Cumulative normal distribution

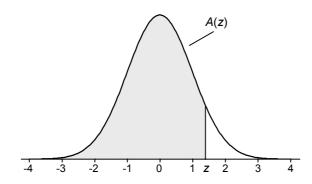
Critical values of the *t* distribution

Critical values of the *F* distribution

Critical values of the chi-squared distribution

Table A.1

Cumulative Standardized Normal Distribution



A(z) is the integral of the standardized normal distribution from  $-\infty$  to z (in other words, the area under the curve to the left of z). It gives the probability of a normal random variable not being more than z standard deviations above its mean. Values of z of particular importance:

| Z     | A(z)   |                                 |
|-------|--------|---------------------------------|
| 1.645 | 0.9500 | Lower limit of right 5% tail    |
| 1.960 | 0.9750 | Lower limit of right 2.5% tail  |
| 2.326 | 0.9900 | Lower limit of right 1% tail    |
| 2.576 | 0.9950 | Lower limit of right 0.5% tail  |
| 3.090 | 0.9990 | Lower limit of right 0.1% tail  |
| 3.291 | 0.9995 | Lower limit of right 0.05% tail |

| Z   | 0.00   | 0.01   | 0.02   | 0.03   | 0.04   | 0.05   | 0.06   | 0.07   | 0.08   | 0.09   |
|-----|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 0.0 | 0.5000 | 0.5040 | 0.5080 | 0.5120 | 0.5160 | 0.5199 | 0.5239 | 0.5279 | 0.5319 | 0.5359 |
| 0.1 | 0.5398 | 0.5438 | 0.5478 | 0.5517 | 0.5557 | 0.5596 | 0.5636 | 0.5675 | 0.5714 | 0.5753 |
| 0.2 | 0.5793 | 0.5832 | 0.5871 | 0.5910 | 0.5948 | 0.5987 | 0.6026 | 0.6064 | 0.6103 | 0.6141 |
| 0.3 | 0.6179 | 0.6217 | 0.6255 | 0.6293 | 0.6331 | 0.6368 | 0.6406 | 0.6443 | 0.6480 | 0.6517 |
| 0.4 | 0.6554 | 0.6591 | 0.6628 | 0.6664 | 0.6700 | 0.6736 | 0.6772 | 0.6808 | 0.6844 | 0.6879 |
| 0.5 | 0.6915 | 0.6950 | 0.6985 | 0.7019 | 0.7054 | 0.7088 | 0.7123 | 0.7157 | 0.7190 | 0.7224 |
| 0.6 | 0.7257 | 0.7291 | 0.7324 | 0.7357 | 0.7389 | 0.7422 | 0.7454 | 0.7486 | 0.7517 | 0.7549 |
| 0.7 | 0.7580 | 0.7611 | 0.7642 | 0.7673 | 0.7704 | 0.7734 | 0.7764 | 0.7794 | 0.7823 | 0.7852 |
| 0.8 | 0.7881 | 0.7910 | 0.7939 | 0.7967 | 0.7995 | 0.8023 | 0.8051 | 0.8078 | 0.8106 | 0.8133 |
| 0.9 | 0.8159 | 0.8186 | 0.8212 | 0.8238 | 0.8264 | 0.8289 | 0.8315 | 0.8340 | 0.8365 | 0.8389 |
| 1.0 | 0.8413 | 0.8438 | 0.8461 | 0.8485 | 0.8508 | 0.8531 | 0.8554 | 0.8577 | 0.8599 | 0.8621 |
| 1.1 | 0.8643 | 0.8665 | 0.8686 | 0.8708 | 0.8729 | 0.8749 | 0.8770 | 0.8790 | 0.8810 | 0.8830 |
| 1.2 | 0.8849 | 0.8869 | 0.8888 | 0.8907 | 0.8925 | 0.8944 | 0.8962 | 0.8980 | 0.8997 | 0.9015 |
| 1.3 | 0.9032 | 0.9049 | 0.9066 | 0.9082 | 0.9099 | 0.9115 | 0.9131 | 0.9147 | 0.9162 | 0.9177 |
| 1.4 | 0.9192 | 0.9207 | 0.9222 | 0.9236 | 0.9251 | 0.9265 | 0.9279 | 0.9292 | 0.9306 | 0.9319 |
| 1.5 | 0.9332 | 0.9345 | 0.9357 | 0.9370 | 0.9382 | 0.9394 | 0.9406 | 0.9418 | 0.9429 | 0.9441 |
| 1.6 | 0.9452 | 0.9463 | 0.9474 | 0.9484 | 0.9495 | 0.9505 | 0.9515 | 0.9525 | 0.9535 | 0.9545 |
| 1.7 | 0.9554 | 0.9564 | 0.9573 | 0.9582 | 0.9591 | 0.9599 | 0.9608 | 0.9616 | 0.9625 | 0.9633 |
| 1.8 | 0.9641 | 0.9649 | 0.9656 | 0.9664 | 0.9671 | 0.9678 | 0.9686 | 0.9693 | 0.9699 | 0.9706 |
| 1.9 | 0.9713 | 0.9719 | 0.9726 | 0.9732 | 0.9738 | 0.9744 | 0.9750 | 0.9756 | 0.9761 | 0.9767 |
| 2.0 | 0.9772 | 0.9778 | 0.9783 | 0.9788 | 0.9793 | 0.9798 | 0.9803 | 0.9808 | 0.9812 | 0.9817 |
| 2.1 | 0.9821 | 0.9826 | 0.9830 | 0.9834 | 0.9838 | 0.9842 | 0.9846 | 0.9850 | 0.9854 | 0.9857 |
| 2.2 | 0.9861 | 0.9864 | 0.9868 | 0.9871 | 0.9875 | 0.9878 | 0.9881 | 0.9884 | 0.9887 | 0.9890 |
| 2.3 | 0.9893 | 0.9896 | 0.9898 | 0.9901 | 0.9904 | 0.9906 | 0.9909 | 0.9911 | 0.9913 | 0.9916 |
| 2.4 | 0.9918 | 0.9920 | 0.9922 | 0.9925 | 0.9927 | 0.9929 | 0.9931 | 0.9932 | 0.9934 | 0.9936 |
| 2.5 | 0.9938 | 0.9940 | 0.9941 | 0.9943 | 0.9945 | 0.9946 | 0.9948 | 0.9949 | 0.9951 | 0.9952 |
| 2.6 | 0.9953 | 0.9955 | 0.9956 | 0.9957 | 0.9959 | 0.9960 | 0.9961 | 0.9962 | 0.9963 | 0.9964 |
| 2.7 | 0.9965 | 0.9966 | 0.9967 | 0.9968 | 0.9969 | 0.9970 | 0.9971 | 0.9972 | 0.9973 | 0.9974 |
| 2.8 | 0.9974 | 0.9975 | 0.9976 | 0.9977 | 0.9977 | 0.9978 | 0.9979 | 0.9979 | 0.9980 | 0.9981 |
| 2.9 | 0.9981 | 0.9982 | 0.9982 | 0.9983 | 0.9984 | 0.9984 | 0.9985 | 0.9985 | 0.9986 | 0.9986 |
| 3.0 | 0.9987 | 0.9987 | 0.9987 | 0.9988 | 0.9988 | 0.9989 | 0.9989 | 0.9989 | 0.9990 | 0.9990 |
| 3.1 | 0.9990 | 0.9991 | 0.9991 | 0.9991 | 0.9992 | 0.9992 | 0.9992 | 0.9992 | 0.9993 | 0.9993 |
| 3.2 | 0.9993 | 0.9993 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9995 | 0.9995 | 0.9995 |
| 3.3 | 0.9995 | 0.9995 | 0.9995 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9997 |
| 3.4 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9998 |
| 3.5 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 |
| 3.6 | 0.9998 | 0.9998 | 0.9999 |        |        |        |        |        |        |        |

