Deep Learning Hands-on Example

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1 Backpropagation

The ANN neurons function is described as below:

$$y = \phi(\sum_{i=1}^{n} w_i X_i + b_i) \tag{1}$$

where w, b, X and $\phi(z)$ are the weight, bias, input and activation function respectively. Sigmoid function $\phi(z) = \frac{1}{1+e^{-z}}$ is widely used as an activation function (outputs 0 - 1).

The error function can also be shown as the following:

$$\mathrm{E} = \frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2 \tag{2}$$

$$y^{(L)} = \phi\left(z^{(L)}\right) \tag{3}$$

The parital derivative of error, with respect of the weight for layer $^{(L)}$ is calculated as below:

$$\frac{\partial E_1}{\partial w^{(L)}} = y^{(L-1)}\phi'\left(z^{(L)}\right) 2\left(y^{(L)} - \hat{y}\right), \tag{4}$$

where the derivative of the sigmoid activation function is $\phi'\left(z^{(L)}\right) = \phi\left(z^{(L)}\right)\left((1 - \phi\left(z^{(L)}\right)\right)$.

For each iteration, the weight can then be updated by the following:

$$w \longleftarrow w - \alpha\frac{\partial E}{\partial w}, \tag{5}$$

where learning rate α dictates how fast the weights need to be adjusted.

2 Example

A Deep Neural Network is shown in Figure 1. The aim of this network is to classify human and dog by using backpropagation discussed in Section 1.

1. Calculate the initial error of the Deep Neural Network by feeding the following input: $X_1 = 5$, $X_2 = 2$. The X_1 indicates the length of the tail and X_2 indicates the number of eyes.
2. Update the layer 2 weights (w_5,w_6,w_7,w_8) of the Deep Neural Network shown in Figure 1 for a single iteration. The learning rate is set to $\alpha = 1$ and there are no biases in the neurons.
3. Recalculate the error with the updated weights of layer 2 and report its improvement with respect to the initial error.
4. (Bonus) Update the layer 1 weights (w_1,w_2,w_3,w_4) of the Deep Neural Network and calculate the improvement to the error.

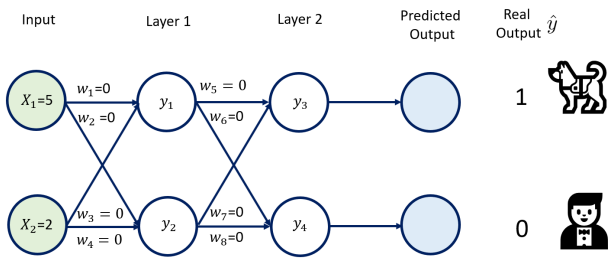

Figure 1: A Deep Neural Network designed for dog or human detection