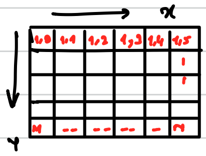
Image Sampling & Quantization

$f(x,y)$ - 2D function ∴ where x and $y \rightarrow$ spatial coordinate.
$f \rightarrow$ amplitude / (intensity level)

Image
- Analogue. $\rightarrow$ x, y & f = continuous
- Digital $\rightarrow$ x, y & f = discrete quantities

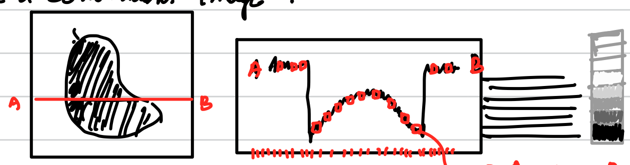


x & Y : spatial coordinates.

Analogue ---- Sampling → Quantization → digital Image

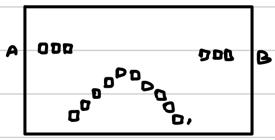
Sampling: Digitization of x & Y
Quantization: digitization of amplitude

Let $f(s,t)$, be a continuous image :



→ Samples : Process is called Sampling.

Resulting image



- <u>Representation of digital image:</u>

$f(s,t) \rightarrow$ be a continuous Image, where s & $t \rightarrow$ continuous Variable

$M \rightarrow$ Rows & N - columns

$f(x,y)$ - Digital Image

where $x = 0, 1, 2 \ldots M-1$ & $Y = 0, 1, 2 \ldots N-1$

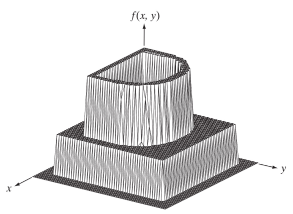


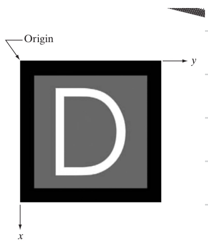
Fig 2.18(a)

Fig 2.18 represented as surface, This image has 3 axes; x & y spatial coordinates & 3rd is the intensity level. For the complex images, this kind of representation is not preferable.

Fig 2.18(b): Displayed as the visual intensity level — most family — TV screens or photographs.

Three levels of intensities $[0 \quad 0.5 \quad 1]$
Black gray white

Fig 2.18(b)



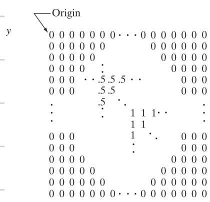


Fig 2.18(c)

(b) This kind of representation is usually used in printing / publishing.

Fig 2.18 (c) : 3rd type of representation as Numerical Array.

$$\begin{bmatrix} 0\ 000 \cdots\cdots 000 \\ 0.5\ 0.5 \cdots \\ 0.5 \cdots \\ 1\ 1\ 1 \cdots\cdots 111 \end{bmatrix}$$

(b) & (c) are mostly used.

$$f(x,y) = \begin{bmatrix} f(0,0) & f(0,1) & f(0,2) & \cdots & f(0,N-1) \\ f(1,0) & f(1,1) & f(1,2) & \cdots & f(1,N-1) \\ \vdots & & & & \\ f(M-1,0) & f(M-1,1) & f(M-1,2) & \cdots & f(M-1,N-1) \end{bmatrix}$$

• Mostly used for developing of algos and

No of the intensity levels is $\boxed{L = 2^k}$ ∵ k = integer

Dynamic intensity range = $\dfrac{\text{Max measurable Intensity level}}{\text{Min detectable " "}}$

→ Saturation

→ noise

intensity level beyond this saturation level is "clipped off" and the " " below " noise " is "not true intensity level.

No of bits (To store the digital image) is $\boxed{b = M \times N \times k}$

if image is having M = N (same rows & columns)

then $\boxed{b = N^2 k}$ ⟶ (2)

TABLE 2.1
Number of storage bits for various values of *N* and *k*. *L* is the number of intensity levels.

