

# NUMERICAL-1

on

RESERVATION TABLE

CALCULATE COLLISION VECTOR,  
STATE DIAGRAM,  
GREEDY CYCLES,  
MAL

# Question 1: Consider the following Reservation Table:

→ Time

	1	2	3	4	5	6	7	8
Stages								
$S_1$	X					X		X
$S_2$		X		X				
$S_3$			X		X		X	

Reservation table for function X

1. What are the forbidden latencies?
2. Draw the state transition diagram.
3. List all the simple cycles and greedy cycles.
4. Determine the minimal average latency (MAL).

# 1. What are the forbidden latencies

S1:  $\{(6-1), (8-6), (8-1)\}$  i.e.  $\{5, 2, 7\}$

S2:  $\{(4-2)\}$  i.e.  $\{2\}$

S3:  $\{(5-3), (7-5), (7-3)\}$  i.e.  $\{2, 2, 4\}$

**Forbidden latencies =  $\{2, 4, 5, 7\}$**

> It causes collision

**Permissible Latency =  $\{1, 3, 6, 8^+\}$**

> It does not causes collision

Maximum forbidden latency (m) = 7,

■ So the collision vector is of 7 bits

**Collision Vector** ~~= 8~~  $\{C_7 \ C_6 \ C_5 \ C_4 \ C_3 \ C_2 \ C_1\}$

0 1 0 1 1 0 1 0

1 = forbidden latency

0 = Permissible latency

**Collision Vector = (1011010)**

**Note:** The *last permissible latency of 8* can be *ignored* because we need only 7 bits to represent Collision Vector

→ Time

	1	2	3	4	5	6	7	8
Stages								
S <sub>1</sub>	X					X		X
S <sub>2</sub>		X		X				
S <sub>3</sub>			X		X		X	

Reservation table for function X

**Initial Collision Vector (ICV) = 1011010**

# 2. Draw State Transition diagram

A **state diagram** is constructed from **Collision Vector**. It specifies the **permissible state transitions** among successive initiations based on the **collision vector**.

## How to draw state transition diagram?

- The **Permissible Latency** =  $\{1, 3, 6, 8^+\}$ , so we have to **find** the next states of collision vector for the transitions 1, 3, 6, 8<sup>+</sup>

- **Method:**

To **find** the collision vector of the next state the collision vector of the present state is **shifted right** for each transitions (1, 3, 6, 8<sup>+</sup>). The shifted collision vector of present state is **bitwise-ORed** with Initial Collision Vector (ICV).

**Initial Collision Vector (ICV) = 1011010**



## Step 1: Calculating next state with Present State (PS) = 1011010

Permissible latencies (or transitions) = 1, 3, 6, 8<sup>+</sup>

Initial Collision Vector (ICV) = 1011010

With latency 1:

$$\begin{array}{r} 1011010 \text{ ICV} \\ + \quad 0101101 \text{ After Right Shift PS by 1} \\ \hline 1111111 \text{ Next state} \end{array}$$

➤ So, the new state 1111111 with **latency 1**

With latency 3:

$$\begin{array}{r} 1011010 \text{ ICV} \\ + \quad 0001011 \text{ After Right Shift PS by 3} \\ \hline 1011011 \text{ Next state} \end{array}$$

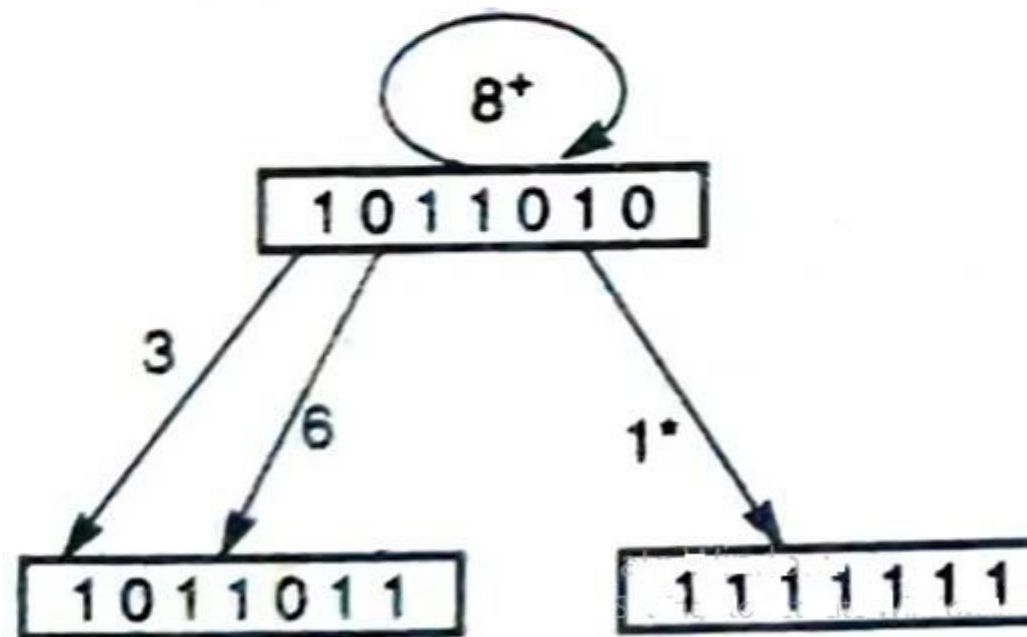
With latency 6:

$$\begin{array}{r} 1011010 \text{ ICV} \\ + \quad 0000001 \text{ After Right Shift PS by 6} \\ \hline 1011011 \text{ Next state} \end{array}$$

□ With latency 8<sup>+</sup>:

$$\begin{array}{r} 1011010 \text{ ICV} \\ + \quad 0000000 \text{ After Right Shift PS by 8} \\ \hline 1011010 \text{ Next state} \end{array}$$

➤ So the **latency 8** returns to **original ICV (1011010)**



Initial State Diagram

Permissible latencies (or transitions) = 3, 6, 8<sup>+</sup>.

□ With latency 3:

Initial Collision Vector (ICV) = 1011010

$$\begin{array}{r}
 1011010 \text{ ICV} \\
 + \quad 0001011 \text{ After right shifting PS by 3} \\
 \hline
 1011011 \text{ Next State}
 \end{array}$$

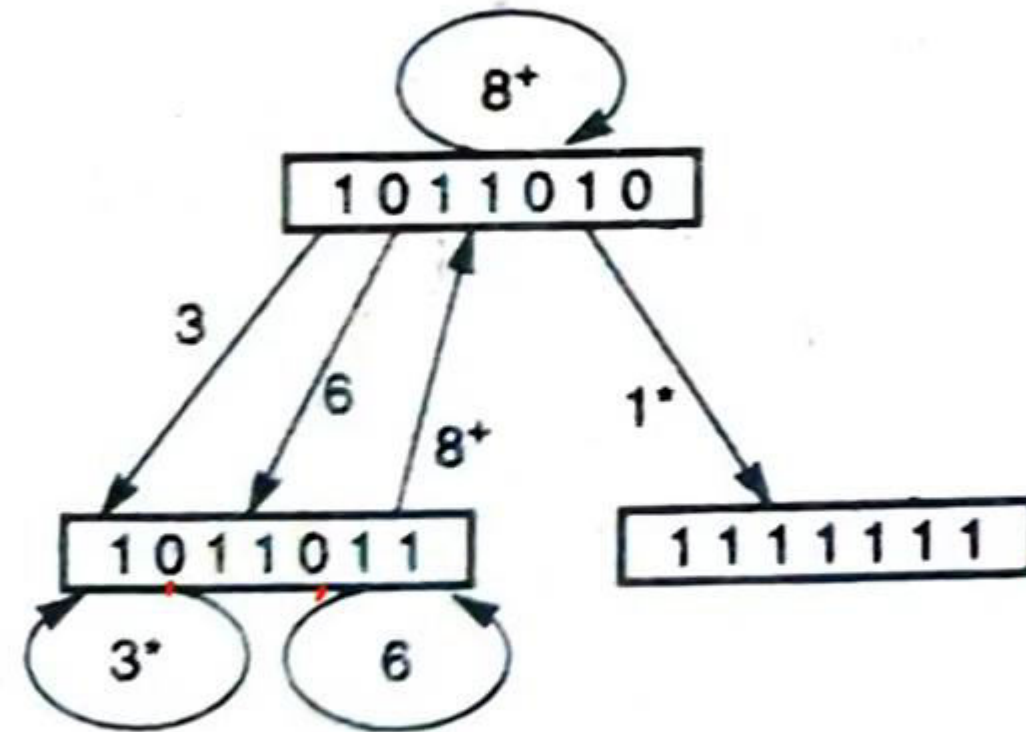
□ With latency 6:

$$\begin{array}{r}
 1011010 \text{ ICV} \\
 + \quad 0000001 \text{ After right shifting PS by 3} \\
 \hline
 1011011 \text{ Next State}
 \end{array}$$

➤ So the **latency 3 & 6** returns to **same state**.

