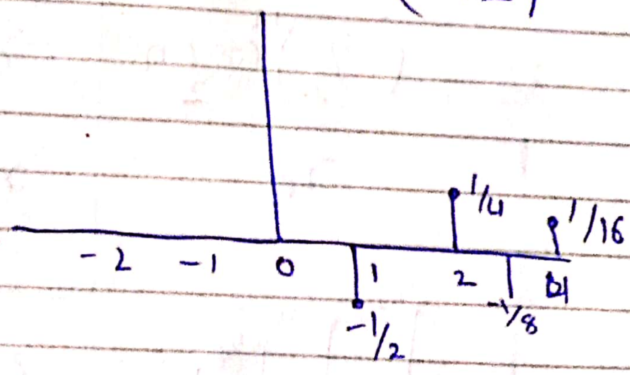


On -ive indexes, values increas..

Case 3:

$$-1 < a < 0$$

$$x[n] = \left(-\tfrac{1}{2}\right)^n$$



$$x[n] = [-2]^n$$

↓ odd values -ive hojaige baqi same signal hoga.

On even index +ive value & vice versa

Jitna power barhtaha utna hi uska Rate barhtaha

a complex:

$$x[n] = a^n$$

$$a = \text{complex}$$

$$a = c + dj = re^{j\theta} \qquad \theta = \tan^{-1}\frac{d}{c}$$

$$x[n] = (re^{j\theta})^n = r^n e^{j\theta n}$$

$$= r^n[\cos\theta n + j\sin\theta n]$$

Exponential Signal to Sinusoidal Signal.

$$= r^n\cos\theta n + jr^n\sin\theta n.$$

For Real:

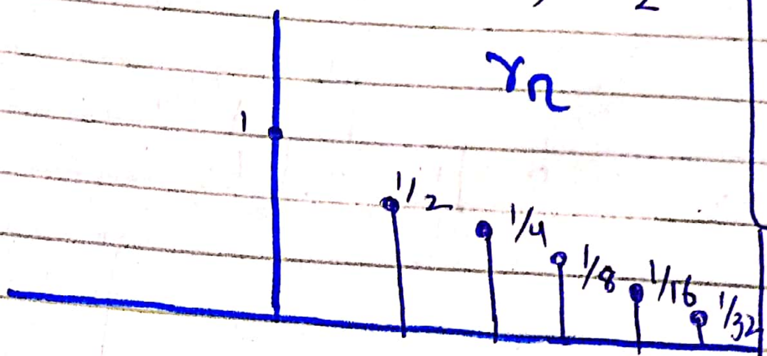
$$x_R[n] = r^n\cos\theta n$$

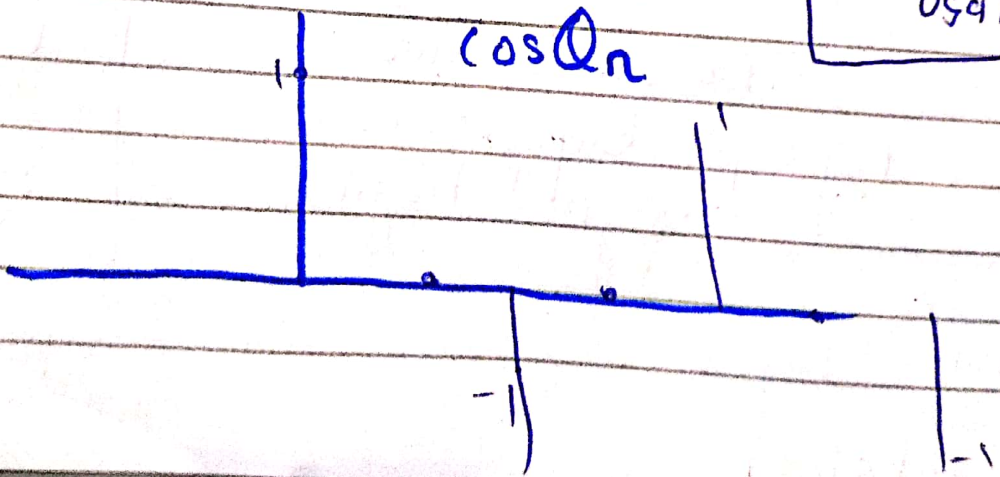
(madeup of 2 signals)
(Exp + Sin)

$$\Rightarrow \left(\tfrac{1}{2}\right)^n \cos\frac{\pi}{2}n$$

Sin
$r > 1$ ↑
$r < 1$ ↓

Exp
θ measures Rate of oscillation.





$\boxed{r^n \cos \Omega n}$

Damping Signal.

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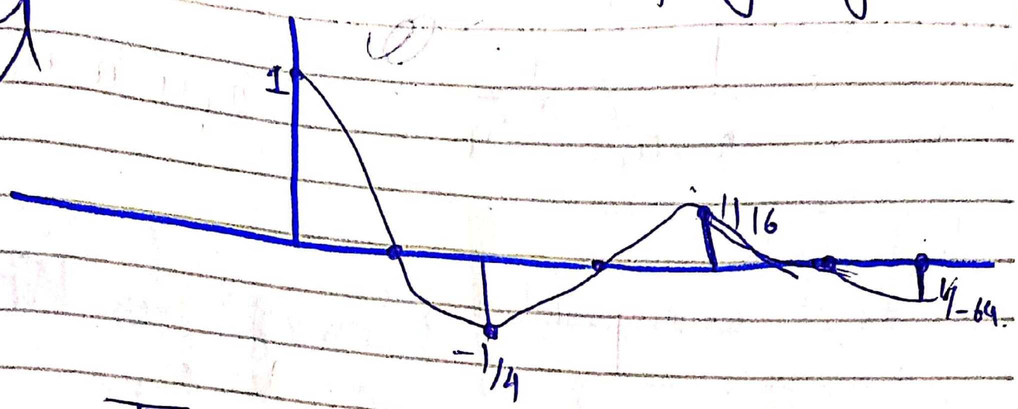
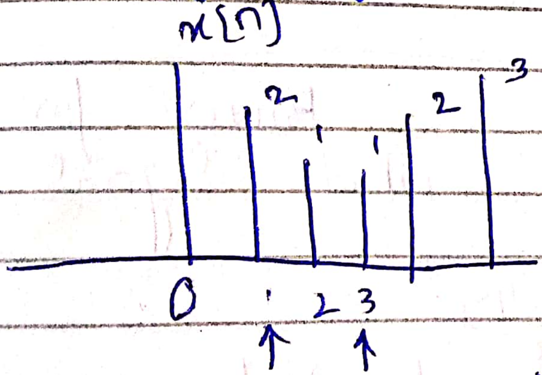


The signal oscillate due to sinusoidal signal & decrease in signal is due to exponential.

2.1.2

Classification of DT Signals:

Energy signal & Power Signal:



$$E = |x[n]|^2$$

$$E_{n_1 \to n_2} = \sum_{n=n_1}^{n_2} [x[n]]^2 \rightarrow \text{finding } E \text{ in interval.}$$

$P \Rightarrow$ Average Energy per unit Sample

$$\underset{n_1 \to n_2}{P} = \frac{E}{\Delta n} = \frac{1}{n_2 - n_1 + 1} \sum_{n=n_1}^{n_2} |x[n]|^2$$

In Discrete
∵ E is in point.
So E is one point less than that of continuous

Energy for Whole Signal:

$$E_\infty = \sum_{n=-\infty}^{\infty} |x[n]|^2$$

$$P_\infty = \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2$$

if

$$\boxed{P_\infty = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2}$$

Power for whole Signal.

On basis of result we classify the signal as Energy or power Signal.

Case 1: $E_\infty < \infty$ (finite), $P_\infty \to 0$.

Case 2: $E_\infty \to \infty$,

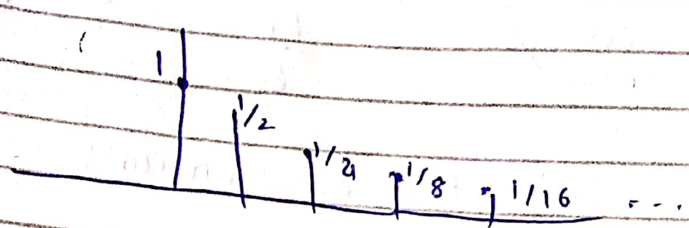
When signal is of finite duration

Q1

$0 \rightarrow 5 \rightarrow$ (no zero portion)

indexes $\rightarrow \infty \rightarrow -\infty$.

Case 2:



$1 + 1/2 + \frac{1}{4} + 1/8 \ldots\ldots \rightarrow 2$

converging

$1+2+4+8+16$

diverging.

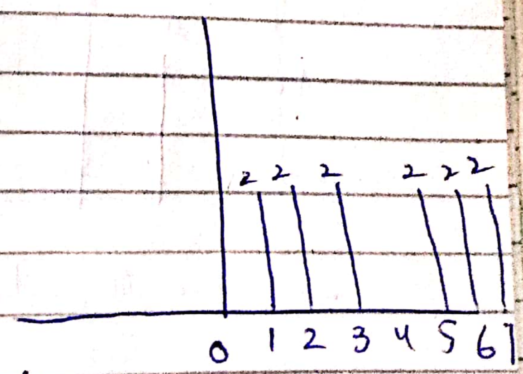
decreasing exp geometric sequ

Infinite signal $\downarrow$ $\rightarrow E \uparrow$

Infinite signal $\uparrow$ $E \downarrow$.

Case 3:

$\Rightarrow \frac{E\infty}{E\infty}$



$= \frac{4+4+0+4+4}{\infty}$

~~Take First period~~

$\Rightarrow \frac{4+4+0}{3} = \frac{8}{3} \Rightarrow 2.6$

$\frac{4+4+0+4+4+0}{6}$

u2

$\frac{16}{6} \Rightarrow 2.6$

Ratio Remains Constant

for ∞ no. of cycles power remain 2.6

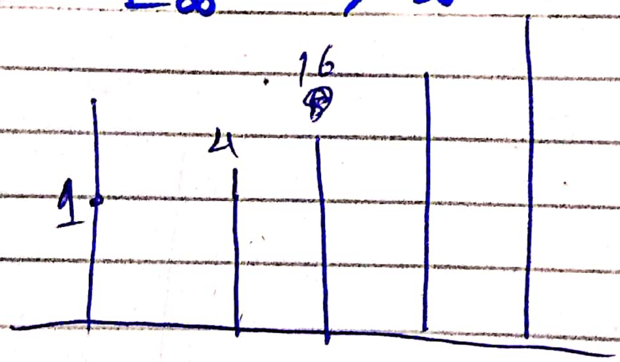
$$\boxed{E \to \infty \quad , P_\infty < \infty}$$

Case 2.

