University of Engineering and Technology, Peshawar

Engineering Economics

CSE-305

(Chapter 03c)

Principles of Money-Time Relationship

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Agenda

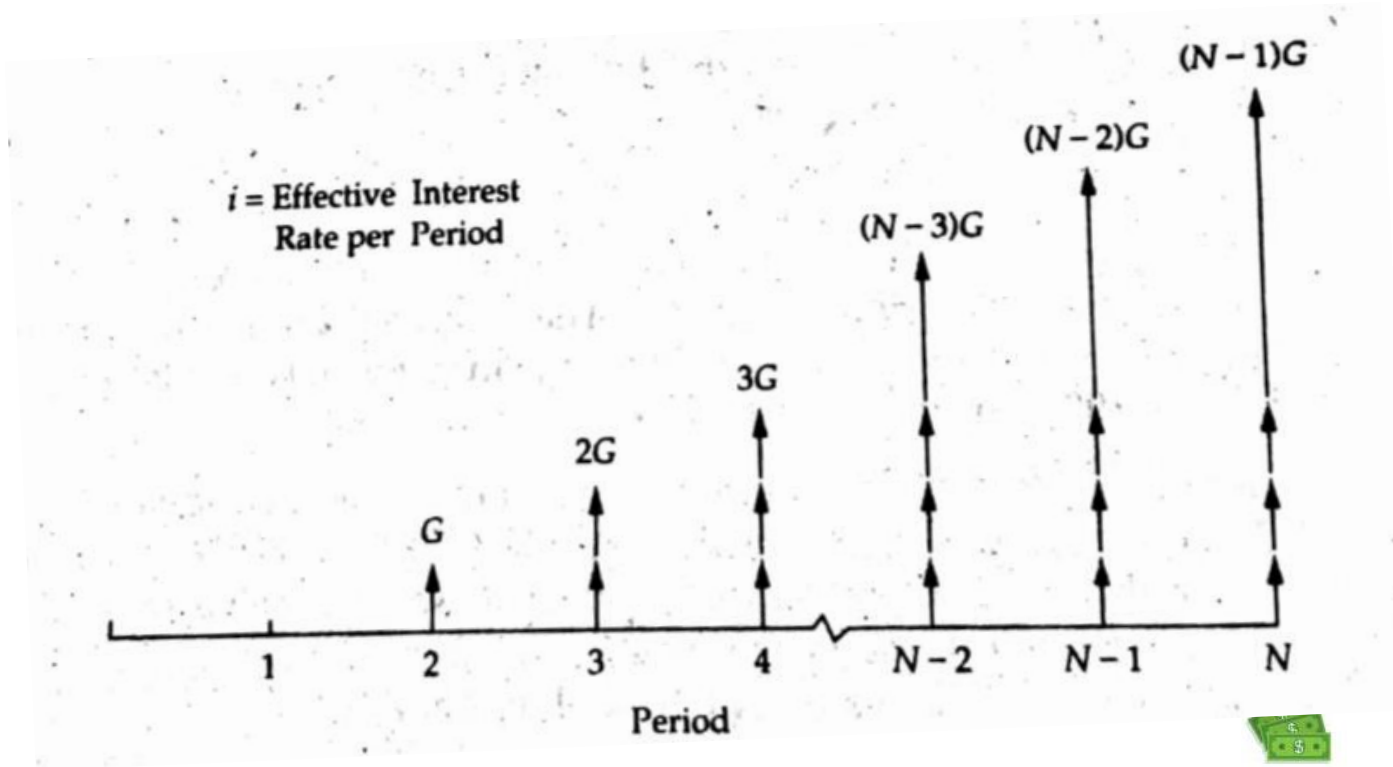
- Arithmetic Uniform Gradient Payment Series
- Cash Flow of Uniform Gradient Series
- Timeline of Cash Flow in Uniform Gradient Series
- Gradient Series: Compound Amount Factor
- Gradient Series: Uniform Series Conversion Factor
- Gradient Series: Present Worth Factor
- Composite Gradient Series

Uniform Gradient Series

- Some Economic Analysis problems involve receipts or expenses that are projected to increase or decrease by a uniform amount each period. Thus, constituting an arithmetic sequence of Cash Flows.

