Chapter-02

2.1 Discrete-time signals:

① Graphical representation:

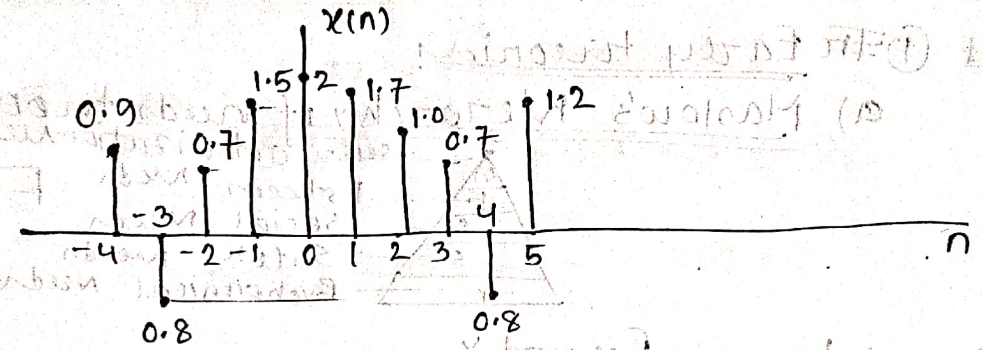


Figure 2.1.1. Graphical representation of a discrete-time signal.

Alternative representations:

① Functional representation, such as

$$x(n) = \begin{cases} 1 & \text{for } n=1,3 \\ 4 & \text{for } n=2 \\ 0, & \text{elsewhere} \end{cases}$$

② Tabular representation, such as

n	----	-2	-1	0	1	2	3	4	5	----
$x(n)$	----	0	0	0	1	4	1	0	0	----

③ Sequence representation:

An infinite-duration signal or sequence with the time origin $(n=0)$ indicated by the symbol $\uparrow$ is represented as,

$$x(n) = \{\ldots 0, 0, 1, \underset{\uparrow}{4}, 1, 0, 0 \ldots\}$$

A sequence $x(n)$, which is zero for $n<0$, can be represented as,

$$x(n) = \{\underset{\uparrow}{0}, 1, 4, 1, 0, 0 \ldots\}$$

A finite-duration ~~signal~~ sequence can be represented as,

$$x(n) = \{3, -1, -2, \underset{\uparrow}{5}, 0, 4, -1\}$$

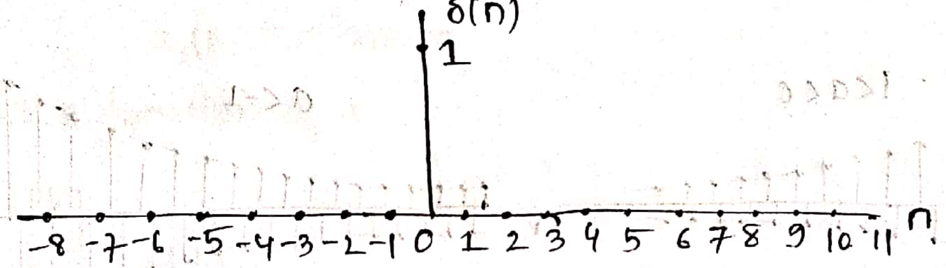
Whereas a finite-duration sequence that satisfies the condition $x(n) = 0$ for $n<0$ can be represented as,

$$x(n) = \{\underset{\uparrow}{0}, 1, 4, 1\}$$

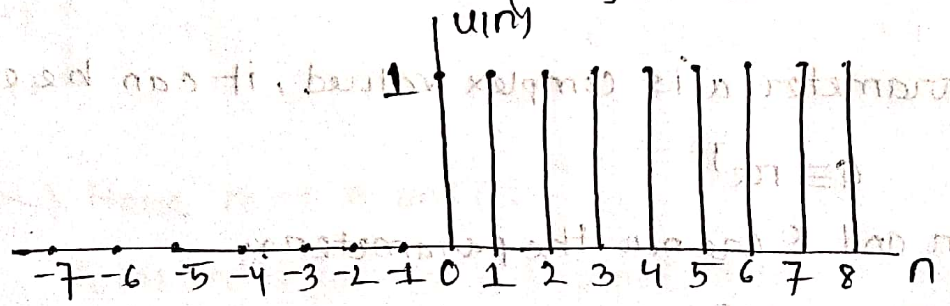
$\boxed{2.1.1}$ Some elementary Discrete-Time signals:

① Unit sample sequence / unit impulse:

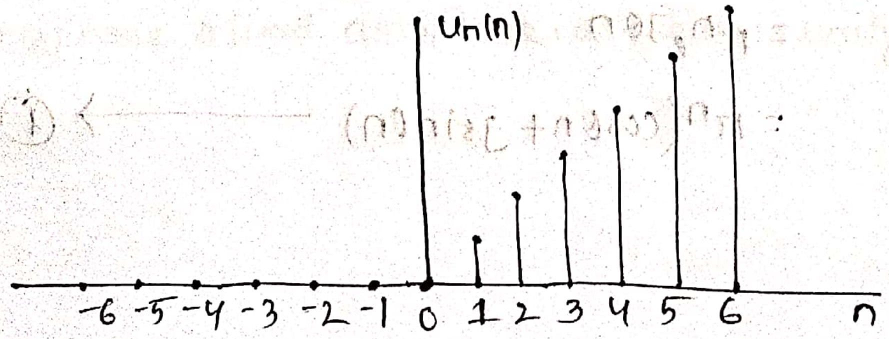
$$\delta(n) \equiv \begin{cases} 1, & \text{for } n=0 \\ 0, & \text{for } n \neq 0 \end{cases}$$



② Unit step signal: $u(n) \equiv \begin{cases} 1, & \text{for } n \geq 0 \\ 0, & \text{for } n<0 \end{cases}$



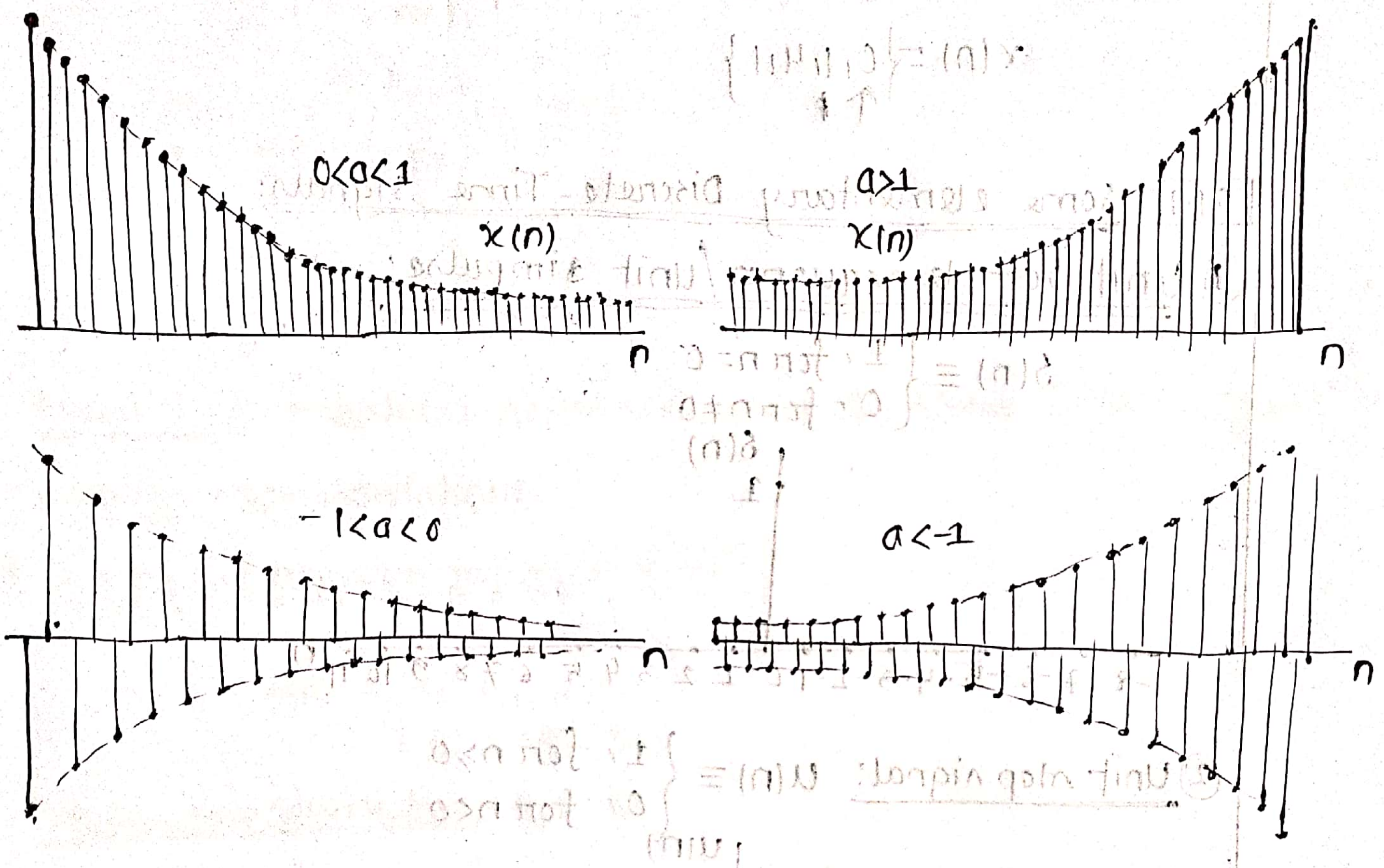
③ Unit ramp signal: $u_r(n) \equiv \begin{cases} n, & \text{for } n \geq 0 \\ 0, & \text{for } n<0 \end{cases}$