Case 3:

useful signal

$E_\infty \to \infty \quad , P_\infty \to \infty$



$\to 1+4+16$ period $\uparrow$

power $\uparrow \to \infty$

$E \uparrow \to \infty$

Lecture # 89

Periodic & Aperiodic:

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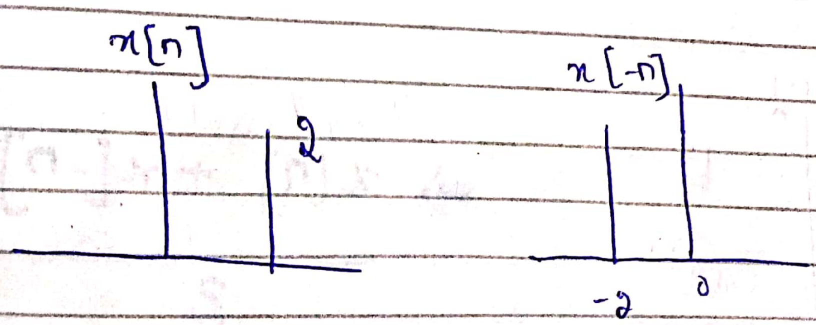
Symmetric Signals & Anti Symmetric:

Symmetric → Even

Anti Symmetric → Odd.



$x[n] \rightarrow x[-n]$

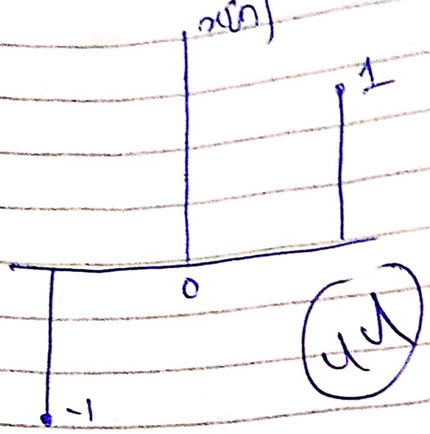
⇒ Based on this reversal, we can classify signals

If $x[n] = x[-n]$

⇒ signal is Symmetric or Even signal.

when signal is opposite to it

If $x[-n] = -x[n] \rightarrow$ Odd signal.

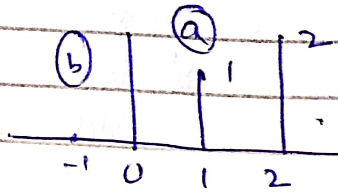


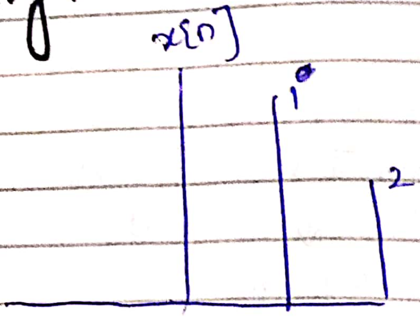
The value at 1 & -1 are equal

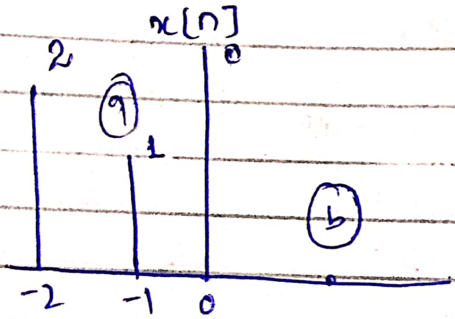
$\Rightarrow$ 0 index par it is a must to have zero value.

else
Asymmetric
↳ No symmetry.

$\Rightarrow x[n] + x[-n]$







Signal clift

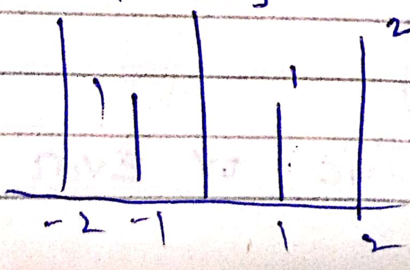
For even:

$$\Rightarrow \frac{x[n] + x[-n]}{2}$$

For odd

$$\frac{x[n] - x[-n]}{2}$$

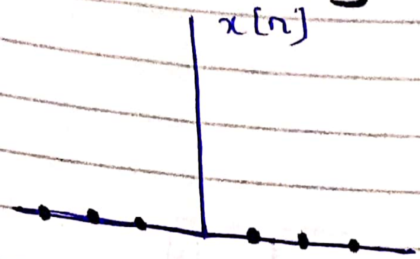
$2 + x[n]$



$$\Rightarrow \frac{x[n]}{2} + \cancel{\frac{x[-n]}{2}} + \frac{x[n]}{2} - \cancel{\frac{x[-n]}{2}}$$

$x[n] \to$ [Even], [Odd]

eg

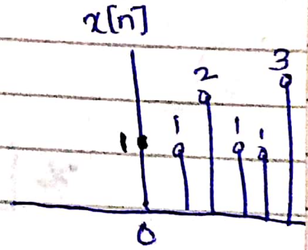


∴ zero signal is even & odd at the same time

2.1.3 Simple manipulations of DT Signal.

① $u[n] \Rightarrow$

$x[n \pm k] \to$ Shifting process.



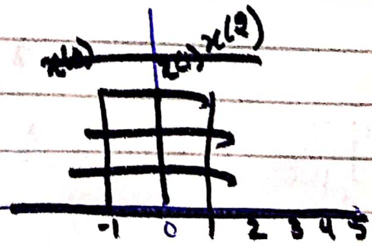
$x[n-1]$

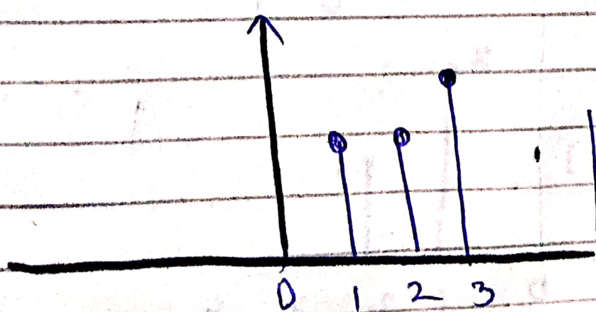
$\Rightarrow 0-1 = -1 \quad x(0)$

$\Rightarrow 1-1 = 0 \quad x(1)$

$\Rightarrow 2-1 = 1 \quad x(2)$

$\Rightarrow 3-1 = 2 \quad x(3)$





Delay of Signal.

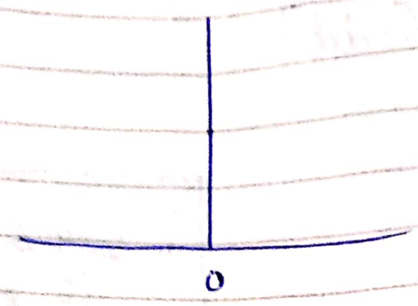
Right Shift

$x(n+1)$

$x(0+1) = 1$ $\qquad x(0)$

$x(1+1) = 2$ $\qquad x(1)$

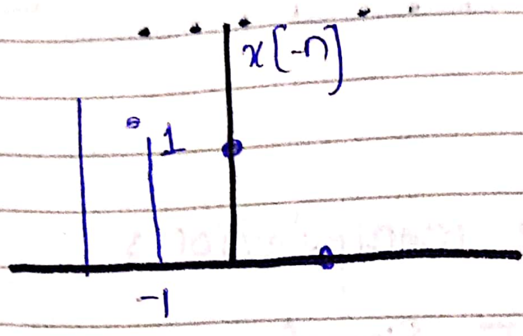
$x(2+1) = 3$ $\qquad x(2)$



4(i)

2

$x[-n]$

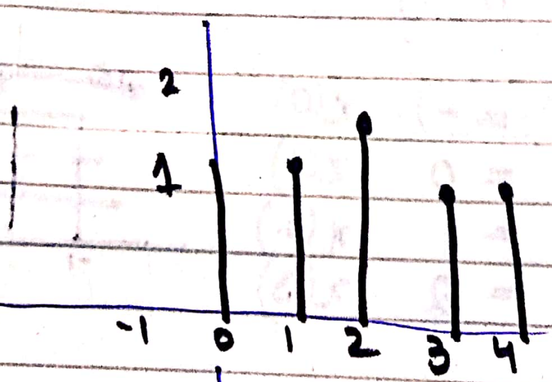


3

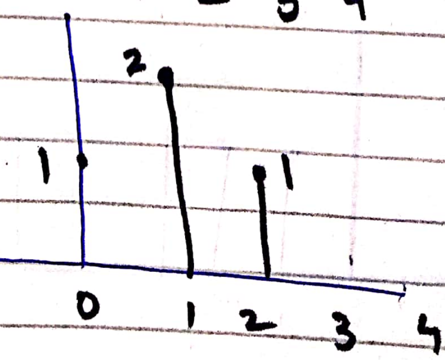
$x[an]$

$x > 1$ (greater than 1)

if we have N samples



then there comes N/2 samples



1 & 3 skip.

Down Sampling

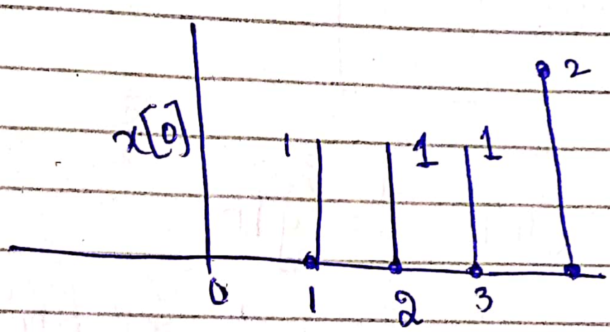
→ Sampling T.P increases
 · inf loss can occur

→ original sample is halfed
 → size of signal reduced.

