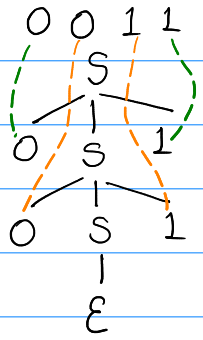
CFG

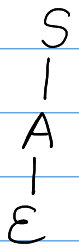
Context Free Grammar

If you spot any errors, please email me at kabbya.kantam@bracu.ac.bd

Qs. $L = \{ \omega \in \{0,1\}^* : \omega = 0^n 1^n, \text{ where } n \geq 0 \}$

$S \rightarrow 0S1 \mid \varepsilon$ ⟶ shortest string $\in L$





$0^n 1^n$

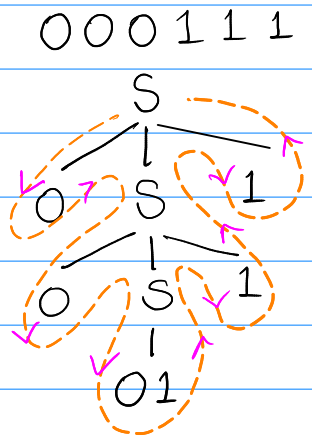
$0\,0^{n-1} 1^{n-1} 1$

$00\,0^{n-2} 1^{n-2} 11$

$000 \ldots 011 \ldots 1$

Qs. $L = \{ \omega \in \{0,1\}^* : \omega = 0^n 1^n, \text{ where } n \geq 1 \}$

$S \rightarrow 0S1 \mid 01$ ⟶ shortest string $\in L$



Qs. $L = \{ w \in \{0,1\}^* : w = 0^{n+1}1^n, \text{ where } n \geq 0 \}$

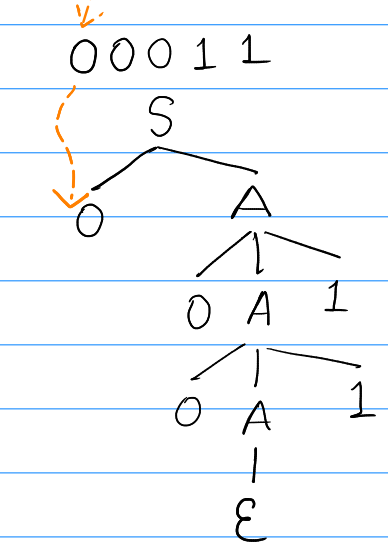
$0^{n+1}1^n$

Sol1:

$\Rightarrow 0\,0^n\,1^n$

$S \rightarrow 0A$

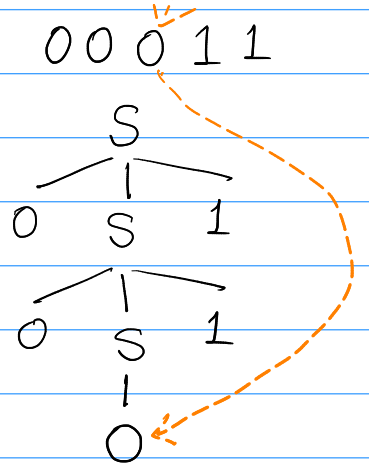
$A \rightarrow 0A1 \mid \varepsilon$



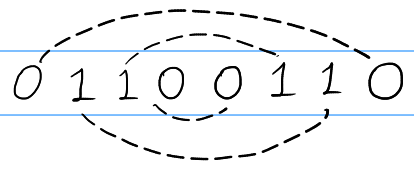
Sol2:

$\Rightarrow 0^n\,0\,1^n$

$S \rightarrow 0S1 \mid 0$



Qs: $L = \{ w \in \{0,1\}^* : w \text{ is a palindrome.} \}$



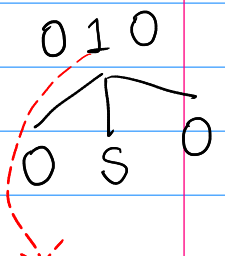
wrong $S \rightarrow 0S0 \mid 1S1 \mid \varepsilon$

