Calculates groebner basis of

- $x^3 + (-2)xy$
- $x^2y + (-2)y^2 + x$

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$S(x^3 + (-2)xy, x^2y + (-2)y^2 + x) = (-1)x^2$.
Not enough. Appends

- $(-1)x^2$

.

$S(x^3 + (-2)xy, (-1)x^2) = (-2)xy$.
$S(x^2y + (-2)y^2 + x, (-1)x^2) = (-2)y^2 + x$.
Not enough. Appends

- $(-2)xy$

.

$S(x^3 + (-2)xy, (-2)xy) = 0$.
$S(x^2y + (-2)y^2 + x, (-2)xy) = (-2)y^2 + x$.
$S((-1)x^2, (-2)xy) = 0$.
Not enough. Appends

- $(-2)y^2 + x$

.

$S(x^3 + (-2)xy, (-2)y^2 + x) = 0$.
$S(x^2y + (-2)y^2 + x, (-2)y^2 + x) = 0$.
$S((-1)x^2, (-2)y^2 + x) = 0$.
$S((-2)xy, (-2)y^2 + x) = 0$.
Enough for groebner basis. Result is

- $x^3 + (-2)xy$
- $x^2y + (-2)y^2 + x$
- $(-1)x^2$
- $(-2)xy$
- $(-2)y^2 + x$

. ■ Minimalizes groebner basis

- $x^3 + (-2)xy$
- $x^2y + (-2)y^2 + x$
- $(-1)x^2$
- $(-2)xy$

- $(-2)y^2 + x$

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$x^3 + (-2)xy$ is removed by $(-1)x^2$.

$x^2y + (-2)y^2 + x$ is removed by $(-1)x^2$.

Minimalized groebner basis is

- x^2
- xy
- $y^2 + (\frac{-1}{2})x$

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