Signals and Systems

Signal: A signal is defined as a function of one or more variables which convey informations. A signal is a physical quantity that varies with time in general. It can be dependent on one or more independent variables.

Noise: Any variables which doesn't convey information is called Noise. unlike signals, noise is usually doesn't carry useful information and is almost always considered undesirable.

One-dimensional signal: when a function depends on a single independent variable to represent the signal, it is said to be one-dimensional signal. Ex: ECG, speech signal.

Two-dimensional signal: when a function depends on two independent variables to represent the signal, it is said to be two-dimensional signal. Ex- two dimensional photograph.

Multi-dimensional signal: when a function depends on more than one independent variables to represent the signal, it is said to be multi-dimensional signal.

CamScanner

Input signal: A signal that enters a system from an external source is referred to as an input signal. Ex- voltage from a function generator, electrocardiogram from heart, temperature from the human body.

Output signal: A signal produced by the system in response to the input signal is called the output signal.

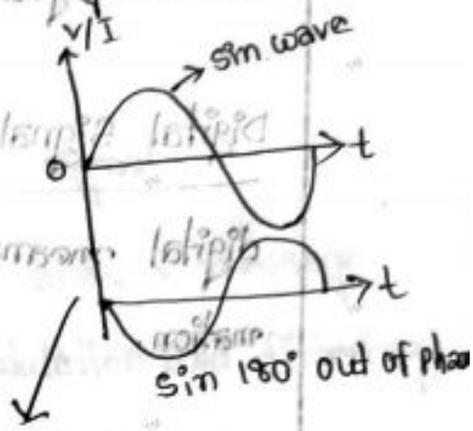
Sampling: Sampling is a process by which a continuous-time signal is converted into a sequence of discrete samples. The sampling can be either uniform or non-uniform sampling. In uniform sampling, the space between any two samples is fixed throughout the signal under consideration. In non-uniform sampling, the space between any two samples varies throughout the signal under consideration based on their characteristics.

System এর ৩ টি part থাকে। Input, Processing, output. Communication system output আকারে input আকারে কিছু হয়। এর part ৩ টি। Transmitter, channel & Receiver

Sine & cosine 90°
α(t) diff. 90°

CLASSIFICATION OF SIGNALS:

1. Continuous-time signal and Discrete-time signal.
2. Periodic signal and Aperiodic signal.
3. Even signal and Odd signal.
4. Deterministic signal and Random signal.
5. Energy signal and power signal.



TYPES OF SIGNALS:

এই ২ signal যোগ করলে 0 হয়।

1. Exponential signal i) Real exponential signal ii) Complex exponential signal
2. Sinusoidal signal
3. Step signal
4. Impulse signal
5. Ramp signal

Energy signal: A signal is said to be an energy signal if and only if its total energy E is finite. That is $0<E<\infty$. For an energy signal, the average power $P=0$. The nonperiodic signals are the examples of energy signals.

Power signal: A signal is said to be a power signal if its average power P is finite, that is $0<P<\infty$. For a power signal, the

total energy $E = \infty$. The periodic signals are the examples of power signals.

Digital signal Processing: DSP is the processing of signals by digital means. A signal in this context means a source of information.

Advantages of digital signal processing over analog signal processing.

- Digital signal processing is a process of analyzing the digital signal. DSP support the processing of complete algorithms at high speed in less time. Using DSP, we can filter signals more accurately.

The advantages of DSP.

1) Physical size of analog systems is quite large while digital processors are more compact and light in weight.

2) Digital components are less sensitive to the environmental changes, noise and distrubances.

3) DSP systems are upgradeable since that are software control

System: A system is an entity that manipulates one or more input signals to perform a function which results in a new output signal.

Input signal → [System] → Output signal

In a communication transmitter, the system accepts message signal (audio, video or data), processes it (modulation and filtering) and gives an acceptable output for communication.

Communication system has 3 parts. Transmitter, Channel, Receiver.

Channel → i) Media, ii) Bandwidth, iii) Noise & Interference (add করে)

Receiver এর কাজ: Transmitter এর opposite কাজ করা।

i) Transmitter এর opposite কাজ করা।

ii) channel effect এর

TV তে use করা হয় both analog & digital modulation.

Radio তে analog modulation.

Mobile এ digital "

Transmitter এর modulation এর প্রকার	
1. Analog Modulation &	2. Digital modulation.

ANALOG MODULATION

1. Amplitude Modulation (Information signal এর respect এ carrier signal এর amplitude change হয়)
2. Frequency " (Information signal এর respect এ carrier signal এর frequency change)
3. Phase " (Information signal এর respect এ " " phase change

$y = A_m \sin(\omega t + \theta)$ f = Linear frequency

↑ Angular frequency

T = Fundamental period and f = Fundamental Frequency.

মূল বিন্দুর সাপেক্ষে Horizontal angle কত degree cut করছে তাই phase (দশা).

Modulation এ ২ টা signal প্রয়োজন।

(Low Frequency signal) i) Message signal / Modulating signal / Baseband signal / Information signal.

(High Frequency signal) ii) Carrier signal. [Modulating signal এর respect এ change করা যাবে]

Message signal and carrier signal মিলে যে signal তৈরি হবে তা modulated signal. Information signal এর কোন property এর উপর ভিত্তি করে carrier signal এর কোন property change হবে।

Digital signal এর complexity Analog এর চেয়ে কম।

DIGITAL MODULATION

1) Amplitude shift keying.
2) Frequency shift keying.
3) Phase shift keying.

Information signal এর উপর ভিত্তি করে carrier signal change হবে না। Bit stream ~~sequence~~ এর উপর ভিত্তি করে change হবে।

এখানে, ৩ টা signal থাকবে।

Information signal, carrier signal, Bit sequence

Analog to Digital

1. Sampling
2. Quantization
3. Coding

[After coding, it results in a result bit & carrier signal change]

Another task of transmitter is compression (সংকোচন)

compression এর মাধ্যমে source থেকে redundant data কে remove করে size ছোট করা হয়।

Compression ২ প্রকার।

1. Lossless [text যা হারায় না/ change করা যায় না]
2. Lossy [video sending/ audio যার quality change হয়]

channel এ coding করে data বাড়ানো হয়। channel এ যে signal পাঠানো তা change করে ফেলা।

ANALOG SIGNAL PROCESSING: এক্ষেত্রে cost কমে।

Resistor, capacitor দিয়ে analog signal process করা হয়। Real time output possible.

continuous: একটি নির্দিষ্ট interval এর মধ্যে time এর respect এ মান নিবে।

Discrete: Fixed interval পর পর t এর respect এ y এর value থাকবে।

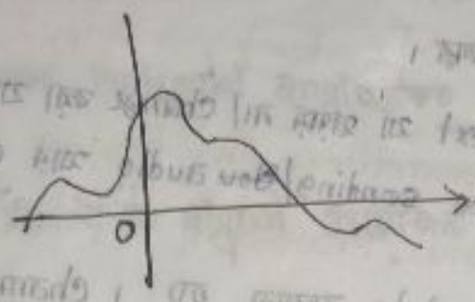
$x(m) = x(mt)$, $m = 0, \pm1, \pm2, \pm3$ (sampling interval) $\downarrow$ m

continuous time signal কে sampling করলে discrete time continuous valued signal পাওয়া যায়।

continuous time continuous amplitude signal কে quantization করলে discrete time discrete valued signal পাওয়া যায়।

Continuous time signal:

A signal $x(t)$ that is defined for all values of time t.



Arises naturally i.e. speech in voice, conversion of sound or light into electrical signal.

Discrete Time Signal: A signal that is defined only at discrete instants of time.

Continuous time & Discrete-time signals

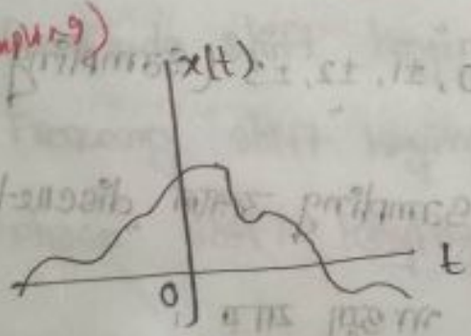


fig-a

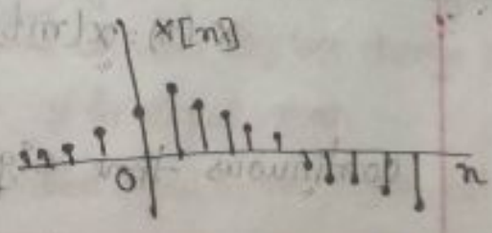


fig-b

Fixed time পর পর fig-a এর sample হচ্ছে fig-b এর নমুনা।

Quantization level: 2 এর power level হচ্ছে 4, 3 → 8, 4 → 16
(sample)

Sampling: Sampling is a process by which a continuous-time signal is converted into a sequence of discrete samples, with each sample representing the amplitude of the signal at a particular instant of time.

(After quantization) Continuous-time and Discrete-time signals
রেজাল্ট

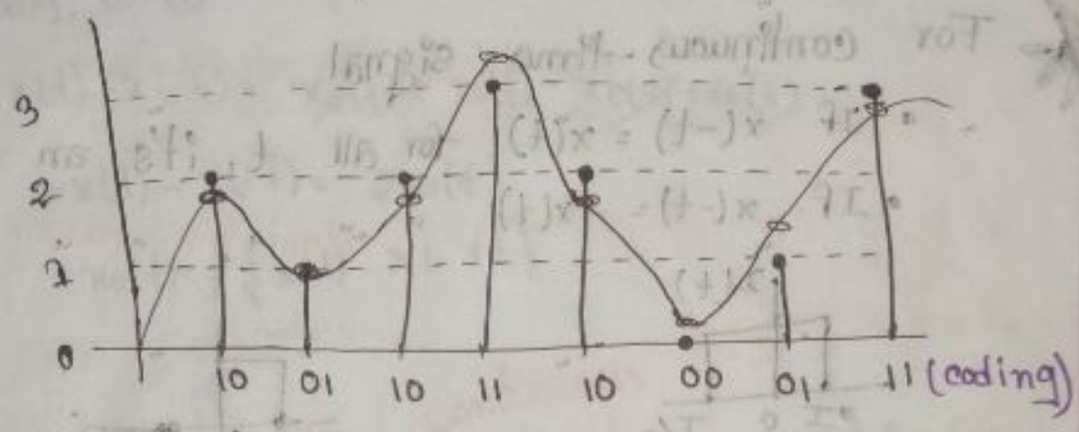


Fig: Quatization

Quantization: Quantization is the process of representing each sampled value to the nearest level selected from a finite number of discrete amplitude levels.

(quantization হলো predefined level এ align করা অর্থাৎ একটি sample কে কত bit দিয়ে represent করবো তার উপর ভিত্তি করে level এসাইন করতে হয়। এই process হই quantization.)

Original signal এর সাথে quantization signal এর যে difference সেই quantization error. Bit যত বাড়াব error তত কমবে।

Hello

Coding: Sampled values are converted to codes.

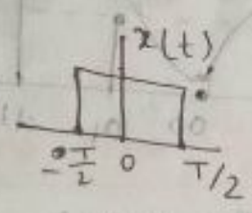
Binary coding: It is the number of bits for representing one sample value is at first decided 8 bit coding, 16 bit coding etc.

Mitu Apu

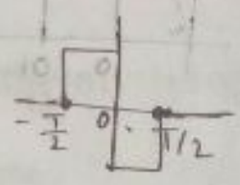
Even and odd signal:

For continuous-time signal

- If $x(-t) = x(t)$ for all t, it's an even signal
- If $x(-t) = -x(t)$ " " " odd "



Even



Odd

Even/odd decomposition of a general signal $x(t)$

Let $x(t)$ be defined as

$$x(t) = x_e(t) + x_o(t) \quad \text{— (i)}$$

here $x_e(t)$ = even and $x_o(t)$ = Odd

Replacing t by $-t$ in (i)

$$x(-t) = x_e(-t) + x_o(-t) \quad \text{— (ii)}$$

We know,

$$x_e(-t) = x_e(t)$$

$$x_o(-t) = -x_o(t)$$

Replacing these values to (ii) we get

$$x(-t) = x_e(t) - x_o(t) \quad \text{—— (iii)}$$

(i) + (iii)

$$x(t) + x(-t) = x_e(t) + \cancel{x_o(t)} + x_e(t) - \cancel{x_o(t)}$$

$$\Rightarrow x(t) + x(-t) = 2x_e(t)$$

$$\therefore x_e(t) = \frac{1}{2}\{x(t) + x(-t)\}$$

Similarly, (i) - (iii)

$$x(t) - x(-t) = \cancel{x_e(t)} + x_o(t) - \cancel{x_e(t)} + x_o(t)$$

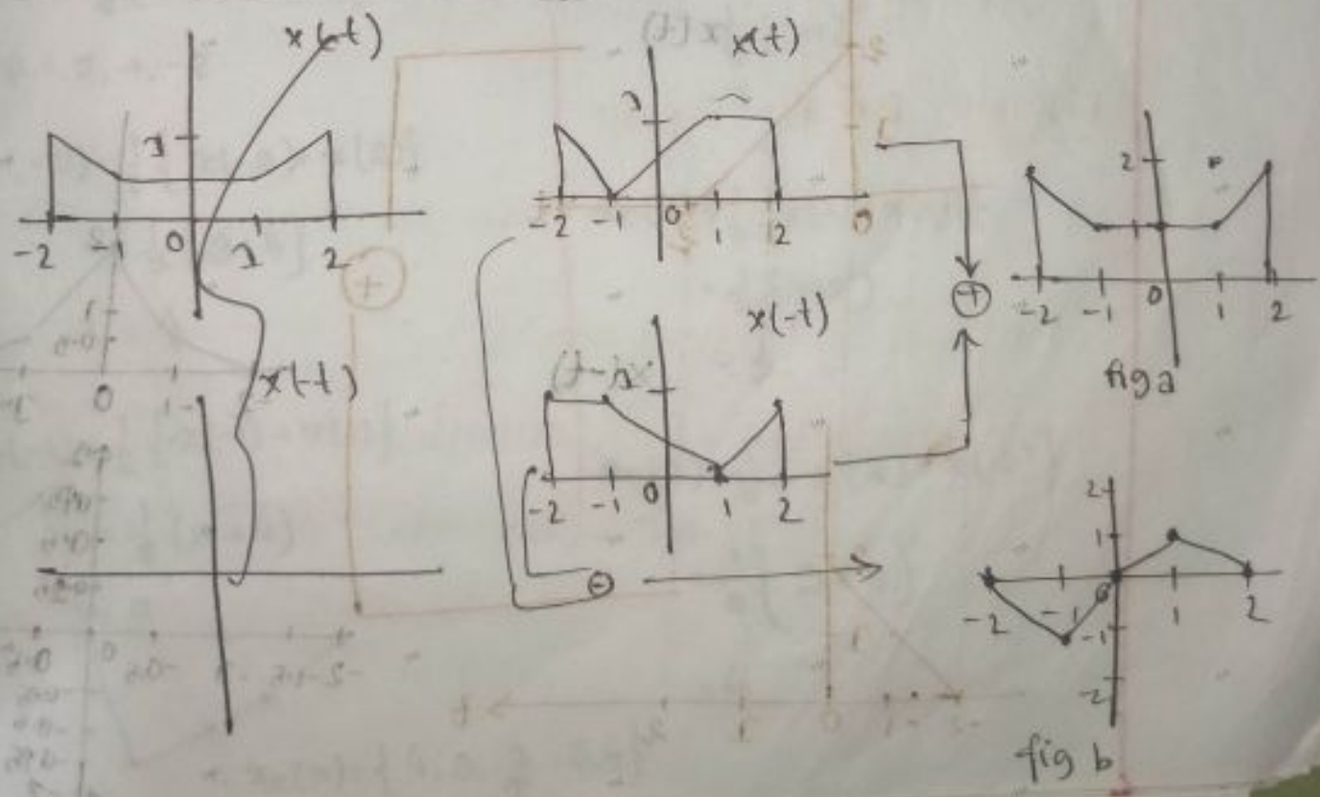
$$\Rightarrow x(t) - x(-t) = 2x_o(t)$$

$$\Rightarrow x_o(t) = \frac{1}{2}\{x(t) - x(-t)\}$$

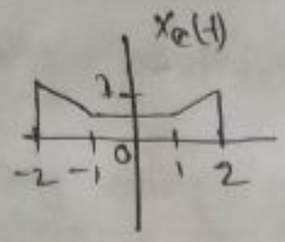
Even / odd decomposition

Continuous

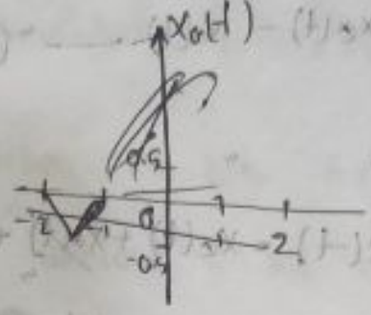
Horizontal বরাবর উল্টায়ে দিলে $-x(-t)$

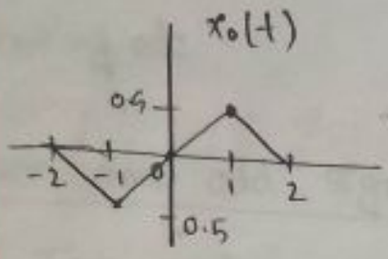


from fig a and b, (dividing verital axis by 2)



and





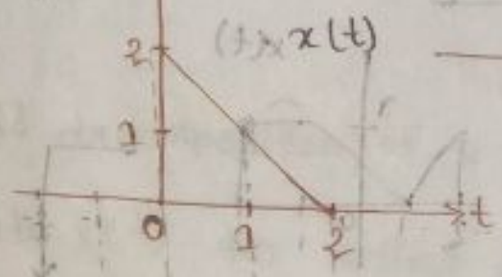
Even/odd Decomposition

Cosider the signal $x(t)$, Sketch the even and odd components of the signal

we know,

$$x_e(t) = \frac{x(t)+x(-t)}{2} \quad \text{(even)}$$

$$x_o(t) = \frac{x(t)-x(-t)}{2} \quad \text{(odd)}$$



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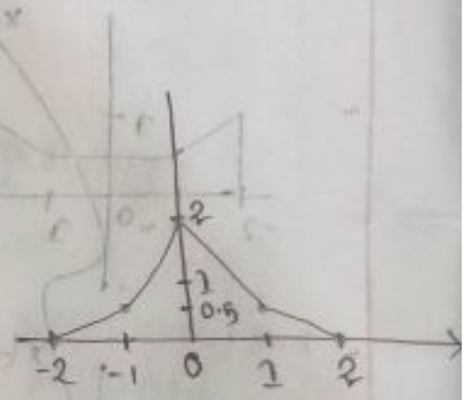