TABLE A.2

t Distribution: Critical Values of t

|               |                  | Significance level |                |                |                |                 |                  |  |
|---------------|------------------|--------------------|----------------|----------------|----------------|-----------------|------------------|--|
| Degrees of    | Two-tailed test: | 10%                | 5%             | 2%<br>1%       | 1%             | 0.2%            | 0.1%             |  |
| freedom       | One-tailed test: | 5%<br>6.314        | 2.5%<br>12.706 | 31.821         | 0.5%<br>63.657 | 0.1%<br>318.309 | 0.05%<br>636.619 |  |
| 1<br>2        |                  | 2.920              | 4.303          | 6.965          | 9.925          | 22.327          | 31.599           |  |
| 3             |                  | 2.353              | 3.182          | 4.541          | 5.841          | 10.215          | 12.924           |  |
| 4             |                  | 2.132              | 2.776          | 3.747          | 4.604          | 7.173           | 8.610            |  |
| 5             |                  | 2.015              | 2.571          | 3.365          | 4.032          | 5.893           | 6.869            |  |
| 6             |                  | 1.943              | 2.447          | 3.143          | 3.707          | 5.208           | 5.959            |  |
| 7<br><b>8</b> |                  | 1.894<br>1.860     | 2.365<br>2.306 | 2.998<br>2.896 | 3.499<br>3.355 | 4.785<br>4.501  | 5.408<br>5.041   |  |
| 9             |                  | 1.833              | 2.262          | 2.821          | 3.250          | 4.297           | 4.781            |  |
| 10            |                  | 1.812              | 2.228          | 2.764          | 3.169          | 4.144           | 4.587            |  |
| 11            |                  | 1.796              | 2.201          | 2.718          | 3.106          | 4.025           | 4.437            |  |
| 12            |                  | 1.782              | 2.179          | 2.681          | 3.055          | 3.930           | 4.318            |  |
| 13            |                  | 1.771              | 2.160          | 2.650          | 3.012          | 3.852           | 4.221            |  |
| 14            |                  | 1.761              | 2.145          | 2.624          | 2.977          | 3.787           | 4.140            |  |
| 15            |                  | 1.753              | 2.131          | 2.602          | 2.947          | 3.733           | 4.073            |  |
| 16            |                  | 1.746              | 2.120          | 2.583          | 2.921          | 3.686           | 4.015            |  |
| 17            |                  | 1.740              | 2.110          | 2.567          | 2.898          | 3.646           | 3.965            |  |
| 18<br>19      |                  | 1.734<br>1.729     | 2.101<br>2.093 | 2.552<br>2.539 | 2.878<br>2.861 | 3.610<br>3.579  | 3.922<br>3.883   |  |
| 20            |                  | 1.725              | 2.086          | 2.528          | 2.845          | 3.552           | 3.850            |  |
| 21            |                  | 1.721              | 2.080          | 2.518          | 2.831          | 3.527           | 3.819            |  |
| 22            |                  | 1.717              | 2.074          | 2.508          | 2.819          | 3.505           | 3.792            |  |
| 23            |                  | 1.714              | 2.069          | 2.500          | 2.807          | 3.485           | 3.768            |  |
| 24            |                  | 1.711              | 2.064          | 2.492          | 2.797          | 3.467           | 3.745            |  |
| 25            |                  | 1.708              | 2.060          | 2.485          | 2.787          | 3.450           | 3.725            |  |
| 26            |                  | 1.706              | 2.056          | 2.479          | 2.779          | 3.435           | 3.707            |  |
| 27            |                  | 1.703              | 2.052          | 2.473          | 2.771          | 3.421           | 3.690            |  |
| 28            |                  | 1.701              | 2.048          | 2.467          | 2.763          | 3.408           | 3.674            |  |
| 29<br>30      |                  | 1.699<br>1.697     | 2.045<br>2.042 | 2.462<br>2.457 | 2.756<br>2.750 | 3.396<br>3.385  | 3.659<br>3.646   |  |
| 32            |                  | 1.694              | 2.037          | 2.449          | 2.738          | 3.365           | 3.622            |  |
| 34            |                  | 1.691              | 2.032          | 2.441          | 2.728          | 3.348           | 3.601            |  |
| 36            |                  | 1.688              | 2.028          | 2.434          | 2.719          | 3.333           | 3.582            |  |
| 38            |                  | 1.686              | 2.024          | 2.429          | 2.712          | 3.319           | 3.566            |  |
| 40            |                  | 1.684              | 2.021          | 2.423          | 2.704          | 3.307           | 3.551            |  |
| 42            |                  | 1.682              | 2.018          | 2.418          | 2.698          | 3.296           | 3.538            |  |
| 44            |                  | 1.680              | 2.015          | 2.414          | 2.692          | 3.286           | 3.526            |  |
| 46<br>48      |                  | 1.679<br>1.677     | 2.013<br>2.011 | 2.410<br>2.407 | 2.687<br>2.682 | 3.277<br>3.269  | 3.515<br>3.505   |  |
| 50            |                  | 1.676              | 2.009          | 2.407          | 2.678          | 3.261           | 3.496            |  |
| 60            |                  | 1.671              | 2.000          | 2.390          | 2.660          | 3.232           | 3.460            |  |
| 70            |                  | 1.667              | 1.994          | 2.381          | 2.648          | 3.211           | 3.435            |  |
| 80            |                  | 1.664              | 1.990          | 2.374          | 2.639          | 3.195           | 3.416            |  |
| 90            |                  | 1.662              | 1.987          | 2.368          | 2.632          | 3.183           | 3.402            |  |
| 100           |                  | 1.660              | 1.984          | 2.364          | 2.626          | 3.174           | 3.390            |  |
| 120           |                  | 1.658              | 1.980          | 2.358          | 2.617          | 3.160           | 3.373            |  |
| 150           |                  | 1.655              | 1.976          | 2.351          | 2.609          | 3.145           | 3.357            |  |
| 200<br>300    |                  | 1.653<br>1.650     | 1.972<br>1.968 | 2.345<br>2.339 | 2.601<br>2.592 | 3.131           | 3.340<br>3.323   |  |
| 400           |                  | 1.630              | 1.966          | 2.339          | 2.588          | 3.118<br>3.111  | 3.323            |  |
| 500           |                  | 1.648              | 1.965          | 2.334          | 2.586          | 3.107           | 3.310            |  |
| 600           |                  | 1.648              | 1.963          | 2.334          | 2.584          | 3.107           | 3.310            |  |
| <b>∞</b>      |                  | 1.645              | 1.960          | 2.326          | 2.576          | 3.090           | 3.291            |  |
|               |                  |                    |                |                |                |                 |                  |  |

٠

Table A.3

F Distribution: Critical Values of F (5% significance level)

