1 are

Calculates groebner basis of

- $(-1)x^2 + x + y$
- $(-1)x^2 + z$
- $3x^2 + (-1)x + w$

.

$\overline{S((-1)x^2 + x + y, (-1)x^2 + z)} = (-1)x + (-1)y + z.$
$\overline{S((-1)x^2 + x + y, 3x^2 + (-1)x + w)} = (\frac{-2}{3})x + (-1)y + (\frac{-1}{3})w.$
$\overline{S((-1)x^2 + z, 3x^2 + (-1)x + w)} = \frac{1}{3}x + (-1)z + (\frac{-1}{3})w.$
Not enough. Appends

- $(-1)x + (-1)y + z$

.

$\overline{S((-1)x^2 + x + y, (-1)x + (-1)y + z)} = y^2 + (-2)yz + z^2 + (-1)z.$
$\overline{S((-1)x^2 + z, (-1)x + (-1)y + z)} = y^2 + (-2)yz + z^2 + (-1)z.$
$\overline{S(3x^2 + (-1)x + w, (-1)x + (-1)y + z)} = y^2 + (-2)yz + \frac{1}{3}y + z^2 + (\frac{-1}{3})z + \frac{1}{3}w.$
Not enough. Appends

- $y^2 + (-2)yz + z^2 + (-1)z$

.

$\overline{S((-1)x^2 + x + y, y^2 + (-2)yz + z^2 + (-1)z)} = 0.$
$\overline{S((-1)x^2 + z, y^2 + (-2)yz + z^2 + (-1)z)} = 0.$
$\overline{S(3x^2 + (-1)x + w, y^2 + (-2)yz + z^2 + (-1)z)} = \frac{7}{3}yz^2 + \frac{2}{3}yzw + \frac{1}{3}yz + (\frac{-4}{3})z^3 + (\frac{-1}{3})z^2w + \frac{4}{3}z^2 + \frac{1}{3}zw.$
$\overline{S((-1)x + (-1)y + z, y^2 + (-2)yz + z^2 + (-1)z)} = 0.$
Not enough. Appends

- $\frac{7}{3}yz^2 + \frac{2}{3}yzw + \frac{1}{3}yz + (\frac{-4}{3})z^3 + (\frac{-1}{3})z^2w + \frac{4}{3}z^2 + \frac{1}{3}zw$

.

$\overline{S((-1)x^2 + x + y, \frac{7}{3}yz^2 + \frac{2}{3}yzw + \frac{1}{3}yz + (\frac{-4}{3})z^3 + (\frac{-1}{3})z^2w + \frac{4}{3}z^2 + \frac{1}{3}zw)} = 0.$
$\overline{S((-1)x^2 + z, \frac{7}{3}yz^2 + \frac{2}{3}yzw + \frac{1}{3}yz + (\frac{-4}{3})z^3 + (\frac{-1}{3})z^2w + \frac{4}{3}z^2 + \frac{1}{3}zw)} = 0.$
$\overline{S(3x^2 + (-1)x + w, \frac{7}{3}yz^2 + \frac{2}{3}yzw + \frac{1}{3}yz + (\frac{-4}{3})z^3 + (\frac{-1}{3})z^2w + \frac{4}{3}z^2 + \frac{1}{3}zw)} = \frac{2}{147}yzw^2 + \frac{3}{49}yzw + \frac{4}{147}yz + \frac{3}{7}z^4 + \frac{8}{49}z^3w + (\frac{-79}{147})z^3 + (\frac{-1}{147})z^2w^2 + (\frac{-4}{21})z^2w + \frac{16}{147}z^2 + \frac{1}{147}zw^2 + \frac{4}{147}zw.$
$\overline{S((-1)x + (-1)y + z, \frac{7}{3}yz^2 + \frac{2}{3}yzw + \frac{1}{3}yz + (\frac{-4}{3})z^3 + (\frac{-1}{3})z^2w + \frac{4}{3}z^2 + \frac{1}{3}zw)} = (\frac{-2}{343})yzw^2 + (\frac{-16}{343})yzw + (\frac{-32}{343})yz + (\frac{-9}{49})z^4 + (\frac{-24}{343})z^3w + \frac{142}{343}z^3 + \frac{1}{343}z^2w^2 + \frac{8}{49}z^2w + (\frac{-79}{343})z^2 + (\frac{-1}{343})zw^2 + (\frac{-32}{343})zw.$
$\overline{S(y^2 + (-2)yz + z^2 + (-1)z, \frac{7}{3}yz^2 + \frac{2}{3}yzw + \frac{1}{3}yz + (\frac{-4}{3})z^3 + (\frac{-1}{3})z^2w + \frac{4}{3}z^2 + \frac{1}{3}zw)} = \frac{2}{343}yzw^2 + \frac{16}{343}yzw + \frac{32}{343}yz + \frac{9}{49}z^4 + \frac{24}{343}z^3w + (\frac{-142}{343})z^3 + (\frac{-1}{343})z^2w^2 + (\frac{-8}{49})z^2w + \frac{79}{343}z^2 + \frac{1}{343}zw^2 + \frac{32}{343}zw.$
Not enough. Appends

