CSE331 Practice Sheet for NFA, RE

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1 NFA Construction

For each DFA Construction problem in Practice Sheet 1, try to come up with an NFA as well. Moreover, construct an NFA for each of the following languages.

1. Construct an NFA for the language $L = \{w \in \{\texttt{0,1}\}^* : w \text{ ends with } \texttt{01}\}$
2. Construct an NFA for the regular expression $(\texttt{a} \cup \texttt{b})^*\texttt{ab}$
3. Construct an NFA for the language $L = \{w \in \{\texttt{a,b}\}^* : w \text{ contains the substring } \texttt{aba}\}$
4. Construct an NFA for the regular expression $\texttt{a}^*\texttt{b}^*$
5. Construct an NFA that accepts all strings over $\{\texttt{0,1}\}$ that start and end with the same symbol.
6. Construct an NFA for the regular expression $(\texttt{ab} \cup \texttt{ba})^*$
7. Construct an NFA for the language $L = \{w \in \{\texttt{a,b}\}^* : w \text{ has at most one } \texttt{b}\}$
8. Construct an NFA for the language $L = \{w \in \{\texttt{a,b}\}^* : \text{ every } \texttt{a} \text{ is immediately followed by a } \texttt{b}\}$
9. Construct an NFA for the regular expression $(\texttt{a} \cup \texttt{b})^*\texttt{a}(\texttt{a} \cup \texttt{b})(\texttt{a} \cup \texttt{b})$
10. Construct an NFA for the regular expression $\varepsilon \cup \texttt{a} \cup \texttt{ab}$
11. Construct an NFA for the language $L = \{w \in \{\texttt{a,b}\}^* : w \text{ has an even number of } \texttt{a} \text{ or ends with } \texttt{bb}\}$
12. Construct an NFA for $L = \{w \in \{\texttt{a,b}\}^* : w \text{ is a palindrome of length} \leq 4\}$
13. Construct an NFA that accepts the union of two regular expressions $R_1 = \texttt{a}^*\texttt{b}, \quad R_2 = \texttt{ab}^*$
14. Design an NFA that accepts all strings over $\{\texttt{a,b}\}$ that contain both substrings $\texttt{ab}$ and $\texttt{ba}$ (in any order).
15. Given $R = (\texttt{a} \cup \texttt{b})^*\texttt{abb}$. Convert this regular expression to an NFA.

2 Regular Expression

1. Write the strings that the following Regular Expressions will generate:
   (a) $(\texttt{0} \cup \texttt{1})^*$
   (b) $\texttt{0} \cup \texttt{1}$
   (c) $(\texttt{00})^*$
   (d) $\texttt{0}^*\texttt{1}^*$
   (e) $(\texttt{0}^*\texttt{1}^*)^*$
   (f) $(\texttt{01}^*)^*$
   (g) $\texttt{0}^*\texttt{1}^* \cup (\texttt{ab})^*$
   (h) $(\texttt{0}^*\texttt{1}^* \cup (\texttt{ab})^*)^*$
   (i) $\texttt{a}(\texttt{0} \cup \texttt{1})\texttt{b}$
   (j) $\texttt{a}(\texttt{0} \cup \texttt{1})^*\texttt{b}$
   (k) $(\texttt{a}(\texttt{0} \cup \texttt{1})^*\texttt{b})^*$
   (l) $\texttt{ab} \cup \texttt{1}^*\texttt{01}^*$

2. Are $(\texttt{0} \cup \texttt{1})^*$ and $(\texttt{0}^*\texttt{1}^*)^*$ the same? Justify your answer.

3. Write the shortest string that will be generated by the regular expression:
   (a) $\texttt{aa}^*\texttt{b}(\texttt{0} \cup \texttt{1}) \cup \texttt{1}^*\texttt{0}(\texttt{baa}^*)$
   (b) $(\texttt{0} \cup \texttt{1})\texttt{1}(\texttt{00} \cup \texttt{0}(\texttt{11})^* \cup \texttt{100}) \cup \texttt{1}^*(\texttt{00}(\texttt{010} \cup \texttt{01}^*\texttt{0}))\texttt{01}$

