Binary Logics:

→ Consist of binary variables and logical operation

- variables
  - → A, B, C, x, y, z
  - → possible values [0, 1]
- operation
  - → AND
  - → OR
  - → NOT
  - → etc.

Logic Gates:

→ Most Basic Digital Device

→ 1 or more input & 1 outputs

→ Output follows a certain logics

Truthtable:

→ A list of all possible inputs and outputs

All logic gates

NOT, AND, OR, XOR

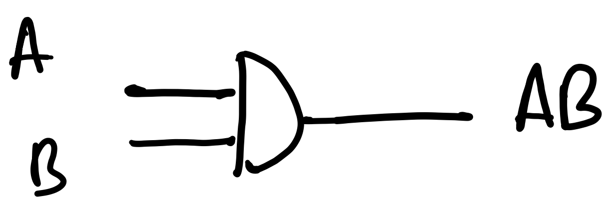
NAND, NOR, XNOR

* NOT, AND, OR are Basic gates.

* NAND & NOR are universal gate.

A universal gate is a gate which can implement any other gate.

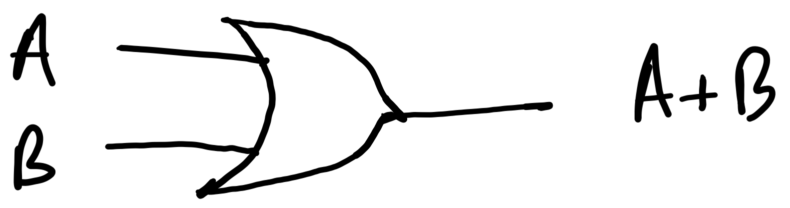
AND :



A	B	AB
0	0	0
0	1	0
1	0	0
1	1	0

* Output 1 when all inputs are 1,

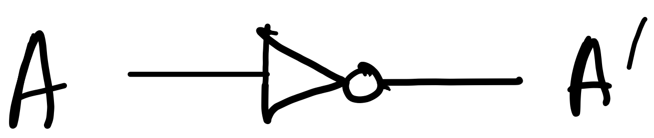
OR :



A	B	A+B
0	0	0
0	1	1
1	0	1
1	1	1

* Output 1 when atleast one input is 1

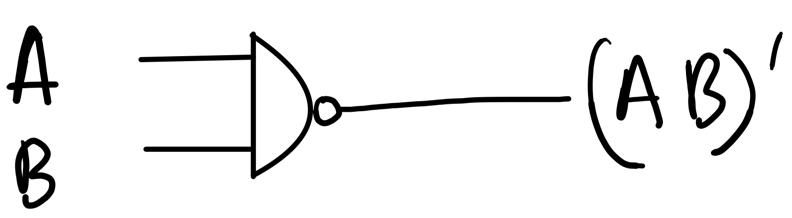
NOT (Inverter)



