

F - TEST

Introduction & formula

Objective of F-Test:

- To find out whether the two independent estimates of population variance differ significantly OR
- To find out whether the two samples may be regarded as drawn from the normal populations having same variance.
- To perform the test of significance, we have to compute the ratio of F.

$$F = \frac{\sigma_1^2}{\sigma_2^2} \quad \text{OR} \quad F = \frac{S_1^2}{S_2^2} \quad (\text{in case of samples})$$

$$\therefore \sigma_1^2 > \sigma_2^2 \quad S_1^2 > S_2^2$$

$$\text{Variance} = \sigma^2 = \frac{\sum (x - \mu)^2}{n}, \quad S^2 = \frac{\sum (x - \bar{x})^2}{n-1}$$

- F can be written as:

$$F = \frac{\text{Larger estimate of variance}}{\text{Smaller estimate of variance}}$$

- Degree of freedom (ν)

$$\nu_1 (\text{Numerator}) = n_1 - 1 \quad \& \quad \nu_2 (\text{denominator}) = n_2 - 1$$

\hookrightarrow (Larger variance) \hookrightarrow (Smaller variance)

- Now the F value will be compared with tabulated F value for ν_1 & ν_2 at 1% or 5% level of significance.

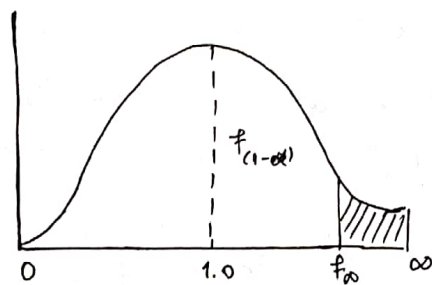
① Calc; F value < Tab; F value: $\Rightarrow H_0$ is accepted & No significant difference between the two variance (S^2)

if Calc F value $>$ Tab; F value then

H_0 is rejected & there will be significant difference between the two variance (S^2).

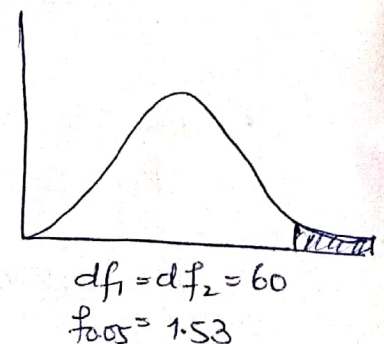
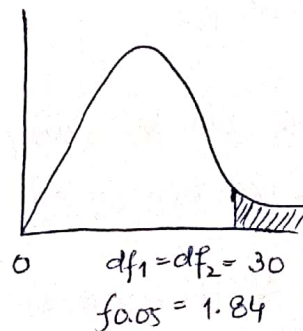
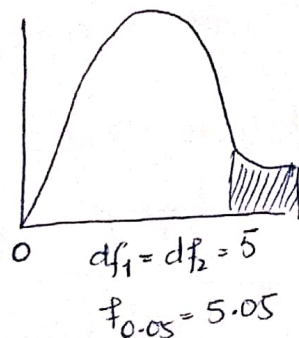
- Hence, F-Test is based on the ratio of the 2 variance, it is also known as the ("Variance Ratio test")

PROPERTIES OF F-DISTRIBUTION



F-distribution curve for n-degree of freedom.

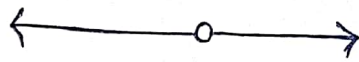
- 1:- F-distribution curve is skewed towards right with range 0 to ∞ and having a median value roughly = 1.0
- 2:- Value of F will always be more than 0.
- 3:- Shape of F-distribution curve is dependent
- dof of Numerator & dof of denominator
- 4:- F-distribution curve is never symmetrical but if dof increases then it will be more similar to the symmetrical shape.



✓ Degree of skewness decreases with increase in degree of freedom (ν) for numerator (ν_1) & denominator (ν_2).

6. Shape of the curve will be more symmetrical with the increase in degree of freedom.

• In F-test variance will be compared from randomly drawn samples and the observations are dependent.