- For instance, the maintenance and operational expenses on an equipment may increase by a relatively constant amount each period.

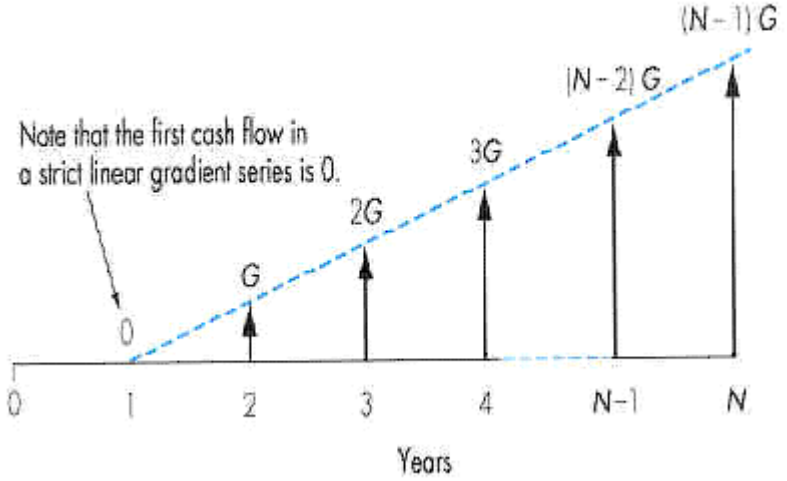
Cash Flows of Uniform Gradient Series

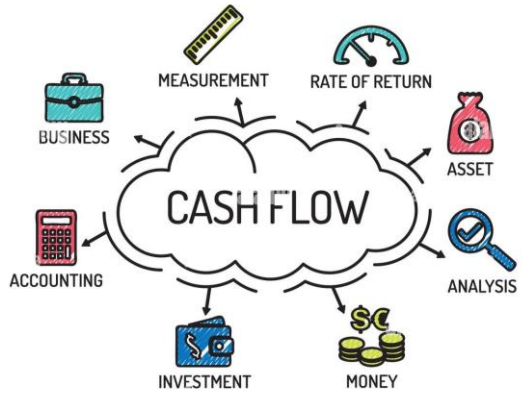


General cash flow diagram relating uniform gradient series increasing by g.

