Test of Hypothesis (1)

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Hypothesis

- A hypothesis is an assumption/statement to be tested.

Statistical hypothesis

- A statistical hypothesis is some statement
- About a population parameter

Getting population data is hard because there's a lot of it.

- Which we want to verify
- On the basis of sample information.

Getting sample data is easy

Statistical hypothesis

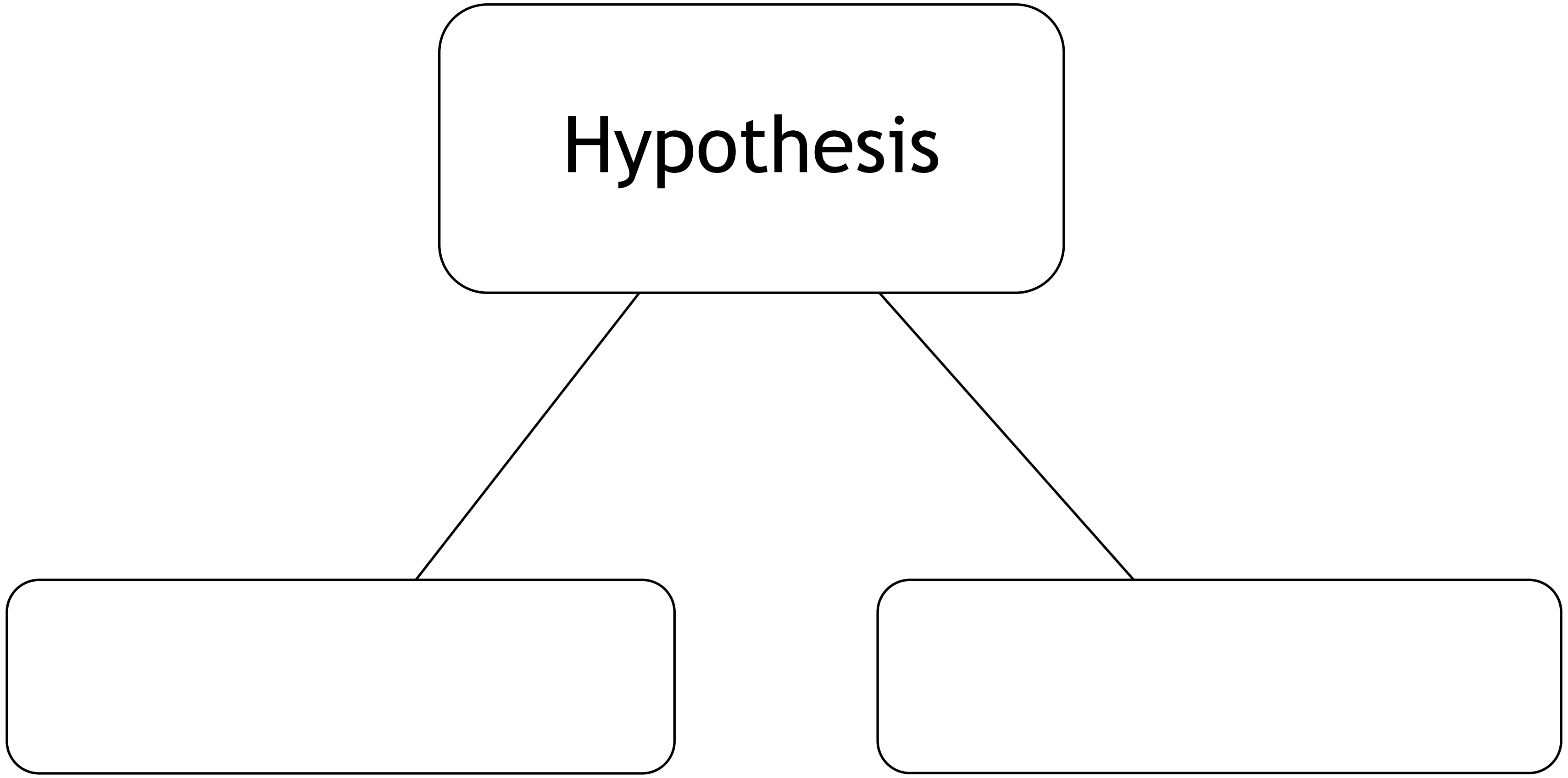
- A statistical hypothesis is some statement about a population parameter or about the probability distribution characterizing a population which we want to verify on the basis of sample information.

