

11

stem	leaf
1	3 8 9
2	1 4 7 8 9
3	0 1 2 3 4 5 8 8 9
4	0 2 4 5 6 7 7
5	7

key: 5|7 means 57

a) Calculate mean, median, and mode

b) Calculate 25th percentile value
75th percentile value

c) Draw box and whisker plot

d) Detect outliers.

Solutions:

$$\text{Mean: } \bar{X} = \frac{13 + 18 + 19 + \dots + 57}{25} = \frac{857}{25} = 34.28$$

Median: Here, $n = 25$, an odd number.

$$\begin{aligned}\therefore \text{Median} &= \left(\frac{n+1}{2}\right)^{\text{th}} \text{ observation} \\ &= \left(\frac{25+1}{2}\right)^{\text{th}} \text{ obs.} \\ &= 13^{\text{th}} \text{ obs.} \\ &= 34\end{aligned}$$

Mode: The most frequent obs. 38 and 37.

25th percentile:

Q₂ or P₂₅

$$\text{Position of } Q_1 = \frac{1 \times 25}{4}$$

$$= 6.25$$

Since, the position of Q_1 is not integer, we go for next integer value.

$$\therefore Q_1 = 7\text{th obs.}$$

$$= 28$$

Ans

75th percentile:

Q₃ or P₇₅

$$\text{Position of } Q_3 = \frac{3 \times 25}{4}$$

$$= 18.75$$

Since the position of Q_3 is not integer,

$$\therefore Q_3 = 19\text{th obs.}$$

$$= 42$$

* Interpretation must *

Box and whisker plot :

For box and whisker plot, we need,

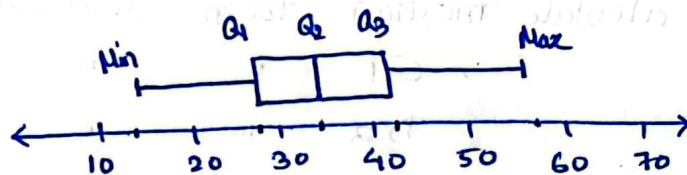
$$\text{Minimum} = 13$$

$$Q_1 = 28$$

$$Q_2 / \text{Median} / 50\text{th percentile} = 34$$

$$Q_3 = 42$$

$$\text{Max} = 57$$



Outliers

$$\begin{aligned} \text{IQR} &= Q_3 - Q_1 \\ &= 14 \end{aligned}$$

$$\begin{aligned} \text{Lower fences} &= Q_1 - 1.5 \text{IQR} \\ &= 28 - (1.5 \times 14) \\ &= 7 \end{aligned}$$

$$\begin{aligned} \text{Upper fences} &= Q_3 + 1.5 \text{IQR} \\ &= 42 + (1.5 \times 14) \\ &= 63 \end{aligned}$$

Since, there is no value outside of this inner fences range.

So, there are no outlier in this data set.

21 Suppose, you are making a table for 15 observations with four classes: (5-9); (9-13); (13-17); and (17-21). The percentage frequencies for first class is 20% and last class is 20%.

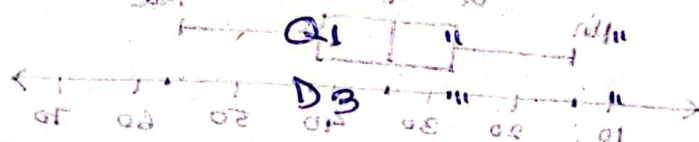
a) Calculate the frequency of middle two classes, when average is 12.6

b) Calculate median/ Q_2 /50th percentile value.

c) Calculate median from cumulative freq. curve

Median from Grouped data

Median from ogive curve



Solution:

Here, $N=15$

Relative freq $\times N = \text{freq.}$
Percent. freq $\times N = \text{freq.}$
100

class	Percent	freq(f_i)	(x_i)	Mid value	$f_i x_i$
5-9	20%	3	7		21
9-13		f_1	11		$11f_1$
13-17		f_2	15		$15f_2$
17-21		3	19		57

Now,

$$\bar{x} = \frac{\sum f_i x_i}{\sum f_i}$$

$$\Rightarrow 12.6 = \frac{21 + 11f_1 + 15f_2 + 57}{15} \quad [\sum f_i = N = 15]$$

$$\Rightarrow 189 = 78 + 11f_1 + 15f_2$$

$$\Rightarrow 11f_1 + 15f_2 = 111 \quad \text{--- (1)}$$

Again,

$$\sum f_i = 15$$

$$\Rightarrow 6 + f_1 + f_2 = 15$$

$$\Rightarrow f_1 + f_2 = 9 \quad \text{--- (11)}$$

$$\therefore f_1 = ???$$

$$\therefore f_2 = ???$$

3) Data set A: 15, 12, 18, 10, 16

Data set B: 11, 17, 14, 20, 13

Hint: Coefficient of variance

a) Which data set shows more consistency?

b) If the last value in data set B is corrected to 10, compute the corrected standard deviation.