P_n ↗ Palindrome length n

$\Rightarrow 0\,P_{n-2}\,0$

$\Rightarrow 0\,1\,P_{n-4}\,1\,0$

$0\,1 \ldots\ldots 1\,0$

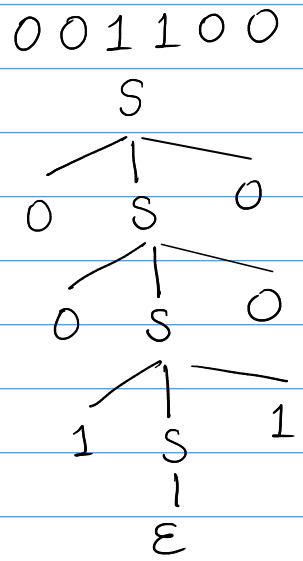


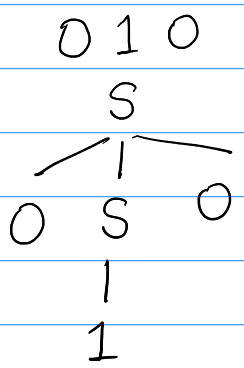
The solution is partially correct. Can you find a string that can't be derived?

↳ odd length palindrome

Correct Solution:

$$S \rightarrow 0S0 \mid 1S1 \mid 0 \mid 1 \mid \varepsilon$$





Qs: $L = \{w \in \{a,b\} : w \text{ is an even length palindrome.}\}$

$$S \rightarrow aSa \mid bSb \mid \varepsilon$$

↳ make sure, our grammar doesn't parse odd length palindrome.

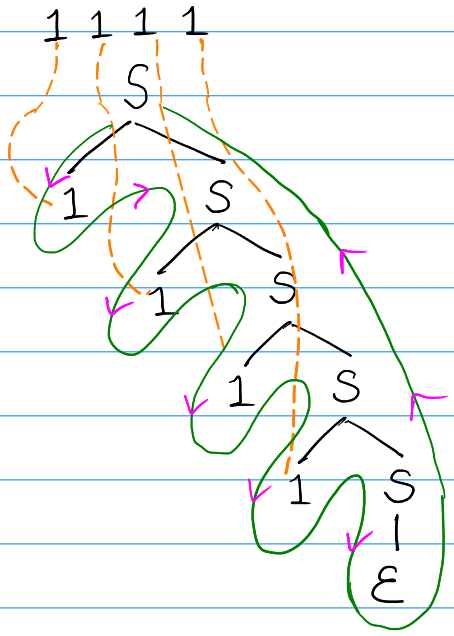
Qs: $L = \{w \in \{a,b\} : w \text{ is a odd length palindrome.}\}$

$$S \rightarrow aSa \mid bSb \mid a \mid b$$

Qs $L = \{ \omega \in \{0,1\}^* : \omega = 1^n, \text{ where } n \geq 0 \}$

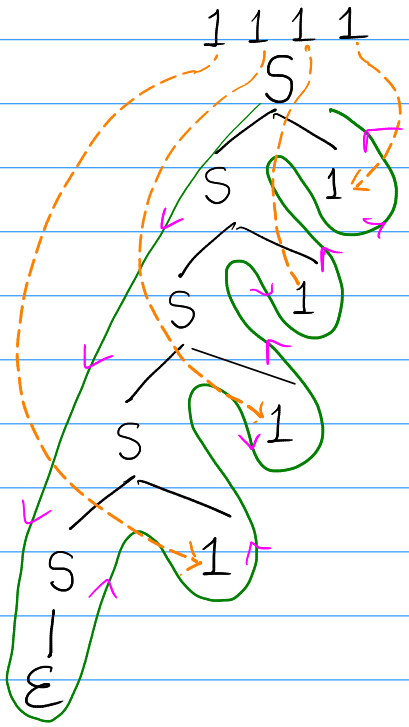
Sol 1 :

$S \rightarrow 1S \mid \varepsilon$



Sol 2 :

$S \rightarrow S1 \mid \varepsilon$

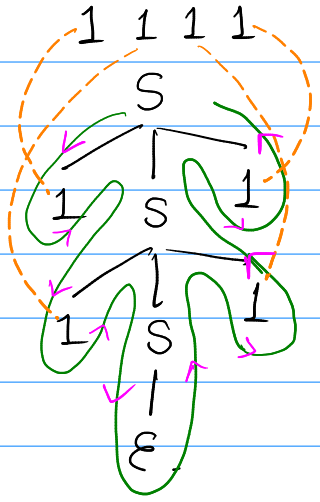


Sol 3 :