| <i>v</i> <sub>1</sub>    | 1  | 2                                       | 3    | 4                                       | 5    | 6                                       | 7    | 8    | 9                                       | 10   | 12                                      | 14    | 16   | 18                                      | 20   |
|--------------------------|--|---|------|---|------|---|------|------|---|------|---|-------|------|---|------|
| v <sub>2</sub> 1 2 3 4 5 | 161.45<br>18.51<br>10.13<br>7.71<br>6.61 | 199.50<br>19.00<br>9.55<br>6.94<br>5.79 |      | 224.58<br>19.25<br>9.12<br>6.39<br>5.19 |      | 233.99<br>19.33<br>8.94<br>6.16<br>4.95 |      |      | 240.54<br>19.38<br>8.81<br>6.00<br>4.77 |      | 243.91<br>19.41<br>8.74<br>5.91<br>4.68 | 19.42 |      | 247.32<br>19.44<br>8.67<br>5.82<br>4.58 |      |
| 6                        | 5.99                                     | 5.14                                    | 4.76 | 4.53                                    | 4.39 | 4.28                                    | 4.21 | 4.15 | 4.10                                    | 4.06 | 4.00                                    | 3.96  | 3.92 | 3.90                                    | 3.87 |
| 7                        | 5.59                                     | 4.74                                    | 4.35 | 4.12                                    | 3.97 | 3.87                                    | 3.79 | 3.73 | 3.68                                    | 3.64 | 3.57                                    | 3.53  | 3.49 | 3.47                                    | 3.44 |
| 8                        | 5.32                                     | 4.46                                    | 4.07 | 3.84                                    | 3.69 | 3.58                                    | 3.50 | 3.44 | 3.39                                    | 3.35 | 3.28                                    | 3.24  | 3.20 | 3.17                                    | 3.15 |
| 9                        | 5.12                                     | 4.26                                    | 3.86 | 3.63                                    | 3.48 | 3.37                                    | 3.29 | 3.23 | 3.18                                    | 3.14 | 3.07                                    | 3.03  | 2.99 | 2.96                                    | 2.94 |
| 10                       | 4.96                                     | 4.10                                    | 3.71 | 3.48                                    | 3.33 | 3.22                                    | 3.14 | 3.07 | 3.02                                    | 2.98 | 2.91                                    | 2.86  | 2.83 | 2.80                                    | 2.77 |
| 11                       | 4.84                                     | 3.98                                    | 3.59 | 3.36                                    | 3.20 | 3.09                                    | 3.01 | 2.95 | 2.90                                    | 2.85 | 2.79                                    |       | 2.70 | 2.67                                    | 2.65 |
| 12                       | 4.75                                     | 3.89                                    | 3.49 | 3.26                                    | 3.11 | 3.00                                    | 2.91 | 2.85 | 2.80                                    | 2.75 | 2.69                                    |       | 2.60 | 2.57                                    | 2.54 |
| 13                       | 4.67                                     | 3.81                                    | 3.41 | 3.18                                    | 3.03 | 2.92                                    | 2.83 | 2.77 | 2.71                                    | 2.67 | 2.60                                    |       | 2.51 | 2.48                                    | 2.46 |
| 14                       | 4.60                                     | 3.74                                    | 3.34 | 3.11                                    | 2.96 | 2.85                                    | 2.76 | 2.70 | 2.65                                    | 2.60 | 2.53                                    |       | 2.44 | 2.41                                    | 2.39 |
| 15                       | 4.54                                     | 3.68                                    | 3.29 | 3.06                                    | 2.90 | 2.79                                    | 2.71 | 2.64 | 2.59                                    | 2.54 | 2.48                                    |       | 2.38 | 2.35                                    | 2.33 |
| 16                       | 4.49                                     | 3.63                                    | 3.24 | 3.01                                    | 2.85 | 2.74                                    | 2.66 | 2.59 | 2.54                                    | 2.49 | 2.42                                    | 2.37  | 2.33 | 2.30                                    | 2.28 |
| 17                       | 4.45                                     | 3.59                                    | 3.20 | 2.96                                    | 2.81 | 2.70                                    | 2.61 | 2.55 | 2.49                                    | 2.45 | 2.38                                    | 2.33  | 2.29 | 2.26                                    | 2.23 |
| 18                       | 4.41                                     | 3.55                                    | 3.16 | 2.93                                    | 2.77 | 2.66                                    | 2.58 | 2.51 | 2.46                                    | 2.41 | 2.34                                    | 2.29  | 2.25 | 2.22                                    | 2.19 |
| 19                       | 4.38                                     | 3.52                                    | 3.13 | 2.90                                    | 2.74 | 2.63                                    | 2.54 | 2.48 | 2.42                                    | 2.38 | 2.31                                    | 2.26  | 2.21 | 2.18                                    | 2.16 |
| 20                       | 4.35                                     | 3.49                                    | 3.10 | 2.87                                    | 2.71 | 2.60                                    | 2.51 | 2.45 | 2.39                                    | 2.35 | 2.28                                    | 2.22  | 2.18 | 2.15                                    | 2.12 |
| 21                       | 4.32                                     | 3.47                                    | 3.07 | 2.84                                    | 2.68 | 2.57                                    | 2.49 | 2.42 | 2.37                                    | 2.32 | 2.25                                    | 2.20  | 2.16 | 2.12                                    | 2.10 |
| 22                       | 4.30                                     | 3.44                                    | 3.05 | 2.82                                    | 2.66 | 2.55                                    | 2.46 | 2.40 | 2.34                                    | 2.30 | 2.23                                    | 2.17  | 2.13 | 2.10                                    | 2.07 |
| 23                       | 4.28                                     | 3.42                                    | 3.03 | 2.80                                    | 2.64 | 2.53                                    | 2.44 | 2.37 | 2.32                                    | 2.27 | 2.20                                    | 2.15  | 2.11 | 2.08                                    | 2.05 |
| 24                       | 4.26                                     | 3.40                                    | 3.01 | 2.78                                    | 2.62 | 2.51                                    | 2.42 | 2.36 | 2.30                                    | 2.25 | 2.18                                    | 2.13  | 2.09 | 2.05                                    | 2.03 |
| 25                       | 4.24                                     | 3.39                                    | 2.99 | 2.76                                    | 2.60 | 2.49                                    | 2.40 | 2.34 | 2.28                                    | 2.24 | 2.16                                    | 2.11  | 2.07 | 2.04                                    | 2.01 |
| 26                       | 4.22                                     | 3.37                                    | 2.98 | 2.74                                    | 2.59 | 2.47                                    | 2.39 | 2.32 | 2.27                                    | 2.22 | 2.15                                    | 2.09  | 2.05 | 2.02                                    | 1.99 |
| 27                       | 4.21                                     | 3.35                                    | 2.96 | 2.73                                    | 2.57 | 2.46                                    | 2.37 | 2.31 | 2.25                                    | 2.20 | 2.13                                    | 2.08  | 2.04 | 2.00                                    | 1.97 |
| 28                       | 4.20                                     | 3.34                                    | 2.95 | 2.71                                    | 2.56 | 2.45                                    | 2.36 | 2.29 | 2.24                                    | 2.19 | 2.12                                    | 2.06  | 2.02 | 1.99                                    | 1.96 |
| 29                       | 4.18                                     | 3.33                                    | 2.93 | 2.70                                    | 2.55 | 2.43                                    | 2.35 | 2.28 | 2.22                                    | 2.18 | 2.10                                    | 2.05  | 2.01 | 1.97                                    | 1.94 |
| 30                       | 4.17                                     | 3.32                                    | 2.92 | 2.69                                    | 2.53 | 2.42                                    | 2.33 | 2.27 | 2.21                                    | 2.16 | 2.09                                    | 2.04  | 1.99 | 1.96                                    | 1.93 |
| 35                       | 4.12                                     | 3.27                                    | 2.87 | 2.64                                    | 2.49 | 2.37                                    | 2.29 | 2.22 | 2.16                                    | 2.11 | 2.04                                    |       | 1.94 | 1.91                                    | 1.88 |
| 40                       | 4.08                                     | 3.23                                    | 2.84 | 2.61                                    | 2.45 | 2.34                                    | 2.25 | 2.18 | 2.12                                    | 2.08 | 2.00                                    |       | 1.90 | 1.87                                    | 1.84 |
| 50                       | 4.03                                     | 3.18                                    | 2.79 | 2.56                                    | 2.40 | 2.29                                    | 2.20 | 2.13 | 2.07                                    | 2.03 | 1.95                                    |       | 1.85 | 1.81                                    | 1.78 |
| 60                       | 4.00                                     | 3.15                                    | 2.76 | 2.53                                    | 2.37 | 2.25                                    | 2.17 | 2.10 | 2.04                                    | 1.99 | 1.92                                    |       | 1.82 | 1.78                                    | 1.75 |
| 70                       | 3.98                                     | 3.13                                    | 2.74 | 2.50                                    | 2.35 | 2.23                                    | 2.14 | 2.07 | 2.02                                    | 1.97 | 1.89                                    |       | 1.79 | 1.75                                    | 1.72 |
| 80                       | 3.96                                     | 3.11                                    | 2.72 | 2.49                                    | 2.33 | 2.21                                    | 2.13 | 2.06 | 2.00                                    | 1.95 | 1.88                                    | 1.82  | 1.77 | 1.73                                    | 1.70 |
| 90                       | 3.95                                     | 3.10                                    | 2.71 | 2.47                                    | 2.32 | 2.20                                    | 2.11 | 2.04 | 1.99                                    | 1.94 | 1.86                                    | 1.80  | 1.76 | 1.72                                    | 1.69 |
| 100                      | 3.94                                     | 3.09                                    | 2.70 | 2.46                                    | 2.31 | 2.19                                    | 2.10 | 2.03 | 1.97                                    | 1.93 | 1.85                                    | 1.79  | 1.75 | 1.71                                    | 1.68 |
| 120                      | 3.92                                     | 3.07                                    | 2.68 | 2.45                                    | 2.29 | 2.18                                    | 2.09 | 2.02 | 1.96                                    | 1.91 | 1.83                                    | 1.78  | 1.73 | 1.69                                    | 1.66 |
| 150                      | 3.90                                     | 3.06                                    | 2.66 | 2.43                                    | 2.27 | 2.16                                    | 2.07 | 2.00 | 1.94                                    | 1.89 | 1.82                                    | 1.76  | 1.71 | 1.67                                    | 1.64 |
| 200                      | 3.89                                     | 3.04                                    | 2.65 | 2.42                                    | 2.26 | 2.14                                    | 2.06 | 1.98 | 1.93                                    | 1.88 | 1.80                                    | 1.74  | 1.69 | 1.66                                    | 1.62 |
| 250                      | 3.88                                     | 3.03                                    | 2.64 | 2.41                                    | 2.25 | 2.13                                    | 2.05 | 1.98 | 1.92                                    | 1.87 | 1.79                                    | 1.73  | 1.68 | 1.65                                    | 1.61 |
| 300                      | 3.87                                     | 3.03                                    | 2.63 | 2.40                                    | 2.24 | 2.13                                    | 2.04 | 1.97 | 1.91                                    | 1.86 | 1.78                                    | 1.72  | 1.68 | 1.64                                    | 1.61 |
| 400                      | 3.86                                     | 3.02                                    | 2.63 | 2.39                                    | 2.24 | 2.12                                    | 2.03 | 1.96 | 1.90                                    | 1.85 | 1.78                                    | 1.72  | 1.67 | 1.63                                    | 1.60 |
| 500                      | 3.86                                     | 3.01                                    | 2.62 | 2.39                                    | 2.23 | 2.12                                    | 2.03 | 1.96 | 1.90                                    | 1.85 | 1.77                                    | 1.71  | 1.66 | 1.62                                    | 1.59 |
| 600                      | 3.86                                     | 3.01                                    | 2.62 | 2.39                                    | 2.23 | 2.11                                    | 2.02 | 1.95 | 1.90                                    | 1.85 | 1.77                                    | 1.71  | 1.66 | 1.62                                    | 1.59 |
| 750                      | 3.85                                     | 3.01                                    | 2.62 | 2.38                                    | 2.23 | 2.11                                    | 2.02 | 1.95 | 1.89                                    | 1.84 | 1.77                                    | 1.70  | 1.66 | 1.62                                    | 1.58 |
| 1000                     | 3.85                                     | 3.00                                    | 2.61 | 2.38                                    | 2.22 | 2.11                                    | 2.02 | 1.95 | 1.89                                    | 1.84 | 1.76                                    | 1.70  | 1.65 | 1.61                                    | 1.58 |