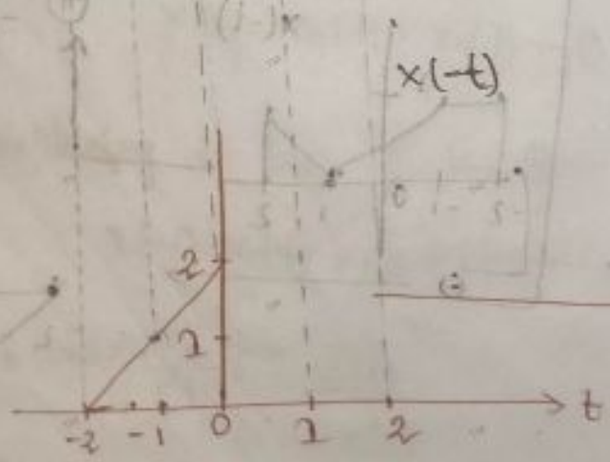

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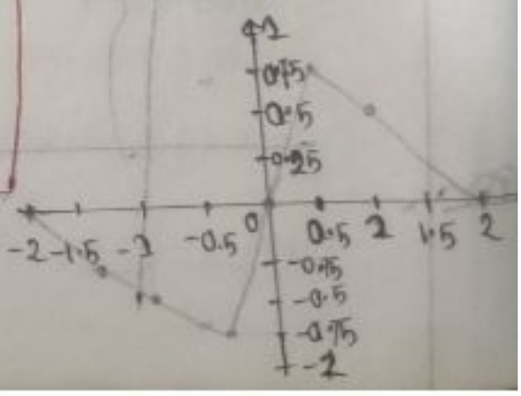


Discrete (Even/odd)

$x(n) = \{3, 2, 1, 4, 5\}$ [without arrow]

$\quad\quad\quad

$x_o(n) = \frac{1}{2}\{x(n) - x(-n)\}$

$x_o(0) = \frac{1}{2}\{x(0) - x(0)\}$

$= 0$

$x_o(-2) = \frac{1}{2}\{x(-2) - x(2)\}$

$= \frac{1}{2}(3-5)$

$= -1$

$x_o(1) = \frac{1}{2}\{x(1) - x(-1)\}$

$= \frac{1}{2}(4-2)$

$= 1$

$x_o(-1) = \frac{1}{2}\{x(-1) - x(1)\}$

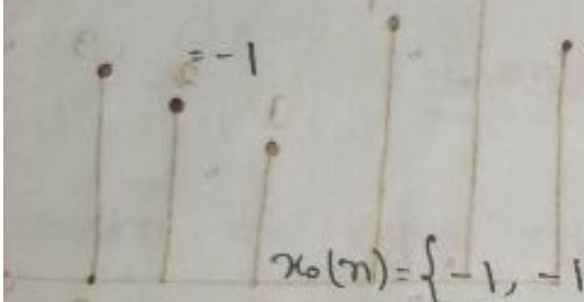
$= \frac{1}{2}(2-4)$

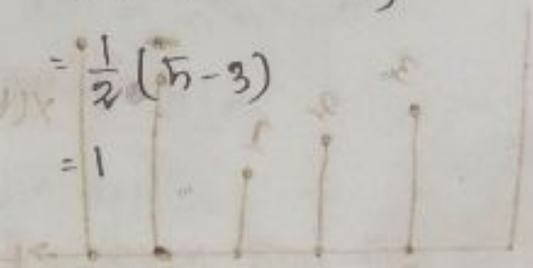
$= -1$

$x_o(2) = \frac{1}{2}\{x(2) - x(-2)\}$

$= \frac{1}{2}(5-3)$

$= 1$





$x_o(n) = \{-1, -1, \underset{\uparrow}{0}, 1, 1\}$

PERIODIC & NON-PERIODIC SIGNAL

$x(t) = x(t+T) \rightarrow$ Periodic signal

Discrete Periodic Signal:

$x(n) = x(n+N)$; here N = Number of samples.

Now, $x(n) = A\cos(\Omega n + \varphi)$

$\Rightarrow x(n+N) = A\cos(\Omega(n+N) + \varphi)$

$\Rightarrow x(n+N) = A\cos(\Omega n + \Omega N + \varphi)$

→ 2 টা signal সমান হবে যদি $\Omega N = 2\pi m$ হয়

check, whether $x(n) = 5\sin(2n)$ periodic or non-periodic

Given, $x(n) = 5\sin(2n)$

$\Omega N = 2\pi m$

$\Rightarrow 2N = 2\pi m$

$\therefore N = \pi m$

Here m is an integer for which N is integer.

But here N is non-integer

So it's non-periodic

check whether $x(n) = 5\sin\left(\frac{6\pi n}{35}\right)$ periodic or not.

$\Omega N = \frac{6\pi}{35} 2\pi m$

$\Rightarrow \frac{6\pi}{35} N = 2\pi m$

$\Rightarrow \frac{3}{35} N = m$

$\Rightarrow N = \frac{35}{3} m$

N can only be periodic if m is any multiplier of 3.

Check whether or not $x(n) = 5\sin(0.2\pi n)$ periodic or not

$\Omega N = 2\pi m$

$\Rightarrow 0.2\pi N = 2\pi m$

$\Rightarrow N = 10m$

$\therefore$ Its periodic & Fundamental period = 10

frequency = 0.1

$x(n) = \sin(2\pi/3\, n)$

$e^{i\theta} = \cos\theta + i\sin\theta$

$2\pi N = 2\pi m$

$\Rightarrow \frac{2\pi}{3} N = 2\pi m$

$\therefore N = 3m$

periodic

2.3) $x(t) = e^{\sin(t)}$

$x(t+T) = e^{\sin(t+T)}$

since, $T = 2\pi$

$e^{\sin(t+2\pi)}$

$= e^{\sin(t)}$ = periodic

Periodic function (continuous-time):

$x(t)$ is periodic if $x(t) = x(t+T)$ for all t, where T is a positive constant.

If this condition satisfied for $T = T_0$, it is also satisfied for $T = 2T_0$, $T = 3T_0$ and so on.

The smallest possible value of T is called fundamental period

$f = 1/T$

$\omega = 2\pi/T$

Periodic Signal (Discerte-time):

$x[n]$ is periodic if $x[n] = x[n+N]$ for all integer n, where

N is a positive integer constant

Fundamental angular frequency is (measured in radians per second and is defined as

$$\Omega = 2\pi/N$$

Prove that the cosine ~~coare~~ signal is periodic with periodicity T,

Let us consider a cosine signal $x(t) = A\cos(\omega t + \phi)$

$$x(t+T) = A\cos\{\omega(t+T) + \phi\}$$

$$= A\cos(\omega t + \omega T + \phi)$$

$$= A\cos(\omega t + 2\pi + \phi)$$

$$= A\cos\{\omega t + \phi\}$$

$$= x(t)$$

Problem 2.3)

1) $x(t) = e^{\sin(t)}$

from definition. $x(t) = x(t+T)$

$$x(t+T) = e^{\sin(t+T)}$$

$$= e^{\sin(2\pi + t)}$$

$$= e^{\sin(t)}$$ so it is periodic

11) $x(t) = t\, e^{\sin(t)}$

~~from~~ Now,

$x(t+T) = (t+T)\, e^{\sin(t+T)}$

$= (t+T)\, e^{\sin(t+2\pi)}$

$= (t+T)\, e^{\sin(t)} \neq x(t)$

so it is non-periodic

$\cos 2\pi = 1$, $\cos\pi = -1$, $\cos \pi/2 = 0$, $\cos(2\pi/3) = -\frac{1}{2}$, $\cos \pi/3 = \frac{1}{2}$

$\cos \pi/6 = 0.866$

2.5) $x(t) = \cos(t + \pi/3)$ ~~→~~ check periodic or not.

Soln:

Given, $x(t) = \cos(t+\pi/3)$ ~~→ (1)~~

From the definition of periodic signal we know,

$x(t) = x(t+T)$ if the signal is periodic

$x(t+T) = \cos(t+T+\pi/3)$

$= \cos(t+2\pi+\pi/3)$

$= \cos(t+\pi/3) = x(t)$

$\therefore$ The signal is periodic, where fundamental period is 2π.

2.6) Test whether the signal is periodic. And find fundamental period.

$$x(t) = \sin\left(\frac{2\pi}{5}\right)t$$

$$\omega = \frac{2\pi}{5}$$

$$\therefore \cancel{\text{frequency}} = T = \frac{2\pi}{\omega} = \frac{2\pi}{2\pi/5} = 2\pi \times \frac{5}{2\pi}$$

$$= 5$$

$\therefore$ The fundamental period for which the given signal exhibits periodicity is 5.

2.7) Find the fundamental period of $x(t) = \cos(\pi/3)t + \sin(\pi/5)t$

Soln: The given signal $x(t)$ is a composite signal which has two component signals $x_1(t)$ and $x_2(t)$.

here, $x_1 = \cos \pi/3$ and $x_2 = \sin \pi/3$

$\omega_1 = \pi/3$ and $\omega_2 = \pi/5$

$$T_1 = \frac{2\pi}{\omega_1}$$

$$\Rightarrow T_1 = \frac{2\pi}{\pi/3} = 6$$

$$\Rightarrow T_2 = \frac{2\pi}{\omega_2}$$

$$\Rightarrow T_2 = \frac{2\pi}{\pi/5}$$

$$= 10$$

The signal $x(t)$ is said to be periodic if and only if the ratio of T_1 and T_2 is a rational.

$$\frac{T_1}{T_2} = \frac{6}{10} = 3/5$$

2.8) Test whether the given signal is periodic or not

$$x(t) = \cos t + \sin\sqrt{2}\,t$$

$$x(t) = \underbrace{\cos t}_{x_1(t)} + \underbrace{\sin\sqrt{2}\,t}_{x_2(t)}$$

Soln:

$\omega_1 = 1$ and $\omega_2 = \sqrt{2}$

$$T_1 = \frac{2\pi}{\omega_1} = 2\pi \qquad T_2 = \frac{2\pi}{\omega_2} = \frac{2\pi}{\sqrt{2}} = \sqrt{2}\pi$$

The signal $x(t)$ is said to be periodic if and only if the ratio of T_1 and T_2 is a rational, that is

$$\frac{T_1}{T_2} = \frac{2\pi}{\sqrt{2}\pi} = \frac{\sqrt{2}}{1} \text{ (irrational)}$$

$\therefore$ The signal is not periodic

2.9) Test for periodicity of $x(t) = j e^{j10t}$

Soln: The frequency of the signal, $\omega_0 = 10$

$$T_0 = \frac{2\pi}{\omega_0} = \frac{2\pi}{10} = \pi/5$$

$\therefore$ The signal is periodic with fundamental period $\pi/5$

2.10) Test whether the given discrete-signal is periodic or not. If so, then find the fundamental period.

$$x_n = \sin\left(\frac{2\pi}{3}\right)n$$

here, $\Omega = \frac{2\pi}{3}$

The signal will be periodic if,

$$\Omega N = 2\pi m$$

$$\Rightarrow \frac{2\pi}{3} N = 2\pi m$$

$\therefore N = 3m$, here m is any integer for which N is integer.

Here N is integer.

$\therefore$ The signal is periodic and fundamental period is 3

2.11) Test for the periodicity of $x(n) = \cos^2(\pi/8)n$

Sol^n:

$$x(n) = \cos^2(\pi/8)n$$

$$= \frac{1}{2}\, 2\cos^2(\pi/8)n$$

$$= \frac{1}{2}\left[1 + \cos 2(\pi/8)n\right]$$

$$= \frac{1}{2}\left[1 + \cos(\pi/4)n\right]$$

$\Omega = \pi/4$

Now, $\Omega N = 2\pi m$

$$\Rightarrow \frac{\pi}{4} N = 2\pi m$$

$$\Rightarrow N = 8m$$

$\therefore$ The signal is periodic and fundamental period is 8.

Problem 2.12) Test for the periodicity of $x(n) = e^{j7\pi n}$

Solⁿ:

Here, $\Omega = 7\pi$

Now,

$$\Omega N = 2\pi m$$

$$\Rightarrow 7\pi N = 2\pi m$$

$$\Rightarrow N = \frac{2}{7} m$$

The signal will be periodic if and only if m is a ~~an~~ ~~ir~~ multiplier of 7.

And the fundamental period will be 2 if $m = 7$

Problem 2.13) Test for the Periodicity of $x(n) = 4e^{j\frac{4\pi(n+1/3)}{5}}$

Solⁿ:

$$x(n) = 4e^{j\frac{4\pi n + \frac{4\pi}{3}}{5}}$$

$$= 4e^{j\frac{4\pi n}{5}} \cdot e^{j\frac{4\pi}{15}}$$

here, $\Omega = \frac{4\pi}{5}$

Now,

$$\Omega N = 2\pi m$$

$$\Rightarrow \frac{4\pi}{5} N = 2\pi m$$

$$\Rightarrow \frac{2}{5} N = m$$

$$\Rightarrow N = \frac{5}{2} m$$

The function will be periodic if m is an multiplier of 2

And the fundamental time is 5.

2.14) Test for the periodicity of $x(n) = 4e^{j\frac{4(n+1/5)}{5}}$

Sol^n:

$$x(n) = 4e^{j\frac{4n}{5}} \cdot e^{4/15}$$

$$\Omega = \frac{4}{5}$$

Now, $\Omega N = 2\pi m$

$$\Rightarrow \frac{4}{5} N = 2\pi m$$

$$\Rightarrow N = \frac{5\pi}{2} m$$

$\therefore$ Signal is non-periodic

2.15) Determine the fundamental period of the signal

$$x(n) = 1 + \underbrace{e^{j\frac{4\pi n}{7}}}_{x_1(t)} - \underbrace{e^{j\frac{2\pi n}{5}}}_{x_2(t)}$$

$$\Omega_1 = \frac{4\pi}{7}$$

$$\Omega_1 N_1 = 2\pi m$$

$$\Rightarrow \frac{\cancel{4}^{2}\pi}{7} N_1 = \cancel{2\pi} m$$

$$\Rightarrow N_1 = \frac{7}{2} m$$

$$\Omega_2 = \frac{2\pi}{5}$$

$$\Rightarrow \Omega_2 N_2 = 2\pi m$$

$$\Rightarrow \frac{2\pi}{5} N_2 = \cancel{2\pi} m$$

$$\Rightarrow N_2 = 5m$$

The signal is said to be periodic if and only if N_1 and N_2 is rational.

$$\frac{N_1}{N_2} = \frac{7/2\, m}{5m} = \frac{7}{10}$$

Problem 2.16) A pair of sinusoidal signals with a common angular frequency is represented by

$$x_1(n) = 3\sin(3\pi n) \text{ and } x_2(n) = \sqrt{3}\sin(3\pi n)$$

Specify the period for which the period N of both $x_1(n)$ and $x_2(n)$ must satisfy them to be periodic.