(u7)

$x[\frac{1}{2}n]$

⇒ $\boxed{\text{upsampling}}$



A) $x(-2n-2)$

⇒ Shifting
⇒ Scaling
⇒ Reversal

$2x[n]$
xing by 2
$-[x[n]]$
-1 xing

Lecture # 10

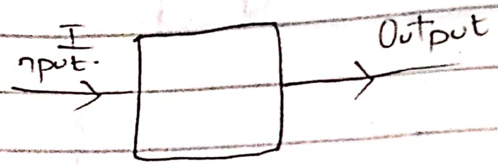
(48)

Section 2.2

Discrete-time Systems:

⇒ Signal is a physical quantity which can measured or calculated.

⇒ System are physical or logical devices which process the signal.



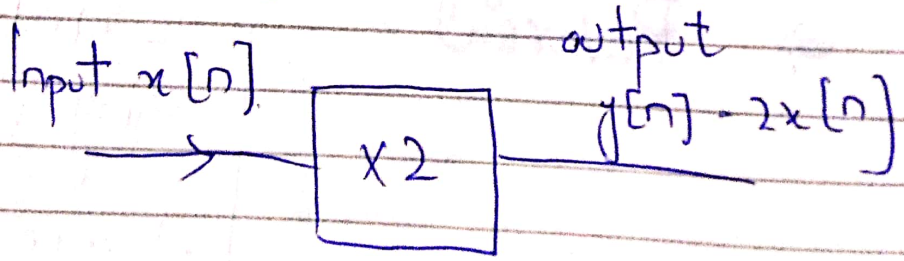
$$x[n] \xrightarrow{T} y[n].$$

T is any operation.

$$T[n] = 2x[n]$$

$$y[n] = x[n-1]$$

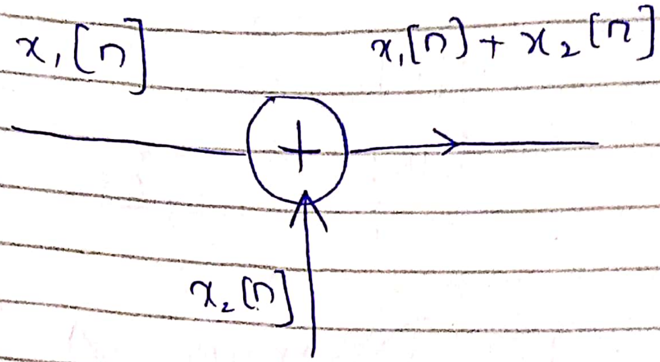
$$y[n] = 3x[n-1]$$



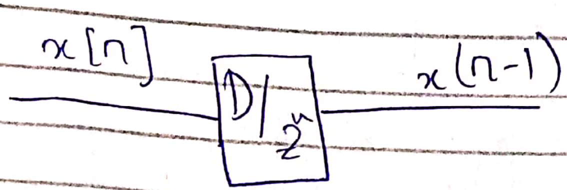
$y[n] + 3y[n-1] = 2x[n]$

49

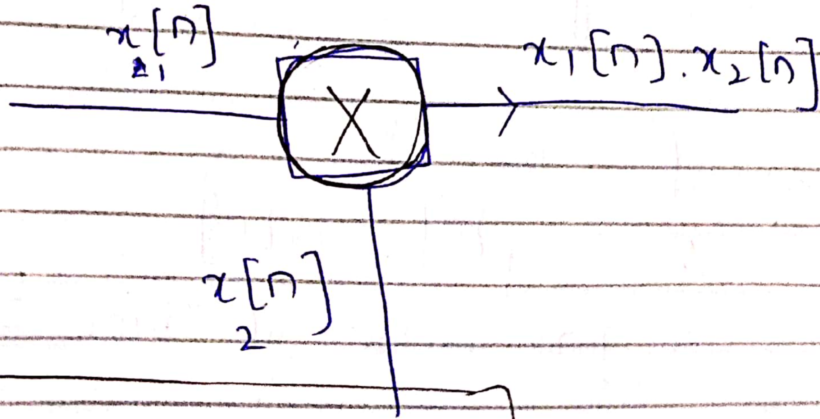
Adder :



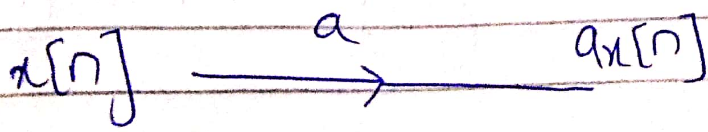
Delay :



Multiply :



Scalar Multiplication

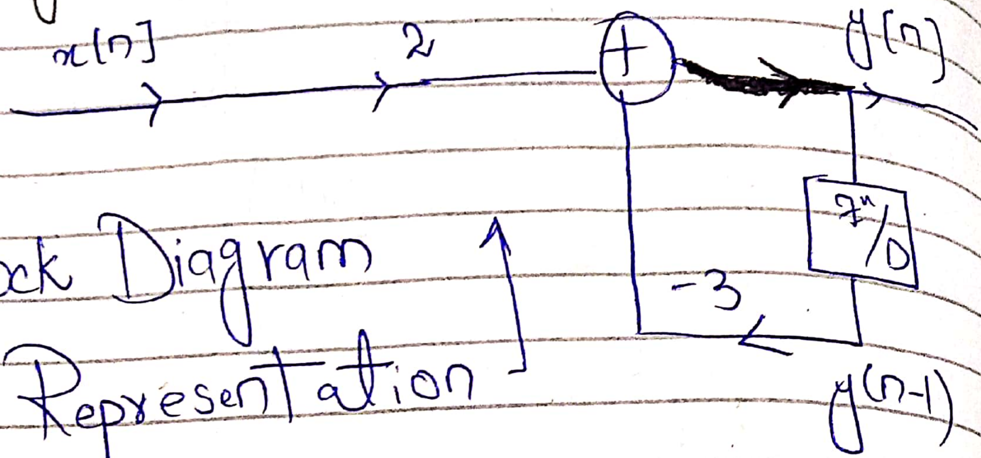


$$y[n] + 3y[n-1] = 2x[n]$$

$$y[n] = 2x[n] - 3y[n-1]$$

EU

$$y[n] =$$



Block Diagram Representation

Classification of Signals.

⇒ Static | Dynamic

(Memoryless) (Memory)

3 3

$$y[n] = 2x[n]$$

to find output at time 3

we need input at " "

$$y[n] = 3x[n-1]$$

For $y(n) = 3x[n-1]$ (51)

⇒ to find output at time 3, we need input at time 2.

⇒ t=2 ke data ko memory me raky & use it at t = 3

⊙ Here we are using previous inputs so this is Dynamic system.

→ If system depends on previous inputs then it is Dynamic (with memory) vice versa.

⇒ Time Invariant and Time-Variant System.

→ Behaviour doesn't change wd time
→ Behavious changes with time.

$$x[n] \longrightarrow y[n]$$

$$x[n-K] \longrightarrow y(n-k)$$

⇒ Linearity:

$$x[n] \longrightarrow y[n]$$

if we doubles input & output also doubles so it follows linearity.

$A\,x[n] \longrightarrow A\,y[n]$

The system is said to be Linear.

$x_1[n] \longrightarrow y_1[n]$

$x_2[n] \longrightarrow y_2[n]$

$A x_1[n] + B x_2[n] \longrightarrow A y_1[n] + B y_2[n]$

⇒ **Causality** :- (57)

$y[n] = 2x[n]$

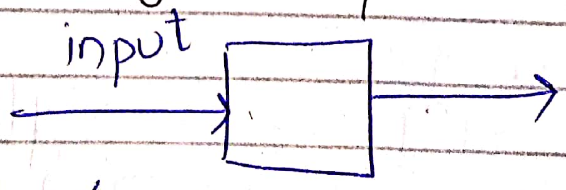
$y[n] = 3x[n] + 2x[n-1]$

$y[n] = 2x[n+1]$

⇒ systems which depend on current or past inputs or current / past that system is known as Causal.

⇒ System which depend on future inputs → non causal

Stable system / Unstable.



⇒ this system is Stable.

$x[n] < \infty$ → $y[n] < \infty$

finite value → finite output.

$x[n] \rightarrow \boxed{} \rightarrow y[n]$ ⇒ this system is unstable.

$y[n] = \Sigma x[n]$

input is finite & output is summing (infinite)

n {3}

→ bridge unstable
↓
It is not something bad but we have ~~as~~ to follow some condition

Lecture # 11

3.3 Analysis of DI LIT Systems.

Naming a system is also analysis.

Describing functionality of system is analysis.

$y[n] = 2x[n]$

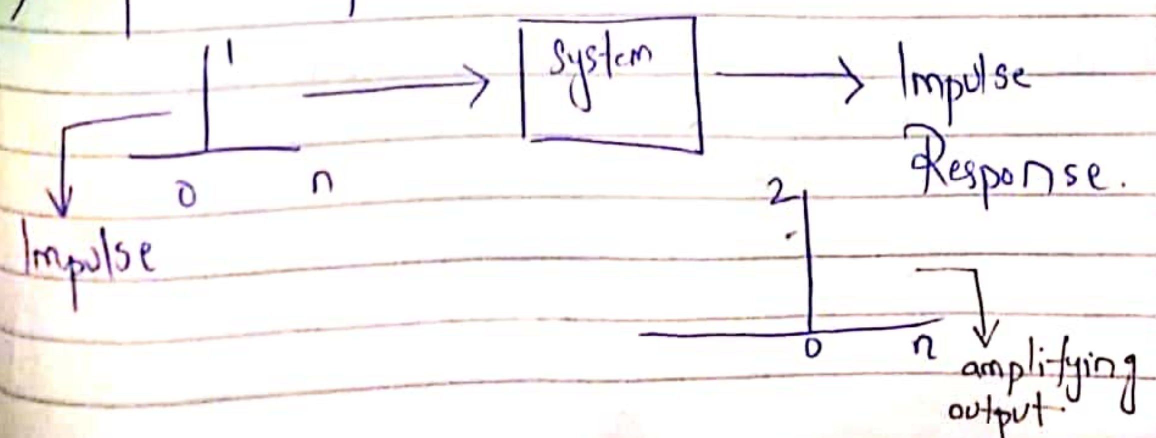
$y[n] - 3x[n] + 2x[n-1]$

$\underbrace{y[n] = 2x[n] - 3y[n-1]}$

Solving this is also Analysis.

