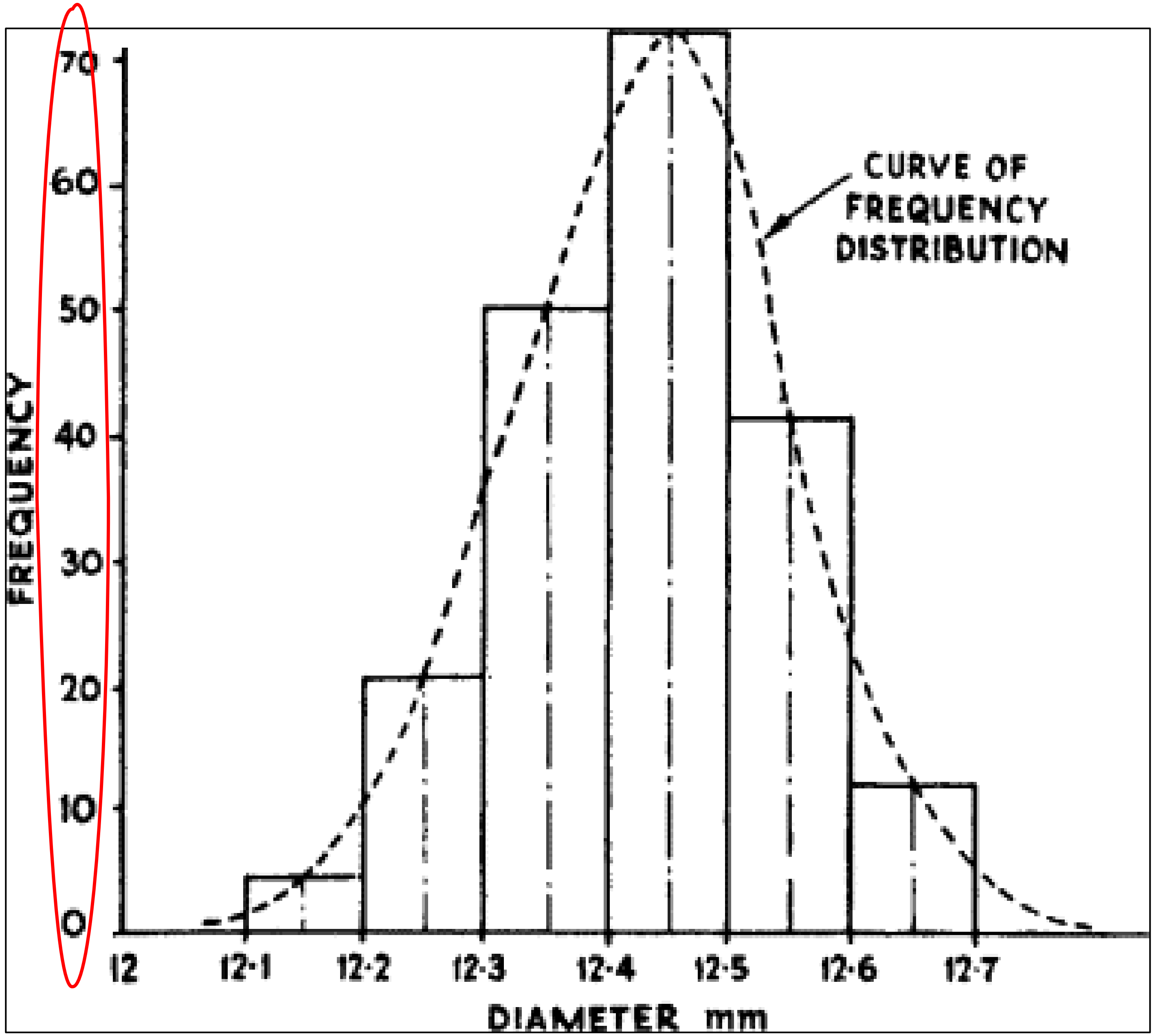
Probability Distribution (2)

