PIECE-WISE LINEAR TRANSFORMATION FUNCTIONS:-

This transformation function is complimentary to the other functions we discussed previously. This function has both adjvantages & disadvantages. It is less complex but it requires more user inputs.

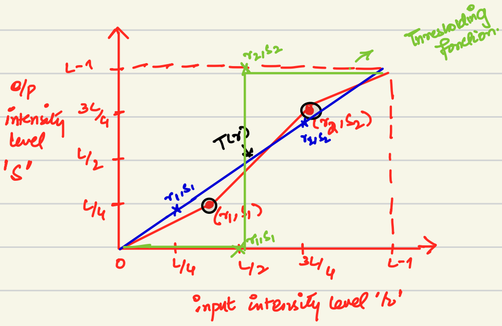
Three different applications:

1. Contrast Stretching.
2. Intensity level slicing.
3. Bit plane slicing.

1. **Contrast Stretching:-** Aquisite image sometimes of very low quality because restriction of the dynamic range of the acquistion system we have used or may be due to poor illumination lightning conditions or may be due to wrong fixing of the aperture. So, those low contrast images should be enhanced.

$\boxed{S = T(r)}$ Transformation function.

$I = [0, L-1]$ Black White



Case I: If $r_1 = r_2$ & $s_1 = s_2$

Identical function, no intensity change.

Case II: if $r_1 = r_2$ & $s_1 = 0$, $s_2 = L-1$

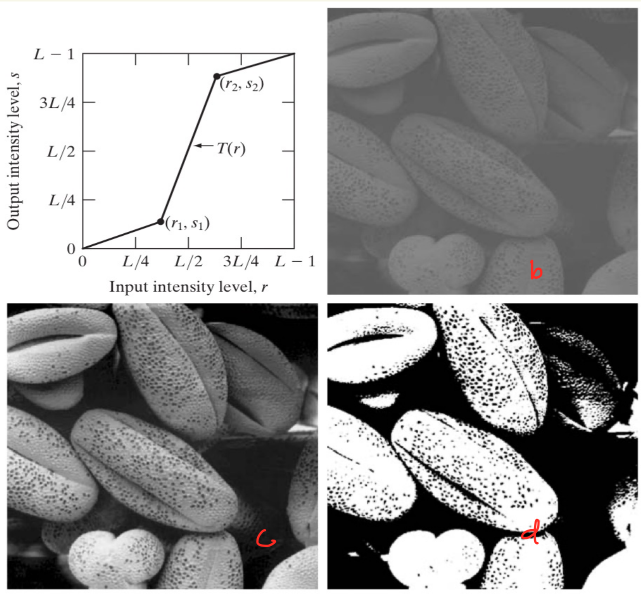
Hard - Thresholding function.

Case III: If $r_1 = r_{min}$ & $r_2 = r_{max}$, $s_1 = 0$, $s_2 = L-1$

contrast stretching

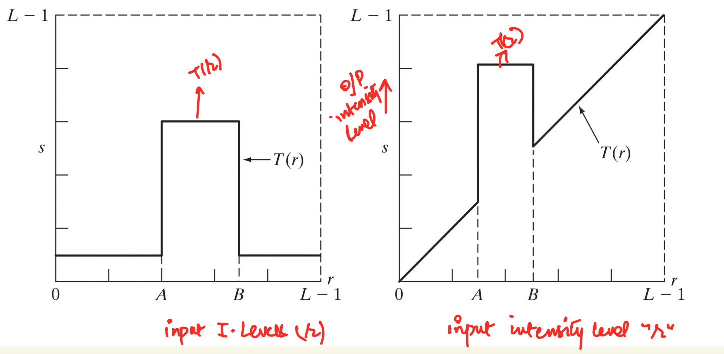
a b
c d

FIGURE 3.10 Contrast stretching. (a) Form of transformation function. (b) A low-contrast image. (c) Result of contrast stretching. (d) Result of thresholding. (Original image courtesy of Dr. Roger Heady, Research School of Biological Sciences, Australian National University, Canberra, Australia.)