□ With latency 8<sup>+</sup>:

$$\begin{array}{r}
 1011010 \text{ ICV} \\
 + \quad 0000000 \text{ After right shifting PS by 8}^+ \\
 \hline
 1011010 \text{ Next state}
 \end{array}$$



Intermediate State Diagram

➤ So the **latency 8** returns to **original ICV (1011010)**



### Step 3: Calculating Next state with Present State (PS) = 1111111

Permissible latencies (or transitions) =  $8^+$

Initial Collision Vector (ICV) = (1011010)

With latency  $8^+$  :

1011010 ICV

+ 0000000 After right shifting PS by  $8^+$

1011010 Next State

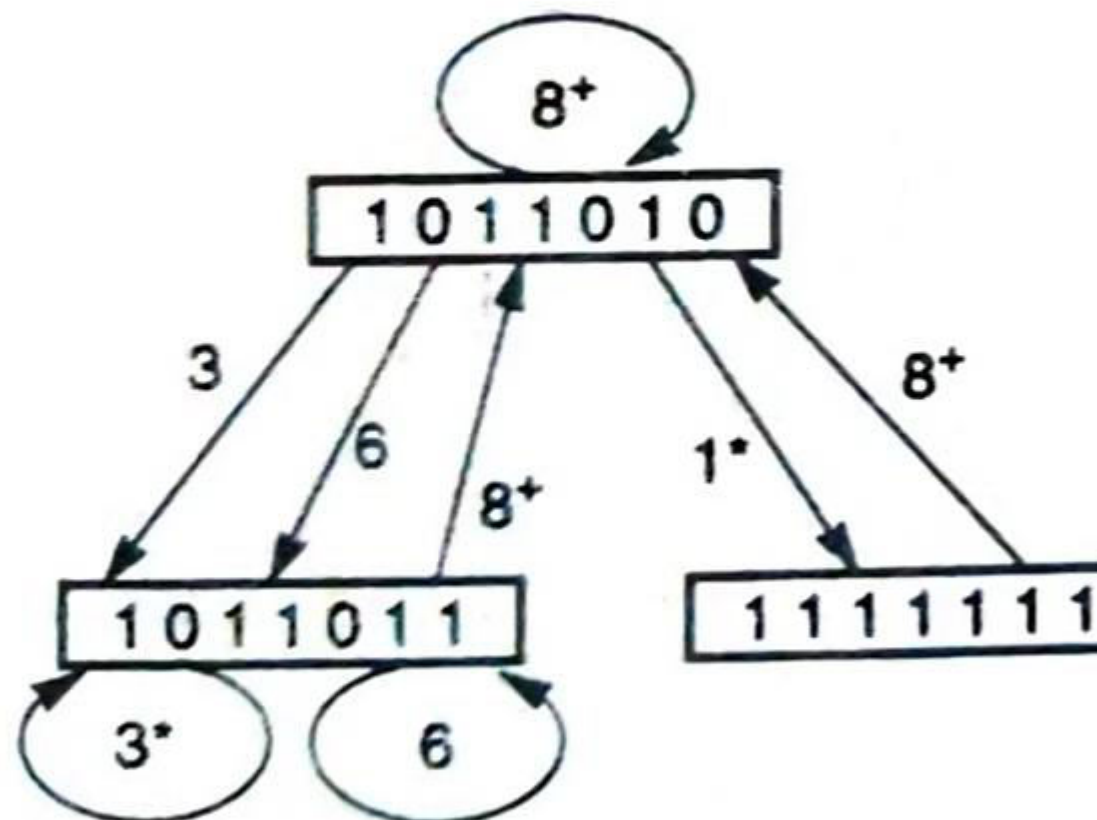
➤ So the **latency 8** returns to **original ICV (1011010)**

Note:

. The **bitwise ORing** of the **shifted version of the present state** with the **initial collision vector (ICV)** is meant to prevent collisions from future initiations.

. When the number of shifts is  $m + 1$  ( $7 + 1 = 8$ ) or greater ( $8^+$ ), all transitions are redirected back to the **initial state**.

- For example, after **eight or more shifts** (denoted as  $8^+$ ), the next  $8^+$  state must be the initial state, regardless of which state the transition starts from.