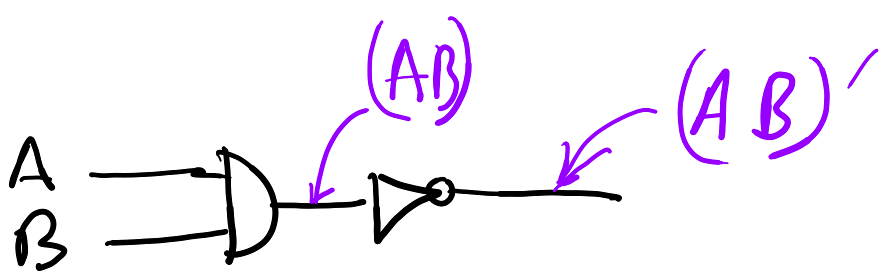
A	A'
0	1
1	0

* Output is inverse of input

NAND



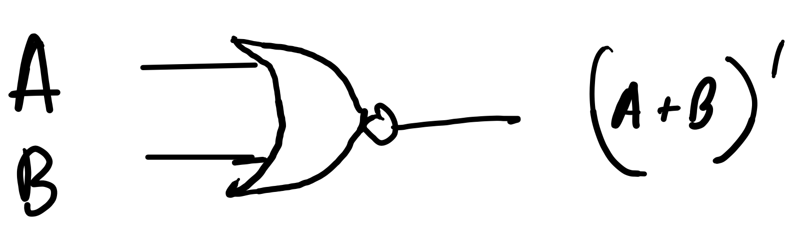
aka



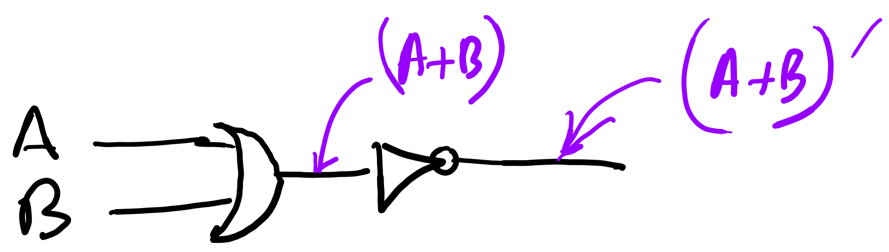
A	B	(A.B)'
0	0	1
0	1	1
1	0	1
1	1	0

* Inverse of AND

NOR



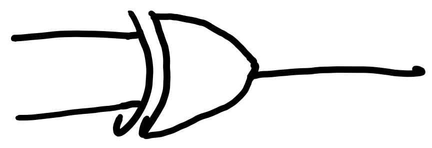
aka



A	B	(A+B)'
0	0	1
0	1	0
1	0	0
1	1	0

* Inverse of OR

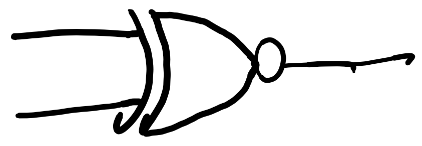
XOR

A, B  $A \oplus B = AB' + A'B$

A	B	$(A \oplus B)'$
0	0	0
0	1	1
1	0	1
1	1	0

Output 1 if odd number of input is 1

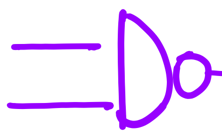
XNOR :

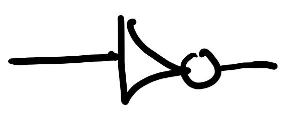
A, B  $A \odot B = AB + A'B$

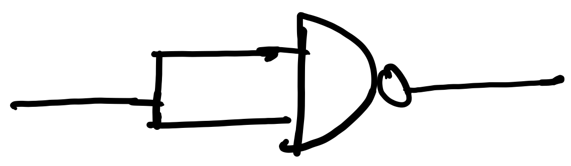
A	B	$(A \odot B)'$
0	0	1
0	1	0
1	0	0
1	1	1

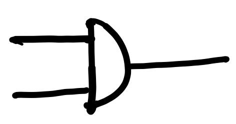
Inverse of XOR

Basic Gates using NAND :

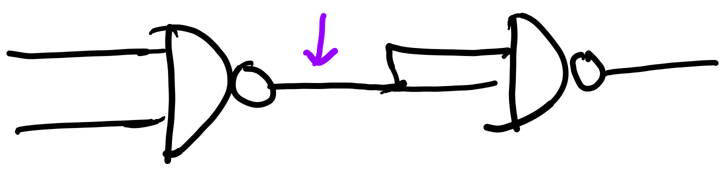
NAND A, B  $(AB)'$

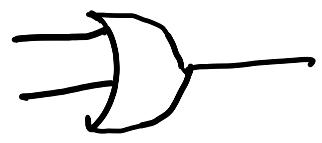
NOT A  A'

