Lecture 4 : SOP & POS

SOP: → Sum of Product

→ Logical sum of several product term

Example: $\underbrace{x}+\underbrace{yz'}$, $\underbrace{xy'}+\underbrace{x'yz}$, $\underbrace{AB}+\underbrace{A'B'}$ each of them are called product term

POS: → Product of Sum

→ Logical product term of several sum terms.

Example: $\underbrace{x}\underbrace{(y+z')}$, $\underbrace{(x+y')}\underbrace{(x'+y+z)}$, $\underbrace{(A+B)}\underbrace{(A'+B')}$ Each of these are called sum term

* Every boolean expression can be expressed using SOP or POS expression.

Min terms:

⇒ For boolean function, minterms of a function are the terms for which, the results is 1.

Max terms:

⇒ For boolean function, minterms of a function are the terms for which, the results is 0.

* Boolean functions can be represent as sum of minterms or product of max terms (like SOP & POS)

Example:

	A	B	C	F
0	0	0	0	1
1	0	0	1	1
2	0	1	0	0
3	0	1	1	0
4	1	0	0	1
5	1	0	1	1
6	1	1	0	0
7	1	1	1	0

Minterms = 0, 1, 4, 5 [000, 001, 100, 101]

$\therefore F = \Sigma(0,1,4,5)$

Maxterms = 2, 3, 6, 7 [010, 011, 110, 111]

$\therefore F = \Pi(2,3,6,7)$

Now, from the minterms & maxterms, we can find out the function in SOP & POS form respectively. The rules are,

Min/SOP	Max/SOP
0 → Prime 1 → No Prime	0 → No Prime 1 → Prime
AND among literals	OR among literals
OR among terms	AND among literal

from the above example:

	A	B	C	F	Min	Max
0	0	0	0	1	$A'B'C'$	
1	0	0	1	1	$A'B'C$	
2	0	1	0	0		$A+B'+C$
3	0	1	1	0		$A+B'+C'$
4	1	0	0	1	$AB'C'$	
5	1	0	1	1	$AB'C$	
6	1	1	0	0		$A'+B'+C$
7	1	1	1	0		$A'+B'+C'$

Minterms = 0, 1, 4, 5

$\therefore F = \Sigma(0,1,4,5)$

Maxterms = 2, 3, 6, 7

$\therefore F = \Pi(2,3,6,7)$

∴ We can write,

$F = A'B'C' + A'B'C + AB'C' + AB'C$ ⟸ SOP form of F

$F = (A+B'+C)(A+B'+C')(A'+B'+C)(A'+B'+C')$ ⟸ POS form of F

* Each Minterm is the complement of the corresponding Maxterm.

x	y	z	minterm	Designation	Maxterms	Designation
0	0	0	$x'y'z'$	m_0	$x+y+z$	M_0
0	0	1	$x'y'z$	m_1	$x+y+z'$	M_1
0	1	0	$x'yz'$	m_2	$x+y'+z$	M_2
0	1	1	$x'yz$	m_3	$x+y'+z'$	M_3
1	0	0	$xy'z'$	m_4	$x'+y+z$	M_4
1	0	1	$xy'z$	m_5	$x'+y+z'$	M_5
1	1	0	xyz'	m_6	$x'+y'+z'$	M_6
1	1	1	xyz	m_7	$x'+y'+z'$	M_7

Example:

$m_1 = x'y'z$ $Dual(m_1) = (x'+y'+z)$ $\therefore (m_1)' = (x+y+z') = M_1$

Similarly we can prove for all minterms & maxterms.

So far, we saw how to derive SOP & POS from given truthtable.

We can also derive SOP & POS from a function.