Table A.3 (continued)

# F Distribution: Critical Values of F (5% significance level)

| <i>v</i> <sub>1</sub>           | 25                                      | 30                                   | 35                                   | 40                                   | 50                                   | 60                                   | 75                                   | 100                                  | 150                                  | 200                                  |
|---------------------------------|---|--------------------------------------|--------------------------------------|--------------------------------------|--------------------------------------|--------------------------------------|--------------------------------------|--------------------------------------|--------------------------------------|--------------------------------------|
| v <sub>2</sub> 1 2 3 4 5        | 249.26<br>19.46<br>8.63<br>5.77<br>4.52 | 19.46<br>8.62<br>5.75                | 19.47<br>8.60<br>5.73                | 19.47<br>8.59                        |                                      | 19.48<br>8.57                        | 8.56<br>5.68                         | 19.49<br>8.55<br>5.66                | 19.49<br>8.54                        |                                      |
| 6<br>7<br>8<br>9<br>10          | 3.83<br>3.40<br>3.11<br>2.89<br>2.73    | 3.08<br>2.86                         | 3.06<br>2.84                         |                                      | 3.75<br>3.32<br>3.02<br>2.80<br>2.64 | 3.74<br>3.30<br>3.01<br>2.79<br>2.62 | 3.29<br>2.99<br>2.77                 | 2.97                                 | 2.96<br>2.74                         |                                      |
| 11<br>12<br>13<br>14<br>15      | 2.60<br>2.50<br>2.41<br>2.34<br>2.28    | 2.38<br>2.31                         | 2.36<br>2.28                         | 2.34<br>2.27                         | 2.51<br>2.40<br>2.31<br>2.24<br>2.18 | 2.49<br>2.38<br>2.30<br>2.22<br>2.16 | 2.37<br>2.28<br>2.21                 | 2.35<br>2.26<br>2.19                 | 2.24<br>2.17                         |                                      |
| 16<br>17<br>18<br>19<br>20      | 2.23<br>2.18<br>2.14<br>2.11<br>2.07    | 2.15<br>2.11<br>2.07                 | 2.12<br>2.08<br>2.05                 |                                      | 2.12<br>2.08<br>2.04<br>2.00<br>1.97 | 2.11<br>2.06<br>2.02<br>1.98<br>1.95 | 2.00<br>1.96                         | 2.02<br>1.98<br>1.94                 | 2.00<br>1.96                         | 2.04<br>1.99<br>1.95<br>1.91<br>1.88 |
| 21<br>22<br>23<br>24<br>25      | 2.05<br>2.02<br>2.00<br>1.97<br>1.96    | 1.98<br>1.96<br>1.94                 | 1.93<br>1.91                         | 1.91<br>1.89                         | 1.94<br>1.91<br>1.88<br>1.86<br>1.84 | 1.92<br>1.89<br>1.86<br>1.84<br>1.82 | 1.87<br>1.84<br>1.82                 | 1.85                                 | 1.80<br>1.78                         | 1.84<br>1.82<br>1.79<br>1.77<br>1.75 |
| 26<br>27<br>28<br>29<br>30      | 1.94<br>1.92<br>1.91<br>1.89<br>1.88    | 1.88<br>1.87<br>1.85                 | 1.84<br>1.83                         | 1.82                                 | 1.82<br>1.81<br>1.79<br>1.77<br>1.76 | 1.80<br>1.79<br>1.77<br>1.75<br>1.74 | 1.76<br>1.75<br>1.73                 | 1.73                                 |                                      | 1.73<br>1.71<br>1.69<br>1.67<br>1.66 |
| 35<br>40<br>50<br>60<br>70      | 1.82<br>1.78<br>1.73<br>1.69<br>1.66    | 1.74<br>1.69<br>1.65                 | 1.72<br>1.66                         |                                      | 1.70<br>1.66<br>1.60<br>1.56<br>1.53 | 1.68<br>1.64<br>1.58<br>1.53<br>1.50 | 1.61<br>1.55                         | 1.59<br>1.52                         |                                      | 1.48                                 |
| 80<br>90<br>100<br>120<br>150   | 1.64<br>1.63<br>1.62<br>1.60<br>1.58    | 1.59<br>1.57<br>1.55                 | 1.57<br>1.55<br>1.54<br>1.52<br>1.50 | 1.54<br>1.53<br>1.52<br>1.50<br>1.48 | 1.51<br>1.49<br>1.48<br>1.46<br>1.44 | 1.48<br>1.46<br>1.45<br>1.43<br>1.41 | 1.45<br>1.44<br>1.42<br>1.40<br>1.38 | 1.43<br>1.41<br>1.39<br>1.37<br>1.34 | 1.39<br>1.38<br>1.36<br>1.33<br>1.31 | 1.38<br>1.36<br>1.34<br>1.32<br>1.29 |
| 200<br>250<br>300<br>400<br>500 | 1.56<br>1.55<br>1.54<br>1.53<br>1.53    | 1.52<br>1.50<br>1.50<br>1.49<br>1.48 | 1.48<br>1.47<br>1.46<br>1.45<br>1.45 | 1.46<br>1.44<br>1.43<br>1.42<br>1.42 | 1.41<br>1.40<br>1.39<br>1.38<br>1.38 | 1.39<br>1.37<br>1.36<br>1.35<br>1.35 | 1.35<br>1.34<br>1.33<br>1.32<br>1.31 | 1.32<br>1.31<br>1.30<br>1.28<br>1.28 | 1.28<br>1.27<br>1.26<br>1.24<br>1.23 | 1.26<br>1.25<br>1.23<br>1.22<br>1.21 |
| 600<br>750<br>1000              | 1.52<br>1.52<br>1.52                    | 1.48<br>1.47<br>1.47                 | 1.44<br>1.44<br>1.43                 | 1.41<br>1.41<br>1.41                 | 1.37<br>1.37<br>1.36                 | 1.34<br>1.34<br>1.33                 | 1.31<br>1.30<br>1.30                 | 1.27<br>1.26<br>1.26                 | 1.23<br>1.22<br>1.22                 | 1.20<br>1.20<br>1.19                 |

Table A.3 (continued)

