

9.3 Probability Distribution (Normal)

1. Let X be a standard normal variable. Find the probability that X will,

- a) Less than 1.5.
- b) Greater than 2.4.
- c) Between 1.5 and 2.14.
- d) Less than 1.32.
- e) Greater than -2.52.
- f) Between -2.52 and 1.64
- g) Between -2.52 and -1.5
- h) Exactly 2.

Hints: As X standard normal variable, the mean of X is zero, and variance of X is one.

$$X \sim N(0,1)$$

2. Suppose that the growth in inches during the tenth year of Bangladeshi boy is a normal random variable with mean 2 inches and standard deviation 1 inches. Find the probability that a randomly selected boy will grow,

- a) Between 1 and 2 inches. (Ans: 0.3413)
- b) More than 3 inches. (Ans: 0.1587)
- c) At least 1 inch. (Ans: 0.8413)
- d) At best 1 inch.

Hints: At best and At most are same.

3. If $X \sim N(8,4)$, find the following probabilities,

- a) $P(X \geq 12)$. Ans: 0.0228
- b) $P(X \leq 12)$. Ans: 0.9772
- c) $P(0 \leq X \leq 8)$. Ans: 0.5
- d) Find the value of " q ", when $P(X > q) = 0.25$. Ans: 9.35

4. Given the normally distributed variable X with mean 18 and standard deviation 2.5. Find the value of k , such that,

- a) $P(X < k) = 0.2578$ [Ans: 16.375]
- b) $P(X > k) = 0.1539$ [20.55]

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5. The normal random variable X having a mean 30 and an unknown variance is distributed that $P(X < 35) = 0.69$. Find the variance of X .

Solution:

Let σ^2 be the variance. Then, we can write,

$$\begin{aligned}P(X < 35) &= P\left(\frac{X - \mu}{\sigma} < \frac{35 - \mu}{\sigma}\right) = 0.69 \\ \Rightarrow P\left(Z < \frac{35 - 30}{\sigma}\right) &= 0.69 \\ \Rightarrow P(Z < 0.5) &= 0.69\end{aligned}$$

Now,

$$\begin{aligned}\frac{35 - 30}{\sigma} &= 0.5 \\ \therefore \sigma &= 10 \\ \therefore \sigma^2 &= 100\end{aligned}$$

6. A certain type of insect survives on the average 3 years with standard deviation of 0.5 years. Assuming that the lives of the insect are normally distributed, find the probability that a given insect will survive less than 2.3 years. [Ans: 0.0808]

7. The Phillips Bangladesh manufactures electric bulbs that have a length of life that is normally distributed with mean equal to 800 hours and standard deviation of 40 hours. Find the probability that a bulb burns between 778 and 834 hours. [Ans: 0.5111]

8. A company pays its employees an average wage of \$5.25 per hour with a standard deviation of 60 cents. If the wages are approximately normally distributed,

- What percentage of the employees receive wages between \$4.75 and \$5.69 per hour? [Ans: 56.4%]
- Calculate the lowest cut off wages value for 10% employees.
- Calculate the cut off wages value for bottom 10% employees.
- Calculate the first and third quartile value.

Hints: $\mu = 5.25, \sigma = 0.6$

b) $P(X > c) = 0.1$

c) $P(X < c) = 0.1$

- First quartile bottom (25% or 0.25)
- Third quartile bottom (75% or 0.75)

9.3 Probability Distribution (Normal)

9. The marks of the students in a certain examination are normally distributed with mean marks as 40 and standard deviation, marks as 20. On this basis, 60% students failed. The result was moderated and 70% students passed. Find the pass marks before and after the moderation. [Ans: 45 and 29.5]

Hints:

Passed before moderation:

$$P(X \geq c_1) = 0.4$$

Passed after moderation:

$$P(X \geq c_2) = 0.7$$

Calculate the value of c_1 and c_2

10. A factory turns out an article by mass production methods. From past experience it appears that 20 articles on an average are rejected out of every batch of 100.

- Find the average of the number of rejects in a batch. [Ans: 20]
- Find the variance of the number of rejects in a batch. [Ans: 16]
- If the mean and variance from (a) and (b) are the parameter of normal distribution, what is the probability that the number of rejects in a batch exceeds 30? [Ans: 0.0062]