1. Hypothesis: A hypothesis is an assumption to be tested.

2. Statistical Hypothesis: A statistical hypothesis is some statement about a population parameter or about the probability distribution characterizing a population which we want to verify on the basis of sample information.

Example: A physician may hypothesize that the recommended drug is effective is 99% cases.

3. Types of hypotheses: Two type of hypothesis,

- ***Null hypothesis:*** A null hypothesis is a statistical hypothesis that contains a statement of equality such as $\leq, =, or \geq$. It is usually denoted by H_0.
- ***Alternative hypothesis:*** A alternative hypothesis is the complement of null hypothesis and it contains a statement of inequality such as $<, \neq, or >$. It is usually denoted by H_1/H_A.

Example:

H_0: CSE students' average programming skills are the same whether or not they attend a coding bootcamp.

H_1: CSE students' average programming skills are different depending on whether or not they attend a coding bootcamp.

Example:

Let us consider a normal distribution with mean μ and variance σ^2. Then the hypothesis that the normal distribution has specified mean 5 i.e., $\mu = 5$ is known as null hypothesis. And, the alternative hypothesis can be written as $H_1: \mu \neq 5$.

4. Write the hypotheses:

- A company collects information on the retail price of books and publishes the data in the website. In 2005, the mean retail price of history books was \$78.01. Suppose that we want to perform a hypothesis test to decide whether these years mean retail price of history books has increases from the 2005 mean. Determine the null and alternative hypotheses.

  Solution: The null and alternative hypotheses can be written as,

$$H_0: \mu = 78.01$$

$$H_1: \mu > 78.01$$

- A software company releases a new version of its operating system in 2022 and claims that the mean boot-up time for computers using this new OS is faster than the mean boot-up time in 2010 when the previous version was released. Determine the null and alternative hypotheses.

  H_0: The mean boot-up time for computers using the new operating system in 2022 is equal to or slower than the mean boot-up time for computers using the previous version in 2010.

  H_1: The mean boot-up time for computers using the new operating system in 2022 is faster than the mean boot-up time for computers using the previous version in 2010.

- A software development team claims that their new algorithm for data compression is more efficient in terms of compression ratio compared to the older algorithm they were using. Determine the null and alternative hypotheses.

  H_0: The mean compression ratio achieved by the new algorithm is equal to or worse than the mean compression ratio achieved by the older algorithm.

  H_1: The mean compression ratio achieved by the new algorithm is better than the mean compression ratio achieved by the older algorithm.

5. Distinguish between null hypothesis and alternative hypothesis:

a) A null hypothesis is a statement of equality.
   On the other side, An alternative hypothesis is statement of inequality.

b) Denoted by H_0.
   Denoted by H_1.

c) Mathematical formulation is equal sign
   Mathematical formulation is unequal sign.

6. Test statistic: A statistic which provides a basis for testing a null hypothesis is called a test statistic. The most commonly test statistic are,

a) $Z\ test$
b) $t\ test$
c) $\chi^2\ test$
d) $F\ test$

7. Steps in Hypothesis Testing

Step 1: Identify the null hypothesis H_0 and the alternate hypothesis H_1.

Step 2: Choose α, level of significance. The value should be small, usually less than 5%. It is important to consider the consequences of both types of errors.

Step 3: Select the test statistic and determine its value from the sample data. This value is called the observed value/calculated value of the test statistic. Remember that a t statistic is usually appropriate for a small number of samples; for larger number of samples, a z statistic can work well if data are normally distributed.

Step 4: Compare the observed value of the statistic to the critical value obtained for the chosen α.

Step 5: Make a decision. If the test statistic falls in the critical region: Reject H_0 in favor of H_1. If the test statistic does not fall in the critical region: Conclude that there is not enough evidence to reject H_0.