④ Exponential signal is a sequence of the form

$$x(n) = a^n \text{ for all } n$$

If the parameter a is real, then $x(n)$ is ~~rea~~ a real signal



when the parameter a is complex valued, it can be expressed as,

$$a \equiv re^{j\theta}$$

where r and θ are now the parameters.

$$\therefore x(n) = a^n$$

$$= (re^{j\theta})^n$$

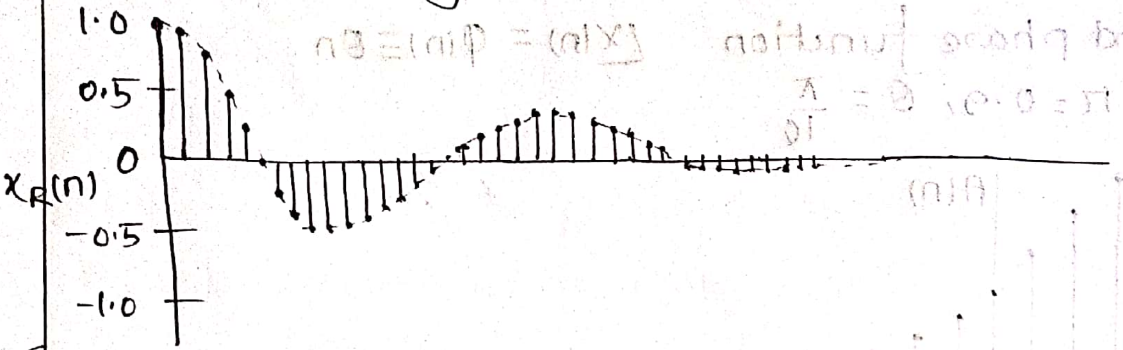
$$= r^n e^{j\theta n}$$

$$= r^n(\cos\theta n + j\sin\theta n) \longrightarrow ①$$

since, $x(n)$ is now complex valued, it (eqn 1) can be represented graphically by plotting the real part

$$x_R(n) \equiv r^n \cos \theta n$$

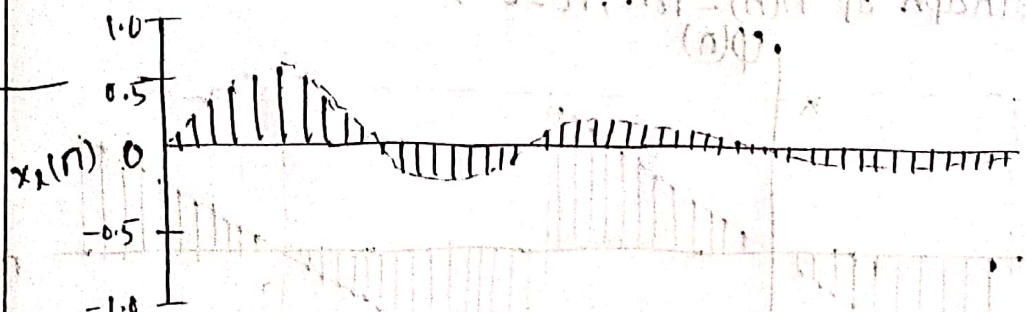
as a function of n



and separately plotting the imaginary part,

$$x_I(n) \equiv r^n \sin \theta n$$

as a function of n.



