Histogram Processing

Histogram of an Image

Histogram provides us with global statistics about an image. Let S be a set and define $|S|$ to be the cardinality of this set, i.e $|S|$ is the number of elements in S. The histogram $h_A(l)$ ($l = 0,...,255$) of the image A is defined as:

In other words, the histogram is a discrete function $p(r_k)$ versus r_k where

$$h_A(l) = |\{(i,\ j) \mid A(i,\ j) = l,\ i = 0,\ ...,\ N-1, j = 0,\ ...,\ M-1\}| \tag{1}$$

$$\sum_{i=0}^{255} h_A(l) = Number\ of\ pixels\ in\ A \tag{2}$$

In other words, a histogram is a discrete function $p(r_k)$ versus r_k where

$$p(r_k) = \frac{n_k}{n} \tag{3}$$

$$r_k = k^{th}\ gray\ level\ (0 \leq k \leq L-1) \tag{4}$$

and n is the total number of pixels in the image. The function $p(r_k)$ estimates the probability of occurrence of gray level r_k. Histogram of an image A 250 x 250, Fig. 6.1 (a), is shown in Fig. 6.1 (b). As we can see that image A has half of its portion as black and half as white and its histogram of this image shows an equal number of occurrences of black and white pixels.

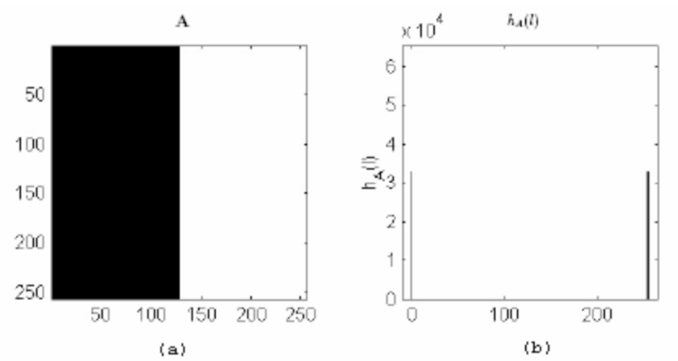

Figure 6.1: (a) Input image A (b) Histogram of image A

The Histogram of an image does not depend upon its shape. It is proved in Fig. 6.2. Again, we have equal parts of the black and white portion, indicated in Fig. 6.2 (a), but its shape is different from the image in Fig. 6.1 (a). Surprisingly, the histogram in Fig. 6.2 (b) is exactly similar to the histogram in Fig. 6.1 (b).

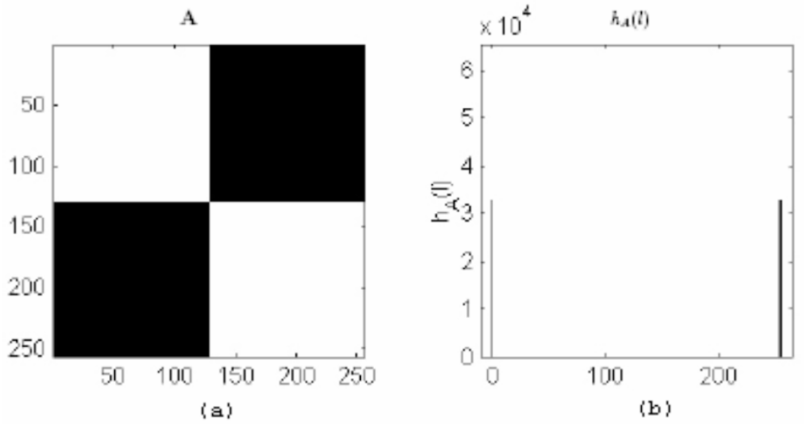

Figure 6.2: (a) Input image A (b) Histogram of image A

Histogram Equalization

Histogram equalization is also called "histogram flattening". In histogram equalization, we transform an image so that each gray level appears an equal number of times i.e. the resulting probability density function is uniformly distributed.

Let us consider continuous, single-valued, monotonically increasing, and normalized gray level transformation function, $T(.)$. Let Eq. 5 perform histogram equalization then it must satisfies the following conditions:

1. T(r) is single-valued and monotonically increasing in the interval $0 \leq r \leq 1$, to preserve the order from black to white

2. *and* $0 \leq T(r) \leq 1$ *for* $0 \leq r \leq 1$

$$s = T(r) \tag{5}$$

Let $p_r(r)$ and $p_s(s)$ be the probabilities of input and output gray levels, respectively. If the above two conditions are true then

$$r = T^{-1}(s) \tag{6}$$

Generally, we know $p_r(r)$ and $T(r)$ and $T(s)$ satisfy condition (1), then the probability density function $p_s(s)$ of the transformed variable s can be obtained using a rather simple formula;

$$p_s(s) = p_r(r) \left| \frac{\partial dr}{\partial ds} \right| \tag{7}$$

In discrete variables, the probability density function is defined by Eq. 3 and Eq. 8 defines histogram equalization or histogram linearization.

$$s_k = T(r_k) = \sum_{j=0}^{k} p_r(r_j) = \sum_{j=0}^{k} \frac{n_j}{n} \tag{8}$$

The output histogram has a dynamic range of gray levels as compared to the histogram of the input image.