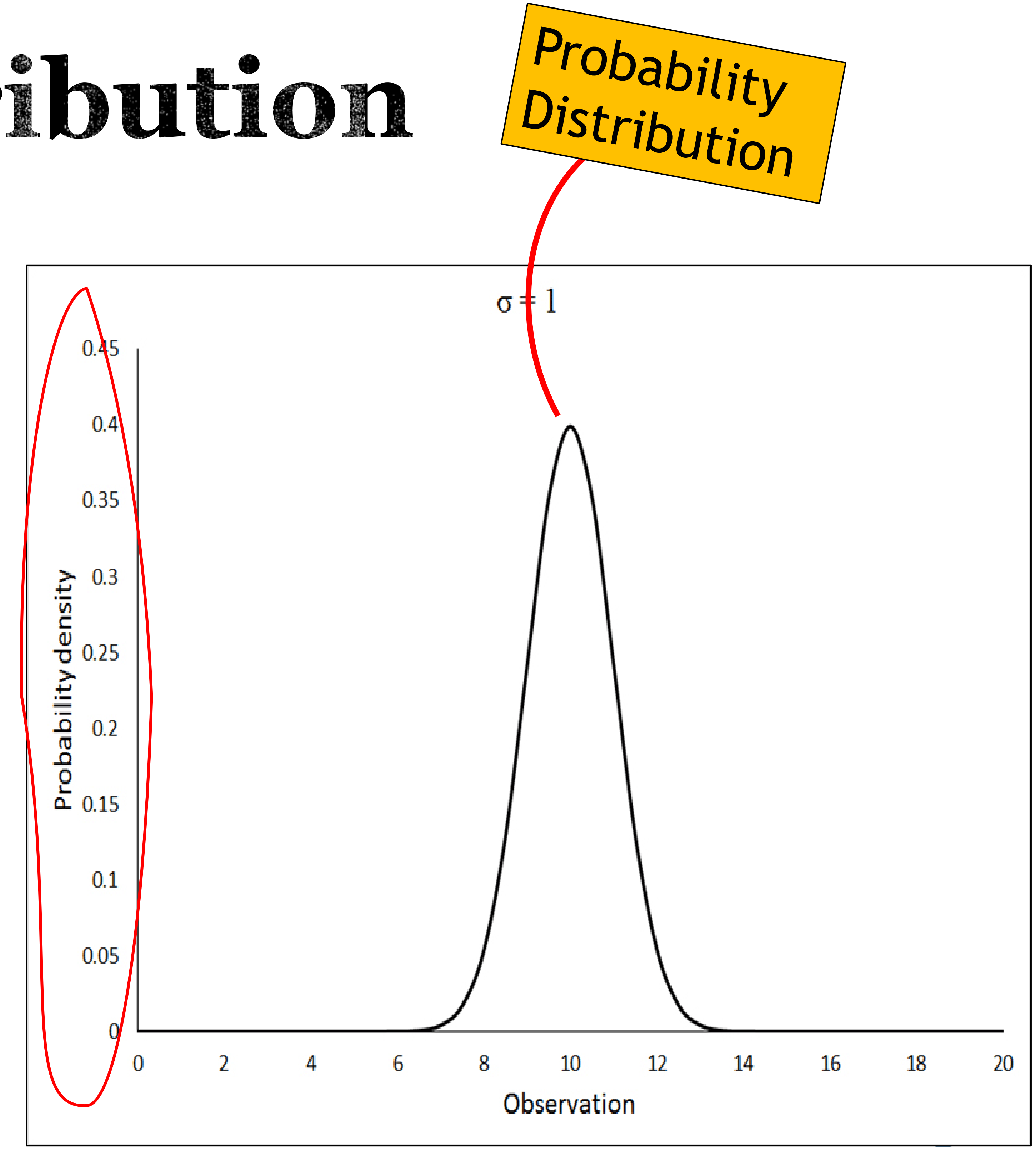
Md. Ismail Hossain Riday

Probability Distribution

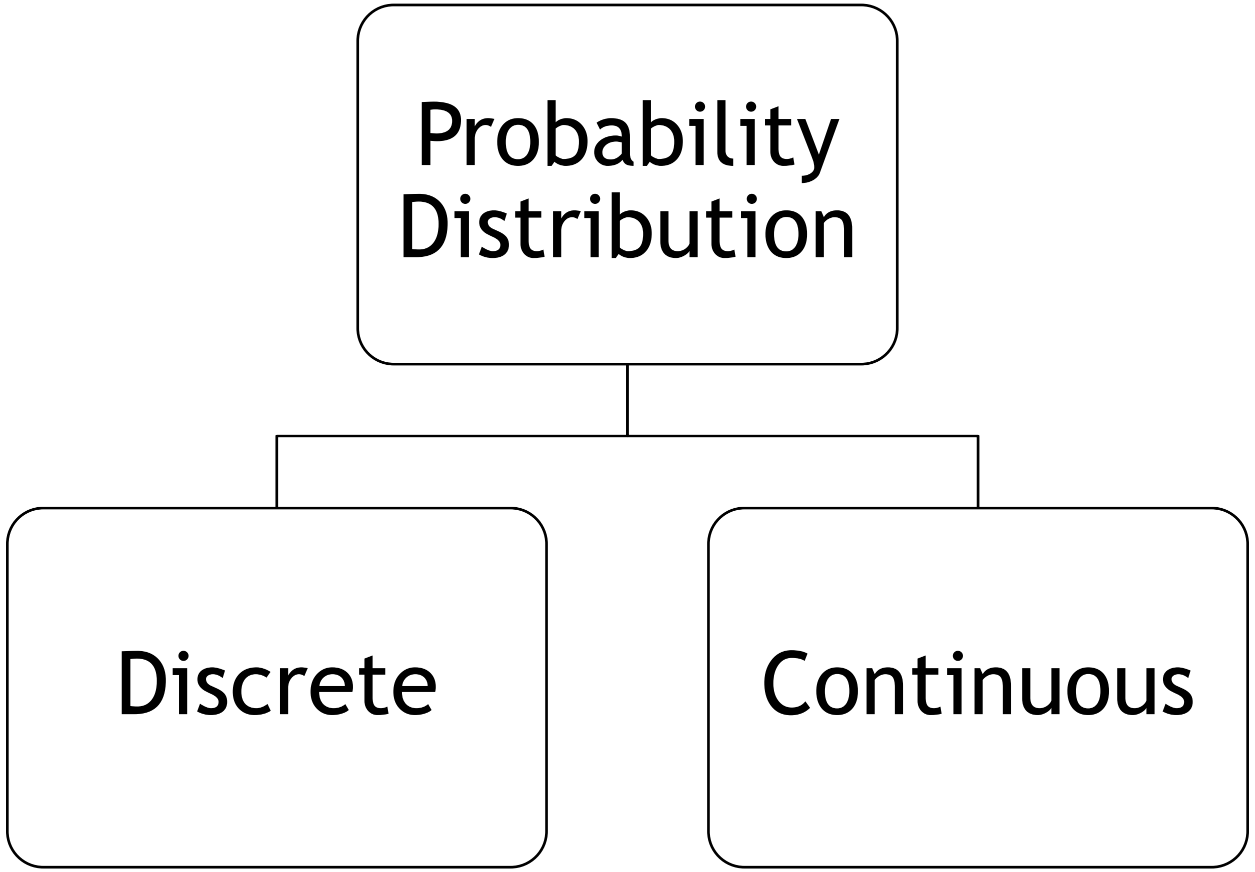
- Distribution of the probabilities among the different values of a random variable.

Probability Distribution





Probability Distribution



Probability Distribution

- Discrete PD
  - Binomial Distribution
  - Poisson Distribution
  - Geometric Distribution
- Continuous PD
  - Uniform Distribution
  - Normal Distribution
  - Exponential Distribution

Binomial Distribution

- First we need to learn about Bernoulli trial.

- Suppose an experiment has only two outcomes
  a. Success
  b. Failure

Bernoulli variable $\Rightarrow$ $X = \begin{cases} 1; if\ succsess\ occurs \\ 0; if\ failure\ occurs \end{cases}$

Binomial Distribution

- “**n**” independent Bernoulli trials are performed
- Each trial has two outcomes: Success and Failure
- Probability of success p and probability of failure $(1 - p)$

Binomial Distribution

- Probability mass function of binomial distribution,

$$p(x) = {}^{n}C_x \, p^x \, (1-p)^{n-x}; x = 0,1,2,3,\ldots,n$$

$$X \sim binomial\ (n, p)$$

"n" and "p" are Parameter

Binomial Distribution

- Mean of the binomial distribution

$$\mu = E(X) = \sum x\, p(x) = np$$

- Variance of the binomial distribution

$$\sigma^2 = E(X^2) - \left(E(X)\right)^2 = np(1-p)$$

Binomial Distribution

A fair coin is tossed 5 times. Find the probability of (a) exactly two heads,(b) at least 4 heads,(c) at most 2 head,(d) no heads, e) Find the mean and variance of that distribution.