- $\frac{2}{147}yzw^2+\frac{3}{49}yzw+\frac{4}{147}yz+\frac{3}{7}z^4+\frac{8}{49}z^3w+(\frac{-79}{147})z^3+(\frac{-1}{147})z^2w^2+(\frac{-4}{21})z^2w+\frac{16}{147}z^2+\frac{1}{147}zw^2+\frac{4}{147}zw$

.

$\overline{S((-1)x^2+x+y, \frac{2}{147}yzw^2+\frac{3}{49}yzw+\frac{4}{147}yz+\frac{3}{7}z^4+\frac{8}{49}z^3w+(\frac{-79}{147})z^3+(\frac{-1}{147})z^2w^2+(\frac{-4}{21})z^2w+\frac{16}{147}z^2+\frac{1}{147}zw^2+}$ $(\frac{-63}{2})z^5+(-21)z^4w+35z^4+(\frac{-7}{2})z^3w^2+21z^3w+(\frac{-7}{2})z^3+\frac{7}{2}z^2w^2.$

$\overline{S((-1)x^2+z, \frac{2}{147}yzw^2+\frac{3}{49}yzw+\frac{4}{147}yz+\frac{3}{7}z^4+\frac{8}{49}z^3w+(\frac{-79}{147})z^3+(\frac{-1}{147})z^2w^2+(\frac{-4}{21})z^2w+\frac{16}{147}z^2+\frac{1}{147}zw^2+\frac{4}{147}zw}$ $(\frac{-63}{2})z^5+(-21)z^4w+35z^4+(\frac{-7}{2})z^3w^2+21z^3w+(\frac{-7}{2})z^3+\frac{7}{2}z^2w^2.$

$\overline{S(3x^2+(-1)x+w, \frac{2}{147}yzw^2+\frac{3}{49}yzw+\frac{4}{147}yz+\frac{3}{7}z^4+\frac{8}{49}z^3w+(\frac{-79}{147})z^3+(\frac{-1}{147})z^2w^2+(\frac{-4}{21})z^2w+\frac{16}{147}z^2+\frac{1}{147}zw^2+}$ $(\frac{-1}{84})yzw+(\frac{-1}{21})yz+(\frac{-63}{2})z^5+(\frac{-39}{2})z^4w+\frac{137}{4}z^4+(\frac{-5}{2})z^3w^2+\frac{113}{6}z^3w+(\frac{-215}{84})z^3+\frac{1}{6}z^2w^3+\frac{29}{12}z^2w^2+$ $\frac{5}{7}z^2w+(\frac{-4}{21})z^2+(\frac{-1}{6})zw^3+\frac{1}{12}zw^2+(\frac{-1}{21})zw.$

$\overline{S((-1)x+(-1)y+z, \frac{2}{147}yzw^2+\frac{3}{49}yzw+\frac{4}{147}yz+\frac{3}{7}z^4+\frac{8}{49}z^3w+(\frac{-79}{147})z^3+(\frac{-1}{147})z^2w^2+(\frac{-4}{21})z^2w+\frac{16}{147}z^2+\frac{1}{147}zw^2+}$ $(\frac{-1}{4})yzw+(-1)yz+(\frac{-27}{2})z^5+(-9)z^4w+\frac{69}{4}z^4+(\frac{-3}{2})z^3w^2+\frac{21}{2}z^3w+(\frac{-7}{4})z^3+\frac{7}{4}z^2w^2+(\frac{-1}{2})z^2w+$ $(-2)z^2+(\frac{-1}{4})zw^2+(-1)zw.$