Soln:

$$x_1(n) = 3\sin(3\pi n)$$

$$x_2(n) = \sqrt{3}\sin(3\pi n)$$

Here, $\Omega = 3\pi$ for both the signal

and phase angle, $\phi = 0$

We know,

$$\Omega N = 2\pi m$$

$$\Rightarrow 3\pi N = 2\pi m$$

$$\Rightarrow N = \frac{2}{3} m$$

For $x_1(n)$ and $x_2(n)$ both to be periodic if their period N must be an integer. This can only be satisfied for m = integers multiples of 3

And the results in period will be integer multiples of 2

Problem 2.17) Determine whether the signal $x(n) = \cos(0.1\pi n)$ is periodic or not.

$$x(n) = \cos(0.1\pi n)$$

$$\Omega = 0.1\pi$$

$$\Omega N = 2\pi m$$

$$\Rightarrow 0.1\pi N = 2\pi m$$

$$\Rightarrow N = \frac{2\times 10}{0\pi} m$$

$$\Rightarrow N = 20m$$

$\therefore$ The signal is periodic and fundamental period will be 20.

ODD & EVEN DISCRETE-TIME SIGNAL

A discrete time signal is said to be even if $x(-n) = \cancel{x(t)}\, x(n)$

and A " " " " " " odd " $\cancel{x(-t) = -x(t)}$ $x(-n) = -x(n)$

A signal $x(n)$ can be decomposed as odd and even signals

$x(n) = \cancel{x(-n)}\ x_e(n) + x_o(n)$ — ① here, $x_e(n)$ is an even signal and $x_o(n)$ " " odd "

Replacing $\cancel{x(n)}$ $n = -n$ in ①, we get

$$x(-n) = x_e(-n) + x_o(-n)$$

$$\Rightarrow x(-n) = x_e(n) - x_o(n) \quad \text{—— (II)}$$

[from the definition of even & odd signals]

① + (II)

$$x(n) + x(-n) = x_e(n) + \cancel{x_o(n)} + x_e(n) - \cancel{x_o(n)}$$

$\Rightarrow x(n)+x(-n) = 2x_e(n)$

$\Rightarrow x_e(n) = \frac{1}{2}\left[x(n)+x(-n)\right]$

(i) − (ii)

$x(n)-x(-n) = \cancel{x_e(n)} + x_o(n) - \cancel{x_e(n)} + x_o(n)$

$\Rightarrow x(n)-x(-n) = 2x_o(n)$

$\Rightarrow x_o(n) = \frac{1}{2}\left[x(n)-x(-n)\right]$

2.18) Find the odd and even components of $x(t) = e^{j2t}$

Soln:

$x(t) = e^{j2t}$

$\therefore x(-t) = e^{j2(-t)}$

$= e^{-j2t}$

Even signal, $x_e(t) = \frac{x(t)+x(-t)}{2}$

$= \frac{e^{j2t}+e^{-j2t}}{2}$

$= \cos 2t$

Odd signal, $x_o(t) = \frac{x(t)-x(-t)}{2}$

$= \frac{e^{j2t}-e^{-j2t}}{2}$

$= j\left[\frac{e^{j2t}-e^{-j2t}}{2j}\right]$

$= j\sin 2t$

$x(t) = x_e(t)+x_o(t) = \cos 2t + j\sin 2t$

2.19) Show that the product of two even signals or two odd signals is an even signal and that the product of an even signal and odd signal is an odd signal.

Soln:

Let us consider, $x_2(t)$ and $x_4(t)$ be two even signals.

$$x(t) = x_2(t) \cdot x_4(t) \quad \text{--- (i)}$$

Replacing t with -t in (i) we get

$$x(-t) = x_2(-t) \cdot x_4(-t)$$

$$\Rightarrow x(-t) = x_2(t)\, x_4(t) \quad \text{--- (ii)}$$ [as they are even signal]

Equation (i) and (ii) are equal

As $x(t) = x(-t)$ so we can say the product of two even signals ~~is~~ is an even signal.

Let $x_1(t)$ and $x_3(t)$ be two odd signals.

$$x(t) = x_1(t) \cdot x_3(t) \quad \text{--- (i)}$$

Replacing t by -t in (i)

$$x(-t) = x_1(-t) \cdot x_3(-t)$$

$$\Rightarrow x(-t) = -x_1(t)\{-x_3(t)\}$$

$$\therefore x(-t) = x_1(t)\, x_3(t) \quad \text{--- (ii)}$$

$$x(t) = x(-t)$$

$\therefore$ Two odd signal's product is an even signal.

Let $x_e(t)$ be an even signal and $x_o(t)$ be odd signal.

By definition,

$$x_e(-t) = x_e(t) \text{ and}$$
$$x_o(-t) = -x_o(t)$$

$$x(t) = x_e(t) \cdot x_o(t) \quad \text{--- (i)}$$

$$\Rightarrow x(-t) = x_e(-t)\, x_o(-t)$$

$$= x_e(t)\{-x_o(t)\}$$

$$\Rightarrow x(-t) = -x_e(t)\, x_o(t) \quad \text{--- (ii)}$$

~~(i) and (ii) are opposite in S~~

$$\therefore x(t) = -x(-t)$$

$\therefore$ The product of an even and an odd signal is odd.

Problem 2.2a) Find the even and odd components of $x(t) = \cos t + \sin t$

Soln:

$$x(t) = \cos t + \sin t$$

$$x(-t) = \cos(-t) + \sin(-t)$$

$$x(-t) = \cos t - \sin t$$

$$x_e(t) = \frac{x(t) + x(-t)}{2} = \frac{\cos t + \cancel{\sin t} + \cos t - \cancel{\sin t}}{2}$$

$$= \frac{2\cos t}{2} = \cos t$$

$$x_o(t) = \frac{x(t) - x(-t)}{2}$$

$$= \frac{\cos t + \sin t - \cos t + \sin t}{2}$$

$$= \frac{2\sin t}{2}$$

$$= \sin t$$

$$x(t) = x_e(t) + x_o(t) = \cos t + \sin t$$

DETERMINISTIC and RANDOM SIGNAL

Deterministic signal:

A signal about which there is a certainty with respect to its value at any time. That is there are completely specified function of time.

Examples: All periodic signals

Random Signal:

A signal about which there are uncertainty before its actual occurance. They often belong to group of signals of different waveforms.

Examples: Noise generated in amplifier, wireless channel ECG signals etc.

ENERGY SIGNALS and POWER SIGNALS

~~Instan~~ Energy Signal :

A signal that have non-zero finite energy i.e. $0<E<\infty$ and zero average power is called energy signal.

Example: Both non-periodic and deterministic signal

Power Signal :

A signal that have non-zero finite average power i.e. $0<P<\infty$ and infinite energy is called power signal.

Example : Periodic signal, random signal.

- Instantaneous power $P(t) = x^2(t)$
- Total Energy of a signal $E = \int_{-\infty}^{\infty} x^2(t)\,dt$

Limitation of the definition of Total Energy :

If the signal doesn't decay with time or if it is a period signal, it means, the signal has finite energy. A signal having finite energy has no meaning as far as comparing it

$E = \int_{-\infty}^{\infty} x^2(t)\,dt$

$P = \frac{1}{T}\int_{-\infty}^{\infty} x^2(t)\,dt$

with another signal is concerned. In this cases the concept of average power helps.

Average Power, $P = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} x^2(t)\,dt$

Average power of a periodic signal, $P = \frac{1}{T}\int_{-T/2}^{T/2} x^2(t)\,dt$

$E = \int_{-\infty}^{\infty} x^2(t)\,dt$

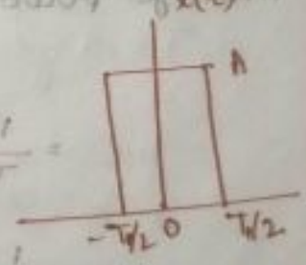
In discrete time approach:

Total Energy $E = \sum_{n=-\infty}^{n=+\infty} x^2[n]$

Average power $P = \lim_{N\to\infty} \frac{1}{2N+1}\sum_{n=-N}^{N} x^2[n]$

Average power of a periodic signal, $P = \frac{1}{N}\sum_{n=0}^{N-1} x^2[n]$

Ex-1: $x(t) = \begin{cases} A, & -T_1/2 \le t \le T_1/2 \\ 0, & \text{otherwise} \end{cases}$



Soln: $E = \int_{-\infty}^{+\infty} x^2(t)\,dt$

$= \int_{-T_1/2}^{T_1/2} A^2\,dt$

$= A^2[T]_{-T_1/2}^{T_1/2}$

$= A^2\left[\frac{T_1}{2} + \frac{T_1}{2}\right]$

$= A^2\frac{2T_1}{2} = A^2 T_1$

$P = \lim_{T\to\infty} \frac{1}{T}\int_{-T_1/2}^{T_1/2} x^2(t)\,dt$

$= \lim_{T\to\infty} \frac{1}{T}\int_{-T_1/2}^{T_1/2} A^2\,dt$

$= \lim_{T\to\infty} \frac{1}{T} A^2[T]_{-T_1/2}^{T_1/2}$

$= \lim_{T\to\infty} \frac{1}{T} A^2\left[T_1/2 + T_1/2\right]$

$= \lim_{T\to\infty} \frac{1}{T} A^2 T_1$

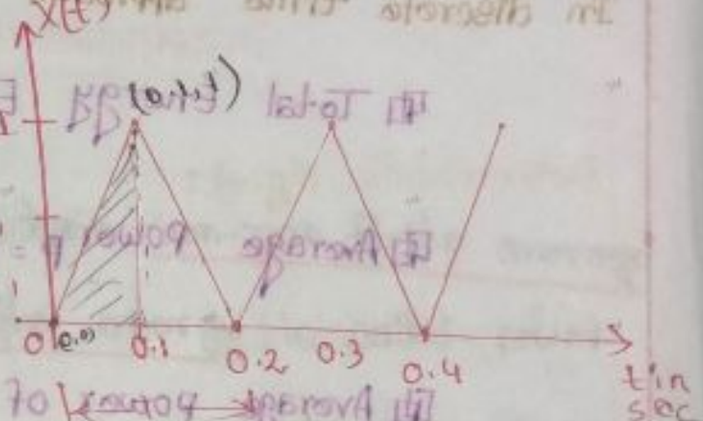
$= 0$

Here this signal has non-periodic zero finite energy and zero average power energy.

$\therefore$ It is an energy signal.

Ex-2:

$$P = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt$$



here,

Average power, $P = \frac{1}{T} \int_{-T/2}^{T/2} x^2(t)\, dt$

$T = 0.2$ $T/2 = 0.1$

$x(t) = \begin{cases} \frac{x_1 - 0}{1 - 0}, & 0 \le x(t) < 0.1 \\ \frac{0 - x_1}{0 - 1}, & 1 \le x(t) < 0.2 \end{cases}$

$= \frac{1}{T} \int_0^T x^2(t)\, dt$

$= \frac{1}{T} 2 \int_0^{0.1} x^2(t)\, d$ $= \frac{1}{T} 2 \int_0^{T/2} x^2(t)\, dt$

$= \frac{1}{T} 2 \int_0^{0.1} \left(\frac{1}{0.1} t\right)^2 dt$

Slope, $\frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{1 - 0}{0.1 - 0}$

$= \frac{1}{0.1}$

here, $T = 0.2$

$= \frac{2}{0.2} \times \frac{1}{(0.1)^2} \int_0^{0.1} t^2 dt$

$= \frac{2}{0.2 \times 0.1 \times 0.1} \left[\frac{t^3}{3}\right]_0^{0.1}$

$$= \frac{\cancel{2} \times 10 \times 10 \times 10}{\cancel{2} \times 1 \times 1} \frac{1}{3}\left[(0.1)^3 - 0\right]$$

$$= \frac{1000}{3}\left(\frac{0.1 \times 0.1 \times 0.1}{1000}\right)$$

$$= \frac{1}{3}$$

$\therefore P = \frac{1}{3}$ = Non-zero finite power

A signal can be neither energy not power

$$E = \lim_{T \to \infty} \int_{-T/2}^{T/2} |x(t)|^2 dt$$

$$P = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt$$

Energy

$$E = \lim_{T \to \infty} P \cdot T$$

$$\cancel{\Rightarrow P = \lim_{T \to \infty} \frac{1}{T} E}$$

$$= \lim_{T \to \infty} \frac{1}{3} \cdot T$$

$$= \infty$$

$\therefore$ power signal

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SOLVED PROBLEMS

2.26) Test whether the given signal is an energy or power signal.

$$x(t) = e^{-2t}\, u(t)$$

$$\boxed{\begin{aligned} \text{Energy} &= \lim_{T\to\infty} P \cdot T \\ \Rightarrow P &= \lim_{T\to\infty} \frac{1}{T} E \end{aligned}}$$

$$E = \lim_{T\to\infty} \int_{-T/2}^{T/2} x^2(t)\, dt$$

$$= \lim_{T\to\infty} \int_{-T/2}^{T/2} \left[e^{-2t} u(t)\right]^2 dt \qquad \left[\begin{array}{ll} u(n) = 1 & \text{for } n \geq 0 \\ 0 & \text{for } n < 1 \end{array}\right.$$

$$= \lim_{T\to\infty} \int_{-T/2}^{-1} \left[e^{-2t} u(t)\right]^2 dt + \sum_{0}^{T/2} \left(e^{-2t} u(t)\right)^2 dt$$

$$= 0 + \sum_{0}^{T/2} \left(e^{-2t}\right)^2 dt$$

$$= \lim_{T\to\infty} \int_{0}^{T/2} \left(e^{-2t} \cdot 1\right)^2 dt$$

$$= \lim_{T\to\infty} \int_{0}^{T/2} e^{-4t}\, dt$$

$$= \lim_{T\to\infty} \left[\frac{e^{-4t}}{-4}\right]_0^{T/2}$$

$$= \lim_{T\to\infty} -\frac{1}{4}\left(e^{-4\cdot\frac{T}{2}} - e^{-0}\right)$$

$$= \lim_{T\to\infty} -\frac{1}{4}\left(e^{-2T} - 1\right)$$

$$= -\frac{1}{4}\left(e^{-2\infty} - 1\right)$$

$$= -\frac{1}{4}(0 - 1)$$

$$= \frac{1}{4} < \infty$$

$\therefore$ Energy is finite

$$P = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} x^2(t)\,dt$$

$$= \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} |e^{-2t}u(t)|^2\,dt$$

$$= \lim_{T\to\infty} \frac{1}{T}\int_{0}^{T/2} |e^{-2t}|^2\,dt \quad [\text{here } u(t)=1]$$

$$= \lim_{T\to\infty} \frac{1}{T}\int_{0}^{T/2} e^{-4t}\,dt$$

$$= \lim_{T\to\infty} \frac{1}{T}\left[\frac{e^{-4t}}{-4}\right]_0^{T/2}$$

$$= \lim_{T\to\infty} \frac{1}{-4T}\left[e^{-4\frac{T}{2}} - e^{0}\right]$$

$$= \lim_{T\to\infty} \frac{1}{-4T}\left[e^{-2T} - e^{0}\right]$$

$$= 0$$

$\therefore$ power is zero

$\therefore$ It is an energy signal

Another Method

$$E = \lim_{T\to\infty} P\cdot T$$

$$\Rightarrow \lim_{T\to\infty} P = \frac{E}{T}$$

$$\Rightarrow P = \lim_{T\to\infty} \frac{E}{T}$$

$$\Rightarrow P = \lim_{T\to\infty} \frac{1/4}{T}$$

$$\Rightarrow P = \lim_{T\to\infty} \frac{1}{4T}$$

$$\therefore P = 0$$

Non-periodic * Periodic = Non-periodic

Periodic signal = P

Energy signal = non-periodic.

e^{-2t} → Non-periodic but e^{jt} = periodic

2.26) Test whether the given signal is an energy or power signal.

$$x(n) = (-0.5)^n u(n)$$

Sol^n:

we know,

$$E = \sum_{-\infty}^{\infty} x^2(n)$$

$$= \sum_{-\infty}^{\infty} \left|(0.5)^n u(n)\right|^2$$

$$= \sum_{0}^{\infty} \left|(0.5)^2\right|^n$$

$$= \sum_{0}^{\infty} (0.25)^n$$

$$= \frac{1}{1-0.25}$$

$$= 1.33 < \infty$$

$$\left[\sum_{0}^{\infty} a^n = \frac{1}{1+a}\right] \quad \left[\sum_{N_1}^{N_2} 1^n = N_2 - N_1 + 1\right]$$

$$\sum_{0}^{N} (0.25)^n = N - 0 + 0.25$$

∴ E is finite

$$\cancel{P = \frac{1}{2N+1}} \quad P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{-N}^{N} \left|x(n)\right|^2$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{-N}^{N} \left|(-0.5)^n u(n)\right|^2$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{0}^{N} \left|(-0.5)^n\right|^2$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{0}^{N} (0.25)^n$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \frac{0.25}{1-0.25} = \frac{1}{2\cdot\infty+1} \frac{0.25}{0.75}$$

$$= \frac{1}{\infty} = 0 \quad \therefore P = 0$$

∴ Energy signal

Problem 2.27) Test whether the given signal is an energy signal or a power signal.

