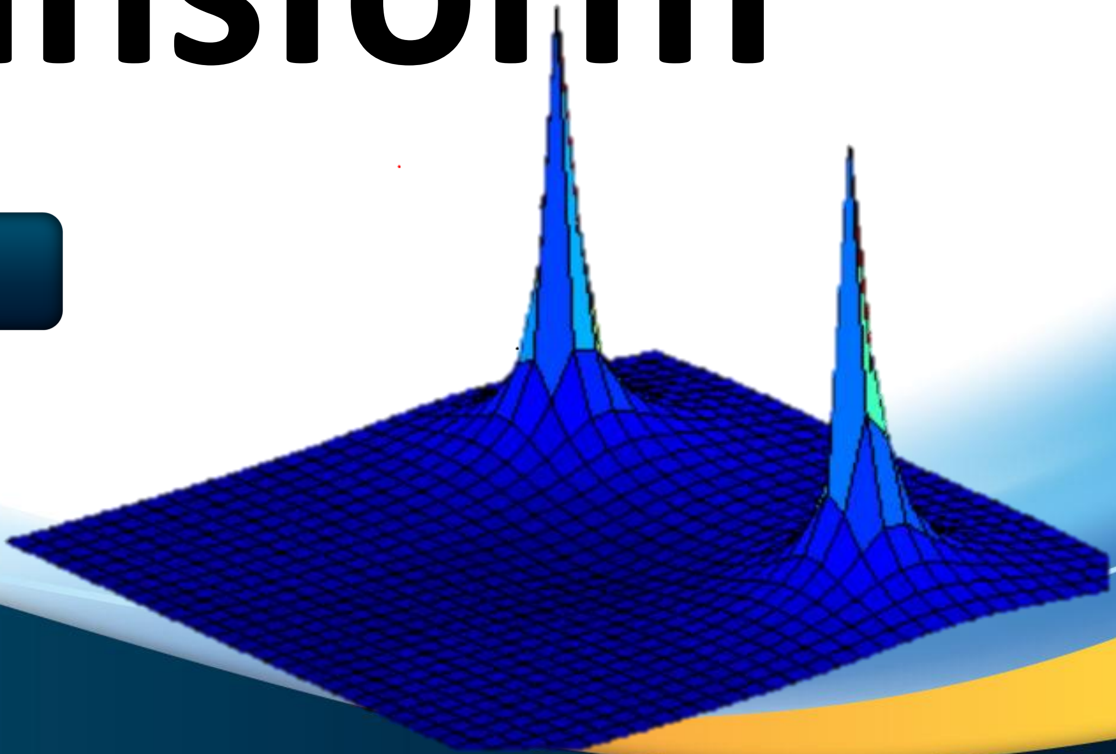
Undergraduate Course in Mathematics

Laplace Transform

Theorems of Laplace Transform



Conducted By

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First Translation Theorem

If $\mathcal{L}\{f(t)\} = F(s)$ and a is any real number, then

$$\boxed{\mathcal{L}\{e^{at} f(t)\} = F(s-a)}$$

We can also write like this

$$\mathcal{L}\{e^{at} f(t)\} = \mathcal{L}\{f(t)\}\Big|_{s \to s-a}$$

Evaluate

$$\mathcal{L}\{t^3 e^{5t}\}$$

$$\therefore \mathcal{L}\{t^3\} = \frac{3!}{s^4} = \frac{6}{s^4}$$

$s \to s-5$

$$\therefore \mathcal{L}\{t^3 \cdot e^{5t}\} = \frac{6}{(s-5)^4}$$

Evaluate

$$\mathcal{L}\{5e^{-3t}\sin 4t\}$$

$$\therefore \mathcal{L}\{5\sin 4t\} = 5\,\mathcal{L}\{\sin 4t\} = 5\cdot\frac{4}{s^2+4^2} = \frac{20}{s^2+16}.$$

$$\therefore \mathcal{L}\{5\sin 4t\cdot e^{-3t}\} = \frac{20}{(s+3)^2+16}.$$

Evaluate

$$\mathcal{L}\{(t+2)^2 e^t\}$$

$$\therefore \mathcal{L}\{(t+2)^2\} = \mathcal{L}\{t^2+4t+4\} = \mathcal{L}\{t^2\} + 4\cdot\mathcal{L}\{t\} + \mathcal{L}\{4\}$$

$$= \frac{2!}{s^3} + 4\cdot\frac{1}{s^2} + \frac{4}{s}$$

$$= \frac{2}{s^3} + \frac{4}{s^2} + \frac{4}{s}.$$

$$\therefore \mathcal{L}\{(t+2)^2\cdot e^t\} = \frac{2}{(s-1)^3} + \frac{4}{(s-1)^2} + \frac{4}{s-1}$$