$S \rightarrow 1S1 \mid 11 \mid \varepsilon$

for odd length

for even length

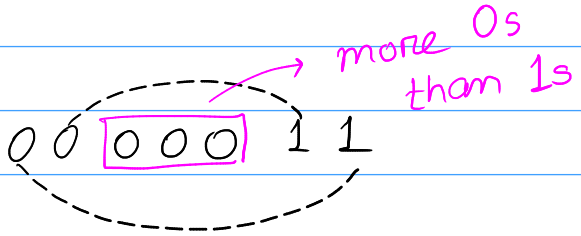


we can say $m \geq 1$

Qs $L = \{ w \in \{0,1\}^* : 0^m 1^n$, where $m > n$ and $n \geq 0 \}$

✓ 0 0 0 1 1

✓ 0 0 0 0 0 1 1

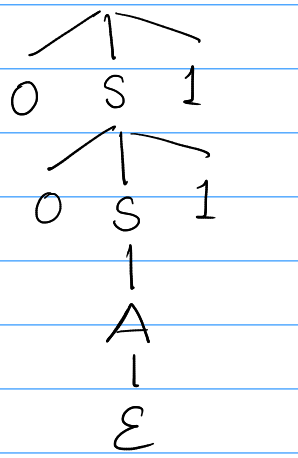


$S \rightarrow 0S1 \mid A$

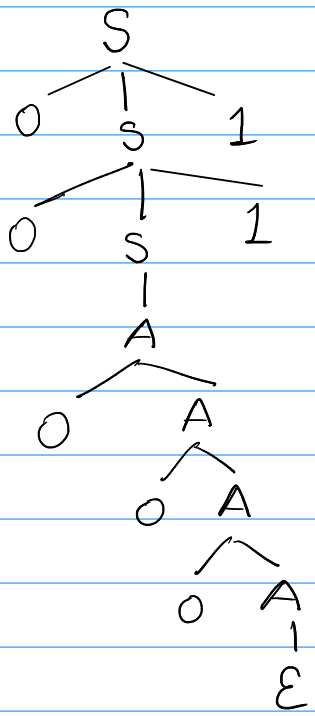
$A \rightarrow 0A \mid \varepsilon$

Wrong. Why?

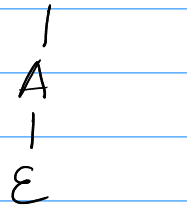
$0\,0\,1\,1 \notin L$



$0\,0\,0\,0\,0\,1\,1\,1 \in L$



$\varepsilon \notin L$



Our grammar can also parse equal amount of 0s & 1s

We have to ensure there is atleast one 0 more than 1s.

Correct Solution:

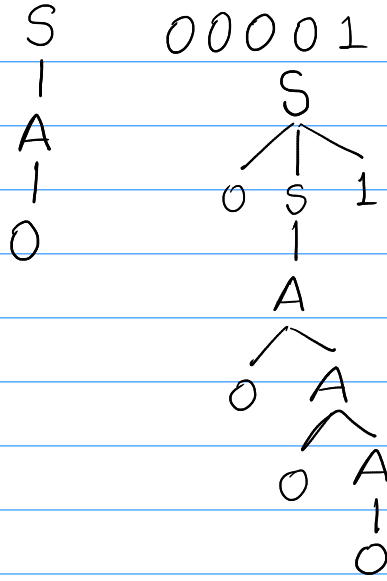
Sol 1:

since we are generating 0 & 1 together, 1 can't be more than 0

$S \rightarrow 0S1 \mid A$

$A \rightarrow 0A \mid 0$

at least one 0 is more

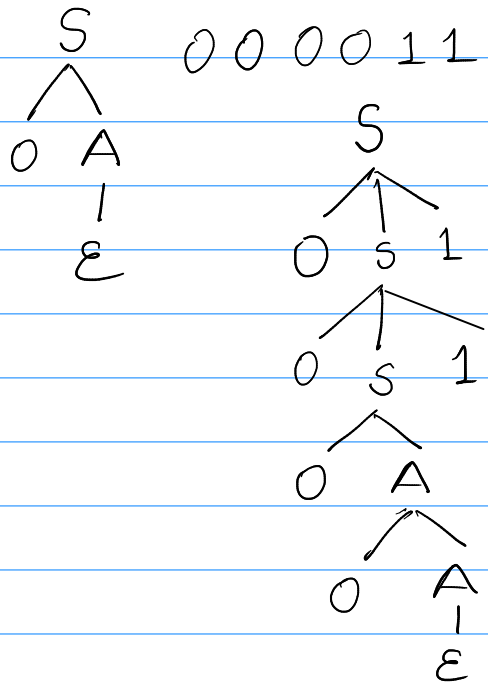


Sol2:

