1. Probability: Probability is the likeliness/chance of occurring any event(s).

For example, consider tossing a coin.

- What is the chance of it landing heads up? (Probability)
- What is the chance of it landing tails up? (Probability)

2. Experiment: An experiment refers to a specific action, process, or phenomenon that leads to observable outcomes.

For example,

- Measuring distance from Dhaka to Chattogram
- Tossing a fair coin

3. Types of experiment: There are two types of experiment.

a) Deterministic experiment: An experiment whose outcome is predictable in advance is called deterministic experiment. For example, “Measuring distance from Dhaka to Chattogram” is deterministic experiment.

b) Random/Probabilistic experiment: An experiment whose outcome is not predictable in advance is called random/probabilistic experiment. For example, “Tossing a fair coin” is random experiment.

4. Distinguish between “Deterministic” and “Probabilistic/Random”. [HOME WORK]

5. Is tossing a coin a random experiment or deterministic experiment?

Ans: Tossing a coin is an experiment. There are two possible outcomes (head or tail). These outcomes are unpredictable before the coin is tossed. Therefore, this is a random experiment.

6. Is drawing a card from well shuffled deck of cards a random experiment or deterministic experiment?

Ans: Drawing a card from a well-shuffled deck of cards is a random experiment. The randomness arises from the uncertainty of which card will be drawn, even though the deck is well-shuffled. The outcome (the specific card drawn) is not predictable beforehand, making it a random event.

7. Is multiplying 4 and 8 on a calculator a random experiment or deterministic experiment?

Ans: Multiplying 4 and 8 on a calculator is a deterministic experiment. In a deterministic experiment, the outcome is certain and predictable based on the given inputs and the rules of the operation. In this case, multiplying 4 and 8 will always result in the same answer: 32. There is no randomness or uncertainty involved in this calculation, making it a deterministic process.

8. Some basic definitions:

a) ***<u>Sample space:</u>*** A sample space associated with an experiment which consist all possible outcomes of the experiment. A sample space is usually denoted by S. The elements of S are called sample points.

b) ***<u>Events:</u>*** Any subset of the sample space is called an event. In other words, an event is a collection of simple events or outcomes.

c) ***<u>Mutually exclusive events/Disjoint events:</u>*** Two events are called mutually exclusive if both the events cannot occur simultaneously in a single trial. In other words, if one of those events occurs, the other event will not occur.

d) ***<u>Mutually exhaustive events:</u>*** Exhaustive events are those, which includes all possible outcomes.

e) ***<u>Equally likely events:</u>*** The events of a random experiment are called equally likely if the chance of occurring those events is all equal.

9. Consider an experiment of tossing three coins simultaneously. Write down the sample space.

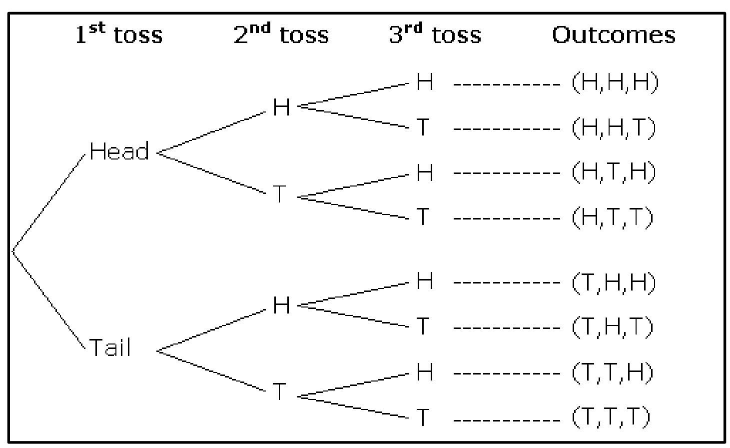
Ans:

$S = \{H,T\} \times \{H,T\} \times \{H,T\}$

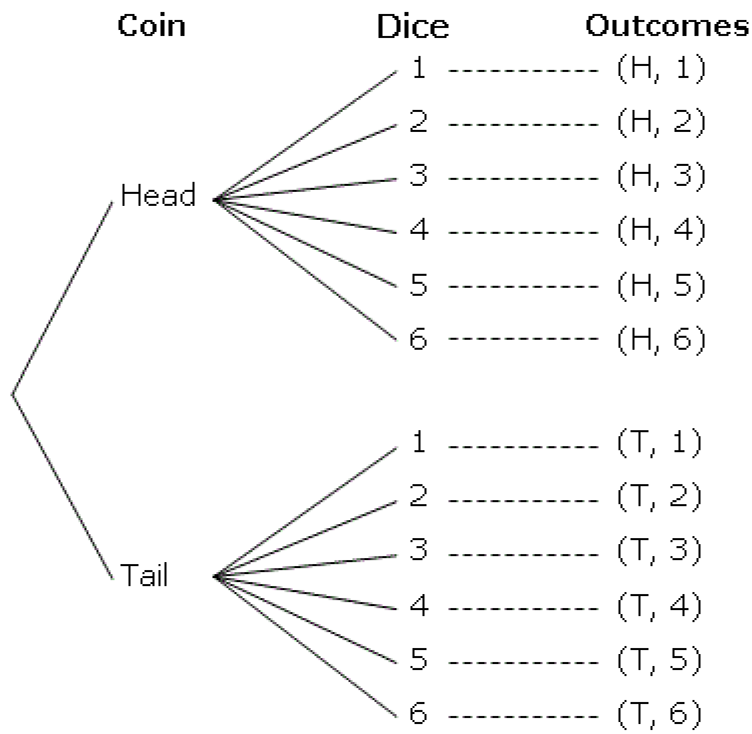
$= \{H,T\} \times \{HH,HT,TH,TT\}$

$= \{HHH,HHT,HTH,HTT,THH,THT,TTH,TTT\}$

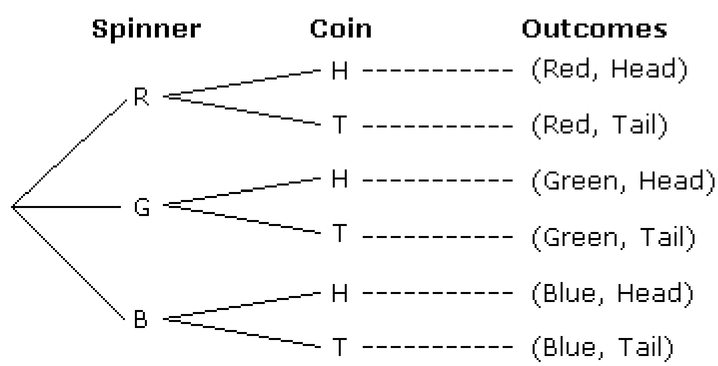
The tree diagram can be drawn as,



10. Write the sample space and tree diagram of toss a coin and rolling a dice simultaneously.



11. A spinner is labeled with three colors: Red, Green and Blue. A person spins the spinner once and tossed a coin once. Write down the sample space.



12. Roll a pair of dice and write down the sample space. [HOME WORK]

13. Tossing three coins at a time. Find the probability of getting:

a) Three tails.

b) Exactly two heads.

c) At least two heads.

Ans:

a) Here, $S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

$\therefore number\ of\ possible\ outcomes, n(S) = 8$

Let, A be the event of three tails, $A = \{TTT\}$

$\therefore number\ of\ event\ outcome, n(A) = 1$

$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{1}{8}$

b) $(Ans: \frac{3}{8})$

c) $\left(Ans: \frac{1}{2}\right)$

14.

Example: A bag contains 4 white and 6 black balls. If one ball is drawn at random from the bag, what is the probability that it is a) black, b) white c) white or black d) red?

Solution:

Total number of balls is 10.

a) Let A be the event that the ball is black, then the number of outcomes favorable to A is 6. Hence,

$$P[A] = \frac{No.of\ Black\ Balls}{Total\ No.of\ Balls} = \frac{6}{10} = 0.6$$

b) Let B be the event that the ball is white, then the favorable outcomes corresponding to B are 4. Therefore,

$$P[A] = \frac{No.of\ White\ Balls}{Total\ No.of\ Balls} = \frac{4}{10} = 0.4$$

c) Let C be the event that the ball is white or black, then the favorable outcomes corresponding to C are 10. Therefore,

$$P[A] = \frac{No.of\ White\ or\ Black\ Balls}{Total\ No.of\ Balls} = \frac{10}{10} = 1$$

d) Let D be the event that the ball is red, then the favorable outcomes corresponding to D are 0. Therefore,

$$P[A] = \frac{No.of\ Red\ Balls}{Total\ No.of\ Balls} = \frac{0}{10} = 0$$

15. In a community of 400 people, 20 people have a particular disease. If a person is selected randomly from that community, what is the probability that he/ she does not have the disease? $(\boldsymbol{Ans}: \mathbf{0.95})$