$\overline{S(y^2+(-2)yz+z^2+(-1)z, \frac{2}{147}yzw^2+\frac{3}{49}yzw+\frac{4}{147}yz+\frac{3}{7}z^4+\frac{8}{49}z^3w+(\frac{-79}{147})z^3+(\frac{-1}{147})z^2w^2+(\frac{-4}{21})z^2w+\frac{16}{147}z^2+}$ $\frac{1}{4}yzw+yz+(-18)z^5+(-12)z^4w+\frac{71}{4}z^4+(-2)z^3w^2+\frac{21}{2}z^3w+(\frac{-7}{4})z^3+\frac{7}{4}z^2w^2+\frac{1}{2}z^2w+2z^2+\frac{1}{4}zw^2+zw.$

$\overline{S(\frac{7}{3}yz^2+\frac{2}{3}yzw+\frac{1}{3}yz+(\frac{-4}{3})z^3+(\frac{-1}{3})z^2w+\frac{4}{3}z^2+\frac{1}{3}zw, \frac{2}{147}yzw^2+\frac{3}{49}yzw+\frac{4}{147}yz+\frac{3}{7}z^4+\frac{8}{49}z^3w+(\frac{-79}{147})z^3+(\frac{-1}{147})}$ $(\frac{-63}{2})z^5+(-21)z^4w+35z^4+(\frac{-7}{2})z^3w^2+21z^3w+(\frac{-7}{2})z^3+\frac{7}{2}z^2w^2.$

Not enough. Appends

- $(\frac{-63}{2})z^5+(-21)z^4w+35z^4+(\frac{-7}{2})z^3w^2+21z^3w+(\frac{-7}{2})z^3+\frac{7}{2}z^2w^2$

.

$\overline{S((-1)x^2+x+y, (\frac{-63}{2})z^5+(-21)z^4w+35z^4+(\frac{-7}{2})z^3w^2+21z^3w+(\frac{-7}{2})z^3+\frac{7}{2}z^2w^2)}=0.$

$\overline{S((-1)x^2+z, (\frac{-63}{2})z^5+(-21)z^4w+35z^4+(\frac{-7}{2})z^3w^2+21z^3w+(\frac{-7}{2})z^3+\frac{7}{2}z^2w^2)}=0.$

$\overline{S(3x^2+(-1)x+w, (\frac{-63}{2})z^5+(-21)z^4w+35z^4+(\frac{-7}{2})z^3w^2+21z^3w+(\frac{-7}{2})z^3+\frac{7}{2}z^2w^2)}=(\frac{-2}{21})yzw+$ $(\frac{-1}{21})yz+\frac{6}{7}z^3+\frac{8}{21}z^2w+(\frac{-4}{21})z^2+(\frac{-1}{21})zw.$

$\overline{S((-1)x+(-1)y+z, (\frac{-63}{2})z^5+(-21)z^4w+35z^4+(\frac{-7}{2})z^3w^2+21z^3w+(\frac{-7}{2})z^3+\frac{7}{2}z^2w^2)}=0.$

$\overline{S(y^2+(-2)yz+z^2+(-1)z, (\frac{-63}{2})z^5+(-21)z^4w+35z^4+(\frac{-7}{2})z^3w^2+21z^3w+(\frac{-7}{2})z^3+\frac{7}{2}z^2w^2)} =$ $0.$

$\overline{S(\frac{7}{3}yz^2+\frac{2}{3}yzw+\frac{1}{3}yz+(\frac{-4}{3})z^3+(\frac{-1}{3})z^2w+\frac{4}{3}z^2+\frac{1}{3}zw, (\frac{-63}{2})z^5+(-21)z^4w+35z^4+(\frac{-7}{2})z^3w^2+21z^3w+(\frac{-7}{2})z^3+}$ $0.$

$\overline{S(\frac{2}{147}yzw^2+\frac{3}{49}yzw+\frac{4}{147}yz+\frac{3}{7}z^4+\frac{8}{49}z^3w+(\frac{-79}{147})z^3+(\frac{-1}{147})z^2w^2+(\frac{-4}{21})z^2w+\frac{16}{147}z^2+\frac{1}{147}zw^2+\frac{4}{147}zw, (\frac{-63}{2})z^5+(}$ $0.$

Not enough. Appends

- $(\frac{-2}{21})yzw+(\frac{-1}{21})yz+\frac{6}{7}z^3+\frac{8}{21}z^2w+(\frac{-4}{21})z^2+(\frac{-1}{21})zw$

.

