Basic concepts of Probability (1)

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Probability

- Probability is the likeliness/chance of occurring any event(s).
- Consider tossing a coin.
- What is the chance of it landing heads up?
- What is the chance of it landing tails up?

Experiment

- A specific action, or
- A process, or
- A phenomenon that leads to observable outcomes

Experiment

- An experiment refers to a specific action, process, or phenomenon that leads to observable outcomes.

- For example,
  - Measuring distance from Dhaka to Chattogram
  - Tossing a fair coin

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Types of Experiment

- Two types of experiment-

1. Deterministic: Whose outcome is predictable in advance

2. Probabilistic/Random: Whose outcome is not predictable

Some examples

- Is tossing a coin a random experiment or deterministic experiment?

- Ans: Tossing a coin is an experiment. There are two possible outcomes (head or tail). These outcomes are unpredictable before the coin is tossed. Therefore, this is a random experiment.

Some examples

- Is drawing a card from well shuffled deck of cards a random experiment or deterministic experiment?

- Ans: Drawing a card from a well-shuffled deck of cards is a random experiment. The randomness arises from the uncertainty of which card will be drawn, even though the deck is well-shuffled. The outcome (the specific card drawn) is not predictable beforehand, making it a random event.

Some examples

- Is multiplying 4 and 8 on a calculator a random experiment or deterministic experiment?

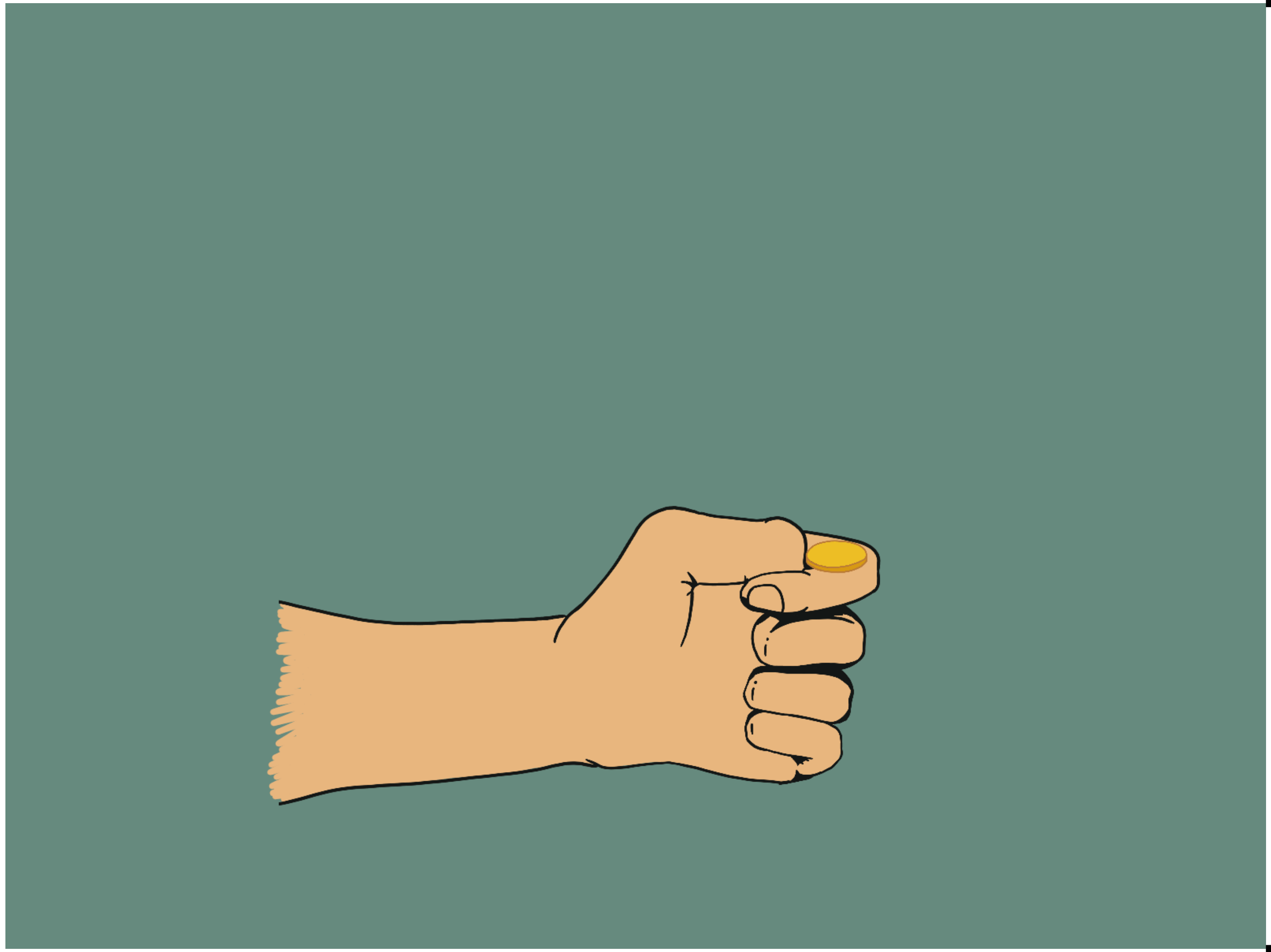
- Ans: Multiplying 4 and 8 on a calculator is a deterministic experiment. In a deterministic experiment, the outcome is certain and predictable based on the given inputs and the rules of the operation. In this case, multiplying 4 and 8 will always result in the same answer: 32. There is no randomness or uncertainty involved in this calculation, making it a deterministic process.

