STEPS FOR FILTERING IN FREQUENCY DOMAIN :

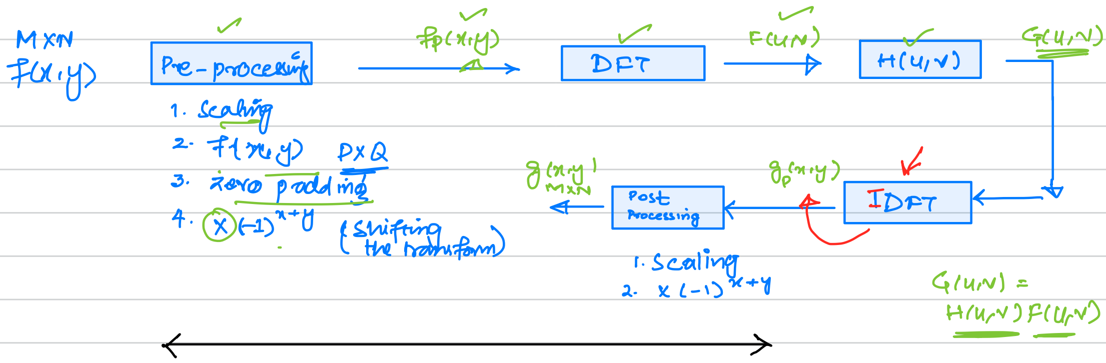
- Given an image $f(x,y)$ of a size $M \times N$, obtain the padding parameters P & Q. Typically ($P = 2M$ & $Q = 2N$)
- Let $f_p(x,y)$ be the zero padded image.
- Form $f_p(x,y)$ of a size $P \times Q$ by appending the required number of zeros to $f(x,y)$.
- Multiply $f_p(x,y)$ by $(-1)^{x+y}$ to center its transform.
- Compute DFT and obtain $F(u,v)$.
- Generate a real symmetric filter function $H(u,v)$ of a size $P \times Q$ with center at co-ordinate $(P/2, Q/2)$. The product obtained is

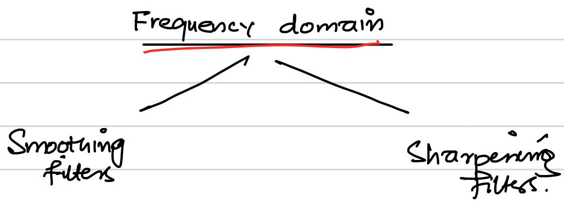
$$G(u,v) = H(u,v)\, F(u,v)$$

- Obtained the processed image :

$$g(x,y) = \{ \text{real} \mid \mathcal{T}^{-1}[G(u,v)] \mid \} (-1)^{x+y}$$

- Obtain final processed image $g(x,y)$ in $M \times N$





Smoothing filters in frequency domain :-

- Edges or sharp transitions like noises should be blurred
- We perform smoothing to obtain this blurness by using "filters"
- Edges / noises / sharp transitions contributes significantly to the high frequency components. We need to remove these high frequency components in order to obtain to smoothen / blur the image.
- We use the filters (To stop High frequencies & allow the low frequencies), such filters are called "low pass filters".

• **LOW-PASS FILTERS** : Passes only the low frequencies and attenuates / block high frequency components ; Thereby providing the blurness / smoothen the image.

LPF

Sharp cut off ← (i) Ideal low pass filter.
order as a parameter, smoothing depends on order. ← (ii) Butterworth LPF &
Smooth transition ← (iii) Gaussian LPF.

} Provides smoothing in various range.

• High order butterworth → ideal filter.
• Low order " " → Gaussian filter.