$$x(n) = u(n)$$

Soln:

$$E = \sum_{-\infty}^{\infty} |x(n)|^2$$

$$= \sum_{-\infty}^{\infty} |u(n)|^2 \qquad \left[\text{Formula, } \sum_{N_1}^{N_2} 1^n = N_2 - N_1 + 1\right]$$

$$= \sum_{0}^{\infty} 1 \qquad \left[\sum_{0}^{\infty} 1 = \infty\right]$$

$$= \infty - 0 + 1$$

$$= \infty$$

$\therefore$ The signal has infinite energy

$$P = \lim_{N \to \infty} \frac{1}{N+1} \sum_{-N}^{N} |x(n)|^2$$

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{-N}^{N} |x(n)|^2$$

$$= \lim_{N \to \infty} \frac{1}{2N+1} \sum_{-N}^{N} |u(n)|^2$$

$$= \lim_{N \to \infty} \frac{1}{2N+1} \sum_{0}^{N} 1$$

$$= \lim_{N \to \infty} \frac{1}{2N+1} (N+1) \qquad \left[\sum_{0}^{N} 1 = N+1\right]$$

$$= \lim_{N \to \infty} \frac{N(1+1/N)}{N(2+\frac{1}{N})}$$

$$= \frac{1+\frac{1}{\infty}}{2+1/\infty} = \frac{1+0}{2+0} = \frac{1}{2} < \infty$$

$\therefore$ The signal is a power sig

2.28) Test whether the given signal is an energy or power signal.

$$x(t) = t\,u(t)$$

$$E = \lim_{T\to\infty} \int_{-T/2}^{T/2} x^2(t)\,dt$$

$$= \lim_{T\to\infty} \int_{-T/2}^{T/2} [t\,u(t)]^2\,dt$$

$$= \lim_{T\to\infty} \int_{-T/2}^{T/2} t^2\,u(t)\,dt$$

$$= \lim_{T\to\infty} \int_{0}^{T/2} t^2\,dt$$

$$= \lim_{T\to\infty} \left[\frac{t^3}{3}\right]_0^{T/2}$$

$$= \lim_{T\to\infty} \frac{1}{3}\left[\frac{T}{2}-0\right]^3$$

$$= \lim_{T\to\infty} \frac{1}{3}\left[\frac{T^3}{8}\right]$$

$$= \frac{1}{24}\,\infty^3$$

$$= \infty$$

$$P = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(t)\,dt$$

$$= \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} t\,u(t)\,dt$$

$$= \lim_{T\to\infty} \frac{1}{T} \int_{0}^{T/2} t\,dt = \lim_{T\to\infty} \frac{1}{T}\left[\frac{t^3}{3}\right]_0^{T/2}$$

$$= \lim_{T\to\infty} \frac{1}{3T}\left[t^3\right]_0^{T/2}$$

$$= \lim_{T\to\infty} \frac{1}{3T}\left[\left(\frac{T}{2}\right)^3-0\right]$$

$$= \lim_{T\to\infty} \frac{1}{3T}\cdot\frac{T^3}{8}$$

$$= \lim_{T\to\infty} \frac{T^2}{24}$$

$$= \infty$$

$\therefore$ The signal is neither a power signal nor an energy signal.

Problem 2.29) Test whether the given signal $x(t) = e^{j(2t+\pi/4)}$ is an energy signal or power signal.

$$x(t) = e^{j(2t+\pi/4)}$$

$$E = \lim_{T\to\infty} \int_{-T/2}^{T/2} x^2(t)\,dt$$

$$= \lim_{T\to\infty} \int_{-T/2}^{T/2} \left|e^{j(2t+\pi/4)}\right|^2 dt \qquad \left[\left|e^{j\theta}\right| = 1\right]$$

$$= \lim_{T\to\infty} \int_{-T/2}^{T/2} 1^2\,dt$$

$$= \lim_{T\to\infty} \int_{-T/2}^{T/2} dt$$

$$= \lim_{T\to\infty} \left[t\right]_{-T/2}^{T/2}$$

$$= \lim_{T\to\infty} \left[\frac{T}{2} + \frac{T}{2}\right]$$

$$= \lim_{T\to\infty} \left[\frac{2T}{2}\right]$$

$$= \lim_{T\to\infty} T$$

$$= \infty$$

The signal has infinite energy

$$P = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(t)\,dt$$

$$= \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} \left[e^{j(2t+\pi/4)}\right]^2 dt$$

$$= \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} 1^2\,dt$$

$$= \lim_{T\to\infty} \frac{1}{T} \left[t\right]_{-T/2}^{T/2}$$

$$= \lim_{T\to\infty} \frac{1}{T} \left[\frac{T}{2} + \frac{T}{2}\right]$$

$$= \lim_{T\to\infty} \frac{1}{T} \left[T\right]$$

$$= \lim_{T\to\infty} 1$$

$$= 1$$

$\therefore$ P is finite

$\therefore$ It's a power signal

problem 2.36) Test whether the given signal is an energy signal or a power signal.

$$x(t) = \cos t$$

$$E = \lim_{T\to\infty} \int_{-T/2}^{T/2} |x(t)|^2 dt.$$

$$= \lim_{T\to\infty} \int_{-T/2}^{T/2} \cos^2 t \, dt$$

$$= \lim_{T\to\infty} \frac{1}{2} \int_{-T/2}^{T/2} 2\cos^2 t \, dt$$

$$= \lim_{T\to\infty} \frac{1}{2} \int_{-T/2}^{T/2} \left[1 + \cos 2t\right] dt$$

$$= \lim_{T\to\infty} \frac{1}{2} \left[\int_{-T/2}^{T/2} dt + \int_{-T/2}^{T/2} \cos 2t \, dt \right]$$

$$= \lim_{T\to\infty} \frac{1}{2} \left\{ \left[t\right]_{-T/2}^{T/2} + \left[\frac{\sin 2t}{2}\right]_{-T/2}^{T/2} \right\}$$

$$= \lim_{T\to\infty} \frac{1}{2} \left[\left(\frac{T}{2} + \frac{T}{2}\right) + \frac{1}{2} \left\{ \sin 2 \cdot \frac{T}{2} - \sin 2\left(-\frac{T}{2}\right) \right\} \right]$$

$$= \lim_{T\to\infty} \frac{1}{2} \left[T + \frac{1}{2} (\sin T + \sin T) \right]$$

$$= \lim_{T\to\infty} \frac{1}{2} \left[T + \sin T \right]$$

$$= \frac{1}{2} (\infty + \sin\infty) = \infty + 0 = \infty$$ ∴ Energy is infinite

$\sin\infty = 0$

$\cos\infty = 0$

$\int_{-T/2}^{T/2} \cos t \, dt = 0$

$$P = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(t)\,dt$$

$$= \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} \cos^2 t\,dt$$

$$\left[\int \cos^2 t = \frac{1}{2}\left[t + \sin t\right]\right]$$

$$= \lim_{T\to\infty} \frac{1}{T}\frac{1}{2}\left[T + \sin T\right]$$

$$= \lim_{T\to\infty} \left(\frac{1}{2} + \frac{\sin T}{T}\right)$$

$$= \frac{1}{2} + \sin\infty$$

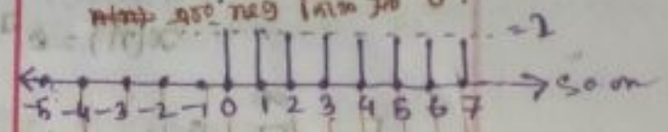
$= \frac{1}{2}$ is a non-zero finite power

$\therefore$ The signal is a power signal

$u(n)$

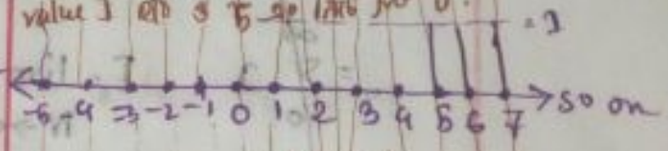
$u(n) = 1$ for $n \geq 0$
$= 0$ for $n < 0$

অর্থাৎ, $u(n) = 1$ যেই 0 এর সমান বা বড় হবে তার value 1 হবে নতুবা 0.

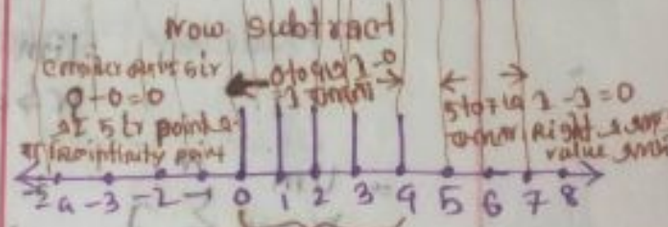


$u(n-5)$ → 1 for $n \geq 5$
0 for $n < 5$

অর্থাৎ, $u(n-5)$ যেই 5 এর সমান বা বড় হবে তার value 1 হবে 5 এর নিচে হবে 0.



Now subtract



$\therefore$ Limit is 0 to 4 as they are only 1. The rests are 0.

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{-N}^{N} x^2(n)$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=0}^{4} 1$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \cdot 5$$

$$= \frac{5}{2\cdot\infty+1}$$

$$= \frac{1}{\infty}$$

$$= 0$$

$\therefore$ Power is zero

$\therefore$ It's an energy signal

2.31) Check energy/power signal.

$$x(n) = u(n) - u(n-5)$$

$$E = \sum_{-\infty}^{\infty} x^2(n)$$

$$= \sum_{-\infty}^{\infty} \left[u(n) - u(n-5)\right]^2$$

$$= \sum_{0}^{4} 1$$

$$\left[\sum_{N_1}^{N_2} 1 = N_2 - N_1 + 1\right]$$

$$= 4 - 0 + 1$$

$$= 5$$

$\therefore E$ is non-zero finite value

2.32) check energy or power signal;

$$x(n) = e^{j\left(\frac{\pi}{2}n + \pi/4\right)}$$

$$\boxed{\left[e^{j\theta}\right| = 1\right]}$$

$$E = \sum_{-\infty}^{\infty} x^2(n)$$

$$= \sum_{-\infty}^{\infty} \left|e^{j(\pi/2\, n + \pi/4)}\right|^2$$

$$= \sum_{-\infty}^{\infty} 1 = \infty$$

$$E = \lim_{N\to\infty} \sum_{-N}^{N} x^2(n)$$

$$= \lim_{N\to\infty} \sum_{-N}^{N} \left|e^{j(\pi/2\, n + \pi/4)}\right|^2$$

$$= \lim_{N\to\infty} \sum_{-N}^{N} 1$$

$$\sum_{-N_1}^{N_2} 1 = N_2 - N_1 + 1$$

$$\Rightarrow \sum_{-N}^{N} 1 = 2N+1$$

$$= \lim_{N\to\infty} (N-(-N)+1)$$

$$= \lim_{N\to\infty} [2N+1]$$

$$= \infty$$

$\therefore$ E is infinite

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{-N}^{N} x^2(n)$$

$$= \lim_{N\to\infty} \frac{1}{\cancel{2N+1}} (\cancel{2N+1})$$

$$= 1$$

$\therefore$ P is finite

The given signal is power signal.

BASIC OPERATIONS ON SIGNALS

Operations performed on dependent variables

- Amplitude Scaling
- Addition
- Multiplication
- Differentiation
- Integration

Operations performed on independent variables.

- Time scaling
- Reflection
- Time shifting

Dalai's Book (155 - 88 page)

Amplitude Scaling: Applying amplitude scaling on x(t); yields

$$y(t) = cx(t)$$

where c is a scalar quantity known as the scaling factor.

For ~~continuous~~ discrete time signal,

$$y[n] = cx[n]$$

Ex - Electronic Amplifier, A resistor (x(t)), current, c resistance y(t) voltage)

Additional of a signal:

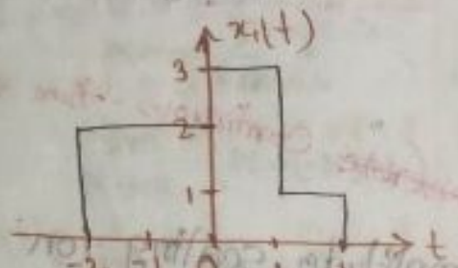
For continuous-time signal, if $x_1(t)$ and $x_2(t)$ are two signals, the signal $y(t)$ obtained by the addition of $x_1(t)$ and $x_2(t)$ is defined by

$$y(t) = x_1(t) + x_2(t)$$

For discrete time signal,

$$y[n] = x_1[n] + x_2[n]$$

Example: Audio mixer



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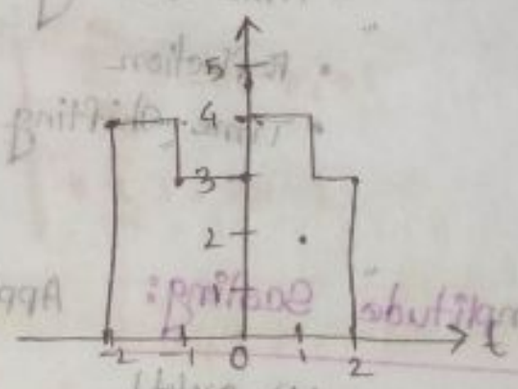

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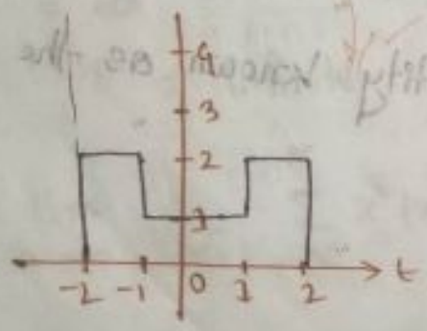


details in 155-34

Multiplication:

For continuous-time signal if $x_1(t)$ and $x_2(t)$ are two signals, the signal $y(t)$ obtained by the multiplication of $x_1(t)$ and $x_2(t)$ is defined by

$$y(t) = x_1(t) \cdot x_2(t)$$

For ~~dist~~ discrete-time signal,

$$y[n] = x_1[n]\, x_2[n]$$

Example: AM modulation, sampling, spreading.

Differentiation:

If $x(t)$ is continuous-time signal, the derivative of $x(t)$ with respect to time is given by,

$$y(t) = \frac{d}{dt} x(t)$$

Ex- Voltage across an inductor of inductance L due to the flow of current $i(t)$

$$v(t) = L \frac{d}{dt} i(t)$$

Integration:

If $x(t)$ is continuous-time signal, the integral of $x(t)$ with respect to time is given by,

$$y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$$

Ex- Voltage across a capacitor of capacitance of C

due to the flow of current $i(t)$.

Time Scaling:

For continuous-time signal, if $x(t)$ is a continuous-time signal, the signal $y(t)$ obtained by the scaling of independent variable t, by the factor a is defined by,

$$y(t) = x(at)$$

If $a > 1$, $y(t)$ is the compressed version of $x(t)$

If $0 < a < 1$, $y(t)$ is the expanded version of $x(t)$.

$x = t$

যদি $x = t$ $[-1, 1]$ range এ

$x = 2t$, range কে 2 দিয়ে ভাগ করে,

$x = [-0.5, 0.5]$

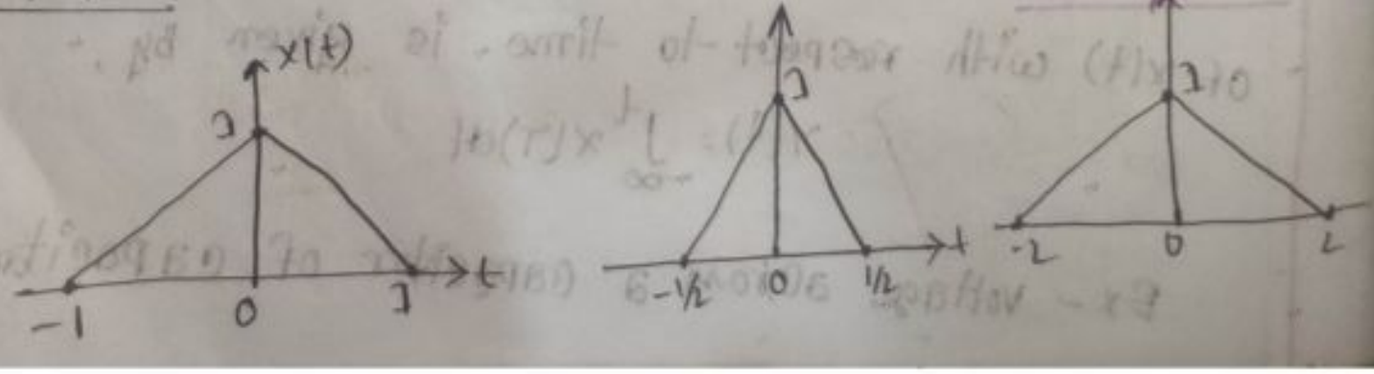
$x = 0.5t$ range কে 2 দিয়ে গুণ করে

For discrete-time signal,

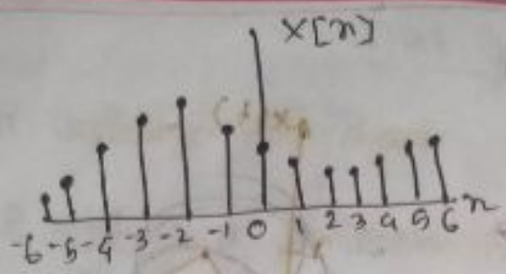
$$y[n] = x[kn], k \geq 1$$

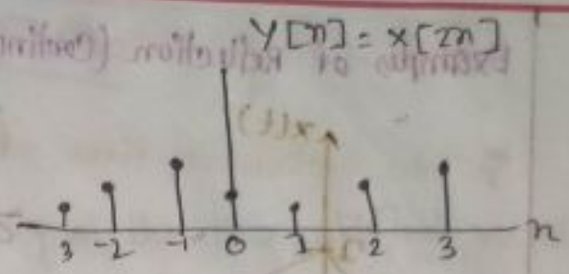
If $k > 1$, $y[n]$ is the compressed version of $x[n]$ and some values of the signal is lost.