Intensity level slicing: ⊙ Used to highlight a particular range of intensity. ⊙ used in medical applications. (x-rays) - Enhance a patch of x-rays where the flaws are present. ⊙ Highlight the range of intensity levels which is of interest. ⊡ Also useful in satellite images.

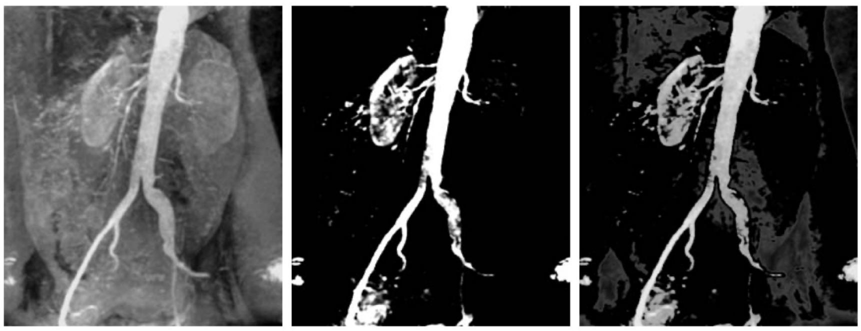
* can be done using two approaches:



a b

FIGURE 3.11 (a) This transformation highlights intensity range $[A, B]$ and reduces all other intensities to a lower level. (b) This transformation highlights range $[A, B]$ and preserves all other intensity levels.

- Both these approaches are useful in the case(s) where we want to extract some object of interest.



a b c

FIGURE 3.12 (a) Aortic angiogram. (b) Result of using a slicing transformation of the type illustrated in Fig. 3.11(a), with the range of intensities of interest selected in the upper end of the gray scale. (c) Result of using the transformation in Fig. 3.11(b), with the selected area set to black, so that grays in the area of the blood vessels and kidneys were preserved. (Original image courtesy of Dr. Thomas R. Gest, University of Michigan Medical School.)

(3) **Bit-plane slicing:** ⊙ Highlight the entire image appearance which is specific to that particular bit plane.

e.g **8-bit image:** $2^8 = 256$

L = [0, 1, 2, 3, 4, 5. . . . 255]

$\boxed{00000010}$

MSB LSB

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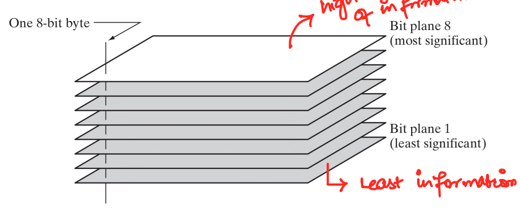
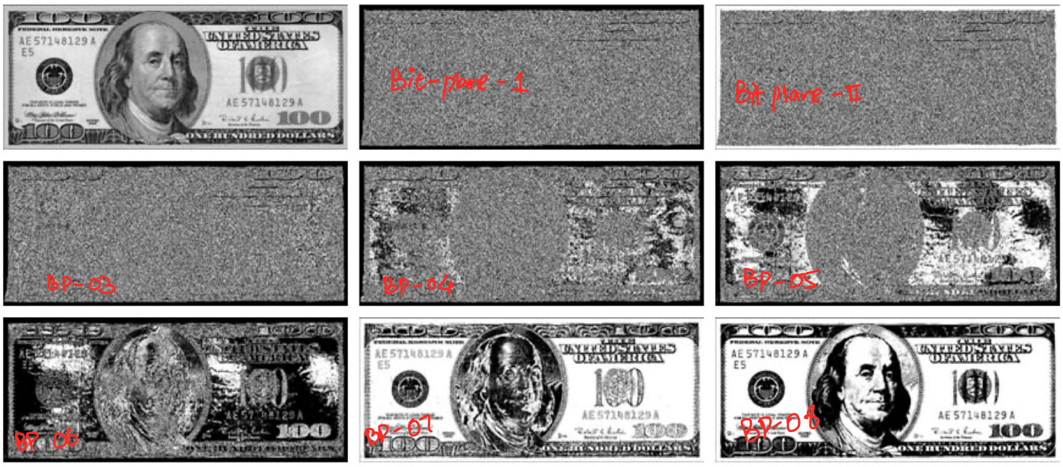
FIGURE 3.13 Bit-plane representation of an 8-bit image.