There are two ways of analysis

1) Impulse Response.



Linearity

NL	L

Time Variant

N	V
no changes with times	changes with time.

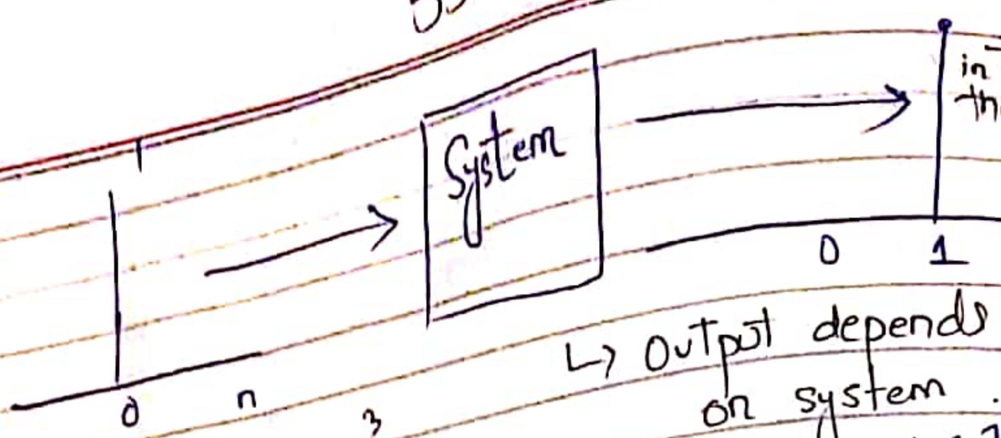
① L I I linear time variant.

② NLTI

③ NL TV

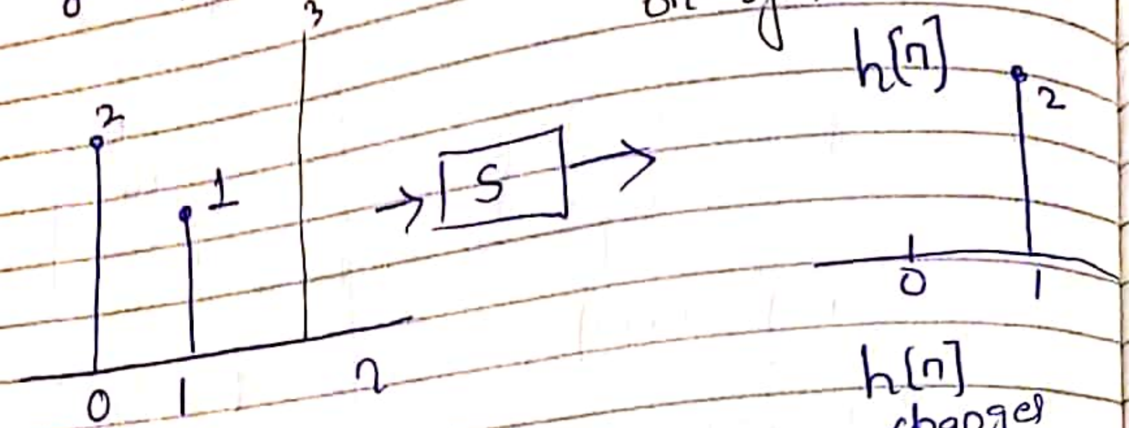
④ LTV

Output $\mathcal{H}$ cause



→ Delaying in the ~~system~~ signal

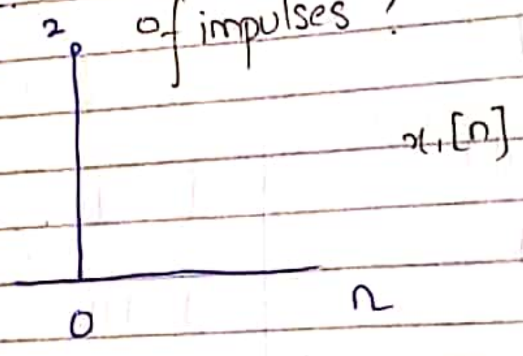
↳ Output depends on system.

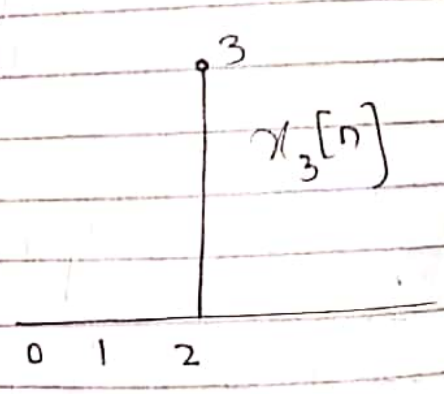


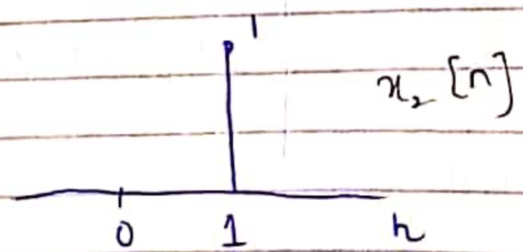
h[n] changes with each system

$\delta[n]$ is a fixed signal.

How to represent this in form of impulses?







We can write signal as sum of sub signal

$$x[n] = x_1[n] + x_2[n].$$
$$= 2\,\delta[n] + \delta[n-1] + 3\,\delta[n-2]$$

⇒ ~~There~~ is no need of shifting at 0 location

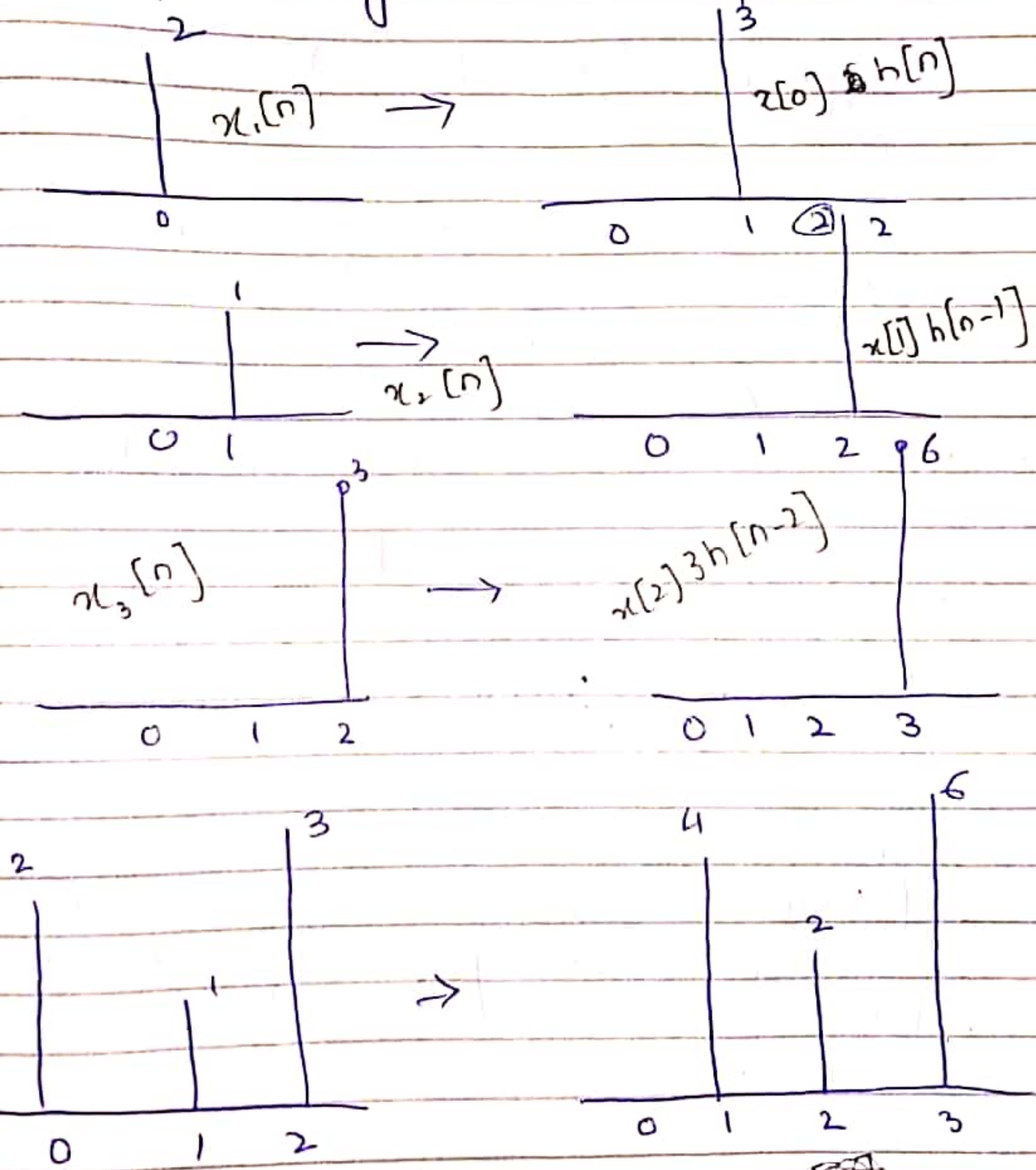
$= x[0]\,\delta[n] + x[1]\,\delta[n-1] + x[2]\,\delta[n-2]$

→ we can represent any signal in the form of shifted & scaled impulses.

h[n] ko hum scale karty ha according to x[n]

Analysis the signal according

Compare signal to Impulse



Any signal in form of Impulse.

$x[n] = x(-1)\,\delta(n+1) + x(0)\delta(n) + x(1)\,\delta(n-1) + x(2)\,\delta(n-2)$

$$\boxed{x[n] = \sum_{k=-\infty}^{\infty} x(k)\;\delta(n+k)}$$

signal

$y[n] = x(0)h[n] + x(1)h(n-1) + x_2(2)h(n-2)$ ..

$$\boxed{y[n] = \sum_{k=-\infty}^{\infty} x(k)\;h(n-k)}$$

diffindexes

normally they both are diff but here we consider both signals

convolution sum formula.

system Impulse.