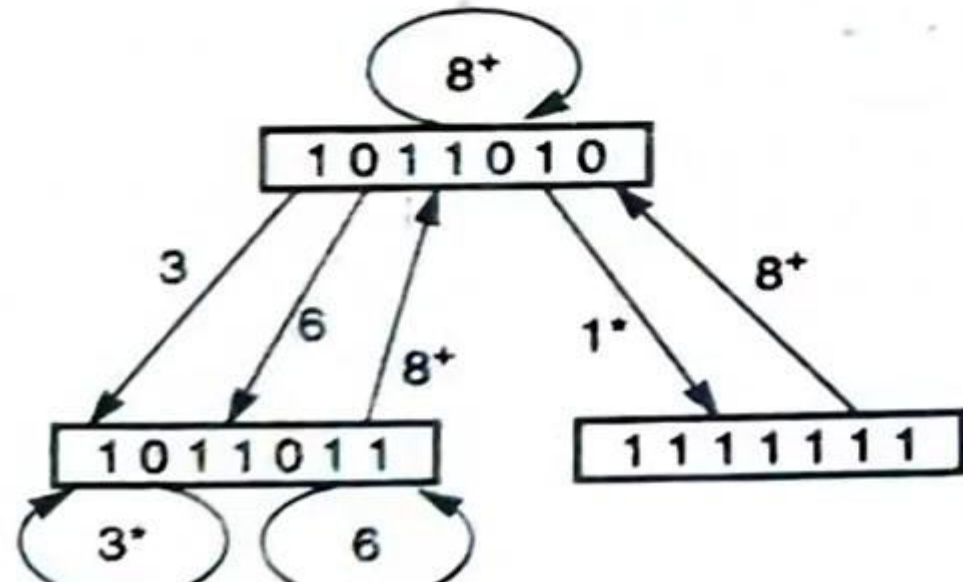
Final State Diagram

# 3. Simple & Greedy Cycles...

**Simple Cycle:** A **Simple Cycle** is a **latency cycle** in which **each state appears only once**.

**Greedy Cycle:** A **Greedy Cycle** is a simple cycle whose edges are all made with **minimum latencies** from their respective starting states.

- ▣ Their **average latencies** must be **lower**



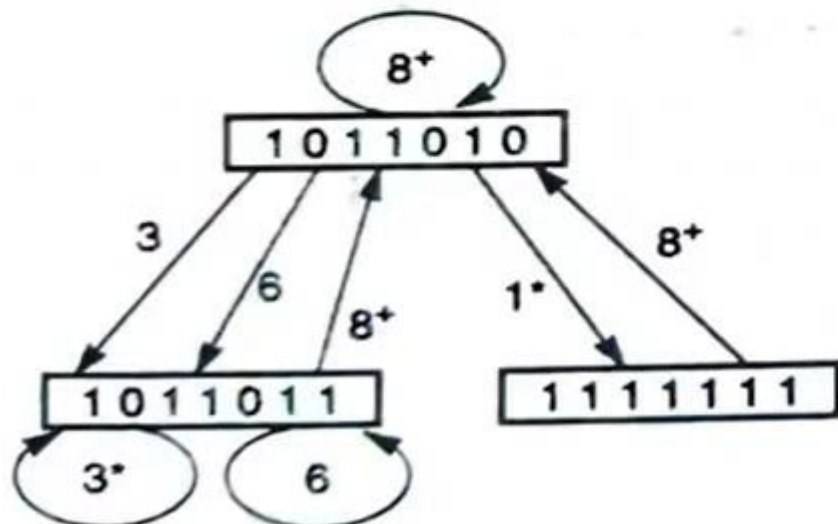
Latency Cycles	Types of cycles	Average Latency
(3)	Simple cycle (Greedy cycle)	3
(6)	Simple cycle	6
(8)	Simple cycle	8
(1,8)	Simple cycle (Greedy cycle)	4.5
(3,8)	Simple cycle	5
(6,8)	Simple cycle	7
(3,6,3)	-----	-----
(3,3,6)	-----	-----
(1,8,3,8)	-----	-----
(1,8,6,8)...	-----	-----



# 4. MAL (Minimum Average Latency)

**MAL (Minimum Average Latency)** is the **minimum average latency** obtained from the **greedy cycle**.

- The minimum-latency edges in the state diagrams are marked with asterisks.
- Atleast one of the greedy cycles will lead to the MAL



Latency Cycles	Types of cycles	Average Latency
(3)	Simple cycle (Greedy cycle)	3 (MAL)
(6)	Simple cycle	6
(8)	Simple cycle	8
(1,8)	Simple cycle (Greedy cycle)	4.5
(3,8)	Simple cycle	5
(6,8)	Simple cycle	7
(3,6,3)	-----	-----
(3,3,6)	-----	-----
(1,8,3,8)	-----	-----
(1,8,6,8)...	-----	-----

## Question 2: Consider the following Reservation Table:

→ Time

	1	2	3	4	5	6	7	8	9
S1	X								X
S2		X	X					X	
S3				X					
S4					X	X			
S5							X	X	

Stages

1. What are the forbidden latencies and initial collision vector?
2. Draw the state transition diagram for scheduling the pipeline.
3. List all the simple cycles and greedy cycles.
4. Determine the minimal average latency (MAL).