Evaluate

$$\mathcal{L}\{e^{-t}(3\sinh 2t - 5\cosh 2t)\}$$

$$\mathcal{L}\{3\sinh 2t - 5\cosh 2t\} = 3\cdot\mathcal{L}\{\sinh 2t\} - 5\,\mathcal{L}\{\cosh 2t\}$$

$$= 3\cdot\frac{2}{s^2-2^2} - 5\cdot\frac{s}{s^2-2^2} = \frac{6-5s}{s^2-4}.$$

$$\therefore \mathcal{L}\{(3\sinh 2t - 5\cosh 2t)\,e^{-t}\} = \frac{6-5(s+1)}{(s+1)^2-4}$$

$$= \frac{1-5s}{(s+1)^2-4} \quad \text{Ans}$$

Evaluate

$$\mathcal{L}\{e^{-4t}\cosh 2t\}$$

$$\mathcal{L}\{\cosh 2t\} = \frac{s}{s^2-2^2}$$

$$\mathcal{L}\{e^{-4t}\cosh 2t\} = \frac{s+4}{(s+4)^2-4}$$

Evaluate

$$\mathcal{L}\{e^{2t}(3\sin 4t - 4\cos 4t)\}$$

$$\mathcal{L}\{3\sin 4t - 4\cos 4t\} = 3\cdot\frac{4}{s^2+4^2} - 4\cdot\frac{s}{s^2+4^2}$$

$$= \frac{12-4s}{s^2+16}.$$

$$\mathcal{L}\{e^{2t}(3\sin 4t - 4\cos 4t)\} = \frac{12-4(s-2)}{(s-2)^2+16}$$

Laplace Transform of the form $\mathcal{L}\{t^n f(t)\}$

If $\boxed{\mathcal{L}\{f(t)\} = F(s)}$ and n is a natural number

$$\mathcal{L}\{t^n f(t)\} = (-1)^n \frac{d^n}{ds^n}(F(s))$$

For $n = 1$,

$$\mathcal{L}\{t\, f(t)\} = -\frac{d}{ds}(F(s))$$

Evaluate

$$\mathcal{L}\{t \sin 2t\}$$

$$\mathcal{L}\{\sin 2t\} = \frac{2}{s^2+2^2} = \frac{2}{s^2+4}.$$

$$\mathcal{L}\{t \cdot \sin 2t\} = -\frac{d}{ds}\left(\frac{2}{s^2+4}\right) = -\frac{(s^2+4)\cdot 0 - 2\cdot 2s}{(s^2+4)^2}$$

$$= \frac{4s}{(s^2+4)^2}.$$

Evaluate

$$\mathcal{L}\{t \underbrace{\sin 2t \cos 5t}\}$$

$$\mathcal{L}\{\sin 2t \cos 5t\}$$

$$= \frac{1}{2}\mathcal{L}\{2 \sin 2t \cos 5t\}$$

$$= \frac{1}{2}\mathcal{L}\{\sin 7t + \sin(-3t)\}$$

$$= \frac{1}{2}\mathcal{L}\{\sin 7t - \sin 3t\}$$

$$= \frac{1}{2}\mathcal{L}\{\sin 7t\} - \frac{1}{2}\mathcal{L}\{\sin 3t\}$$

$$= \frac{1}{2} \cdot \frac{7}{s^2+49} - \frac{1}{2}\frac{3}{s^2+9}.$$

$$\mathcal{L}\left\{t \cdot \sin 2t \cos 5t\right\}$$

$$= -\frac{d}{ds}\left(\frac{\frac{7}{2}}{s^2+49} - \frac{\frac{3}{2}}{s^2+9}\right)$$

$$= -\frac{-\frac{7}{2}\cdot 2s}{(s^2+49)^2} + \frac{-\frac{3}{2}\cdot 2s}{(s^2+9)^2}$$

$$= \frac{7s}{(s^2+49)^2} - \frac{3s}{(s^2+9)^2}.$$

Evaluate

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$$\mathcal{L}\{te^{-2t}\sin 3t\}$$

$$\therefore \mathcal{L}\{\sin 3t\} = \frac{3}{s^2+9}$$

$$\therefore \mathcal{L}\{t \cdot \sin 3t\} = -\frac{d}{ds}\left(\frac{3}{s^2+9}\right)$$

$$= -\frac{(s^2+9)\cdot 0 - 3\cdot 2s}{(s^2+9)^2} = \frac{6s}{(s^2+9)^2}$$

$$\mathcal{L}\{t \cdot \sin 3t \cdot e^{-2t}\}$$

$$= \frac{6(s+2)}{\left[(s+2)^2+9\right]^2}.$$

Evaluate

$$\mathcal{L}\{te^{2t}(2\sinh 3t + 5\cosh 4t)\}$$

$$\mathcal{L}\{2\sinh 3t + 5\cosh 4t\}$$

$$= 2\cdot\frac{3}{s^2-9} + 5\cdot\frac{4}{s^2-16}$$

$$= \frac{6}{s^2-9} + \frac{20}{s^2-16}$$

$$\therefore \mathcal{L}\{t\cdot(2\sinh 3t + 5\cosh 4t)\}$$

$$= -\frac{d}{ds}\left(\frac{6}{s^2-9} + \frac{20}{s^2-16}\right)$$

$$= -\frac{(s^2-9)\cdot 0 - 6\cdot 2s}{(s^2-9)^2} - \frac{(s^2-16)\cdot 0 - 20\cdot 2s}{(s^2-16)^2}$$

$$= \frac{12s}{(s^2-9)^2} + \frac{40s}{(s^2-16)^2}.$$

$$\mathcal{L}\left\{ t\cdot\left(2\sinh 3t + 5\cosh 4t\right) e^{2t} \right\}$$

$$= \frac{12(s-2)}{\left[(s-2)^2-9\right]^2} + \frac{40(s-2)}{\left[(s-2)^2-16\right]^2}$$