Here $r = 0.9$ and $\theta = \frac{\pi}{10}$

$x_R(n)$ and $x_I(n)$ are a damped (decaying ~~to~~ exponential) cosine function and a damped sine function.

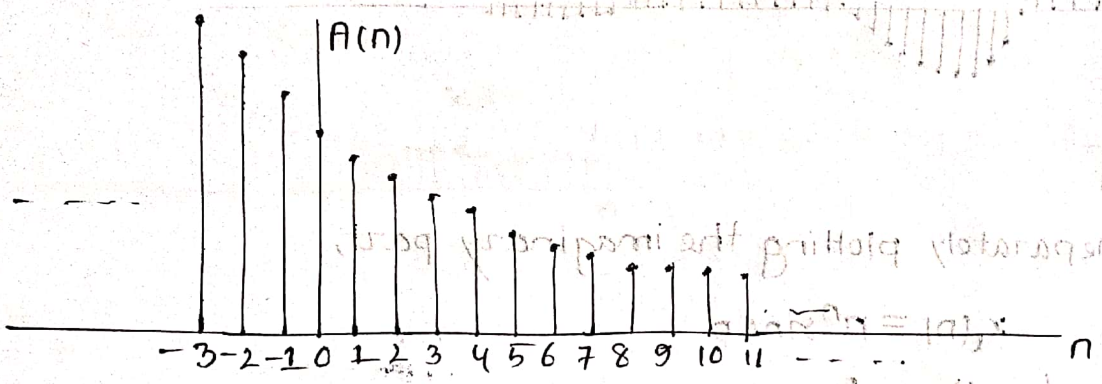
if $r=1$, the damping disappears and $x_R(n)$, $x_I(n)$ and $x(n)$ have a fixed amplitude, ~~or~~ which is unity

Alternatively, $x(n)$, eqn ① can be represented graphically by the amplitude function.

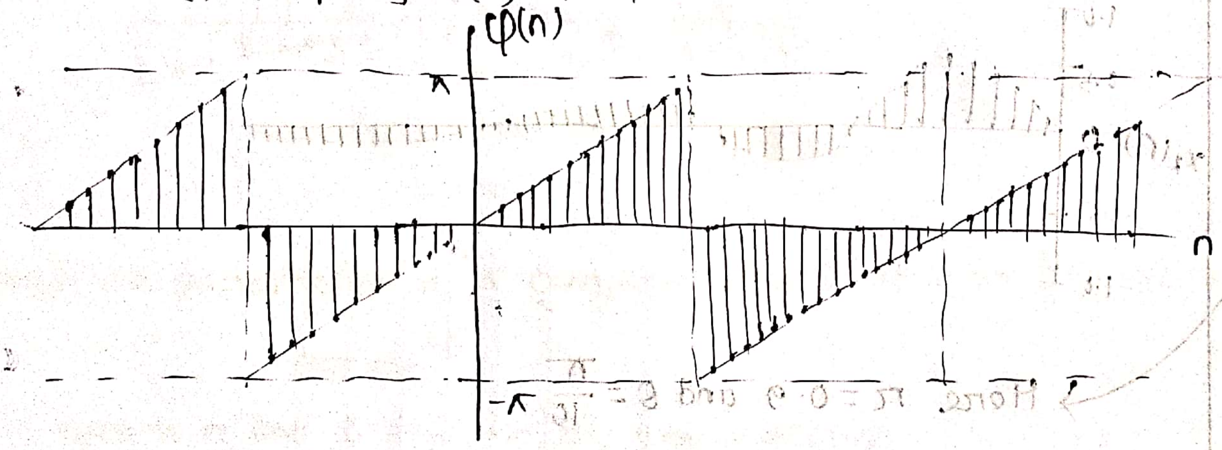
$$|x(n)| = A(n) \equiv r^n$$

and phase function $\angle x(n) = \varphi(n) \equiv \theta n$

Here, $r = 0.9$, $\theta = \frac{\pi}{10}$



(a) Graph of $A(n) = r^n$, $r = 0.9$



(b) Graph of $\varphi(n) = \frac{\pi}{10} n$