Example:



Remuneration





Reflection:

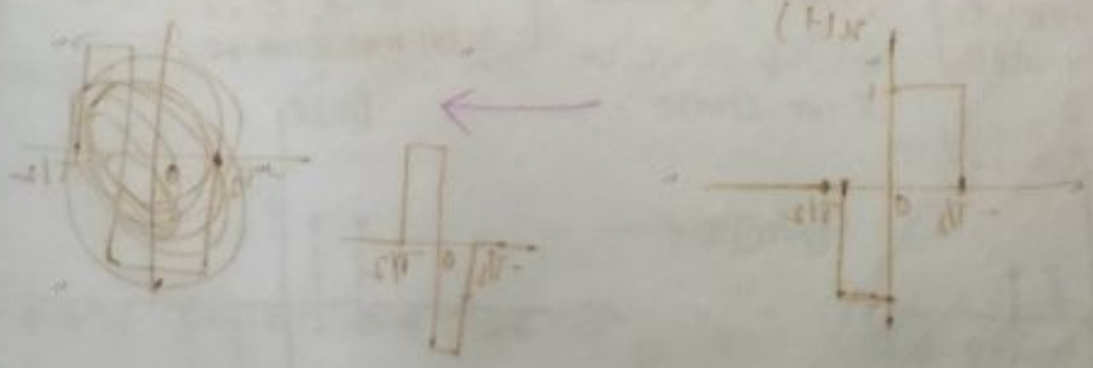
For continuous-time signal, if $x(t)$ is a continuous time signal, the signal $y(t)$ obtained by replacing t by $-t$ is called the reflected version of $x(t)$ i.e.

$$y(t) = x(-t)$$

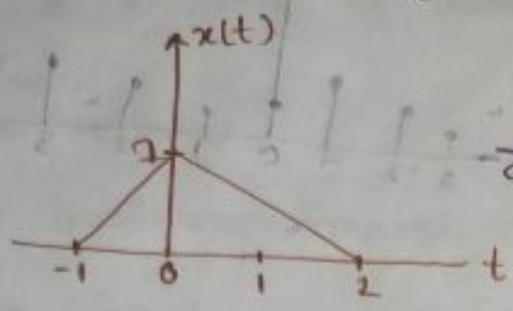
Special cases:

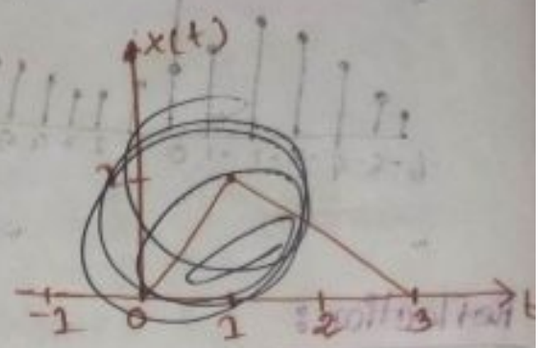
For even-signals, since $x(-t) = x(t)$ for all t, an even signal is same as its reflected version.

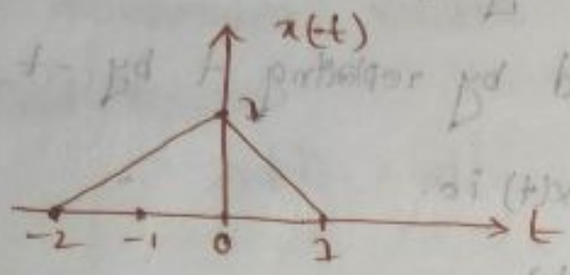
For odd-signals, since $x(-t) = -x(t)$ for all t, an odd signal is '-' ve of its reflected version.



Examples of Reflection (Continuous)







(Odd)

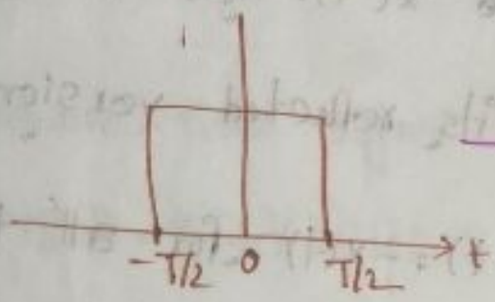
Reflection of odd signal can be possible.

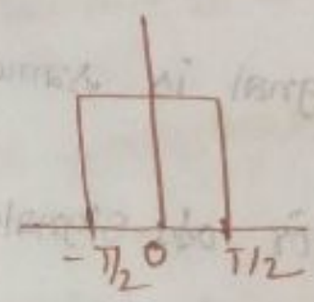
$x(-t) = -x(t)$

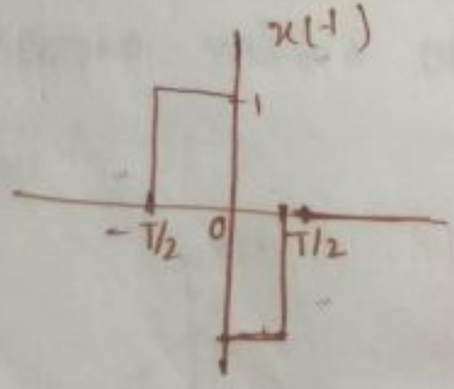
Even signals reflection is ~~same as~~ as same as the signal

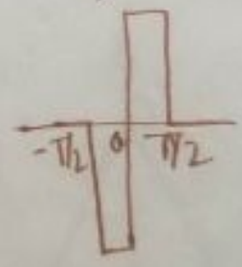
ex. $x(-t) = x(t)$

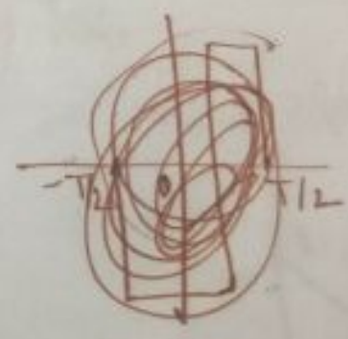
Even











TIME SHIFTING

For continuous-time signal:

If $x(t)$ is a continuous-time signal, the time shifted version of $x(t)$ defined by

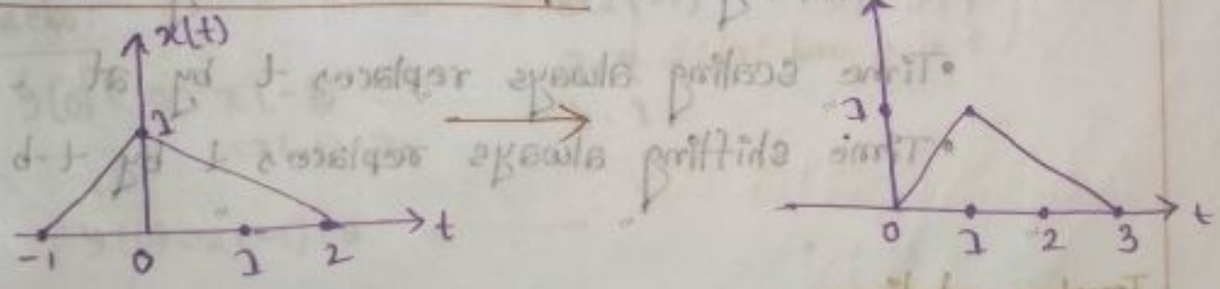
$$y(t) = x(t - t_0), \text{ here } t_0 \text{ is the time shift.}$$

- If $t_0 > 0$, the waveform of $x(t)$ is shifted to the right.
- If $t_0 < 0$, the waveform of $x(t)$ is shifted to the left.

For discrete-time signal:

$y[n] = x[n-m]$, here m is the shift which must be an integer.

Example of Time shifting

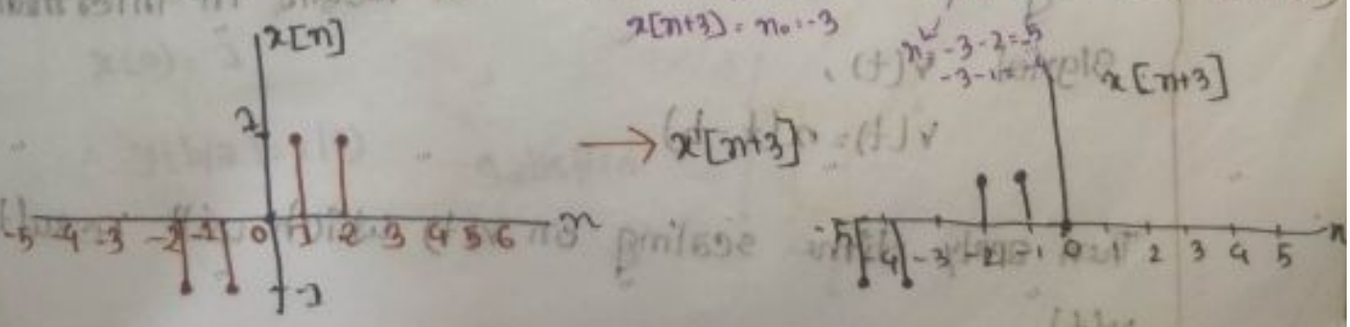


$$x[n] = \begin{cases} 1, & n = 1, 2 \\ -1, & n = -1, -2 \\ 0, & n = 0 \text{ and } |n| > 2 \end{cases} \longrightarrow x[n+3] = \begin{cases} 1, & n = -2, -1 \\ -1, & n = -4, -5 \\ 0, & n = -3, n < -5 \text{ and } n > -1 \end{cases}$$

$x[n+3] = n_0 = -3$

$n_0 = -3-2 = -5$

$-3-1 = -4$



Time shifting & Time scaling

For continuous-time signal

If $x(t)$ is a continuous-time signal, the time shifted and time scaled version of $x(t)$ is defined by,

$$y(t) = x(at-b)$$

Relation between $y(t)$ and $x(t)$ satisfies the following conditions

1. $y(0) = x(-b)$
2. $y(b/a) = x(0)$

Precedence Rule:

First time shifting and then time scaling.

- Time scaling always replaces t by at
- Time shifting always replaces t by $t-b$

Implementation:

Apply time shift operation that results in intermediate signal $v(t)$,

$$v(t) = x(t-b)$$

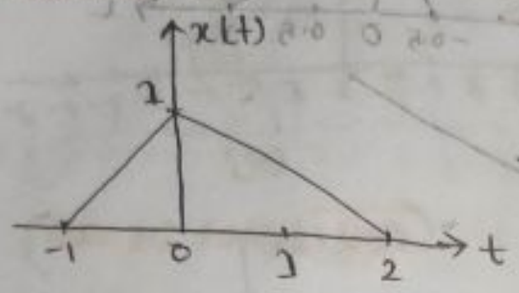
Then apply time scaling on $v(t)$ which will result in $y(t)$

$y(t) = x(at)$

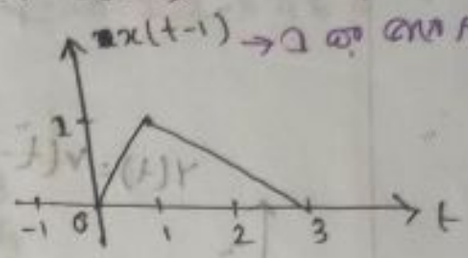
Time shifting and Time scaling:

1st shifting
2nd scaling

Objective is to find $x(2t-1)$

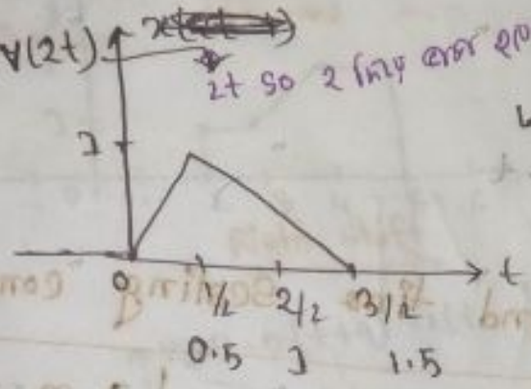


$v(t) = x(t-1)$ → right shift



$y(t) = v(2t)$

2t so 2 দিয়ে ভাগ হবে



$y(t) = x(at-b)$

$y(0) = x(-b)$

$y(b/a) = x(0)$

Checks:

- $y(0) = 0$, $x(-b)$
  $= x(-1) = 0$

  $\therefore y(0) = x(-1) = 0$

$x(2t-1)$

$a = 2$

$b = 1$

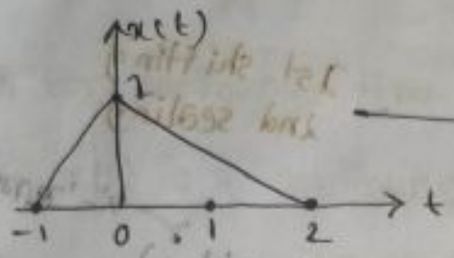
- $y(b/a) = y(1/2) = y(0.5) = 1$

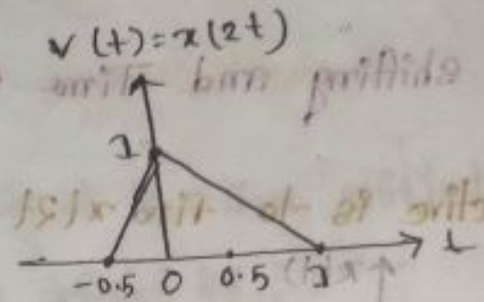
  $x(0) = 1$

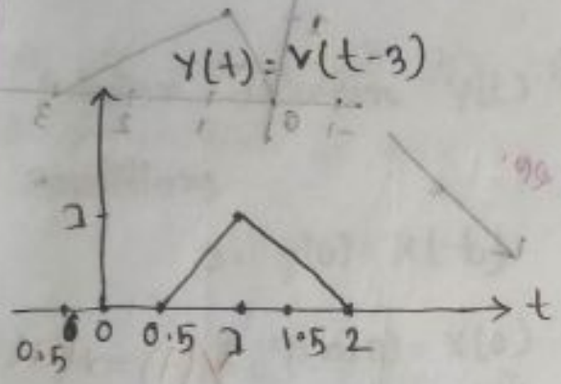
  $\therefore y(b/a) = x(0)$

Satisfied

If Precedence rule if not followed

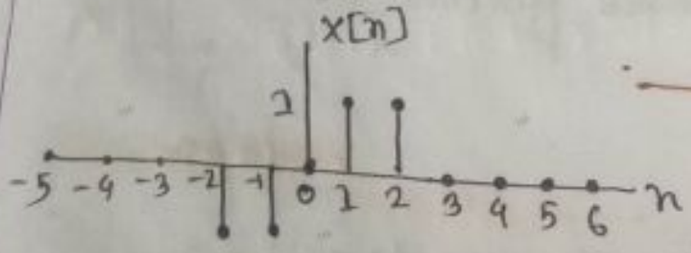


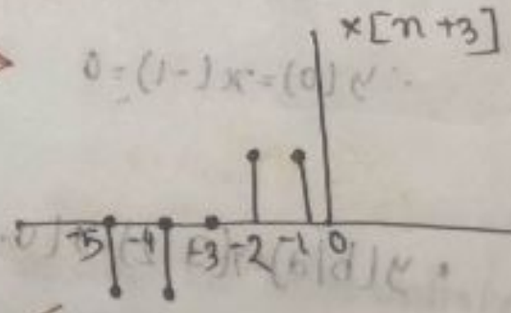


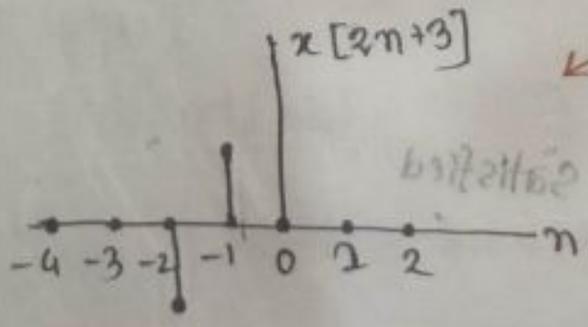


Time shifting and time scaling continuous

$$x[n] = \begin{cases} 1, & n = 1, 2 \\ -1, & n = -1, 2 \\ 0, & n = 0 \text{ and } |n| > 2 \end{cases} \longrightarrow x[n+3] = \begin{cases} 1, & n = -1, -2 \\ -1, & n = -4, -5 \\ 0, & n = -3, n < -5 \text{ and } n > -1 \end{cases}$$