# 1. What are the forbidden latencies & initial collision vector?

## Forbidden latencies

S1: {(9-1)}	i.e. 8
S2: {(3-2), (8-3), (8-2)}	i.e. 1, 5, 6
S3: 0	i.e. 0
S4: {(6-5)}	i.e. 1
S5: {(8-7)}	i.e. 1

	Time →								
	1	2	3	4	5	6	7	8	9
S1	X								X
S2		X	X					X	
S3				X					
S4					X	X			
S5							X	X	

**Forbidden latencies = {1, 5, 6, 8}**

**It causes collision**

**Permissible Latency = {2, 3, 4, 7, 9<sup>+</sup>}**

**It does not causes collision**

Maximum forbidden latency ( $m$ ) = 8,

So the collision vector is of 8 bits

**Collision Vector : {C<sub>8</sub> C<sub>7</sub> C<sub>6</sub> C<sub>5</sub> C<sub>4</sub> C<sub>3</sub> C<sub>2</sub> C<sub>1</sub>} = {10110001}**

**1 0 1 1 0 0 0 1**

**1 = forbidden latency**

**0 = Permissible latency**

**Initial Collision Vector (ICV) = 10110001**



# 2. Draw State Transition diagram

## How to draw state transition diagram?

- The **Permissible Latency** =  $\{2, 3, 4, 7, 9^+\}$ , so we have to **find** the next states of collision vector for the transitions 2, 3, 4, 7, 9<sup>+</sup>

- **Method:**

To **find** the collision vector of the next state the collision vector of the present state is **shifted right** for each transitions (2, 3, 4, 7, 9<sup>+</sup>). The shifted collision vector of present state is **bitwise-ORed** with Initial Collision Vector (ICV).

**Initial Collision Vector (ICV) = 10110001**

## pt 1: Calculating next state with Present state (PS) = 10110001

**Permissible latencies (or transitions) = 2, 3, 4, 7, 9<sup>+</sup>**

### With latency 2:

$$\begin{array}{r} 10110001 \text{ ICV} \\ + \quad \underline{00101100} \text{ After Right Shift PS by 2} \\ \hline 10111101 \text{ Next state} \end{array}$$

➤ So, the **new state is 10111101** with **latency 2**

### With latency 3:

$$\begin{array}{r} 10110001 \text{ ICV} \\ + \quad \underline{00010110} \text{ After Right Shift PS by 3} \\ \hline 10110111 \text{ Next state} \end{array}$$

➤ So, the **new state is 10110111** with **latency 3**

### With latency 4:

$$\begin{array}{r} 10110001 \text{ ICV} \\ + \quad \underline{00001011} \text{ After Right Shift PS by 4} \\ \hline 10111011 \text{ Next state} \end{array}$$

➤ So, the **new state is 10111011** with **latency 4**

**Initial Collision Vector (ICV) = 10110001**

### With latency 7:

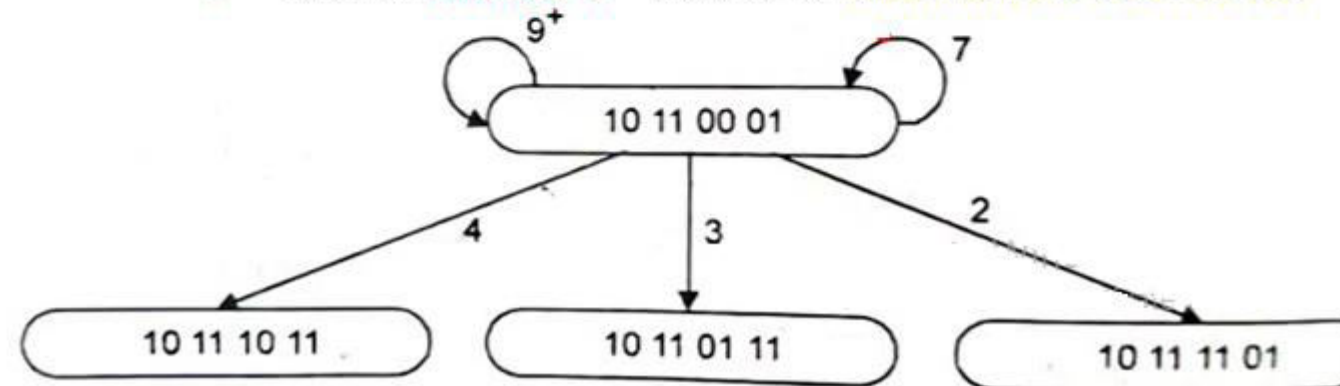
$$\begin{array}{r} 10110001 \text{ ICV} \\ + \quad \underline{00000001} \text{ After Right Shift PS by 7} \\ \hline 10110001 \text{ Next state} \end{array}$$