$H(u,v)$
$\therefore u = 0, 1 \ldots P-1$
$v = 0, 1, \ldots Q-1$

Radially symmetric about the origin ←

(1). **IDEAL LOW PASS FILTER** : The transfer function of ideal low pass filter for 2-D images in frequency domain can be represented as

$$H(u,v) = \begin{cases} 1, & D(u,v) \le D_0 \\ 0, & D(u,v) > D_0 \end{cases} \rightarrow (1)$$

where D_0 is a positive constant, $D(u,v)$ is the distance from the center of the frequency rectangle.

$$D(u,v) = \left[(u - P/2)^2 + (v - P/2)^2 \right]^{1/2} \rightarrow (2)$$

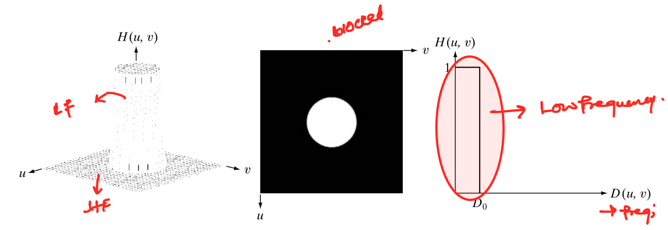


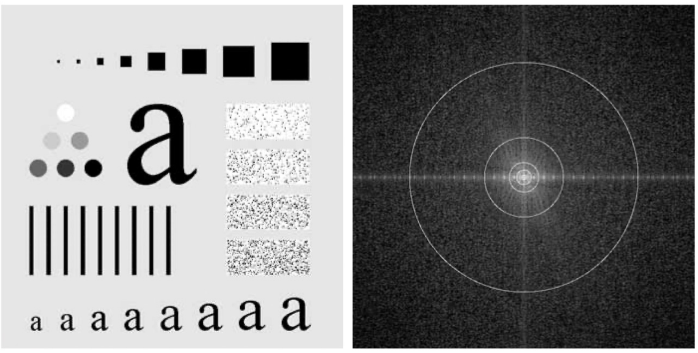
FIGURE 4.40 (a) Perspective plot of an ideal lowpass-filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross section.

To decide the standard cut off frequency, which is allowed to pass through the the LPF. So, we need to consider the specified power which is enclosed in the circle. In order to decide the cut off in Fig 4.40(c) to pass the low frequency components. we have to compute the power which is enclosed in circle shown in Fig 4.40(a). It can be done with the help of "padded image".

$$\boxed{P_T = \sum_{u=0}^{P-1}\sum_{v=0}^{Q-1} P(u,v)} \longrightarrow 3$$

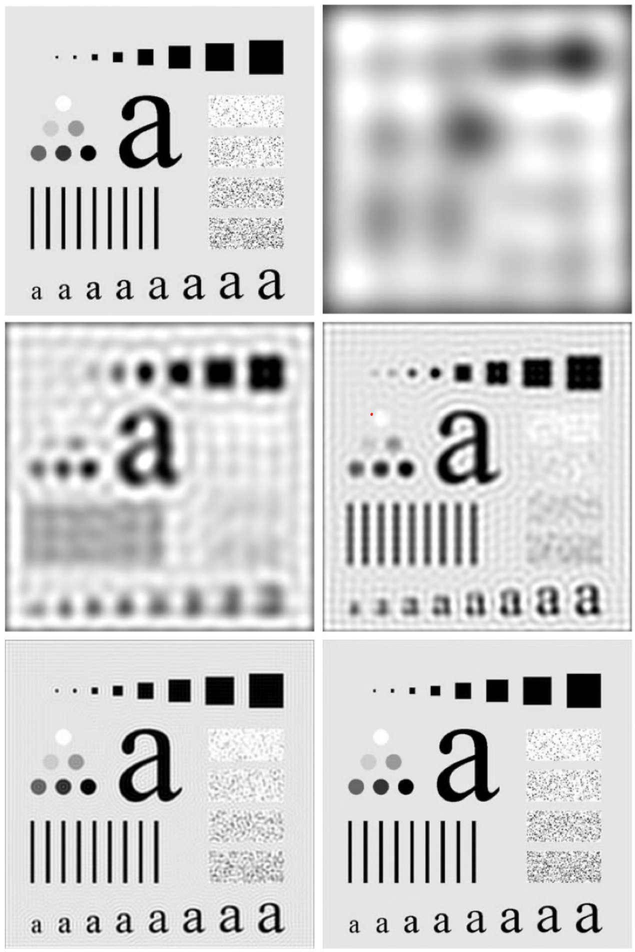
: $u = 0,1,2,\ldots, P-1$
$v = 0,1,2,\ldots, Q-1$

where $P(u,v) = |F(u,v)|^2$
$= R^2(u,v) + I^2(u,v)$.