N/k	1 (L = 2)	2 (L = 4)	3 (L = 8)	4 (L = 16)	5 (L = 32)	6 (L = 64)	7 (L = 128)	8 (L = 256)
32	1,024	2,048	3,072	4,096	5,120	6,144	7,168	8,192
64	4,096	8,192	12,288	16,384	20,480	24,576	28,672	32,768
128	16,384	32,768	49,152	65,536	81,920	98,304	114,688	131,072
256	65,536	131,072	196,608	262,144	327,680	393,216	458,752	524,288
512	262,144	524,288	786,432	1,048,576	1,310,720	1,572,864	1,835,008	2,097,152
1024	1,048,576	2,097,152	3,145,728	4,194,304	5,242,880	6,291,456	7,340,032	8,388,608
2048	4,194,304	8,388,608	12,582,912	16,777,216	20,971,520	25,165,824	29,369,128	33,554,432
4096	16,777,216	33,554,432	50,331,648	67,108,864	83,886,080	100,663,296	117,440,512	134,217,728
8192	67,108,864	134,217,728	201,326,592	268,435,456	335,544,320	402,653,184	469,762,048	536,870,912

• Spatial and Gray level Resolution:

• Spatial resolution is the smallest discernable detail in an image

• pixel per unit distance, dots per unit distance or line pair per unit distance

In US → $\boxed{\text{dpi}}$ dot per inch – widely used in publishing / printing

A newspaper have a resolution of 75dpi
A magazine " " " " 133 "
A glossy brochure " " " 175 "

A 20MP camera provides a better quality (high resolution) compare to 8MP " . If no of pixel per unit distance (dpi) increases, the quality and the resolution gets increases.

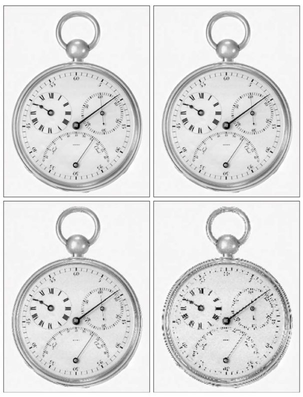
Intensity resolution: Gray level resolution similarly refers to the smallest discernable change in the intensity / gray level; keep in mind that measuring discernable changes in gray level is highly subjective process. The intensity level $L = 2^K$, most widely 8 bits, 16 bits, 32 bits (rarely)

depends on x & y
depends on f

The Spatial resolution depends on the no; of Samples whereas " intensity or gray level " " " " " " intensity levels (L)

Example of spatial resolution:

Image quality is related to the spatial resolution
→ reduction in the spatial resolution causes reduction of quality of the image.



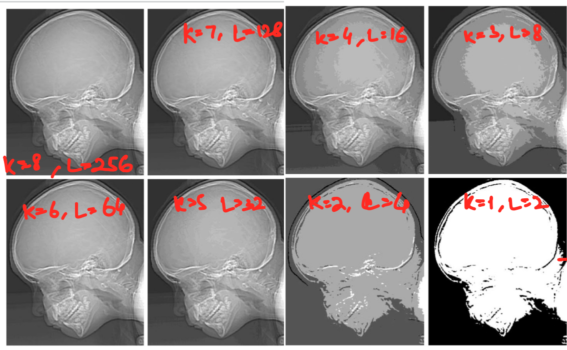
a b
c d

FIGURE 2.20 Typical effects of reducing spatial resolution. Images shown at: (a) 1250 dpi, (b) 300 dpi, (c) 150 dpi, and (d) 72 dpi. The thin black borders were added for clarity. They are not part of the data.

Example of gray level resolution:

a b
c d

FIGURE 2.21 (a) 452 × 374, 256-level image. (b)–(d) Image displayed in 128, 64, and 32 intensity levels, while keeping the image size constant.



e f
g h

FIGURE 2.21 (*Continued*) (e)–(h) Image displayed in 16, 8, 4, and 2 intensity levels. (Original courtesy of Dr. David R. Pickens, Department of Radiology & Radiological Sciences, Vanderbilt University Medical Center.)

K =

K=1 [0, 1]

binary image.
0 = black
1 = white

Zooming may be viewed as over sampling and Shrinking may be under-Sampling.

Zooming requires two step: the creation of new pixel locations and assignment of gray levels to those.