- Example: A physician may hypothesize that the recommended drug is effective is 99% cases.

Statistical hypothesis

Types of Statistical hypothesis

- Two types of statistical hypothesis



Types of Statistical hypothesis

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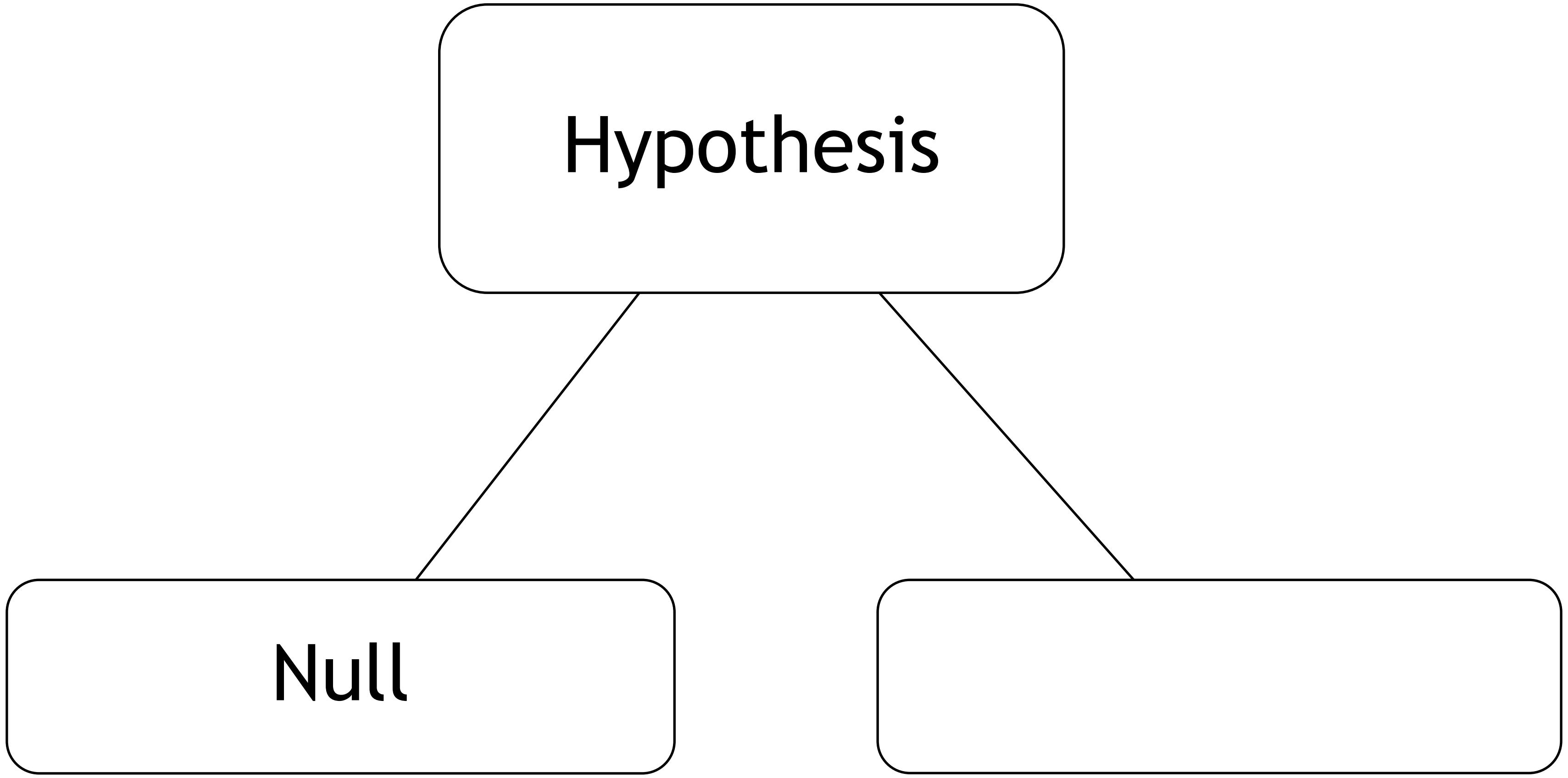