a b

FIGURE 4.41 (a) Test pattern of size 688 × 688 pixels, and (b) its Fourier spectrum. The spectrum is double the image size due to padding but is shown in half size so that it fits in the page. The superimposed circles have radii equal to 10, 30, 60, 160, and 460 with respect to the full-size spectrum image. These radii enclose 87.0, 93.1, 95.7, 97.8, and 99.2% of the padded image power, respectively.



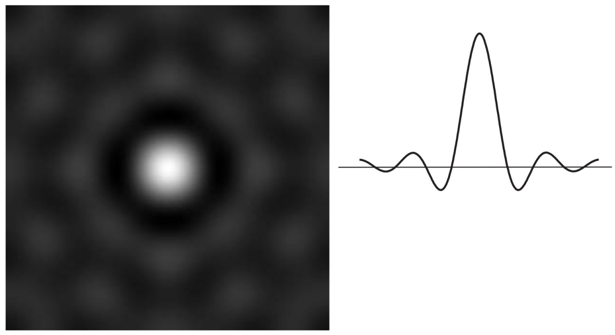
a b
c d
e f

FIGURE 4.42 (a) Original image. (b)–(f) Results of filtering using ILPFs with cutoff frequencies set at radii values 10, 30, 60, 160, and 460, as shown in Fig. 4.41(b). The power removed by these filters was 13, 6.9, 4.3, 2.2, and 0.8% of the total, respectively.

2. **BUTTERWORTH LOW PASS FILTERS:** The transfer function of butterworth LPF of order 'n' and the distance 'D_0' can be represented as

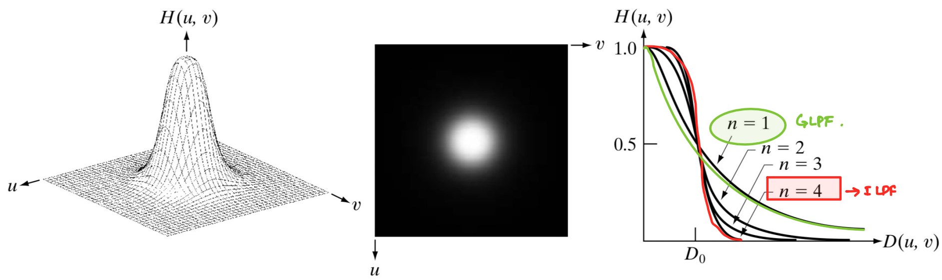
$$\boxed{H(u,v) = \frac{1}{1+\left[\frac{D(u,v)}{D_0}\right]^{2n}}} \rightarrow (1)$$

* The cut-off of this filter depends on order of the filter. If the order is higher then the BWLPF approaches towards the ideal filter and for the lower order it behaves like Gaussian LPF.



