1

Calculates groebner basis of

- $x^3 + (-2)xy$
- $x^2y + (-2)y^2 + x$

.
$\overline{S(x^3 + (-2)xy, x^2y + (-2)y^2 + x)} = (-1)x^2$.
Not enough. Appends

- $(-1)x^2$

.
$\overline{S(x^3 + (-2)xy, (-1)x^2)} = (-2)xy$.
$\overline{S(x^2y + (-2)y^2 + x, (-1)x^2)} = (-2)y^2 + x$.
Not enough. Appends

- $(-2)xy$

.
$\overline{S(x^3 + (-2)xy, (-2)xy)} = 0$.
$\overline{S(x^2y + (-2)y^2 + x, (-2)xy)} = (-2)y^2 + x$.
$\overline{S((-1)x^2, (-2)xy)} = 0$.
Not enough. Appends

- $(-2)y^2 + x$

.
$\overline{S(x^3 + (-2)xy, (-2)y^2 + x)} = 0$.
$\overline{S(x^2y + (-2)y^2 + x, (-2)y^2 + x)} = 0$.
$\overline{S((-1)x^2, (-2)y^2 + x)} = 0$.
$\overline{S((-2)xy, (-2)y^2 + x)} = 0$.
Enough for groebner basis. Result is

- $x^3 + (-2)xy$
- $x^2y + (-2)y^2 + x$
- $(-1)x^2$
- $(-2)xy$
- $(-2)y^2 + x$

. ■ Minimalizes groebner basis

- $x^3 + (-2)xy$

- $x^2y + (-2)y^2 + x$
- $(-1)x^2$
- $(-2)xy$
- $(-2)y^2 + x$

.

$x^3 + (-2)xy$ is removed by $(-1)x^2$.

$x^2y + (-2)y^2 + x$ is removed by $(-1)x^2$.

Minimalized groebner basis is

- x^2
- xy
- $y^2 + (\frac{-1}{2})x$

. $\blacksquare$

Reduce groebner basis

- x^2
- xy
- $y^2 + (\frac{-1}{2})x$

.

Reducing: $\overline{x^2} = x^2$.

Reducing: $\overline{xy} = xy$.

Reducing: $\overline{y^2 + (\frac{-1}{2})x} = y^2 + (\frac{-1}{2})x$.

Reduced groebner basis is

- $y^2 + (\frac{-1}{2})x$
- xy
- x^2

. $\blacksquare$

Reduce groebner basis

- $x^2 + xy$
- xy
- $y^2 + (\frac{-1}{2})x$

.

Reducing: $\overline{x^2 + xy} = x^2$.

Reducing: $\overline{xy} = xy$.

Reducing: $\overline{y^2 + (\frac{-1}{2})x} = y^2 + (\frac{-1}{2})x$.

Reduced groebner basis is

- $y^2 + (\frac{-1}{2})x$

- xy
- x^2

. ■

2

Calculates groebner basis of

- $x^2y + (-1)$
- $xy^2 + (-1)x$

.

$\overline{S(x^2y + (-1), xy^2 + (-1)x)} = x^2 + (-1)y$.
Not enough. Appends

- $x^2 + (-1)y$

.

$\overline{S(x^2y + (-1), x^2 + (-1)y)} = y^2 + (-1)$.
$\overline{S(xy^2 + (-1)x, x^2 + (-1)y)} = y^3 + (-1)y$.
Not enough. Appends

- $y^2 + (-1)$

.

$\overline{S(x^2y + (-1), y^2 + (-1))} = 0$.
$\overline{S(xy^2 + (-1)x, y^2 + (-1))} = 0$.
$\overline{S(x^2 + (-1)y, y^2 + (-1))} = 0$.
Enough for groebner basis. Result is

- $x^2y + (-1)$
- $xy^2 + (-1)x$
- $x^2 + (-1)y$
- $y^2 + (-1)$

. ■ Minimalizes groebner basis

- $x^2y + (-1)$
- $xy^2 + (-1)x$
- $x^2 + (-1)y$
- $y^2 + (-1)$

.

$x^2y + (-1)$ is removed by $x^2 + (-1)y$.

$xy^2 + (-1)x$ is removed by $y^2 + (-1)$.
Minimalized groebner basis is

- $x^2 + (-1)y$
- $y^2 + (-1)$

. ■

Reduce groebner basis

- $x^2 + (-1)y$
- $y^2 + (-1)$

.