Solution: Let the number of heads be random variate X which can take values 0, 1,2,3,4,and 5. X is a binomial variate with probability $\frac{1}{2}$ and $n = 5$.

Binomial Distribution

(a) exactly two heads

$$p(x) = {}^{n}C_x \, p^x \, (1-p)^{n-x}$$

$$p(X = 2) = {}^{5}C_2 \left(\frac{1}{2}\right)^2 \left(1 - \frac{1}{2}\right)^{5-2}$$

$$\therefore p(X = 2) = 0.3125$$

(b) at least 4 heads

$$p(x) = {}^{n}C_x \, p^x \, (1-p)^{n-x}$$

$$p(X \geq 4) = P(X = 4) + P(X = 5)$$

$$\therefore p(X = 4) = {}^{5}C_4 \left(\frac{1}{2}\right)^4 \left(1 - \frac{1}{2}\right)^{5-4}$$

$$\therefore p(X = 5) = {}^{5}C_5 \left(\frac{1}{2}\right)^5 \left(1 - \frac{1}{2}\right)^{5-5}$$

$$\therefore P(X = 4) + P(X = 5) = 0.1875$$

Binomial Distribution

(c) at most 2 head

$$p(x) = {}^{n}C_{x}\, p^{x}\, (1-p)^{n-x}$$

$$p(X \le 2) = P(X = 0) + P(X = 1) + P(X = 2)$$

$$\therefore p(X \le 2) = 0.5$$

(d) no heads

$$p(x) = {}^{n}C_{x}\, p^{x}\, (1-p)^{n-x}$$

$$p(X = 0) = {}^{5}C_{0} \left(\frac{1}{2}\right)^{0} \left(1 - \frac{1}{2}\right)^{5-0}$$

$$\therefore p(X = 0) = 0.03$$

Binomial Distribution

- $Mean = np = 5 \times \frac{1}{2} = 2.5$

- $Variance = np(1-p) = 5 \times \frac{1}{2} \times \frac{1}{2} = 1.25$

$$\boxed{Mean = np}$$

$$\boxed{Variance = np(1-p)}$$

Binomial Distribution

- In a community, the probability that a newly born child will be boy $\frac{2}{5}$. Among the 4 newly born children in that community, what is the probability that

  a. All the four boys (Ans: 0.0256)
  b. No boys (Ans: 0.1296)
  c. Exactly one boy. (Ans: 0.3456)

Binomial Distribution

- The incidence of occupational disease in an industry is such that the workers have a 20% chance of suffering from it. What is the probability that out of 6 workers

a. 4 or more will contract disease (Ans: 0.01696)
b. exactly 2 workers will contract disease? (Ans:0.24576)

Binomial Distribution

- The mean and standard deviation of binomial variable are 20 and 2, respectively. Find the parameters and probability function of the distribution.

$$Ans: n = 25 \ and \ p = 0.8$$

$$p(x) = {}^{25}C_x \, 0.8^x \, (1 - 0.8)^{25-x}$$

Poisson Distribution

- When data represents the number of occurrence of a specific event in a fixed period of time.

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Poisson Distribution

Binomial dist. describe the distribution of binary data from **finite** sample

- When data represents the number of occurrence of a specific event in a fixed period of time.

Poisson dist. describe the distribution of binary data from **infinite** sample $(n \geq 100)$

- For example,
- "The number of cars arrived at a toll gate per minute"
- "The number of traffic accidents during a given time day"

Poisson Distribution

- The probability mass function of Poisson distribution:

$$\lambda = np$$

$$P(x) = \frac{e^{-\lambda}\lambda^{x}}{x!}; x = 0,1,2,\ldots$$

$$Mean, E(X) = \lambda = np$$
$$Variance, Var(X) = \lambda = np$$

$$X \sim Poisson\ (\lambda)$$

Parameter of Poisson dist.

Poisson Distribution

- Suppose that the number of emergency patients in a given day at a certain hospital is a Poisson variable X with parameter $\lambda = 20$. What is the probability that in a given day there will be

a. 15 emergency patients.
b. 8 emergency patients.
c. At least 3 emergency patients.
d. More than 20 but less than 25 patients.