Hint: Assignment Question.

Best measures of dispersion = Standard deviation

Over the years geometric growth/decrease = Geometric mean

Distance vs speed = Harmonic mean

For comparison = Coefficient of variance

Position of Q_i, D_i or P_i is not integer = Go to next integer value

$$\begin{aligned} 6.75 &= 7\text{th obs.} \\ 8.25 &= 9\text{th obs.} \end{aligned}$$

Position of Q_i, D_i or P_i is integer = take average.

$$\begin{aligned} Q_i = 6 & \text{ Then, } \frac{6\text{th obs.} + 7\text{th obs.}}{2} \\ Q_i = 2 & \text{ Then, } \frac{2\text{nd obs.} + 3\text{rd obs.}}{2} \end{aligned}$$

Lowest 25% = Q_1 or P_{25}

Highest 25% = Q_3 or P_{75}

Lowest 30% = P_3 or P_{30}

Highest 30% = D_7 or P_{70}

4) Sales Revenue: 7, 8, 9, 11, 12, 14, 15

Advertising cost: 32, 35, 39, 44, 50, 52, 55

Here sales revenue influenced by the ad. cost.

a) Determine the Direction and strength of relationship.

b) Determine the fitted regression model

c) Estimate sales revenue when ad. cost

is 60.

d) Estimate ad. cost when sales rev. is 10.

e) How precise your prediction.

f) If the last ad. value is corrected to 50, calculate the corrected regression parameter.

Solution:

Here,
So,

sales rev. influence by ad. cost.

Sales revenue is Y

Advertising cost is X

Table 1:

Sales Revenue (y_i)	Ad. cost (x_i)	x_i^2	y_i^2	$x_i y_i$
7	32	1024	49	224
8	35	1225	64	280
9	39	1521	81	351
11	44	1936	121	484
12	50	2500	144	600
14	52	2704	196	728
15	55	3025	225	825
$\Sigma y_i = 76$	$\Sigma x_i = 307$	$\Sigma x_i^2 = 13925$	$\Sigma y_i^2 = 880$	$\Sigma x_i y_i = 3492$

a) From table 1,

$$\begin{aligned}\sum x_i &= 307, & \sum y_i^2 &= 880 \\ \sum y_i &= 76, & \sum x_i y_i &= 3492 \\ \sum x_i^2 &= 13935,\end{aligned}$$

$$\therefore r = \frac{\sum x_i y_i - \frac{\sum x_i \sum y_i}{n}}{\sqrt{\sum x_i^2 - \frac{(\sum x_i)^2}{n}} \sqrt{\sum y_i^2 - \frac{(\sum y_i)^2}{n}}} = ???$$

b) from table 1,

...

$$\therefore \hat{\beta} = \frac{n \sum x_i y_i - \sum x_i \sum y_i}{n \sum x_i^2 - (\sum x_i)^2} = \frac{(7 \times 3492) - (307 \times 76)}{(7 \times 13935) - (307)^2} = 0.34$$

$$\therefore \hat{\alpha} = \bar{y} - \hat{\beta} \bar{x} = -4.05$$

$$\therefore \hat{y}_i = -4.05 + 0.34 x_i$$

c) $x = 60$; $\hat{y}_i = ??$

d) $y_i = 10$; $x = ???$

e) $R^2 = 1 - \frac{SSE}{SST}$; [Ref: lecture slide]

f)

h

Uncorrected part:

$$\sum x_i = 307$$

$$\sum y_i = 76$$

$$\sum x_i^2 = 13935$$

$$\sum y_i^2 = 880$$

$$\sum x_i y_i = 3492$$

Corrected part:

$$\sum x_i = 307 - 55 + 50 = ???$$

$$\sum y_i = 76$$

$$\sum x_i^2 = 13935 - 55^2 + 50^2 = ???$$

$$\sum y_i^2 = 880$$

$$\sum x_i y_i = 3492 - (15 \times 55) + (15 \times 50) = ???$$

$$\hat{\beta}_{\text{corrected}} =$$

$$\hat{\alpha}_{\text{corrected}} =$$

$$[2022 \text{ Solution : 11.9}] :$$

Strength of association = Karl Pearson.

Direction of association = Scatter plot

Direction of and Strength = Karl Pearson

Y on X = Before "on" Dependent variable
After "on" Independent variable