$$7xy^2 + xy \tag{1}$$

$$3xy^2 + xy \tag{2}$$

$$3xy^2 + \frac{7}{2} \tag{3}$$

0 (4)

$$21x^2y^4 + 3x^2y^3 + \frac{49}{2}xy^2 + \frac{7}{2}xy\tag{5}$$

$$4x_0x_1^2x_2^7x_3^3x_4 + 3x_0x_1^2x_2^3x_3^4x_4^5 + x_0x_1$$

$$\tag{6}$$

Lex (PlainMonomial [3,2,0])

$$2x + 2 \tag{7}$$

$$(-1)x + (-1)$$
 (8)

$$x+1 (9)$$

$$0 \tag{10}$$

1. Start: calculates $x^2 + 2x + 1$:

• x + 1,

- 2. Division: x + 1 divides stock. stock is x + 1 .
- 3. Division: x+1 divides stock. stock is 0 .
- 4. Completed: quotients are
 - x + 1,
 - . remainder is 0. \blacksquare

$$xy^2 + 1\tag{11}$$

$$xy + 1 \tag{12}$$

$$y+1 \tag{13}$$

Lex (PlainMonomial [0,1])Lex (PlainMonomial [0,1])True

$$2 \tag{14}$$

- 1. Start: calculates $xy^2 + 1 \div$
 - xy + 1,
 - y + 1,

.

- 2. Division: xy + 1 divides stock. stock is (-1)y + 1 .
- 3. Division: y+1 divides stock. stock is 2 .
- 4. Remainder: 2 moved to remainder.
- 5. Completed: quotients are
 - *y*,
 - (-1),
 - . remainder is 2. \blacksquare

- 1. Start: calculates $x^2y + xy^2 + y^2 \div$
 - xy + (-1),
 - $y^2 + (-1)$,

.

- 2. Division: xy + (-1) divides stock. stock is $xy^2 + x + y^2$.
- 3. Division: xy + (-1) divides stock. stock is $x + y^2 + y$.
- 4. Remainder: x moved to remainder.
- 5. Division: $y^2 + (-1)$ divides stock. stock is y + 1.
- 6. Remainder: y moved to remainder.
- 7. Remainder: 1 moved to remainder.
- 8. Completed: quotients are
 - $\bullet \ x+y,$
 - 1,
 - . remainder is x + y + 1.
- 1. Start: calculates $x^2y + xy^2 + y^2 \div$
 - $y^2 + (-1)$,
 - xy + (-1),

.

- 2. Division: xy + (-1) divides stock. stock is $xy^2 + x + y^2$.
- 3. Division: $y^2 + (-1)$ divides stock. stock is $2x + y^2$.
- 4. Remainder: 2x moved to remainder.
- 5. Division: $y^2 + (-1)$ divides stock. stock is 1.
- 6. Remainder: 1 moved to remainder.
- 7. Completed: quotients are
 - x + 1,
 - *x*,
 - . remainder is 2x + 1.
- 1. Start: calculates $x^2 + 2x + 1 \div$
 - x + 1,

.

- 2. Division: x + 1 divides stock. stock is x + 1.
- 3. Division: x + 1 divides stock. stock is 0.
- 4. Completed: quotients are
 - x + 1,
 - . remainder is 0. \blacksquare