$\overline{S((-1)x^2+x+y, (\frac{-2}{21})yzw+(\frac{-1}{21})yz+\frac{6}{7}z^3+\frac{8}{21}z^2w+(\frac{-4}{21})z^2+(\frac{-1}{21})zw)}=0.$

$\overline{S((-1)x^2+z, (\frac{-2}{21})yzw+(\frac{-1}{21})yz+\frac{6}{7}z^3+\frac{8}{21}z^2w+(\frac{-4}{21})z^2+(\frac{-1}{21})zw)}=0.$

$\overline{S(3x^2+(-1)x+w, (\frac{-2}{21})yzw+(\frac{-1}{21})yz+\frac{6}{7}z^3+\frac{8}{21}z^2w+(\frac{-4}{21})z^2+(\frac{-1}{21})zw)} = \frac{1}{12}yz+\frac{3}{2}z^4+z^3w+$ $(\frac{-5}{3})z^3+\frac{1}{6}z^2w^2+(-1)z^2w+\frac{1}{3}z^2+(\frac{-1}{6})zw^2+\frac{1}{12}zw.$

$\overline{S((-1)x+(-1)y+z,(\frac{-2}{21})yzw+(\frac{-1}{21})yz+\frac{6}{7}z^3+\frac{8}{21}z^2w+(\frac{-4}{21})z^2+(\frac{-1}{21})zw)} = (\frac{-1}{4})yz + (\frac{-1}{2})z^2 + (\frac{-1}{4})zw.$

$\overline{S(y^2+(-2)yz+z^2+(-1)z,(\frac{-2}{21})yzw+(\frac{-1}{21})yz+\frac{6}{7}z^3+\frac{8}{21}z^2w+(\frac{-4}{21})z^2+(\frac{-1}{21})zw)} = \frac{1}{4}yz + \frac{1}{2}z^2 + \frac{1}{4}zw.$

$\overline{S(\frac{7}{3}yz^2+\frac{2}{3}yzw+\frac{1}{3}yz+(\frac{-4}{3})z^3+(\frac{-1}{3})z^2w+\frac{4}{3}z^2+\frac{1}{3}zw,(\frac{-2}{21})yzw+(\frac{-1}{21})yz+\frac{6}{7}z^3+\frac{8}{21}z^2w+(\frac{-4}{21})z^2+(\frac{-1}{21})zw)} = 0.$

$\overline{S(\frac{2}{147}yzw^2+\frac{3}{49}yzw+\frac{4}{147}yz+\frac{3}{7}z^4+\frac{8}{49}z^3w+(\frac{-79}{147})z^3+(\frac{-1}{147})z^2w^2+(\frac{-4}{21})z^2w+\frac{16}{147}z^2+\frac{1}{147}zw^2+\frac{4}{147}zw,(\frac{-2}{21})yzw+}$
$\frac{63}{2}z^4+21z^3w+(\frac{-7}{2})z^3+\frac{7}{2}z^2w^2.$

$\overline{S((\frac{-63}{2})z^5+(-21)z^4w+35z^4+(\frac{-7}{2})z^3w^2+21z^3w+(\frac{-7}{2})z^3+\frac{7}{2}z^2w^2,(\frac{-2}{21})yzw+(\frac{-1}{21})yz+\frac{6}{7}z^3+\frac{8}{21}z^2w+(\frac{-4}{21})z^2+}$
$0.$

Not enough. Appends

- $\frac{1}{12}yz+\frac{3}{2}z^4+z^3w+(\frac{-5}{3})z^3+\frac{1}{6}z^2w^2+(-1)z^2w+\frac{1}{3}z^2+(\frac{-1}{6})zw^2+\frac{1}{12}zw$

.