Types of Statistical hypothesis

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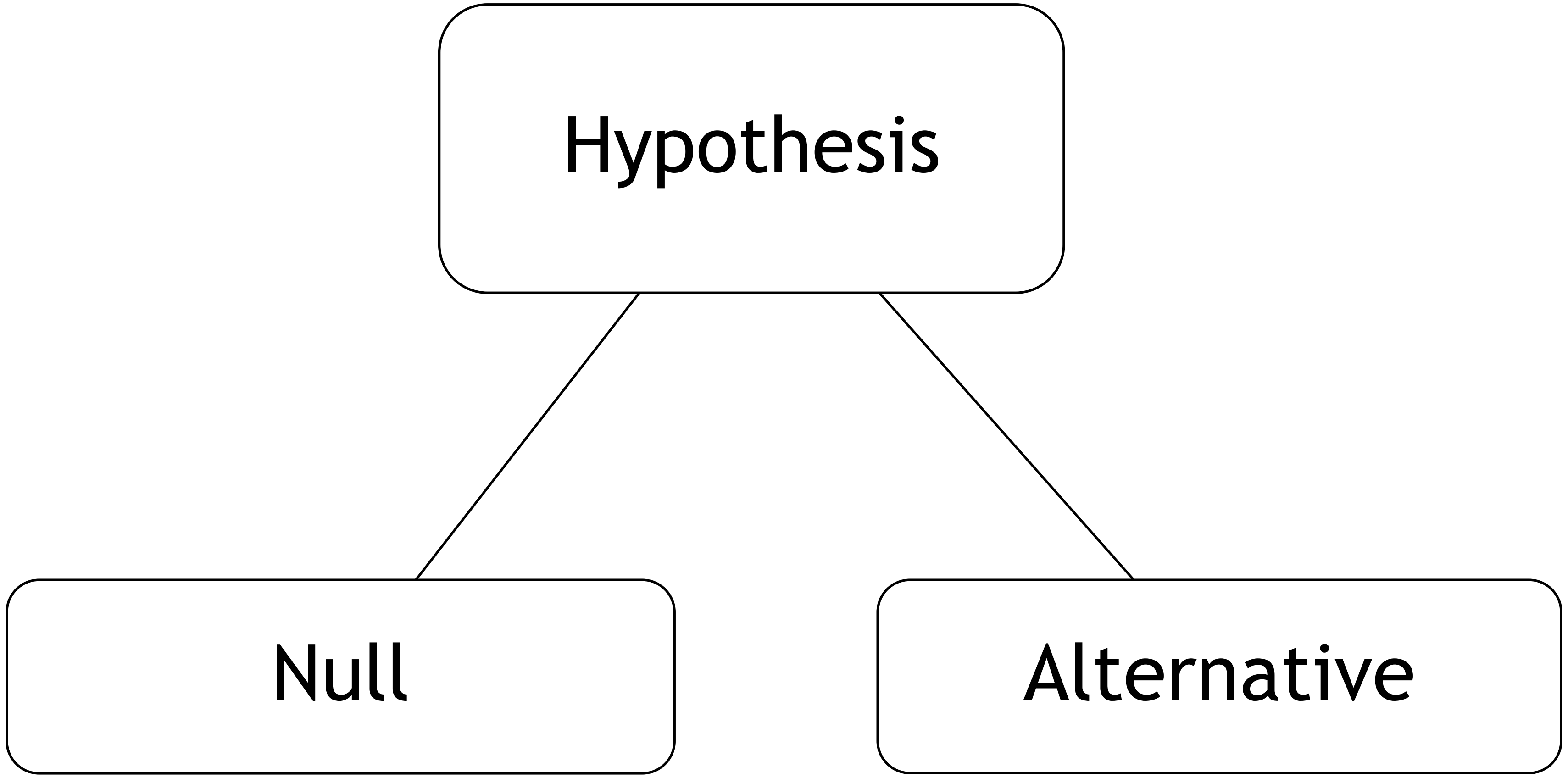


Null hypothesis

- A null hypothesis is a statistical hypothesis that contains a statement of equality
- $\leq or = or \geq$
- Denoted by H_0