Evaluate

$$\mathcal{L}\{te^{-3t}\underbrace{\sin 2t\sin 5t}\}$$

$$\therefore \mathcal{L}\{\sin 2t\cdot\sin 5t\}$$

$$= \frac{1}{2}\mathcal{L}\{2\sin 2t\,\sin 5t\}$$

$$= \frac{1}{2}\mathcal{L}\{\cos(-3t)-\cos(7t)\}$$

$$= \frac{1}{2}\mathcal{L}\{\cos 3t-\cos 7t\}$$

$$= \frac{1}{2}\mathcal{L}\{\cos 3t\} - \frac{1}{2}\mathcal{L}\{\cos 7t\}$$

$$= \frac{1}{2}\cdot\frac{s}{s^2+9} - \frac{1}{2}\,\frac{s}{s^2+49}.$$

$$\mathcal{L}\{t\cdot \sin 2t\cdot \sin 5t\}$$

$$= -\frac{d}{ds}\left(\frac{1}{2}\frac{s}{s^2+9} - \frac{1}{2}\frac{s}{s^2+49}\right)$$

$$= -\frac{1}{2}\frac{(s^2+9)\cdot 1 - s\cdot 2s}{(s^2+9)^2} + \frac{1}{2}\frac{(s^2+49)\cdot 1 - s\cdot 2s}{(s^2+49)^2}$$

$$= \frac{1}{2}\frac{s^2-9}{(s^2+9)^2} + \frac{1}{2}\frac{49-s^2}{(s^2+49)^2}$$

$$\mathcal{L}\{t\cdot e^{-3t}\sin 2t\ \sin 5t\}$$

$$= \frac{1}{2}\frac{(s+3)^2-9}{\left[(s+3)^2+9\right]^2} + \frac{1}{2}\frac{49-(s+3)^2}{\left[(s+3)^2+49\right]^2}$$

Evaluate

$$\mathcal{L}\{te^{2t}\cos 2t\cos 5t\}$$

HW

Evaluate

$$\mathcal{L}\{t\, e^{-2t} \underbrace{\sin 5t \cos 7t}\}$$

Hw

Trigonometric Identity

$$2\ \sin \boldsymbol{A}\ \sin \boldsymbol{B} = \cos(\boldsymbol{A}-\boldsymbol{B}) - \cos(\boldsymbol{A}+\boldsymbol{B})$$

$$2\ \cos \boldsymbol{A}\ \cos \boldsymbol{B} = \cos(\boldsymbol{A}+\boldsymbol{B}) + \cos(\boldsymbol{A}-\boldsymbol{B})$$

$$2\ \sin \boldsymbol{A}\ \cos \boldsymbol{B} = \sin(\boldsymbol{A}+\boldsymbol{B}) + \boldsymbol{sin}(\boldsymbol{A}-\boldsymbol{B})$$

$$\left.\begin{aligned}\boldsymbol{sin}(-\boldsymbol{\theta}) &= -\boldsymbol{sin}(\boldsymbol{\theta})\\ \boldsymbol{cos}(-\boldsymbol{\theta}) &= \boldsymbol{cos}(\boldsymbol{\theta})\end{aligned}\right\}$$

মুখস্থবিদ্যা প্রতিভাকে ধ্বংস করে কিন্তু সফলতাকে ত্বরান্বিত করে।

Laplace Transform of the form $\mathcal{L}\left\{\frac{f(t)}{t}\right\}$

If $\mathcal{L}\{ f(t)\} = F(s)$

Exam এ আসবেই

$$\mathcal{L}\{t \cdot f(t)\} = -\frac{d}{ds}(F(s))$$

$$\mathcal{L}\left\{\frac{f(t)}{t}\right\} = \int_s^{\infty} F(u)du$$

Evaluate each of the following:

i) $\mathcal{L}\left\{\frac{\sin 2t}{t}\right\}$ ii) $\mathcal{L}\left\{\frac{\sin 3t}{t}e^{2t}\right\}$

$\mathcal{L}\{\sin 2t\}$

$= \frac{2}{s^2+2^2}$

$F(s) = \frac{2}{s^2+4}$

$\mathcal{L}\left\{\frac{\sin 2t}{t}\right\}$

$= \int_s^\infty F(u)\,du$

$= \int_s^\infty \frac{2}{u^2+4}\,du$

$= 2\int_s^\infty \frac{du}{u^2+2^2}$

$= \cancel{2}\cdot\left[\frac{1}{\cancel{2}}\cdot \tan^{-1}\left(\frac{u}{2}\right)\right]_s^\infty$

