Lecture 2

Topics:

1. Conditional Statements
2. Converse, Inverse, and Contrapositives
3. Biconditional Statements
4. Truth tables of Conditional Statements

Conditional Statements

Definition 5 Let p and q be propositions. The conditional statement $p \to q$ is the proposition "if p, then q." The conditional statement $p \to q$ is false when p is true and q is false, and true otherwise. In the conditional statement $p \to q$, p is called the hypothesis (or antecedent or premise) and q is called the conclusion (or consequence)

TABLE 5 The Truth Table for the Conditional Statement

p	**q**	**$p \to q$**
T	T	T
T	F	F
F	T	T
F	F	T

Other ways to express conditional statements

"if p, then q"
"p implies q"
"if p, q"
"p only if q"
"p is sufficient for q"
"a sufficient condition for q is p"
"q if p"
"q whenever p"
"q when p"
"q is necessary for p"
"a necessary condition for p is q"
"q follows from p"
"q unless $\neg$p"
"q provided that p"

“If I am elected, then I will lower taxes.”

“If you get 100% on the final, then you will get an A.”

Remark: Because some of the different ways to express the implication p implies q can be confusing, we will provide some extra guidance. To remember that “p only if q” expresses the same thing as “if p, then q,” note that “p only if q” says that p cannot be true when q is not true. That is, the statement is false if p is true, but q is false. When p is false, q may be either true or false, because the statement says nothing about the truth value of q.

“You can receive an A in the course only if your score on the final is at least 90%.

EXAMPLE 10 Let p be the statement “Maria learns discrete mathematics” and q the statement “Maria will find a good job.” Express the statement $p \to q$ as a statement in English.”

There are many other ways to express this conditional statement in English. Among the most natural of these are “Maria will find a good job when she learns discrete mathematics.” “For Maria to get a good job, it is sufficient for her to learn discrete mathematics.” and “Maria will find a good job unless she does not learn discrete mathematics.”

“If Juan has a smartphone, then 2 + 3 = 5” is true from the definition of a conditional statement, because its conclusion is true. (The truth value of the hypothesis does not matter then.) The conditional statement “If Juan has a smartphone, then 2 + 3 = 6”

CONVERSE, CONTRAPOSITIVE, AND INVERSE

We can form some new conditional statements starting with a conditional statement $p \to q$. In particular, there are three related conditional statements that occur so often that they have special names. The proposition $q \to p$ is called the converse of $p \to q$. The contrapositive of $p \to q$ is the proposition $\neg q \to \neg p$. The proposition $\neg p \to \neg q$ is called the inverse of $p \to q$. We will see that of these three conditional statements formed from $p \to q$, only the contrapositive always has the same truth value as $p \to q$.

We first show that the contrapositive, $\neg q \rightarrow \neg p$, of a conditional statement $p \rightarrow q$ always has the same truth value as $p \rightarrow q$. To see this, note that the contrapositive is false only when $\neg p$ is false and $\neg q$ is true, that is, only when p is true and q is false. We now show that neither the converse, $q \rightarrow p$, nor the inverse, $\neg p \rightarrow \neg q$, has the same truth value as $p \rightarrow q$ for all possible truth values of p and q. Note that when p is true and q is false, the original conditional statement is false, but the converse and the inverse are both true.

EXAMPLE 12 Find the contrapositive, the converse, and the inverse of the conditional statement Extra Examples “The home team wins whenever it is raining.”

Solution: Because “q whenever p” is one of the ways to express the conditional statement $p \rightarrow q$, the original statement can be rewritten as “If it is raining, then the home team wins.” Consequently, the contrapositive of this conditional statement is “If the home team does not win, then it is not raining.” The converse is “If the home team wins, then it is raining.” The inverse is “If it is not raining, then the home team does not win.” Only the contrapositive is equivalent to the original statement.

Biconditional Statement

TABLE 6 The Truth Table for the Biconditional $p \leftrightarrow q$

p	**q**	$\mathbf{p \leftrightarrow q}$
T	T	T
T	F	F
F	T	F
F	F	T

Definition 6 Let p and q be propositions. The biconditional statement $p \leftrightarrow q$ is the proposition "p if and only if q." The biconditional statement $p \leftrightarrow q$ is true when p and q have the same truth values, and is false otherwise. Biconditional statements are also called bi-implications.

"p is necessary and sufficient for q" "if p then q, and conversely" "p iff q." "p exactly when q."