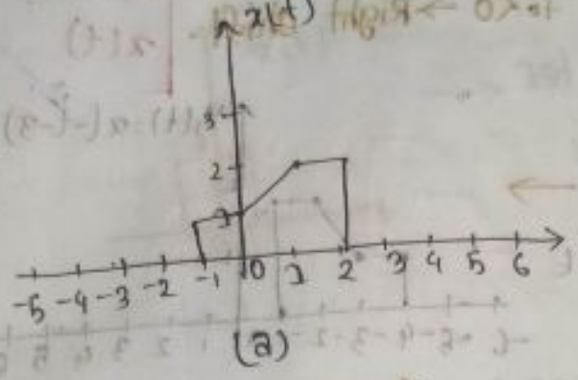




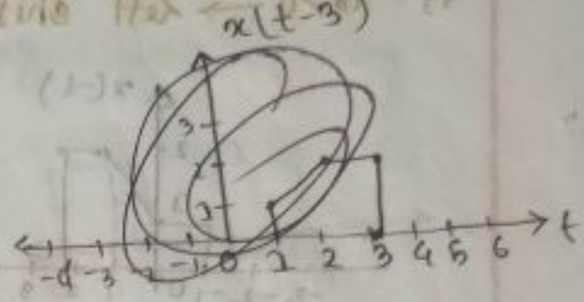
Here the signal $x(t)$ is positive

TIME SHIFTING:

$x(t-3)$ $\quad t_0 = +3 \rightarrow$ Right shift



(a)

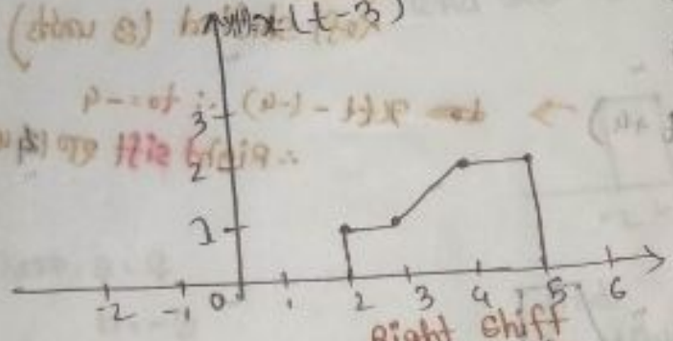


$-1+3=2$

$0+3=3$

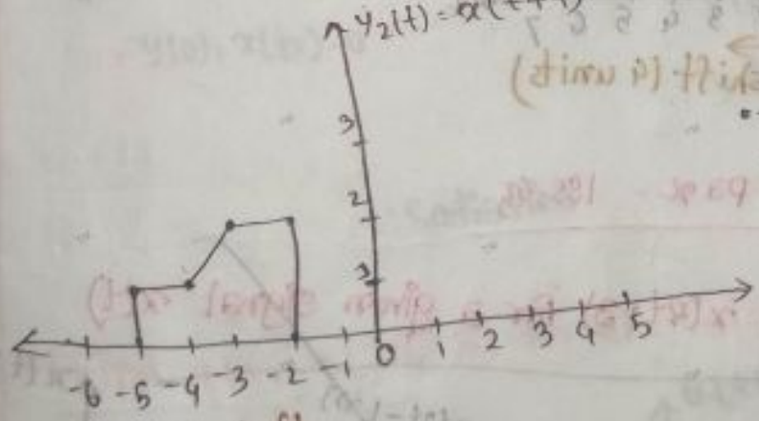
$1+3=4$

$2+3=5$



Right shift

$y_2(t) = x(t+4) = x(t-(-4))$ $\quad t_0 = -4 \rightarrow$ Left shift



$-5 = 1-4 \rightarrow 1$

$-4 = 0-4 \rightarrow 1$

$-3 = 1-4 \rightarrow 2$

$-2 = 2-4 \rightarrow 2$

Left shift

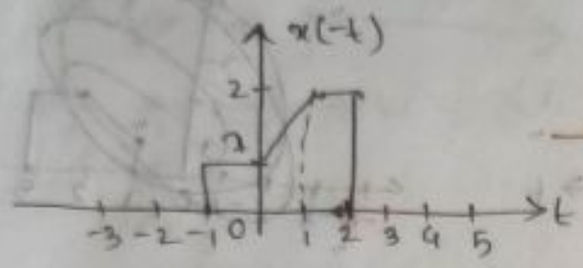
$t_0 = +3$, so left shift

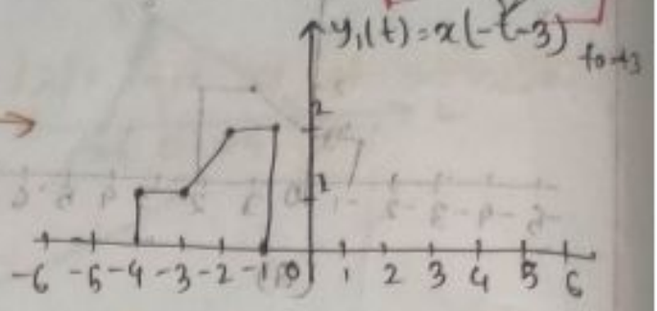
$[-t-(+3)]$

For the signal $x(-t)$ is negative

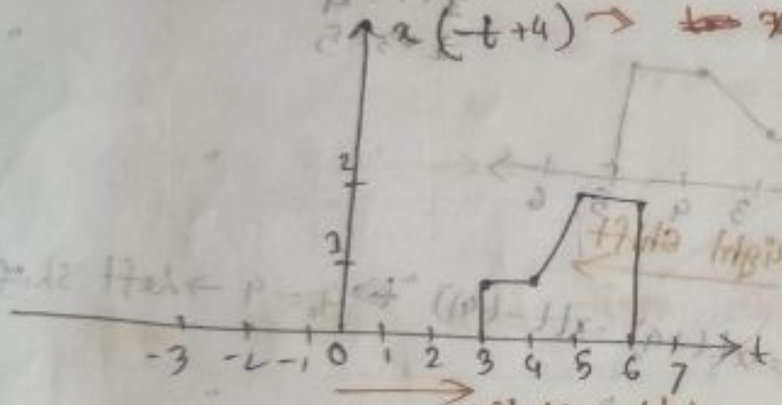
if $t_0 > 0 \rightarrow$ left shift if $t_0 < 0 \rightarrow$ Right shift.

Reverse of $x(t)$





Left shifted (3 units)



$x(t-(-4))$ $\therefore t_0 = -4$

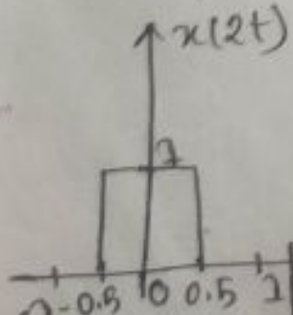
$\therefore$ Right sift (4 units)

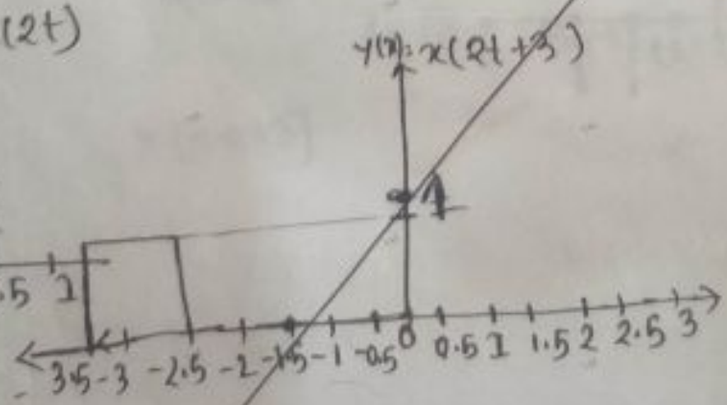
Right shift (4 units)

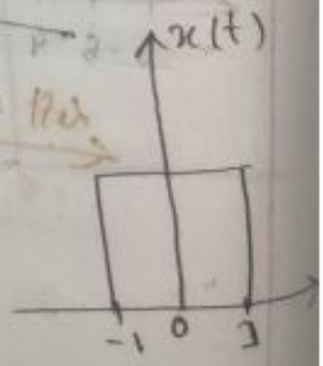
Page - 155-45

Problem-2.33) i) Find $x(2t+3)$ for a given signal $x(t)$

~~x(2t)~~







$a = 2$

$b = -3$

$y(0) = x(-b) = 0$

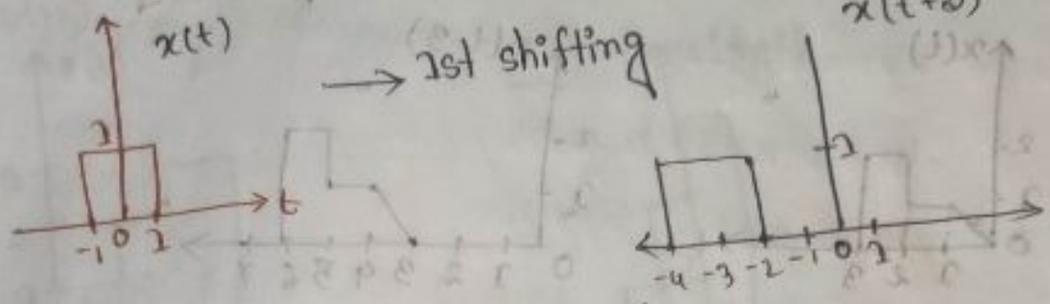
$y(b/a) = x(0)$

Page- 155-45

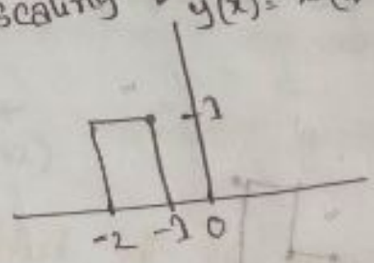
~~Elements~~

1st shift
2nd scale

2.33) i) $x(t)$ we have to find $x(2t+3)$



2nd scaling



here, $a = 2$

$b = -3$

$y(0) = 0$, $x(b) = x(-3) = 0$

$\therefore y(0) = x(b) = 0$

And,

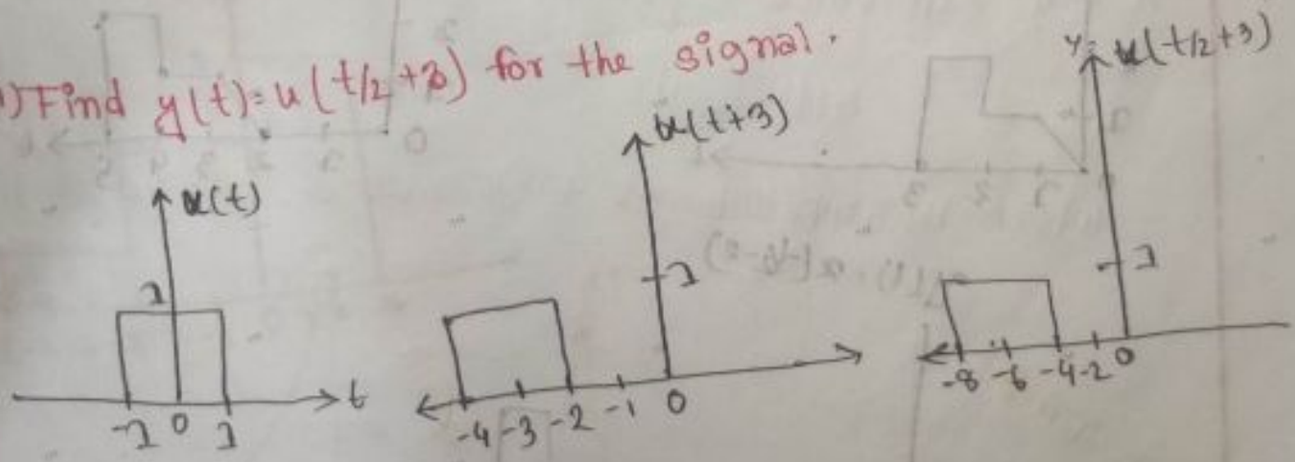
$y(b/a) = y(-3/2) = y(-1.5) = 1$

$x(0) = 1$

$\therefore y(a/b) = x(0) = 1$

$\therefore$ Satisfied

ii) Find $y(t) = u(t/2 + 3)$ for the signal.

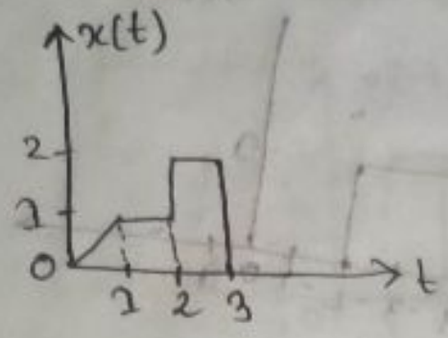


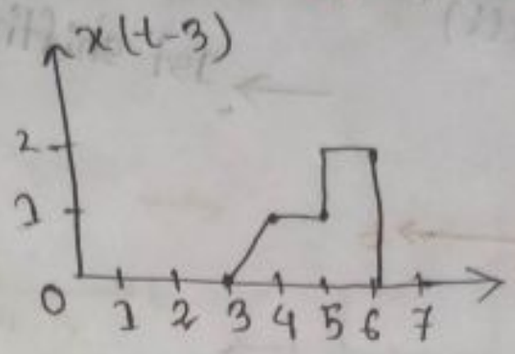
$y(0) = 0$ $u(b) = 0$ $\therefore y(0) = u(b)$

$y(b/a) = y(\frac{-6}{2}) = y(-3) = 1$ $x(0) = 1$

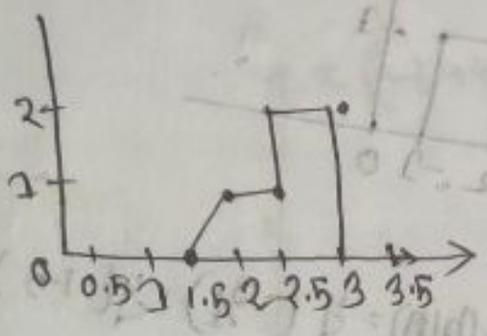
$\therefore y(b/a) = x(0)$

Problem 2.34) Find $x(2t-3)$ for the given signal $x(t)$



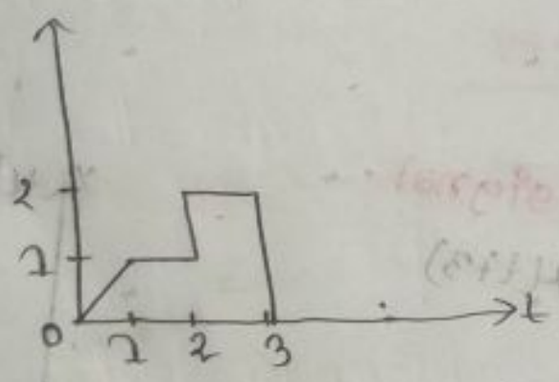


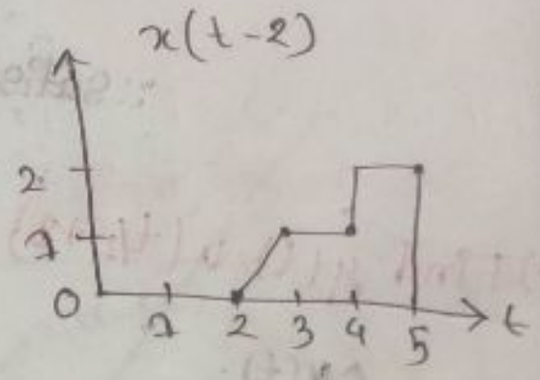
$x(2t-3)$



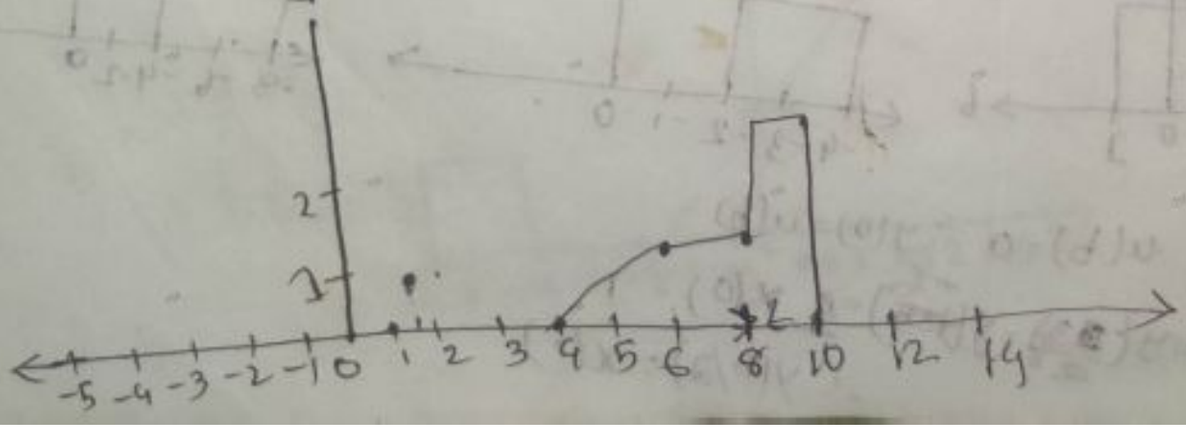
2.35) Find $x\left(\frac{t-4}{2}\right)$

$x(t/2-2)$



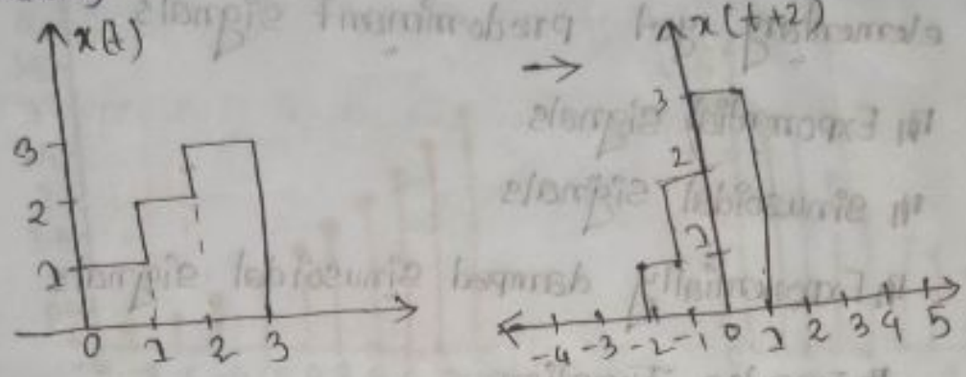


$y(t) = x(t/2-2)$

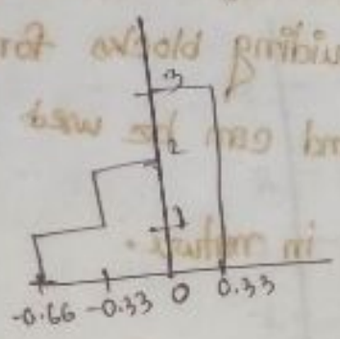


2.36) Find $x(3t+2)$ and $x(t/3+2)$ for the given signal.