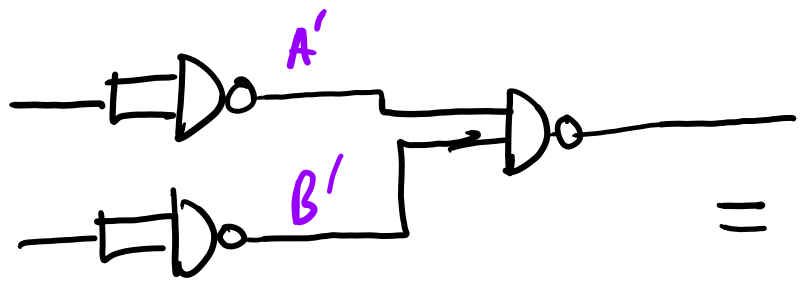
A  $(AA)' = A'$

AND A, B  AB

$(AB)'$

A, B  $((AB)')' = AB$

OR A, B  $A+B$

A, B 

$(A'B')'$

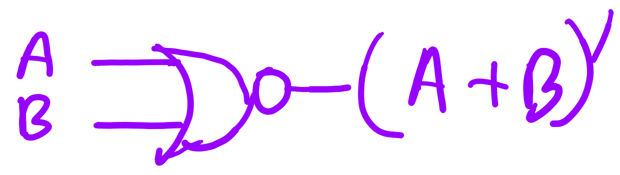
$= (A')' + (B')'$

$= A + B$

Using DeMorgan's law

Basic Gates using NOR :