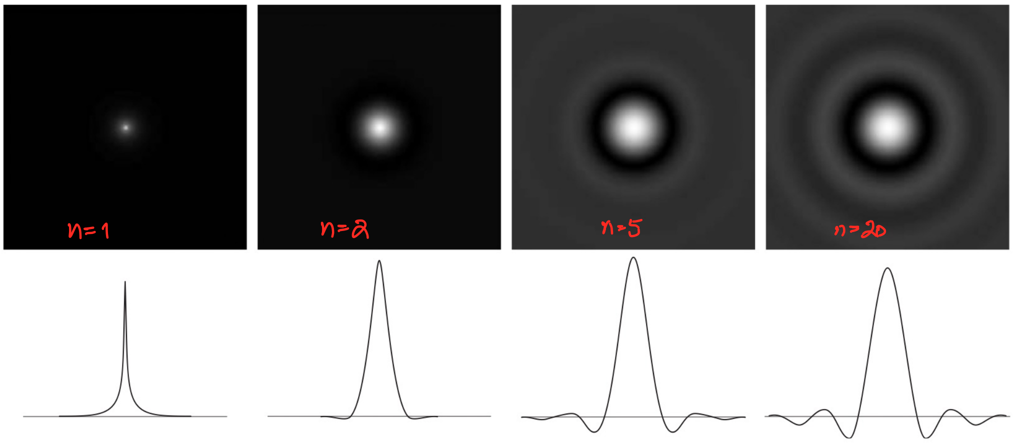
a b

FIGURE 4.43 (a) Representation in the spatial domain of an ILPF of radius 5 and size 1000 × 1000. (b) Intensity profile of a horizontal line passing through the center of the image.



a b c

FIGURE 4.44 (a) Perspective plot of a Butterworth lowpass-filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections of orders 1 through 4.



a b c d

FIGURE 4.46 (a)–(d) Spatial representation of BLPFs of order 1, 2, 5, and 20, and corresponding intensity profiles through the center of the filters (the size in all cases is 1000 × 1000 and the cutoff frequency is 5). Observe how ringing increases as a function of filter order.

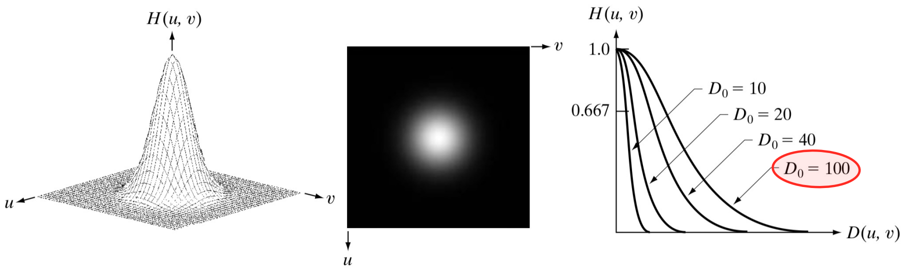
GAUSSIAN LOW PASS FILTER: The transfer function of Gaussian low pass filter for 2-D images in frequency domain is represented as

$$\boxed{H(u,v) = e^{\frac{-D^2(u,v)}{2\delta^2}}} \longrightarrow (1)$$

If $\delta = D_0$ then

$$\boxed{H(u,v) = e^{\frac{-D^2(u,v)}{2D_0^2}}} \longrightarrow (2)$$

↳ Provides smooth transition.



a b c

FIGURE 4.47 (a) Perspective plot of a GLPF transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections for various values of D_0.

Image Sharpening in Frequency domain:

One can acheive image sharpening by using high pass filters (HPF). It is used to block the low frequency components and allow the high frequency components.
We use the discrete filter function i.e., $H(u,v)$ of size $P \times Q$ where $u = 0,1,2, \dots P-1$ & $v = 0,1,2, \dots Q-1$.

$$H_{HP}(u,v) = 1 - H_{LP}(u,v)$$

where $H_{LP}(u,v) \rightarrow$ Transfer function of LPF.

