Computer Security

Lecture 10: Diffie-Hellman Key Exchange

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Diffie-Hellman Key Exchange

Lecture Outline

- Diffie-Hellman Key Exchange

Diffie-Hellman Key Exchange

- First PKC offered by Diffie and Hellman in 1976
- still in commercial use
- purpose is secure key-exchange
  - actually key “agreement”
  - both parties agree on a session key without releasing this key to a third party
    - to be used for further communication using symmetric crypto
- Security is in the hardness of the discrete logarithm problem
  - given g^x mod p, g and p, it is computationally infeasible to find out x if p is large enough prime number

Cappuccino Recipe

Easy

+

=

Hard

Diffie-Hellman Key exchange

- Requires two large numbers, one prime (P), and (G), a primitive root of P

Primitive root

3 is a primitive root of 5:

If the set of remainders in the third column reproduces the set of integers in the first (the order need not be identical), then 3 is a primitive root of 5. It looks like 3 is indeed a primitive root of 5.

n	3^n	$3^n \bmod 5$
1	3	3
2	9	4
3	27	2
4	81	1

4 on the other hand is not,

because we won't get the values 1 through 4 when we repeat the above process.

x	4^x	$4^x \bmod 5$
1	4	4
2	16	1
3	64	4
4	256	1

Implementation

- P and G are both publicly available numbers
  - P is at least 512 bits
- Users pick private values a and b
- Compute public values
  - $A = g^a \bmod p$
  - $B = g^b \bmod p$
- Public values A and B are exchanged

Implementation

- Both users Compute shared, private key
  - s = B ^ a mod p
  - s = A ^ b mod p

- Algebraically it can be shown that both s are equal.
  - Thus, Users now have a symmetric secret key to encrypt

Example

- Alice and Bob agree to use a prime number p=23 and base g=5.
- Alice chooses a secret integer $\boldsymbol{a}$=**6**, then sends Bob A = g^{a} mod p
  - A = 5^{6} mod 23
  - A = 15,625 mod 23
  - A = 8
- Bob chooses a secret integer $\boldsymbol{b}$=**15**, then sends Alice B = g^{b} mod p
  - B = 5^{15} mod 23
  - B = 30,517,578,125 mod 23
  - B = 19
- Alice computes **s** = B^{a} mod p
  - **s** = 19^{6} mod 23
  - **s** = 47,045,881 mod 23
  - **s** = **2**

Example-contd..

- Bob computes **s** = A^{b} mod p
  - **s** = 8^{15} mod 23
  - **s** = 35,184,372,088,832 mod 23
  - **s** = **2**
- Alice and Bob now share a secret: **s** = **2**. This is because 6*15 is the same as 15*6. So somebody who had known both these private integers might also have calculated s as follows:
  - **s** = 5^{6*15} mod 23
  - **s** = 5^{15*6} mod 23
  - **s** = 5^{90} mod 23
  - **s** = 807,793,566,946,316,088,741,610,050,849,573,099,185,363,389,551,639,556,884,765,625 mod 23
  - **s** = **2**

- Both Alice and Bob have arrived at the same value, because $(g^a)^b$ and $(g^b)^a$ are equal mod p. Note that only a, b and $g^{ab} = g^{ba}$ mod p are kept secret. All the other values – p, g, g^a *mod* p, and g^b *mod* p – are sent in the clear

Example - Diffie-Hellman Key exchange

	Alice	Evil Eve	Bob
	Alice and Bob exchange a Prime (P) and a Generator (G) in clear text, such that P > G and G is Primitive Root of P G = 7, P = 11	Evil Eve sees G = 7, P = 11	Alice and Bob exchange a Prime (P) and a Generator (G) in clear text, such that P > G and G is Primitive Root of P G = 7, P = 11
Step 1	Alice generates a random number: X_A X_A=6 (Secret)		Bob generates a random number: X_B X_B=9 (Secret)
Step 2	$Y_A=G^{X_A}(\text{mod } P)$ $Y_A = 7^6 \ (\text{mod } 11)$ $Y_A = 4$		$Y_B=G^{X_B}(\text{mod } P)$ $Y_B = 7^9 \ (\text{mod } 11)$ $Y_B = 8$
Step 3	Alice receives Y_B = 8 in clear-text	Evil Eve sees $Y_A = 4$, $Y_B = 8$	Bob receives Y_A = 4 in clear-text
Step 4	Secret Key $=Y_B^{X_A}(\text{mod } P)$ Secret Key = 8^6 (mod 11) **Secret Key = 3**		Secret Key $=Y_A^{X_B}(\text{mod } P)$ Secret Key = 4^9 (mod 11) **Secret Key = 3**