Poisson Distribution

a) 15 emergency patients

$$P(x) = \frac{e^{-\lambda}\lambda^x}{x!}; \lambda = 20$$

$$P(X = 15) = \frac{e^{-20}20^{15}}{15!}$$

$$P(X = 15) = 0.0516$$

b) 8 emergency patients

$$P(x) = \frac{e^{-\lambda}\lambda^x}{x!}; \lambda = 20$$

$$P(X = 8) = \frac{e^{-20}20^{8}}{8!}$$

$$P(X = 8) = ???$$

Poisson Distribution

c) At least 3 patients

$\lambda = 20$

$P(X \geq 3)$

$P(X \geq 3) = 1 - P(X < 3)$

$P(X \geq 3) = 1$

$P(X = 0) + P(X = 1) + P(X = 2)$

$$P(x) = \frac{e^{-\lambda}\lambda^{x}}{x!}$$

Poisson Distribution

c) >20 but <25 patients

$\lambda = 20$

$P(20 < X < 25)$

$P(X = 21) + P(X = 22) + P(X = 23) + P(X = 24)$

$P(20 < X < 25) = 0.2841$

$$P(x) = \frac{e^{-\lambda}\lambda^{x}}{x!}$$

Poisson Distribution

- The average number of errors on a page of a certain magazine is 0.2. What is the probability that the next page (or a randomly selected page) you read contains

  a. 0 (zero) error? (Ans: 0.8187)
  b. 2 or more errors? (Ans: 0.01756)
  c. What is the average error per page? (Ans: 0.2)
  d. Also, find standard deviation of the number of errors. (Ans: 0.45)

Poisson Distribution

$$P(x) = \frac{e^{-\lambda}\lambda^{x}}{x!}$$

- Imagine you're employed as a transportation engineer, tasked with assessing the likelihood of vehicular accidents. On a highly trafficked segment of road, the probability of a single car accident occurring per hour is 0.001. If 2000 cars traverse this stretch of road in one hour, what is the probability that precisely three car accidents will occur on this road within that time frame? (Ans: 0.180)

$P(X = 3)$

$p = 0.001$

$n = 2000$

$np = \lambda = 2000 \times 0.001$

Poisson Distribution

$\lambda = 3$

- A telephone operator receives 3 telephone calls on average from 9AM to 10AM. Find the probability that in a given time interval of a day, the operator receives,

  a. No call
  b. At least two calls
  c. At best two calls (At most two calls)
  d. Two or three calls

a) $P(X = 0)$

b) $P(X \geq 2)$

c) $P(X \leq 2)$

d) $P(2 \leq X \leq 3)$

$$P(x) = \frac{e^{-\lambda}\lambda^{x}}{x!}$$

Geometric Distribution

- When we want to know how many how many trials required before achieving the first success in repeated Bernoulli trials, we may used geometric probability distribution.

Geometric Distribution

- Toss a coin and the event is “Head”
- First trial = Tail
- Second trial = Tail
- Third trial = Tail
- Fourth trial = Head

Geometric Distribution

- Toss a coin and the event is “Head”

- First trial = Tail $(1-P)$
- Second trial = Tail $(1-P)$
- Third trial = Tail $(1-P)$
- Fourth trial = Head P

$P(1-P)^{4-1}$

$P(1-P)^{5-1}$

$P(1-P)^{6-1}$

$P(1-P)^{x-1}$

Geometric Distribution

- The probability mass function for geometric distribution

$$p(x) = p(1-p)^{x-1} \;\; ; x = 1,2,\ldots$$

- $Mean = \frac{1}{p}$

- $Variance = \frac{1-p}{p^2}$

Geometric Distribution

- Two person decides that they will take balls until a red ball. If the probability of red ball is $\frac{1}{3}$.

a. What is the probability that the fourth ball is red.
b. Find the mean number of balls to get a red ball.

Geometric Distribution

$P(X = 4)$

$P(X = 4) = \frac{1}{3} \times \left(\frac{2}{3}\right)^{4-1}$

$P(X = 4) = \frac{1}{3} \times \left(\frac{2}{3}\right)^{3} = \frac{8}{81}$

$Mean$

$Mean = \frac{1}{p}$

$Mean = \frac{1}{\frac{1}{3}}$

$Mean = 3$

Mathematical exercise

To access additional mathematical problems, please refer to the PDF lecture notes.

Thank You