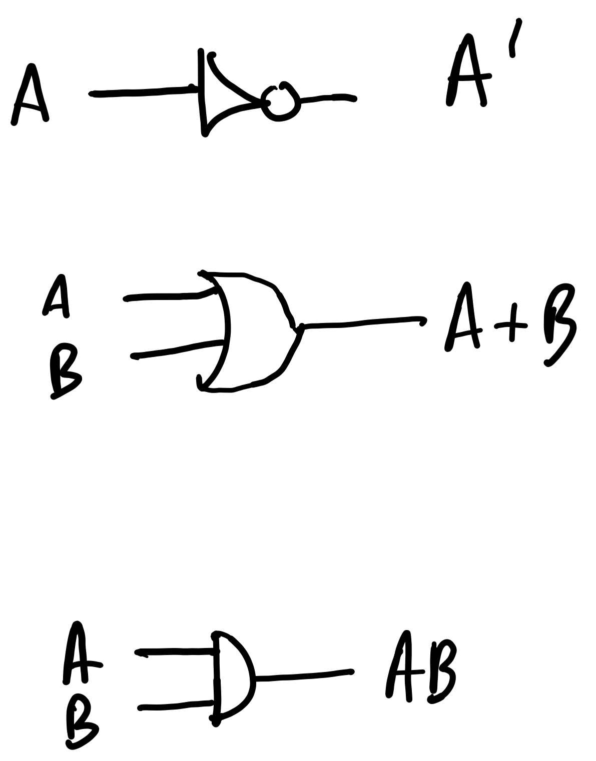
NOR

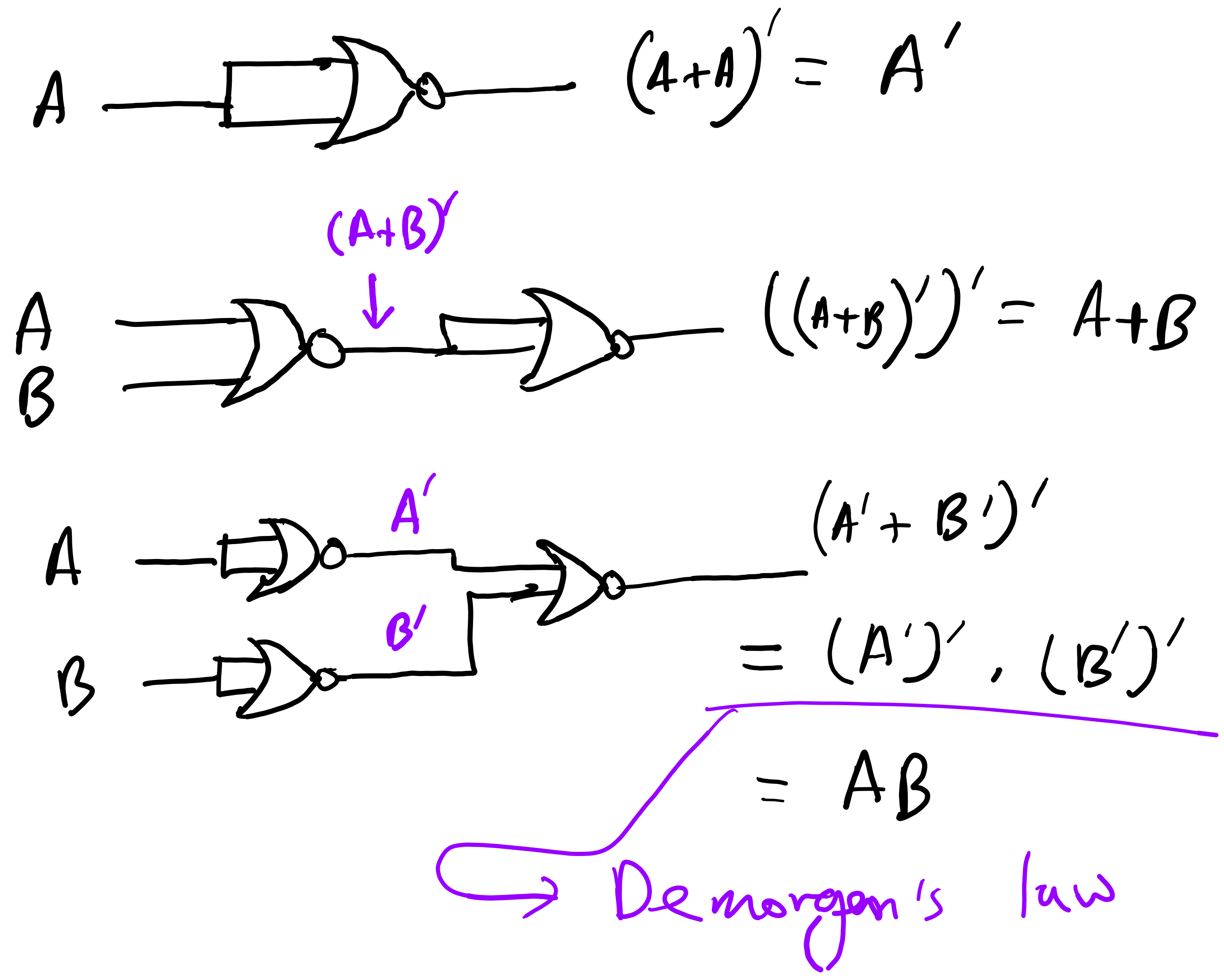


NOT

OR

AND





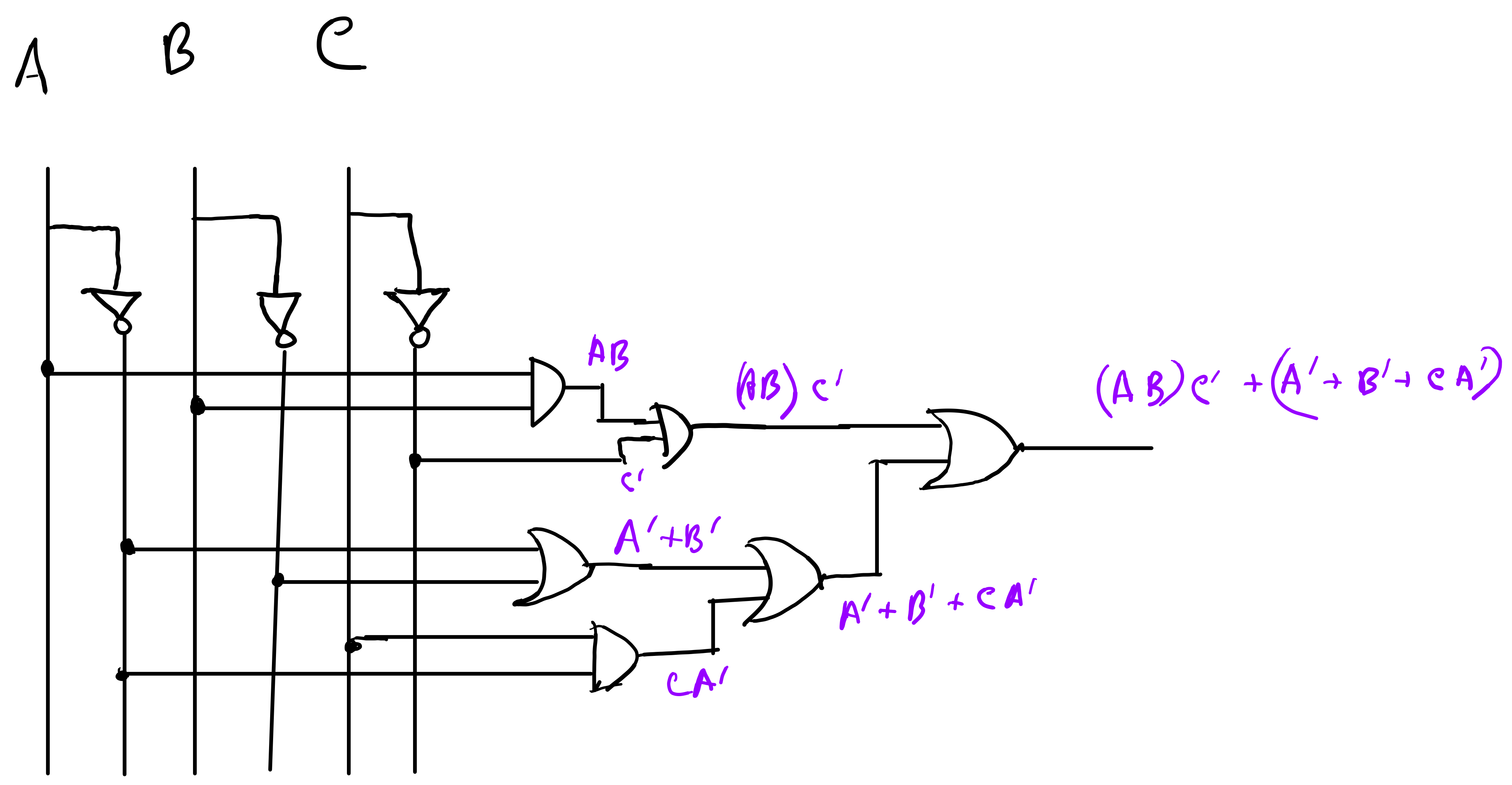
Proof using Truth table:

✱ Proof that $\underbrace{x \cdot (y+z)}_{\text{LHS}} = \underbrace{(x \cdot y) + (x \cdot z)}_{\text{RHS}}$

x	y	z	$y+z$	$x \cdot (y+z)$	$x \cdot y$	$x \cdot z$	$(x \cdot y) + (x \cdot z)$
0	0	0	0	0	0	0	0
0	0	1	1	0	0	0	0
0	1	0	1	0	0	0	0
0	1	1	1	0	0	0	0
1	0	0	0	0	0	0	0
1	0	1	1	1	0	1	1
1	1	0	1	1	1	0	1
1	1	1	1	1	1	1	1

$\therefore$ LHS = RHS.

Implement $(AB)C'+(A'+B'+CA')$ using Basic Gates



Boolean Algebra

Basic theorem of Boolean Algebra

* Theorem can be proved using Truthtable or Algebric Manipulation

Basic Theorems are:

$\Rightarrow x + 0 = x$	$x \cdot 0 = x$	identity
$\Rightarrow x + x' = 1$	$x \cdot x' = 0$	complement
$\Rightarrow x + x = x$	$x \cdot x = x$	
$\Rightarrow x + 1 = 1$	$x \cdot 0 = 0$	
$\Rightarrow (x')' = x$		
$\Rightarrow x+y = y+x$	$x \cdot y = y \cdot x$	commutative
$\Rightarrow x(yz) = (xy)z$	$x+(y+z) = (x+y)+z$	
* $\Rightarrow x(y+z) = xy+xz$	* $x+yz = (x+y)(x+z)$	Distributive
* $\Rightarrow (x+y)' = x'y'$	* $(xy)' = x'+y'$	De Morgan
* $\Rightarrow x + xy = x$	* $x(x+y) = x$	Absorption

