The Laplacian in the frequency domain:

Just like the laplacian in spatial domain, we can use it in the frequency domain to improve the quality of the image. we already know that the laplacian is a second-order derivative and is used to enhance the image. The laplacian is implemented using the filter:

$$\boxed{H(u,v) = -4\pi^2(u^2+v^2)} \rightarrow (1)$$

with respect to the center of the frequency rectangle, this can be represented as:

$$\boxed{H(u,v) = -4\pi^2[(u-P/2)^2 - (v-Q/2)^2]} \rightarrow (2)$$

since,

$$D(u,v) = [(u-P/2)^2 - (v-Q/2)^2]^{1/2}$$

The eq (2) can be represented as:

$$\boxed{H(u,v) = -4\pi^2 . D^2(u,v)} \rightarrow (3)$$

The laplacian for an image can be represented as:

$$\nabla^2 f(x,y) = \mathfrak{I}^{-1}[H(u,v).F(u,v)] \rightarrow (4)$$

where: $F(u,v)$ is the fourier transform of $f(x,y)$.

The enhancement in the image in the can be obtained as:

$$\boxed{g(x,y) = f(x,y) + c\nabla^2 f(x,y)} \rightarrow (5)$$

$$g(x,y) = \mathfrak{I}^{-1}[F(u,v) - H(u,v)F(u,v)]$$

$$g(x,y) = \mathfrak{I}^{-1}[(1 - H(u,v))F(u,v)] \rightarrow (6)$$

Substitute eq (3) in eq (6), we get.

$$\boxed{g(x,y) = \mathfrak{I}^{-1}[(1+4\pi^2 D^2(u,v))F(u,v)]} \rightarrow (7)$$

Processed image.

Here $g(x,y)$ is the processed image using Laplacian filter.

a b

FIGURE 4.58 (a) Original, blurry image. (b) Image enhanced using the Laplacian in the frequency domain. Compare with Fig. 3.38(e).

Homomorphic Filtering :-

In frequency domain, we can ehance the image by using the illumination and reflectance model. We can simultaneously perform intensity range compression as well as the contrast enhancement.

Let an image be:

$$\boxed{f(x,y) = i(x,y)\,r(x,y)} \rightarrow (1)$$

$i(x,y) \rightarrow$ illumination component.

$r(x,y) \rightarrow$ reflectance " .

We need to perform the filtering operation in frequency domain for which we have to compute the fourier transform of $f(x,y)$ but we can't apply the fourier on eq (1) as :

$$\Im[f(x,y)] \neq \Im[i(x,y)] \cdot \Im[r(x,y)].$$

So, we will apply the logrithmic function first before computing the fourier transform.

$$\begin{aligned} z(x,y) &= \ln f(x,y) \\ &= \ln[i(x,y).r(x,y)] \\ &= \ln i(x,y) + \ln r(x,y) \end{aligned}$$

$$\Im[z(x,y)] = \Im[\ln i(x,y)] + \Im[\ln r(x,y)]$$

$$\boxed{Z(u,v) = F_i(u,v) + F_r(u,v)} \longrightarrow (2)$$

where $F_i(u,v) = \Im[\ln i(x,y)]$
$F_r(u,v) = \Im[\ln r(x,y)]$.

using the filter $H(u,v)$,

$$\boxed{S(u,v) = Z(u,v).H(u,v)} \rightarrow (3)$$

From eq (2),

$$\boxed{S(u,v) = F_i(u,v)H(u,v) + F_r(u,v)H(u,v)} \rightarrow (4)$$

This is in frequency domain, we should bring back the result in spatial domain by applying inverse fourier transform.

To represent in spatial domain:

$$g(x,y) = \Im^{-1}[S(u,v)] \rightarrow (5)$$

Now, substitute $S(u,v)$ from eq (4) in eq (5), we get.

$$g(x,y) = \underbrace{\Im^{-1}[F_i(u,v)H(u,v)]}_{i'(x,y)} + \underbrace{\Im^{-1}[F_r(u,v)H(u,v)]}_{r'(x,y)}$$

in spatial domain

$$\boxed{\therefore g(x,y) = i'(x,y) + r'(x,y)} \rightarrow (6)$$

Note that the image we obtained in eq (6) is not in the original form because we applied ln to the original image before applying the Fourier transform. We need to perform the reverse operations on the obtained image which is done by taking exponential.
Such as

$$g(x,y) = e^{i'(x,y)+r'(x,y)}$$
$$= e^{i'(x,y)} + e^{r'(x,y)}.$$

$$\boxed{g(x,y) = i_0(x,y) + r_0(x,y)}$$

This is processed image in spatial domain

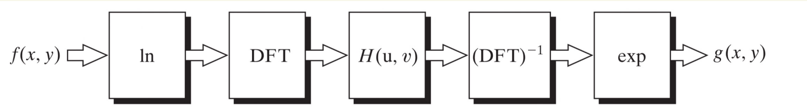
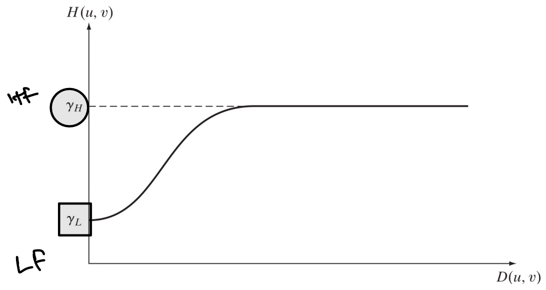


FIGURE 4.60
Summary of steps in homomorphic filtering.

The selection of $H(u,v)$ in homomorphic filter is a challenging task, we need to choose such $H(u,v)$ that performs both dynamic range compression & contrast enhancement simultaneously.

FIGURE 4.61 Radial cross section of a circularly symmetric homomorphic filter function. The vertical axis is at the center of the frequency rectangle and $D(u, v)$ is the distance from the center.



$\gamma_L < 1$ and $\gamma_H > 1$

$$H(u, v) = (\gamma_H - \gamma_L)\left[1 - e^{-c[D^2(u,v)/D_0^2]}\right] + \gamma_L$$

- **Unsharp Masking, Highboost Filtering, and High-Frequency-Emphasis Filtering**

Just like spatial domain filtering, the unsharp mask is generated by subtracting the smoothing image from the original image.

$$\boxed{g_{mask}(x,y) = f(x,y) - f_{LP}(x,y)} \longrightarrow (1)$$

$$f_{LP}(x,y) = \mathcal{F}^{-1}[H_{LP}(u,v)F(u,v)]$$

$$\boxed{g(x,y) = f(x,y) + k * g_{mask}(x,y)} \longrightarrow (2)$$

where $k=1 \longrightarrow$ unsharp masking
$k>1 \rightarrow$ highboost filtering.

To obtain high freq; emphasis filtering.

$$g(x,y) = \mathcal{F}^{-1}\{[1 + k * [1 - H_{LP}(u,v)]]F(u,v)\}$$

$$g(x,y) = \mathcal{F}^{-1}\{[1 + k * H_{HP}(u,v)]F(u,v)\}$$

$$g(x,y) = \mathcal{F}^{-1}\{[k_1 + k_2 * H_{HP}(u,v)]F(u,v)\}$$

where $k_1 \geq 0$ offset from origin
where $k_2 \geq 0$ contributes to HF filtering.