$= \tan^{-1}(\infty) - \tan^{-1}\left(\frac{s}{2}\right)$

$= \frac{\pi}{2} - \tan^{-1}\left(\frac{s}{2}\right)$ Ans

Evaluate each of the following:

ii) $\mathcal{L}\left\{\frac{\sin 3t}{t}e^{2t}\right\}$

$\mathcal{L}\{\sin 3t\}$

$f(s) = \frac{3}{s^2+9}$

$\mathcal{L}\left\{\frac{\sin 3t}{t}\right\}$

$= \int_s^\infty f(u)\,du$

$= \int_s^\infty \frac{3}{u^2+9}\,du$

$= 3 \cdot \int_s^\infty \frac{du}{u^2+3^2}$

$= 3 \cdot \frac{1}{3} \cdot \left[\tan^{-1}\left(\frac{u}{3}\right)\right]_s^\infty$

$= \tan^{-1}(\infty) - \tan^{-1}\left(\frac{s}{3}\right) = \frac{\pi}{2} - \tan^{-1}\left(\frac{s}{3}\right).$

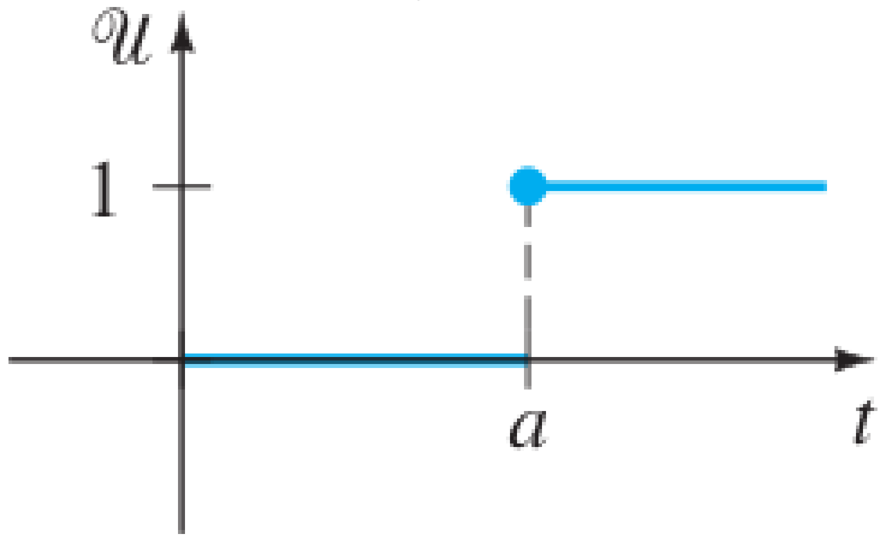
$\therefore \mathcal{L}\left\{\frac{\sin 3t}{t} \cdot e^{2t}\right\} = \frac{\pi}{2} - \tan^{-1}\left(\frac{s-2}{3}\right)$