QNO:01 Given data

A: 16, 17, 25, 26, 32, 34, 38, 40, 42 $n=9$

B: 14, 16, 24, 28, 32, 35, 37, 42, 43, 45, 47 $n=11$

$$H_0: \sigma_1^2 = \sigma_2^2 \quad ; \quad H_a: \sigma_1^2 > \sigma_2^2$$

$$F = \frac{S_1^2}{S_2^2}, \quad S^2 = \frac{\sum (x - \bar{x})^2}{n-1}$$

As we studied above that s_1^2 will be greater than s_2^2 for F value greater than 0. so alternative hypothesis $H_a = s_1^2 > s_2^2$

A	$(x - \bar{x}_A)$	$(x - \bar{x}_A)^2$	B	$(x - \bar{x}_B)$	$(x - \bar{x}_B)^2$
16	-14	196	14	-19	361
17	-13	169	16	-17	289
25	-5	25	24	-9	81
26	-4	16	28	-5	25
32	2	4	32	-1	01
34	4	16	35	2	4
38	8	64	37	4	16
40	10	100	42	9	81
42	12	144	43	10	100
			45	12	144
			47	14	196

$$\Sigma A = 270$$

$$\bar{x}_A = 270/9 = 30$$

$$\Sigma (x - \bar{x}_A)^2 = 734$$

$$\Sigma B = 363$$

$$\bar{x}_B = 363/11 = 33$$

$$\Sigma (x - \bar{x}_B)^2 = 1298$$

$$S_2^2 = 734/9-1 = 91.75$$

$$S_1^2 = \frac{1298}{10} = 129.8$$

$$F = \frac{129.8}{91.75} = 1.4147 \text{ (calculated F value)}$$

Degree of Freedom (ν) = ?

$$\nu_1 = 11-1 = 10$$

$$\nu_2 = 9-1 = 8$$

$$F_{0.05} = 3.35 \text{ (Tabulated Value)}$$

Calculated F value < Tabulated F value.

H_0 is correct & accepted. We can say that the two populations have same variance.

Q No. 02

Given data.

$$n=9 ; \sum (x-\bar{x})^2 = 64 \text{ (sum of The squared deviation)}$$

$$n=11 ; \sum (x-\bar{x})^2 = 88$$

Hypothesis

$$H_0 : \sigma_1^2 = \sigma_2^2 ; H_a : \sigma_1^2 > \sigma_2^2$$

$$F = \frac{S_1^2}{S_2^2} , S_1^2 > S_2^2$$

$$S^2 = \frac{\sum (x-\bar{x})^2}{n-1} \Rightarrow S_2^2 = \frac{64}{9-1} = \frac{64}{8} = 8$$

$$S_1^2 = \frac{88}{11-1} = \frac{88}{10} = 8.8$$

$$F = \frac{8.8}{8} = 1.1 \text{ (calculated F value)}$$

Degree of freedom (ν):?

$$\nu_1 = 10 \quad \nu_2 = 8$$

$$F_{0.05} = 3.35 \text{ (Tabulated F value)}$$

* H_0 is accepted. No significance in the variance.



QUESTION

Given data:

$$S_1^2 = 130$$

$$n_1 = 61 \text{ (no. of women)}$$

$$S_2^2 = 70$$

$$n_2 = 31 \text{ (no. of men)}$$

Hypothesis:

$$H_0 : \sigma^2_{(\text{women})} = \sigma^2_{(\text{men})}$$

$$H_a : \sigma^2_{(\text{women})} > \sigma^2_{(\text{men})}$$

$$F = S_1^2 / S_2^2 \Rightarrow F = 130 / 70 = 1.857 \text{ (calculated F-value)}$$

Degree of freedom (ν) = ?

$$\nu_1 = n_1 - 1 = 60$$

$$\nu_2 = n_2 - 1 = 30$$

$$F_{(0.05)} = 1.74 \text{ (Tabulated value)}$$

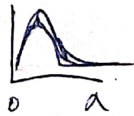
H_a is accepted & H_0 is rejected;

Hence, the result of the research support the

belief that women have a greater variation in attitude towards political issues than men.

Properties :

1) Range: $0 \rightarrow \infty$



2) $\mu = \frac{V_1}{V_2 - 2} \quad \therefore V_2 > 2$

3) $\sigma^2 = \frac{2V_2^2(V_1 + V_2 - 2)}{V_1(V_2 - 2)^2(V_2 - 4)}$ for $V_2 > 4$

4) Homogeneity of several means.

5) Unimodal

6) Skewed to the right

CI