Alternative hypothesis

- An alternative hypothesis is a statistical hypothesis that contains a statement of inequality
- $< or \neq or >$
- Denoted by H_1

Example

CSE students' average programming skills are the same whether or not they attend a coding bootcamp. → H_0

CSE students' average programming skills are different depending on whether or not they attend a coding bootcamp → H_1

Example

- A company collects information on the retail price of books and publishes the data in the website. In 2005, the mean retail price of history books was $78.01. Suppose that we want to perform a hypothesis test to decide whether these years mean retail price of history books has increases from the 2005 mean. Determine the null and alternative hypotheses.

$$\boxed{H_0: \mu = 78.01}$$

$$\boxed{H_1: \mu > 78.01}$$

Difference

1. A null hypothesis is a statement of equality. On the other side, An alternative hypothesis is statement of inequality.

2. Denoted by H_0. Denoted by H_1.

3. Mathematical formulation is equal sign. Mathematical formulation is unequal sign.

Test Statistic

Verify on the basis of sample information

Population function → Parameter

Sample function → Statistic → Test Statistic

- Z test → $n \geq 30$
- t test → $n < 30$
- F test
- χ^2 test

Example

- The mean life time of a sample of 100 light tubes produced by a company is found to be 1570 hours with standard deviation of 80 hours. Test the hypothesis that the mean life time of the tubes produced by the company is 1600 hours.

$$\boxed{H_0: \mu = 1600}$$

$$\boxed{H_1: \mu \neq 1600}$$

$$\boxed{z\ test}$$

Example

- Is the temperature required to damage a computer on the average less than 110 degrees? Because of the price of testing, twenty computers were tested to see what minimum temperature will damage the computer. The damaging temperature averaged 109 degrees with a standard deviation of 3 degrees.