High pass Filters

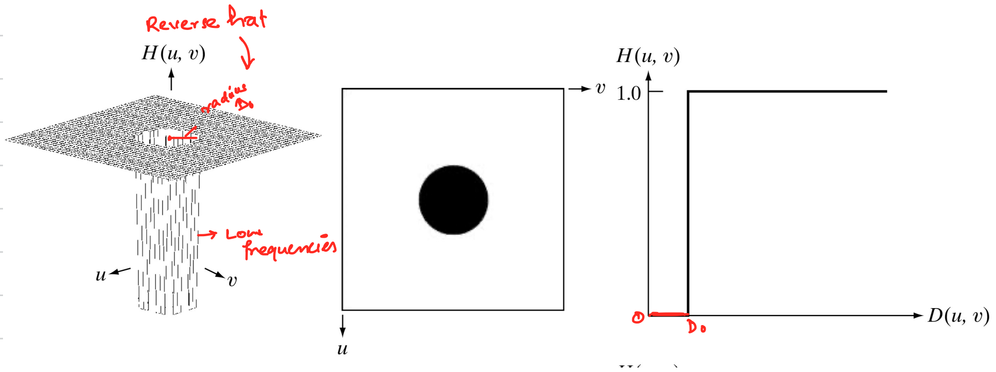
1. Ideal HPF
2. Butterworth HPF
3. Gaussian HPF.

1. **Ideal high pass Filter:** The 2-D function of the ideal HPF can be represented as:

$$H(u,v) = \begin{cases} 0, & \text{if } D(u,v) \leq D_0 \\ 1, & \text{if } D(u,v) > D_0 \end{cases} \longrightarrow (1)$$

where D_0 is the cut-off frequency & $D(u,v)$ is the distance from the center of the frequency rectangle.

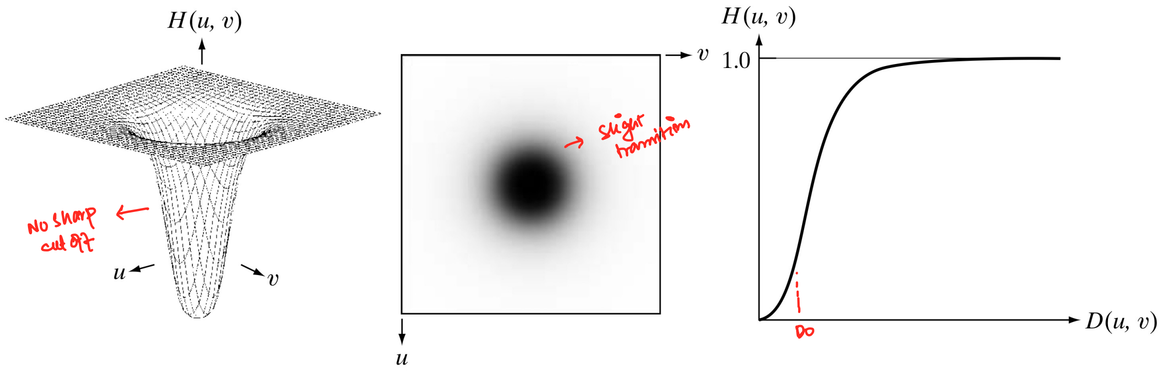
This ideal high pass filter behaves opposite to the ideal low pass filter. This HPF blocks all the frequencies that lies inside the circle (D_0) and allows the rest of the frequencies.



- **Butterworth high pass filter:** The 2-D function of the BWHPL of order "n" and cut-off frequency of "D_0" is represented as:

$$\boxed{H(u,v) = \frac{1}{1 + [D_0/D(u,v)]^{2n}}} \longrightarrow (1)$$

$$D(u,v) = \left[(u-P/2)^2 + (v-Q/2)^2\right]^{1/2}$$



a b c

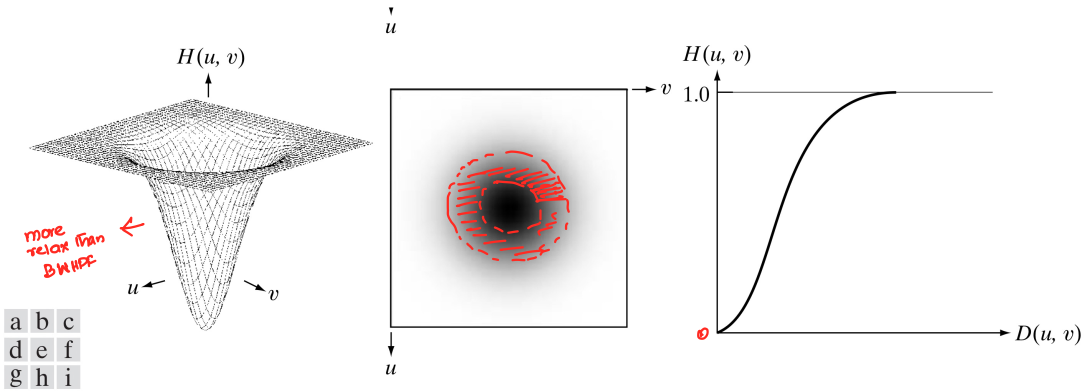
FIGURE 4.54 Results of highpass filtering the image in Fig. 4.41(a) using an IHPF with $D_0 = 30$, 60, and 160.