➤ So the **latency 7** returns to **original ICV (1011010)**

### With latency 9<sup>+</sup>:

$$\begin{array}{r} 10110001 \text{ ICV} \\ + \quad \underline{00000000} \text{ After Right Shift PS by 9} \\ \hline 10110001 \text{ Next state} \end{array}$$

➤ So the **latency 9<sup>+</sup>** returns to **original ICV (1011010)**



## Step 2: Calculating Next state with Present State (PS) = 10111101

Permissible latencies (or transitions) = 2, 7, 9<sup>+</sup>.

### □ With latency 2:

Initial Collision Vector (ICV) = 10110001

$$\begin{array}{r} 10110001 \text{ ICV} \\ + \quad 00101111 \text{ After right shifting PS by 2} \\ \hline 10111111 \text{ Next State} \end{array}$$

➤ So, the new state is 10111111 with latency 2

### □ With latency 7:

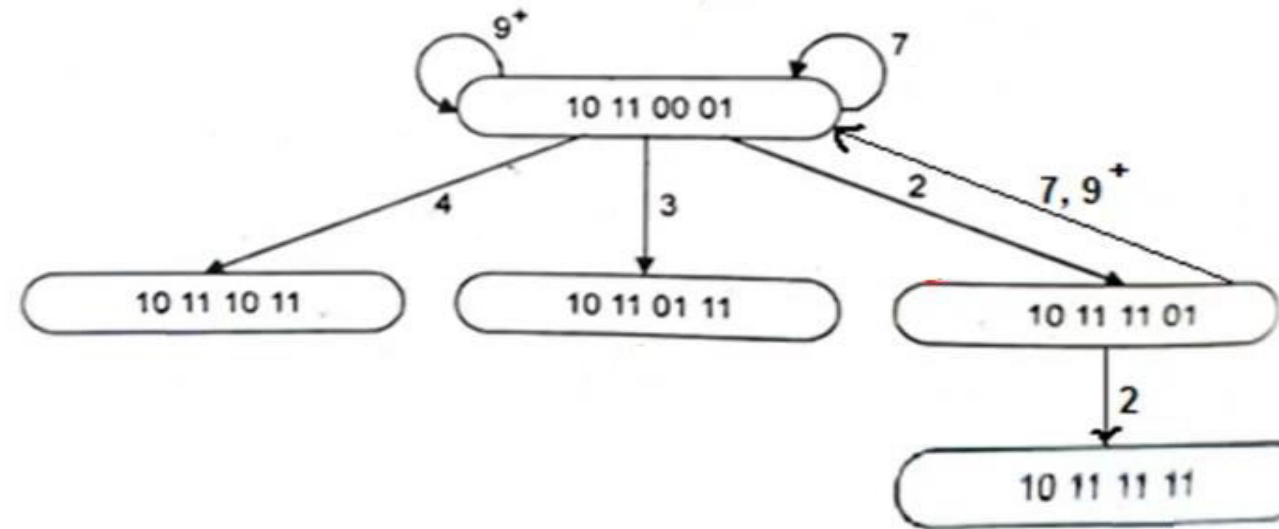
$$\begin{array}{r} 10110001 \text{ ICV} \\ + \quad 00000001 \text{ After right shifting PS by 7} \\ \hline 10110001 \text{ Next State} \end{array}$$

➤ So the latency 7 returns to original ICV (10110001)

### □ With latency 9<sup>+</sup>:

$$\begin{array}{r} 10110001 \text{ ICV} \\ + \quad 00000000 \text{ After right shifting PS by 9}^+ \\ \hline 10110001 \text{ Next state} \end{array}$$

➤ So the latency 9<sup>+</sup> returns to original ICV (10110001)



### Intermediate State Transition Diagram

**Note:** The process of shifting continues by taking the collision vector of new state, shifting and ORing with the initial collision vector (ICV).