Reducing: $\overline{x^2 + (-1)y} = x^2 + (-1)y$.
Reducing: $\overline{y^2 + (-1)} = y^2 + (-1)$.
Reduced groebner basis is

- $y^2 + (-1)$
- $x^2 + (-1)y$

. ■

3

Calculates groebner basis of

- $x^2y + (-1)$
- $xy^2 + (-1)x$

.

$\overline{S(x^2y + (-1), xy^2 + (-1)x)} = x^2 + (-1)y$.
Not enough. Appends

- $x^2 + (-1)y$

.

$\overline{S(x^2y + (-1), x^2 + (-1)y)} = y^2 + (-1)$.
$\overline{S(xy^2 + (-1)x, x^2 + (-1)y)} = y^3 + (-1)y$.
Not enough. Appends

- $y^2 + (-1)$

.

$\overline{S(x^2y + (-1), y^2 + (-1))} = 0$.
$\overline{S(xy^2 + (-1)x, y^2 + (-1))} = 0$.
$\overline{S(x^2 + (-1)y, y^2 + (-1))} = 0$.

Enough for groebner basis. Result is

- $x^2y + (-1)$
- $xy^2 + (-1)x$
- $x^2 + (-1)y$
- $y^2 + (-1)$

. ■ Minimalizes groebner basis

- $x^2y + (-1)$
- $xy^2 + (-1)x$
- $x^2 + (-1)y$
- $y^2 + (-1)$

.

$x^2y + (-1)$ is removed by $x^2 + (-1)y$.
$xy^2 + (-1)x$ is removed by $y^2 + (-1)$.
Minimalized groebner basis is

- $x^2 + (-1)y$
- $y^2 + (-1)$

. ■

Reduce groebner basis

- $x^2 + (-1)y$
- $y^2 + (-1)$

.

Reducing: $\overline{x^2 + (-1)y} = x^2 + (-1)y$.
Reducing: $\overline{y^2 + (-1)} = y^2 + (-1)$.
Reduced groebner basis is

- $y^2 + (-1)$
- $x^2 + (-1)y$

. ■

4 b-lex

Calculates groebner basis of

- $x^2 + y$
- $x^4 + 2x^2y + y^2 + 3y$

.

$\overline{S(x^2+y, x^4+2x^2y+y^2+3y)} = (-3)y$.

Not enough. Appends

- $(-3)y$

.

$\overline{S(x^2+y, (-3)y)} = 0$.

$\overline{S(x^4+2x^2y+y^2+3y, (-3)y)} = 0$.

Enough for groebner basis. Result is

- x^2+y
- $x^4+2x^2y+y^2+3y$
- $(-3)y$

. ■ Minimalizes groebner basis

- x^2+y
- $x^4+2x^2y+y^2+3y$
- $(-3)y$

.

$x^4+2x^2y+y^2+3y$ is removed by x^2+y.

Minimalized groebner basis is

- x^2+y
- y

. ■

Reduce groebner basis

- x^2+y
- y

.

Reducing: $\overline{x^2+y} = x^2$.

Reducing: $\overline{y} = y$.

Reduced groebner basis is

- y
- x^2

. ■

5 b-grlex

Calculates groebner basis of

- $x^2 + y$
- $x^4 + 2x^2y + y^2 + 3y$

.

$\overline{S(x^2 + y, x^4 + 2x^2y + y^2 + 3y)} = (-3)y$.

Not enough. Appends

- $(-3)y$

.

$\overline{S(x^2 + y, (-3)y)} = 0$.

$\overline{S(x^4 + 2x^2y + y^2 + 3y, (-3)y)} = 0$.

Enough for groebner basis. Result is

- $x^2 + y$
- $x^4 + 2x^2y + y^2 + 3y$
- $(-3)y$

. ■ Minimalizes groebner basis

- $x^2 + y$
- $x^4 + 2x^2y + y^2 + 3y$
- $(-3)y$

.

$x^4 + 2x^2y + y^2 + 3y$ is removed by $x^2 + y$.

Minimalized groebner basis is

- $x^2 + y$
- y

. ■

Reduce groebner basis

- $x^2 + y$
- y

.

Reducing: $\overline{x^2 + y} = x^2$.

Reducing: $\overline{y} = y$.

Reduced groebner basis is

- y
- x^2

. ■

6 c-lex

Calculates groebner basis of

- $x + (-1)z^4$
- $y + (-1)z^5$

.