a b c

FIGURE 4.55 Results of highpass filtering the image in Fig. 4.41(a) using a BHPF of order 2 with $D_0 = 30$, 60, and 160, corresponding to the circles in Fig. 4.41(b). These results are much smoother than those obtained with an IHPF.

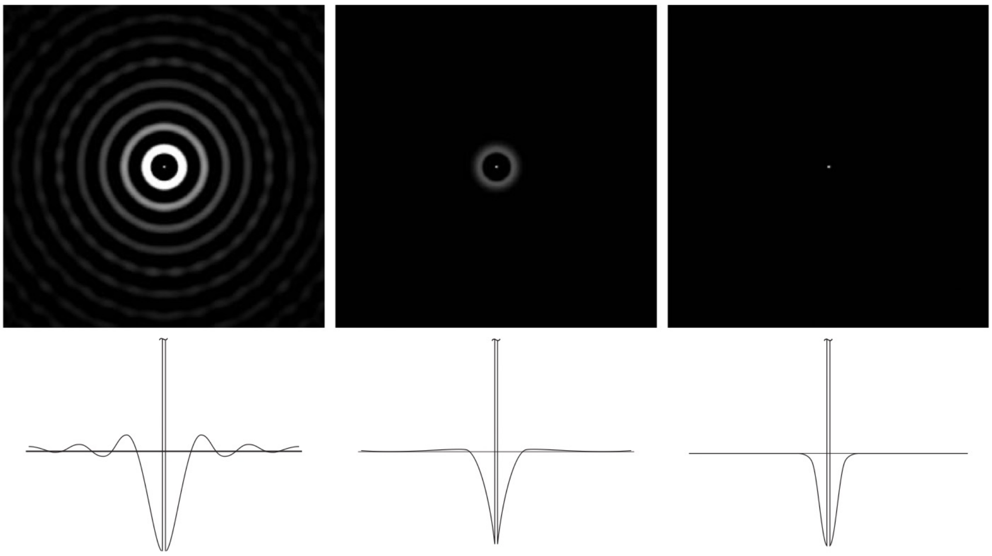
3. Gaussian HPF : The Transfer function of the Gaussian high pass filter is represented by :

$$\boxed{H(u,v) = 1 - e^{-D^2(u,v)/2D_0^2}} \rightarrow (1)$$



a b c

FIGURE 4.56 Results of highpass filtering the image in Fig. 4.41(a) using a GHPF with $D_0 = 30$, 60, and 160, corresponding to the circles in Fig. 4.41(b). Compare with Figs. 4.54 and 4.55.



a b c

FIGURE 4.53 Spatial representation of typical (a) ideal, (b) Butterworth, and (c) Gaussian frequency domain highpass filters, and corresponding intensity profiles through their centers.

TABLE 4.5
Highpass filters. D_0 is the cutoff frequency and n is the order of the Butterworth filter.

Ideal	Butterworth	Gaussian
$H(u,v) = \begin{cases} 1 & \text{if } D(u,v) \le D_0 \\ 0 & \text{if } D(u,v) > D_0 \end{cases}$	$H(u,v) = \dfrac{1}{1 + [D_0/D(u,v)]^{2n}}$	$H(u,v) = 1 - e^{-D^2(u,v)/2D_0^2}$