a b c
d e f
g h i

FIGURE 3.14 (a) An 8-bit gray-scale image of size 500 × 1192 pixels. (b) through (i) Bit planes 1 through 8, with bit plane 1 corresponding to the least significant bit. Each bit plane is a binary image.

- Used in compression techniques. We can avoid lower order bit plane and exploit the higher order bit plane for the reconstruction of the image.

<-->

Histogram Processing:

- Basis of various spatial domain processing technique
- Used to improve the quality of the image
- Used in image compression, enhancement and segmentation.

- $L = [0, L-1]$, Histogram is nothing but the "frequency of occurrence of each gray level in the image"

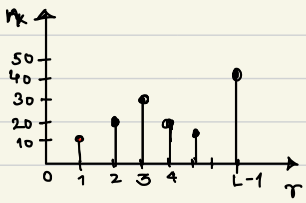
- Graphically represented as two types of plots:

I: x-axis : gray level & y-axis : no. of pixels in the image in each gray level.

$$\boxed{h(r_k) = n_k}$$

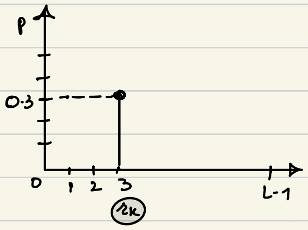
$\therefore$ r_k = k^{th} intensity level

n_k = no. of pixel values in the image with the intensity level.



II : x-axis → gray level & y-axis → Probability of occurrance of pixels in each gray level

$$P(r_k) = \frac{n_k}{MN}$$



$\frac{30}{100} = 0.3$

r_k = k^{th} intensity level

n_k = no. of pixels in the image with intensity level (r_k)

↳ "normalized histogram"

MN = Total no. of pixels in an image.

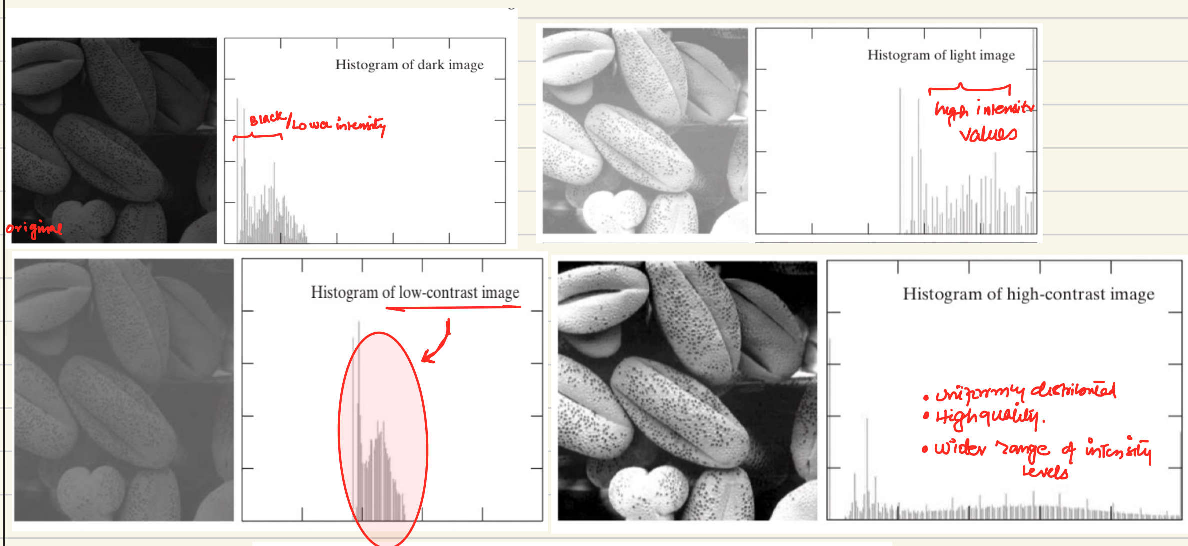
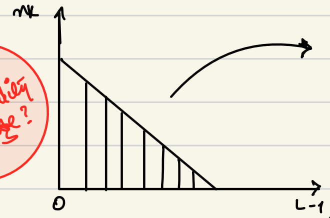