$x[n] * h[n]$

Just applicable LTI system.

Linearity ~~pro~~ property is maintained. if we 2 add to input so 2 is also added to output

If time changes, so signal doesnt change, only shifting

is

fi

r

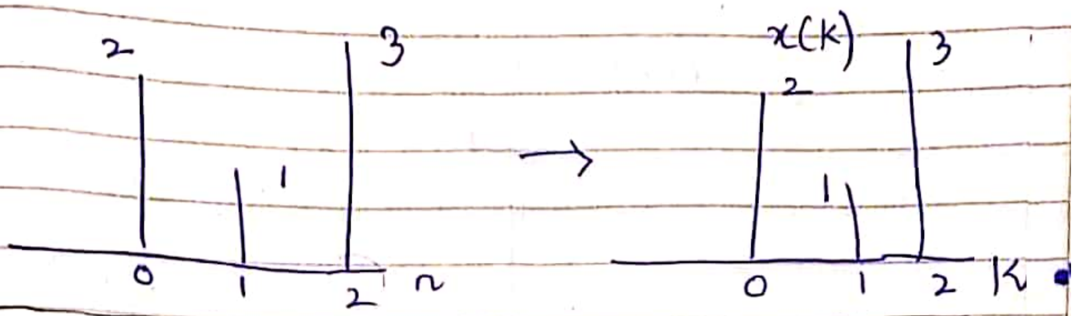
hoi ha

$$y[n] = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$ 58

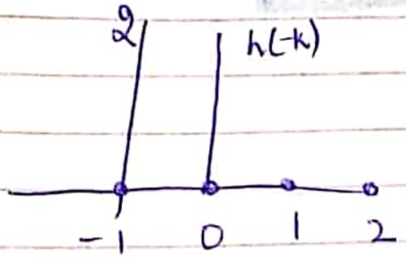
$= x(0)\, h[n] + x(1)\, h[n-1] + x(2)\, h(n-2)$

we have two signals , we don't divide it .

$x[n] \longrightarrow x[k]$

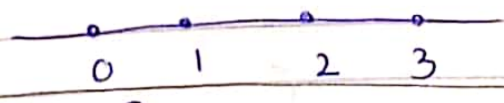


$h(-k)$ $\boxed{\therefore\ n=0}$



$x(k)\, h(-k)$

$\downarrow$

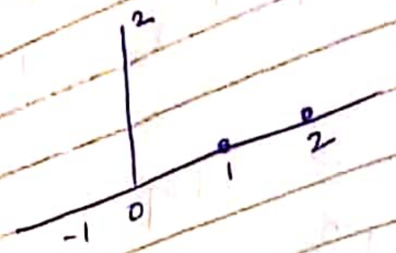


$y(0) = 0$

$$y(1) = \sum_{k=-\infty}^{\infty} x(k)\, h(1-k)$$

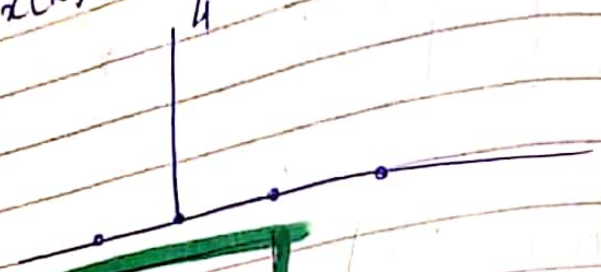
$h(1-k)$

$h(-k+1)$



$x(k)$

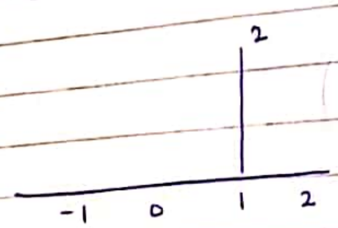
$x(k)\,h(1-k)$



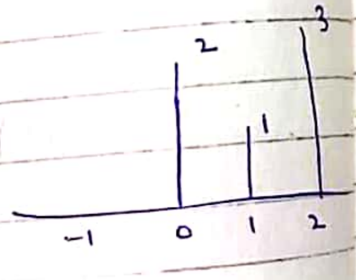
$$\boxed{y(1) = 4}$$

$$y(2) = \sum_{k=-\infty}^{\infty} x(k)\,h(2-k)$$

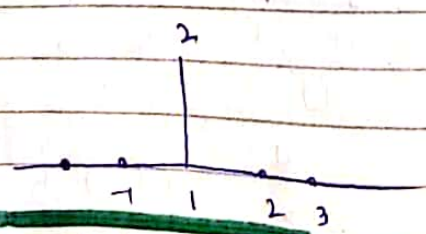
$h(-k+2)$



$x(k)$



$x(k)h(-k+2)$



$$\boxed{y(2) = 2}$$

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60

look at $x(k)$ — For calculating value of output we need to consider the whole input signal.

Shift the signal according to it

if

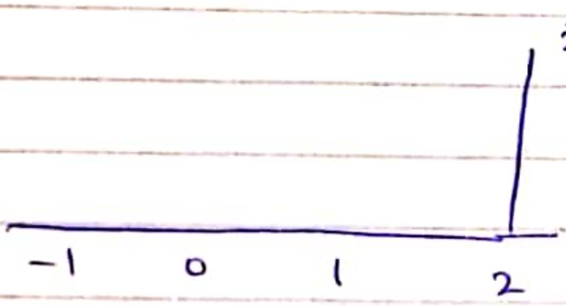
$x(k) = 0, 1, 2, 3$

then

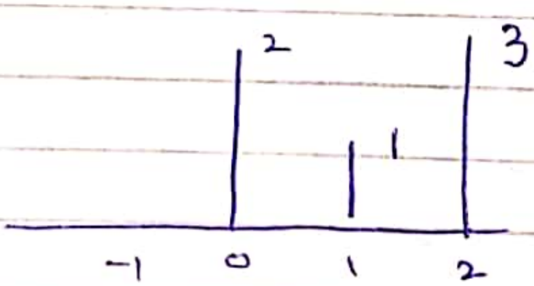
after 3 ($h(k)$) no need to shift it anymore)

$$y(3) = \sum_{k=-\infty}^{\infty} x(k)\, h(3-k)$$

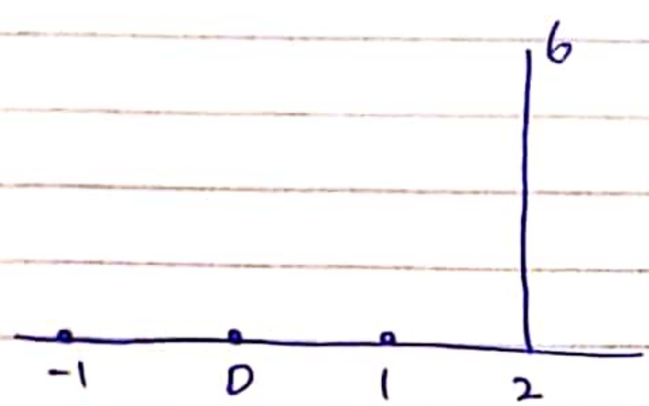
$h(-k+3)$

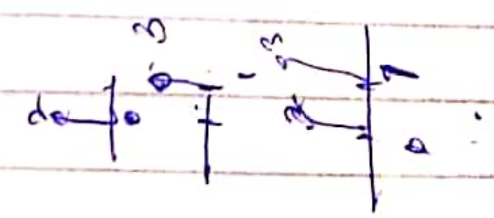


$x(k)$



$x(k)\; h(-k+3)$





$y(0)$ $y(1)$ $y(2)$

$y(2) = 0 + 4 + 9 + 6$

Analysis of LTI sys. 2.3.3.

1st November 2019.

$$y[n] = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

$$y[n] = \cdots + x(0)h[n] + x(1)\, h(n-1) + \cdots\cdots$$

Convolution:

$x[n]$, $h[n]$

1) Change index of $x[n]$ & $h[n]$ to get $x(k)$ & $h(k)$.
2) Reverse $h(k)$ to get $h(-k)$.
3) Shift $h(-k)$ to get $h(n-k)$.
4) Multiply $x(k)$ & $h(n-k)$
5) Sum the product signal from $k = -\infty$ → $+\infty$:)
6) Repeat steps 3-5 for all values of $n(-\infty, \infty)$.

$y(n) = 0$

Is convolution commutative?

Commulative Property:

Properties:

$$x[n] * h[n] = h[n] * x[n]$$

if Commutative Law is true

then

$$h[n] * x[n] = \sum_{k=-\infty}^{\infty} h(k)\, x(x-k).$$

$$y[n] = x[n] * h[n]$$

or

$$= \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

$$= \sum_{r=-\infty}^{\infty} h(r)\, x(n-r)$$

$n-k=r$ ∴ $k=-\infty$, $r=\infty$

$n-r=k$

∴ $k=+\infty$, $r=-\infty$

convention

$- \rightarrow +$

other

$+ \rightarrow -$

is okay.

$$= h(n) * x(n)$$

So, it is commutative.

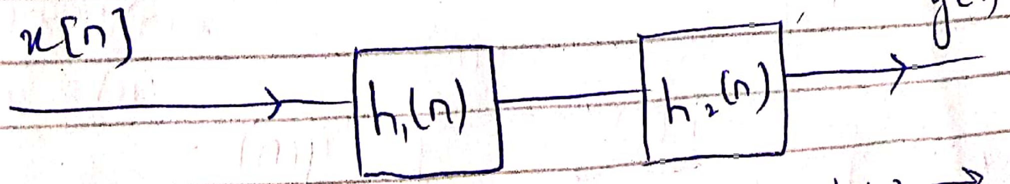
Associative Property:

$$\left[x_1[n] * x_2[n]\right] * x_3[n]$$

$$= x_1[n] * \left[x_2[n] * x_3[n]\right]$$

⇒ Convolution obeys Associative Law.

if two systems are connected in series, so, we take its overall convolution.