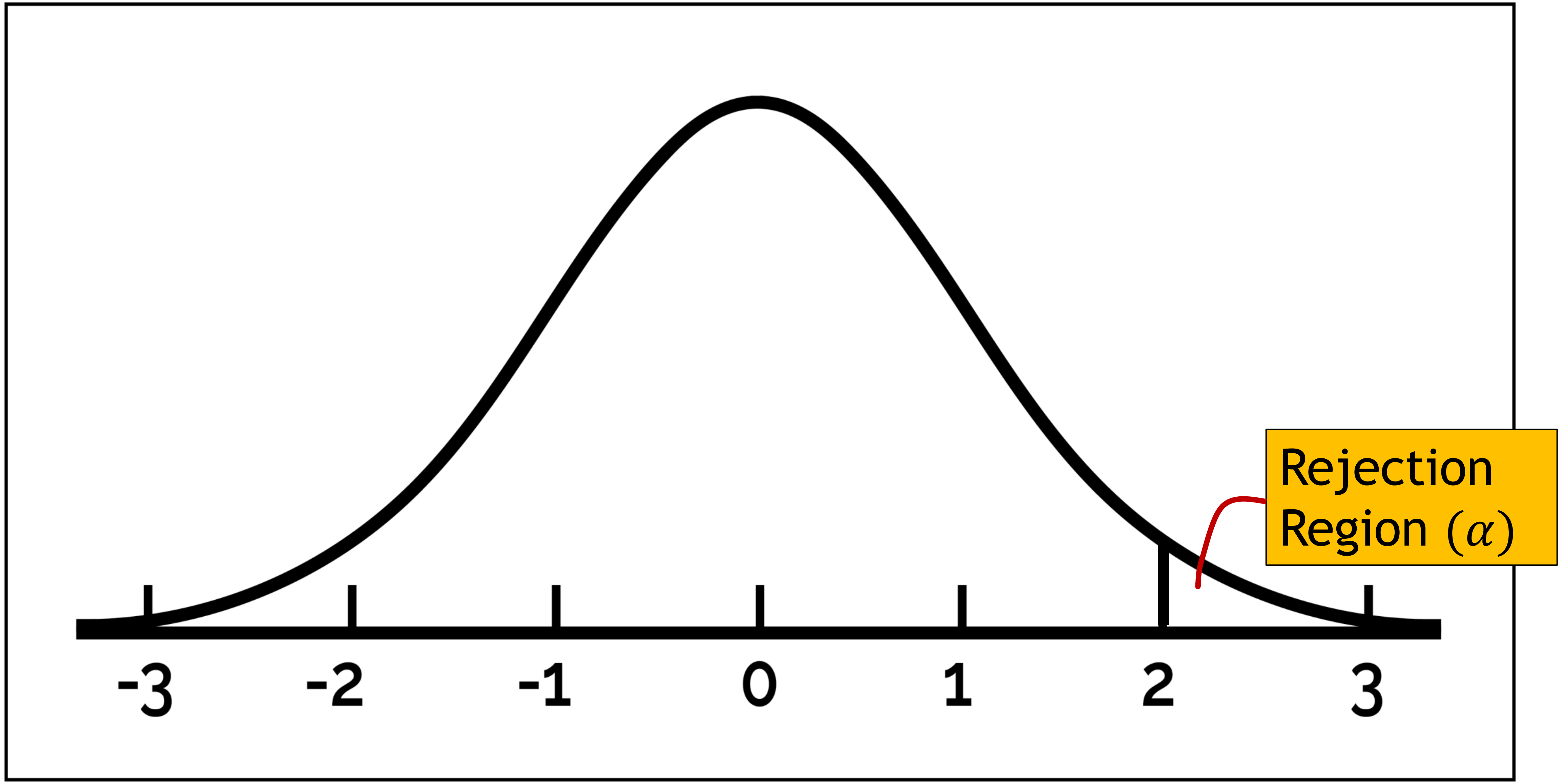
$H_0: \mu \geq 110$

$t\ test$

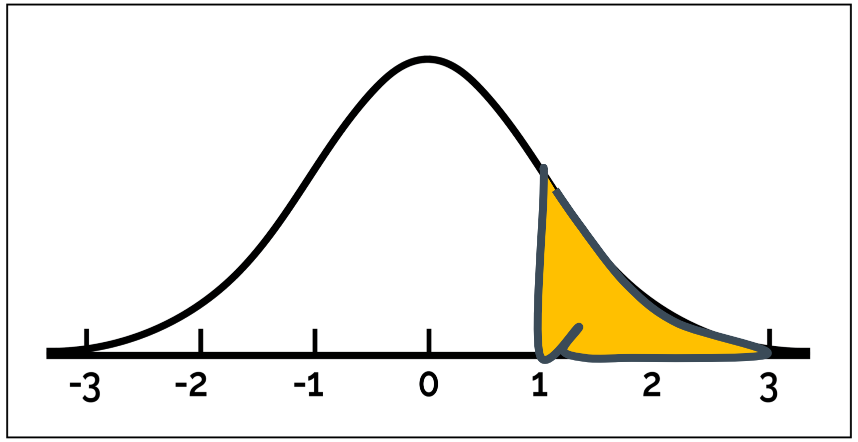
$H_1: \mu < 110$

Rejection Region

The region where we may reject our hypothesis.



Type of tailed



Right tailed test

$H_1: \mu > \mu_0$

- There are two types:

  a. One tailed test

  Entire rejection region lies one side

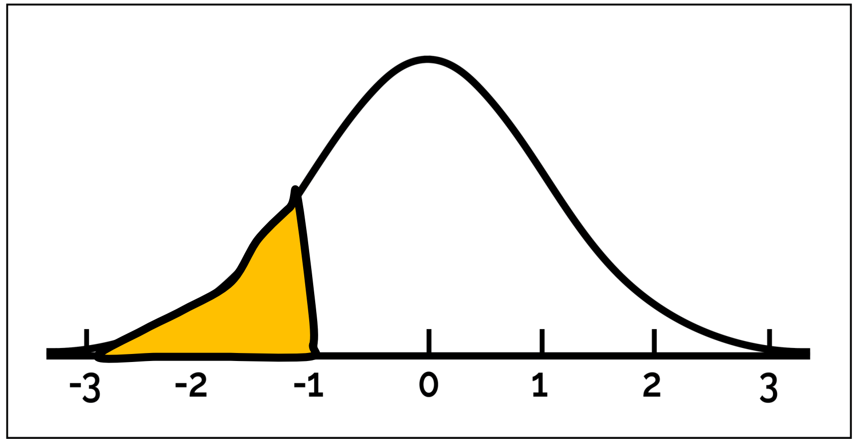
  $H_1: \mu \neq \mu_0$

  b. Two tailed test

  Entire rejection region lies two side

Left tailed test

$H_1: \mu < \mu_0$



Example

- The mean life time of a sample of 100 light tubes produced by a company is found to be 1570 hours with standard deviation of 80 hours. Test the hypothesis that the mean life time of the tubes produced by the company is 1600 hours.