$S \rightarrow 0S1 \mid 0A$

$A \rightarrow 0A \mid \varepsilon$

at least one 0 is more



Another way to mapping:

000011

Sol3:

$S \rightarrow 0S \mid 0A$

$A \rightarrow 0A1 \mid \varepsilon$

at least one 0 more

Sol4:

at least one 0 more

$S \rightarrow 0A$

$A \rightarrow 0A \mid B$

$B \rightarrow 0B1 \mid \varepsilon$

Now, for the same problem you have two more solutions.

$L = \{ \omega \in \{0,1\}^* : \omega = 0^m 1^n$, where $m > n$ & $n \geq 0 \}$

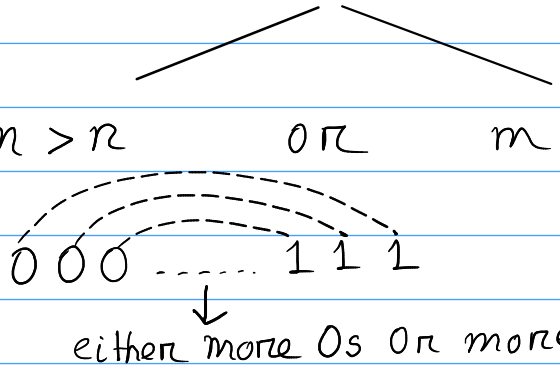
Sol: $S \rightarrow 0S \mid 0S1 \mid 0$ → correct or wrong ?

Sol: $S \rightarrow 0SR$
$R \rightarrow 1 \mid \varepsilon$
→ correct or wrong ?

Qs: $L = \{ \omega \in \{0,1\}^* : \omega = 0^m 1^n$, where $m < n$ & $n \geq 0 \}$

$S \rightarrow 1S \mid 0S1 \mid 0$ → correct or wrong ?

Qs: $L = \{ w \in \{0,1\}^* : w = 0^m 1^n$, where $m \neq n$ and $m, n \geq 0 \}$



Sol1:

$S \rightarrow 0S1 \mid A \mid B$

$A \rightarrow 0A \mid \cancel{\varepsilon} \mid 0$

$B \rightarrow 1A \mid \cancel{\varepsilon} \mid 1$

Sol2:

$S \rightarrow 0S1 \mid 0A \mid 1B$

$A \rightarrow 0A \mid \varepsilon$

$B \rightarrow 1B \mid \varepsilon$

why ε to be correct here?

Sol3:

$S \rightarrow P \mid Q$

$P \rightarrow 0P1 \mid 0P \mid 0$

$Q \rightarrow 0Q1 \mid \cancel{1Q}\ Q1 \mid 1$

Why can't we write 1Q?

either more 0s

0 0 0 1 1 1

0 0 0 1 1 1 - - - -

or more 1s

Sol4:

$S \rightarrow AP \mid \cancel{BP}\ PB$

$A \rightarrow 0A \mid 0$

$B \rightarrow 1B \mid 1$

$P \rightarrow 0P1 \mid \varepsilon$

Why writing 1B or B1 both are correct?

why can't we write BP?

Now, for the same problem you have two more solutions.

Qs: $L = \{ w \in \{0,1\}^* : w = 0^m 1^n$, where $m \neq n$ and $m, n \geq 0 \}$

Sol: $S \rightarrow 0S1 \mid 0S \mid 1S \mid 0 \mid 1 \longrightarrow$ Correct or wrong ?

Sol: $S \rightarrow 0S \mid 1S \mid 0A \mid A1$
$A \rightarrow 0A1 \mid \varepsilon$ $\longrightarrow$ Correct or wrong ?

