CSE331 Practice Sheet for DFA

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1 DFA Construction

For each of the following languages, construct a DFA that recognizes the language.

1. $L = \{w \in \{\texttt{0}, \texttt{1}\}^* : w$ does not contain three consecutive 0s$\}$.
2. $L = \{w \in \{\texttt{a}, \texttt{b}, \texttt{c}\}^* : w$ contains bbac as a subsequence$\}$.
3. $L = \{w \in \{\texttt{a}, \texttt{b}\}^* : w$ starts and ends with different symbols$\}$.
4. $L = \{w \in \{\texttt{a}, \texttt{b}\}^* : w$ starts and ends with the same symbol$\}$.
5. $L = \{w \in \{\texttt{0}, \texttt{1}\}^* : w$ ends with the substring 0101$\}$.
6. $L = \{w \in \{\texttt{x}, \texttt{y}\}^* : w$ ends with the substring yxxy$\}$.
7. $L = \{w \in \{\texttt{0}, \texttt{1}\}^* : |w| \equiv 2 \pmod 4\}$.
8. $L = \{w \in \{\texttt{0}, \texttt{1}\}^* : \#\texttt{1}\text{s in } w \equiv 2 \pmod 4\}$.
9. $L = \{w \in \{\texttt{0}, \texttt{1}\}^* : w$ (interpreted as a binary number) is divisible by 5$\}$.
10. $L = \{w \in \{\texttt{0}, \texttt{1}, \texttt{\#}\}^* : \texttt{\#} \notin w$ and the number of 0s is not divisible by 3$\}$. You can also do it by showing $L = L_1 \cap L_2$, where:

    $$L_1 = \{w \in \{\texttt{0}, \texttt{1}, \texttt{\#}\}^* : \texttt{\#} \notin w\}, \quad L_2 = \{w \in \{\texttt{0}, \texttt{1}\}^* : \#\texttt{0}\text{s} \not\equiv 0 \pmod 3\};$$

    and then performing the cross-product construction for intersection.
11. $L = \{w \in \{\texttt{a}, \texttt{b}, \texttt{c}\}^* :$ ba is not a substring and w ends with cb$\}$.
12. $L = \{w \in \{\texttt{0}, \texttt{1}\}^* : w$ contains at least three 0s or exactly two 1s.$\}$
13. $L = \{w \in \{\texttt{a}, \texttt{b}\}^* :$ the second last symbol of w is a$\}$.
14. $L = \{w \in \{\texttt{0}, \texttt{1}\}^* :$ the third last symbol of w is 1$\}$.
15. $L = \{w \in \{\texttt{a}, \texttt{b}\}^* :$ the last letter appears at least twice in $w\}$.

2 More DFA

For each of the following languages, construct a DFA that recognizes the language.