FIGURE 3.16 Four basic image types: dark, light, low contrast, high contrast, and their corresponding histograms.

Histogram Equalization:-

To obtain the flattern histogram, we can use histogram equalization. It is a technique to improve the quality of the image. It helps to obtain the uniformly distributed histogram over the entire intensity levels from 0 to L-1.

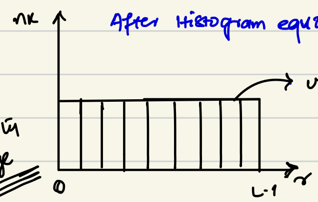
Poor or good quality image?



It's a darker intensity image and of a poor quality.

we need to perform histogram equalization.

After Histogram equalization.



uniform distribution.

good quality image

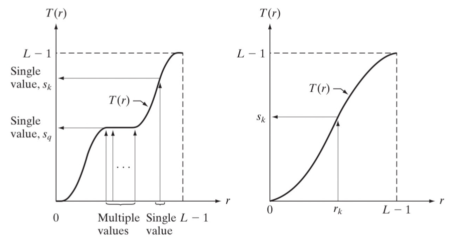
$S = T(r) \longrightarrow (1)$

To transform the intensity levels of the input image to the new intensity levels we usually used the transformation functions e.g. using eq (1). The mapping of "r" to "S" depends on "T". This "T" can be anything like summation etc.

To perform histogram equalizations there are two important conditions:

(i) $T(r)$ must be a single value & monotonically increasing function in nature for the interval $0 \le r \le L-1$.

(ii) $0 \le T(r) \le L-1$ for $0 \le r \le L-1$.



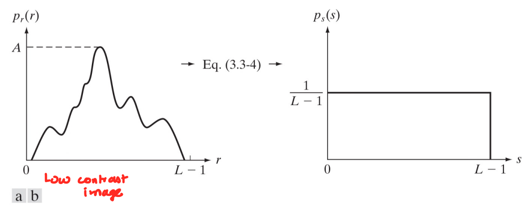
a b

FIGURE 3.17 (a) Monotonically increasing function, showing how multiple values can map to a single value. (b) Strictly monotonically increasing function. This is a one-to-one mapping, both ways.

cummulative result

[†]Recall that a function $T(r)$ is *monotonically increasing* if $T(r_2) \ge T(r_1)$ for $r_2 > r_1$. $T(r)$ is a *strictly monotonically increasing* function if $T(r_2) > T(r_1)$ for $r_2 > r_1$. Similar definitions apply to monotonically decreasing functions.

An image with a continuous intensity values shown in Fig 3.18 a (probably a low contrast image) - a poor quality image requires to flatten the histogram producing a good quality image.



a b

FIGURE 3.18 (a) An arbitrary PDF. (b) Result of applying the transformation in Eq. (3.3-4) to all intensity levels, r. The resulting intensities, s, have a uniform PDF, independently of the form of the PDF of the r's.

$P_r(r)$ is a PDF of "r" → input image

$P_s(s)$ is a PDF of "s" → output (transformed) Image

Using basics of Probability theory

$$P_s(s) = P_r(r)\left|\frac{dr}{ds}\right| \longrightarrow eq (2)$$

Let "T" be a particular transformation

$$T(r) = (L-1)\int_0^r P_r(w)\,dw \longrightarrow (3)$$

where "w" is a dummy variable for integration.