Histogram Specification

Histogram specification is also called histogram matching. In histogram specification an the input image is transformed according to the specified gray-level histogram. While the case of the histogram equalization method generates only one result i.e. the output image with approximately uniform histogram (without any flexibility). Fig. 6.3 shows the implementation of histogram matching and has the following steps.

1. Obtain transformation function $T(r_k)$ *Eq.*9
2. Obtain transformation function $G(z_k)$ *Eq.* 10
3. Obtain the inverse function G
4. Finally, obtain output image z_k *Eq.* 11

$$s_k = T(r_k) = \sum_{j=0}^{k} p_r(r_j) = \sum_{j=0}^{k} \frac{n_j}{n}, \forall k = 0, \dots, L-1 \tag{9}$$

$$v_k = G(z_k) = \sum_{j=0}^{k} p_z(z_j) = s_k, \forall k = 0, \dots, L-1 \tag{10}$$

$$z_k = G^{-1}(z_k) = G^{-1}[T(r_k)], \forall k = 0, \dots, L-1 \tag{11}$$

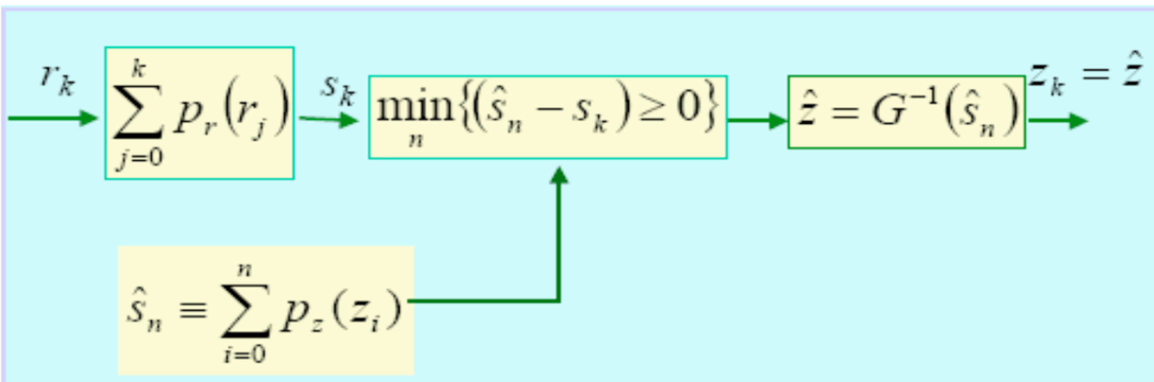

Figure 6.3: Histogram specification

Activity No.1

Prove that the images in Fig. 6.1(a) and Fig. 6.2(a) have the same histogram.

Activity No.2

Prove that dark, bright, low contrast, and high contrast images have histograms similar to the histograms in Fig. 6.4 (a), Fig. 6.4 (b), Fig. 6.4 (c), and Fig. 6.4(d), respectively.

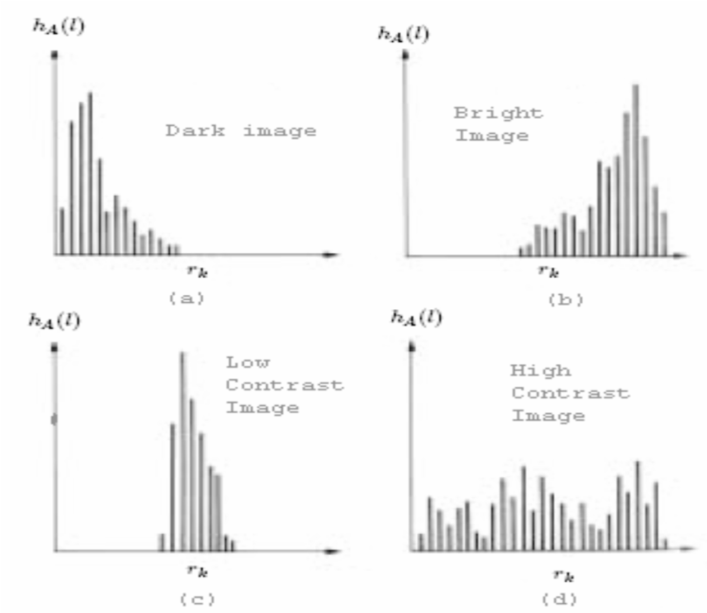

Figure 6.4: (a) Histogram of dark image (b) Histogram of bright image (c) Histogram of low contrast image (d) Histogram of high contrast image

Activity No.3

Apply histogram specification on a low-contrast image. Sketch and compare the histogram of the input and output images.