Timeline of Cash Flows of Uniform Gradient Series

End of Period	Payments
1	0
2	G
3	2G
.	.
.	.
.	.
N − 1	(N − 2)G
N	(N − 1)G



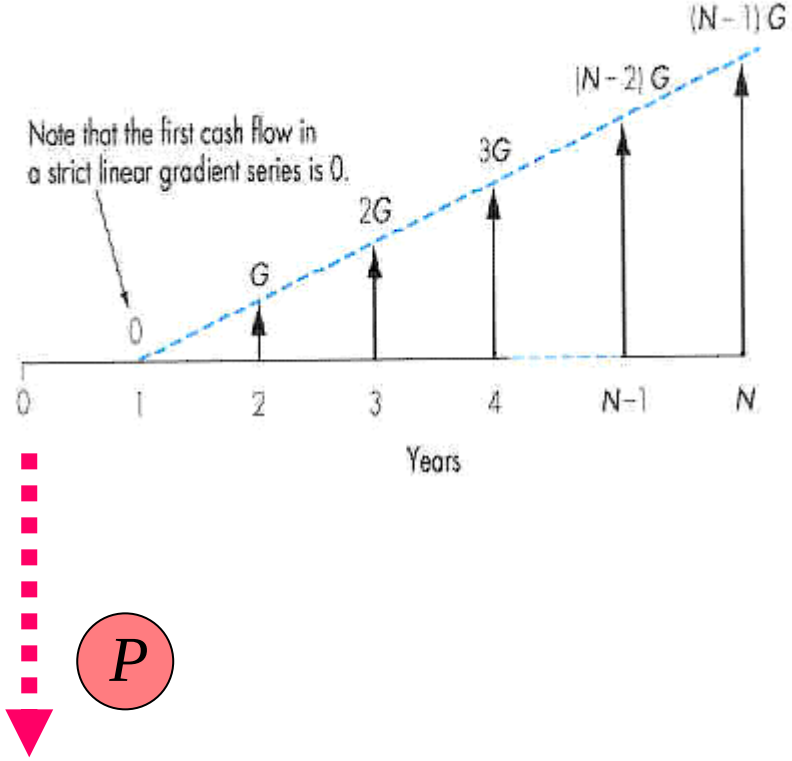


Uniform Linear Gradient Series

The present worth of uniform linear Gradient Series

$$P = G\frac{i(1+i)^N - iN - 1}{i^2(1+i)^N}$$

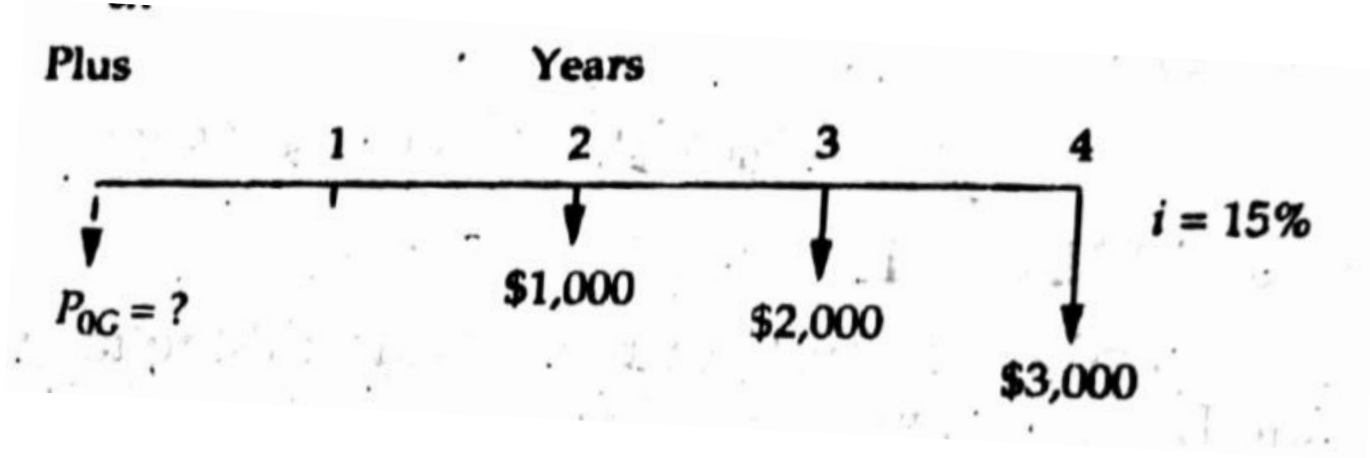
$$= G(P/G, i, N)$$



The term in braces is called the Gradient Series Present Worth Factor

Uniform Linear Gradient Series: Present Worth Factor

A certain end-of-year cash flows are expected to be $1000 for the second year, $2000 for the third year, and $3000 for the fourth year. If the interest rate is 15% and it is desired to find **the present worth** at beginning of the first year.



$$P_{0G} = G\,(P/G, 15\%, 4)$$

$$= \$1000\,(3.79)$$

$$= \$3790$$

Gradient to Uniform Series Conversion Factor

$$A = \frac{G}{i} - \frac{NG}{i}\left[\frac{i}{(1+i)^N - 1}\right]$$

$$= G\left[\frac{1}{i} - \frac{N}{(1+i)^N - 1}\right]$$

$$A = G\,(A/G, i, N)$$

The term in braces is called the **Gradient to Uniform Series Conversion Factor**

Gradient to Uniform Series Conversion Factor: Example

Taking the previous example i.e., certain end-of-year cash flows are expected to be $1000 for the second year, $2000 for the third year and $3000 for the fourth year. If the interest rate is 15% and it is desired to find **the uniform annual worth** at the end of each year.