* All are important but student tend to forget star marked ones while simplifying

Duality Principle:

* If we interchange operators and elements as follows,

$$\boxed{(\text{OR}) \; + \rightleftharpoons \cdot \; (\text{AND})}$$

$$\boxed{0 \rightleftharpoons 1}$$

Example:

if $a + (b \cdot c) + 0 = (a+b) \cdot (a+c) \cdot 1$

then we can find its dual expression

$$a \cdot (b+c) \cdot 1 = (a \cdot b) + (b \cdot c) + 0$$

* Duality Gives us free theorems

If a theorem/expression for example,

$x + 1 = 1$ is valid/Proved.

then its dual expression,

$x \cdot 0 = 0$ is also valid/proved

Operator Precedence:

Parenthesis $\longrightarrow$ NOT $\longrightarrow$ AND $\longrightarrow$ OR

Highest   Lowest

Boolean Expression simplification: (Using Boolean Algebric Manipulation)

* Simplify: $(x'y'z) + (x'yz) + (xy')$

$$= x'z(y+y') + xy'$$

$$= x'z \cdot 1 + xy'$$

$$= x'z + xy'$$

* Simplify, $xy + xy'$

$$= x(y+y') = x \cdot 1 = x$$

* Simplify, $BC + AB' + AB + BCD$

$$= BC(1+D) + AB' + AB$$

$$= BC \cdot 1 + AB' + AB$$

$$= BC + A(B'+B)$$

$$= BC + A \cdot 1$$

$$= BC + A$$

Prove, $(A \oplus B)' = A \odot B$ using Boolean Manipulation:

$$(A \oplus B)'$$
$$= (AB' + A'B)'$$
$$= (AB')' \cdot (A'B)' \quad \longrightarrow \text{Applied Demorgan}$$
$$= (A' + B'') \cdot (A'' + B') \quad \longrightarrow \text{"} \quad \text{"}$$
$$= (A' + B) \cdot (A + B') \quad \longrightarrow$$
$$= \underbrace{AA'}_{0} + A'B' + BA + \underbrace{BB'}_{0}$$
$$= A'B' + BA$$
$$= AB + A'B'$$
$$= A \odot B$$

Practice more & more.

Complement of a function:

if $F = (x+y)$, then its complement is $F' = (x+y)'$

However, we can also find complement of a function by,

1. Taking Dual of the function
2. Complement each variables

Example:

$$F = x'yz' + x'y'z$$

1. Dual $= (x' + y + z') \cdot (x' + y' + z)$
2. Complement each var $= (x + y' + z) \cdot (x + y + z') = F$

* Find complement of $x(y'z' + yz)$

1. Dual $= x + ((y' + z') \cdot (y + z))$

2. Complement $= x' + (y + z) \cdot (y' + z')$

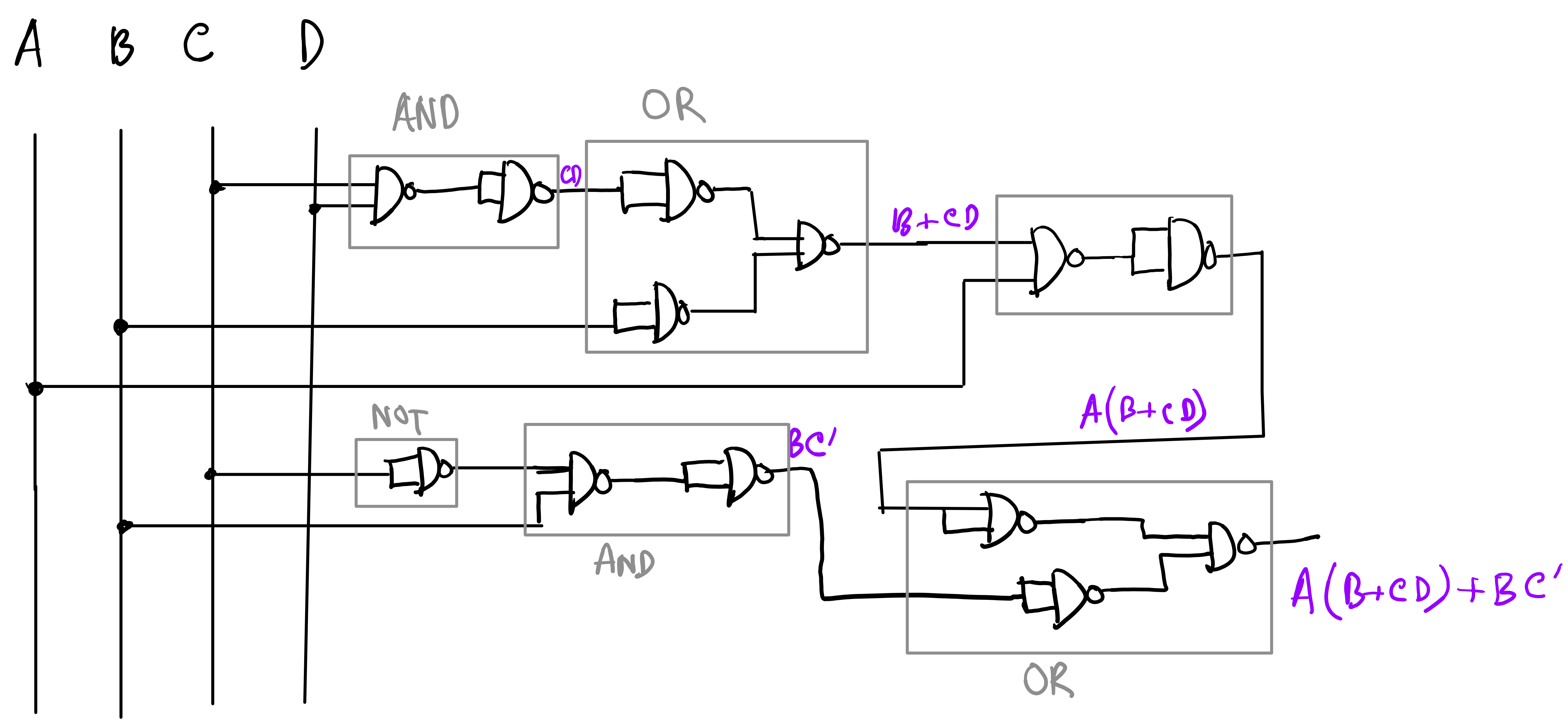
$\therefore \quad F' = x' + (y+z)(y'+z')$

Using NAND or NOR to Draw circuit of a function:

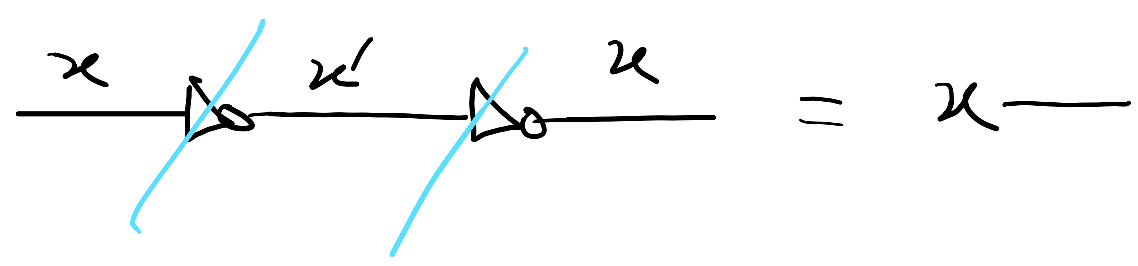
Steps:

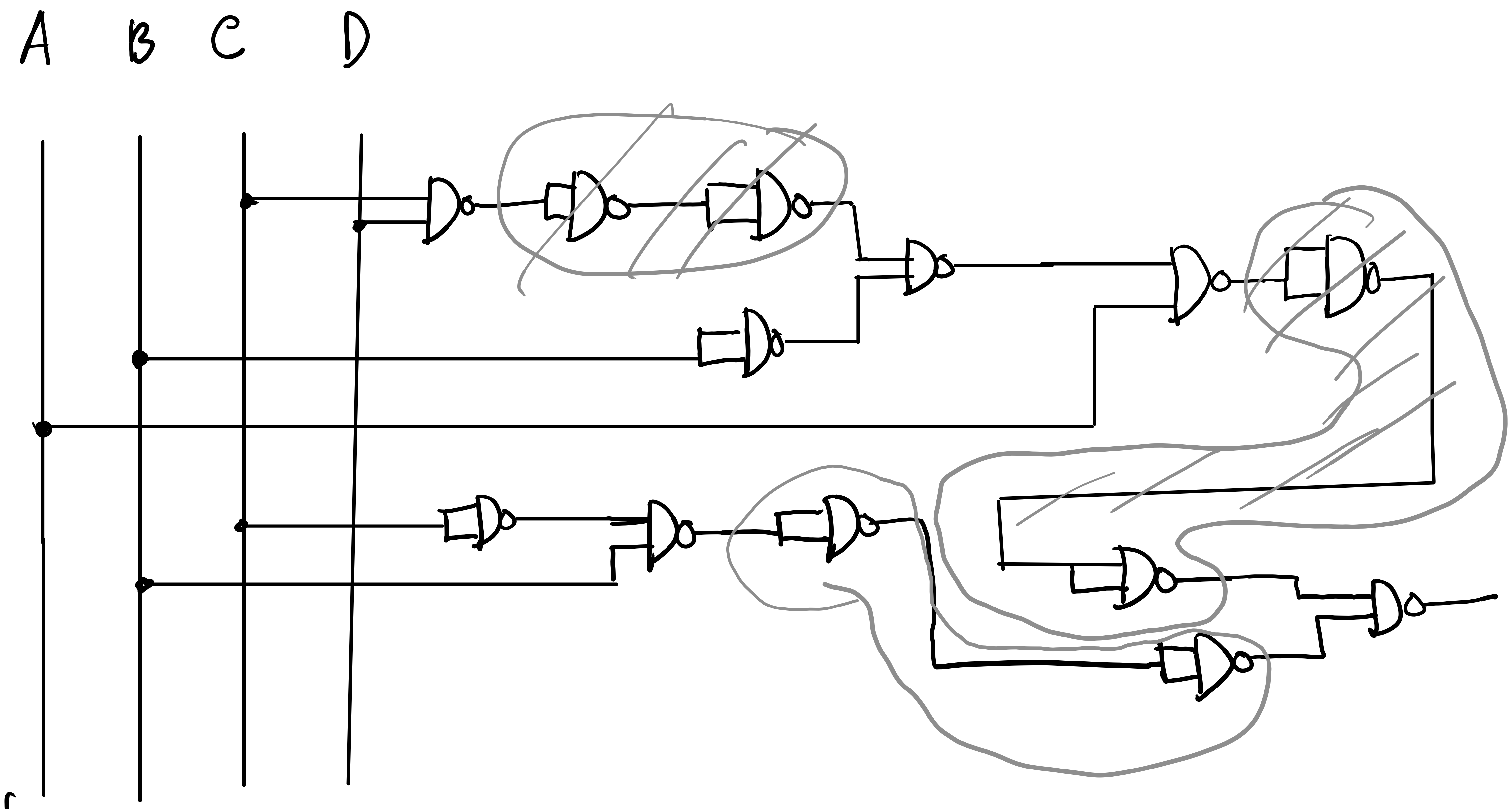
1. Represent the expression using AND, OR, NOT gate.
2. Draw each gate with equavalent NAND / NOR representation.
3. Remove any 2 cascading inverter.
4. Remove inverter from single input connection and replace input with it's complement. (Only if there is a constraint

Implement $F = A(B+CD) + BC'$ using NAND only.



Now, if we want to reduce number of gates, we can remove cascading inverters.





Therefore