1. A coin tossed three times in which the probability of head is twice as the probability of tail. If the number of head is a random variable, find the probability mass function of the random variable. Also find,

a) $P(X \geq 1)$

b) $P(X = 2)$

c) $P(X \leq 1)$

Solution:

Let, H be the head of the coin, and T be the tail of the coin. The sample space of the experiment is,

$$S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$$

Since, the probability of head is twice as the probability of tail,

$$P(H) = \frac{2}{3}$$

$$P(T) = \frac{1}{3}$$

So,

$$P(HHH) = P(H) \times P(H) \times P(H) = \frac{8}{27}$$

$$P(HHT) = \frac{4}{27}$$

$$P(HTH) = \frac{4}{27}$$

$$P(THH) = \frac{4}{27}$$

$$P(HTT) = \frac{2}{27}$$

$$P(THT) = \frac{2}{27}$$

$$P(TTH) = \frac{2}{27}$$

$$P(TTT) = \frac{1}{27}$$

Hence, the probability mass function is,

X:	0	1	2	3
$P(X)$:	1/27	6/27	12/27	8/27

a) $P(X \geq 1) = P(X = 1) + P(X = 2) + P(X = 3) = 26/27$

b) $P(X = 2) = \frac{12}{27}$

c) $P(X \leq 1) = 7/27$

2. Let X be a random variable with PMF defined by P(-2)=1/10, P(0)=2/10, P(4)=4/10, P(11)=3/10. Find,

a) $P(-2 \leq X \leq 4)$

b) $P(X > 0)$

c) $P(X \leq 4)$

3. A random variable X has following function,

X:	0	1	2	3	4	5	6	7	8
$P(X)$:	a	3a	5a	7a	9a	11a	13a	15a	17a

a) Determine the values of a. [Hints:$\sum P(X) = 1$]

4. The probability function of X can be expressed as,

$$P(x) = \frac{x-1}{10}; x = 2,3,4,5$$

Calculate $P(X \geq 3), P(2 \leq X \leq 3)$

5. $f(x) = \begin{cases} Ax; 0 \leq X \leq 5 \\ A(10-x); 5 \leq X \leq 10 \\ 0; Otherwise \end{cases}$

a) Find the value of A. $[Hints: \int f(x)dx = 1]\ ans: \frac{1}{25}$

b) Calculate probability

   a. More than 5

   b. Less than 2

   c. Between 2.5 and 7.5

6. A continuous random variable X has PDF

$$f(x) = 3x^2; 0 \leq X \leq 1$$

Find "a" such that $P(X \leq a) = P(X > a)$

Solution:

Since, $P(X \leq a) = P(X > a)$ each must be equal to $1/2$, because total probability always 1.

Now,

$$P(X \leq a) = \frac{1}{2}$$

$$\Rightarrow \int_0^a f(x)dx = \frac{1}{2}$$

$$\Rightarrow \int_0^a 3x^2 dx = \frac{1}{2}$$

$$\Rightarrow a^3 = \frac{1}{2}; [After\ integration]$$

$$\therefore a = \left(\frac{1}{2}\right)^3$$

7.

Baier's Electronics manufactures computer parts that are supplied to many computer companies. Despite the fact that two quality control inspectors at Baier's Electronics check every part for defects before it is shipped to another company, a few defective parts do pass through these inspections undetected. Let x denote the number of defective computer parts in a shipment of 400. The following table gives the probability distribution of x.

x	0	1	2	3	4	5
$P(x)$	.02	.20	.30	.30	.10	.08

Compute the standard deviation of x.

[Ans: 1.204 defective computer parts]

8. Loraine Corporation is planning to market a new makeup product. According to the analysis made by the financial department of the company, it will earn an annual profit of \$4.5 million if this product has high sales, it will earn an annual profit of \$1.2 million if the sales are mediocre, and it will lose \$2.3 million a year if the sales are low. The probabilities of these three scenarios are .32, .51, and .17, respectively.

a) Let x be the profits (in millions of dollars) earned per annum from this product by the company. Write the probability distribution of x.

(a) The table below lists the probability distribution of x. Note that because x denotes profits earned by the company, the loss is written as a *negative profit* in the table.

x	$P(x)$
4.5	.32
1.2	.51
−2.3	.17

b) Calculate the mean and standard deviation of x. (Ans: \$1.661 million, \$2.314 million)

Interpretation: Thus, it is expected that Loraine Corporation will earn an average of \$1.661 million in profits per year from the new product, with a standard deviation of \$2.314 million.

9. Let X be a random variable with probability function,

$$f(x) = \begin{cases} \frac{1}{9}(4x - x^2); for\ 1 \le x \le 4 \\ 0; Other\ wise \end{cases}$$

Find,

a) $E(X); ans: \frac{81}{36}$

b) $E(X^2); ans: \frac{252}{45}$

c) $V(X)$

d) $E(X-1); ans: 1.25$

10. Given that,

$$f(x) = \frac{3}{4}x(2-x); 0 \leq x \leq 2$$

Find mean, median, and comment about its asymmetrical characteristics.

Solution:

$$Mean, E(x) = \int_0^2 xf(x)dx = 1$$

Median: As median is 50% of the observation, we can write,

$Median = 50\%\ or\ 0.5\ or \frac{1}{2}$

$Q_1 = Lowest\ 25\%\ or\ 0.25\ or \frac{1}{4}$

$Q_3 = Highest\ 25\% = Lowest\ 75\% = \frac{3}{4}$

$$\int_0^M f(x)dx = 0.5$$

$$\Rightarrow \frac{3}{4}\int_0^M x(2-x)dx = 0.5$$

$$\Rightarrow (M-1)(M^2 - 2M^2 - 2) = 0; [On\ simplification. You\ must\ be\ solve\ step\ by\ step]$$

$$\therefore M = 1$$

Here, $Mean = Median$, so the distribution is symmetrical.