$$A = G\,(A/G, 15\%, 4)$$
$$= \$1000\,(1.3263) = \$1,326.3$$

As we have already evaluated the PW in the previous example, hence the uniform AW can be calculated as:

$$A = PoG\,(A/P, 15\%, 4)$$
$$= \$3790\,(0.3503) = \$1326.3$$

Gradient Series: Compound Amount Factor

Finding F given G:

$$F = \frac{G}{i}\left[\sum_{k=0}^{N-1}(1+i)^k\right] - \frac{NG}{i}$$

$$= \frac{G}{i}(F/A, i\%, N) - \frac{NG}{i}$$

$F = G\,(F/G, i, N)$

The term in braces is called the **Compound amount Conversion Factor**

Gradient Series: Compound Amount Factor

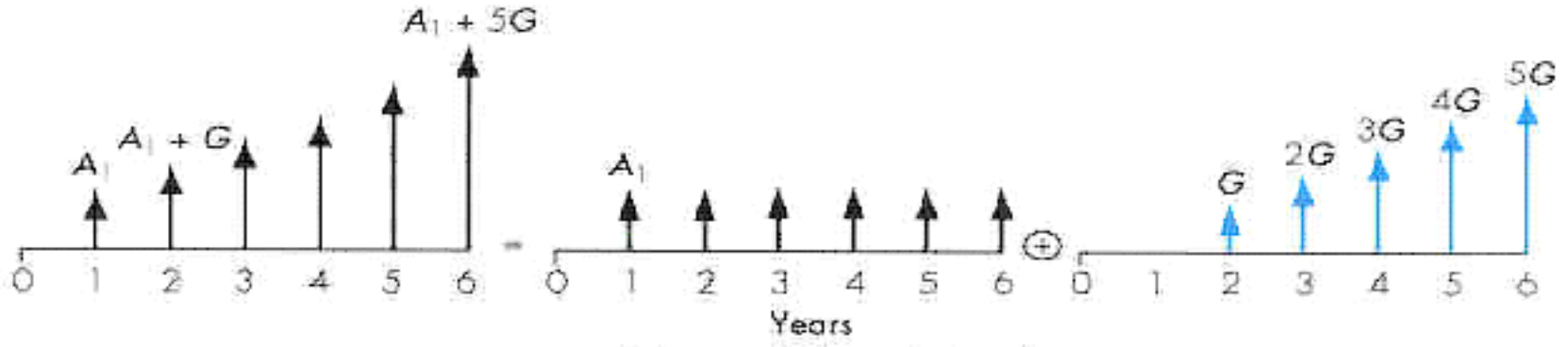
Taking the previous example for evaluating **Compound Amount:**

$$F_{4G} = G\,(F/G, 15\%, 4)$$
$$= G/i\;(F/A, 15\%, 4) - NG/I$$

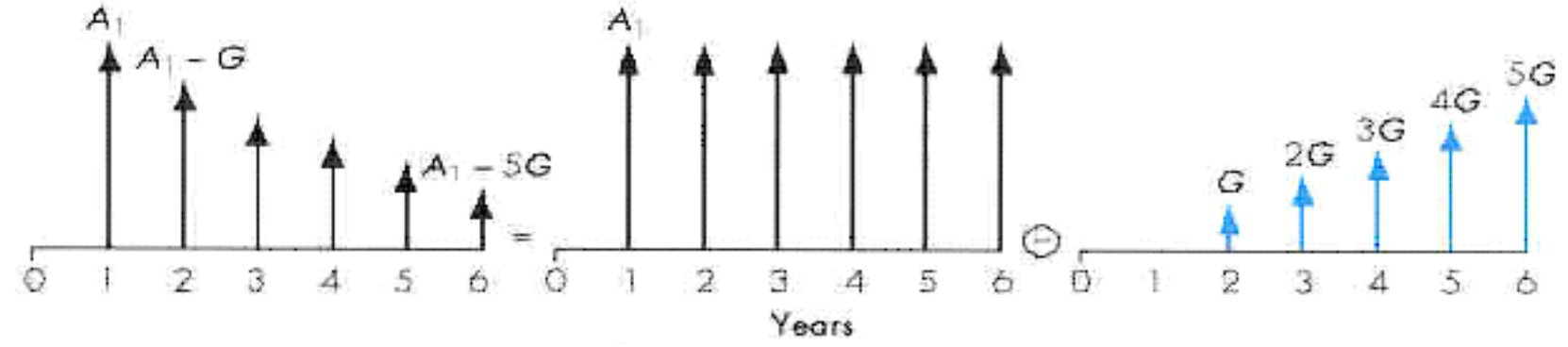
Using Annuity formula and solving:

$$= (((\$1000 / 0.15) * (4.99)) - ((4 * \$1000)/0.15))$$
$$= \$6600$$

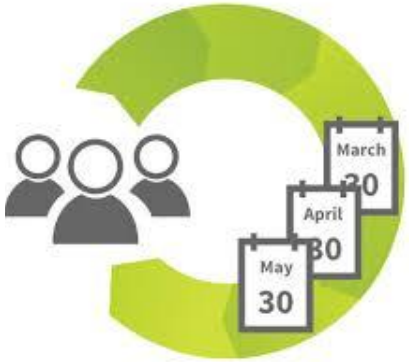
Gradient Series: Composite Series



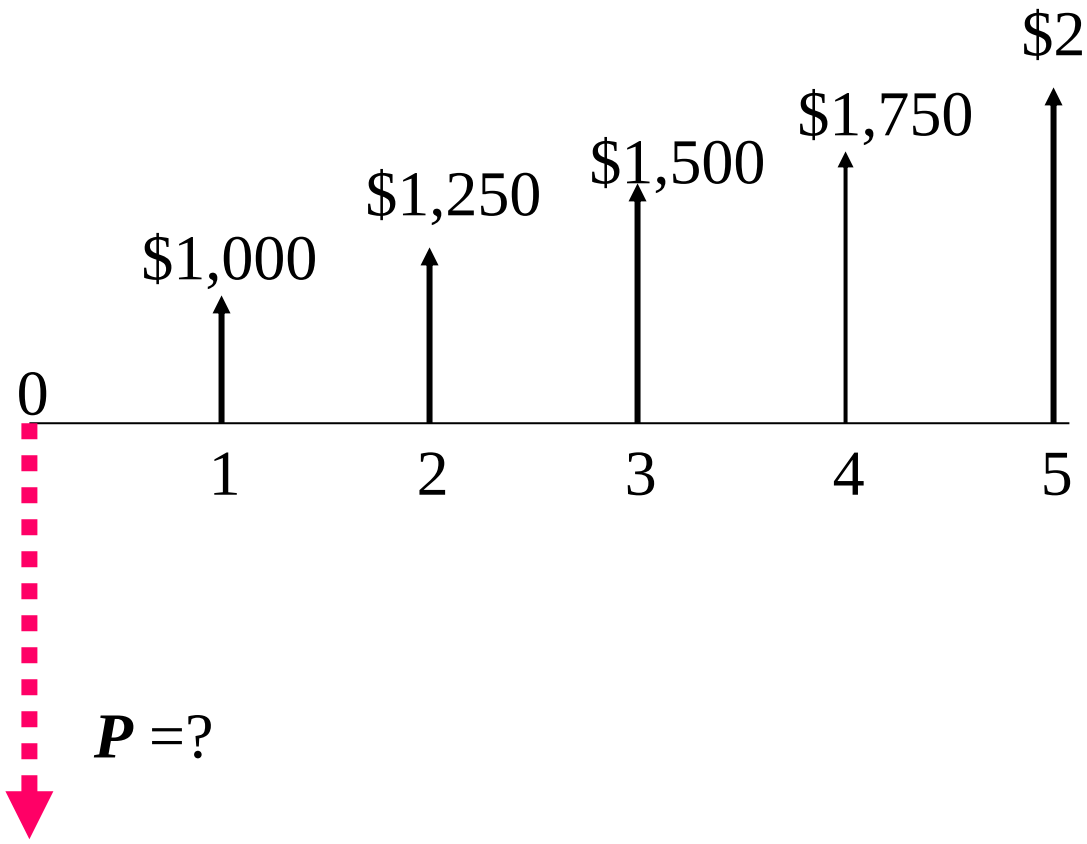
(a) Increasing gradient series



(b) Decreasing gradient series