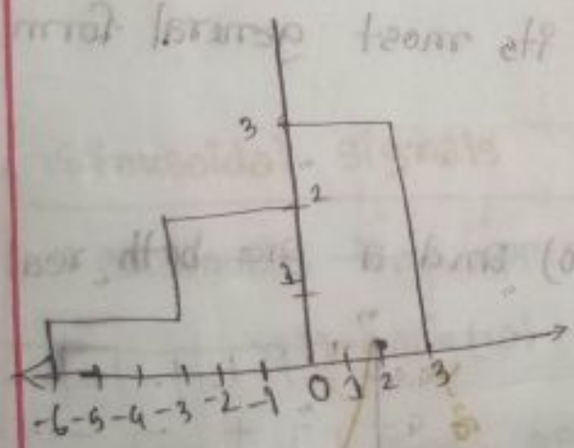
Ans $x(3t+2)$



~~x(t/3+~~
$x(3t+2)$



$x(t/3+2)$



TYPES OF SIGNALS

Some elementary yet predominant signals

- Exponential signals
- Sinusoidal signals
- Exponentially damped sinusoidal signals
- Impulse Function.
- Ramp Function.

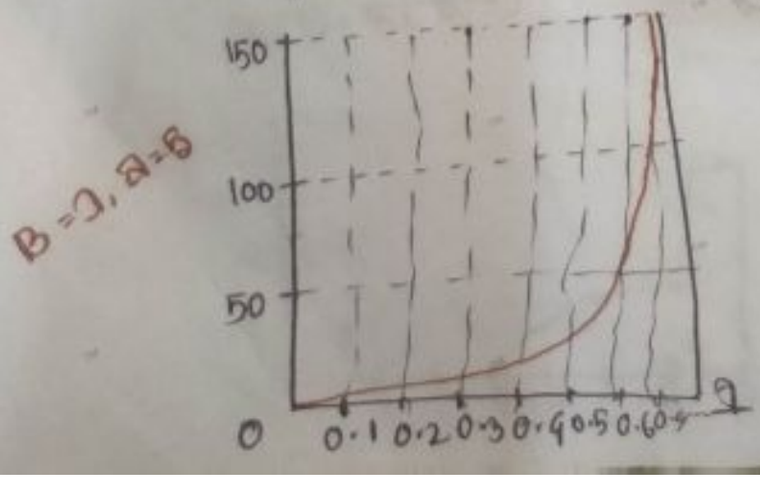
This signals serve as the buiding blocks for the construction of more complex signals and can be used to model many physical signals that occur in nature.

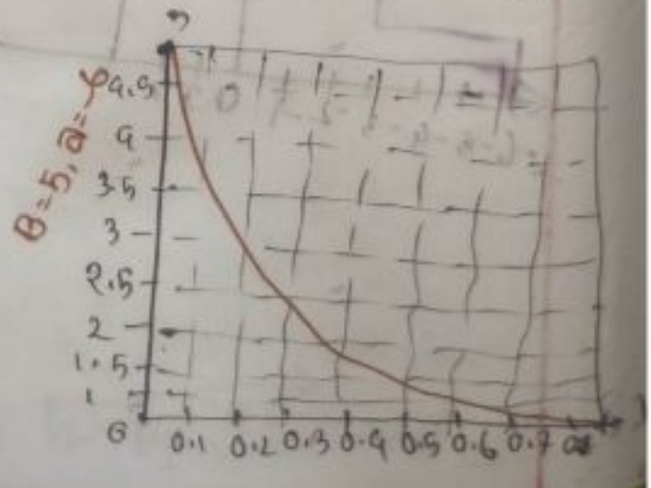
Exponential Signals:

A real exponential signal in its most general form is

$$x(t) = Be^{at}$$

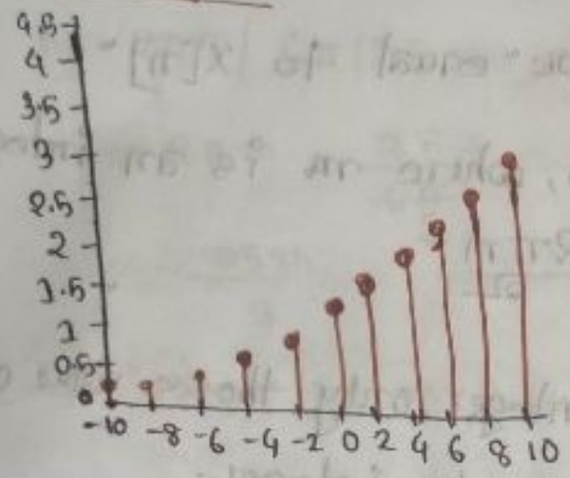
where B (amplitude at $t=0$) and a are both real.



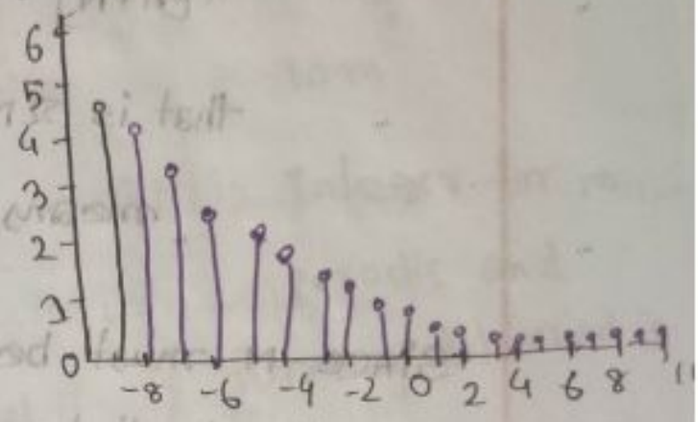


sinusoid signal is a function that describes a smooth periodic oscillation. Cos signal is also a sine signal with phase dif $\pi/2$

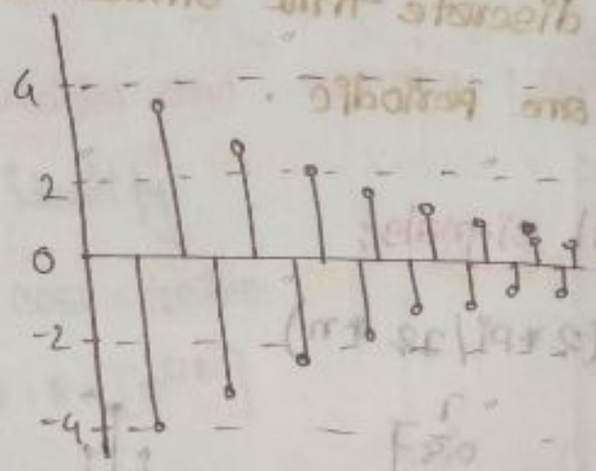
Exponential signals: discrete-time



$B=1,\ r=1.15$



$B=1,\ r=0.85$



$B=1,\ r=-0.85$

Sinusoidal signals

Continuous time approach,

$$x(t)=A\cos(\omega t+\varphi)$$

Discrete time approach,

$$x[n]=A\cos(\Omega n+\varphi)$$

Again $x[n+N]=A\cos(\Omega n+\Omega N+\varphi)$

To be periodic with period of N samples,

$x[n+N]$ must be equal to $x[n]$

that is $\Omega N = 2\pi m$, where m is an integer,

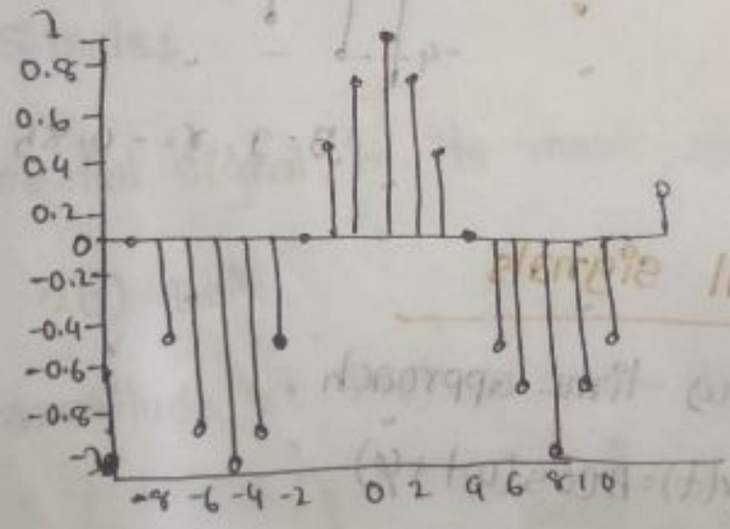
means $N = \frac{2\pi m}{\Omega}$

Since N must be an integer, only those value of Ω are allowed that makes $2\pi m/\Omega$ integer.

Not all discrete time sinusoidals with arbitary value of Ω are periodic.

Sinusoidal signals:

cos(2*pi/12*n)



$x[n] = 5\sin(2n)$

Here, $N = \frac{2\pi m}{2}$

N can't be integer for any integer value of m.

∴ Non-periodic

$\Omega N = 2\pi m$

$x[n] = 5\sin(6\pi n/35)$ $\quad \Omega = \frac{6\pi}{35}$

$N = 2\pi m / 6\pi/35$

$= 2\pi m \times \frac{35}{6\pi}$

$\frac{35m}{3}$

$\frac{6\pi}{35} n = 2\pi m$

$n = \frac{35}{3} m$

N is integer for $m = 3, 6, 9 \ldots$

$\therefore$ Periodic and fundamental period is 35

$x[n] = 5\sin(0.2\pi n)$

$N = \frac{2\pi m}{0.2\pi}$

$= 10m$

$\therefore$ N is integer for $m = 1, 2 \ldots$

$\therefore$ Periodic and fundamental period is 10.

Relation between sinusoidal and complex Exponential signals.

From Euler's identity

$$e^{j\theta} = \cos\theta + j\sin\theta$$

$$\Rightarrow \cos\theta = Re[e^{j\theta}]$$

Similarly,

$$A\cos(\omega t + \varphi) = A\,Re[e^{j(\omega t + \varphi)}]$$

$$\Rightarrow A\cos(\omega t + \varphi) = Re[Be^{j\omega t}]$$

where, $B = Ae^{j\varphi}$

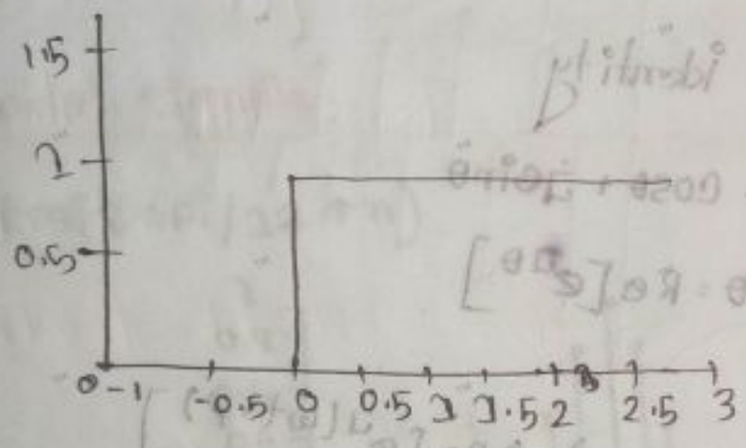
Similarly, $A\sin(\omega t + \varphi) = Im[Be^{j\omega t}]$

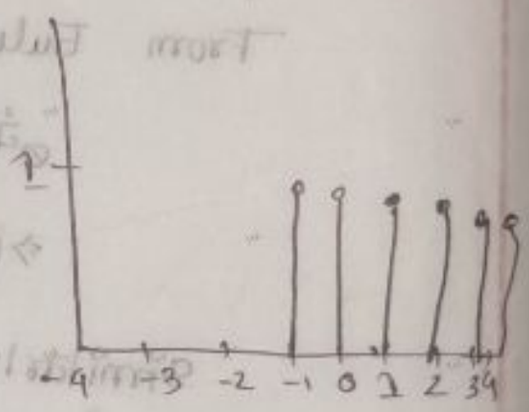
Unit step Function:

Defined by,

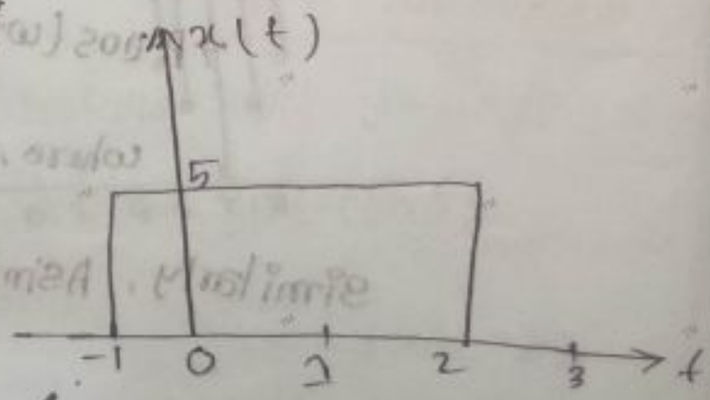
$$u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

used for testing and defining other signals.
For example, when different shifted versions of the step function are multiplied by other signals, one can select a certain portion of the signal and zero out the rest.