## F Distribution: Critical Values of F (1% significance level)

| <i>v</i> <sub>1</sub>    | 1   | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9   | 10             | 12   | 14   | 16   | 18   | 20   |
|--------------------------|---|---|---|---|---|---|---|---|---|----------------|--|--|--|--|------|
| v <sub>2</sub> 1 2 3 4 5 | 4052.18<br>98.50<br>34.12<br>21.20<br>16.26 | 4999.50<br>99.00<br>30.82<br>18.00<br>13.27 | 5403.35<br>99.17<br>29.46<br>16.69<br>12.06 | 5624.58<br>99.25<br>28.71<br>15.98<br>11.39 | 5763.65<br>99.30<br>28.24<br>15.52<br>10.97 | 5858.99<br>99.33<br>27.91<br>15.21<br>10.67 | 5928.36<br>99.36<br>27.67<br>14.98<br>10.46 | 5981.07<br>99.37<br>27.49<br>14.80<br>10.29 | 6022.47<br>99.39<br>27.35<br>14.66<br>10.16 | 99.40<br>27.23 | 6106.32<br>99.42<br>27.05<br>14.37<br>9.89 | 6142.67<br>99.43<br>26.92<br>14.25<br>9.77 | 6170.10<br>99.44<br>26.83<br>14.15<br>9.68 | 6191.53<br>99.44<br>26.75<br>14.08<br>9.61 |      |
| 6                        | 13.75                                       | 10.92                                       | 9.78  | 9.15  | 8.75  | 8.47  | 8.26  | 8.10  | 7.98  | 7.87           | 7.72                                       | 7.60                                       | 7.52                                       | 7.45                                       | 7.40 |
| 7                        | 12.25                                       | 9.55  | 8.45  | 7.85  | 7.46  | 7.19  | 6.99  | 6.84  | 6.72  | 6.62           | 6.47                                       | 6.36                                       | 6.28                                       | 6.21                                       | 6.16 |
| 8                        | 11.26                                       | 8.65  | 7.59  | 7.01  | 6.63  | 6.37  | 6.18  | 6.03  | 5.91  | 5.81           | 5.67                                       | 5.56                                       | 5.48                                       | 5.41                                       | 5.36 |
| 9                        | 10.56                                       | 8.02  | 6.99  | 6.42  | 6.06  | 5.80  | 5.61  | 5.47  | 5.35  | 5.26           | 5.11                                       | 5.01                                       | 4.92                                       | 4.86                                       | 4.81 |
| 10                       | 10.04                                       | 7.56  | 6.55  | 5.99  | 5.64  | 5.39  | 5.20  | 5.06  | 4.94  | 4.85           | 4.71                                       | 4.60                                       | 4.52                                       | 4.46                                       | 4.41 |
| 11                       | 9.65  | 7.21  | 6.22  | 5.67  | 5.32  | 5.07  | 4.89  | 4.74  | 4.63  | 4.54           | 4.40                                       | 4.29                                       | 4.21                                       | 4.15                                       | 4.10 |
| 12                       | 9.33  | 6.93  | 5.95  | 5.41  | 5.06  | 4.82  | 4.64  | 4.50  | 4.39  | 4.30           | 4.16                                       | 4.05                                       | 3.97                                       | 3.91                                       | 3.86 |
| 13                       | 9.07  | 6.70  | 5.74  | 5.21  | 4.86  | 4.62  | 4.44  | 4.30  | 4.19  | 4.10           | 3.96                                       | 3.86                                       | 3.78                                       | 3.72                                       | 3.66 |
| 14                       | 8.86  | 6.51  | 5.56  | 5.04  | 4.69  | 4.46  | 4.28  | 4.14  | 4.03  | 3.94           | 3.80                                       | 3.70                                       | 3.62                                       | 3.56                                       | 3.51 |
| 15                       | 8.68  | 6.36  | 5.42  | 4.89  | 4.56  | 4.32  | 4.14  | 4.00  | 3.89  | 3.80           | 3.67                                       | 3.56                                       | 3.49                                       | 3.42                                       | 3.37 |
| 16                       | 8.53  | 6.23  | 5.29  | 4.77  | 4.44  | 4.20  | 4.03  | 3.89  | 3.78  | 3.69           | 3.55                                       | 3.45                                       | 3.37                                       | 3.31                                       | 3.26 |
| 17                       | 8.40  | 6.11  | 5.18  | 4.67  | 4.34  | 4.10  | 3.93  | 3.79  | 3.68  | 3.59           | 3.46                                       | 3.35                                       | 3.27                                       | 3.21                                       | 3.16 |
| 18                       | 8.29  | 6.01  | 5.09  | 4.58  | 4.25  | 4.01  | 3.84  | 3.71  | 3.60  | 3.51           | 3.37                                       | 3.27                                       | 3.19                                       | 3.13                                       | 3.08 |
| 19                       | 8.18  | 5.93  | 5.01  | 4.50  | 4.17  | 3.94  | 3.77  | 3.63  | 3.52  | 3.43           | 3.30                                       | 3.19                                       | 3.12                                       | 3.05                                       | 3.00 |
| 20                       | 8.10  | 5.85  | 4.94  | 4.43  | 4.10  | 3.87  | 3.70  | 3.56  | 3.46  | 3.37           | 3.23                                       | 3.13                                       | 3.05                                       | 2.99                                       | 2.94 |
| 21                       | 8.02  | 5.78  | 4.87  | 4.37  | 4.04  | 3.81  | 3.64  | 3.51  | 3.40  | 3.31           | 3.17                                       | 3.07                                       | 2.99                                       | 2.93                                       | 2.88 |
| 22                       | 7.95  | 5.72  | 4.82  | 4.31  | 3.99  | 3.76  | 3.59  | 3.45  | 3.35  | 3.26           | 3.12                                       | 3.02                                       | 2.94                                       | 2.88                                       | 2.83 |
| 23                       | 7.88  | 5.66  | 4.76  | 4.26  | 3.94  | 3.71  | 3.54  | 3.41  | 3.30  | 3.21           | 3.07                                       | 2.97                                       | 2.89                                       | 2.83                                       | 2.78 |
| 24                       | 7.82  | 5.61  | 4.72  | 4.22  | 3.90  | 3.67  | 3.50  | 3.36  | 3.26  | 3.17           | 3.03                                       | 2.93                                       | 2.85                                       | 2.79                                       | 2.74 |
| 25                       | 7.77  | 5.57  | 4.68  | 4.18  | 3.85  | 3.63  | 3.46  | 3.32  | 3.22  | 3.13           | 2.99                                       | 2.89                                       | 2.81                                       | 2.75                                       | 2.70 |
| 26                       | 7.72  | 5.53  | 4.64  | 4.14  | 3.82  | 3.59  | 3.42  | 3.29  | 3.18  | 3.09           | 2.96                                       | 2.86                                       | 2.78                                       | 2.72                                       | 2.66 |
| 27                       | 7.68  | 5.49  | 4.60  | 4.11  | 3.78  | 3.56  | 3.39  | 3.26  | 3.15  | 3.06           | 2.93                                       | 2.82                                       | 2.75                                       | 2.68                                       | 2.63 |
| 28                       | 7.64  | 5.45  | 4.57  | 4.07  | 3.75  | 3.53  | 3.36  | 3.23  | 3.12  | 3.03           | 2.90                                       | 2.79                                       | 2.72                                       | 2.65                                       | 2.60 |
| 29                       | 7.60  | 5.42  | 4.54  | 4.04  | 3.73  | 3.50  | 3.33  | 3.20  | 3.09  | 3.00           | 2.87                                       | 2.77                                       | 2.69                                       | 2.63                                       | 2.57 |
| 30                       | 7.56  | 5.39  | 4.51  | 4.02  | 3.70  | 3.47  | 3.30  | 3.17  | 3.07  | 2.98           | 2.84                                       | 2.74                                       | 2.66                                       | 2.60                                       | 2.55 |
| 35                       | 7.42  | 5.27  | 4.40  | 3.91  | 3.59  | 3.37  | 3.20  | 3.07  | 2.96  | 2.88           | 2.74                                       | 2.64                                       | 2.56                                       | 2.50                                       | 2.44 |
| 40                       | 7.31  | 5.18  | 4.31  | 3.83  | 3.51  | 3.29  | 3.12  | 2.99  | 2.89  | 2.80           | 2.66                                       | 2.56                                       | 2.48                                       | 2.42                                       | 2.37 |
| 50                       | 7.17  | 5.06  | 4.20  | 3.72  | 3.41  | 3.19  | 3.02  | 2.89  | 2.78  | 2.70           | 2.56                                       | 2.46                                       | 2.38                                       | 2.32                                       | 2.27 |
| 60                       | 7.08  | 4.98  | 4.13  | 3.65  | 3.34  | 3.12  | 2.95  | 2.82  | 2.72  | 2.63           | 2.50                                       | 2.39                                       | 2.31                                       | 2.25                                       | 2.20 |
| 70                       | 7.01  | 4.92  | 4.07  | 3.60  | 3.29  | 3.07  | 2.91  | 2.78  | 2.67  | 2.59           | 2.45                                       | 2.35                                       | 2.27                                       | 2.20                                       | 2.15 |
| 80                       | 6.96  | 4.88  | 4.04  | 3.56  | 3.26  | 3.04  | 2.87  | 2.74  | 2.64  | 2.55           | 2.42                                       | 2.31                                       | 2.23                                       | 2.17                                       | 2.12 |
| 90                       | 6.93  | 4.85  | 4.01  | 3.53  | 3.23  | 3.01  | 2.84  | 2.72  | 2.61  | 2.52           | 2.39                                       | 2.29                                       | 2.21                                       | 2.14                                       | 2.09 |
| 100                      | 6.90  | 4.82  | 3.98  | 3.51  | 3.21  | 2.99  | 2.82  | 2.69  | 2.59  | 2.50           | 2.37                                       | 2.27                                       | 2.19                                       | 2.12                                       | 2.07 |
| 120                      | 6.85  | 4.79  | 3.95  | 3.48  | 3.17  | 2.96  | 2.79  | 2.66  | 2.56  | 2.47           | 2.34                                       | 2.23                                       | 2.15                                       | 2.09                                       | 2.03 |
| 150                      | 6.81  | 4.75  | 3.91  | 3.45  | 3.14  | 2.92  | 2.76  | 2.63  | 2.53  | 2.44           | 2.31                                       | 2.20                                       | 2.12                                       | 2.06                                       | 2.00 |
| 200                      | 6.76  | 4.71  | 3.88  | 3.41  | 3.11  | 2.89  | 2.73  | 2.60  | 2.50  | 2.41           | 2.27                                       | 2.17                                       | 2.09                                       | 2.03                                       | 1.97 |
| 250                      | 6.74  | 4.69  | 3.86  | 3.40  | 3.09  | 2.87  | 2.71  | 2.58  | 2.48  | 2.39           | 2.26                                       | 2.15                                       | 2.07                                       | 2.01                                       | 1.95 |
| 300                      | 6.72  | 4.68  | 3.85  | 3.38  | 3.08  | 2.86  | 2.70  | 2.57  | 2.47  | 2.38           | 2.24                                       | 2.14                                       | 2.06                                       | 1.99                                       | 1.94 |
| 400                      | 6.70  | 4.66  | 3.83  | 3.37  | 3.06  | 2.85  | 2.68  | 2.56  | 2.45  | 2.37           | 2.23                                       | 2.13                                       | 2.05                                       | 1.98                                       | 1.92 |
| 500                      | 6.69  | 4.65  | 3.82  | 3.36  | 3.05  | 2.84  | 2.68  | 2.55  | 2.44  | 2.36           | 2.22                                       | 2.12                                       | 2.04                                       | 1.97                                       | 1.92 |
| 600                      | 6.68  | 4.64  | 3.81  | 3.35  | 3.05  | 2.83  | 2.67  | 2.54  | 2.44  | 2.35           | 2.21                                       | 2.11                                       | 2.03                                       | 1.96                                       | 1.91 |
| 750                      | 6.67  | 4.63  | 3.81  | 3.34  | 3.04  | 2.83  | 2.66  | 2.53  | 2.43  | 2.34           | 2.21                                       | 2.11                                       | 2.02                                       | 1.96                                       | 1.90 |
| 1000                     | 6.66  | 4.63  | 3.80  | 3.34  | 3.04  | 2.82  | 2.66  | 2.53  | 2.43  | 2.34           | 2.20                                       | 2.10                                       | 2.02                                       | 1.95                                       | 1.90 |

