

Calculates groebner basis of

- $x^3 + (-2)xy$
- $x^2y + (-2)y^2 + x$

$$S(x^3 + (-2)xy, x^2y + (-2)y^2 + x) = (-1)x^2.$$

Not enough. Appends

- $(-1)x^2$

$$S(x^3 + (-2)xy, (-1)x^2) = (-2)xy.$$

$$S(x^2y + (-2)y^2 + x, (-1)x^2) = (-2)y^2 + x.$$

Not enough. Appends

- $(-2)xy$
- $(-2)y^2 + x$

$$S(x^3 + (-2)xy, (-2)xy) = (-2)xy^2.$$

$$S(x^3 + (-2)xy, (-2)y^2 + x) = (-2)xy^3 + 2y^2 + (-1)x.$$

$$S(x^2y + (-2)y^2 + x, (-2)xy) = (-2)y^2 + x.$$

$$S(x^2y + (-2)y^2 + x, (-2)y^2 + x) = (-2)y^3 + 2xy.$$

$$S((-1)x^2, (-2)xy) = 0.$$

$$S((-1)x^2, (-2)y^2 + x) = xy.$$

$$S((-2)xy, (-2)y^2 + x) = 0.$$

Not enough. Appends

- $(-2)xy^2$
- $(-2)xy^3 + 2y^2 + (-1)x$
- $(-2)y^2 + x$
- $(-2)y^3 + 2xy$
- xy

$$S(x^3 + (-2)xy, (-2)xy^2) = 0.$$

$$S(x^3 + (-2)xy, (-2)xy^3 + 2y^2 + (-1)x) = 0.$$

$$S(x^3 + (-2)xy, (-2)y^2 + x) = 0.$$

$$S(x^3 + (-2)xy, (-2)y^3 + 2xy) = 0.$$

$$S(x^3 + (-2)xy, xy) = 0.$$

$$S(x^2y + (-2)y^2 + x, (-2)xy^2) = 0.$$

$$S(x^2y + (-2)y^2 + x, (-2)xy^3 + 2y^2 + (-1)x) = 0.$$

$$S(x^2y + (-2)y^2 + x, (-2)y^2 + x) = 0.$$

$$\begin{aligned}
S(x^2y + (-2)y^2 + x, (-2)y^3 + 2xy) &= 0. \\
S(x^2y + (-2)y^2 + x, xy) &= 0. \\
S((-1)x^2, (-2)xy^2) &= 0. \\
S((-1)x^2, (-2)xy^3 + 2y^2 + (-1)x) &= 0. \\
S((-1)x^2, (-2)y^2 + x) &= 0. \\
S((-1)x^2, (-2)y^3 + 2xy) &= 0. \\
S((-1)x^2, xy) &= 0. \\
S((-2)xy, (-2)xy^2) &= 0. \\
S((-2)xy, (-2)xy^3 + 2y^2 + (-1)x) &= 0. \\
S((-2)xy, (-2)y^2 + x) &= 0. \\
S((-2)xy, (-2)y^3 + 2xy) &= 0. \\
S((-2)xy, xy) &= 0. \\
S((-2)y^2 + x, (-2)xy^2) &= 0. \\
S((-2)y^2 + x, (-2)xy^3 + 2y^2 + (-1)x) &= 0. \\
S((-2)y^2 + x, (-2)y^2 + x) &= 0. \\
S((-2)y^2 + x, (-2)y^3 + 2xy) &= 0. \\
S((-2)y^2 + x, xy) &= 0. \\
S((-2)xy^2, (-2)xy^3 + 2y^2 + (-1)x) &= 0. \\
S((-2)xy^2, (-2)y^2 + x) &= 0. \\
S((-2)xy^2, (-2)y^3 + 2xy) &= 0. \\
S((-2)xy^2, xy) &= 0. \\
S((-2)xy^3 + 2y^2 + (-1)x, (-2)y^2 + x) &= 0. \\
S((-2)xy^3 + 2y^2 + (-1)x, (-2)y^3 + 2xy) &= 0. \\
S((-2)xy^3 + 2y^2 + (-1)x, xy) &= 0. \\
S((-2)y^2 + x, (-2)y^3 + 2xy) &= 0. \\
S((-2)y^2 + x, xy) &= 0. \\
S((-2)y^3 + 2xy, xy) &= 0.
\end{aligned}$$

Enough for groebner basis. Result is

- $x^3 + (-2)xy$
- $x^2y + (-2)y^2 + x$
- $(-1)x^2$
- $(-2)xy$
- $(-2)y^2 + x$
- $(-2)xy^2$
- $(-2)xy^3 + 2y^2 + (-1)x$
- $(-2)y^2 + x$
- $(-2)y^3 + 2xy$
- xy

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