1. Probability Laws: Two basic rules/laws of probability theory-

a. Additive Law
b. Multiplicative Law

2. Additive Law: Let, A and B be two events. If the events are joint events, then according to additive law, we can write

$$P(A\ or\ B) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

If the events are mutually exclusive events, then the term $P(A \cap B) = 0$. Then we can write, $P(A \cup B) = P(A) + P(B)$

3. In a company, 60% of the employees have motorcycle, 40% has private car and 20% has both. What is the probability that the employee has either motorcycle or private car?

Solution: Let,

$M = Event\ of\ employee\ have\ motorcycle$

$C = Event\ of\ employee\ have\ private\ car$

Here, $P(M) = 0.6; P(C) = 0.4; and\ P(M \cap C) = 0.2$

According to the additive law, $P(M \cup C) = P(M) + P(C) - P(M \cap C) = 0.6 = 0.4 - 0.2 = 0.8$

4. Mr. Ali feels that the probability that he will pass mathematics is 2/3 and statistics is 5/6. If the probability that he will pass both the course is 3/5, what is the probability that he will pass at least one of the courses? $\left(Ans: \frac{9}{10}\right)$

5. In a sample of 500 students, 320 said they had a stereo, 175 said they had a TV, and 100 said they had both. 5 said they had neither. If a student is selected at random, what is the probability that the student has only a stereo or TV? What is the probability that the student has both a stereo and TV? $(Ans: 0.79, 0.20)$

6. A and B are two weak students in Statistics. A can answer correctly 15% of the questions and B can answer correctly 10% of the questions. A and B both can answer 2% of the questions. A question is selected at random. Find the probability that (a) at least one of them can answer correctly, (b) No one can answer correctly, (c) only one can answer correctly.

Solution: Let us denote the events,

$A: A\ can\ answer\ the\ question\ correctly$

$B: B\ can\ answer\ the\ question\ correctly$

Here, $P(A) = 0.15; P(B) = 0.10; P(A \cap B) = 0.02$

a) $P(at\ least\ one) = P(A \cup B) = P(A) + P(B) = P(A \cap B) = ???$
b) $P(No\ one) = P(\overline{A \cup B}) = 1 - P(A \cup B) = ???$
c) $P(Only\ one) = P(A \cap \bar{B}) + P(B \cap \bar{A}) = ???$

7. Consider a dartboard with a bullseye and two concentric rings around it. The probability of hitting the bullseye is 0.15, the probability of hitting the inner ring is 0.30, and the probability of hitting the outer ring is 0.40.

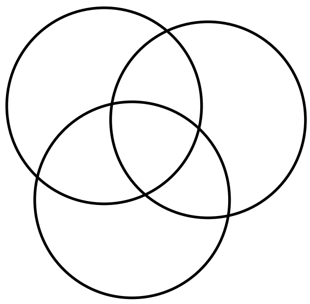
a) What is the probability that the projectile hits the target?
b) What is the probability that it misses the target?

8. Nadia feels that the probability that she will get 4.00 in "Algorithm" is 3/4, 4.00 in "Statistics" is 4/5, 4.00 in both the courses is 3/5. What is the probability that Nadia will get

a) At least one 4.00?
b) Only one 4.00?
c) No 4.00?

9. A sample survey was undertaken to investigate which papers A, B, and C people read. In survey of 100 people following results were obtained: 60 people read A, 40 people read B, 70 people read C, 32 people read A and B, 45 people read A and C, 38 people read B and C, 30 people read A, B, C. If a person is selected at random, find the probability,

a) Read only newspaper A.
b) Read only one newspaper.
c) Read at least one newspaper.
d) Read at most one newspaper.



10. Multiplicative Law: Let, A and B be two events. If they are dependent, then we can write,

$$P(A \cap B) = P(A|B)\,P(B)$$

$$P(A \cap B) = P(B|A)P(A)$$

If they are independent events,

$$P(A \cap B) = P(A) \times P(B)$$

11. Conditional probability: Conditional probability refers to the chances that some outcome occurs given that another event has also occurred. Let, $A = One\ event\ occured$, $B = Another\ event\ already\ occured$, then the conditional probability can be written as,

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Again, Let, $A = One\ event\ already\ occured$, $B = Another\ event\ occured$, then the conditional probability can be written as,

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

12. In a company, 60% of the employees have motorcycle, 40% has private car and 20% has both. If it is known that the employee has a motorcycle, then what is the probability that the employee also has a car?

Solution: Let,

$$M = Event\ of\ motorcycle$$

$$C = Event\ of\ private\ car$$

Here, $P(M) = 0.6; P(C) = 0.4; P(M \cap C) = 0.2$

$$\therefore P(C|M) = \frac{P(M \cap C)}{P(M)} = \frac{0.2}{0.6} = \frac{1}{3} = 0.33$$

Hence, 33% employee has a car who has motorcycle before.

13. Suppose a balanced die is rolled once.

a. Find the probability that a number divisible by 3 is rolled given that the die comes up even.

b. Find the probability that the die comes up even given that a number divisible by 3 is rolled.

c. Find the probability that a number divisible by 3 is rolled given that die comes up at most 4.

d. Find the probability that the die comes up at most 4 given that a number divisible by 3 is rolled.