Qs: $L = \{ w \in \{0,1\}^* : w = 0^m 1^n$, where $m \neq n$ and $m, n \geq 1 \}$

$T \rightarrow 0S1 \rightsquigarrow n, m \geq 1$
$S \rightarrow 0S1 \mid A \mid B$
$A \rightarrow 0A \mid 0$
$B \rightarrow 1B \mid 1$

$L = \{ \omega \in \{(,)\}^* : \omega \text{ is a valid parenthesis} \}$

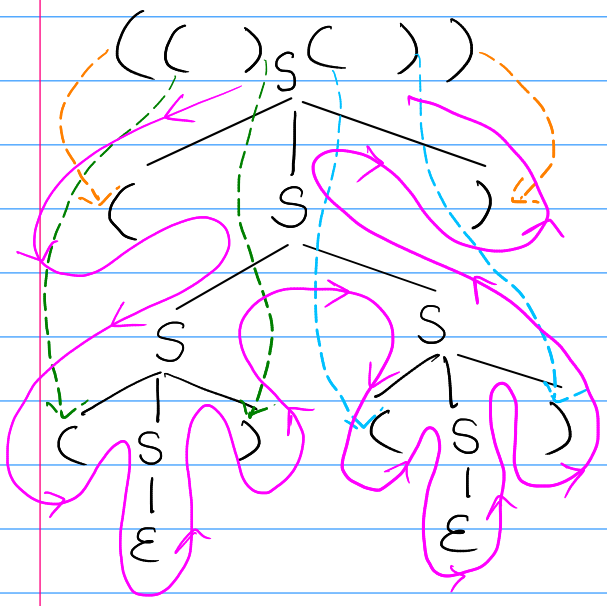
$S \rightarrow (S) \mid SS \mid \varepsilon$

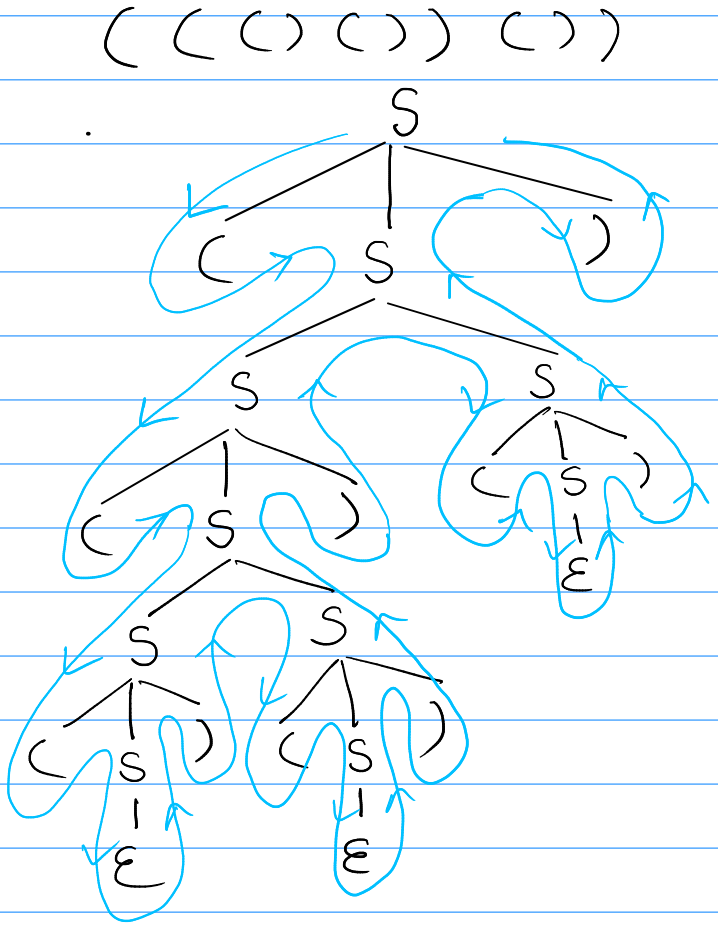
Using the production rule of the grammar draw a parse tree for the string

( ) (( ) ( )) (( ) (( ) ( )))

↳ H.W.

Parse tree example:





Qs. $L = \{ w \in \{0,1,2\}^* : w = 0^i 2^K 1^j$, where $i = j$ and $i, j, K \geq 0 \}$