4. For each of the following problems, write a regular expression that generates the language:
   (a) $L = \{w \in \{\texttt{0},\texttt{1}\}^* : w \text{ contains } \texttt{101} \text{ as a substring}\}$
   (b) $L = \{w \in \{\texttt{0},\texttt{1}\}^* : w \text{ starts with } \texttt{101}\}$
   (c) $L = \{w \in \{\texttt{0},\texttt{1}\}^* : w \text{ ends with } \texttt{101}\}$
   (d) $L = \{w \in \{\texttt{0},\texttt{1}\}^* : w \text{ doesn't start with } \texttt{1}\}$
   (e) $L_1 = \{w \in \{\texttt{0},\texttt{1}\}^* : w \text{ contains } \texttt{00} \text{ or } \texttt{11}\}$
   (f) $L_2 = \{w \in \{\texttt{0},\texttt{1}\}^* : w \text{ contains both } \texttt{00} \text{ and } \texttt{11}\}$
   (g) Write regular expressions for both $\overline{L_1}$ and $\overline{L_2}$.
   (h) $L = \{w \in \{\texttt{0},\texttt{1}\}^* : w \text{ contains exactly two } \texttt{1}\text{s}\}$
   (i) $L = \{w \in \{\texttt{0},\texttt{1}\}^* : w \text{ contains at least two } \texttt{1}\text{s}\}$
   (j) $L = \{w \in \{\texttt{0},\texttt{1}\}^* : w \text{ contains at most two } \texttt{1}\text{s}\}$
   (k) $L = \{w \in \{\texttt{0},\texttt{1}\}^* : |w| \text{ is even}\}$
   (l) $L = \{w \in \{\texttt{0},\texttt{1}\}^* : |w| \text{ is odd}\}$
   (m) $L = \{w \in \{\texttt{0},\texttt{1}\}^* : |w| \equiv 0 \pmod 3\}$
   (n) $L = \{w \in \{\texttt{0},\texttt{1}\}^* : |w| \equiv 2 \pmod 4\}$
   (o) $L = \{w \in \{\texttt{0},\texttt{1}\}^* : |w| \not\equiv 0 \pmod 3\}$
   (p) $L = \{w \in \{\texttt{0},\texttt{1}\}^* : \text{number of } \texttt{1}\text{s is divisible by } 3\}$

(q) $L = \{w \in \{\mathtt{a,b}\}^* : w \text{ starts and ends with different symbols}\}$

(r) $L = \{w \in \{\mathtt{a,b}\}^* : w \text{ starts and ends with the same symbol}\}$

(s) $L = \{w \in \{\mathtt{0,1}\}^* : w \text{ doesn't end with } \mathtt{01}\}$

(t) $L = \{w \in \{\mathtt{0,1}\}^* : \mathtt{0} \text{ and } \mathtt{1} \text{ alternate in } w\}$

(u) $L = \{w \in \{\mathtt{0,1}\}^* : w \text{ doesn't contain } \mathtt{00}\}$

(v) $L = \{w \in \{\mathtt{0,1}\}^* : w \text{ doesn't contain } \mathtt{11}\}$

(w) $L = \{w \in \{\mathtt{0,1}\}^* : w \text{ doesn't contain } \mathtt{111}\}$

(x) $L = \{w \in \{\mathtt{0,1}\}^* : w \text{ doesn't contain } \mathtt{10}\}$

(y) $L = \{w \in \{\mathtt{0,1}\}^* : w \text{ doesn't contain } \mathtt{00} \text{ and } \mathtt{11}\}$

(z) $L = \{w \in \{\mathtt{0,1}\}^* : \mathtt{0} \text{ occurs in every third position}\}$. Construct an RE for $\overline{L}$ as well.

5. Consider the following languages over $\Sigma = \{\mathtt{0}, \mathtt{1}\}$.

$$L_1 = \{w : w \text{ does not contain } \mathtt{11}\}$$
$$L_2 = \{w : \text{ every } \mathtt{1} \text{ in } w \text{ is followed by at least one } \mathtt{0}\}$$
$$L_3 = \{w : \text{ the number of times } \mathtt{1} \text{ appears in } w \text{ is even}\}$$

Now solve the following problems.

(a) Give a regular expression for the language L_1.

(b) Your friend claims that $L_1 = L_2$. Prove her wrong by writing down a five-letter string in $L_1 \setminus L_2$. [Recall that $L_1 \setminus L_2$ contains all strings that are in L_1 but not in L_2.]

(c) Give a regular expression for the language $L_1 \setminus L_2$.

(d) Give a regular expression for the language L_3.

(e) Give a regular expression for the language $L_2 \setminus L_3$.

3 Conversion Problems (NFA to DFA, DFA to RE)

These are mechanical processes and you just need to be careful. So, whenever you encounter an NFA, try converting it into a DFA. And if you have a DFA (or NFA, because the construction we did works for NFA as well), try constructing a regular expression that generates the same language.