Basic terms

- Sample space:

Basic terms

The result of an experiment is known as outcomes



Outcome

Sample space

Head (H) and Tail (T)

All possible outcomes

Outcome

Basic terms

- Sample space: A sample space associated with an experiment which consist all possible outcomes of the experiment.

- Denoted by S

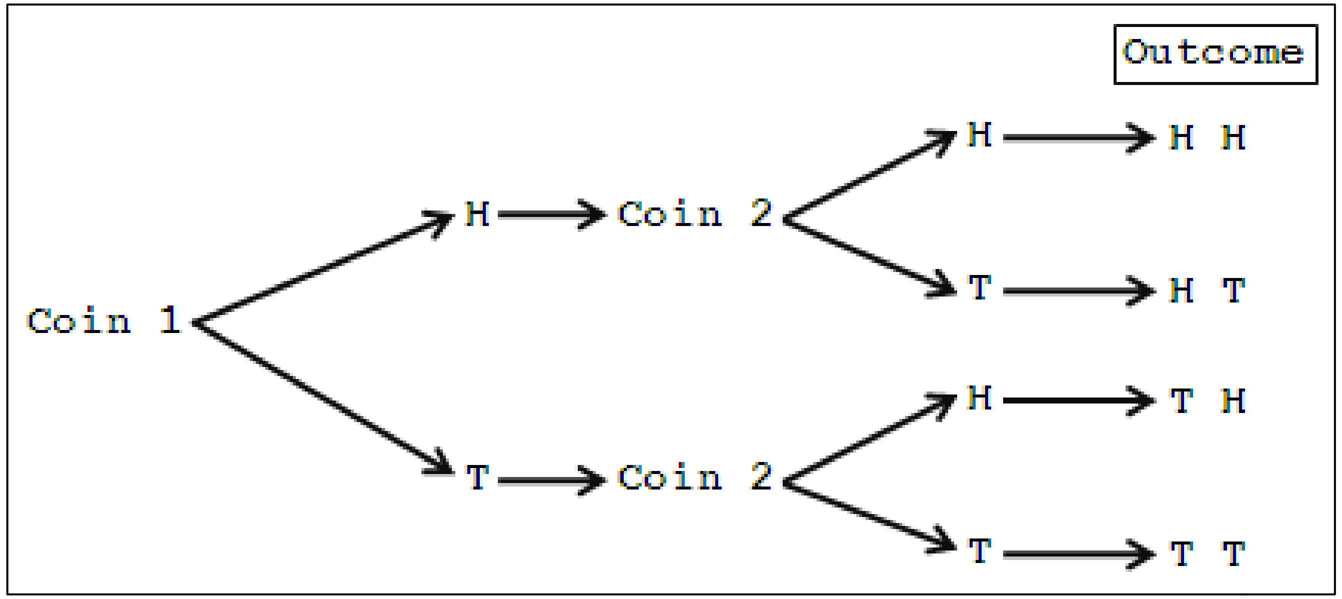
- $S = \{H, T\}; \; from\ previous\ example$

Basic terms

- For example, consider the experiment of tossing two coins. Write the sample space of this experiment.

	$\boldsymbol{Head\ (H)}$	$\boldsymbol{Tail\ (T)}$
$\boldsymbol{Head\ (H)}$	H,H	H,T
$\boldsymbol{Tail\ (T)}$	T,H	T,T

- $S = \{(H,H),(H,T),(T,H),(T,T)\}$



Basic terms

- Event:

Basic terms

Is this a random experiment?

