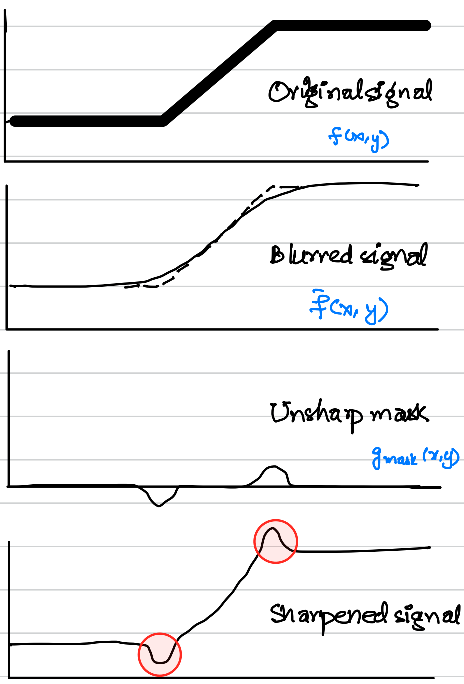
Sharpening Spatial Filters:

- Unsharp, masking and highboost Filtering:



Unsharp mask → get after subtracting blurred image from original image

Sharpened signal → get by adding unsharp image to the original image.

Sharpen. The sharp transitions, at these particular points these are enhanced. by sharpening the image.

Let the original image be $f(x,y)$ and blurred image be $\bar{f}(x,y)$.

$$g_{mask}(x,y) = f(x,y) - \bar{f}(x,y) \longrightarrow (1)$$

The sharpen image is represented as :

$$\boxed{g(x,y) = f(x,y) + k * g_{mask}(x,y)} \longrightarrow (2)$$

$\therefore$ k = weight $(k > 0)$

if $k = 1$ → unsharp mask

if $k > 1$ → highboost filtering

if $k < 1$ → Emphasizes the construction of unsharp masking.

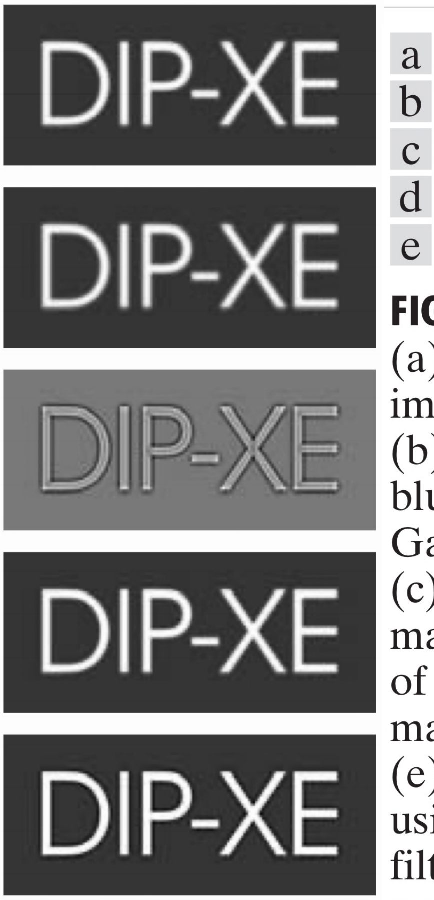


FIGURE 3.40 (a) Original image. (b) Result of blurring with a Gaussian filter. (c) Unsharp mask. (d) Result of using unsharp masking. (e) Result of using highboost filtering.

Using First-Order Derivatives for (Nonlinear) Image Sharpening—The Gradient:

For the function $f(x,y)$, the gradient of "f" at the coordinates (x,y) can be represented as the 2D column vectors:

$$\nabla f = \text{grad}[f(x,y)] = \begin{bmatrix} g_x \\ g_y \end{bmatrix} = \begin{bmatrix} \partial f/\partial x \\ \partial f/\partial y \end{bmatrix} \rightarrow ①$$

The magnitude of the vector ∇f is defined as:
$M(x,y)$ & is given by:

$$M(x,y) = \text{Magnitude}[\nabla f] = \sqrt{g_x^2 + g_y^2}$$

$$\boxed{M(x,y) \approx |g_x| + |g_y|} \rightarrow ②$$

$$\begin{bmatrix} f(x-1,y-1) & f(x-1,y) & f(x-1,y+1) \\ f(x,y-1) & f(x,y) & f(x,y+1) \\ f(x+1,y-1) & f(x+1,y) & f(x+1,y+1) \end{bmatrix} \quad \begin{bmatrix} z_1 & z_2 & z_3 \\ z_4 & z_5 & z_6 \\ z_7 & z_8 & z_9 \end{bmatrix}$$

