AS PER LATEST NTA NOTIFICATION

CHAPTER WISE TEST

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MATHEMATICS JEE MAIN (2025)

Topic – Continuity and Differentiability

Time Allowed: 60 mins **Maximum Marks: 100**

Instructions for the Candidate:

The Paper consist of 25(Twenty-Five) Question, which are divided in to two Sections
(a) Section A (MCQ's) Shall consist of 20 (Twenty) Questions. In which all are compulsory.
(b) Section B (NTA) Shall Consist of 5 (Five) Questions. In which all are compulsory.

Section – A (MCQ's)

1. If $f(x) = \begin{cases} \dfrac{\sin(a+2)x + \sin x}{x} & ; \quad x < 0 \\ b & ; \quad x = 0 \\ \dfrac{\left(x + 3x^2\right)^{1/3} - x^{1/3}}{x^{4/3}} & ; \quad x > 0 \end{cases}$ is continuous at x = 0, then a + 2b is equal to:
(A) 1 (B) – 2
(C) 0 (D) – 1

2. Let f be a composite function of x defined by $f(u) = \dfrac{1}{u^2 + u - 2}$, $u(x) = \dfrac{1}{x-1}$.
Then the number of points x where f is discontinuous is:
(A) 3 (B) 1
(C) 2 (D) 4

3. Let f, g: R → R be two functions defined by $f(x) = \begin{cases} x \sin\left(\dfrac{1}{x}\right), x \neq 0 \\ 0, \quad x = 0 \end{cases}$, and g(x) = x f(x)
Statement I: f is a continuous function at x = 0.
Statement II: g is a differentiable function at x = 0.
(A) Statement I is false, statement II is true.
(B) Statement I is true, statement II is false.
(C) Both statement I and II are false.
(D) Both statement I and II are true.

4. Let f(x) = x^2 + {x}, where {} represents the fractional part. Then
(A) f is periodic function
(B) f is not continuous at integral values of x
(C) $\lim_{x \to 3}$ f(x) exists
(D) $\lim_{x \to \frac{3}{2}} f(x) = 3$

5. If $f(x) = \begin{cases} \left(p^2 - 1\right)\left(\{x\} + 2[x]\right) - 2 & -2 < x \leq -1 \\ q\left(\dfrac{e^x + e^{-x}}{2}\right) + |p|(x-1), & -1 < x < 2,\ p, q \in R \end{cases}$
is continuous in (– 2, 2) then $f\left(f\left(f\left(\dfrac{-1}{2}\right)\right)\right)$ is:
[Note: [k] denotes greatest integer function less than or equal to k and {k}denotes fractional part function of k.]
(A) – 1 (B) 0
(C) – 2 (D) not defined

6. If the equation $x^4 + 8x^3 + 18x^2 + 8x + a = 0$ has four distinct real roots, then the range of a is:
(A) (0, 9) (B) (– 8, 1)
(C) (– 9, 0) (D) (– 1, 8)

7. Let f(x) = ||x – 1| – 1|. Then, which of the following is a wrong statement?
(A) $\lim_{x\to 1^-} \frac{f(x)-f(1)}{x-1}$ and $\lim_{x\to 1^+} \frac{f(x)-f(1)}{x-1}$ exist
(B) f is not differentiable at x = 1
(C) f is differentiable on R – {0, 1}
(D) f is continuous at x = 1

8. Let $f(x) = mx - 1 + \frac{1}{x}$. Then the smallest value of the constant m such that $f(x) \geq 0$ for every $x > 0$ is:
(A) $\frac{1}{2}$ (B) 1
(C) $\frac{1}{4}$ (D) $\frac{1}{8}$

9. If $y = x^2 + \cfrac{1}{x^2 + \cfrac{1}{x^2 + \cfrac{1}{x^2 + \ldots..\infty}}}$, then $\frac{dy}{dx}$ =
(A) $\frac{2xy}{2y - x^2}$ (B) $\frac{xy}{y + x^2}$
(C) $\frac{2xy}{2 + \frac{x^2}{y}}$ (D) $\frac{xy}{y - x^2}$

10. If $y = (\sin x)^{(\sin x)^{(\sin x)\ldots\infty}}$, then $\frac{dy}{dx}$ =
(A) $\frac{y\cot x}{1 + y\log(\sin x)}$ (B) $\frac{y^2\cot x}{1 + y\log(\sin x)}$
(C) $\frac{y\cot x}{1 - y\log(\sin x)}$ (D) $\frac{y^2\cot x}{1 - y\log(\sin x)}$