8. One sample mean test:

$Z\ test$	$t\ test$
a) Large sample $(n \geq 30)$ ***b)*** Variance (σ^2) is known or estimated from large sample.	***a)*** Large sample $(n < 30)$ ***b)*** Variance (σ^2) is unknown and estimated from small sample.
Hypothesis: $H_0: \mu = \mu_0\ and\ H_1: \mu \neq \mu_0$	***Hypothesis:*** $H_0: \mu = \mu_0\ and\ H_1: \mu \neq \mu_0$
Test statistic: $Z = \frac{(\bar{X} - \mu_0)}{\frac{\sigma}{\sqrt{n}}}$	***Test statistic:*** $t = \frac{(\bar{X} - \mu_0)}{\frac{s}{\sqrt{n}}}$
Significance level: $\alpha = 0.05\ or\ 0.01$	***Significance level:*** $\alpha = 0.05\ or\ 0.01$
Critical value: When $\alpha = 0.05$ ***a)*** For one tailed: $Z_\alpha = 1.645$ ***b)*** For two tailed: $Z_{\frac{\alpha}{2}} = 1.96$ ***Critical value:*** When $\alpha = 0.01$ ***a)*** For one tailed: $Z_\alpha = 2.33$ ***b)*** For two tailed: $Z_{\frac{\alpha}{2}} = 2.56$	***Critical value:*** For $\alpha = 0.05\ or\ 0.01$, the critical value depends on the number of degrees of freedom. In this case, degrees of freedom can be written as, $(n-1)$
Decision: We may reject H_0 if, ***a)*** For left tailed: $Z_{cal} \leq -(Z_\alpha)$ ***b)*** For right tailed: $Z_{cal} \geq +(Z_\alpha)$ ***c)*** For two tailed: $\lvert Z_{cal} \rvert \geq Z_{\frac{\alpha}{2}}$	***Decision:*** We may reject H_0 if, ***a)*** For left tailed: $t_{cal} \leq -(t_{\alpha,(n-1)})$ ***b)*** For right tailed: $t_{cal} \geq +(t_{\alpha,(n-1)})$ ***c)*** For two tailed: $\lvert t_{cal} \rvert \geq t_{\frac{\alpha}{2},(n-1)}$

1. **The mean life time of a sample of 100 light tubes produced by a company is found to be 1570 hours with standard deviation of 80 hours. Test the hypothesis that the mean life time of the tubes produced by the company is 1600 hours (consider 5% significance level, Tabulated value = 1.96).**

Solution: We consider the following hypotheses,

$$H_0: \mu = 1600$$

$$H_1: \mu \neq 1600$$

Given that the significance level $\alpha = 5\% = 0.05$

Since the sample size is large, the test statistic is,

$$Z_{cal} = \frac{(\bar{X} - \mu_0)}{\frac{\sigma}{\sqrt{n}}} = \frac{(1570 - 1600)}{\frac{80}{\sqrt{100}}} = -3.75$$

Here, the critical value/tabulated value for $\alpha = 0.05$ is $Z_{tab} = 1.96$. Since, $|Z_{cal}| > Z_{tab}$, so we may reject the null hypothesis. By rejecting null hypothesis, we can conclude that, mean life time of the tubes produces by the company is not 1600 hours.

2. **A random sample of 10 boys had the following I. Q's: 70, 120, 110, 101, 88, 83, 95, 98, 107, 100. Do these data support the assumption of a population mean I. Q. of 100? Tabulated value = 2.26**

Solution: We consider the following hypotheses,

$$H_0: \mu = 100$$

$$H_1: \mu \neq 100$$

Given that the significance level $\alpha = 5\% = 0.05$

Since the sample size is large, the test statistic is,

$$t_{cal} = \frac{(\bar{X} - \mu_0)}{\frac{s}{\sqrt{n}}} = \frac{(97.2 - 100)}{\frac{14.27}{\sqrt{10}}} = -0.62; \left[\bar{X} = \frac{\sum X_i}{n} = 97.2, s = \sqrt{\frac{\sum(X_i - \bar{X})^2}{n-1}} = 14.27\right]$$

Here, the critical value for $\alpha = 0.05$ and 9 df is $t_{tab} = 2.26$. Since, $|t_{cal}| < t_{tab}$, so we may not reject the null hypothesis. By rejecting null hypothesis, we can conclude that, the data support that the population mean I. Q is 100.