Table A.3 (continued)

# F Distribution: Critical Values of F (1% significance level)

| <i>v</i> <sub>1</sub>           | 25   | 30   | 35                                   | 40                                   | 50                                   | 60                                   | 75                                   | 100                                  | 150                                  | 200                                  |
|---------------------------------|--|--|--------------------------------------|--------------------------------------|--------------------------------------|--------------------------------------|--------------------------------------|--------------------------------------|--------------------------------------|--------------------------------------|
| v <sub>2</sub> 1 2 3 4 5        | 6239.83<br>99.46<br>26.58<br>13.91<br>9.45 | 6260.65<br>99.47<br>26.50<br>13.84<br>9.38 | 99.47<br>26.45<br>13.79              | 99.47<br>26.41                       | 99.48<br>26.35<br>13.69              | 99.48<br>26.32                       | 26.28                                | 99.49<br>26.24<br>13.58              | 99.49<br>26.20                       | 99.49<br>26.18<br>13.52              |
| 6<br>7<br>8<br>9<br>10          | 7.30<br>6.06<br>5.26<br>4.71<br>4.31       | 7.23<br>5.99<br>5.20<br>4.65<br>4.25       | 5.15<br>4.60                         | 5.12                                 | 5.86<br>5.07                         | 7.06<br>5.82<br>5.03<br>4.48<br>4.08 | 5.00                                 | 4.96<br>4.41                         | 5.72                                 | 4.91                                 |
| 11<br>12<br>13<br>14<br>15      | 4.01<br>3.76<br>3.57<br>3.41<br>3.28       | 3.94<br>3.70<br>3.51<br>3.35<br>3.21       | 3.65<br>3.46                         | 3.43<br>3.27                         | 3.81<br>3.57<br>3.38<br>3.22<br>3.08 | 3.78<br>3.54<br>3.34<br>3.18<br>3.05 | 3.31<br>3.15                         | 3.47<br>3.27                         | 3.24<br>3.08                         | 3.06                                 |
| 16<br>17<br>18<br>19<br>20      | 3.16<br>3.07<br>2.98<br>2.91<br>2.84       | 3.10<br>3.00<br>2.92<br>2.84<br>2.78       | 2.96<br>2.87<br>2.80                 | 2.84<br>2.76                         | 2.71                                 |                                      |                                      | 2.76<br>2.68<br>2.60                 | 2.64                                 |                                      |
| 21<br>22<br>23<br>24<br>25      | 2.79<br>2.73<br>2.69<br>2.64<br>2.60       | 2.72<br>2.67<br>2.62<br>2.58<br>2.54       | 2.62<br>2.57<br>2.53                 | 2.58<br>2.54<br>2.49                 | 2.53<br>2.48<br>2.44                 |                                      | 2.46<br>2.41<br>2.37                 | 2.42<br>2.37<br>2.33                 | 2.38<br>2.34<br>2.29                 | 2.36<br>2.32                         |
| 26<br>27<br>28<br>29<br>30      | 2.57<br>2.54<br>2.51<br>2.48<br>2.45       | 2.50<br>2.47<br>2.44<br>2.41<br>2.39       | 2.42<br>2.39<br>2.36                 | 2.35                                 | 2.36<br>2.33<br>2.30<br>2.27<br>2.25 | 2.26                                 | 2.23                                 | 2.22<br>2.19<br>2.16                 | 2.15                                 | 2.19<br>2.16<br>2.13<br>2.10<br>2.07 |
| 35<br>40<br>50<br>60<br>70      | 2.35<br>2.27<br>2.17<br>2.10<br>2.05       | 2.28<br>2.20<br>2.10<br>2.03<br>1.98       | 2.15<br>2.05                         | 2.01                                 | 2.06<br>1.95<br>1.88                 | 2.10<br>2.02<br>1.91<br>1.84<br>1.78 | 1.98<br>1.87                         | 1.94<br>1.82<br>1.75                 | 1.90                                 | 1.76                                 |
| 80<br>90<br>100<br>120<br>150   | 2.01<br>1.99<br>1.97<br>1.93<br>1.90       | 1.94<br>1.92<br>1.89<br>1.86<br>1.83       | 1.89<br>1.86<br>1.84<br>1.81<br>1.77 | 1.85<br>1.82<br>1.80<br>1.76<br>1.73 | 1.79<br>1.76<br>1.74<br>1.70<br>1.66 |                                      | 1.65                                 | 1.65<br>1.62<br>1.60<br>1.56<br>1.52 | 1.61<br>1.57<br>1.55<br>1.51<br>1.46 | 1.58<br>1.55<br>1.52<br>1.48<br>1.43 |
| 200<br>250<br>300<br>400<br>500 | 1.87<br>1.85<br>1.84<br>1.82<br>1.81       | 1.79<br>1.77<br>1.76<br>1.75<br>1.74       | 1.74<br>1.72<br>1.70<br>1.69<br>1.68 | 1.69<br>1.67<br>1.66<br>1.64<br>1.63 | 1.63<br>1.61<br>1.59<br>1.58<br>1.57 | 1.58<br>1.56<br>1.55<br>1.53<br>1.52 | 1.53<br>1.51<br>1.50<br>1.48<br>1.47 | 1.48<br>1.46<br>1.44<br>1.42<br>1.41 | 1.42<br>1.40<br>1.38<br>1.36<br>1.34 | 1.39<br>1.36<br>1.35<br>1.32<br>1.31 |
| 600<br>750<br>1000              | 1.80<br>1.80<br>1.79                       | 1.73<br>1.72<br>1.72                       | 1.67<br>1.66<br>1.66                 | 1.63<br>1.62<br>1.61                 | 1.56<br>1.55<br>1.54                 | 1.51<br>1.50<br>1.50                 | 1.46<br>1.45<br>1.44                 | 1.40<br>1.39<br>1.38                 | 1.34<br>1.33<br>1.32                 | 1.30<br>1.29<br>1.28                 |

Table A.3 (continued)