1. $L = \{w \in \{\texttt{0}, \texttt{1}\}^* : \#\texttt{0} - \#\texttt{1} \equiv 0 \pmod 3\}$.
2. $L = \{w \in \{\texttt{0}, \texttt{1}\}^* : \text{every third symbol of } w \text{ is } \texttt{1}\}$
3. $L = \{w \in \{\texttt{0}, \texttt{1}\}^* : \text{every symbol at an even index of } w \text{ is } \texttt{0}\}$
4. $L = \{w \in \{\texttt{a}, \texttt{b}\}^* : w \text{ contains exactly one occurrence of the substring } \texttt{ab}\}$
5. $L = \{w \in \{\texttt{a}, \texttt{b}\}^* : w \text{ contains exactly two occurrences of the substring } \texttt{ab}\}$
6. $L = \{w \in \{\texttt{0}, \texttt{1}\}^* : w \text{ contains at least two occurrences of the substring } \texttt{00}\}$
7. $L = \{w \in \{\texttt{0}, \texttt{1}\}^* : w \text{ contains exactly two occurrences of the substring } \texttt{00}\}$
8. $L = \{w \in \{\texttt{0}, \texttt{1}\}^* : w \text{ contains at most two occurrences of the substring } \texttt{00}\}$
9. $L = \{w \in \{\texttt{0}, \texttt{1}\}^* : \text{an even number of } \texttt{0}\text{s follow the last } \texttt{1} \text{ in } w\}$
10. $L = \{w \in \{\texttt{a}, \texttt{b}\}^* : \text{every } \texttt{b} \text{ in } w \text{ is followed by at least one } \texttt{a}\}$
11. $L = \{w \in \{\texttt{0}, \texttt{1}\}^* : \text{the number of } \texttt{0}\text{s between any two successive } \texttt{1}\text{s is even}\}$
12. $L = \{w \in \{\texttt{0}, \texttt{1}\}^* : \texttt{00} \text{ does not occur as a substring before the first } \texttt{11}\}$.
13. $L = \{w \in \{\texttt{0}, \texttt{1}\}^* : \texttt{00} \text{ does not occur as a subsequence before the first } \texttt{11}\}$.
14. $L = \{w \in \{\texttt{0}, \texttt{1}\}^* : w \text{ contains the substring } \texttt{0}\texttt{1}^m\texttt{0},\ m \equiv 0 \pmod 3\}$,
15. $L = \{w \in \{\texttt{0}, \texttt{1}\}^* : w \text{ contains the substring } \texttt{0}\texttt{1}^m\texttt{0},\ m \equiv 2 \pmod 3\}$.
16. $L = \{w = \texttt{0}^m\texttt{1}^n : m, n \text{ are odd}\}$,
17. $L = \{w = \texttt{0}^m\texttt{1}^n : m, n \text{ are even}\}$,. Alternatively, prove $L = L_1 \circ L_2$ is regular, where:

$$L_1 = \{\texttt{0}^m : m \text{ even}\}, \quad L_2 = \{\texttt{1}^n : n \text{ even}\}.$$

3 Further DFA

Problem 1. Let $\Sigma = \{\texttt{0}, \texttt{1}\}$, and define:

$$L_1 = \{\texttt{1}^m : m \text{ is odd}\}$$
$$L_2 = \{w \in \Sigma^* : \text{no substring of } w \text{ belongs to } L_1\}$$

(a) Give a length-6 string that belongs to L_2.

(b) Draw the state diagram of a DFA that accepts L_1.

(c) Draw the state diagram of a DFA that accepts L_2.

(d) Draw a DFA that accepts $L_1 \cap L_2$ (you may use product construction or simplify).

Problem 2. Define the symmetric difference:

$$L_1 \triangle L_2 = (L_1 \cup L_2) \setminus (L_1 \cap L_2)$$

Let $\Sigma = \{\texttt{0}, \texttt{1}\}$ and define:

$$A = \{w \in \Sigma^* : 3 \leq |w| \leq 5\}$$
$$B = \{w \in \Sigma^* : 2 \leq |w| \leq 4\}$$
$$C = \{w \in \Sigma^* : |w| \text{ is odd}\}$$

(a) Construct a DFA for A.

(b) Construct a DFA for B.

(c) Construct a DFA for $A \triangle B$.

(d) Using product construction, how many states does the DFA for $(A \triangle B) \cup C$ have?

(e) Give a 5-state DFA that recognizes $(A \triangle B) \cup C$.

Problem 3. Let $\Sigma = \{\texttt{0}, \texttt{1}\}$ and define:

$$L_1 = \{w \in \Sigma^* : \text{every second symbol is } \texttt{0}\}$$
$$L_2 = \{w \in \Sigma^* : \text{every third symbol is } \texttt{1}\}$$

(a) Give a length-5 string in $L_1 \cap L_2$.

(b) Construct a DFA for L_1.

(c) Construct a DFA for L_2.

(d) Construct a DFA for $L_1 \cap L_2$.

Problem 4. Let $\Sigma = \{\texttt{0}, \texttt{1}\}$ and define:

$$L_1 = \{\texttt{0}, \texttt{10}\}$$
$$L_2 = L_1^*$$
$$L_3 = \{w \in \Sigma^* : |w| = 4\}$$

(a) List all strings in $L_2 \cap L_3$.

(b) Draw a DFA for L_1.

(c) Draw a DFA for L_2.