3. **A sample of 400 male students is found to have a mean height 67.47 inches. Can it be reasonably regarded as a sample from a large population with mean height 67.39 inches and standard deviation 1.30 inches? Test at 5% level of significance.**

   Solution: (Follow same steps) $[Z_{cal} = 1.23, Z_{tab} = 1.96, May\ not\ reject\ H_0]$

4. **Suppose that it is known from experience that the standard deviation of the weight of 8 ounces packages of cookies made by a certain bakery is 0.16 ounces. To check its production is under control on a given day, the true average of the packages is 8 ounces, they select a random sample of 40 packages and find their mean weight is 8.122 ounces. Test whether the production is under control or not at 5% level of significance.**

   Solution: (Follow same steps) $[Z_{cal} = 4.43, Z_{tab} = 1.96, May\ reject\ H_0]$

5. **Is the temperature required to damage a computer on the average less than 110 degrees? Because of the price of testing, twenty computers were tested to see what minimum temperature will damage the computer. The damaging temperature averaged 109 degrees with a standard deviation of 3 degrees. (Use α = 0.05)** $[t_{cal} = -1.49, May\ not\ reject\ H_0]$

6. The specimen of copper wires drawn from a large lot has the following breaking strength (in Kg. weight): 578, 572, 570, 568, 572, 578, 570, 572, 596, 544. Test whether the mean breaking strength of the lot may be taking to be 578 Kg. weights by using 10% level of significance.

   Solution: $[t_{cal} = -1.49, May\ not\ reject\ H_0]$

7. **A bulb manufacturing company claims that the average longevity of their bulb is 3.65 years with a standard deviation of 0.16 years. A random sample of 36 bulbs gave a mean longevity of 3.45 years. Does the sample mean justify the claim of the manufacturer? Use a 5% level of significance.** $[Z_{cal} = -7.5, May\ reject\ the\ null\ hypothesis]$

8. **The mean and standard deviation of GPA scores obtained from a random sample of 40 students of a college were 2.8 and 0.35, respectively. Can we conclude that the sample has come from the entire group of students, which has mean score of 2.4? Use 1% level of significance.**

Solution: $[Z_{cal} = 7.22, May\ reject\ the\ null\ hypothesis]$

9. **Suppose that a steel manufacturing company wishes to know whether the tensile strength of the steel wire has an overall average of 120 pounds. A sample of 25 units of steel wire produces by the company yields a mean strength of 110 pounds and a variance of 144 pounds. Should the company conclude that the strength is not 120 pounds with $\alpha = 0.05$?**

Solution: $[t_{cal} = -4.17, df = 24, May\ reject\ the\ null\ hypothesis]$

10. **Suppose that a sociologist selects a sample of 20 records of duration of marriage (in years (ending in divorce among the women in a community. The records were as follows: "10.1, 21.2, 13.8, 11.1, 10.9, 9.2, 6.6, 12.3, 7.8, 15.1, 2.6, 14.3, 14.9, 5.4, 8.7, 4.8, 19.4, 26.3, 24.5, 21.6". By setting up an appropriate null hypothesis and alternative hypothesis, determine whether these data provide evidence, at 5% level of significance, that the average duration of marriage ending in divorce in the community has decreased from an earlier value of 14.9 years.**

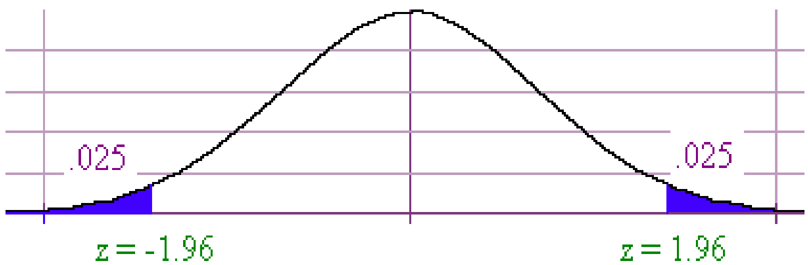
Solution: $[\bar{x} = 13.03, SD = 6.7, t_{cal} = -1.248, df = 19, May\ not\ reject\ the\ null\ hypothesis]$