$\therefore \frac{\partial f}{\partial x} = f(x+1) - f(x)$

As we know that

$$z_5 = f(x,y)$$

$$g_x = \frac{\partial f}{\partial x} = f(x+1,y) - f(x,y)$$

$$\boxed{g_x = z_8 - z_5}$$

$$g_y = \frac{\partial f}{\partial y} = f(x,y+1) - f(x,y)$$

$$\boxed{g_y = z_6 - z_5}$$

$$M(x,y) = |g_x| + |g_y|$$

$$\boxed{M(x,y) = |z_8 - z_5| + |z_6 - z_5|}$$

Masks →

-1	0
1	0

g_x (vertical)

-1	1
0	0

g_y (Horizontal)

Robert proposes cross differences:

$$g_x = z_9 - z_5, \quad g_y = z_8 - z_6$$

$$M(x,y) = |z_9 - z_5| + |z_8 - z_6|$$

-1	0
0	1

g_x

0	-1
1	0

g_y

2x2 cross difference mask

In these 2x2 masks, there is no center symmetry. To achieve center symmetry, we have to consider the odd mask and the smallest odd mask we can have 3x3. Sobel proposes 3x3 mask

$$\left.\begin{aligned} g_x &= (z_7 + 2z_8 + z_9) - (z_1 + 2z_2 + z_3) \\ g_y &= (z_3 + 2z_6 + z_9) - (z_1 + 2z_4 + z_7) \end{aligned}\right\} \text{Sobel's operator}$$

-1	-2	-1
0	0	0
1	2	1

g_x

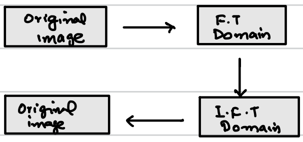
-1	0	1
-2	0	2
-1	0	1

g_y

Sobel masks

$$\boxed{M(x,y) = |(z_7 + 2z_8 + z_9) - (z_1 + 2z_2 + z_3)| - |(z_3 + 2z_6 + z_9) - (z_1 + 2z_4 + z_7)|.}$$

FREQUENCY DOMAIN :



Let a digital image be:

$f(x,y) \xleftrightarrow{D.F.T} F(u,v)$

$F(u,v) \xleftrightarrow{I.D.F.T} f(x,y)$

The 2-D discrete fourier transform (DFT) is given by:

$$\boxed{F(u,v) = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)e^{-j\frac{2\pi}{M}ux} \cdot e^{-j\frac{2\pi}{N}vy}} \rightarrow (1)$$

where $f(x,y)$ is a digital image of $M \times N$ sample, u and v are the frequency domain variables.

$\therefore u = 0, 1, 2 \ldots M-1$ & $v = 0, 1, 2 \ldots N-1$.

To obtain $f(x,y)$ from $F(u,v)$, we need to perform Inverse discrete fourier Transform (IDFT)

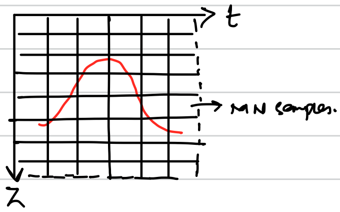
$$\boxed{f(x,y) = \frac{1}{MN}\sum_{u=0}^{M-1}\sum_{v=0}^{N-1} F(u,v)e^{j\frac{2\pi}{M}ux} \cdot e^{j\frac{2\pi}{M}vx}} \rightarrow (2)$$

Properties of 2-D discrete Fourier Transform:

1) Relationship between spatial and frequency intervals:

Let $f(t,z) \rightarrow$ continuous image in spatial domain.

$f(x,y) \rightarrow$ Digital image having $M \times N$ Samples.



The frequency domain variables

$$\boxed{\Delta u = \frac{1}{M \cdot \Delta T}} \rightarrow (1)$$

seperation b/w the samples

$$\boxed{\Delta v = \frac{1}{N \cdot \Delta z}} \rightarrow (2)$$

Where ΔT and Δz represents the seperation between the samples.