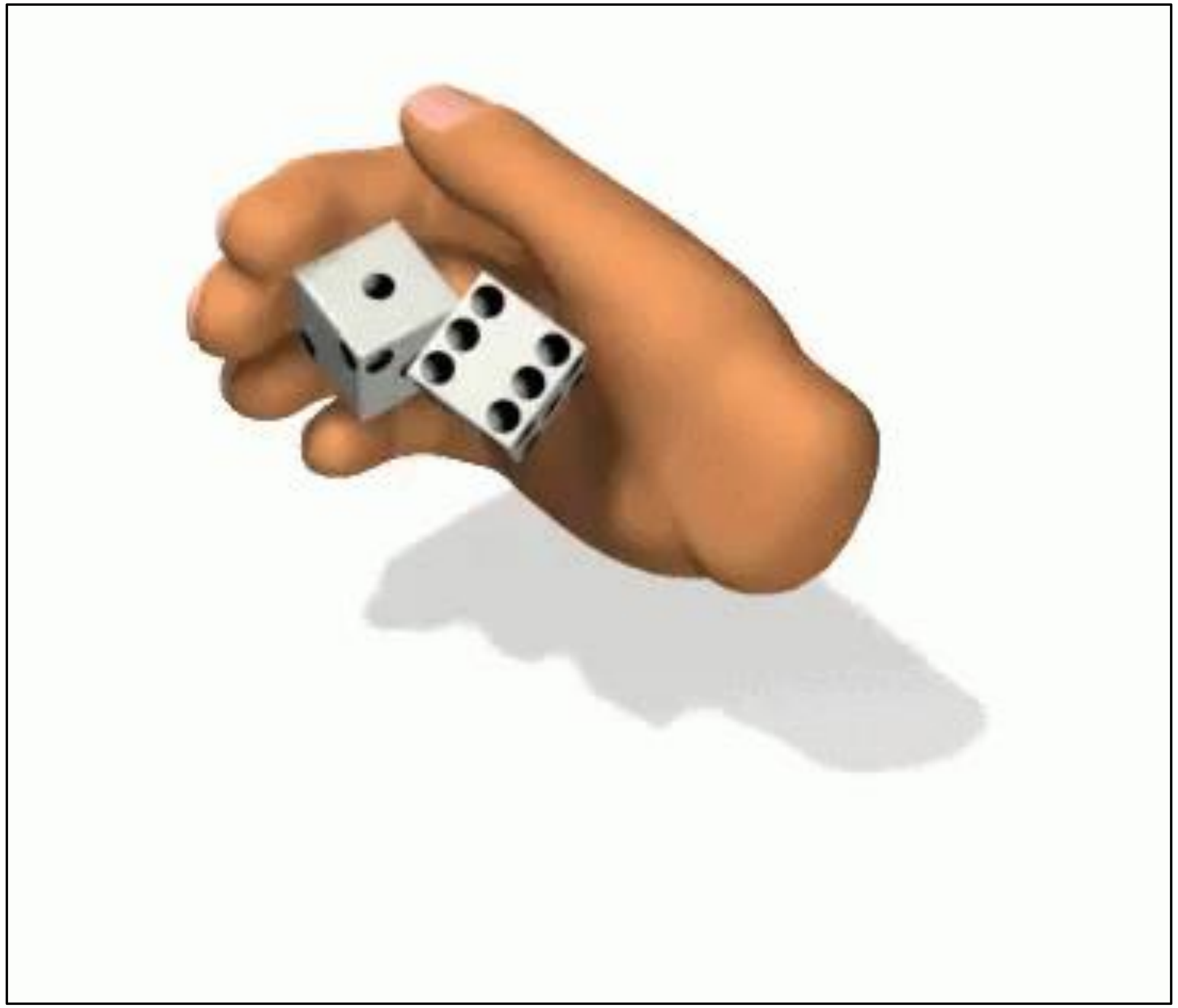
Yes, sir!

Can you write the sample space?

$S = \{1,2,3,4,5,6\}$

Split the odd number...

An Event $\rightarrow$ $A = \{1,3,5\}$



Basic terms

- Event: Any subset of the sample space

Basic terms

- Event: Any subset of the sample space
- Different types of events:

Exclusive events: Two events are called mutually exclusive if both the events cannot occur simultaneously in a single trial. In other words, if one of those events occurs, the other event will not occur.

Exhaustive events: Exhaustive events are those, which includes all possible outcomes.

Equally likely events: The events of a random experiment are called equally likely if the chance of occurring those events are all equal.

Union of events $(A \cup B)$

Intersection of events $(A \cap B)$

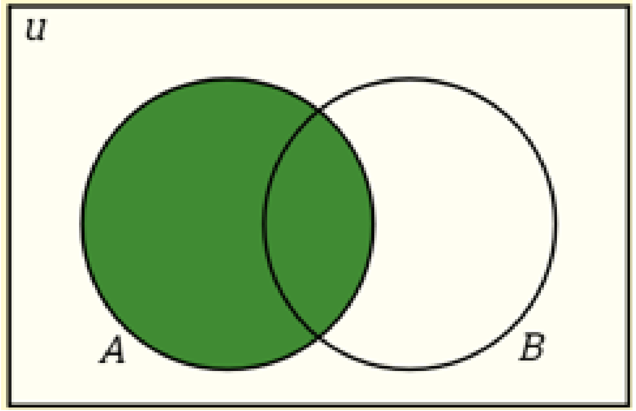
Complement of an event $(\bar{A}\ or\ A')$