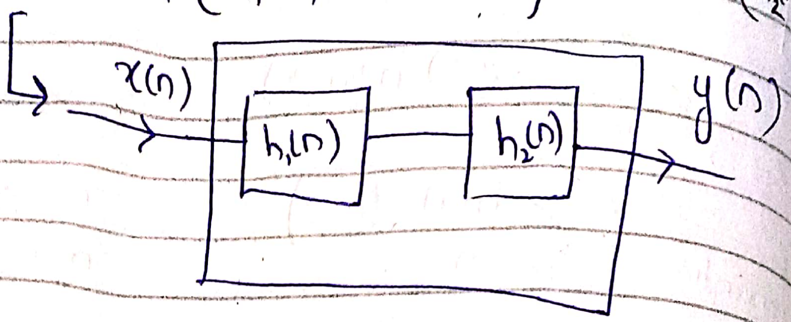
output $\rightarrow y_1(n) * h_1(n) \rightarrow$

$$y_1(n) = x(n) * h_1(n)$$

$$y(n) = y_1(n) * h_2(n)$$

$$= (x(n) * h_1(n)) * h_2(n)$$

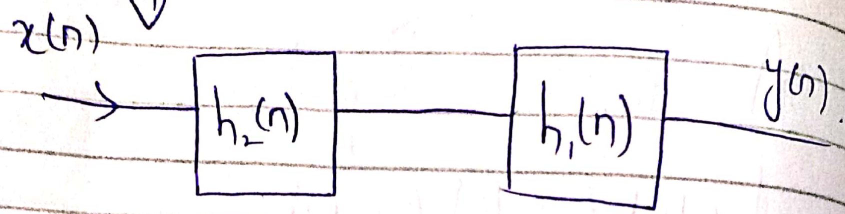
$$= x(n) * (h_1(n) * h_2(n)) \rightarrow x(n) * (h_2(n) * h_1(n))$$



$$h(n) = h_1(n) * h_2(n).$$

$$(x(n) * h_2(n)) * h_1(n).$$

$$\Downarrow$$



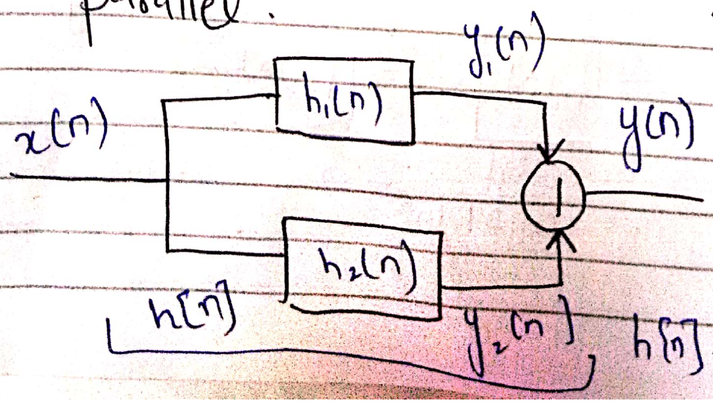
interconnections of LTI systems → Properties tell us

Distributive Law : $(x_1(n) * (x_2(n) + x_3(n))$

$$\Downarrow$$

$$x_1(n) * x_2(n) + x_1(n) * x_3(n).$$

- If two systems are connected in parallel.



$h[n] = h_1[n] + h_2[n]$

64

$$y(n) = y_1(n) + y_2(n)$$

$$y(n) = x(n) * h_1(n) + x(n) * h_2(n)$$

$$y(n) = x(n) * (h_1(n) + h_2(n))$$

↳ tells us equivalent system in parallel

Convolution ⇓ Series

Summation ⇓ parallel.

Analysis of LTI sys 2.3.3

1st November 2019

$$y[n] = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

$$y[n] = \dots + x(0)h[n] + x(1)h(n-1) + \dots$$

Convolution 6

$x[n]$, $h[n]$

1) Change index of $x[n]$ & $h[n]$ to get $x(K)$ & $h(K)$.

2) Reverse $h(k)$ to get $h(-k)$

3) Shift $h(-k)$ to get $h(n-k)$

4) Multiply $x(k)$ & $h(n-k)$:)

5) Sum the product signal from $k = -\infty$ + ∞

6) Repeat steps 3-5 for all values of $n(-\infty, \infty)$

$y(n) = 0$

Is convolution commutative?

Commulative Property:

Properties:

$$x[n] * h[n] = h[n] * x[n]$$

if

then

$y[n] =$

6

convert

other

is oka

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if Commutative law is true

then

$$h[n] * x[n] = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k).$$

$$y[n] = x[n] * h[n]$$

$$= \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

$$= \sum_{r=-\infty}^{\infty} h(r)\, x(n-r)$$

convention
$- \to +$
other
$+ \to -$
is okay.

$n-k=r \therefore k=-\infty$, $r=\infty$

$n-r=k$

$\therefore k=+\infty$, $r=-\infty$

$$= h(n) * x(n)$$

So, it is commutative.

Associative Property:

$$[x_1[n] * x_2[n]] * x_3[n]$$

$$= x_1[n] * [x_2[n] * x_3[n]]$$

$\Rightarrow$ Convolution obeys Associative law.

if two systems are connected in series. so, we take its overall convolution

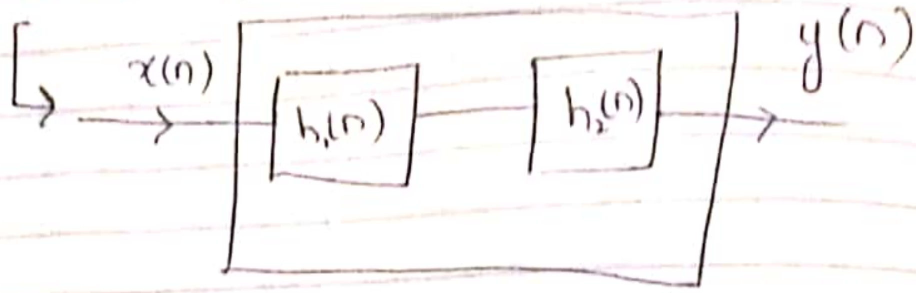
output $\longrightarrow y_1(n) * h_2(n) \rightarrow$

$y_1(n) = x(n) * h_1(n)$

$* \; y(n) = y_1(n) * h_2(n)$

$= (x(n) * h_1(n)) * h_2(n)$

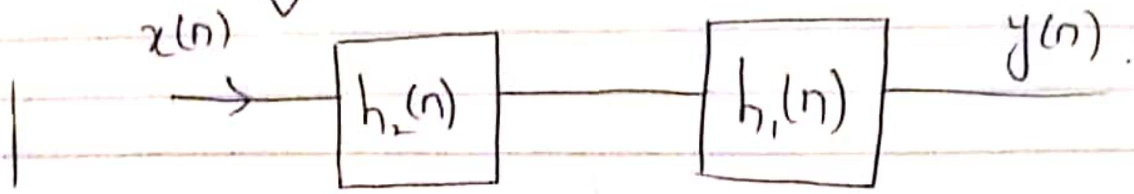
$= x(n) * (h_1(n) * h_2(n)) \rightarrow x(n) * (h_2(n) * h_1(n))$



$h(n) = h_1(n) * h_2(n)$.

$\rightarrow (x(n) * h_2(n)) * h_1(n)$.

$\Downarrow$



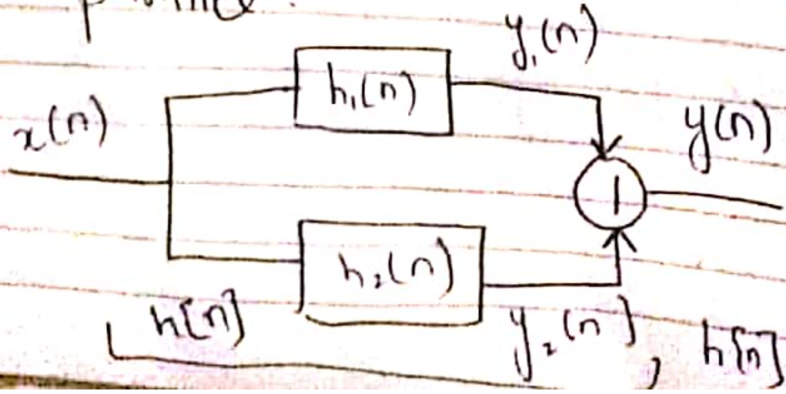
interconnections of LTI systems $\rightarrow$ Properties tell us

Distributive Law: $(x_1(n) * (x_2(n) + x_3(n))$

$\Downarrow$

$x_1(n) * x_2(n) + x_1(n) * x_3(n)$.

- If two systems are connected in parallel.



, $h[n] = h_1[n] + h_2[n]$

$y(n) = y_1(n) + y_2(n)$

$y(n) = x_0(n) * h_1(n) + x_0(n) * h_2(n)$

$y(n) = x(n) * (h_1(n) + h_2(n))$

↳ tells us equivalent system in parallel

Convolution ⇓ Series

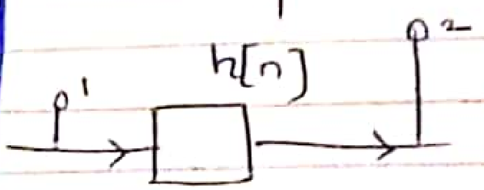
Summation ⇓ parallel.

2.3.5 Causal LTI System

➔ h[n] tells us about the system.
what the system will do.

exact outputs depends on input.

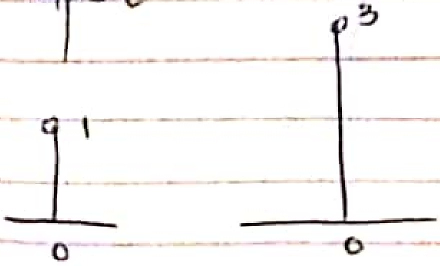
Impulse response



$y[n] = x[n] * h[n]$

$= \sum_{k=-\infty}^{\infty} x(k)\,h(n-k)$

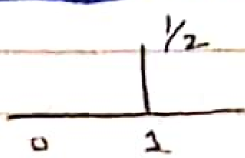
(Amplification)



➔ output at 0 time is dependent on output at 0 time.