## Step 5: Present State (PS) = 10111111

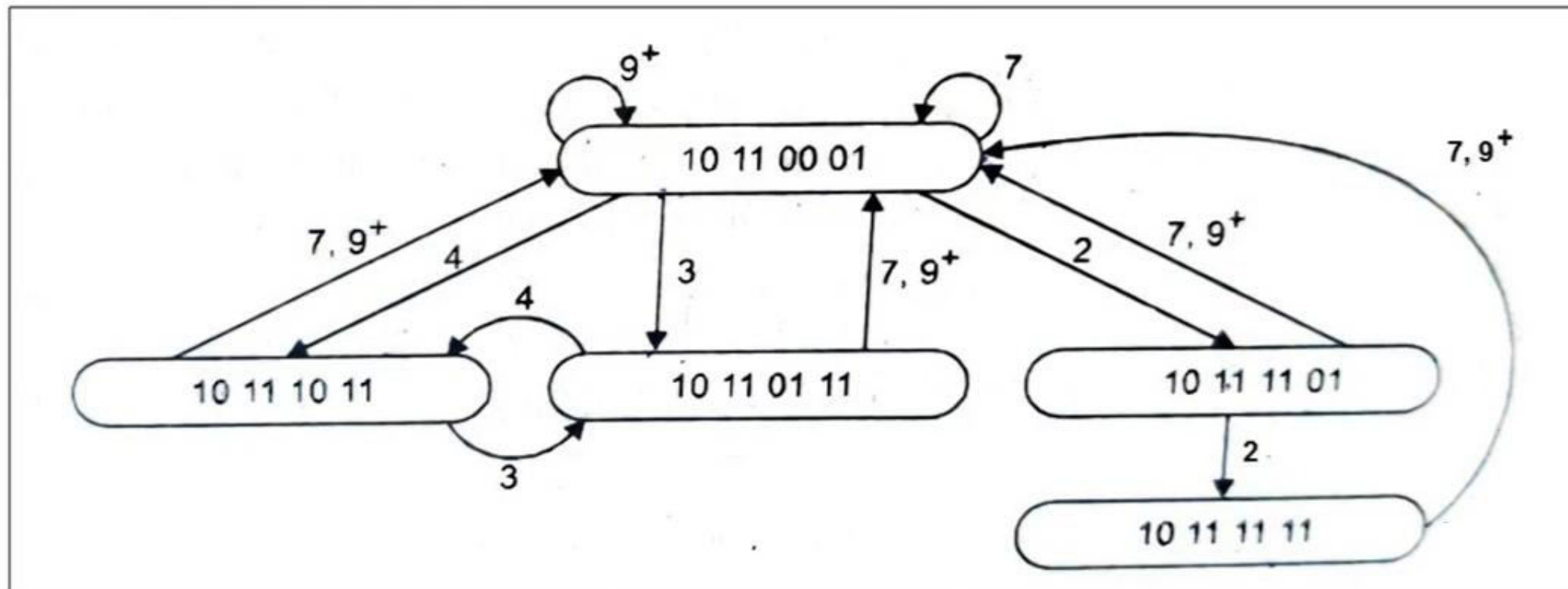
Permissible latencies (or transitions) = 7, 9<sup>+</sup>.

□ With latency 7:

$$\begin{array}{r} 10110001 \text{ ICV} \\ + \quad \underline{00000001} \text{ After right shifting PS by 7} \\ \hline 10110001 \text{ Next State} \end{array}$$

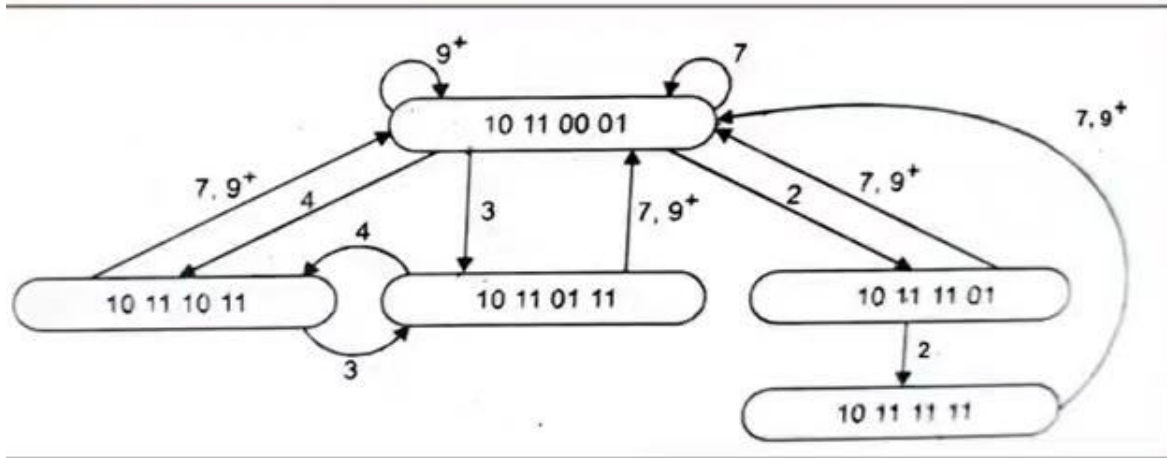
□ With latency 9<sup>+</sup>:

$$\begin{array}{r} 10110001 \text{ ICV} \\ + \quad \underline{00000000} \text{ After right shifting PS by 9}^+ \\ \hline 10110001 \text{ Next state} \end{array}$$



Final State Transition Diagram

# 3. Simple & Greedy Cycles



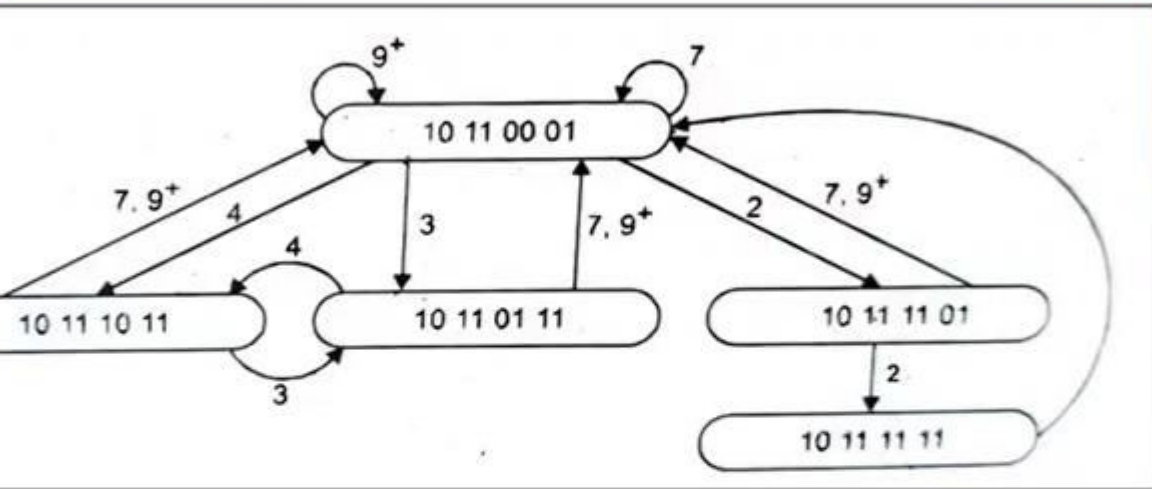
Latency Cycles	Types of cycles	Average Latency
(7)	Simple cycle (Constant Cycle)	7
(2, 7)	Simple cycle	4.5
(2, 2, 7)	<b>Simple cycle (Greedy Cycle)</b>	<b>3.6</b>
(3, 7)	Simple cycle	5
(3, 4)	<b>Simple cycle (Greedy Cycle)</b>	<b>3.5</b>
(4, 7)	Simple cycle	5.5
(4, 3, 7)	Simple cycle	4.6
(3, 4, 7)	Simple cycle	4.6

➤ A **Simple Cycle** is a latency cycle in which **each state appears only once**.

➤ A **Greedy Cycle** is a simple cycle whose edges are all made with **minimum latencies** from their respective starting states.

➤ Their **average latencies** must be **lower**

# 4. MAL Minimum Average Latency



Latency Cycles	Types of cycles	Average Latency
(7)	Simple cycle (Constant Cycle)	7
(2, 7)	Simple cycle	4.5
(2, 2, 7)	<b>Simple cycle (Greedy Cycle)</b>	<b>3.6</b>
(3, 7)	Simple cycle	5
<b>(3, 4)</b>	<b>Simple cycle (Greedy Cycle)</b>	<b>3.5 (MAL)</b>
(4, 7)	Simple cycle	5.5
(4, 3, 7)	Simple cycle	4.6
(3, 4, 7)	Simple cycle	4.6

**MAL (Minimum Average Latency)** is the **minimum average latency** obtained from the **greedy cycle**.

The **greedy cycle (3, 4)** leads to **MAL = 3.5**