Exclusive event

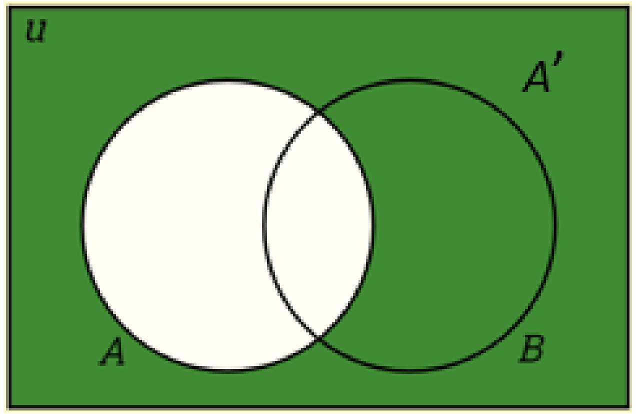
Exhaustive event

Equally likely event

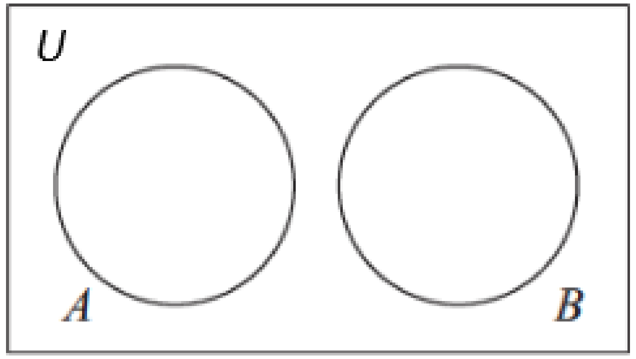
Basic terms



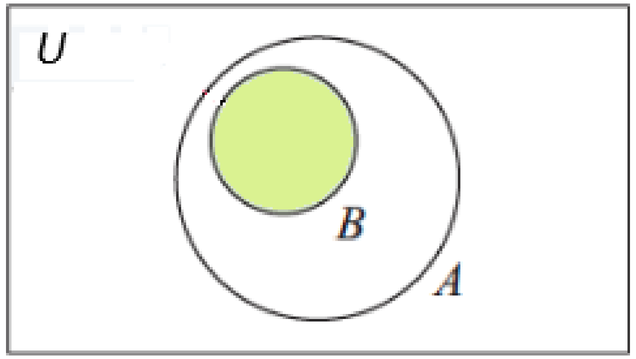
Set *A*



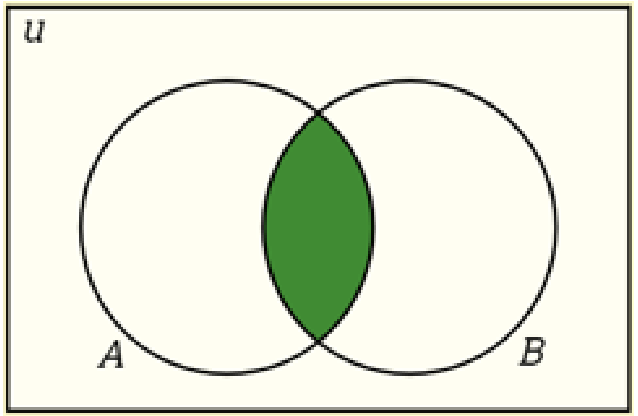
A′ the complement of *A*



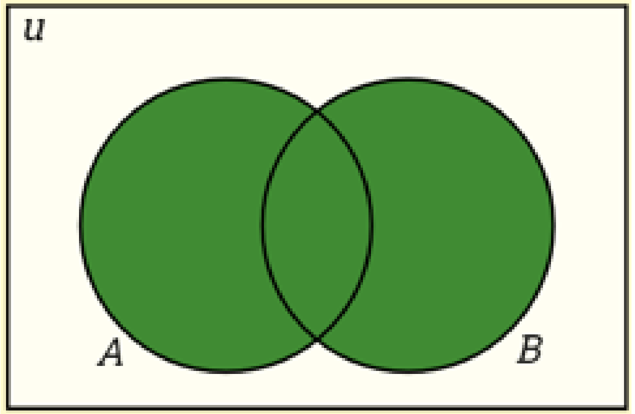
A and *B* are disjoint sets



B is proper subset of *A* $B \subset A$



Both A and B
A intersect *B* $A \cap B$



Either A or B
A union *B* $A \cup B$

Approaches of assigning probability

- At first we identify the sample space S of the random experiment.

- We then define our favorable event and assign probability to the event using one of the following 3 basic approaches-

Classical

Frequency

Subjective

Classical approach

$$P(Event) = \frac{Number\ of\ occurance/outcome\ in\ the\ event}{Total\ number\ of\ outcomes\ in\ the\ sample\ space}$$

- Assumption-
  1. Equally likely outcome
  2. Mutually exclusive outcome
  3. Mutually exhaustive outcome

$S = \{1, 2, 3, 4, 5, 6\}$

$A = \{2, 4, 6\}$ ➡ $P(A) = \frac{3}{6}$

$B = \{1, 3, 5\}$ ➡ $P(B) = \frac{3}{6}$

Classical approach

- Example: A committee consists of five executives of which three women (W_1, W_2, W_3) and two men (M_1, M_2). A random sample of two executives needs to be selected at random without replacement from which chairman and a secretary would be selected. Set up the sample space and find the probability that

a) W_1 and W_2 will be selected

b) M_1 will be selected

c) M_1 will not be selected

d) W_1 or M_1 will be selected

Classical approach

a) W_1 and W_2 will be selected

- Solution: Sample space of this experiment is

- $S = \{(W_1W_2), (W_1W_3), (W_1M_1), (W_1M_2), (W_2W_3), (W_2M_1), (W_2M_2), (W_3M_1), (W_3M_2), (M_1M_2)\}$

- Total number of outcomes in the experiment = 10

a) Let, $A = Event\ of\ selecting\ W_1\ and\ W_2 = \{W_1W_2\}$

$$\therefore P(A) = \frac{1}{10} = 0.1$$

Classical approach

b) M_1 will be selected

- Solution: Sample space of this experiment is

- $S = \{(W_1W_2), (W_1W_3), (W_1M_1), (W_1M_2), (W_2W_3), (W_2M_1), (W_2M_2), (W_3M_1), (W_3M_2), (M_1M_2)\}$

- Total number of outcomes in the experiment = 10

b) Let, $B = Event\ of\ selecting\ M_1 = \{W_1M_1, W_2M_1, W_3M_1, M_1M_2\}$

$$\therefore P(B) = \frac{4}{10} = 0.4$$

Classical approach

c) M_1 will not be selected

- Solution: Sample space of this experiment is
- $S = \{(W_1W_2), (W_1W_3), (W_1M_1), (W_1M_2), (W_2W_3), (W_2M_1), (W_2M_2), (W_3M_1), (W_3M_2), (M_1M_2)\}$
- Total number of outcomes in the experiment = 10