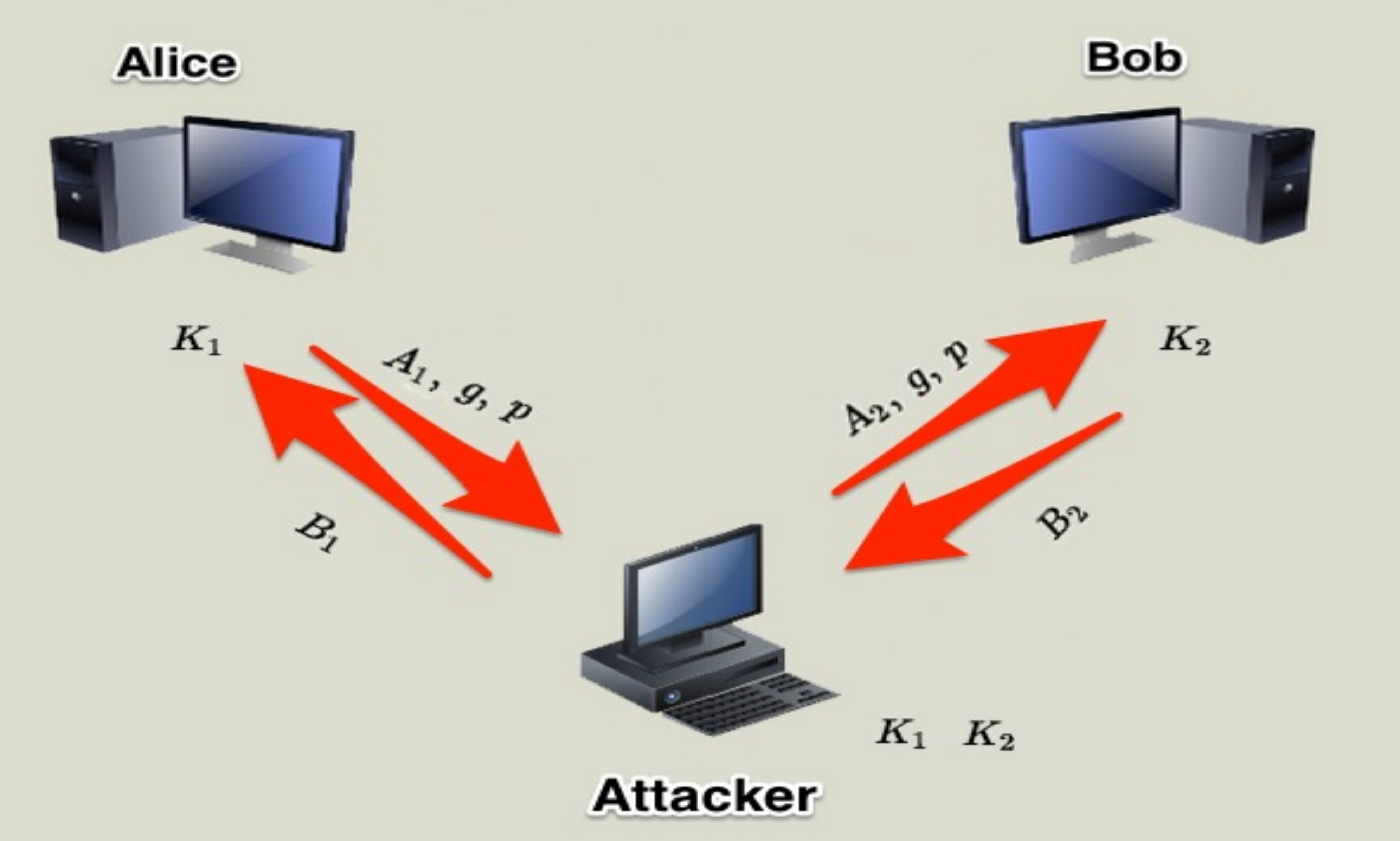
Diffie-Hellman Example

- users Alice & Bob who wish to swap keys:
- agree on prime $p=353$ and $g=3$
- select random secret keys:
  - A chooses $x_A=97$, B chooses $x_B=233$
- compute public keys:
  - $y_A=3^{97} \bmod 353 = 40$ (Alice)
  - $y_B=3^{233} \bmod 353 = 248$ (Bob)
- compute shared session key as:

$K_{AB}= y_B^{x_A} \bmod 353 = 248^{97} = 160$ (Alice)

$K_{AB}= y_A^{x_B} \bmod 353 = 40^{233} = 160$ (Bob)

D-H Key Exchange – Man in the middle attack



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