1. The probability that a patient recovers from a delicate heart operation is 0.9. What is the probability that exactly five of the seven patients undergoing this operation will survive?

Solution: Here, $P = 0.9$ and $n = 7$

$$\therefore P(X=5) = {}^{n}C_x \; P^x(1-P)^{n-x} = P(X=x) = {}^{7}C_5 \; (0.9)^5(1-0.9)^{7-5} = 0.124$$

So, the probability that exactly five of the seven patients undergoing this operation will survive is 0.124

2. It is known that 75% of the mice inoculated with a serum are protected from a certain disease. If three mice are inoculated, what is the probability that at most two of the mice will be protected from the disease? Exactly two will contact the disease? $\left[Ans: \frac{37}{64}; \frac{9}{64}\right]$

3. The probability that a person recovers from a rare blood disease is 0.4. If 15 people are known are known to have contacted this disease, what is the probability that,

a) At least 10 people survive. $[Ans: 0.0338]$

b) From 3 to 8 survive. $[Ans: 0.0271]$

c) Exactly 5 survive. $[0.1859]$

4. 5% of the couple who adopted Family Planning method. A random of 30 couples is taken.

a) Determine the probability function. $Ans: P(X=x) = {}^{30}C_x \; 0.5^x(1-0.5)^{30-x}$

b) Find the probability at least 3 couple adopted FP method. $[Ans: 0.8763]$

5. In a binomial distribution, the mean and the standard deviation are 36 and 4.8.

a) Find the value of n and p. $[Ans: n = 100; p = 0.36]$

b) If the value of $n = 100$ and $p = 0.4$, find the mean and standard deviation.

6. Twenty percent of the TVs produced in an industry are defective. If 4 TVs are placed in a box for marketing, out of 2000 boxes, how many boxes do you expect to have,

a) One defective. $[Ans: 819\ boxes]$

b) Two defectives. $[307\ boxes]$

c) At most 2 defectives. $[Ans: 1946\ boxes]$

<u>**Hints:**</u> *Calculate the probability using the Binomial Distribution function, then multiply it by 2000.*

7. A fair six-sided die is rolled n times. Find the number of trials needed for the probability of obtaining at least one 6 to be $\frac{1}{2}$.

Solution: Here, $p = \frac{1}{6}; 1 - p = \frac{5}{6}$

Now,

$$P(X \geq 1) = 1 - P(X < 1) = 1 - P(0) = 1 - \left(\frac{5}{6}\right)^n$$

Now,

$$\therefore 1 - \left(\frac{5}{6}\right)^n = \frac{1}{2}$$

$$\Rightarrow \left(\frac{5}{6}\right)^n = \frac{1}{2}$$

$$\Rightarrow nlog\left(\frac{5}{6}\right) = \log\left(\frac{1}{2}\right)$$

$$\therefore n = 3.8 \approx 4$$

Therefore, approximately 4 trials will be needed.

8. Suppose that a lot of 5000 electrical fuses contains 5% defectives. If a sample of 5 fuses is tested, find the probability of observing at least one defective. $[Ans: 0.226]$

9. A biologist estimates that the chance of germination for a type of bean seed is 70%. A student was given 6 seeds. Assuming that the germination of seeds is independent, explain the distribution function of random variable with distribution parameters. Also find the probabilities,

a) All seeds germinated.
b) Just one seed not germinated.
c) At most four seeds germinated.
d) At least four seeds germinated.

__Hints:__ Binomial Distribution Function

10. A biologist estimates that the chance of germination for a type of bean seed is 0.5%. A student was given 120 seeds. Assuming that the germination of seeds is independent, find the probabilities, a) At least three seeds are germinated. b) Suppose 6 students have conducted the experiment independently, then what is the probability that 3 students will get at most three seeds germinated?

__Hints:__ First, use Poisson distribution (as $n > 100$), then use Binomial distribution.

11. Suppose that the number of emergency patients in a given day at a certain hospital is a Poisson variable X with parameter $\lambda = 20$. What is the probability that in a given day there will be

a) 15 emergency patients.
b) 8 emergency patients.
c) At least 3 emergency patients.
d) More than 20 but less than 25 patients.

12. The average number of errors on a page of a certain magazine is 0.2. What is the probability that the next page (or a randomly selected page) you read contains

a) 0 (zero) error? (Ans: 0.8187)
b) 2 or more errors? (Ans: 0.01756)
c) What is the average error per page? (Ans: 0.2)
d) Also, find standard deviation of the number of errors. (Ans: 0.45)

Hints: *Poisson Distribution Function*

13. Imagine you're employed as a transportation engineer, tasked with assessing the likelihood of vehicular accidents. On a highly trafficked segment of road, the probability of a single car accident occurring per hour is 0.001. If 2000 cars traverse this stretch of road in one hour, what is the probability that precisely three car accidents will occur on this road within that time frame? (Ans: 0.180).

Hints: *Poisson Distribution Function*

14. A telephone operator receives 3 telephone calls on average from 9AM to 10AM. Find the probability that in a given time interval of a day, the operator receives,

a) No call
b) At least two calls
c) At best two calls (At most two calls)
d) Two or three calls

Hints: *Poisson Distribution Function*

15. A manufacturer of cotter pins knows that 5% of his product is defective. If he sells cotter pins in boxes of 100 and guarantees that not more than 5 pins will be defective, what is the probability that a box will fail to meet the guaranteed quality.

Hints: *Calculate the probability of failing to meet the guaranteed quality,* $P(X > 5)$

16. X is a Poisson variable such that,

$$4P(X=2) = 3P(X=1) + 2P(X=3)$$

Find the probability function and standard deviation.

Ans:

Parameter, $\lambda = 3$

Probability function,

$$P(x) = \frac{e^{-\lambda}\lambda^x}{x!}$$

17. Two person decides that they will take balls until a red ball. If the probability of red ball is 1/3.

a) What is the probability that the fourth ball is red.

b) Find the mean number of balls to get a red ball.