## F Distribution: Critical Values of F (0.1% significance level)

| <i>v</i> <sub>1</sub> | 1                | 2                | 3                         | 4                         | 5                | 6                         | 7                         | 8                | 9                         | 10                        | 12  | 14               | 16               | 18     | 20               |
|-----------------------|------------------|------------------|---------------------------|---------------------------|------------------|---------------------------|---------------------------|------------------|---------------------------|---------------------------|---|------------------|------------------|--------|------------------|
|                       | 998.50<br>167.03 | 999.00<br>148.50 | 999.17<br>141.11<br>56.18 | 999.25<br>137.10<br>53.44 | 999.30<br>134.58 | 999.33<br>132.85<br>50.53 | 999.36<br>131.58<br>49.66 | 999.37<br>130.62 | 999.39<br>129.86<br>48.47 | 999.40<br>129.25<br>48.05 | 6.11e05<br>999.42<br>128.32<br>47.41<br>26.42 | 999.43<br>127.64 | 999.44<br>127.14 | 999.44 | 999.45<br>126.42 |
| 6                     | 35.51            | 27.00            | 23.70                     | 21.92                     | 20.80            | 20.03                     | 19.46                     | 19.03            | 18.69                     | 18.41                     | 17.99   | 17.68            | 17.45            | 17.27  | 17.12            |
| 7                     | 29.25            | 21.69            | 18.77                     | 17.20                     | 16.21            | 15.52                     | 15.02                     | 14.63            | 14.33                     | 14.08                     | 13.71   | 13.43            | 13.23            | 13.06  | 12.93            |
| 8                     | 25.41            | 18.49            | 15.83                     | 14.39                     | 13.48            | 12.86                     | 12.40                     | 12.05            | 11.77                     | 11.54                     | 11.19   | 10.94            | 10.75            | 10.60  | 10.48            |
| 9                     | 22.86            | 16.39            | 13.90                     | 12.56                     | 11.71            | 11.13                     | 10.70                     | 10.37            | 10.11                     | 9.89                      | 9.57  | 9.33             | 9.15             | 9.01   | 8.90             |
| 10                    | 21.04            | 14.91            | 12.55                     | 11.28                     | 10.48            | 9.93                      | 9.52                      | 9.20             | 8.96                      | 8.75                      | 8.45  | 8.22             | 8.05             | 7.91   | 7.80             |
| 11                    | 19.69            | 13.81            | 11.56                     | 10.35                     | 9.58             | 9.05                      | 8.66                      | 8.35             | 8.12                      | 7.92                      | 7.63  | 7.41             | 7.24             | 7.11   | 7.01             |
| 12                    | 18.64            | 12.97            | 10.80                     | 9.63                      | 8.89             | 8.38                      | 8.00                      | 7.71             | 7.48                      | 7.29                      | 7.00  | 6.79             | 6.63             | 6.51   | 6.40             |
| 13                    | 17.82            | 12.31            | 10.21                     | 9.07                      | 8.35             | 7.86                      | 7.49                      | 7.21             | 6.98                      | 6.80                      | 6.52  | 6.31             | 6.16             | 6.03   | 5.93             |
| 14                    | 17.14            | 11.78            | 9.73                      | 8.62                      | 7.92             | 7.44                      | 7.08                      | 6.80             | 6.58                      | 6.40                      | 6.13  | 5.93             | 5.78             | 5.66   | 5.56             |
| 15                    | 16.59            | 11.34            | 9.34                      | 8.25                      | 7.57             | 7.09                      | 6.74                      | 6.47             | 6.26                      | 6.08                      | 5.81  | 5.62             | 5.46             | 5.35   | 5.25             |
| 16                    | 16.12            | 10.97            | 9.01                      | 7.94                      | 7.27             | 6.80                      | 6.46                      | 6.19             | 5.98                      | 5.81                      | 5.55  | 5.35             | 5.20             | 5.09   | 4.99             |
| 17                    | 15.72            | 10.66            | 8.73                      | 7.68                      | 7.02             | 6.56                      | 6.22                      | 5.96             | 5.75                      | 5.58                      | 5.32  | 5.13             | 4.99             | 4.87   | 4.78             |
| 18                    | 15.38            | 10.39            | 8.49                      | 7.46                      | 6.81             | 6.35                      | 6.02                      | 5.76             | 5.56                      | 5.39                      | 5.13  | 4.94             | 4.80             | 4.68   | 4.59             |
| 19                    | 15.08            | 10.16            | 8.28                      | 7.27                      | 6.62             | 6.18                      | 5.85                      | 5.59             | 5.39                      | 5.22                      | 4.97  | 4.78             | 4.64             | 4.52   | 4.43             |
| 20                    | 14.82            | 9.95             | 8.10                      | 7.10                      | 6.46             | 6.02                      | 5.69                      | 5.44             | 5.24                      | 5.08                      | 4.82  | 4.64             | 4.49             | 4.38   | 4.29             |
| 21                    | 14.59            | 9.77             | 7.94                      | 6.95                      | 6.32             | 5.88                      | 5.56                      | 5.31             | 5.11                      | 4.95                      | 4.70  | 4.51             | 4.37             | 4.26   | 4.17             |
| 22                    | 14.38            | 9.61             | 7.80                      | 6.81                      | 6.19             | 5.76                      | 5.44                      | 5.19             | 4.99                      | 4.83                      | 4.58  | 4.40             | 4.26             | 4.15   | 4.06             |
| 23                    | 14.20            | 9.47             | 7.67                      | 6.70                      | 6.08             | 5.65                      | 5.33                      | 5.09             | 4.89                      | 4.73                      | 4.48  | 4.30             | 4.16             | 4.05   | 3.96             |
| 24                    | 14.03            | 9.34             | 7.55                      | 6.59                      | 5.98             | 5.55                      | 5.23                      | 4.99             | 4.80                      | 4.64                      | 4.39  | 4.21             | 4.07             | 3.96   | 3.87             |
| 25                    | 13.88            | 9.22             | 7.45                      | 6.49                      | 5.89             | 5.46                      | 5.15                      | 4.91             | 4.71                      | 4.56                      | 4.31  | 4.13             | 3.99             | 3.88   | 3.79             |
| 26                    | 13.74            | 9.12             | 7.36                      | 6.41                      | 5.80             | 5.38                      | 5.07                      | 4.83             | 4.64                      | 4.48                      | 4.24  | 4.06             | 3.92             | 3.81   | 3.72             |
| 27                    | 13.61            | 9.02             | 7.27                      | 6.33                      | 5.73             | 5.31                      | 5.00                      | 4.76             | 4.57                      | 4.41                      | 4.17  | 3.99             | 3.86             | 3.75   | 3.66             |
| 28                    | 13.50            | 8.93             | 7.19                      | 6.25                      | 5.66             | 5.24                      | 4.93                      | 4.69             | 4.50                      | 4.35                      | 4.11  | 3.93             | 3.80             | 3.69   | 3.60             |
| 29                    | 13.39            | 8.85             | 7.12                      | 6.19                      | 5.59             | 5.18                      | 4.87                      | 4.64             | 4.45                      | 4.29                      | 4.05  | 3.88             | 3.74             | 3.63   | 3.54             |
| 30                    | 13.29            | 8.77             | 7.05                      | 6.12                      | 5.53             | 5.12                      | 4.82                      | 4.58             | 4.39                      | 4.24                      | 4.00  | 3.82             | 3.69             | 3.58   | 3.49             |
| 35                    | 12.90            | 8.47             | 6.79                      | 5.88                      | 5.30             | 4.89                      | 4.59                      | 4.36             | 4.18                      | 4.03                      | 3.79  | 3.62             | 3.48             | 3.38   | 3.29             |
| 40                    | 12.61            | 8.25             | 6.59                      | 5.70                      | 5.13             | 4.73                      | 4.44                      | 4.21             | 4.02                      | 3.87                      | 3.64  | 3.47             | 3.34             | 3.23   | 3.14             |
| 50                    | 12.22            | 7.96             | 6.34                      | 5.46                      | 4.90             | 4.51                      | 4.22                      | 4.00             | 3.82                      | 3.67                      | 3.44  | 3.27             | 3.41             | 3.04   | 2.95             |
| 60                    | 11.97            | 7.77             | 6.17                      | 5.31                      | 4.76             | 4.37                      | 4.09                      | 3.86             | 3.69                      | 3.54                      | 3.32  | 3.15             | 3.02             | 2.91   | 2.83             |
| 70                    | 11.80            | 7.64             | 6.06                      | 5.20                      | 4.66             | 4.28                      | 3.99                      | 3.77             | 3.60                      | 3.45                      | 3.23  | 3.06             | 2.93             | 2.83   | 2.74             |
| 80                    | 11.67            | 7.54             | 5.97                      | 5.12                      | 4.58             | 4.20                      | 3.92                      | 3.70             | 3.53                      | 3.39                      | 3.16  | 3.00             | 2.87             | 2.76   | 2.68             |
| 90                    | 11.57            | 7.47             | 5.91                      | 5.06                      | 4.53             | 4.15                      | 3.87                      | 3.65             | 3.48                      | 3.34                      | 3.11  | 2.95             | 2.82             | 2.71   | 2.63             |
| 100                   | 11.50            | 7.41             | 5.86                      | 5.02                      | 4.48             | 4.11                      | 3.83                      | 3.61             | 3.44                      | 3.30                      | 3.07  | 2.91             | 2.78             | 2.68   | 2.59             |
| 120                   | 11.38            | 7.32             | 5.78                      | 4.95                      | 4.42             | 4.04                      | 3.77                      | 3.55             | 3.38                      | 3.24                      | 3.02  | 2.85             | 2.72             | 2.62   | 2.53             |
| 150                   | 11.27            | 7.24             | 5.71                      | 4.88                      | 4.35             | 3.98                      | 3.71                      | 3.49             | 3.32                      | 3.18                      | 2.96  | 2.80             | 2.67             | 2.56   | 2.48             |
| 200                   | 11.15            | 7.15             | 5.63                      | 4.81                      | 4.29             | 3.92                      | 3.65                      | 3.43             | 3.26                      | 3.12                      | 2.90  | 2.74             | 2.61             | 2.51   | 2.42             |
| 250                   | 11.09            | 7.10             | 5.59                      | 4.77                      | 4.25             | 3.88                      | 3.61                      | 3.40             | 3.23                      | 3.09                      | 2.87  | 2.71             | 2.58             | 2.48   | 2.39             |
| 300                   | 11.04            | 7.07             | 5.56                      | 4.75                      | 4.22             | 3.86                      | 3.59                      | 3.38             | 3.21                      | 3.07                      | 2.85  | 2.69             | 2.56             | 2.46   | 2.37             |
| 400                   | 10.99            | 7.03             | 5.53                      | 4.71                      | 4.19             | 3.83                      | 3.56                      | 3.35             | 3.18                      | 3.04                      | 2.82  | 2.66             | 2.53             | 2.43   | 2.34             |
| 500                   | 10.96            | 7.00             | 5.51                      | 4.69                      | 4.18             | 3.81                      | 3.54                      | 3.33             | 3.16                      | 3.02                      | 2.81  | 2.64             | 2.52             | 2.41   | 2.33             |
| 600                   | 10.94            | 6.99             | 5.49                      | 4.68                      | 4.16             | 3.80                      | 3.53                      | 3.32             | 3.15                      | 3.01                      | 2.80  | 2.63             | 2.51             | 2.40   | 2.32             |
| 750                   | 10.91            | 6.97             | 5.48                      | 4.67                      | 4.15             | 3.79                      | 3.52                      | 3.31             | 3.14                      | 3.00                      | 2.78  | 2.62             | 2.49             | 2.39   | 2.31             |
| 1000                  | 10.89            | 6.96             | 5.46                      | 4.65                      | 4.14             | 3.78                      | 3.51                      | 3.30             | 3.13                      | 2.99                      | 2.77  | 2.61             | 2.48             | 2.38   | 2.30             |