$\overline{S((-1)x^2+x+y,\frac{1}{12}yz+\frac{3}{2}z^4+z^3w+(\frac{-5}{3})z^3+\frac{1}{6}z^2w^2+(-1)z^2w+\frac{1}{3}z^2+(\frac{-1}{6})zw^2+\frac{1}{12}zw)} = 0.$

$\overline{S((-1)x^2+z,\frac{1}{12}yz+\frac{3}{2}z^4+z^3w+(\frac{-5}{3})z^3+\frac{1}{6}z^2w^2+(-1)z^2w+\frac{1}{3}z^2+(\frac{-1}{6})zw^2+\frac{1}{12}zw)} = 0.$

$\overline{S(3x^2+(-1)x+w,\frac{1}{12}yz+\frac{3}{2}z^4+z^3w+(\frac{-5}{3})z^3+\frac{1}{6}z^2w^2+(-1)z^2w+\frac{1}{3}z^2+(\frac{-1}{6})zw^2+\frac{1}{12}zw)} = 3z^4+2z^3w+(\frac{-10}{3})z^3+\frac{1}{3}z^2w^2+(-2)z^2w+\frac{1}{3}z^2+(\frac{-1}{3})zw^2.$

$\overline{S((-1)x+(-1)y+z,\frac{1}{12}yz+\frac{3}{2}z^4+z^3w+(\frac{-5}{3})z^3+\frac{1}{6}z^2w^2+(-1)z^2w+\frac{1}{3}z^2+(\frac{-1}{6})zw^2+\frac{1}{12}zw)} = 9z^4+6z^3w+(-10)z^3+z^2w^2+(-6)z^2w+z^2+(-1)zw^2.$

$\overline{S(y^2+(-2)yz+z^2+(-1)z,\frac{1}{12}yz+\frac{3}{2}z^4+z^3w+(\frac{-5}{3})z^3+\frac{1}{6}z^2w^2+(-1)z^2w+\frac{1}{3}z^2+(\frac{-1}{6})zw^2+\frac{1}{12}zw)} = (-9)z^4+(-6)z^3w+10z^3+(-1)z^2w^2+6z^2w+(-1)z^2+zw^2.$

$\overline{S(\frac{7}{3}yz^2+\frac{2}{3}yzw+\frac{1}{3}yz+(\frac{-4}{3})z^3+(\frac{-1}{3})z^2w+\frac{4}{3}z^2+\frac{1}{3}zw,\frac{1}{12}yz+\frac{3}{2}z^4+z^3w+(\frac{-5}{3})z^3+\frac{1}{6}z^2w^2+(-1)z^2w+\frac{1}{3}z^2+(}$
$0.$

$\overline{S(\frac{2}{147}yzw^2+\frac{3}{49}yzw+\frac{4}{147}yz+\frac{3}{7}z^4+\frac{8}{49}z^3w+(\frac{-79}{147})z^3+(\frac{-1}{147})z^2w^2+(\frac{-4}{21})z^2w+\frac{16}{147}z^2+\frac{1}{147}zw^2+\frac{4}{147}zw,\frac{1}{12}yz+\frac{3}{2}z^4}$
$(-18)z^4w^2+36z^4+(-12)z^3w^3+20z^3w^2+15z^3w+(-4)z^3+(-2)z^2w^4+12z^2w^3+(-4)z^2w^2+z^2w+2zw^4+(-1)zw^3.$

$\overline{S((\frac{-63}{2})z^5+(-21)z^4w+35z^4+(\frac{-7}{2})z^3w^2+21z^3w+(\frac{-7}{2})z^3+\frac{7}{2}z^2w^2,\frac{1}{12}yz+\frac{3}{2}z^4+z^3w+(\frac{-5}{3})z^3+\frac{1}{6}z^2w^2+(-1)z^2}$
$0.$

$\overline{S((\frac{-2}{21})yzw+(\frac{-1}{21})yz+\frac{6}{7}z^3+\frac{8}{21}z^2w+(\frac{-4}{21})z^2+(\frac{-1}{21})zw,\frac{1}{12}yz+\frac{3}{2}z^4+z^3w+(\frac{-5}{3})z^3+\frac{1}{6}z^2w^2+(-1)z^2w+\frac{1}{3}z^2+(}$
$(-18)z^4w+(-9)z^4+(-12)z^3w^2+14z^3w+z^3+(-2)z^2w^3+11z^2w^2+(-2)z^2w+2zw^3.$

Not enough. Appends

- $3z^4+2z^3w+(\frac{-10}{3})z^3+\frac{1}{3}z^2w^2+(-2)z^2w+\frac{1}{3}z^2+(\frac{-1}{3})zw^2$

.