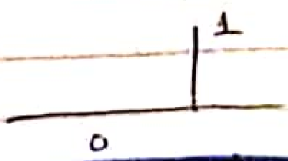
No need of Memory

it just doubles the

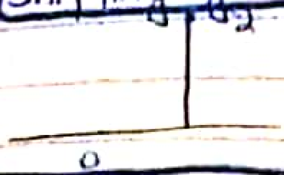


Reverbration.

Reflection of echo system.



Shifting system



Shifting + Amplication.

$\Rightarrow$ 1-time output is dependant on 0-time input.

This system need a memory.

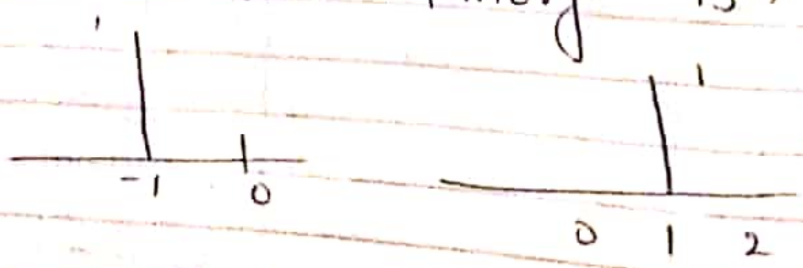
$h[n] = k\,\delta[n] \Rightarrow$

$h[n] = 0 \quad , n \neq 0.$

value on 0

(Memory Less)

When Output is coming later on input that is memory is needed.



For Causal

$$h[n] = 0 \quad , n < 0.$$

For Non-causal

1st imd nahi → read data par hosakta.

Filters used in musical instrument
Recorder inf ke andar we can use.

b) Averaging:

Too short & too long sentences can be compressed in best possible way

Jo points sequence se bahir ho wo un use karta ha.

physical not suitable appearance fo

$$y[n] = h[n] * x[n]$$

$$= + h_0\, x(n-0) + h(1)\, x(n-1) + \dots$$

$h(k)\, x(n-k)$, $k > 0$, 0

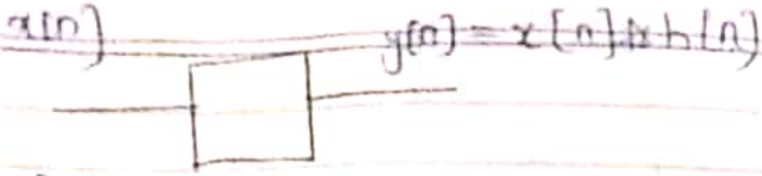
$$= \sum_{k=0}^{\infty} h(k)\, x(n-k).$$

Causal don't have negative value.

2.3 Stability of LTI Systems.

Those systems for which for a finite input is also finite (stable)

Those system for which a finite input gives infinite (Unstable).



$$y[n] = h[n] * x[n]$$

$$= \sum_{k=-\infty}^{\infty} h[k]\, x(n-k)$$

Condition

if input → finite

output → finite.

$$\left| \sum_{k=-\infty}^{\infty} h(k)\, x(n-k) \right| < \infty$$

(stable)

other wise

unstable.

x(0)	2	y(0)	5
x(1)		y(1)	3
x(2)	-3	y(2)	3
	4		

$$= \sum_{k=-\infty}^{\infty} |h(k)|\, |x(n-k)|$$

Absolute helps us in determing

positive & negative both.

Replace (xy) with Highest

value

$$= \sum_{k=-\infty}^{\infty} |h(k)|\, B$$

$$= B \sum_{-\infty}^{\infty} hk$$

if $B \times h(k)$ = finite

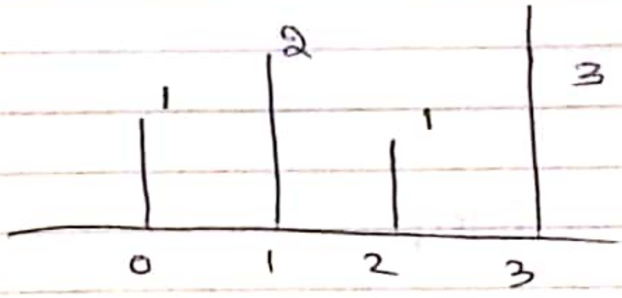
68 then

$B \sum_{k=-\infty}^{\infty} h(k) \rightarrow$ system finite

$$\sum_{n=-\infty}^{\infty} h[n] < \infty$$

then $B \sum_{k=-\infty} h(k) \Rightarrow$ finite system.

System's Impulse Response



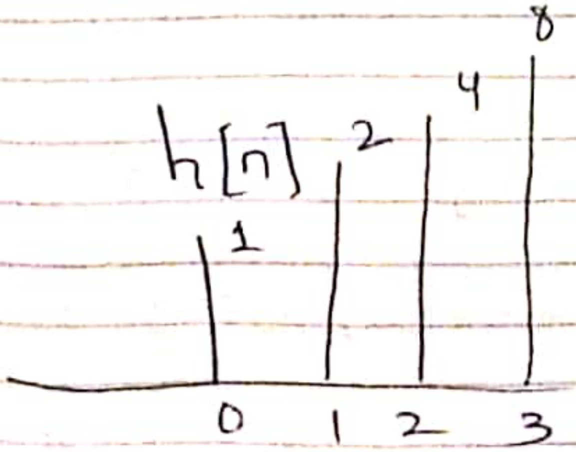
When finite no. of values the system is stable.

Addition of finite $\rightarrow$ finite system.

finite duration $\rightarrow$ finite sum.

$\Downarrow$

stable.



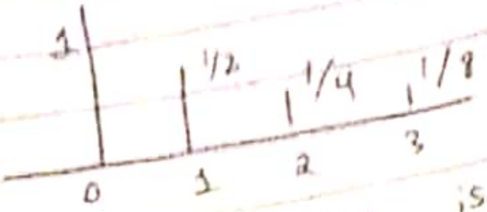
$1 + 2 + 4 + 8 \ldots$

divergent series

When infinite no of values the system is unstable

Scanned by CamScanner

h[n]



Impulse response is going on to infinity

$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} \cdots \rightarrow$ convergent series

$\downarrow$

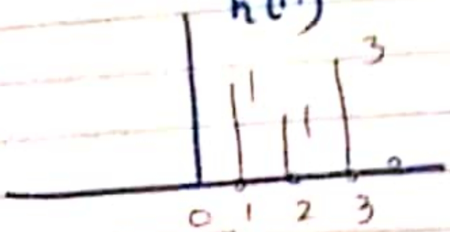
$\frac{1}{1-\frac{1}{2}}$

its sum is finite

so stable system

system $\rightarrow$ duration finite

h[n]



$\boxed{\text{FIR}}$

finite Impulse Response

$\downarrow$

Stable

IIR

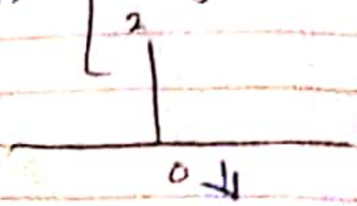
$\downarrow$

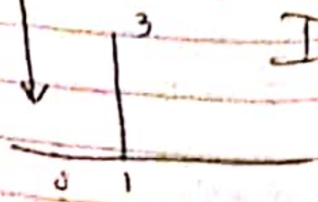
stable / unstable

1 form of representation of system $\leftarrow$

$y[n] = 2x[n] \rightarrow$ double

$y[n] = 3x[n-1] \rightarrow$ three times + Delay





Another form. (eqtn form)

Also known as
derivative system.

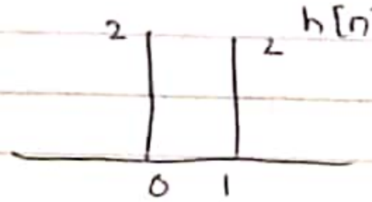
$y[n] = 2x[n] + 3x[n-1] + 5x(n-5) + 2y(n-2)$

$$\boxed{y[n] = F\big(x(n-k),\ y(n-k)\big)}$$

↓

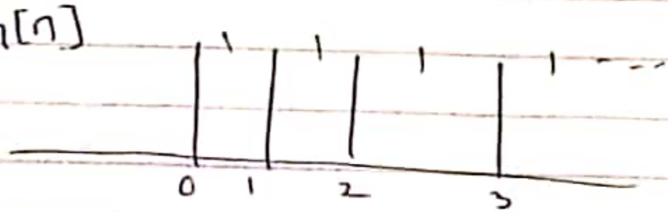
An eqtn which contain $x, y, n-k$ this is known as differential equation.

h[n]



$y[n] = 2x[n] + 2x[n-1]$

h[n]



$y[n] = x[n] + 1x[n-1] + 1x[n-2] + 1x(n-3) + \cdots$

$y[n-1] = x[n-1] + x[n-2] + x[n-3] + x(n-4) + \cdots$

→ $y[n] = x[n] + y[n-1]$

⇒ The system which contains $y[n-1]$ so this is know as recursive system.

↓ output used

FIR IIR

↓ ↓

Non Recur Recursi

8/Nov/19

$x(n-k) \to y(n-k)$

$y[n] = 2x[n] + 3x[n-1] + 5y[n-2]$

- co-efficient → constant
- Power → 1 (linear)
- Degree → highest derivative
- Order
- → Representing LTI system.

(1)

* → Impulse Response
→ Constant co-efficient linear LTI system

$$y[n] = y_n[n] + y_p[n] \quad \text{— (1)}$$

$$y(n) + 2y[n-1] = 3x(n)$$ Equation of syst.

↳ solution → to find $y[n]$.

$y[n]$ → It is the response of system when input is zero.

For $y_h[n] = x[n] - 0$. $x[n] = 2^n u[n]$ input syst

$$y[n] + 2y[n-1] = 0 \quad \text{— (n)}$$

2d Solution

$$y_n[n] = C\lambda^n$$

$$C\lambda^n + 2C\lambda^{n-1} = 0$$

$C \lambda^{n-1} [\lambda + 2] = 0$

$\lambda + 2 = 0$

$\boxed{\lambda = -2}$ $\quad C(-2)^n$

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$C(-2)^0 = 3$

$\boxed{C = 3}$

$\Rightarrow 3(-2)^n \rightarrow$ homogenous Solution.