(2) Translation & Rotation :

The F. Transform pair satisfies the translation & rotation property & and can be represented as:

$$f(x,y)e^{j2\pi\left(\frac{u_0 x}{M} + \frac{v_0 y}{N}\right)} \Longleftrightarrow f(u-u_0, v-v_0) \rightarrow (5)$$

$$f(x-x_0, y-y_0) \Longleftrightarrow F(u,v)\, e^{-j2\pi\left(\frac{x_0 u}{M} + \frac{y_0 v}{N}\right)} \rightarrow (6)$$

In polar coordinates, this can be represented as.

$$x = r\cos\theta,\ y = r\sin\theta,\ u = w\cos\phi,\ v = w\sin\phi.$$

The result obtained is

$$\boxed{f(r, \theta+\theta_0) \Leftrightarrow F(w, \phi+\theta_0)} \rightarrow (7)$$

If $f(x,y)$ is rotated by θ_0, Then, $F(u,v)$ is also rotated by θ_0.

3) **Periodicity :**

The Fourier transform & its inverse are infinitly periodic in u & v direction.

$$F(u,v) = F(u+k_1M, v) = F(u, v+k_2N)$$

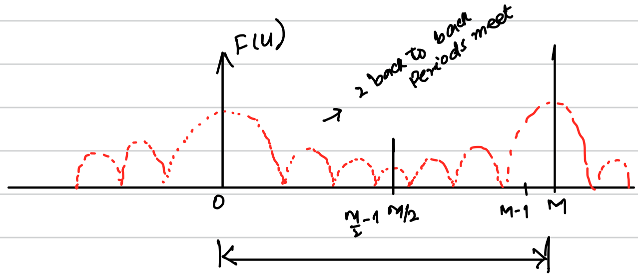
$$\boxed{F(u,v) = F(u+k_1M, v+k_2N)} — (8)$$

$$f(x,y) = f(x+k_1M, y) = f(x, y+k_2N)$$

$$\boxed{f(x,y) = f(x+k_1M, y+k_2N)} \rightarrow 9$$

where k_1 and k_2 are integers.

Lets assume a 1-D signal

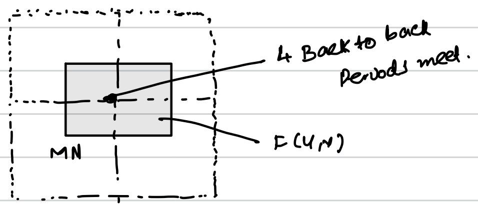


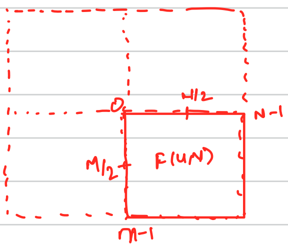
$$f(x)\, e^{j2\pi\left(\frac{u_0 x}{M}\right)} = F(u-u_0) \rightarrow 9$$

$\times$ by $(-1)^x$

$$\boxed{f(x)(-1)^x = F(u - M/2)} \rightarrow 10$$

For 2-D DFT.





$$\boxed{f(x,y)\cdot(-1)^{x+y} = F(u-M/2,\ v-N/2)} \rightarrow (11)$$

(4) Symmetry : Any real and complex functions are the sum of even & odd parts.

$$\boxed{w(x,y) = W_e(x,y) + w_o(x,y)} \rightarrow (1)$$

we know that

$$W_e(x,y) = \frac{w(x,y) + w(-x,-y)}{2} \rightarrow (2a)$$

$$W_o(x,y) = \frac{w(x,y) - w(-x,-y)}{2} \rightarrow (2b)$$

Substitute (2a) & (2b) in 1, we get identity i.e.

$$w(x,y) \triangleq w(x,y) \rightarrow (3)$$

we know that the even components are symmetric & odd components are antisymmetric.

$$W_e(x,y) = W_e(-x,-y) \rightarrow (4a)$$
$$W_o(x,y) = -W_o(-x,-y) \rightarrow (4b)$$

we also know that the product of W_e & W_o gives zero

$$\boxed{\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} W_e(x,y)\, w_o(-x,-y) = 0} \rightarrow (5)$$

(5) **2D Convolutions** The circular convolution of 2D DFT is given as:

$$f(x,y) \cdot h(x,y) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} f(m,n) \cdot h(x-m, y-n) \rightarrow (1)$$

where $x = 0, 1, 2 \ldots M-1$, $y = 0, 1, 2 \ldots N-1$.

For 2D- convolution theorem is given by:

$$\boxed{f(x,y) * h(x,y) = F(u,v) \cdot H(u,v)} \longrightarrow (2)$$

Conversely,

$$\boxed{F(u,v) * H(u,v) = f(x,y)\, h(x,y)} \longrightarrow (3)$$