One Minute Break

What is Unit Step Function

The unit step function $u(t-a)$ is defined to be

$$u(t-a) = \begin{cases} 0, & 0 \le t < a \\ 1, & t \ge a \end{cases}$$



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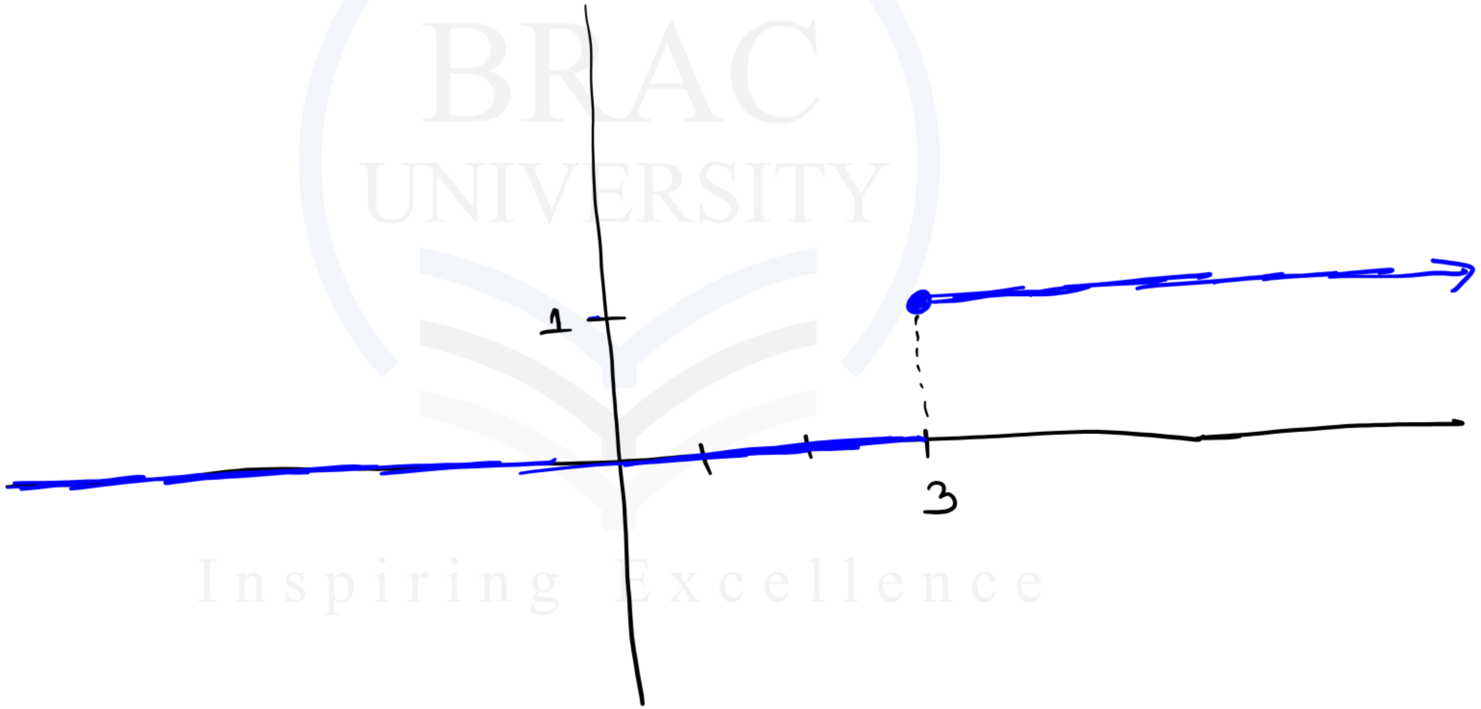
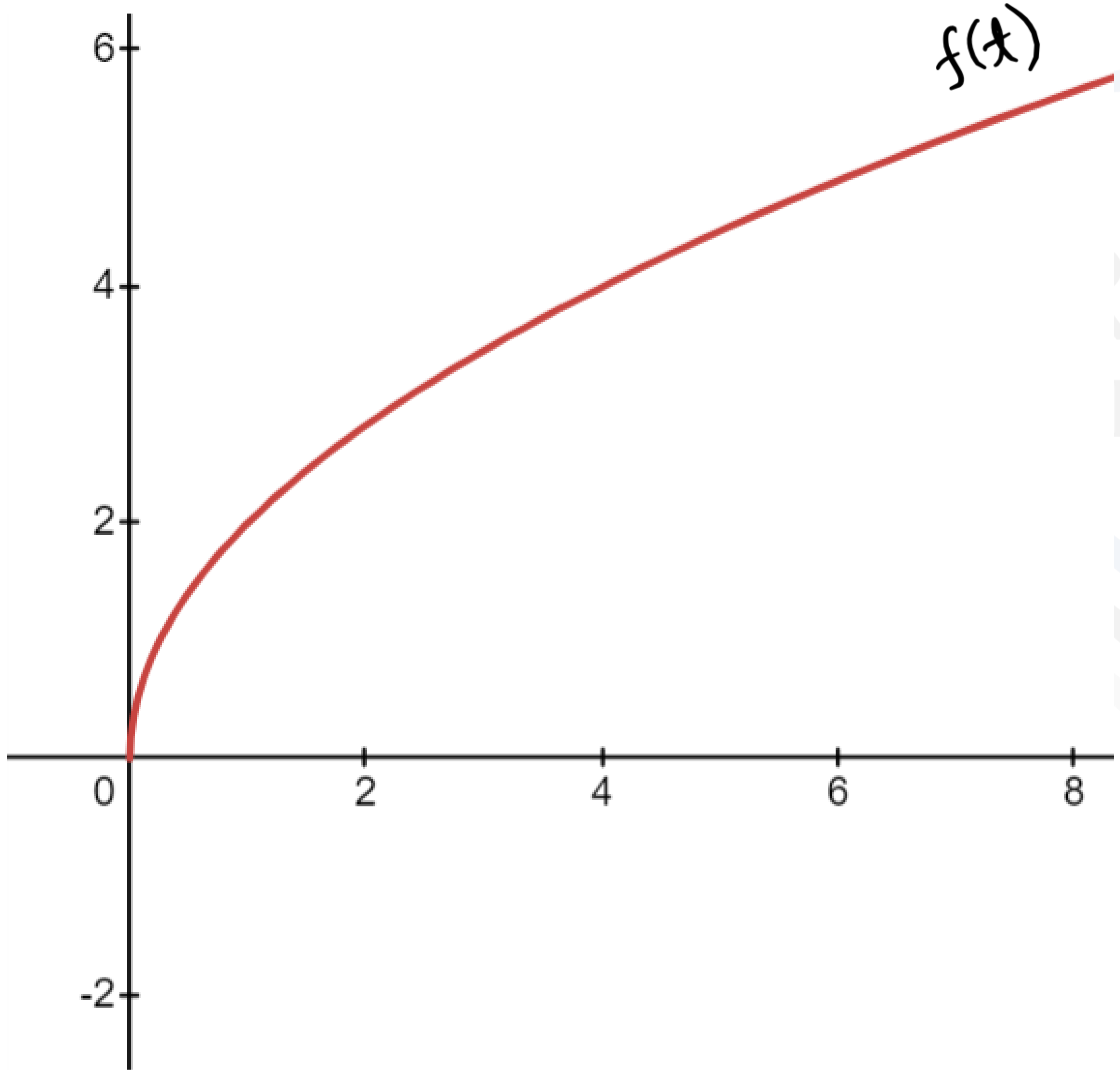
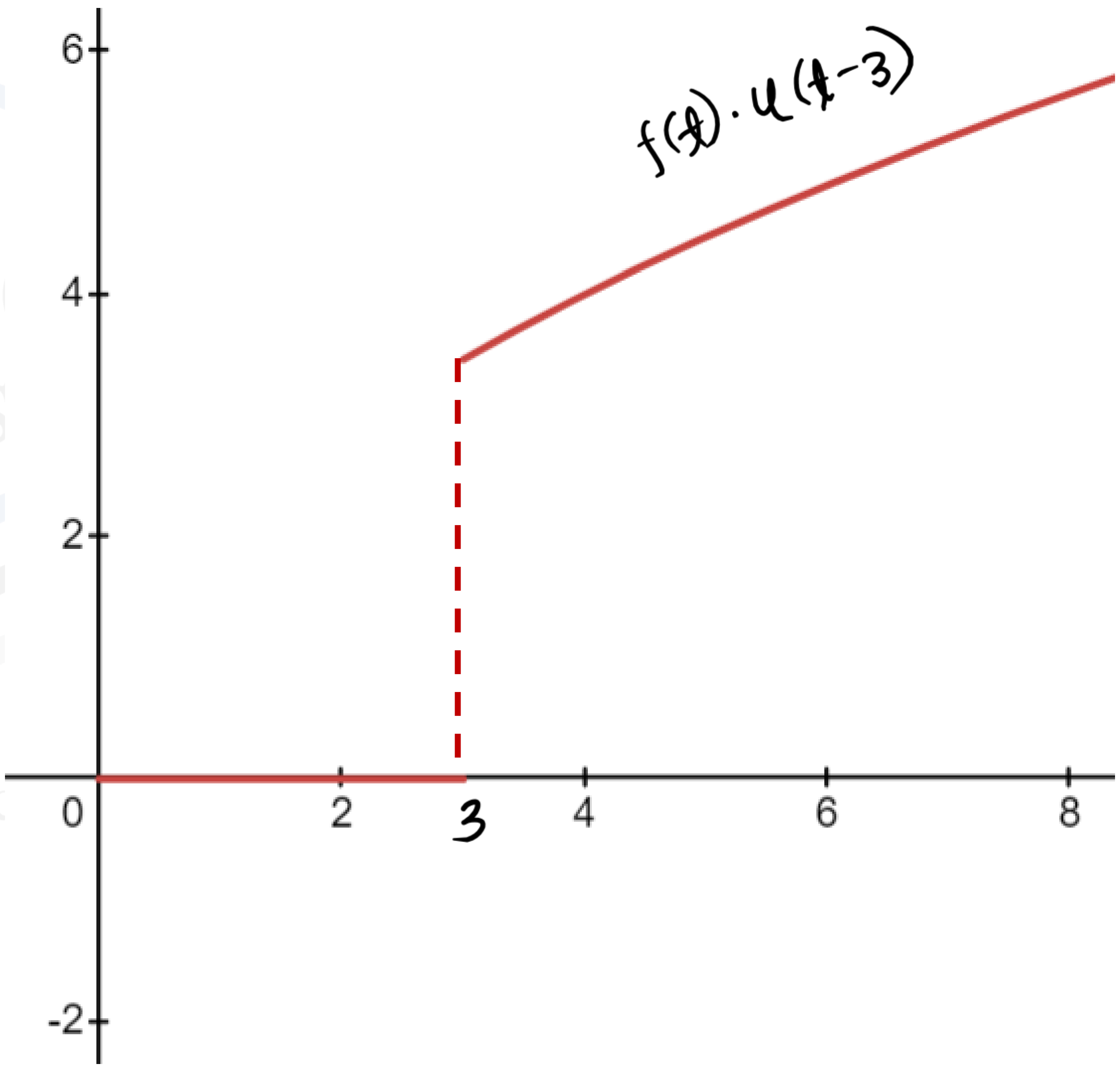
$u(t-3)$