c) Let, $B = Event\ of\ selecting\ M_1 = \{W_1M_1, W_2M_1, W_3M_1, M_1M_2\}$

$$\therefore P(\bar{B}) = 1 - P(B) = 1 - \frac{4}{10} = 0.6$$

d) W_1 or M_1 will be selected

Classical approach

- Solution: Sample space of this experiment is

- $S = \{(W_1W_2), (W_1W_3), (W_1M_1), (W_1M_2), (W_2W_3), (W_2M_1), (W_2M_2), (W_3M_1), (W_3M_2), (M_1M_2)\}$

- Total number of outcomes in the experiment = 10

d) Let,

$$C = Event\ of\ selecting\ W_1 = \{(W_1W_2), (W_1W_3), (W_1M_1), (W_1M_2)\}$$

$$B = Event\ of\ selecting\ M_1 = \{W_1M_1, W_2M_1, W_3M_1, M_1M_2\}$$

$$C\ or\ B = C \cup B = \{(W_1W_2), (W_1W_3), (W_1M_1), (W_1M_2), W_2M_1, W_3M_1, M_1M_2\}$$

$$P(C \cup B) = \frac{7}{10} = 0.7$$

Frequency approach

- A random experiment is repeated n times under same condition
- An event “A” occurs m times
- According to frequency approach
- Probability of A, $P(A) = \lim_{n \to \infty} \frac{m}{n}$

Frequency approach

- For example;

- In a dice throwing experiment, S= {1, 2, 3, 4, 5, 6}

- And our favorable event is E= {2}

- Let, 2 occurred a total of 998 times out of total 6000 trials. Therefore $P(E) = \lim_{n \to \infty} \frac{998}{6000} = \frac{1}{6}$

Subjective approach

- Based on the judgement (personal experience, prior information and belief etc.), one can assign probability to an event E of a random experiment.

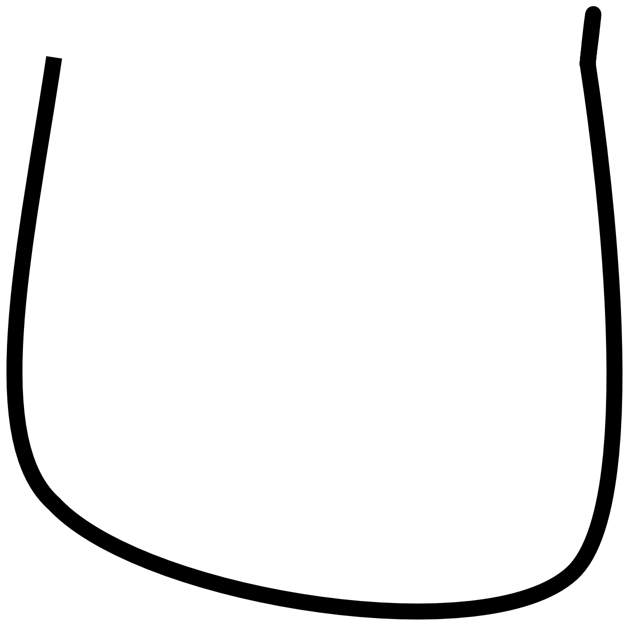
- For example; on a day of summer someone made a statement on probability that rain will occur on that day is .70, based on his previous experience.

Axioms of probability

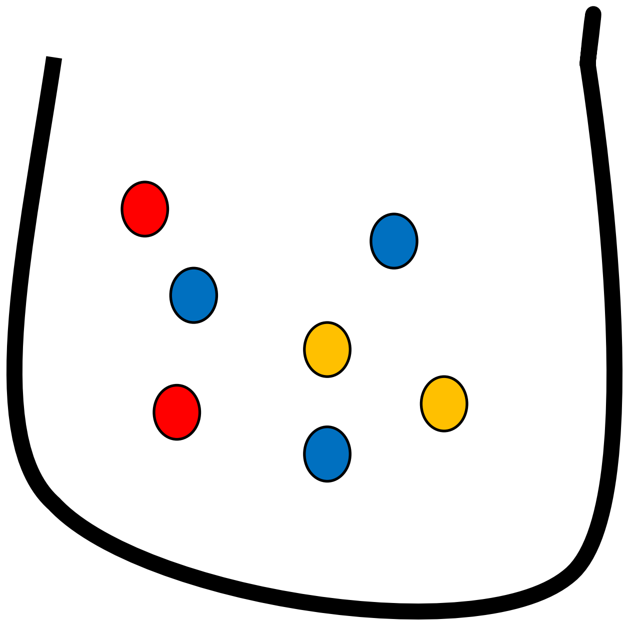
- Probability of an event follows three axioms:

1. $P(A) \geq 0$ (Axiom of positivizes)
2. $P(S) = 1$ (Axiom of certainty) $S = \{1,2,3\}$
3. $P(A_1 \cup A_2 \cup \cdots) = P(A_1) + P(A_2) + \cdots = \sum P(A_i)$

