Control Systems - Week 5

Block Reduction of Complex Systems

Dr. Salman Ahmed

Contents that we have covered till now

We studied the following topics till now:

- Converting state-space to transfer function using formula
- Converting transfer functions to state-space models using canonical forms
- Analyzing step responses of first order systems (time constant and dc-gain)
- stability in TF state and space state model

We will study the following topics before mid term exam

- Block reduction of complex systems (today lecture)
- Analyzing step responses of second order systems (underdamped, undamped, over damped, critically damped)

Block reduction algebra

First we analyze a simple transfer function block. T.f block

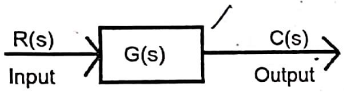


Figure: Transfer function block

The input signal is denoted by $R(s)$ and output signal by $C(s)$. We can write the following:

$$C(s) = G(s)R(s)$$

Sometimes, we skip the term (s) and write the following abusive notation:

$$C = GR$$

Block reduction algebra

There are 3 types of interconnections in control systems:

- Series Interconnection
- Parallel Interconnection
- Feedback Interconnection

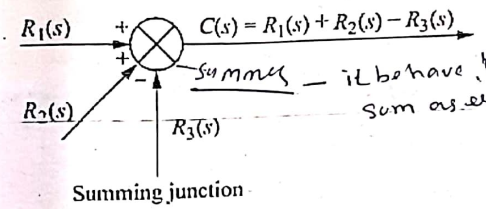
Besides, there are 4 operations which are as follows:

- Moving summing junction after transfer function
- Moving summing junction before transfer function
- Moving before pickoff point
- Moving after pickoff point

Let us introduce a summing junction or summer first, and then pick-off point

Block reduction algebra - Summer or Summing Junction

A summer or summing junction adds (or subtracts) two or more signals. The default sign is + in a summer or summing junction.

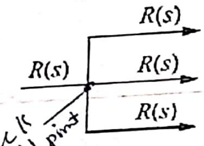


Summer — it behave since sum as well substraction

Figure: Summing Junction Symbol

Block reduction algebra - Pick off point

Pick off point: A point where the same signal has to propagate through more than one paths



pick off point

Figure: Pick Off point

↳ signal passes through multiple part.

First interconnection: Series Interconnection

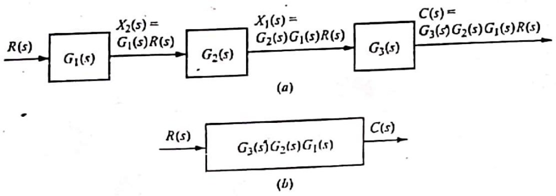


Figure: Series Interconnection of transfer functions

We can write $G_e = G_3 G_2 G_1$ ✓

Second interconnection: Parallel Interconnection

all input have common point & common summer

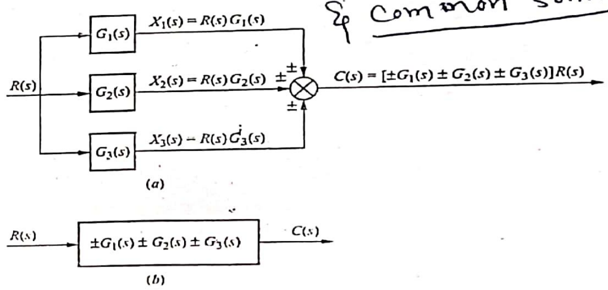


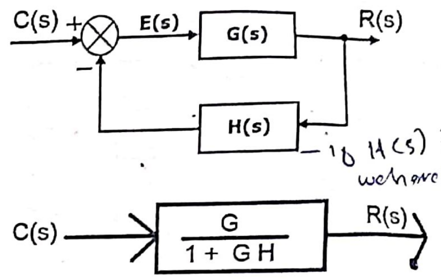
Figure: Parallel Interconnection of transfer functions

We can write $G_e = \pm G_3 \pm G_2 \pm G_1$

...w Important Points

- Series interconnection involves product of transfer functions.
- In parallel interconnection, be careful to identify the transfer functions correctly.
- Two blocks are in parallel if they have same input signal and the output goes towards same summing junction.
- Parallel interconnection involves sum or different of transfer functions.