Considering eq ① $S = T(r)$

Differentiating w.r.t 'r' on both sides

$$\frac{ds}{dr} = \frac{dT(r)}{dr}$$

$$\frac{ds}{dr} = \frac{d}{dr}\left[(L-1)\int_0^r P_r(\omega)d\omega\right]$$

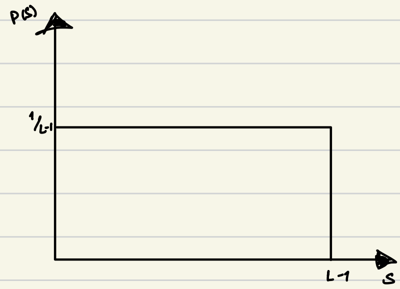
$$= (L-1)\frac{d}{dr}\int_0^r P_r(\omega)d\omega$$

From Leibnitz rule

$$\frac{ds}{dr} = (L-1)P_r(r) \longrightarrow ④$$

Substitute ④ in eq ③ $\quad P_s(s) = \cancel{P_r(r)}\left|\frac{1}{(L-1)\cancel{P_r(r)}}\right|$

$$\boxed{P_s(s) = 1/L-1} - ⑤ \quad 0 \le s \le L-1$$



- Suppose a continuous valued image is considered having

$$P_r(r) = \begin{cases} \frac{2r}{(L-1)^2}, & 0 \le r \le L-1 \\ 0 & \text{otherwise} \end{cases}$$

$$P_s(s) = P_r(r)\left|\frac{dr}{ds}\right|$$

Transform function: $S = T(r)$

Let $T(r) = (L-1)\int_0^r P_r(\omega)d\omega$

$$= (L-1)\int_0^r \frac{2\omega}{(L-1)^2}d\omega \Rightarrow \frac{2\cancel{(L-1)}}{(L-1)^{\cancel{2}}}\int_0^r \omega d\omega$$

$$T(r) = \frac{\cancel{2}}{L-1}\left(\frac{r^2}{\cancel{2}}\right) \Rightarrow \boxed{T(r) = \frac{r^2}{L-1}}$$

Differentiating w.r.t 'r' on both sides

$$\frac{ds}{dr} = \frac{d}{dr}[T(r)]$$

$$= \frac{d}{dr}\left(\frac{r^2}{L-1}\right) \Rightarrow \frac{ds}{dr} = \frac{2r}{L-1} \quad - ⑥$$

Substitute eq ⑥ in eq ② we get

$$P_s(S) = \frac{\cancel{2r}}{(L-1)^{\cancel{2}}}\left|\frac{\cancel{(L-1)}}{\cancel{2r}}\right|$$

$$\boxed{P_s(S) = \frac{1}{L-1}} \rightarrow ⑦$$

The given signal is converted to the good quality image by applying a histogram equalization.

$P_s(s) = \frac{1}{L-1}$ produces a uniform histogram for a range $0 \le s \le L-1$

For Discrete Values :

$$P_r(r_k) = \frac{n_k}{MN} \rightarrow (1)$$

Where $r_k \rightarrow k^{th}$ intensity value, $n_k \rightarrow$ no; of pixels with the intensity value "r_k", and $MN \rightarrow$ Total no; of pixels in an image

$$\boxed{S_k = T(r_k) \rightarrow (2)}$$ Transformation function.

Let $T(r_k) = (L-1) \sum_{j=0}^{k} P_r(r_j) \rightarrow (3)$

Substitute (1) in (3) we get

$$T_r(r_k) = (L-1) \sum_{j=0}^{k} \frac{n_j}{(MN)}$$

$$\boxed{S = T(r_k) = \frac{(L-1)}{MN} \sum_{j=0}^{k} n_j} \rightarrow (4)$$

↳ Histogram equalization or "histogram linearization".