What is $f(t) \cdot u(t-a)$





Second Translation Theorem

If $\mathcal{L}\{f(t)\} = F(s)$ and a is any real number, then

$$\mathcal{L}\{f(t-a)\cdot u(t-a)\} = F(s)\cdot e^{-as}$$

We can also write like this

$$\mathcal{L}\{f(t)\cdot u(t-a)\} = \mathcal{L}\{f(t+a)\}\cdot e^{-as}$$

Find the Laplace Transforms

$$\mathcal{L}\{(2t-3)\cdot u(t-1)\}$$

$f(t) = 2t - 3 \qquad a = 1$

$f(t+a) = f(t+1) = 2(t+1) - 3$

$= 2t - 1.$

$\mathcal{L}\{f(t)\cdot u(t-a)\} = \mathcal{L}\{f(t+a)\}\cdot e^{-as}$

$= \mathcal{L}\{2t-1\}\cdot e^{-s} = \left(2\cdot\frac{1}{s^2} - \frac{1}{s}\right)e^{-s} = \frac{2-s}{s^2}\cdot e^{-s}$

Find the Laplace Transforms

$$\mathcal{L}\{e^{-2t} \cdot u(t-1)\}$$

$f(t) = e^{-2t}$ $\quad a = 1.$

$f(t+1) = e^{-2(t+1)} = e^{-2t-2}$

$\mathcal{L}\{f(t)\, u(t-a)\}$

$= \mathcal{L}\{f(t+a)\}\, e^{-as}$

$= \mathcal{L}\{e^{-2t-2}\}\, e^{-s}$

$= \mathcal{L}\{e^{-2t} \cdot e^{-2}\}\, e^{-s}$

$= e^{-2} \cdot \mathcal{L}\{e^{-2t}\}\, e^{-s}$

$= e^{-2} \cdot \frac{1}{s+2} \cdot e^{-s} = e^{-2-s} \cdot \frac{1}{s+2}$

Find the Laplace Transforms

$$\mathcal{L}\{\cos 2t \cdot u(t-\pi)\}$$

$f(t) = \cos 2t \qquad a = \pi$

$f(t+\pi) = \cos(2(t+\pi))$

$= \cos(2t+2\pi)$

$= \cos(2t)$

$\mathcal{L}\{f(t)\cdot u(t-a)\}$

$= \mathcal{L}\{f(t+a)\}\, e^{-as}$

$= \mathcal{L}\{\cos 2t\}\, e^{-\pi s}$

$= \dfrac{s}{s^2+4}\, e^{-\pi s}$

Converting Piecewise to Step Function

$$f(t) = \begin{cases} \_\_ & \_\_ \\ \_\_ & \_\_ \\ \_\_ & \_\_ \\ \_\_ & \_\_ \end{cases}$$

A Shortcut Method

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$$f(t) = \begin{cases} \text{---} & \text{---} \\ \text{---} & \text{---} \\ g(t) & a \le t < b \\ \text{---} & \text{---} \\ \text{---} & \text{---} \end{cases}$$

$u(t-0) = 1.$

$u(t-\infty) = 0.$

$$= \text{----} + g(t) \cdot \left(u(t-a) - u(t-b) \right) + \text{--}$$

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Find the Laplace Transform of

$$f(t) = \begin{cases} 0, & 0 \le t < \pi \\ \cos(t), & t \ge \pi \end{cases}$$

$$= 0 \cdot [u(t-0) - u(t-\pi)] + \cos t \, [u(t-\pi) - u(t-\infty)]$$

$$= 0 \cdot (1 - u(t-\pi)) + \cos t \cdot (u(t-\pi) - 0)$$

$$= \cos t \cdot u(t-\pi) \cdot$$

$$\mathcal{L}\{\cos t \cdot u(t-\pi)\}$$

$$= \mathcal{L}\{\cos(t+\pi)\} \cdot e^{-\pi s}$$

$$= \mathcal{L}\{-\cos t\}\, e^{-\pi s}$$

$$= -\frac{s}{s^2+1}\, e^{-\pi s} \quad \checkmark$$

Find the Laplace Transform of

$$f(t) = \begin{cases} 5\sin(t), & 0 \le t < \pi \\ 0, & t \ge \pi \end{cases}$$

$$= 5\sin t \cdot \left[u(t-0) - u(t-\pi) \right] + 0 \cdot \left[u(t-\pi) - u(t-\infty) \right]$$

$$= 5\sin t \left(1 - u(t-\pi) \right) + 0$$

$$= 5\sin t - 5\sin t \cdot u(t-\pi).$$

$$\mathcal{L}\left\{5\sin t - 5\sin t \cdot u(t-\pi)\right\}$$

$$= 5 \cdot \mathcal{L}\{\sin t\} - 5 \cdot \mathcal{L}\{\sin t \cdot u(t-\pi)\}$$

$$= 5 \cdot \frac{1}{s^2+1} - 5 \cdot \mathcal{L}\{\sin(t+\pi)\} \cdot e^{-\pi s}$$

$$= \frac{5}{s^2+1} - 5 \cdot \mathcal{L}\{-\sin t\}\, e^{-\pi s} = \frac{5}{s^2+1} + 5 \cdot \frac{1}{s^2+1} e^{-\pi s}$$