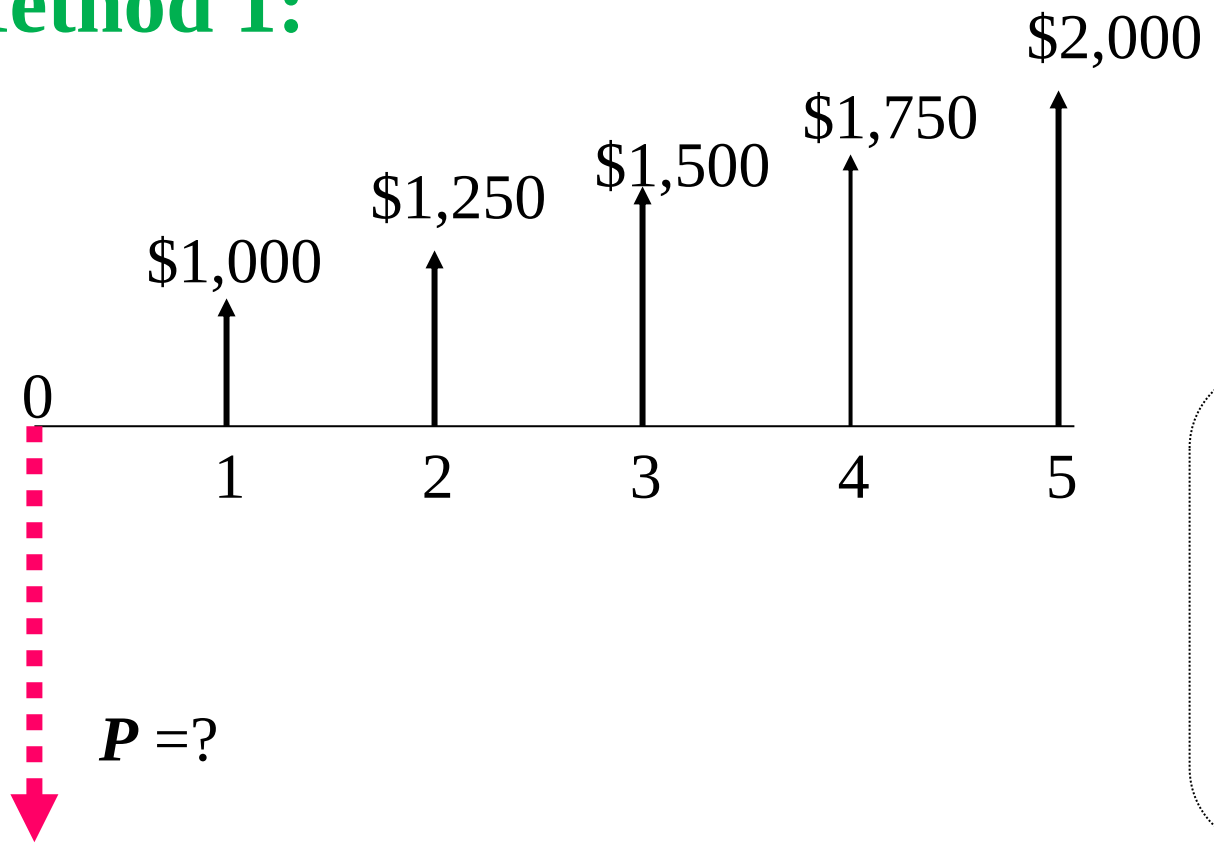
Present Worth For Gradient Series



How much do you have to <u>deposit</u> now in a savings account that earns a 12% annual interest, if you want to withdraw the annual series as shown in the figure?