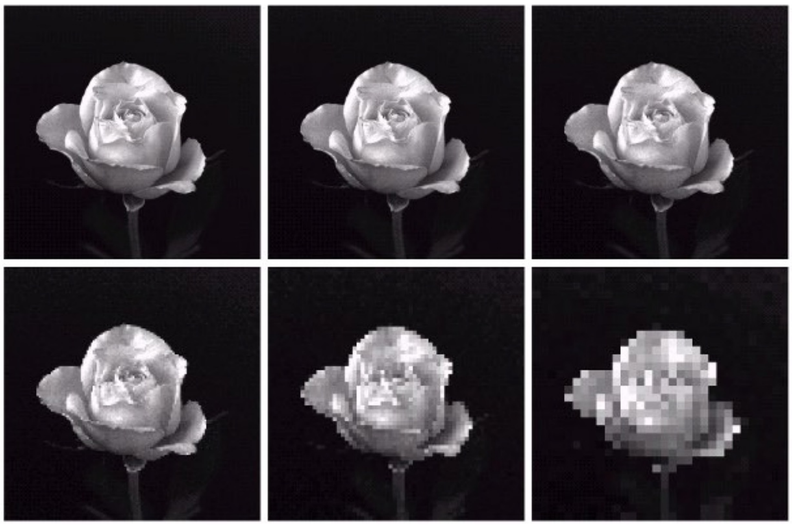
Suppose we have an image of size 500×500 pixels and to enlarge it 1.5 times to 750×750 pixels.

Laying an imaginary grid of 750×750 over the original image.

→ Assignment of gray level to the new pixels is called nearest neighbour interpolation.

• Pixel replication.

<u>Image Interpolation</u> : A tool use to resize the image eg., zooming, shrinking rotating or geometrical correction, etc. It is also called as resampling of an image. In order to resize, we have to resample the image.



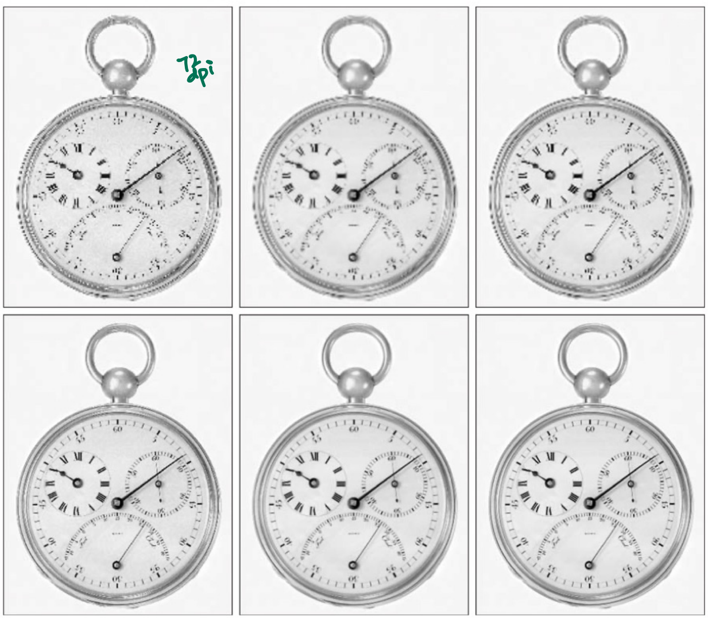
a b c
d e f

FIGURE 2.20 (a) 1024 × 1024, 8-bit image. (b) 512 × 512 image resampled into 1024 × 1024 pixels by row and column duplication. (c) through (f) 256 × 256, 128 × 128, 64 × 64, and 32 × 32 images resampled into 1024 × 1024 pixels.

New pixels get added and will be assigned with intensity level by performing interpolation (Nearest neighbor interpolation)

<u>NN Interpolation</u> → Bilinear interpolation $v(x,y) = ax+by+cxy+d$
4 NN

→ Bi-cubic " $v(x,y) = \sum_{i=0}^{3}\sum_{j=0}^{3} a_{ij}\, x^i y^j$
16 NN



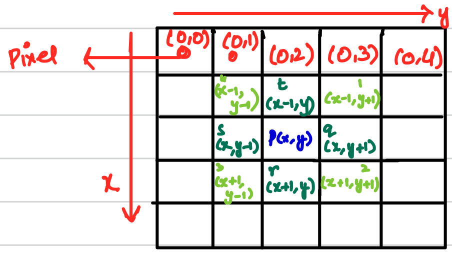
a b c
d e f

FIGURE 2.24 (a) Image reduced to 72 dpi and zoomed back to its original size (3692 × 2812 pixels) using nearest neighbor interpolation. This figure is the same as Fig. 2.20(d). (b) Image shrunk and zoomed using bilinear interpolation. (c) Same as (b) but using bicubic interpolation. (d)–(f) Same sequence, but shrinking down to 150 dpi instead of 72 dpi [Fig. 2.24(d) is the same as Fig. 2.20(c)]. Compare Figs. 2.24(e) and (f), especially the latter, with the original image in Fig. 2.20(a).