$\overline{S((-1)x^2+x+y,3z^4+2z^3w+(\frac{-10}{3})z^3+\frac{1}{3}z^2w^2+(-2)z^2w+\frac{1}{3}z^2+(\frac{-1}{3})zw^2)} = 0.$

$\overline{S((-1)x^2+z,3z^4+2z^3w+(\frac{-10}{3})z^3+\frac{1}{3}z^2w^2+(-2)z^2w+\frac{1}{3}z^2+(\frac{-1}{3})zw^2)} = 0.$

$\overline{S(3x^2+(-1)x+w,3z^4+2z^3w+(\frac{-10}{3})z^3+\frac{1}{3}z^2w^2+(-2)z^2w+\frac{1}{3}z^2+(\frac{-1}{3})zw^2)} = 0.$

$\overline{S((-1)x+(-1)y+z,3z^4+2z^3w+(\frac{-10}{3})z^3+\frac{1}{3}z^2w^2+(-2)z^2w+\frac{1}{3}z^2+(\frac{-1}{3})zw^2)} = 0.$

$\overline{S(y^2+(-2)yz+z^2+(-1)z, 3z^4+2z^3w+(\frac{-10}{3})z^3+\frac{1}{3}z^2w^2+(-2)z^2w+\frac{1}{3}z^2+(\frac{-1}{3})zw^2)} = 0.$

$\overline{S(\frac{7}{3}yz^2+\frac{2}{3}yzw+\frac{1}{3}yz+(\frac{-4}{3})z^3+(\frac{-1}{3})z^2w+\frac{4}{3}z^2+\frac{1}{3}zw, 3z^4+2z^3w+(\frac{-10}{3})z^3+\frac{1}{3}z^2w^2+(-2)z^2w+\frac{1}{3}z^2+(\frac{-1}{3})zw^2}$

0.

$\overline{S(\frac{2}{147}yzw^2+\frac{3}{49}yzw+\frac{4}{147}yz+\frac{3}{7}z^4+\frac{8}{49}z^3w+(\frac{-79}{147})z^3+(\frac{-1}{147})z^2w^2+(\frac{-4}{21})z^2w+\frac{16}{147}z^2+\frac{1}{147}zw^2+\frac{4}{147}zw, 3z^4+2z^3w}$

0.

$\overline{S((\frac{-63}{2})z^5+(-21)z^4w+35z^4+(\frac{-7}{2})z^3w^2+21z^3w+(\frac{-7}{2})z^3+\frac{7}{2}z^2w^2, 3z^4+2z^3w+(\frac{-10}{3})z^3+\frac{1}{3}z^2w^2+(-2)z^2w+\frac{1}{3}}$

0.

$\overline{S((\frac{-2}{21})yzw+(\frac{-1}{21})yz+\frac{6}{7}z^3+\frac{8}{21}z^2w+(\frac{-4}{21})z^2+(\frac{-1}{21})zw, 3z^4+2z^3w+(\frac{-10}{3})z^3+\frac{1}{3}z^2w^2+(-2)z^2w+\frac{1}{3}z^2+(\frac{-1}{3})zw^2)}$

0.

$\overline{S(\frac{1}{12}yz+\frac{3}{2}z^4+z^3w+(\frac{-5}{3})z^3+\frac{1}{6}z^2w^2+(-1)z^2w+\frac{1}{3}z^2+(\frac{-1}{6})zw^2+\frac{1}{12}zw, 3z^4+2z^3w+(\frac{-10}{3})z^3+\frac{1}{3}z^2w^2+(-2)z^2}$

0.