Some 0s — Some 2s — Some 1s

#i 0s — #K 2s — #i 1s

⇓

Since $i = j$

✓ 00022111 ✗ 0101

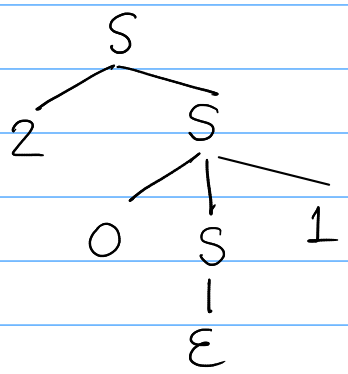
✓ 0011 ✗ 1220

✓ 222 ✗ 00221111

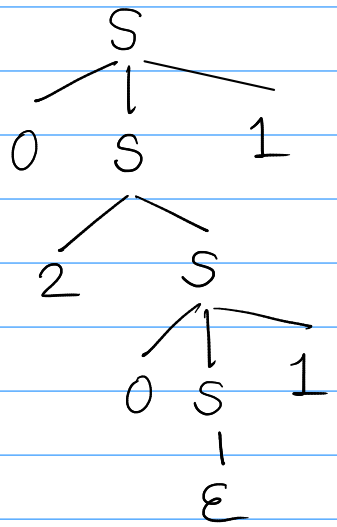
✓ ε

Sol: $S \to 0S1 \mid 2 \mid \varepsilon$ ⟶ wrong, because can't parse 0221.

Sol: $S \to 0S1 \mid 2S \mid \varepsilon$ ⟶ wrong



⇒ 201



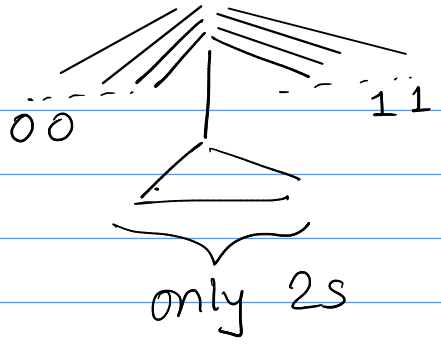
⇒ 02011

not maintaining the given order, so this is a wrong solution.

Correct solution:

$S \rightarrow 0S1 \mid A$

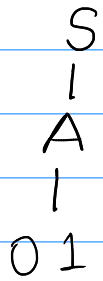
$A \rightarrow 2A \mid \varepsilon$



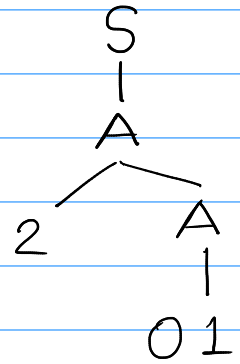
Qs. $L = \{ w \in \{0,1,2\}^* : w = 0^i 2^k 1^j$, where $i = j$, $i.j \geq 1$ and $k \geq 0 \}$

$S \rightarrow 0S1 \mid A$

$A \rightarrow 2A \mid 01$ ⟶ wrong.



we will get atleast one 0 and 1, however



$\Rightarrow 201$ ⟶ wrong sequence

Qs. $L = \{ w \in \{0,1,2\}^* : w = 0^i 2^K 1^j$, where $i = j$, $i, j \geq 1$ and $K \geq 0 \}$

Sol1: ensuring $i, j \geq 1$

$S \rightarrow 0S1 \mid 0A1$

$A \rightarrow 2A \mid \varepsilon$

Sol2: ensuring $i, j \geq 1$

$T \rightarrow 0S1$

$S \rightarrow 0S1 \mid A$

$A \rightarrow 2A \mid \varepsilon$

Qs. $L = \{ w \in \{0,1,2\}^* : w = 0^i 2^K 1^j$, where $i = j$, $i, j \geq 1$ and $K \geq 3 \}$

Sol1: $i, j \geq 1$

$S \rightarrow 0S1 \mid 0A1$

$A \rightarrow 2A \mid 222$

$K \geq 3$

Sol2: $i, j \geq 1$

$S \rightarrow 0S1 \mid 0A1$

$A \rightarrow 222B$, $K \geq 3$

$B \rightarrow 2B \mid \varepsilon$

Sol3: $i, j \geq 1$ & $K \geq 3$

$S \rightarrow 0S1 \mid 0222A1$

$A \rightarrow 2A \mid \varepsilon$

Now, for the same problem you have another Solution:

Qs. $L = \{ w \in \{0,1,2\}^* : w = 0^i 2^K 1^j$, where $i = j$, $i, j \geq 1$ and $K \geq 3 \}$

Sol:
$T \rightarrow 0S1$
$S \rightarrow 0S1 \mid A$ $\longrightarrow$ correct or wrong ?
$A \rightarrow 222B$
$B \rightarrow 2B \mid \varepsilon$

Sol
$S \rightarrow 0S1 \mid 0A1$
$A \rightarrow 222A \mid 222$ $\longrightarrow$ correct or wrong ?