9. Z test tabulated value:

	$\boldsymbol{\alpha = 0.05}$	$\boldsymbol{\alpha = 0.01}$	$\boldsymbol{\alpha = 0.025}$
Left-tailed test	$Z_\alpha = -1.645$	$Z_\alpha = -2.33$	$Z_\alpha = -1.96$
Right-tailed test	$Z_\alpha = 1.645$	$Z_\alpha = 2.33$	$Z_\alpha = 1.96$
Two-tailed test	$Z_\alpha = 1.96$	$Z_\alpha = 2.56$	$Z_\alpha = 2.24$

10. Rejection region: The region of rejection of H_0 when H_0 is true is that region of outcome set where H_0 is rejected. This region is called rejection region.

Suppose that $\alpha = 0.05$. We can draw the appropriate picture and find the Z score for -0.025 and 0.025. We call the outside regions the rejection regions.



We call the blue areas the rejection region since if the value of   falls in these regions, we can say that the null hypothesis is very unlikely so we can reject the null hypothesis.

Example: 50 smokers were questioned about the number of hours they sleep each day. We want to test the hypothesis that the smokers need less sleep than the general public which needs an average of 7.7 hours of sleep. We follow the steps below:

a) Compute a rejection region for a significance level of 0.05.
b) If the sample mean is 7.5 and the standard deviation is 0.5, what can you conclude?

Solution: First, we write down the null and alternative hypotheses

$$H_0: \mu = 7.7 \text{ and } H_1: \mu < 7.7$$

a) This is a left tailed test. The Z-score that corresponds to 0.05 is -1.96. The critical region is the area that lies to the left of -1.96. If the z-value is less than -1.96 there we will reject the null hypothesis and accept the alternative hypothesis. If it is greater than -1.96, we will fail to reject the null hypothesis and say that the test was not statistically significant.

b) [*Similar as previous maths*]

11. Critical value/ Significant value: The values of the test statistic that separated the acceptance region and rejection region are called critical value. It depends on the,

- The level of significance used (that is, the value of α)
- The types of alternative hypothesis whether it is one or two-tailed test

  i. ***One tailed test:*** A test for which the entire rejection region lies in only one of the two tails either in the right tail or in the left tail of the distribution curve of the test statistic is called one tailed test or one-sided test. For example, if we want to test that the population has specific mean μ_0, then
  Null hypothesis, $H_0: \mu = \mu_0$
  Alternative hypothesis, $H_1: \mu < \mu_0; Left\ tailed\ test$
  $H_1: \mu > \mu_0; Right\ tailed\ test$

  ii. ***Two-tailed test:*** A test for which the entire rejection region lies in the two tails of the distribution curve of the test statistic is called two tailed test or two-sided test. For example, if we want to test that the population has specific mean μ_0, then,
  Null hypothesis, $H_0: \mu = \mu_0$
  Alternative hypothesis, $H_1: \mu \neq \mu_0; Two\ tailed\ test$

12. Errors involved in a test of hypothesis:

Any decision we make based on a hypothesis test may be incorrect because we have used partial information obtained from a sample to draw conclusion about the entire population.

So, in case of reaching decision in a test of significance two types of error can be made,

a) ***Type I error:*** It is made if the null hypothesis is rejected, when it is true. The probability of committing Type I error is denoted by α. It is also known as tolerable error.

b) ***Type II error:*** It is made if the null hypothesis is accepted when it is false. The probability of Type II error is denoted by β.

		Actual situation	
		H_0 is true	***H_0 is false***
Decision from sample	***H_0 accepted***	No error	Type II error
	H_0 rejected	Type I error	No error