Present Worth For Gradient Series

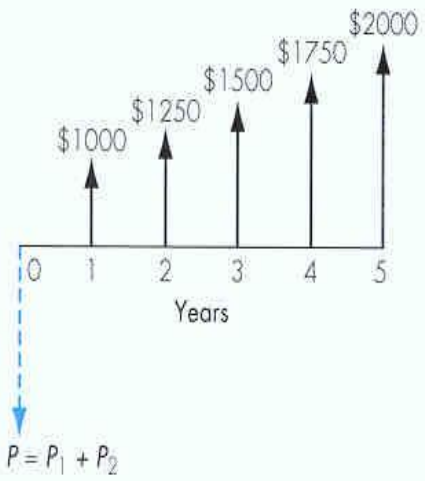
Method 1:



$1,000(*P/F*, 12%, 1) = $892.86
$1,250(*P/F*, 12%, 2) = $996.49
$1,500(*P/F*, 12%, 3) = $1,067.67
$1,750(*P/F*, 12%, 4) = $1,112.16
$2,000(*P/F*, 12%, 5) = $1,134.85
$5,204.03

Present Worth For Gradient Series

Method 2:



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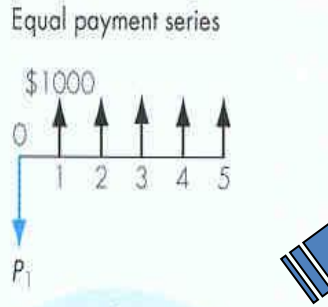

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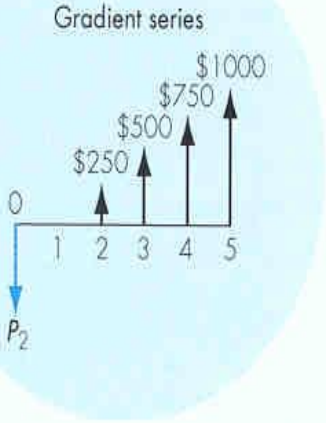
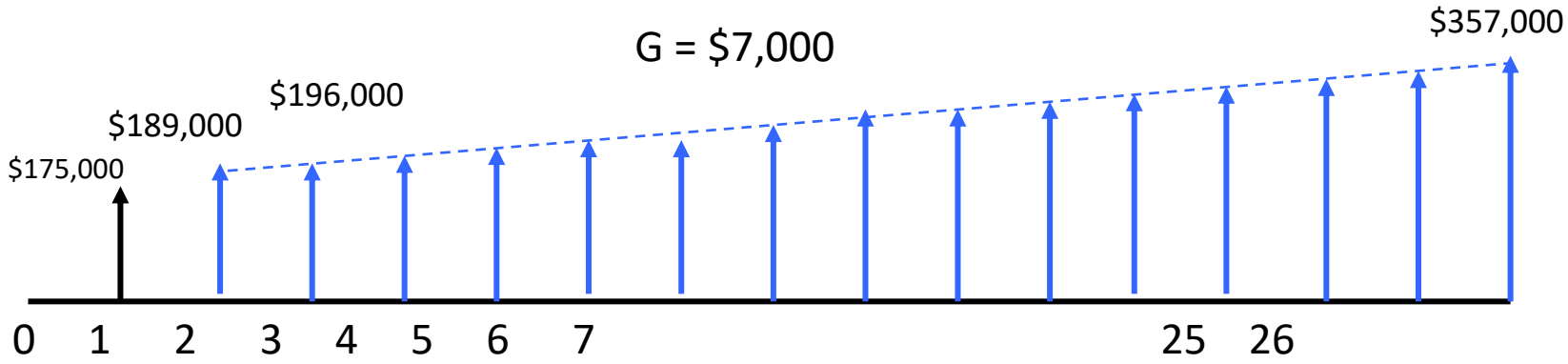