Table A.3 (continued)

## F Distribution: Critical Values of F (0.1% significance level)

|     | 25    | 30  | 35     | 40     | 50     | 60               | 75     | 100    | 150              | 200   |
|-----|-------|---|--------|--------|--------|------------------|--------|--------|------------------|-------|
|     |       | 6.26e05<br>999.47<br>125.45<br>45.43<br>24.87 | 999.47 | 124.96 | 999.48 | 999.48<br>124.47 | 999.49 | 999.49 | 999.49<br>123.87 |       |
| 6   | 16.85 | 16.67   | 16.54  | 16.44  | 16.31  | 16.21            | 16.12  | 16.03  | 15.93            | 15.89 |
| 7   | 12.69 | 12.53   | 12.41  | 12.33  | 12.20  | 12.12            | 12.04  | 11.95  | 11.87            | 11.82 |
| 8   | 10.26 | 10.11   | 10.00  | 9.92   | 9.80   | 9.73             | 9.65   | 9.57   | 9.49             | 9.45  |
| 9   | 8.69  | 8.55  | 8.46   | 8.37   | 8.26   | 8.19             | 8.11   | 8.04   | 7.96             | 7.93  |
| 10  | 7.60  | 7.47  | 7.37   | 7.30   | 7.19   | 7.12             | 7.05   | 6.98   | 6.91             | 6.87  |
| 11  | 6.81  | 6.68  | 6.59   | 6.52   | 6.42   | 6.35             | 6.28   | 6.21   | 6.14             | 6.10  |
| 12  | 6.22  | 6.09  | 6.00   | 5.93   | 5.83   | 5.76             | 5.70   | 5.63   | 5.56             | 5.52  |
| 13  | 5.75  | 5.63  | 5.54   | 5.47   | 5.37   | 5.30             | 5.24   | 5.17   | 5.10             | 5.07  |
| 14  | 5.38  | 5.25  | 5.17   | 5.10   | 5.00   | 4.94             | 4.87   | 4.81   | 4.74             | 4.71  |
| 15  | 5.07  | 4.95  | 4.86   | 4.80   | 4.70   | 4.64             | 4.57   | 4.51   | 4.44             | 4.41  |
| 16  | 4.82  | 4.70  | 4.61   | 4.54   | 4.45   | 4.39             | 4.32   | 4.26   | 4.19             | 4.16  |
| 17  | 4.60  | 4.48  | 4.40   | 4.33   | 4.24   | 4.18             | 4.11   | 4.05   | 3.98             | 3.95  |
| 18  | 4.42  | 4.30  | 4.22   | 4.15   | 4.06   | 4.00             | 3.93   | 3.87   | 3.80             | 3.77  |
| 19  | 4.26  | 4.14  | 4.06   | 3.99   | 3.90   | 3.84             | 3.78   | 3.71   | 3.65             | 3.61  |
| 20  | 4.12  | 4.00  | 3.92   | 3.86   | 3.77   | 3.70             | 3.64   | 3.58   | 3.51             | 3.48  |
| 21  | 4.00  | 3.88  | 3.80   | 3.74   | 3.64   | 3.58             | 3.52   | 3.46   | 3.39             | 3.36  |
| 22  | 3.89  | 3.78  | 3.70   | 3.63   | 3.54   | 3.48             | 3.41   | 3.35   | 3.28             | 3.25  |
| 23  | 3.79  | 3.68  | 3.60   | 3.53   | 3.44   | 3.38             | 3.32   | 3.25   | 3.19             | 3.16  |
| 24  | 3.71  | 3.59  | 3.51   | 3.45   | 3.36   | 3.29             | 3.23   | 3.17   | 3.10             | 3.07  |
| 25  | 3.63  | 3.52  | 3.43   | 3.37   | 3.28   | 3.22             | 3.15   | 3.09   | 3.03             | 2.99  |
| 26  | 3.56  | 3.44  | 3.36   | 3.30   | 3.21   | 3.15             | 3.08   | 3.02   | 2.95             | 2.92  |
| 27  | 3.49  | 3.38  | 3.30   | 3.23   | 3.14   | 3.08             | 3.02   | 2.96   | 2.89             | 2.86  |
| 28  | 3.43  | 3.32  | 3.24   | 3.18   | 3.09   | 3.02             | 2.96   | 2.90   | 2.83             | 2.80  |
| 29  | 3.38  | 3.27  | 3.18   | 3.12   | 3.03   | 2.97             | 2.91   | 2.84   | 2.78             | 2.74  |
| 30  | 3.33  | 3.22  | 3.13   | 3.07   | 2.98   | 2.92             | 2.86   | 2.79   | 2.73             | 2.69  |
| 35  | 3.13  | 3.02  | 2.93   | 2.87   | 2.78   | 2.72             | 2.66   | 2.59   | 2.52             | 2.49  |
| 40  | 2.98  | 2.87  | 2.79   | 2.73   | 2.64   | 2.57             | 2.51   | 2.44   | 2.38             | 2.34  |
| 50  | 2.79  | 2.68  | 2.60   | 2.53   | 2.44   | 2.38             | 2.31   | 2.25   | 2.18             | 2.14  |
| 60  | 2.67  | 2.55  | 2.47   | 2.41   | 2.32   | 2.25             | 2.19   | 2.12   | 2.05             | 2.01  |
| 70  | 2.58  | 2.47  | 2.39   | 2.32   | 2.23   | 2.16             | 2.10   | 2.03   | 1.95             | 1.92  |
| 80  | 2.52  | 2.41  | 2.32   | 2.26   | 2.16   | 2.10             | 2.03   | 1.96   | 1.89             | 1.85  |
| 90  | 2.47  | 2.36  | 2.27   | 2.21   | 2.11   | 2.05             | 1.98   | 1.91   | 1.83             | 1.79  |
| 100 | 2.43  | 2.32  | 2.24   | 2.17   | 2.08   | 2.01             | 1.94   | 1.87   | 1.79             | 1.75  |
| 120 | 2.37  | 2.26  | 2.18   | 2.11   | 2.02   | 1.95             | 1.88   | 1.81   | 1.73             | 1.68  |
| 150 | 2.32  | 2.21  | 2.12   | 2.06   | 1.96   | 1.89             | 1.82   | 1.74   | 1.66             | 1.62  |
| 200 | 2.26  | 2.15  | 2.07   | 2.00   | 1.90   | 1.83             | 1.76   | 1.68   | 1.60             | 1.55  |
| 250 | 2.23  | 2.12  | 2.03   | 1.97   | 1.87   | 1.80             | 1.72   | 1.65   | 1.56             | 1.51  |
| 300 | 2.21  | 2.10  | 2.01   | 1.94   | 1.85   | 1.78             | 1.70   | 1.62   | 1.53             | 1.48  |
| 400 | 2.18  | 2.07  | 1.98   | 1.92   | 1.82   | 1.75             | 1.67   | 1.59   | 1.50             | 1.45  |
| 500 | 2.17  | 2.05  | 1.97   | 1.90   | 1.80   | 1.73             | 1.65   | 1.57   | 1.48             | 1.43  |
| 600 | 2.16  | 2.04  | 1.96   | 1.89   | 1.79   | 1.72             | 1.64   | 1.56   | 1.46             | 1.41  |
| 750 | 2.15  | 2.03  | 1.95   | 1.88   | 1.78   | 1.71             | 1.63   | 1.55   | 1.45             | 1.40  |
| 000 | 2.14  | 2.02  | 1.94   | 1.87   | 1.77   | 1.69             | 1.62   | 1.53   | 1.44             | 1.38  |

Table A.4  $\chi^{2} \mbox{ (Chi-Squared) Distribution: Critical Values of } \chi^{2}$ 

|                       |        | Significance lev | el     |
|-----------------------|--------|------------------|--------|
| Degrees of<br>freedom | 5%     | 1%               | 0.1%   |
| 1                     | 3.841  | 6.635            | 10.828 |
| 2                     | 5.991  | 9.210            | 13.816 |
| 3                     | 7.815  | 11.345           | 16.266 |
| 4                     | 9.488  | 13.277           | 18.467 |
| 5                     | 11.070 | 15.086           | 20.515 |
| 6                     | 12.592 | 16.812           | 22.458 |
| 7                     | 14.067 | 18.475           | 24.322 |
| 8                     | 15.507 | 20.090           | 26.124 |
| 9                     | 16.919 | 21.666           | 27.877 |
| 10                    | 18.307 | 23.209           | 29.588 |