Calculates groebner basis of

- $x^3 + (-2)xy$
- $x^2y + (-2)y^2 + x$

.

$\overline{S(x^3 + (-2)xy, x^2y + (-2)y^2 + x)} = (-1)x^2$.

Not enough. Appends

- $(-1)x^2$

.

$\overline{S(x^3 + (-2)xy, (-1)x^2)} = (-2)xy$.

$\overline{S(x^2y + (-2)y^2 + x, (-1)x^2)} = (-2)y^2 + x$.

Not enough. Appends

- $(-2)xy$

.

$\overline{S(x^3 + (-2)xy, (-2)xy)} = 0$.

$\overline{S(x^2y + (-2)y^2 + x, (-2)xy)} = (-2)y^2 + x$.

$\overline{S((-1)x^2, (-2)xy)} = 0$.

Not enough. Appends

- $(-2)y^2 + x$

.

$\overline{S(x^3 + (-2)xy, (-2)y^2 + x)} = 0$.

$\overline{S(x^2y + (-2)y^2 + x, (-2)y^2 + x)} = 0$.

$\overline{S((-1)x^2, (-2)y^2 + x)} = 0$.

$\overline{S((-2)xy, (-2)y^2 + x)} = 0$.

Enough for groebner basis. Result is

- $x^3 + (-2)xy$
- $x^2y + (-2)y^2 + x$
- $(-1)x^2$
- $(-2)xy$
- $(-2)y^2 + x$

. ■ Minimalizes groebner basis

- $x^3 + (-2)xy$
- $x^2y + (-2)y^2 + x$
- $(-1)x^2$
- $(-2)xy$

- $(-2)y^2 + x$

.

$x^3 + (-2)xy$ is removed by $(-1)x^2$.

$x^2y + (-2)y^2 + x$ is removed by $(-1)x^2$.

Minimalized groebner basis is

- x^2
- xy
- $y^2 + (\frac{-1}{2})x$

. ■

Reduce groebner basis

- x^2
- xy
- $y^2 + (\frac{-1}{2})x$

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Reducing: $\overline{x^2} = x^2$.

Reducing: $\overline{xy} = xy$.

Reducing: $\overline{y^2 + (\frac{-1}{2})x} = y^2 + (\frac{-1}{2})x$.

Reduced groebner basis is

- $y^2 + (\frac{-1}{2})x$
- xy
- x^2

. ■

Reduce groebner basis

- $x^2 + xy$
- xy
- $y^2 + (\frac{-1}{2})x$

.

Reducing: $\overline{x^2 + xy} = x^2$.

Reducing: $\overline{xy} = xy$.

Reducing: $\overline{y^2 + (\frac{-1}{2})x} = y^2 + (\frac{-1}{2})x$.

Reduced groebner basis is

- $y^2 + (\frac{-1}{2})x$
- xy
- x^2

. ■