Third interconnection: Feedback Interconnection

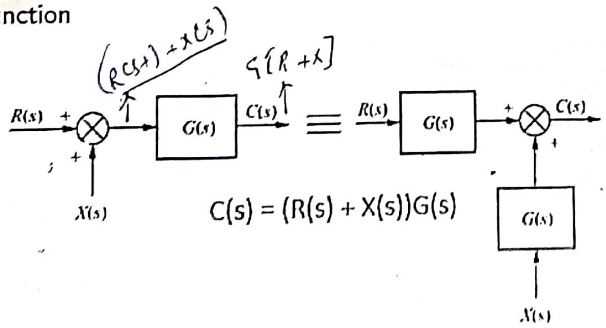


– if H(s) is missing we have to put T.f of unity $\boxed{1}$

Figure: Feedback Interconnection of transfer functions

We can write $G_e = \frac{G}{1 \pm GH}$

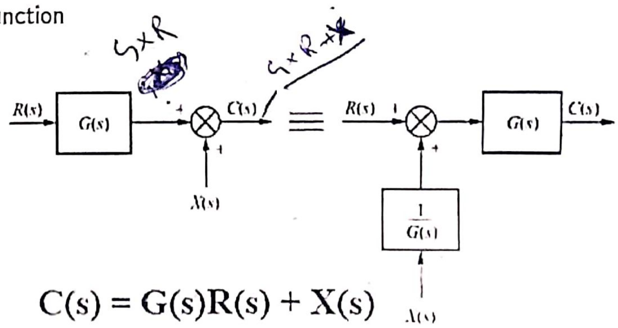
Operation 1: Moving summing junction after transfer function



$C(s) = (R(s) + X(s))G(s)$

Figure: Moving a summing junction after transfer function

Operation 2: Moving summing junction before transfer function



$C(s) = G(s)R(s) + X(s)$

Figure: Moving a summing junction before transfer function

Operation 3: Moving before pickoff point

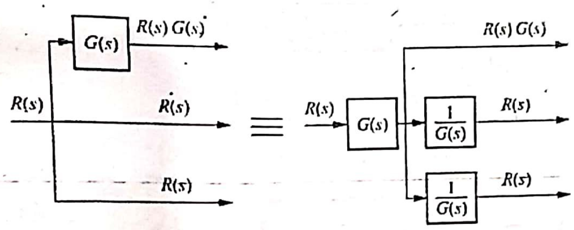


Figure: Moving before a pick-off point

Operation 4: Moving after pickoff point

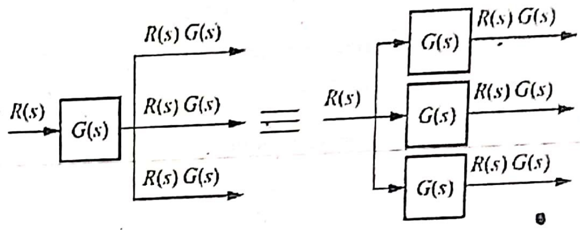


Figure: Moving after a pick-off point

Summary of block reduction rules

We will use the knowledge about these 3 interconnections, and 4 operations to reduce complex systems.

You will be given a complex interconnection schematic, plus input and output, and will be asked to apply this knowledge to reduce or simplify complex systems.