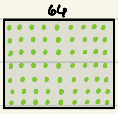
Histogram Equalization :

Example : Perform histogram equalization on a 3-bit image (L=8) of size 64x64 pixels. The intensity distribution of the image is given below :

Given levels (r_k)	0	1	2	3	4	5	6	7
No of pixels (n_k)	790	1023	850	656	329	245	122	81

= 4096

64

64 

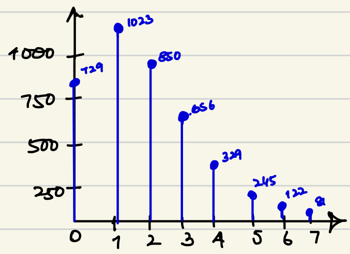
$L = 2^3 = 8$ $[0, L-1]$ i.e., $[0 \dots 7]$

$M = 64, N = 64$

$MN = 64 \times 64 = 4096$ pixels

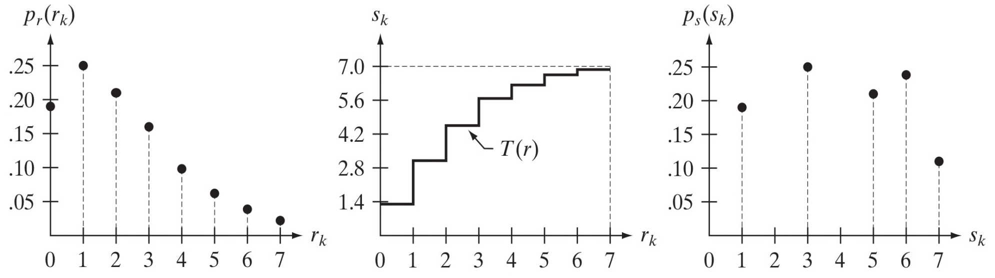
TABLE 3.1 Intensity distribution and histogram values for a 3-bit, 64 × 64 digital image.

r_k	n_k	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02



For discrete intensity values

$$\boxed{S_k = T(r_k) = (L-1) \sum_{j=0}^{k} P_r(r_j)}$$



a b c

FIGURE 3.19 Illustration of histogram equalization of a 3-bit (8 intensity levels) image. (a) Original histogram. (b) Transformation function. (c) Equalized histogram.

For $k=0$, $s_0 = T(r_0) = 7\sum_{j=0}^{0} p_r(r_j) = 7 \cdot p_r(r_0) = 7 \times 0.19 = 1.33$

For $k=1$, $s_1 = T(r_1) = 7\sum_{j=0}^{1} p_r(r_j) = 7(p_r(r_0) + p_r(r_1)) = 7(0.19 + 0.25)$

$= 1.33 + 1.75 = 3.08$

For $k=2$, $s_2 = T(r_2) = 7\sum_{j=0}^{2} p_r(r_j) = 7 \times p_r(r_0) + 7\,p_r(r_1) + 7\,p_r(r_2)$

$= 1.33 + 1.75 + 1.47 = 4.55$

For $k=3$, $s_3 = T(r_3) = 7\sum_{j=0}^{3} p_r(r_j) = 7 \times p_r(r_0) + 7\,p_r(r_1) + 7\,p_r(r_2) + 7\,p_r(r_3)$

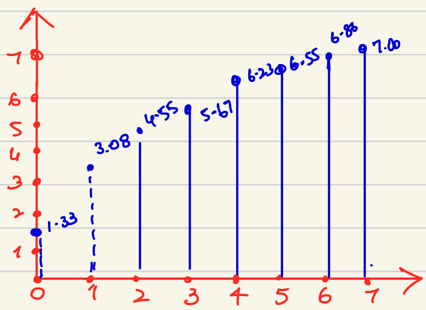
$= 4.55 + 7 \times 0.16 = 5.67$

For $k=4$, $s_4 = T(r_4) = 7\sum_{j=0}^{4} p_r(r_j) = 5.67 + 7 \times 0.08 = 6.23$

For $k=5$, $s_5 = T(r_5) = 7\sum_{j=0}^{5} p_r(r_j) = 6.23 + 7 \times 0.06 = 6.65$

