CSE331 - Practice Sheet on Pushdown Automata and Turing Machine

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1 Pushdown Automata

Problem 1. For each of the following languages, design a Pushdown Automata that recognizes the language:

1. $L = \{\mathtt{0}^n\mathtt{1}^n \ : \ n \geq 0\}$.
2. $L = \left\{\mathtt{0}^{2n}\mathtt{1}^n \ : \ n \geq 0\right\}$.
3. $L = \{\mathtt{0}^n\mathtt{1}^n \ : \ n \text{ is even}\}$.
4. $L = \left\{\mathtt{0}^i\mathtt{1}^j\mathtt{2}^k \ : i = j \text{ or } j = k\right\}$.
5. $L = \left\{\mathtt{0}^{3n}\mathtt{1}^{2n} \ : \ n \geq 0\right\}$.
6. $L = \{w_1\mathtt{\#}w_2 \ : \ w_1, w_2 \in \{\mathtt{0},\mathtt{1}\}^*, \text{ and the number of } \mathtt{00} \text{ in } w_1 \text{ is equal to the number of } \mathtt{11} \text{ in } w_2\}$.
7. $L = \left\{w\mathtt{\#}x \ : \ w, x \in \{\mathtt{0},\mathtt{1}\}^* \text{ and } w^R \text{ is a substring of } x\right\}$.
8. $L = \{\mathtt{0}^n\mathtt{1}^m \ : \ n, m \geq 0 \text{ and } 2n = 3m\}$.
9. $L = \{\mathtt{0}^n\mathtt{1}^m\mathtt{2}^m\mathtt{3}^n \ : \ n, m \geq 0\}$.
10. $L = \{w = \mathtt{0}^i\mathtt{1}^j\mathtt{2}^k \ i, j, k \geq 0 \text{ and } j < i + k\}$.
11. $L = \{\mathtt{0}^n\mathtt{1}^m \ : \ n \geq m, \ m \geq 0\}$
12. $L = \{w \in \{\mathtt{0},\mathtt{1}\}^* \ : \text{ every prefix of } w \text{ has at least as many } \mathtt{0}\text{s as } \mathtt{1}\text{s}\}$.
13. $L = \{x\#y \ : \ x, y \in \{\mathtt{0},\mathtt{1}\}^*, \ |x| = |y|\}$.
14. $L = \{w \in \{\mathtt{0},\mathtt{1}\}^* \ : \ w \text{ has twice as many } \mathtt{1}\text{s as } \mathtt{0}\text{s}\}$.
15. $L = \{\mathtt{0}^n\mathtt{1}^m\mathtt{0}^n \ : \ n, m \geq 0\}$.
16. $L = \{\mathtt{a}^m\mathtt{b}^n\mathtt{c}^p\mathtt{d}^q \ : \ m + n = p + q, \text{ and } m, n, p, q \geq 1\}$.
17. $L = \{\mathtt{a}^i\mathtt{b}^j\mathtt{c}^k\mathtt{d}^l \ : \ i = k \text{ or } j = l, \text{ and } i, j \geq 1\}$.
18. $L = \{w \in \{\mathtt{0},\mathtt{1}\}^* \ : \ w \text{ is a palindrome}\}$.

19. The language of balanced parentheses, i.e. $L = \{w \in \{\ (\ ,\ )\ \}^* \ :\ w \text{ is balanced}\}$.

Problem 2. Can a Deterministic Pushdown Automata (DPDA) recognize the language consistsing of all palindromic strings over $\Sigma = \{0, 1\}$? Why or why not?

Problem 3. We know that a Pushdown Automata can recognize the language

$$\left\{w\#w^R \ :\ w \in \{0,1\}^*\right\}.$$

Can a Pushdown Automata recognize the following language?

$$\left\{w\#w \ :\ w \in \{0,1\}^*\right\}.$$

What goes wrong in this case?

2 Turing Machine

Problem 4. For each of the following languages, design a Turing Machine that recognizes the language:

1. $L = \{0^n1^n2^n \ :\ n \geq 0\}$.
2. $L = \{w \in \{0,1\}^* \ :\ w \text{ is a palindrome}\}$.
3. $L = \{w \in \{0,1\}^* \ :\ w \text{ is an even-length palindrome}\}$.
4. $L = \{w\#w \ :\ w \in \{0,1\}^*\}$.
5. $L = \{ww \ :\ w \in \{0,1\}^*\}$.
6. $L = \left\{0^{2^n} \ :\ n \geq 0\right\}$.
7. $L = \left\{0^i1^j2^{i+j} \ :\ i,j \geq 0\right\}$.
8. $L = \left\{0^i1^j2^{ij} \ :\ i,j \geq 0\right\}$.

Problem 5. Think about how a Turing Machine is more powerful than a PDA. If we modify a PDA to have two stacks, is it as powerful as a Turing Machine? If we use a queue instread of a stack, is the PDA as powerful as a Turing Machine?