Qs. $L = \{ w \in \{0,1,2\}^* : w = 0^i 2^K 1^j$, where $i = j$, $i, j \geq 0$ and K is a multiple of three $\}$

$\Downarrow$

0, 3, 6, 9, 12 .....

we can rewrite the question as

$w = 0^i 2^{3K} 1^j$, where $i = j$ and $i, j, K \geq 0$

$\Downarrow$

what if $K \geq 1$

$S \rightarrow 0S1 \mid A$
$A \rightarrow 222A \mid \varepsilon$

Qs: $L = \{ w \in \{0,1,2\}^* : w = 0^i 2^K 1^j$, where

i, J is multiple of two and

K is multiple of three $\}$

We can rewrite the Qs

$$w = \underbrace{0^{2i}}_{\text{even 0s}} \underbrace{2^{3K}}_{\text{2 multiple of three}} \underbrace{1^{2j}}_{\text{even 1s}} \quad \text{where } i, J, K \geq 0$$

Note: $i = J$ is not mentioned

$S \rightarrow ABC$

$A \rightarrow 00A \mid \varepsilon$

$B \rightarrow 222B \mid \varepsilon$

$C \rightarrow 11C \mid \varepsilon$

Qs: $L = \{ w \in \{0,1,2\}^* : w = 0^i 2^k 1^j$, where $i, j \geq 1$, i, j is multiple of two k is multiple of three $\}$

$S \rightarrow ABC$

$A \rightarrow 00A \mid 00$

$B \rightarrow 222B \mid \varepsilon$

$C \rightarrow 11C \mid 11$

Qs: $L = \{ w \in \{0,1,2\}^* : w = 0^i 2^k 1^j$, where $i, j \geq 1$, $i = j$, i, j is multiple of two k is multiple of three $\}$

$S \rightarrow 00S11 \mid 00A11$

$A \rightarrow 222A \mid \varepsilon$

Qs: $L = \{ w \in \{0,1,2\}^* : 0^i 1^j 2^k$ where $k = i + j$ and $i, j, k \geq 0 \}$

// Check CFG handnotes 2

Qs: $L = \{ w \in \{0,1\}^* : w = 0^i 1^j 0^k,$ where $j = i + k$ and $i, j, k \geq 0 \}$

// Check CFG handnotes 2

Qs: $L = \{ w \in \{0,1\}^* : w = 0^i 1^j 2^k$ where $i = j$ or $j = k$ and $i, j, k \geq 0 \}$

// Check CFG handnotes 2

Qs: $L = \{ w \in \{0,1\}^* : w$ is any combination of 0s and 1s$\}$

Sol 1:

$S \to 0S \mid 1S \mid \varepsilon$

Sol 2: → important

$(0+1)^*$

$M \to XM \mid \varepsilon$

$X \to 0 \mid 1$

Qs: $L = \{ w \in \{0,1\}^* : w$ is any combination 01 and 10$\}$

$S \to 01S \mid 10S \mid \varepsilon$

Qs: $L = \{ w \in \{0,1\}^* : w$ contains 00 or 11$\}$

$\Sigma^* (00+11) \Sigma^*$

$S \to XAX$

$X \to 0X \mid 1X \mid \varepsilon$

$A \to 00 \mid 11$

Qs: $L = \{ w \in \{0,1\}^* : w$ contains 00 and 11$\}$

$\Sigma^* 00 \Sigma^* 11 \Sigma^* + \Sigma^* 11 \Sigma^* 00 \Sigma^*$

$S \to A \mid B$

$A \to X00X11X$

$B \to X11X00X$

$X \to 0X \mid 1X \mid \varepsilon$

Qs: $L = \{w \in \{0,1\}^* : \text{length of } w \text{ is even}\}$

Sol 1:

$$S \to 00S \mid 01S \mid 10S \mid 11S \mid \varepsilon$$

Sol 2:

$$S \to 0S0 \mid 0S1 \mid 1S0 \mid 1S1 \mid \varepsilon$$

Sol 3:

$$S \to XXS \mid \varepsilon$$
$$S \to 0 \mid 1 \mid \cancel{\varepsilon}$$

↳ Why can't we give ε here.

Qs: $L = \{w \in \{0,1\}^* : \text{length of } w \text{ is multiple of six}\}$

$$S \to XXXXXXS \mid \varepsilon$$
$$X \to 0 \mid 1$$

// Check CFG handnotes 3 for more problems.