$\overline{S(x + (-1)z^4, y + (-1)z^5)} = 0.$

Enough for groebner basis. Result is

- $x + (-1)z^4$
- $y + (-1)z^5$

. ■ Minimalizes groebner basis

- $x + (-1)z^4$
- $y + (-1)z^5$

.

Minimalized groebner basis is

- $x + (-1)z^4$
- $y + (-1)z^5$

. ■

Reduce groebner basis

- $x + (-1)z^4$
- $y + (-1)z^5$

.

Reducing: $\overline{x + (-1)z^4} = x + (-1)z^4$.

Reducing: $\overline{y + (-1)z^5} = y + (-1)z^5$.

Reduced groebner basis is

- $y + (-1)z^5$
- $x + (-1)z^4$

. ■

7 c-grlex

Calculates groebner basis of

- $(-1)z^4 + x$
- $(-1)z^5 + y$

.

$\overline{S((-1)z^4 + x, (-1)z^5 + y)} = (-1)xz + y$.

Not enough. Appends

- $(-1)xz + y$

.

$\overline{S((-1)z^4 + x, (-1)xz + y)} = yz^3 + (-1)x^2$.

$\overline{S((-1)z^5 + y, (-1)xz + y)} = 0$.

Not enough. Appends

- $yz^3 + (-1)x^2$

.

$\overline{S((-1)z^4 + x, yz^3 + (-1)x^2)} = 0$.

$\overline{S((-1)z^5 + y, yz^3 + (-1)x^2)} = 0$.

$\overline{S((-1)xz + y, yz^3 + (-1)x^2)} = (-1)y^2z^2 + x^3$.

Not enough. Appends

- $(-1)y^2z^2 + x^3$

.

$\overline{S((-1)z^4 + x, (-1)y^2z^2 + x^3)} = 0$.

$\overline{S((-1)z^5 + y, (-1)y^2z^2 + x^3)} = 0$.

$\overline{S((-1)xz + y, (-1)y^2z^2 + x^3)} = x^4 + (-1)y^3z$.

$\overline{S(yz^3 + (-1)x^2, (-1)y^2z^2 + x^3)} = 0$.

Not enough. Appends

- $x^4 + (-1)y^3z$

.

$\overline{S((-1)z^4 + x, x^4 + (-1)y^3z)} = 0$.

$\overline{S((-1)z^5 + y, x^4 + (-1)y^3z)} = 0$.

$\overline{S((-1)xz + y, x^4 + (-1)y^3z)} = 0$.

$\overline{S(yz^3 + (-1)x^2, x^4 + (-1)y^3z)} = 0$.

$\overline{S((-1)y^2z^2 + x^3, x^4 + (-1)y^3z)} = 0$.

Enough for groebner basis. Result is

- $(-1)z^4 + x$
- $(-1)z^5 + y$
- $(-1)xz + y$
- $yz^3 + (-1)x^2$

- $(-1)y^2z^2 + x^3$
- $x^4 + (-1)y^3z$

. ■ Minimalizes groebner basis

- $(-1)z^4 + x$
- $(-1)z^5 + y$
- $(-1)xz + y$
- $yz^3 + (-1)x^2$
- $(-1)y^2z^2 + x^3$
- $x^4 + (-1)y^3z$

.

$(-1)z^5 + y$ is removed by $(-1)z^4 + x$.
Minimalized groebner basis is

- $z^4 + (-1)x$
- $xz + (-1)y$
- $yz^3 + (-1)x^2$
- $y^2z^2 + (-1)x^3$
- $x^4 + (-1)y^3z$

. ■

Reduce groebner basis

- $z^4 + (-1)x$
- $xz + (-1)y$
- $yz^3 + (-1)x^2$
- $y^2z^2 + (-1)x^3$
- $x^4 + (-1)y^3z$

.

Reducing: $\overline{z^4 + (-1)x} = z^4 + (-1)x$.
Reducing: $\overline{xz + (-1)y} = xz + (-1)y$.
Reducing: $\overline{yz^3 + (-1)x^2} = yz^3 + (-1)x^2$.
Reducing: $\overline{y^2z^2 + (-1)x^3} = y^2z^2 + (-1)x^3$.
Reducing: $\overline{x^4 + (-1)y^3z} = x^4 + (-1)y^3z$.
Reduced groebner basis is

- $x^4 + (-1)y^3z$
- $y^2z^2 + (-1)x^3$
- $yz^3 + (-1)x^2$
- $xz + (-1)y$
- $z^4 + (-1)x$

. ■