Example 1 - Problem to solve

Can you obtain the transfer function, $\frac{C(s)}{R(s)}$?

simply

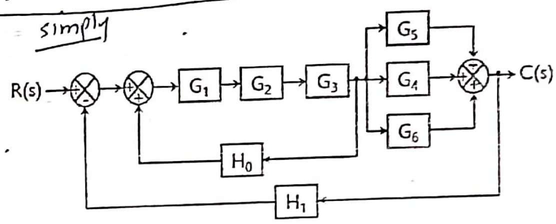
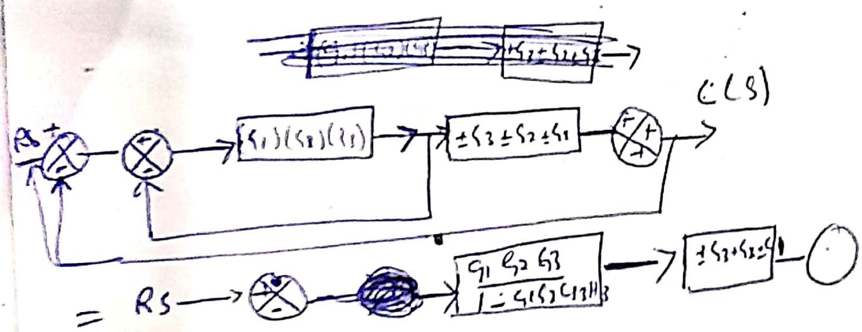


Figure: Example 1

xample 1 - Solution part a - Simplify series interconnection



Example 1 - Solution part b - Simplify parallel interconnection

Example 1 - Solution part c - Simplify inner feedback interconnection

Example 1 - Solution part d

Example 1 - Final Solution

Example 2 - Problem to solve

Can you obtain the transfer function, $\frac{C(s)}{R(s)}$?

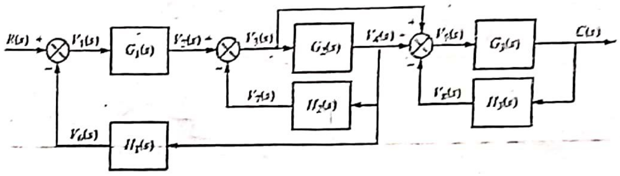
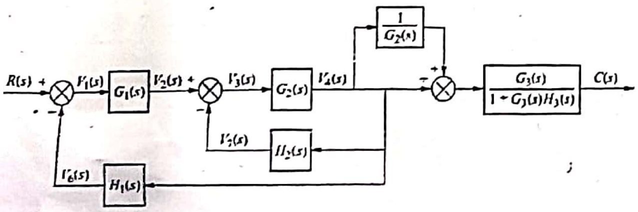
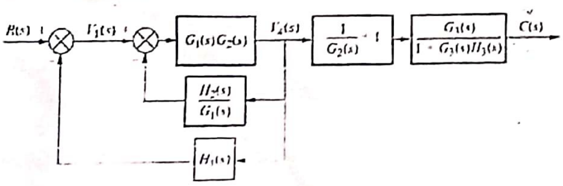


Figure: Example 2

Example 2 - Condensed Solution Part 1



(a)



(b)

Example 2 - Condensed Solution Part 2

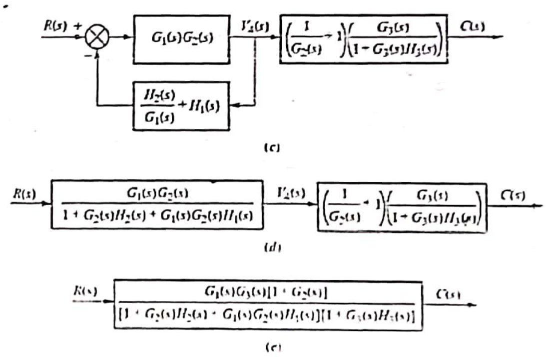


Figure: Example 2 - Final Solution

eal life example of feedback interconnection

Let us consider a water-tank level control systems. The objective is to ensure that the water level remains the same. Can you draw a block diagram of this system?

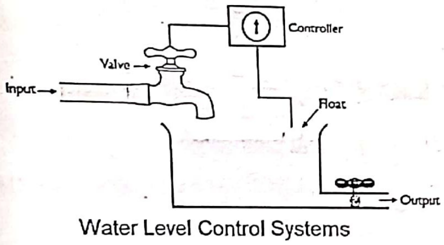


Figure: Example of Water-Level Control Systems

Let us first differentiate between real-world input and output AND control-systems input and output

Real life example of feedback interconnection