14. In rainy season, it rains 70% of the days in Bangladesh. When it rains, 80% times it makes thunderstorms. What is the probability that, in a particular day of rainy season, it will rain and it will thunderstorm?

15. Bayes Theorem: Let, events T_1 and T_2 form partition of S. Let R be an event with $P(R) > 0$. Then,

$$P(T_1|R) = \frac{P(T_1 \cap R)}{P(R)} = \frac{P(T_1)P(R|T_1)}{P(T_1)P(R|T_1) + P(T_2)P(R|T_2)}$$

$$P(T_2|R) = \frac{P(T_2 \cap R)}{P(R)} = \frac{P(T_2)P(R|T_2)}{P(T_2)P(R|T_2) + P(T_1)P(R|T_1)}$$

General formula:

Let, events $T_1, T_2, \ldots$ form partition of S. Let R be an event with $P(R) > 0$. Then,

$$P(T_j|R) = \frac{P(T_j \cap R)}{P(R)} = \frac{P(T_j)P(R|T_j)}{P(T_1)P(R|T_1) + P(T_2)P(R|T_2) + \cdots} = \frac{P(T_j)P(R|T_j)}{\sum P(T_j)P(R|T_j)}$$

16. 60% of the students in a class are male. 5% of the males and 10% of the females are in the photography club. If a student is randomly selected from the class.

a. What is the probability that the student is in photography club?

b. If the randomly selected student is in the photography club, what is the chance that the student is male?

Solution

60% of the students in a class are male

40% of the students in a class are female

5% of the males are in the photography club

10% of the females are in the photography club

- $P(M) = 0.6$
- $P(F) = 0.4$
- $P(C|M) = 0.05$
- $P(C|F) = 0.10$

a) What is the probability that the student is in photography club?

$$P(C) = P(M)P(C|M) + P(F)P(C|F) = 0.07$$

b) If the randomly selected student is in the photography club, what is the chance that the student is male?

$$P(M|C) = \frac{P(M)P(C|M)}{P(C)} = 0.43$$

17. Dr. X diagnoses cancer correctly 80% cases. The chance that a patient will die by his treatment after correct diagnosis is 30%, and the chance of death by wrong diagnosis is 90%. A patient of Dr. X who had cancer died. What is the probability that his diagnosis was wrong?

Example

$P(D|B_1) = 0.3$

$P(D|B_2) = 0.9$

- Dr. X diagnoses cancer correctly 80% cases. The chance that a patient will die by his treatment after correct diagnosis is 30%, and the chance of death by wrong diagnosis is 90%. A patient of Dr. X who had cancer died. What is the probability that his diagnosis was wrong?

$P(B_2) = 0.2$

$B_1 = Cancer\ is\ correctly\ diagnosis$

$B_2 = Cancer\ is\ not\ correctly\ diagnosis$

$P(B_1) = 0.8$

$D = Cancer\ patient\ died$

$P(B_2|D) = ????$

$$P(B_2|D) = \frac{P(B_2)P(D|B_2)}{P(B_1)P(D|B_1) + P(B_2)P(D|B_2)}$$

18. In a class of 120 students, 60 are studying English, 50 are studying French and 20 are studying both. If a student is selected at random from this class, what is the probability that he/she is studying English if it is given that he/she is studying French. (Ans: 0.425)

19. Contingency table: A contingency table, also known as a cross-tabulation or crosstab, is a statistical table used to analyze and display the relationship between two categorical variables. For example, the question, "Do you like watching TV?" was asked of 100 people. Results are shown in the table.

	Yes	**No**	**Total**
Male	19	41	60
Female	12	28	40
Total	31	69	100

20.

Example

- Below given a contingency table for Smoking status and Cancer status.

	Cancer	Healthy	Total
Smoker	7860	1530	9390
Non smoker	5390	11580	16970
Total	13250	13110	26360

1. What is the probability that a randomly selected person is a smoker? (0.36)
2. What is the probability that a randomly selected person has cancer? (0.50)
3. What is the probability that a randomly selected person is both smoker and has cancer? (0.298)
4. If a person is smoker, what is the probability that he also has cancer? (0.84)

21. Information of 200 engineers,

Age	BSc Only	MSc.	Total
<30		10	100
30 to 40	20		50
>40		10	
Total			200

If one engineer is selected at random from the company, find,

a) The probability that he has only a BSc degree. (Ans: 3/4)

b) The probability that he has a MSc degree given that he is over 40. (Ans: 1/5)

c) The probability that he is under 30 given that he has only a BSc degree (Ans: 3/5)

22. Machine always do mistake. In a bolt factory "Machine A" produces 45% of the output and "Machine B" produces the rest. On average 9 items in 1000 produces by "Machine A" are defective, and 2 items in 500 produced by "Machine B" are defective. In a day run, the two machine produce 20000 items. An item is drawn at random from a day's output and is found to be defective. What is the probability that,

a) Defective item was produced by "Machine A"? (Ans: 81/125)

b) Defective item was produced by "Machine B"? (Ans: 44/125)

[Hints: $B_1 = Item\ produced\ by\ Machine\ A; B_2 = Item\ produced\ by\ Machine\ B; A = Defective\ item$]