Enough for groebner basis. Result is

- $(-1)x^2+x+y$
- $(-1)x^2+z$
- $3x^2+(-1)x+w$
- $(-1)x+(-1)y+z$
- $y^2+(-2)yz+z^2+(-1)z$
- $\frac{7}{3}yz^2+\frac{2}{3}yzw+\frac{1}{3}yz+(\frac{-4}{3})z^3+(\frac{-1}{3})z^2w+\frac{4}{3}z^2+\frac{1}{3}zw$
- $\frac{2}{147}yzw^2+\frac{3}{49}yzw+\frac{4}{147}yz+\frac{3}{7}z^4+\frac{8}{49}z^3w+(\frac{-79}{147})z^3+(\frac{-1}{147})z^2w^2+(\frac{-4}{21})z^2w+\frac{16}{147}z^2+\frac{1}{147}zw^2+\frac{4}{147}zw$
- $(\frac{-63}{2})z^5+(-21)z^4w+35z^4+(\frac{-7}{2})z^3w^2+21z^3w+(\frac{-7}{2})z^3+\frac{7}{2}z^2w^2$
- $(\frac{-2}{21})yzw+(\frac{-1}{21})yz+\frac{6}{7}z^3+\frac{8}{21}z^2w+(\frac{-4}{21})z^2+(\frac{-1}{21})zw$
- $\frac{1}{12}yz+\frac{3}{2}z^4+z^3w+(\frac{-5}{3})z^3+\frac{1}{6}z^2w^2+(-1)z^2w+\frac{1}{3}z^2+(\frac{-1}{6})zw^2+\frac{1}{12}zw$
- $3z^4+2z^3w+(\frac{-10}{3})z^3+\frac{1}{3}z^2w^2+(-2)z^2w+\frac{1}{3}z^2+(\frac{-1}{3})zw^2$

. ■ Minimalizes groebner basis

- $(-1)x^2+x+y$
- $(-1)x^2+z$
- $3x^2+(-1)x+w$
- $(-1)x+(-1)y+z$
- $y^2+(-2)yz+z^2+(-1)z$
- $\frac{7}{3}yz^2+\frac{2}{3}yzw+\frac{1}{3}yz+(\frac{-4}{3})z^3+(\frac{-1}{3})z^2w+\frac{4}{3}z^2+\frac{1}{3}zw$
- $\frac{2}{147}yzw^2+\frac{3}{49}yzw+\frac{4}{147}yz+\frac{3}{7}z^4+\frac{8}{49}z^3w+(\frac{-79}{147})z^3+(\frac{-1}{147})z^2w^2+(\frac{-4}{21})z^2w+\frac{16}{147}z^2+\frac{1}{147}zw^2+\frac{4}{147}zw$
- $(\frac{-63}{2})z^5+(-21)z^4w+35z^4+(\frac{-7}{2})z^3w^2+21z^3w+(\frac{-7}{2})z^3+\frac{7}{2}z^2w^2$
- $(\frac{-2}{21})yzw+(\frac{-1}{21})yz+\frac{6}{7}z^3+\frac{8}{21}z^2w+(\frac{-4}{21})z^2+(\frac{-1}{21})zw$
- $\frac{1}{12}yz+\frac{3}{2}z^4+z^3w+(\frac{-5}{3})z^3+\frac{1}{6}z^2w^2+(-1)z^2w+\frac{1}{3}z^2+(\frac{-1}{6})zw^2+\frac{1}{12}zw$
- $3z^4+2z^3w+(\frac{-10}{3})z^3+\frac{1}{3}z^2w^2+(-2)z^2w+\frac{1}{3}z^2+(\frac{-1}{3})zw^2$

.

$(-1)x^2+x+y$ is removed by $(-1)x^2+z$.

$(-1)x^2 + z$ is removed by $3x^2 + (-1)x + w$.

$3x^2 + (-1)x + w$ is removed by $(-1)x + (-1)y + z$.

$\frac{7}{3}yz^2 + \frac{2}{3}yzw + \frac{1}{3}yz + (\frac{-4}{3})z^3 + (\frac{-1}{3})z^2w + \frac{4}{3}z^2 + \frac{1}{3}zw$ is removed by $\frac{1}{12}yz + \frac{3}{2}z^4 + z^3w + (\frac{-5}{3})z^3 + \frac{1}{6}z^2w^2 + (-1)z^2w + \frac{1}{3}z^2 + (\frac{-1}{6})zw^2 + \frac{1}{12}zw$.