16. A committee consists of five executives of which three women (W_1, W_2, W_3) and two men (M_1, M_2). A random sample of two executives needs to be selected at random without replacement from which chairman and a secretary would be selected. Set up the sample space and find the probability that

a) **W_1 and W_2 will be selected**
b) **M_1 will be selected**
c) **M_1 will not be selected**
d) **W_1 or M_1 will be selected**

17. A balanced coin is tossed until head appears; it is tossed maximum 4 times. Construct the sample space of the experiment. [Ans: $S = \{H, TH, TTH, TTTH, TTTT\}$]

18. Let $S = \{1,2,3,4,5,6\}$, $A = \{1,3,5\}$, $B = \{4,6\}$ and $C = \{1,4\}$ find the probability of,

a) $\boldsymbol{A \cap B}$
b) $\boldsymbol{B \cup C}$
c) $\boldsymbol{A \cup (B \cap C)}$
d) $\boldsymbol{(A \cup B)'}$

- $A \cup (B \cup C) = (A \cup B) \cup C$
- $A \cap (B \cap C) = (A \cap B) \cap C$
- $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
- $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
- $A \cap \bar{B} = A - \bar{B}$
- $\overline{(A \cup B)} = \bar{A} \cap \bar{B}$

19. Two balanced dice, one black and one red are thrown and the number of dots on their upper faces are noted. Let, “b” be the outcomes of the black die, and “r” the outcomes of the red die, and both “b”; “r” varies from 1 to 6. Find,

a) **The probability of throwing a double. (Ans: 1/6)**
b) **The probability that the sum is 5, i.e., $b + r = 5$ (Ans: 1/9)**
c) **Probability that the sum is even. (Ans: 1/2)**
d) **The probability that $r \leq 2$ or $b \leq 3$. (Ans: 2/3)**

20.

Statement	**Meaning in terms of set theory**
At least one of A or B occurs	$A \cup B$
Both A and B occurs	$A \cap B$
Neither A nor B occurs	$(\bar{A} \cap \bar{B})$
Event A occurs but B does not	$A \cap \bar{B}$
Not more than A or B occurs	$(A \cap \bar{B}) \cup (\bar{A} \cap B) \cup (\bar{A} \cap \bar{B})$
Complementary of A	$\bar{A}$

20. A box contains seven balls- 2 red, 3 blue, and 2 yellow. Consider an experiment that consists of drawing a ball from the box.

a) What is the probability that the first ball drawn is yellow?

Ans: $P(Yellow\ on\ the\ first\ draw) = \frac{Number\ of\ yellow\ balls}{Total\ number\ of\ balls} = \frac{2}{7}$

b) What is the probability that the same-colored ball is drawn twice without replacement?

Ans:

If the first ball is red,

$$P(Red\ on\ both\ draws) = \frac{2}{7} \times \frac{1}{6}$$

If the first ball is blue,

$$P(Blue\ on\ both\ draws) = \frac{3}{7} \times \frac{2}{6}$$

If the first ball is yellow,

$$P(Yellow\ on\ both\ draws) = \frac{2}{7} \times \frac{1}{6}$$

$$\therefore P(Same\ colored\ ball\ twice\ with\ out\ rep.) = \left(\frac{2}{7} \times \frac{1}{6}\right) + \left(\frac{3}{7} + \frac{2}{6}\right) + \left(\frac{2}{7} + \frac{1}{6}\right)$$

c) What is the probability that the same-colored ball is drawn twice with replacement? [Ans:0.35]

21. A box contains 20 bulbs of which 5 are defective. If 3 of the bulbs are selected at random without replacement, what is the probability that all three bulbs are defective? (Ans: 0.0088)

22. A box contains 10 articles of which just 3 are defectives. If a random sample of five is drawn from this box without replacement, calculate the probability that the sample contains,

a) **Just one defective.**

   Ans: Here, sampling is drawn without replacement and unordered. We can use the theorem of combination. The 5 articles can be taken from 10 articles in ${}^{10}C_5 \ \ ways.$ One defective from 3 and 4 non-defective from 7 can be drawn as $\left({}^{3}C_1 \ \times \ {}^{7}C_4 \right) ways$

$$\therefore P(Just\ one\ defective) = \frac{{}^{3}C_1 \ \times \ {}^{7}C_4}{{}^{10}C_5} = \frac{5}{12}$$

b) **At most one defective. (Ans: 1/2)**

c) **No defective. (Ans: 1/12)**

d) **At least one defective. (Ans: 11/12)**