$$xy \tag{1}$$

$$xz^3 + (-1)yz^2 \tag{2}$$

Checking if following bases are Groebner basis. Divisors and bases are

- $x + (-1)z^2$
- $y + (-1)z^3$

.

1. $S(x + (-1)z^2, y + (-1)z^3) = xz^3 + (-1)yz^2$. Calculation is
   a   Start: calculates $xz^3 + (-1)yz^2 \div$
      - $x + (-1)z^2$,
      - $y + (-1)z^3$,

      .

   b   Division: $x + (-1)z^2$ divides stock. stock is $(-1)yz^2 + z^5$.
   c   Division: $y + (-1)z^3$ divides stock. stock is 0 .
   d   Completed: quotients are
      - z^3,
      - $(-1)z^2$,
      . remainder is 0. ■
   . ■

Checking if following bases are Groebner basis. Divisors and bases are

- $x^2 + (-1)y$
- $x^3 + (-1)z$

.

1. $S(x^2 + (-1)y, x^3 + (-1)z) = (-1)xy + z$. Calculation is
   a   Start: calculates $(-1)xy + z \div$
      - $x^2 + (-1)y$,
      - $x^3 + (-1)z$,

      .

   b   Remainder: $(-1)xy$ moved to remainder.
   c   Remainder: z moved to remainder.
   d   Completed: quotients are
      - 0,
      - 0,
      . remainder is $(-1)xy + z$. ■

. ■

Checking if following bases are Groebner basis. Divisors and bases are

- $(-1)y + x^2$
- $(-1)z + x^3$

.

1. $S((-1)y + x^2, (-1)z + x^3) = (-1)x^2z + x^3y$. Calculation is
   a Start: calculates $(-1)x^2z + x^3y \div$
      - $(-1)y + x^2$,
      - $(-1)z + x^3$,

      .

   b Division: $(-1)z + x^3$ divides stock. stock is $x^3y + (-1)x^5$.
   c Division: $(-1)y + x^2$ divides stock. stock is 0 .
   d Completed: quotients are
      - $(-1)x^3$,
      - x^2,
      . remainder is 0. ■
   . ■

Checking if following bases are Groebner basis. Divisors and bases are

- $xy^2 + (-1)xz + y$
- $xy + (-1)z^2$
- $x + (-1)yz^4$

.

1. $S(xy^2 + (-1)xz + y, xy + (-1)z^2) = (-1)xz + yz^2 + y$. Calculation is
   a Start: calculates $(-1)xz + yz^2 + y \div$
      - $xy^2 + (-1)xz + y$,
      - $xy + (-1)z^2$,
      - $x + (-1)yz^4$,

      .

   b Division: $x + (-1)yz^4$ divides stock. stock is $(-1)yz^5 + yz^2 + y$.
   c Remainder: $(-1)yz^5$ moved to remainder.
   d Remainder: yz^2 moved to remainder.
   e Remainder: y moved to remainder.
   f Completed: quotients are
      - 0,
      - 0,
      - $(-1)z$,

. remainder is $(-1)yz^5 + yz^2 + y.$ ∎

. ∎

2. $S(xy^2 + (-1)xz + y, x + (-1)yz^4) = (-1)xz + y^3z^4 + y.$ Calculation is

   a Start: calculates $(-1)xz + y^3z^4 + y \div$

- $xy^2 + (-1)xz + y,$
- $xy + (-1)z^2,$
- $x + (-1)yz^4,$

.

   b Division: $x + (-1)yz^4$ divides stock. stock is $y^3z^4 + (-1)yz^5 + y$.

   c Remainder: y^3z^4 moved to remainder.

   d Remainder: $(-1)yz^5$ moved to remainder.

   e Remainder: y moved to remainder.

   f Completed: quotients are

- $0,$
- $0,$
- $(-1)z,$

. remainder is $y^3z^4 + (-1)yz^5 + y.$ ∎

. ∎

3. $S(xy + (-1)z^2, x + (-1)yz^4) = y^2z^4 + (-1)z^2.$ Calculation is

   a Start: calculates $y^2z^4 + (-1)z^2 \div$

- $xy^2 + (-1)xz + y,$
- $xy + (-1)z^2,$
- $x + (-1)yz^4,$

.

   b Remainder: y^2z^4 moved to remainder.

   c Remainder: $(-1)z^2$ moved to remainder.

   d Completed: quotients are

- $0,$
- $0,$
- $0,$

. remainder is $y^2z^4 + (-1)z^2.$ ∎

. ∎