$y_h(n) = 3(-2)^n$ —— (i)

Particular Solution:

(input)

$x[n] = 2^n u[n]$

$y_p[n] = K 2^n u[n] \rightarrow$ form of output.

input = constant $\quad$ output = K.

input = Am^n $\quad$ output $\rightarrow KM^n$

cos, sin $\quad$ sinusoidal

put in

$K 2^n u[n] + 2K 2^{n-1} u[n-1] = 3 \cdot 2^n u[n]$

$n = 0 \Rightarrow$ not right

$K 2^0 u[0] + 2K 2^{0-1} u(0-1) = 3 \cdot 2^0 u[0]$

$n = 1$

$K\, 2 u(1) + 2K\, 2^{1-1} u[1-1] = 3 \cdot 2 u(1)$

$2K + 2K = 6.$

$6 = 4K = 6 \quad K = 3/2.$

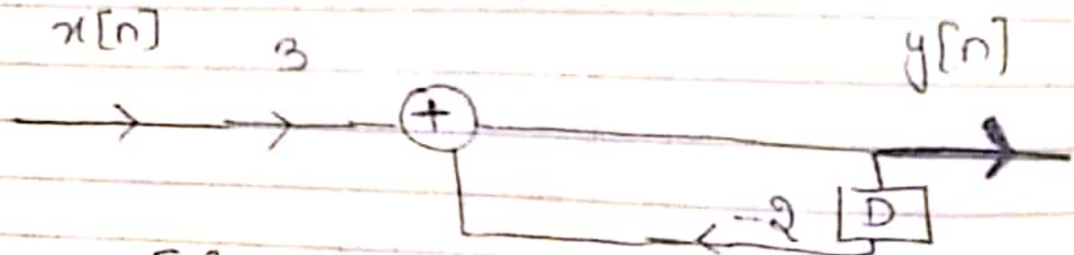
$$y_p[n] = \frac{3}{2} 2^n u[n].$$

$$y[n] = y_h[n] + y_p[n]$$

$$= \left(3(-2)^n + \frac{3}{2} \cdot 2^n\right) u[n]$$

Implementation of Discrete Time Systems.

$$y[n] + 2y[n-1] = 3x[n]$$



$$y[n] = 3x[n] - 2y[n-1]$$

<u>Generic form.</u>

$$a_0 y[n] + a_1 y[n-1] + a_2 y[n-2] + \cdots a_N y(n-N) = b_0 x[n] + b_1 x[n-1] + \cdots b_m x(n-m]$$

(N^{th} order).

$$a_0 y[n] = b_0 x[n] + b_1 x(n-1) + \dots b_m x(n-m) - a_1 y[n-1] - a_2 y[n-2] \dots a_N y(n-N)$$

$$y[n] = b_0 x(n) + b_1 x(n-1) + \dots b_m x(n-m) - a_1 y[n-1] - a_2 y[n-2] \dots a_N y(n-N)$$

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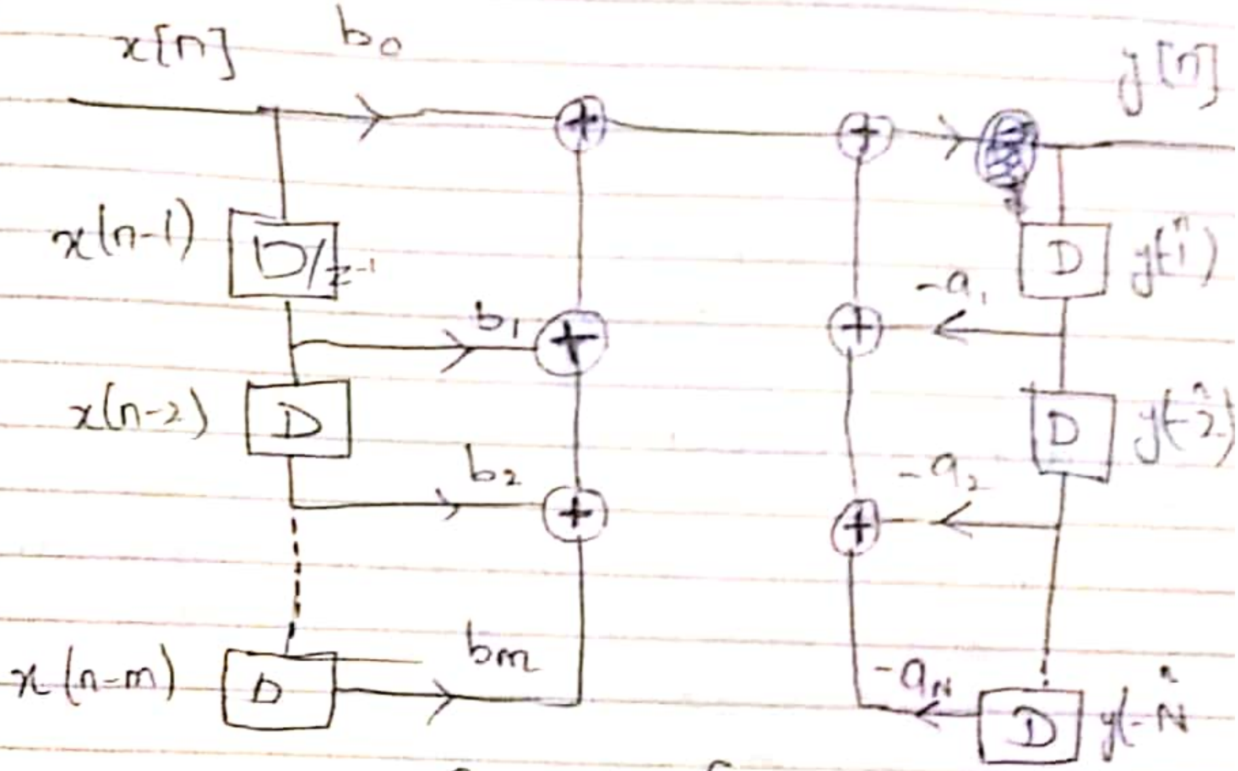
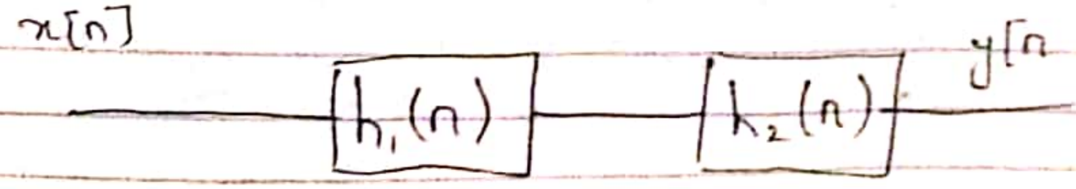


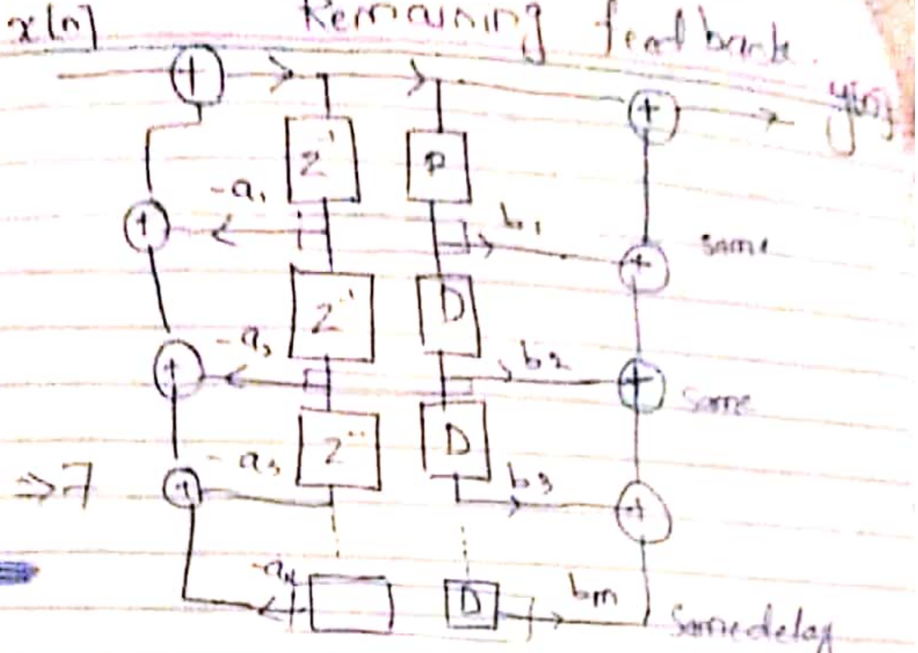
Direct form I structure

Signal flow for generic Equation.



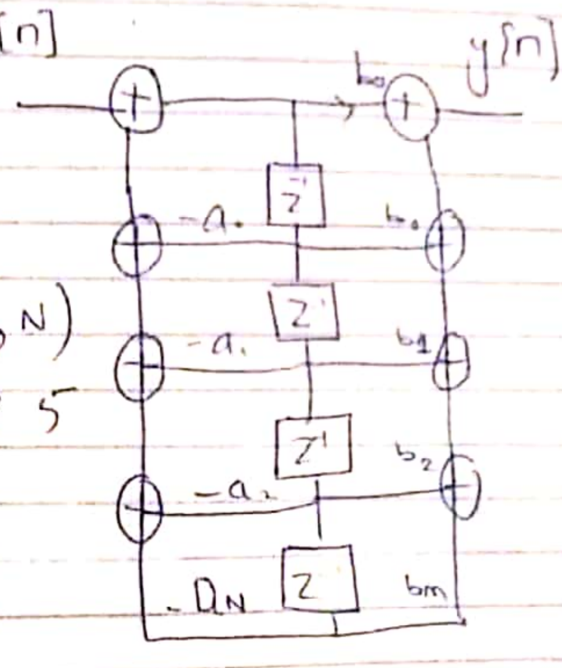
⇒ if the two system connected in series (LTI system) we can reverse them.

One feed forward
Remaining feed back.



M+N
2+5 => 7

After this end result will be equal



max(M+1, N)
2, 5 => 5

efficient

n = m
5 5

5 delay

n > m
7 5

13 delay
6 merge
1 as it is.