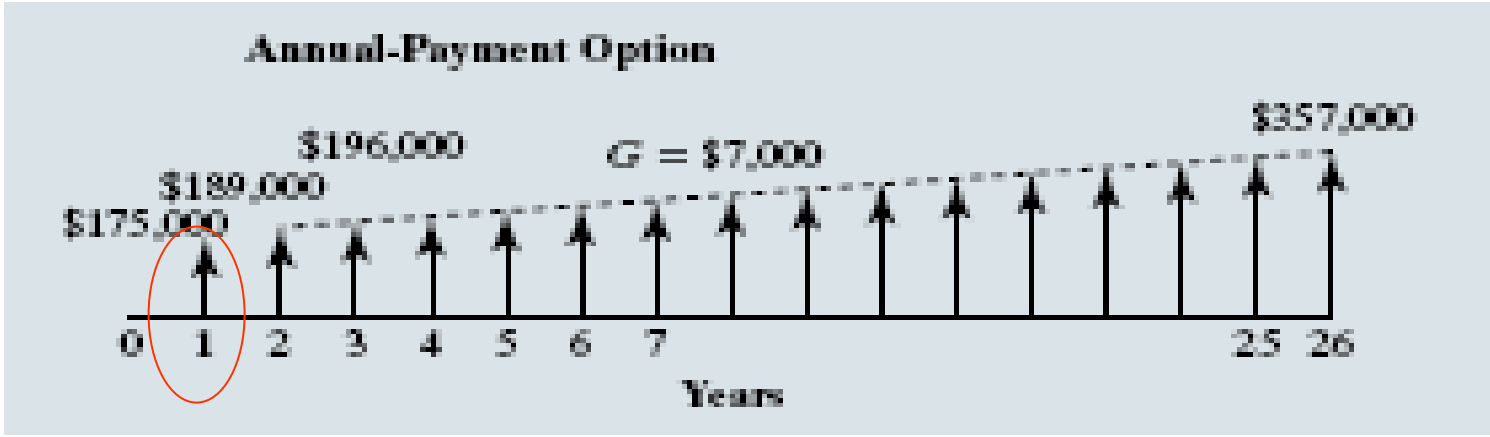
$$P_1 = \$1{,}000(P/A,12\%,5)$$
$$= \$3{,}604.80$$

$$P_2 = \$250(P/G,12\%,5)$$
$$= \$1{,}599.20$$

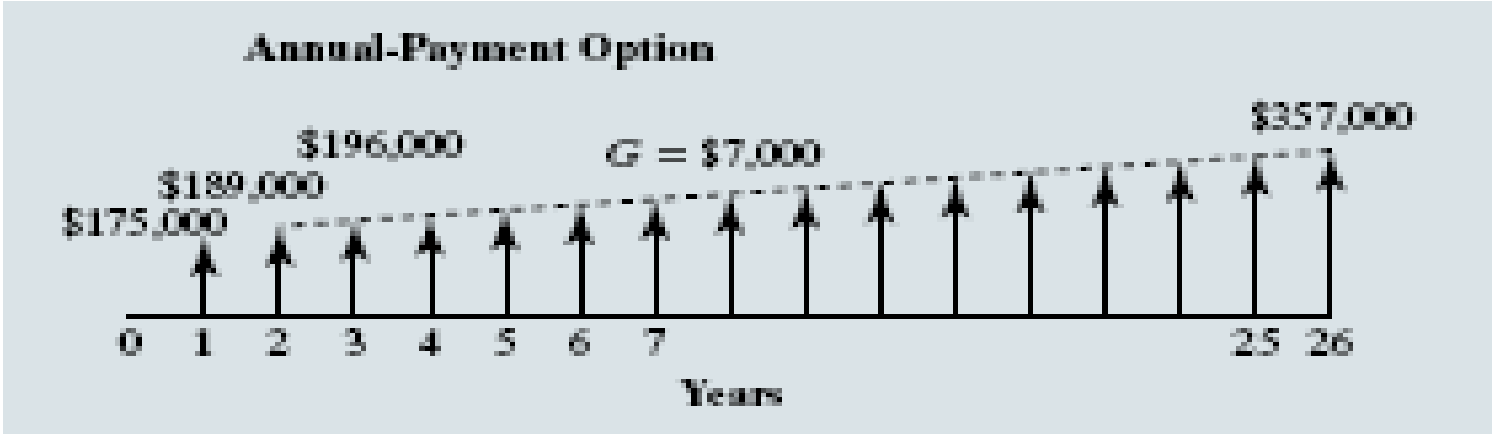
$$P = \$3{,}604.08 + \$1{,}599.20$$
$$= \$5{,}204$$

Gradient Series: Composite Series





Equivalent Present Value of Composite Gradient Series



$$P = [\$175,000 + \$189,000(P/A, 4.5\%, 25) + \$7,000(P/G, 4.5\%, 25)](P/F, 4.5\%, 1)$$

$$= \$3,818,363$$

Also Check book Examples: 3.12 , 3.13. 3.14

Summary

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