With and Without Replacement



With and Without Replacement



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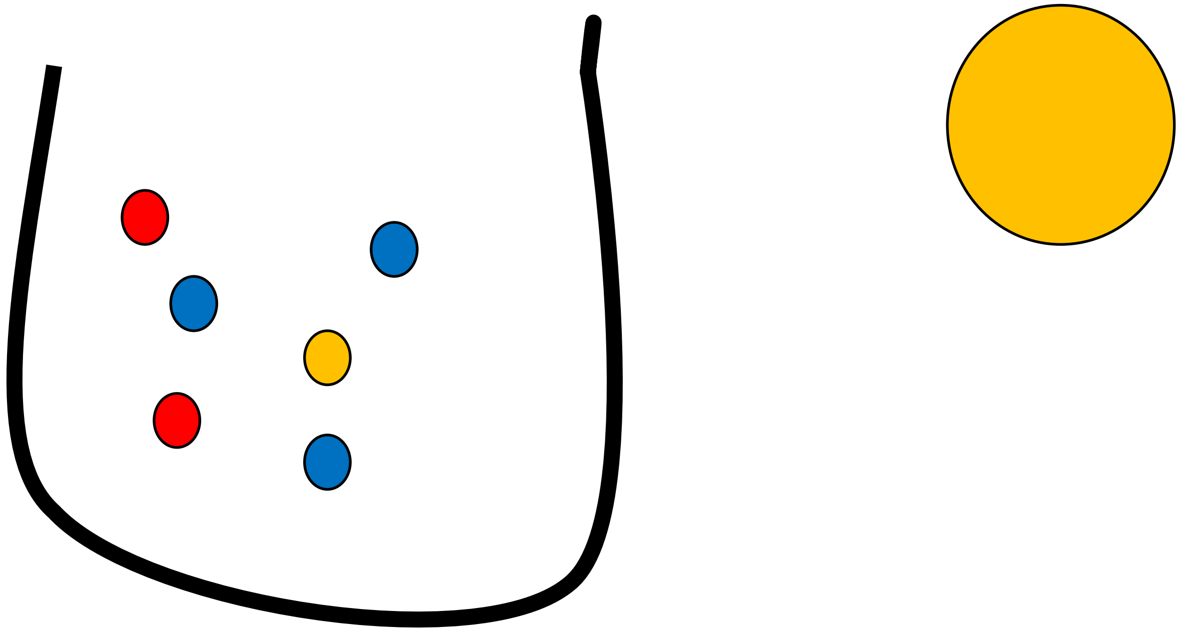
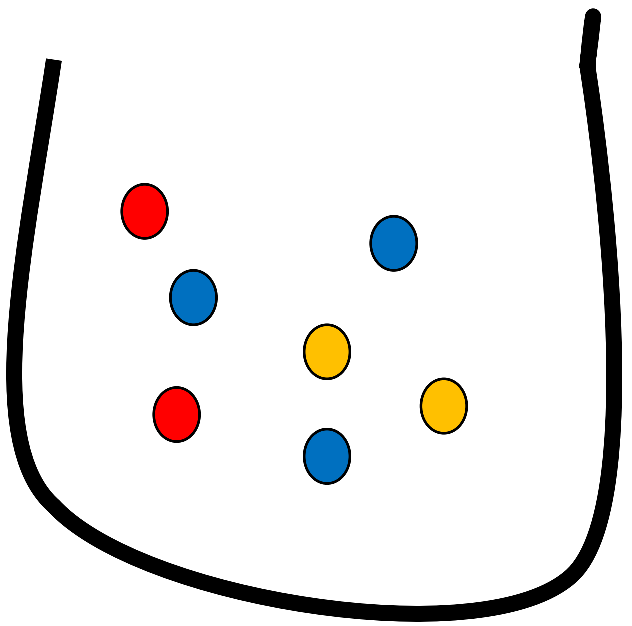
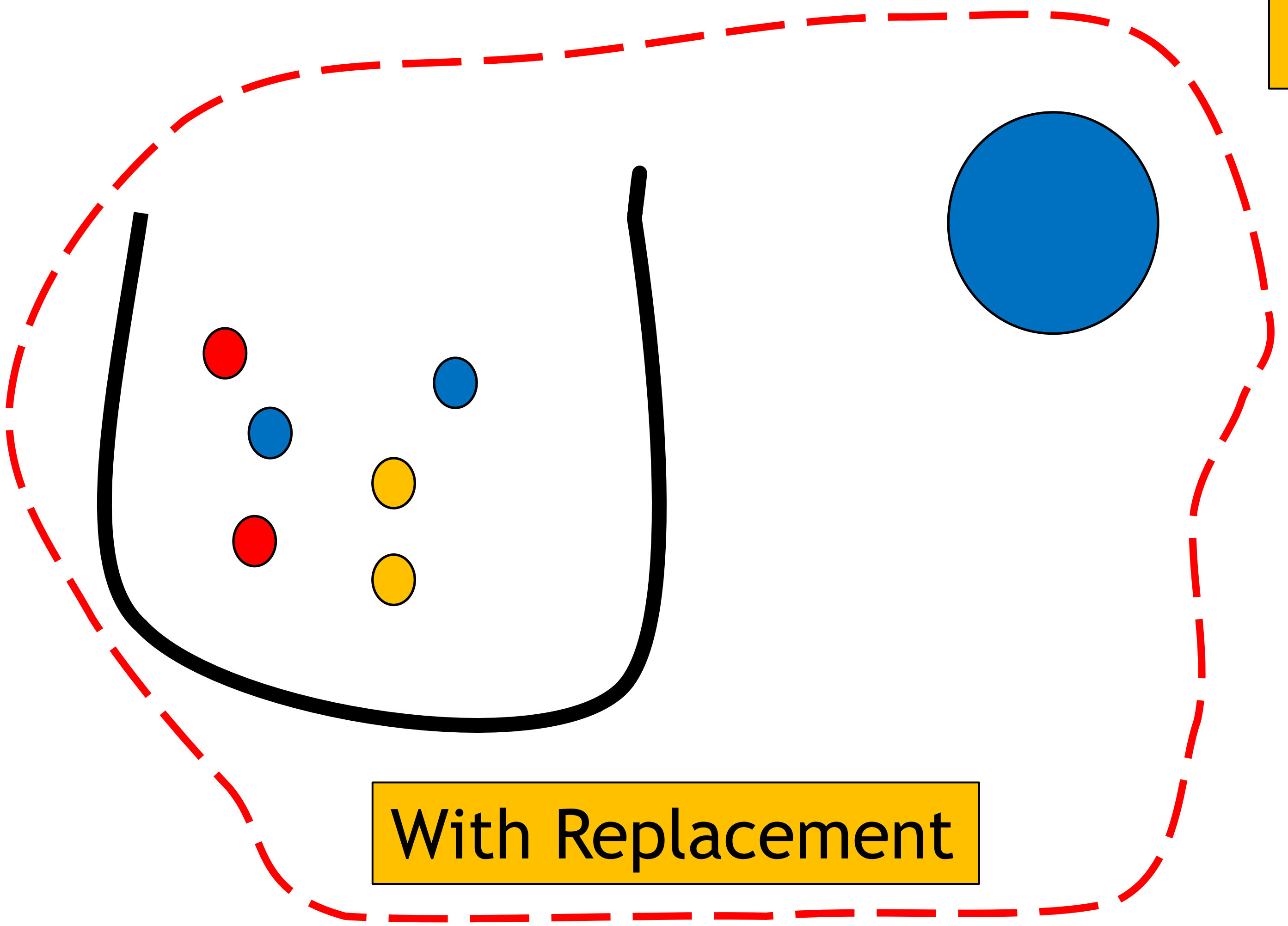