11. If $y = \sqrt{\log x + \sqrt{\log x + \sqrt{\log x + \ldots..\infty}}}$, then $\frac{dy}{dx}$
(A) $\frac{x}{2y+1}$ (B) $\frac{1}{x(1-2y)}$
(C) $\frac{x}{2y-1}$ (D) $\frac{1}{x(2y-1)}$

12. Let $y = \sin(\cos^{-1}(\sin(\cos^{-1} x)))$, then:
(A) $y'\left(\frac{1}{2}\right) = -1$ (B) $y'\left(\frac{3}{4}\right) = 1$
(C) $y'\left(-\frac{3}{4}\right) = 1$ (D) $y'\left(-\frac{1}{2}\right) = 1$

13. If $y = \sec(\tan^{-1} x)$, then $\frac{dy}{dx}$ at x = 1 is equal to
(A) $\frac{1}{\sqrt{2}}$ (B) $\sqrt{2}$
(C) 1 (D) $\frac{1}{2}$

14. If $f(x) = e^x g(x)$, g(0) = 2, g'(0) = 1, then f'(0) is:
(A) 2 (B) 3
(C) 1 (D) 0

15. Functions f and g are such that f is even and g is odd. Given $f(x) + g(x) = e^{-x}$, for all x. The value of f'(1) is:
(A) $\frac{1}{2e}$ (B) $\frac{e}{2}$
(C) $\frac{e - e^{-1}}{2}$ (D) $\frac{e + e^{-1}}{2}$

16. The value of $\log_e 2 \frac{d}{dx}(\log_{\cos x} \operatorname{cosec} x)$ at $x = \frac{\pi}{4}$ is
(A) $2\sqrt{2}$ (B) 4
(C) – 4 (D) $-2\sqrt{2}$

17. The derivative of $y = \tan^{-1}\left(\frac{x}{1+\sqrt{1-x^2}}\right)$ with respect to $z = \sin^{-1} x$ is:
(A) 1 (B) $\frac{3}{2}$
(C) 2 (D) $\frac{1}{2}$

18. If g(x) = $x^2 + \frac{1}{x^2}$ and f(g(x)) = $x^6 + \frac{1}{x^6}$, then f ”(– 2) equals:
(A) – 12 (B) – 4
(C) – 2 (D) 0

19. If y = (A + Bx)e^{nx} + (m – 1)$^{-2}$ e^x then $\frac{d^2y}{dx^2} - 2m\frac{dy}{dx} + m^2y$ is equal to :
(A) e^{nx} (B) $e^{(1-m)}$ x
(C) e^x (D) e^{-nx}

20. For the curve $32x^3 y^2 = (x + y)^5$, the value of $\frac{d^2y}{dx^2}$ at P (1, 1) is equal to:
(A) – 1 (B) $\frac{1}{2}$
(C) 1 (D) 0

Section – B (NTA)

21. Find the set of values of a for which the function f(x) = $|x^2 + (a - 2)| \times |-2a|$ is non – differentiable.

22. Let f(x) = ax^9 + b sin x + cx^2 sgn (x) + $\frac{\left(e^x - e^{-x}\right)}{\left(e^x + e^{-x}\right)}$ be defined on set of real numbers, (a > 0, b, c, $\in$ R). If f(– 5) = 5, f(– 2) = – 3, then find the minimum number of zeros of the equation f(x) = 0.

23. If y = y(x) is an implicit function of x such that $\log_e (x + y) = 4xy$, then $\frac{d^2y}{dx^2}$ at x = 0 is equal to

24. If y = $\sum_{k=1}^{6} k\cos^{-1}\left\{\frac{3}{5}\cos kx - \frac{4}{5}\sin kx\right\}$, then $\frac{dy}{dx}$ at x = 0 is _____.

25. Two continuous and differentiable functions f(x) and g(x) are related as
f(x) = $\begin{cases} \frac{g(x)-4}{x-2}, & x \neq 2 \\ k, & x = 2 \end{cases}$. If equation of tangent to the curve y = f(x) at x = 2 be y = x + 3, then find the value of f(2) + f' (2) +g(2) + g' (2) + g''(2).

SPACE FOR ROUGH WORK

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