(a) degraded image. (72dpi) (b) zoomed using bilinear interpolation
(c) zoomed using Bicubic

(e) degraded image (150dpi). (f) Bilinear (c) bicubic

- Some Basic relationships between pixels:



(a) **Neighbours of pixels:**

$N_4(P) = \{q, r, s, t\}$

$N_4(p) = \{(x-1,y), (x,y-1), (x+1,y) (x,y+1)\}$

diagonal pixel $N_D(P) = \{1, 2, 3, 4\}$

$= \{(x-1, y-1), (x+1, y-1), (x-1, y+1), (x+1, y+1)\}$

$N_8(P) = N_4(P) \cup N_D(P)$

$N_8(P) = \{q, r, s, t, 1, 2, 3, 4\}$

(b) **Adjacency, connectivity, region & boundary:**

a) 4- adjacency
b) 8- adjacency
c) m- adjacency

Example:

a 0	b 1	c 0
d 0	P 1 e	f 1 q
g 0	h 0	i 1

binary image with 2 intensity Level $\{0,1\}$

→ p is adjacent to q

(1) $V = \{1\}$ ∴ p and q have the same values.
p and q should be having the subset of V.

(2) Should be in the set of $N_4(P)$

$N_4(P) = \{b, d, f, h\}$ (q under f)

8-adjacency: (1) P & q have the value of subset V
(2) q should present in $N_8(P)$

$N_8(P) = \{a, b, c, d, e, f, g, h, i\}$ (q above f) 8-adjacent

(c) **m-adjacency:** 1) $N_4(P) = q$ and
2) $N_D(P)$

$\boxed{N_4(P) \cap N_D(P)}$

- If we have two paths between P & q, then we prefer 4-adj; path

$$\begin{matrix} 0 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}$$

m-adjacency.

- P and q are considered as connective if there exist a path. The path can be of any type like 4-adjacent path, 8-adjacent & m-adjacent.

- **Region :**

$$\left.\begin{matrix} 0 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{matrix}\right\} R_i$$

$$\left.\begin{matrix} 0 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{matrix}\right\} R_j$$

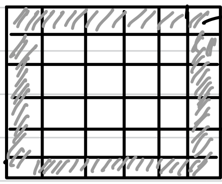
path

R_i and R_j are said to be adjacent region.

Region is defined as, it is a subset of pixel, which is present in the entire image. The subset of pixels which forms connectivity among themselves is represented by a region.

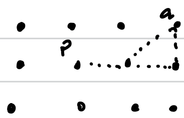
* If there is no connectivity b/w R_i & R_j (not 4-adjacency, no 8-adjacency or no m-adjacency then we called it disjoint regions.

- **Boundary :**



A region is a boundary region if that region lies at the border or the contour of that image, if this region contains atleast 1 pixel of background intensity, we called that as boundary regions.

- **Distance measures :** Let $P(x,y)$, $q(s,t)$ & $z(u,v)$



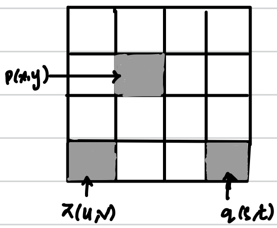
Properties of distance (D)

(i) $D(P,q) \geq 0$

(ii) $D(P,q) = 0$ if $P = q$

(iii) $D(P,q) = D(q,P)$

(iv) $D(P,z) \leq D(P,q) + D(q,z)$



Distance measure :

(i) Euclidean : $D_E(P,q) = [(x-s)^2 + (y-t)^2]^{1/2}$

(ii) cityblock : $D_4(P,q) = |x-s| + |y-t|$

(iii) chess board : $D_8(P,q) = \max\{|x-s|, |y-t|\}$

Example :

(a) $D_E(p,q) = [(2-4)^2 + (2-4)^2]^{1/2}$
$= (8)^{1/2}$

(b) $D_4(p,q) = |2-4| + |2-4| = 2+2 = 4$

4	3	2	3	4
3	2	1	2	3
2	1	0	1	2
3	2	1	2	3
4	3	2	3	4

(c) $D_8(p,q) = \max\{|2-4|, |2-4|\}$
$\max\{2, 2\} = 2$

2	2	2	2	2
2	1	1	1	2
2	1	0	1	2
2	1	1	1	2
2	2	2	2	2