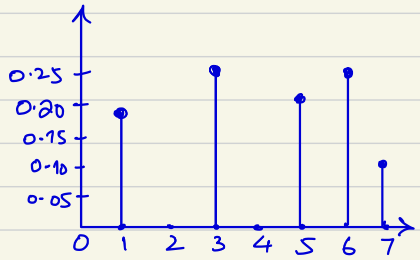
For $k=6$, $s_6 = T(r_6) = 7\sum_{j=0}^{6} p_r(r_j) = 6.65 + 7 \times 0.03 = 6.86$

For $k=7$, $s_7 = T(r_7) = 7\sum_{j=0}^{7} p_r(r_j) = 6.86 + 7 \times 0.02 = 7.00$



• Transformed function.

r_k	n_k	$p_r(r_k) = n_k/MN$	$s_k = T(r_k)$	round off	Histogram equalization $p_s(s_k)$
$r_0 = 0$	790	0.19	1.33	1	0.19
$r_1 = 1$	1023	0.25	3.08	3	0.25
$r_2 = 2$	850	0.21	4.55	5	0.21
$r_3 = 3$	656	0.16	5.67	6	$\frac{(656+329)}{4096} = 0.24$
$r_4 = 4$	329	0.08	6.23	6	
$r_5 = 5$	245	0.06	6.65	7	$\frac{(245+122+81)}{4096} = 0.10$
$r_6 = 6$	122	0.03	6.86	7	
$r_7 = 7$	81	0.02	7.00	7	



Perform the histogram equalization for the following image.

$f(x,y) =$ input image.

$$\begin{bmatrix} 1 & 2 & 1 & 1 & 1 \\ 2 & 5 & 3 & 5 & 2 \\ 2 & 5 & 5 & 5 & 2 \\ 2 & 5 & 3 & 5 & 2 \\ 1 & 1 & 1 & 2 & 1 \end{bmatrix}$$

max value = 5

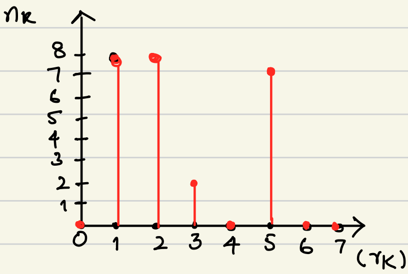
$2^0 = 1$, $2^1 = 2$, $2^2 = 4$

$2^3 = 8$

$L = 8$, $L-1 = 7$

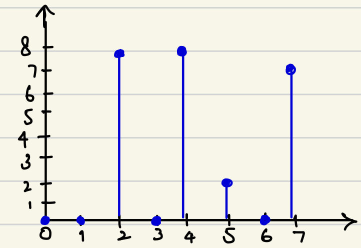
Gray levels (r_k)	0	1	2	3	4	5	6	7
No of pixels (n_k)	0	8	8	2	0	7	0	0

Gray Levels (r_k)	No of pixels (n_k)	$P(r_k) = n_k/n$ (PDF)	S_k (CDF)	S_k	Histogram of Eq. level
0	0	0	0	0	0
1	8	0.32	0.32	2.24	2
2	8	0.32	0.64	4.48	4
3	2	0.08	0.72	5.04	5
4	0	0	0.72	5.04	5
5	7	0.28	1.0	7	7
6	0	0	1.0	7	7
7	0	0	1.0	7	7
	$n = 25$				



Histogram of the input image

Gray levels	0	2	4	5	7
No; of Pixels	0	8	8	2	7



Histogram of the equalized image

$$\begin{bmatrix} 1 & 2 & 1 & 1 & 1 \\ 2 & 5 & 3 & 5 & 2 \\ 2 & 5 & 5 & 5 & 2 \\ 2 & 5 & 3 & 5 & 2 \\ 1 & 1 & 1 & 2 & 1 \end{bmatrix}$$

$$\downarrow$$

output image

$$\begin{bmatrix} 2 & 4 & 2 & 2 & 2 \\ 4 & 7 & 5 & 7 & 4 \\ 4 & 7 & 7 & 7 & 4 \\ 4 & 7 & 5 & 7 & 4 \\ 2 & 2 & 2 & 4 & 2 \end{bmatrix}$$