Find the Laplace Transform of

$$f(t) = \begin{cases} 5\sin(t), & 0 \le t < \pi \\ -4\cos(2t) & t \ge \pi \end{cases}$$

$$= 5\sin t\left[u(t-0) - u(t-\pi)\right] - 4\cos(2t)\left[u(t-\pi) - u(t-\infty)\right]$$

$$= 5\sin t\left(1 - u(t-\pi)\right) - 4\cos 2t\left(u(t-\pi) - 0\right)$$

$$= 5\sin t - 5\sin t \cdot u(t-\pi) - 4\cos t \cdot u(t-\pi)$$

$$\mathcal{L}\{5\sin t - 5\sin t\cdot u(t-\pi) - 4\cos t\cdot u(t-\pi)\}$$

$$= 5\cdot\mathcal{L}\{\sin t\} - 5\,\mathcal{L}\{\sin t\cdot u(t-\pi)\} - 4\cdot\mathcal{L}\{\cos t\cdot u(t-\pi)\}$$

$$= 5\cdot\frac{1}{s^2+1} - 5\cdot\mathcal{L}\{\sin(t+\pi)\}e^{-\pi s} - 4\cdot\mathcal{L}\{\cos(t+\pi)\}e^{-\pi s}$$

$$= \frac{5}{s^2+1} - 5\cdot\mathcal{L}\{-\sin t\}e^{-\pi s} - 4\cdot\mathcal{L}\{-\cos t\}e^{-\pi s}$$

$$= \frac{5}{s^2+1} + 5\cdot\frac{1}{s^2+1}e^{-\pi s} + 4\cdot\frac{s}{s^2+1}e^{-\pi s}$$

Find the Laplace Transform of

$$f(t) = \begin{cases} 0, & 0 \le t < \pi \\ \sin(t), & \pi \le t < 2\pi \\ 0, & t \ge 2\pi \end{cases}$$

$$= 0 \cdot [u(t-0) - u(t-\pi)] + \sin t\,[u(t-\pi) - u(t-2\pi)] + 0 \cdot [u(t-2\pi) - u(t-\infty)]$$

$$= 0 + \sin t \cdot u(t-\pi) - \sin t \cdot u(t-2\pi) + 0$$

$$= \sin t \cdot u(t-\pi) - \sin t \cdot u(t-2\pi).$$

$$\mathcal{L}\left\{\sin t \cdot u(t-\pi) - \sin t \cdot u(t-2\pi)\right\}$$

$$= \mathcal{L}\left\{\sin t \cdot \underline{\underline{u(t-\pi)}}\right\} - \mathcal{L}\left\{\sin t \cdot \underline{\underline{u(t-2\pi)}}\right\}$$

$$= \mathcal{L}\left\{\sin(t+\pi)\right\} \cdot e^{-\pi s} - \mathcal{L}\left\{\sin(t+2\pi)\right\} e^{-2\pi s}$$

$$= \mathcal{L}\left\{-\sin t\right\} e^{-\pi s} - \mathcal{L}\left\{\sin t\right\} e^{-2\pi s}$$

$$= -\frac{1}{s^2+1} e^{-\pi s} - \frac{1}{s^2+1} e^{-2\pi s}$$

Find the Laplace Transform of

$$f(t) = \begin{cases} 0, & 0 \le t < 2 \\ 3, & 2 \le t < 4 \\ e^t, & t \ge 4 \end{cases}$$

$$= 0 \cdot [u(t-0) - u(t-2)] + 3 \cdot [u(t-2) - u(t-4)] + e^t \cdot [u(t-4) - \underbrace{u(t-\infty)}_{0}]$$

$$= 3 \cdot u(t-2) - 3 \cdot u(t-4) + e^t \cdot u(t-4)$$

$$\mathcal{L}\left\{3 \cdot u(t-2) - 3 \cdot u(t-4) + e^{t} \cdot u(t-4)\right\}$$

$$= \mathcal{L}\left\{3 \cdot u(t-2)\right\} - \mathcal{L}\left\{3 \cdot u(t-4)\right\} + \mathcal{L}\left\{e^{t}\, u(t-4)\right\}$$

$$= \mathcal{L}\{3\} \cdot e^{-2s} - \mathcal{L}\{3\}\, e^{-4s} + \mathcal{L}\left\{e^{t+4}\right\} e^{-4s}$$

$$= \frac{3}{s} e^{-2s} - \frac{3}{s} e^{-4s} + \mathcal{L}\left\{e^{t} e^{4}\right\} e^{-4s}$$

$$= \frac{3}{s} e^{-2s} - \frac{3}{s} e^{-4s} + e^{4} \mathcal{L}\{e^{t}\} e^{-4s}$$

$$= \frac{3}{s} e^{-2s} - \frac{3}{s} e^{-4s} + e^{4} \cdot \frac{1}{s-1} e^{-4s}$$

Trigonometric Identity

$$\sin(\boldsymbol{\theta} \pm \boldsymbol{odd} \cdot \boldsymbol{\pi}) = -\boldsymbol{sin}(\boldsymbol{\theta})$$
$$\sin(\boldsymbol{\theta} \pm \boldsymbol{even} \cdot \boldsymbol{\pi}) = \boldsymbol{sin}(\boldsymbol{\theta})$$

$$\cos(\boldsymbol{\theta} \pm \boldsymbol{odd} \cdot \boldsymbol{\pi}) = -\boldsymbol{cos}(\boldsymbol{\theta})$$
$$\cos(\boldsymbol{\theta} \pm \boldsymbol{even} \cdot \boldsymbol{\pi}) = \boldsymbol{cos}(\boldsymbol{\theta})$$

$$\sin(5t - 4\pi) = \sin(5t)$$

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