SOP Rule:

1. Check if each term contains all variables, if not AND $(x+x')$ if x is the missing terms.

2. Minimize the redundant term.

Example:

In this term, A missing.

$$F(A,B,C) = A + B'C$$

In this term B and C missing

$$= A(B+B')(C+C') + B'C(A+A')$$

$$= A(BC + B'C + BC' + B'C') + AB'C + A'B'C$$

$$= ABC + \underbrace{AB'C} + ABC' + AB'C' + \underbrace{AB'C} + A'B'C$$ Redundant

$$= ABC + AB'C + ABC' + AB'C' + A'B'C$$

in SOP, $0 \to x'$, $1 \to x$

$$= \underbrace{111}_{7} + \underbrace{101}_{5} + \underbrace{110}_{6} + \underbrace{100}_{4} + \underbrace{001}_{1}$$

$$= \Sigma(7, 5, 6, 4, 1)$$

$$= \Sigma(1,4,5,6,7)$$

Similarly POS Rule:

1. Check if we can apply, $(x+yz) = (x+y)(x+z)$
2. When can't apply, add missing term x by OR xx' $+ xx'$
3. Go to step 1 again
4. Minimize redundant terms.

Example: $F(A,B,C) = \underbrace{A}_{x} + \underbrace{B'C}_{y\ z}$

$$= \underbrace{(A+B')}_{C \text{ missing}} \underbrace{(A+C)}_{b} \text{ missing}$$

$$= (\underbrace{A+B'}_{x} + \underbrace{C}_{y}\underbrace{C'}_{z})(\underbrace{A+C}_{x} + \underbrace{B}_{y}\underbrace{B'}_{z})$$

$$= (\underbrace{A}_{x} + \underbrace{B'+C}_{y})(\underbrace{A+B'}_{x} + \underbrace{C'}_{z})(\underbrace{A+B}_{x} + \underbrace{C}_{y})(\underbrace{A+B'}_{x} + \underbrace{C}_{z})$$ redundant

$$= (A+B'+C)(A+B'+C')(A+B+C)$$

$$= (0+1+0)(0+1+1)(0+0+0)$$

$$= \Pi(2,3,0)$$

$$= \Pi(0,2,3)$$

More Examples:

* Express $F(w,x,y,z) = wy + x'z$ in cannonical SOP.

$$= wy(x+x')(z+z') + x'z(w+w')(y+y')$$

$$= wy(xz + x'z + xz' + x'z') + x'z(wy + wy' + w'y + w'y')$$

$$= wxyz + wx'yz + wxyz' + wx'yz' + wx'yz + wx'y'z + w'x'yz + w'x'y'z$$

$$= wxyz + wx'yz + wxyz' + wx'yz' + wx'y'z + w'x'yz + w'x'y'z$$

$$= 1111 + 1011 + 1110 + 1010 + 1001 + 0011 + 0001$$

$$= \Sigma(15, 11, 14, 10, 9, 3, 1)$$

$$= \Sigma(1, 3, 9, 10, 11, 14, 15)$$

* Express $F(w,x,y,z) = wy + x'z$ in cannonical POS

Do it yourself. Solution can be found in Slides.

★ Sometimes, a function may not have all literals.

$$F(A,B,C) = AB + A'$$

In such case, we have to consider 'C' as missing variable.

$$F(A,B,C) = AB(C+C') + A'(B+B')(C+C')$$

$$= ABC + ABC' + A'(BC + BC' + B'C + B'C')$$

$$= \underset{111}{ABC} + \underset{110}{ABC'} + \underset{011}{A'BC} + \underset{010}{A'BC'} + \underset{001}{A'B'C} + \underset{000}{A'B'C'}$$

$$= \Sigma(7, 6, 3, 2, 1, 0)$$

Similarly,

$$F(A,B,C,D) = A + BC$$

$$= (A+B)(A+C)$$

$$= (A+B+CC')(A+C+BB')$$

$$= (A+B+C)(A+B+C')(A+B+C)(A+B'+C)$$

$$= (A+B+C)(A+B'+C)(A+B+C')$$

$$= (A+B+C+DD')(A+B'+C+DD')(A+B+C'+DD')$$

$$= \underset{0000}{(A+B+C+D)}\underset{0001}{(A+B+C+D')}\underset{0100}{(A+B'+C+D)}\underset{0101}{(A+B'+C+D')}$$

$$\underset{0010}{(A+B+C'+D)}\underset{0011}{(A+B+C'+D')}$$

$$= \Pi(0, 1, 4, 5, 2, 3)$$