✔ QUICK CHECK EXERCISES 7.8 (See page 557 for answers.)

1. In each part, determine whether the integral is improper, and if so, explain why. Do not evaluate the integrals.

   (a) $\displaystyle\int_{\pi/4}^{3\pi/4} \cot x\,dx$ (b) $\displaystyle\int_{\pi/4}^{\pi} \cot x\,dx$

   (c) $\displaystyle\int_{0}^{+\infty} \frac{1}{x^2+1}\,dx$ (d) $\displaystyle\int_{1}^{+\infty} \frac{1}{x^2-1}\,dx$

2. Express each improper integral in Quick Check Exercise 1 in terms of one or more appropriate limits. Do not evaluate the limits.

3. The improper integral

$$\int_{1}^{+\infty} x^{-p}\,dx$$

converges to _____ provided _____.

4. Evaluate the integrals that converge.

   (a) $\displaystyle\int_{0}^{+\infty} e^{-x}\,dx$ (b) $\displaystyle\int_{0}^{+\infty} e^{x}\,dx$

   (c) $\displaystyle\int_{0}^{1} \frac{1}{x^3}\,dx$ (d) $\displaystyle\int_{0}^{1} \frac{1}{\sqrt[3]{x^2}}\,dx$

EXERCISE SET 7.8 Graphing Utility C CAS

1. In each part, determine whether the integral is improper, and if so, explain why.

   (a) $\displaystyle\int_{1}^{5} \frac{dx}{x-3}$ (b) $\displaystyle\int_{1}^{5} \frac{dx}{x+3}$ (c) $\displaystyle\int_{0}^{1} \ln x\,dx$

   (d) $\displaystyle\int_{1}^{+\infty} e^{-x}\,dx$ (e) $\displaystyle\int_{-\infty}^{+\infty} \frac{dx}{\sqrt[3]{x-1}}$ (f) $\displaystyle\int_{0}^{\pi/4} \tan x\,dx$

2. In each part, determine all values of p for which the integral is improper.

   (a) $\displaystyle\int_{0}^{1} \frac{dx}{x^p}$ (b) $\displaystyle\int_{1}^{2} \frac{dx}{x-p}$ (c) $\displaystyle\int_{0}^{1} e^{-px}\,dx$

3–32 Evaluate the integrals that converge.

3. $\displaystyle\int_{0}^{+\infty} e^{-2x}\,dx$
4. $\displaystyle\int_{-1}^{+\infty} \frac{x}{1+x^2}\,dx$
5. $\displaystyle\int_{3}^{+\infty} \frac{2}{x^2-1}\,dx$
6. $\displaystyle\int_{0}^{+\infty} xe^{-x^2}\,dx$
7. $\displaystyle\int_{e}^{+\infty} \frac{1}{x\ln^3 x}\,dx$
8. $\displaystyle\int_{2}^{+\infty} \frac{1}{x\sqrt{\ln x}}\,dx$
9. $\displaystyle\int_{-\infty}^{0} \frac{dx}{(2x-1)^3}$
10. $\displaystyle\int_{-\infty}^{3} \frac{dx}{x^2+9}$
11. $\displaystyle\int_{-\infty}^{0} e^{3x}\,dx$
12. $\displaystyle\int_{-\infty}^{0} \frac{e^x\,dx}{3-2e^x}$
13. $\displaystyle\int_{-\infty}^{+\infty} x\,dx$
14. $\displaystyle\int_{-\infty}^{+\infty} \frac{x}{\sqrt{x^2+2}}\,dx$
15. $\displaystyle\int_{-\infty}^{+\infty} \frac{x}{(x^2+3)^2}\,dx$
16. $\displaystyle\int_{-\infty}^{+\infty} \frac{e^{-t}}{1+e^{-2t}}\,dt$
17. $\displaystyle\int_{0}^{4} \frac{dx}{(x-4)^2}$
18. $\displaystyle\int_{0}^{8} \frac{dx}{\sqrt[3]{x}}$
19. $\displaystyle\int_{0}^{\pi/2} \tan x\,dx$
20. $\displaystyle\int_{0}^{4} \frac{dx}{\sqrt{4-x}}$
21. $\displaystyle\int_{0}^{1} \frac{dx}{\sqrt{1-x^2}}$
22. $\displaystyle\int_{-3}^{1} \frac{x\,dx}{\sqrt{9-x^2}}$
23. $\displaystyle\int_{\pi/3}^{\pi/2} \frac{\sin x}{\sqrt{1-2\cos x}}\,dx$
24. $\displaystyle\int_{0}^{\pi/4} \frac{\sec^2 x}{1-\tan x}\,dx$
25. $\displaystyle\int_{0}^{3} \frac{dx}{x-2}$
26. $\displaystyle\int_{-2}^{2} \frac{dx}{x^2}$
27. $\displaystyle\int_{-1}^{8} x^{-1/3}\,dx$
28. $\displaystyle\int_{0}^{1} \frac{dx}{(x-1)^{2/3}}$
29. $\displaystyle\int_{0}^{+\infty} \frac{1}{x^2}\,dx$
30. $\displaystyle\int_{1}^{+\infty} \frac{dx}{x\sqrt{x^2-1}}$
31. $\displaystyle\int_{0}^{1} \frac{dx}{\sqrt{x}(x+1)}$
32. $\displaystyle\int_{0}^{+\infty} \frac{dx}{\sqrt{x}(x+1)}$

33–36 True–False Determine whether the statement is true or false. Explain your answer.

33. $\displaystyle\int_{1}^{+\infty} x^{-4/3}\,dx$ converges to 3.
34. If f is continuous on $[a, +\infty)$ and $\lim_{x\to+\infty} f(x) = 1$, then $\int_{a}^{+\infty} f(x)\,dx$ converges.
35. $\displaystyle\int_{1}^{2} \frac{1}{x(x-3)}\,dx$ is an improper integral.
36. $\displaystyle\int_{-1}^{1} \frac{1}{x^3}\,dx = 0$

37–40 Make the u-substitution and evaluate the resulting definite integral.

37. $\displaystyle\int_{0}^{+\infty} \frac{e^{-\sqrt{x}}}{\sqrt{x}}\,dx;\ u = \sqrt{x}$ [*Note:* $u \to +\infty$ as $x \to +\infty$.]
38. $\displaystyle\int_{12}^{+\infty} \frac{dx}{\sqrt{x}(x+4)};\ u = \sqrt{x}$ [*Note:* $u \to +\infty$ as $x \to +\infty$.]
39. $\displaystyle\int_{0}^{+\infty} \frac{e^{-x}}{\sqrt{1-e^{-x}}}\,dx;\ u = 1 - e^{-x}$ [*Note:* $u \to 1$ as $x \to +\infty$.]