$$H_0: \mu = 1600$$

$$H_1: \mu \neq 1600$$

Two Tailed Test

Example

- Is the temperature required to damage a computer on the average less than 110 degrees? Because of the price of testing, twenty computers were tested to see what minimum temperature will damage the computer. The damaging temperature averaged 109 degrees with a standard deviation of 3 degrees.

$\boxed{H_0: \mu \geq 110}$

$\boxed{Left\ Tailed\ Test}$

$\boxed{H_1: \mu < 110}$

Steps (Z Test)

$$t = \frac{\bar{X} - \mu_0}{\frac{s}{\sqrt{n}}}$$

$$Z = \frac{\bar{X} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$$

1. Identify the hypotheses
2. Choose the value of α; level of significance
3. Appropriate test statistic $(Z \ or \ t)$ and determine the value.
4. Compare this value with critical value obtained by using α
5. Decision

Decision: We may reject H_0 if,

a) For left tailed: $Z_{cal} \leq -(Z_\alpha)$

b) For right tailed: $Z_{cal} \geq +(Z_\alpha)$

c) For two tailed: $|Z_{cal}| \geq Z_{\frac{\alpha}{2}}$

Critical value: When $\alpha = 0.05$

a) For one tailed: $Z_\alpha = 1.645$

b) For two tailed: $Z_{\frac{\alpha}{2}} = 1.96$

Critical value: When $\alpha = 0.01$

a) For one tailed: $Z_\alpha = 2.33$

b) For two tailed: $Z_{\frac{\alpha}{2}} = 2.56$

Example

$$H_0: \mu = 1600$$

$$H_1: \mu \neq 1600$$

$$\alpha = 0.05$$

- The mean life time of a sample of 100 light tubes produced by a company is found to be 1570 hours with standard deviation of 80 hours. Test the hypothesis that the mean life time of the tubes produced by the company is 1600 hours.

$$Z_{cal} = \frac{\bar{X} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$$

$$Z_{cal} = \frac{1570 - 1600}{\frac{80}{\sqrt{100}}} = -3.75$$

Example

$H_0: \mu = 1600$

$H_1: \mu \neq 1600$

$\alpha = 0.05$

$$Z_{cal} = \frac{1570 - 1600}{\frac{80}{\sqrt{100}}} = -3.75$$

- The mean life time of a sample of 100 light tubes produced by a company is found to be 1570 hours with standard deviation of 80 hours. Test the hypothesis that the mean life time of the tubes produced by the company is 1600 hours.

Critical value: When $\alpha = 0.05$

a) For one tailed: $Z_\alpha = 1.645$

b) For two tailed: $Z_{\frac{\alpha}{2}} = 1.96$

Critical value: When $\alpha = 0.01$

a) For one tailed: $Z_\alpha = 2.33$

b) For two tailed: $Z_{\frac{\alpha}{2}} = 2.56$

$Z_{tab} = 1.96$

$|Z_{cal}| > Z_{tab}$

$\mathit{Reject}\ H_0$

Decision: We may reject H_0 if,

a) For left tailed: $Z_{cal} \leq -Z_\alpha$

b) For right tailed: $Z_{cal} \geq +Z_\alpha$

c) For two tailed: $|Z_{cal}| \geq Z_{\frac{\alpha}{2}}$

Example

- A sample of 400 male students is found to have a mean height 67.47 inches. Can it be reasonably regarded as a sample from a large population with mean height 67.39 inches and standard deviation 1.30 inches? Test at 5% level of significance.

Example

- Suppose that it is known from experience that the standard deviation of the weight of 8 ounces packages of cookies made by a certain bakery is 0.16 ounces. To check its production is under control on a given day, the true average of the packages is 8 ounces, they select a random sample of 40 packages and find their mean weight is 8.122 ounces. Test whether the production is under control or not at 5% level of significance.

Mathematical exercise

To access additional mathematical problems,

please refer to the PDF lecture notes.

Thank You