With and Without Replacement

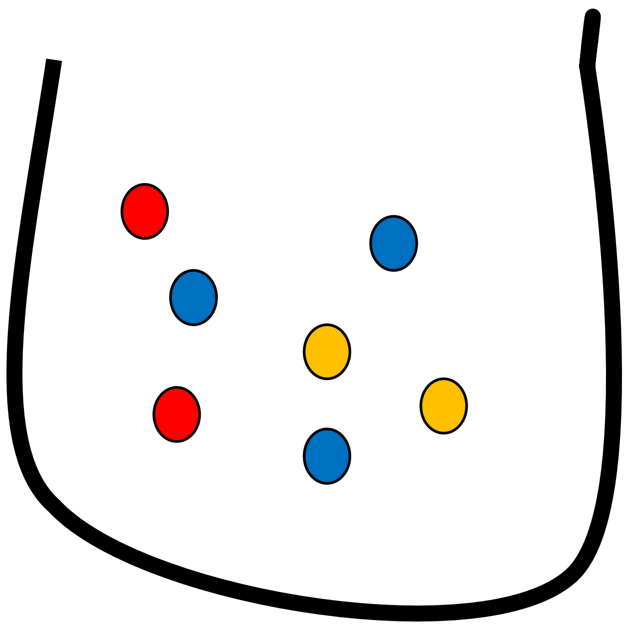


With and Without Replacement

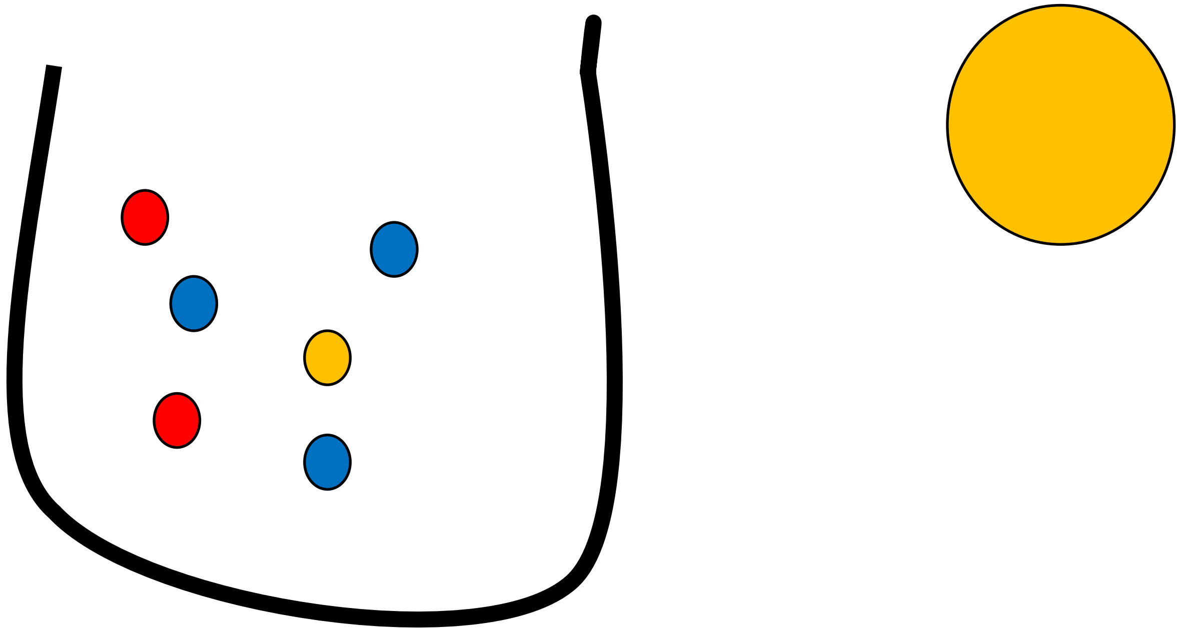
If the elements of a sample are drawn randomly one by one and after each draw the element is returned to the population = With Replacement