$\frac{2}{147}yzw^2 + \frac{3}{49}yzw + \frac{4}{147}yz + \frac{3}{7}z^4 + \frac{8}{49}z^3w + (\frac{-79}{147})z^3 + (\frac{-1}{147})z^2w^2 + (\frac{-4}{21})z^2w + \frac{16}{147}z^2 + \frac{1}{147}zw^2 + \frac{4}{147}zw$ is removed by $(\frac{-2}{21})yzw + (\frac{-1}{21})yz + \frac{6}{7}z^3 + \frac{8}{21}z^2w + (\frac{-4}{21})z^2 + (\frac{-1}{21})zw$.

$(\frac{-63}{2})z^5 + (-21)z^4w + 35z^4 + (\frac{-7}{2})z^3w^2 + 21z^3w + (\frac{-7}{2})z^3 + \frac{7}{2}z^2w^2$ is removed by $3z^4 + 2z^3w + (\frac{-10}{3})z^3 + \frac{1}{3}z^2w^2 + (-2)z^2w + \frac{1}{3}z^2 + (\frac{-1}{3})zw^2$.

$(\frac{-2}{21})yzw + (\frac{-1}{21})yz + \frac{6}{7}z^3 + \frac{8}{21}z^2w + (\frac{-4}{21})z^2 + (\frac{-1}{21})zw$ is removed by $\frac{1}{12}yz + \frac{3}{2}z^4 + z^3w + (\frac{-5}{3})z^3 + \frac{1}{6}z^2w^2 + (-1)z^2w + \frac{1}{3}z^2 + (\frac{-1}{6})zw^2 + \frac{1}{12}zw$.

Minimalized groebner basis is

- $x + y + (-1)z$
- $y^2 + (-2)yz + z^2 + (-1)z$
- $yz + 18z^4 + 12z^3w + (-20)z^3 + 2z^2w^2 + (-12)z^2w + 4z^2 + (-2)zw^2 + zw$
- $z^4 + \frac{2}{3}z^3w + (\frac{-10}{9})z^3 + \frac{1}{9}z^2w^2 + (\frac{-2}{3})z^2w + \frac{1}{9}z^2 + (\frac{-1}{9})zw^2$

. ■

Reduce groebner basis

- $x + y + (-1)z$
- $y^2 + (-2)yz + z^2 + (-1)z$
- $yz + 18z^4 + 12z^3w + (-20)z^3 + 2z^2w^2 + (-12)z^2w + 4z^2 + (-2)zw^2 + zw$
- $z^4 + \frac{2}{3}z^3w + (\frac{-10}{9})z^3 + \frac{1}{9}z^2w^2 + (\frac{-2}{3})z^2w + \frac{1}{9}z^2 + (\frac{-1}{9})zw^2$

.

Reducing: $\overline{x + y + (-1)z} = x + y + (-1)z$.

Reducing: $\overline{y^2 + (-2)yz + z^2 + (-1)z} = y^2 + 5z^2 + 2zw + (-1)z$.

Reducing: $\overline{yz + 18z^4 + 12z^3w + (-20)z^3 + 2z^2w^2 + (-12)z^2w + 4z^2 + (-2)zw^2 + zw} = yz + 2z^2 + zw$.

Reducing: $\overline{z^4 + \frac{2}{3}z^3w + (\frac{-10}{9})z^3 + \frac{1}{9}z^2w^2 + (\frac{-2}{3})z^2w + \frac{1}{9}z^2 + (\frac{-1}{9})zw^2} = z^4 + \frac{2}{3}z^3w + (\frac{-10}{9})z^3 + \frac{1}{9}z^2w^2 + (\frac{-2}{3})z^2w + \frac{1}{9}z^2 + (\frac{-1}{9})zw^2$.

Reduced groebner basis is

- $z^4 + \frac{2}{3}z^3w + (\frac{-10}{9})z^3 + \frac{1}{9}z^2w^2 + (\frac{-2}{3})z^2w + \frac{1}{9}z^2 + (\frac{-1}{9})zw^2$
- $yz + 2z^2 + zw$
- $y^2 + 5z^2 + 2zw + (-1)z$
- $x + y + (-1)z$

. ■