With and Without Replacement

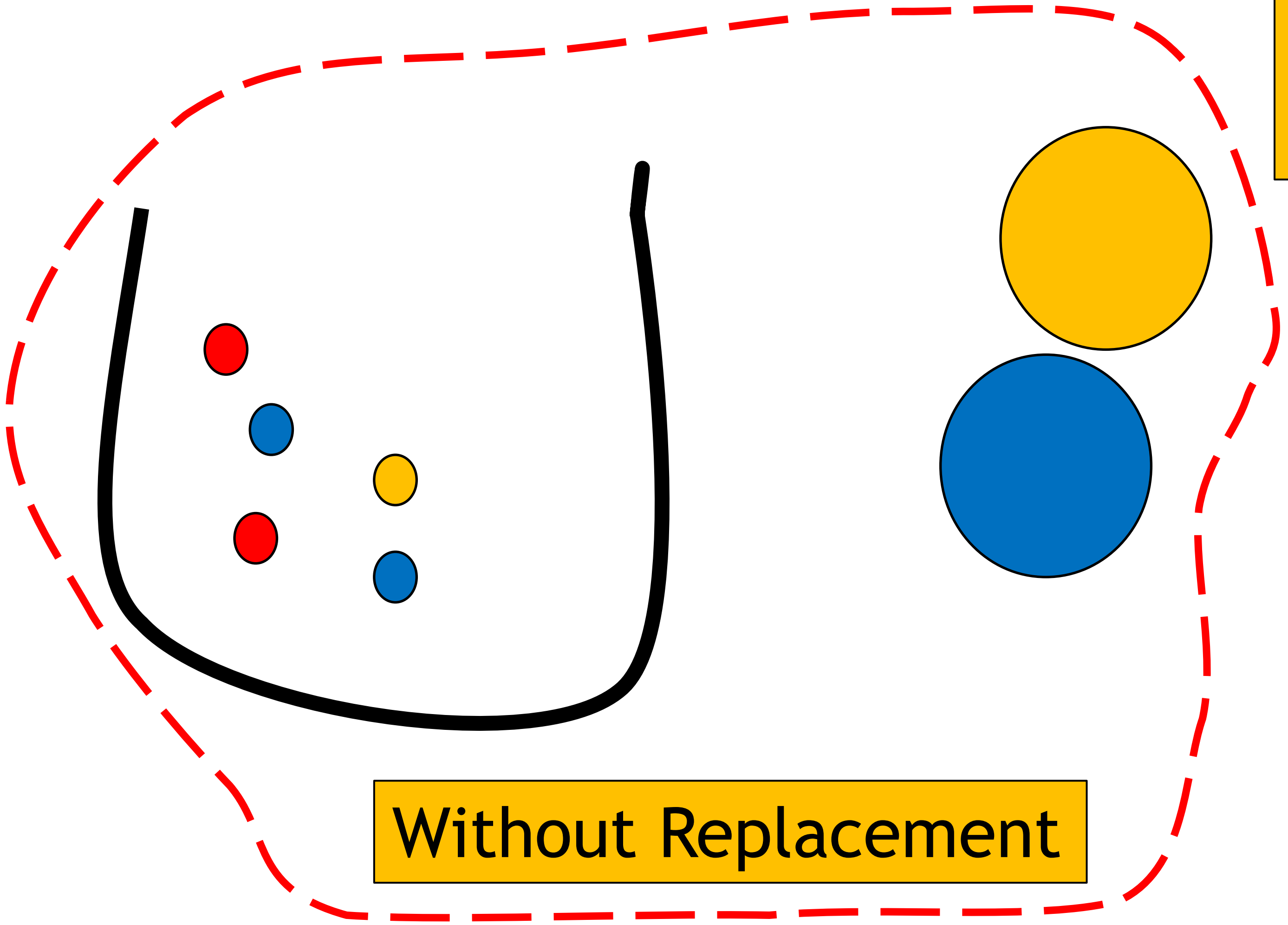


With and Without Replacement



With and Without Replacement

If the elements of a sample are drawn randomly one by one and after each draw the element is not returned to the population = With Out Replacement



With and Without Replacement

- A box contains 20 bulbs, of which 5 are defective. If 3 of the bulbs are selected at random without replacement, what is the probability that all three bulbs are defective?

Solution:

Probability of bulb is defective, $P(D) = \frac{5}{20} \times \frac{4}{19} \times \frac{3}{18} = 0.0088$

With and Without Replacement

- A box contains 20 bulbs, of which 5 are defective. If 3 of the bulbs are selected at random without replacement, what is the probability that all three bulbs are defective?

Another Solution:

$\therefore$ 3 bulbs out of 20 bulbs can be draw in ${}^{20}C_3 = 1140$ ways

$\therefore$ 3 defective bulbs out of 5 defective bulbs can e draw in ${}^{5}C_3 = 10$ ways

$\therefore$ Probability of defective bulbs, $P(D) = \frac{10}{1140} = 0.0088$

Mathematical exercise

To access additional mathematical problems, please refer to the PDF lecture notes.

Thank You