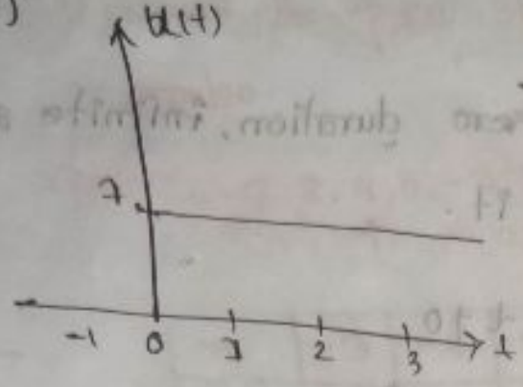


$$x(t) = \begin{cases} 5, & -1 \leq t \leq 2 \\ 0, & \text{otherwise} \end{cases}$$

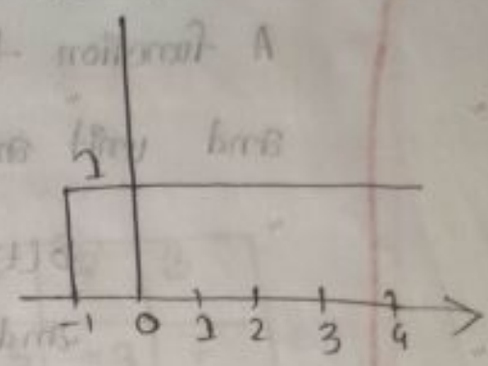


Express it with step func

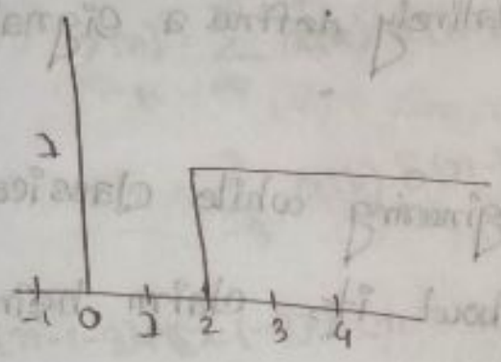
$u(t)$



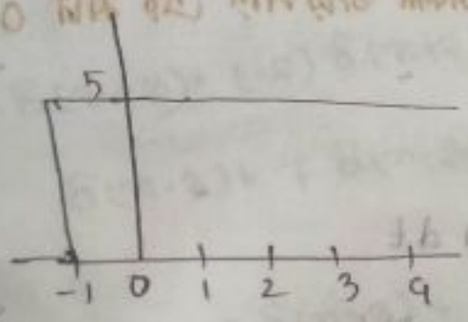
$u(t+1)$



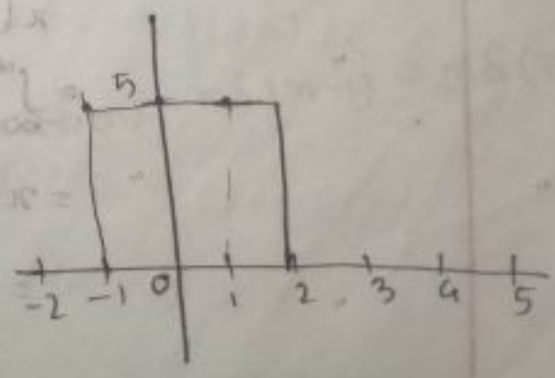
$u(t-2)$



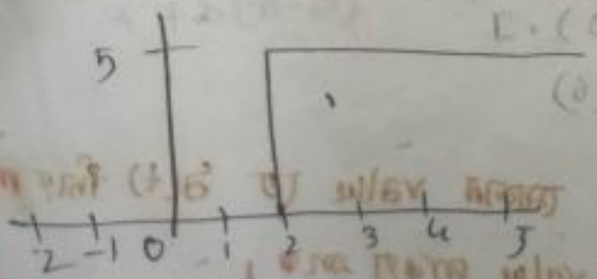
$5*u(t+1)$



$\rightarrow 5*u(t+1) - 5*u(t-2)$



$5*u(t-2)$



Impulse Function:

A function that has zero duration, infinite amplitude and unit area under it.

$$\delta(t) = 0 \quad \text{for } t \neq 0$$

and $$\int_{-\infty}^{+\infty} \delta(t)\,dt = 1$$

- It is used to quantitatively define a signal that occured momentarily.
- used heavily in engineering while classical mathematics still doubt about its claim being a function.

Impulse function এ 0 ব্যতীত সকল জায়গায় এর মান 0.

$\delta(t)$ এর মান 1.

$$x(t)\,\delta(t)$$
$$= \int_{-\infty}^{\infty} x(0)\,\delta(0)\,dt$$
$$= x(0) \int_{-\infty}^{\infty} \delta(0)\,dt$$
$$= x(0) \cdot 1$$
$$= x(0)$$

কোন signal $x(t)$ কে t এর যেকোন value তে $\delta(t)$ দিয়ে multiply করলে $x(t)$ এর ঐ point এর value পাওয়া যাবে।

Problem 3.1) Express the given signal sequence as a time-shifted impulse.

$x(n) = \{1, -2, 8, 4, 5, -3, 7\}$

Soln:

n	-3	-2	-1	0	1	2	3
$x(n)$	1	-2	8	4	5	-3	7

$$x(n) = \sum_{-\infty}^{\infty} x(k)\,\delta(n-k)$$

$$= \sum_{-3}^{3} x(k)\,\delta(n-k)$$

$$= x(-3)\delta(n-(-3)) + x(-2)\delta(n-(-2)) + x(-1)\delta(n-(-1)) + x(0)\delta(n-0) + x(1)\delta(n-1) + x(2)\delta(n-2) + x(3)\delta(n-3)$$

$$= 1\,\delta(n+3) + (-2)\,\delta(n+2) + 8\,\delta(n+1) + 4\,\delta(n) + 5\,\delta(n-1) + (-3)\,\delta(n-2) + 7\,\delta(n-3)$$

$$= \delta(n+3) - 2\,\delta(n+2) + 8\,\delta(n+1) + 4\,\delta(n) + 5\,\delta(n-1) - 3\,\delta(n-2) + 7\,\delta(n-3)$$

problem 3.2) Express the given signal sequence as time-shifted impulse.

$$x(n) = \{2, 3, \underset{\uparrow}{0}, 7, 8, -15, 18, 20\}$$

Sol^n:

n	-2	-1	0	1	2	3	4	5
$x(n)$	2	3	0	7	8	-15	18	20

$$x(n) = \sum_{k=-\infty}^{\infty} \cancel{x(k)\,\delta(x-n)}\; x(k)\,\delta(n-k)$$

$$= x(-2)\,\delta(n-(-2)) + x(-1)\,\delta(n-(-1)) + x(0)\,\delta(n-0) + x(1)\,\delta(n-1)$$

$$+ x(2)\,\delta(n-2) + x(3)\,\delta(n-3) + x(4)\,\delta(n-4) + x(5)$$

$$\delta(n-5)$$

$$= 2\,\delta(n+2) + 3\,\delta(n+1) + 0\,\delta(n) + 7\,\delta(n-1) + 8\,\delta(n-2)$$

$$+ (-15)\,\delta(n-3) + 18\,\delta(n-4) + 20\,\delta(n-5)$$

$$= 2\delta(n+2) + 3\delta(n+1) + 7\delta(n-1) + 8\delta(n-2) - 15\delta(n-3)$$

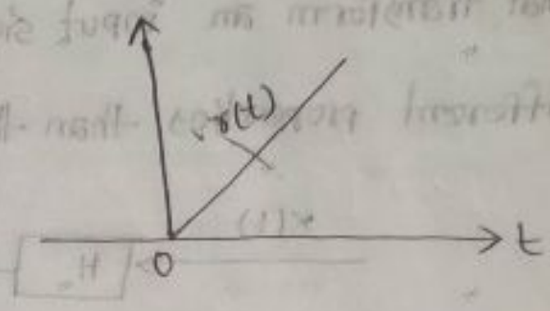
$$+ 18\,\delta(n-4) + 20\,\delta(n-5)$$

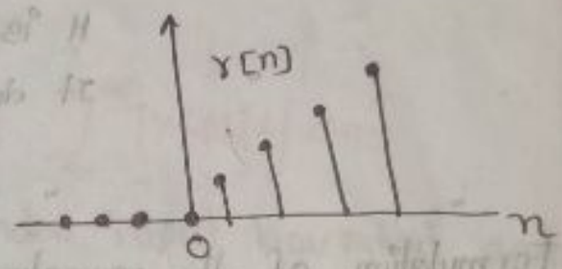
RAMP FUNCTION:

Ramp function is designed by $r(t)$ or $r[n]$ where,

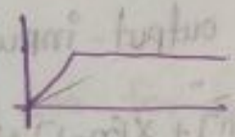
$$r(t) = \begin{cases} t, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

$$r[n] = \begin{cases} n, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

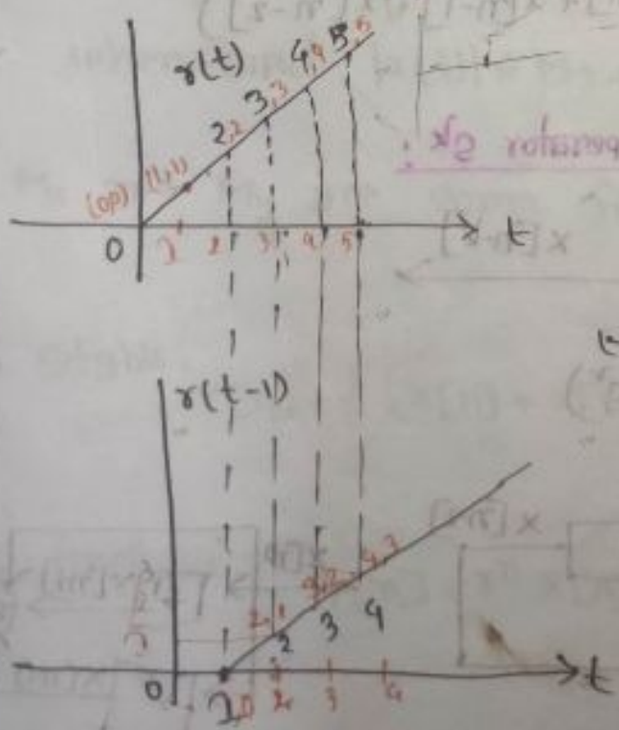


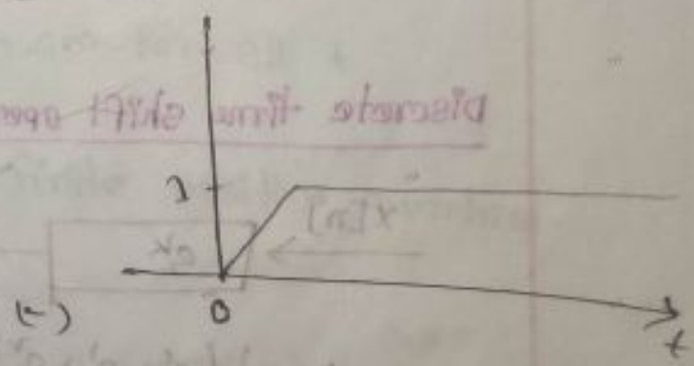


#২ ramp function fig,



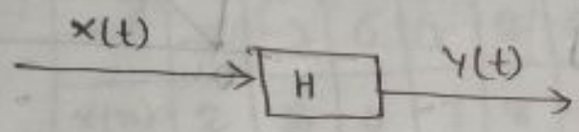
signal produce করে।





System viewed as interconnection of operations:

A system can be viewed as interconnections of operations that transform an input signal into an output signal having different properties than the input signal.

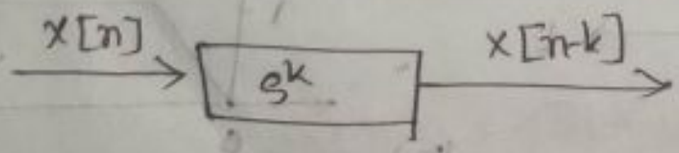


H is the overall operator.
It denotes the action of the system

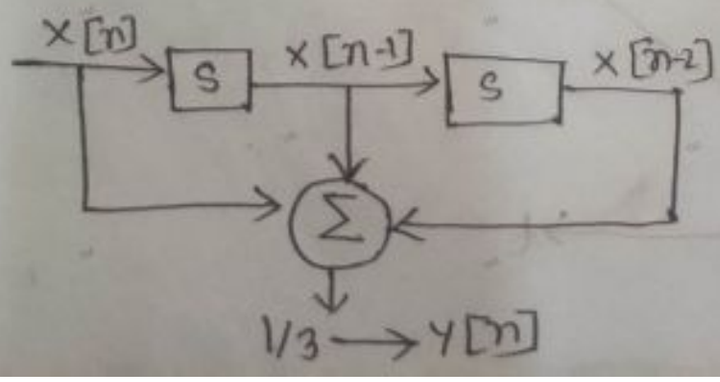
Formulation of the operator H for discrete-time system that has the following output-input relationship

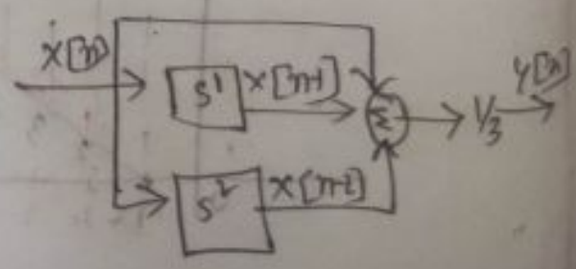
$$y[n] = \frac{1}{3}\left(x[n] + x[n-1] + x[n-2]\right)$$

Discrete time shift operator S^k:



$$H = \frac{1}{3}\left(S^0 + S^1 + S^2\right)$$





Properties of systems:

1. Stability.
2. Memory
3. Causality
4. Invertibility
5. Time invariance
6. Linearity

$$\sum_{-\infty}^{\infty} |y(n)| < \infty \Rightarrow \int_{-\infty}^{\infty} |y(t)dt| < \infty$$

Stability: A system is bounded input bounded output (BIBO) stable, A system is said to be stable if every bounded input results in bounded output.

Output $|y(t)| \leq M_y < \infty$ for all t

when input $|x(t)| \leq M_x < \infty$ for all t

where M_x and M_y are some finite positive numbers.

BIBO stable, $y[n] = \frac{1}{3}\left(x[n] + x[n-1] + x[n-2]\right)$

BIBO unstable, $y[n] = r^n x[n], r > 1$

Example 3.3) Find whether the system with impulse response $h(n) = 2e^{-2|n|}$ is stable or not.

Soln:

The condition for stability is,

$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty$$

$$\Rightarrow \sum_{-\infty}^{\infty} |h(n)| = \sum_{-\infty}^{\infty} 2e^{-2|n|}$$

$$= 2\left(\sum_{n=-\infty}^{-1} e^{-2(-n)} + \sum_{n=0}^{\infty} e^{-2(n)}\right)$$

$$= 2\left(\sum_{n=1}^{\infty}\right.$$

Memory

A system is said to possess memory if the output signal depends on the past values of the input signal.

Ex:

- voltage across a capacitor or current through an inductor.

$$v(t) = \frac{1}{C}\int_{-\infty}^{t} i\,dT$$

- system having the following output-input relation.

$$y[n] = \frac{1}{3}\{x(n) + x(n-1) + x[n-2]\}$$

In a memoryless system the output signal depends only on the present value of the input signal.

- Current or voltage of a resistor
- System having the following output-input relationship.

$$y[n] = 3x[n]$$

Causality

Causal system: The output signal depends only on the present or past values of the input signal.

Ex:

- voltage across a capacitor or current through an inductor.
- System having the following output-input relationship.

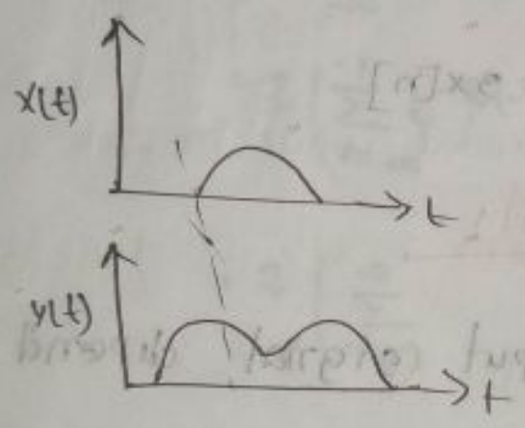
$$y[n] = \frac{1}{3}(x[n] + x[n-1] + x[n-2])$$

Causal ?° case 0, 1, -1 imp check.

Non-causal system: The output signal depends on future values of the input signal. No physical or real systems are non causal.

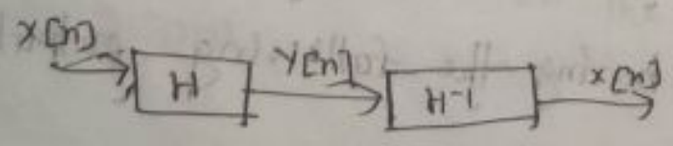
Ex: system having the following input-output relationship.

$$y[n] = \frac{1}{3}(x[n] + x[n+1] + x[n-1])$$



Invertibility

A system is invertible if the ~~ou~~ input can be reconstructed from the output



$$H^{-1}\{y[n]\} = H^{-1}\{H\{x[n]\}\}$$

$$= H^{-1}H\{x[n]\}$$

To be invertible we require that

$$H^{-1}H = I \quad \text{where } I \text{ is the identity operator}$$

$y[n] = x^2[n]$ [noninvertible]

$y(t) = \frac{d}{dt}\{x(t)\} \rightarrow$ noninvertible

$y[n] = x^3[n] \rightarrow$ invertible

Time invariance

A system is time-invariant if a time shift in the input signal leads to an identical time shift in the output signal.

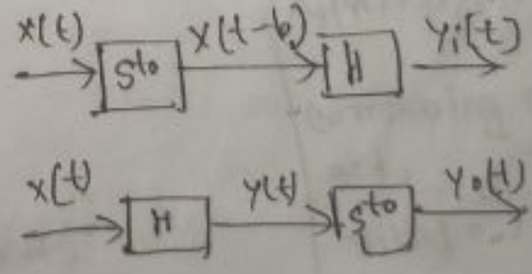
Considering a system where $y(t) = H\{x(t)\}$

Now if the signal is delayed by t_0, the new input will be $x(t - t_0)$.

Let $y_i(t)$, be the new output

$y_i(t) = H\{S^{t_0}\{x(t)\}\}$

$y_i(t) = HS^{t_0}\{x(t)\}$



Now let's assume $y_0(t)$ is the output of the original system shifted by t_0

$y_0(t) = S^{t_0}\{y(t)\}$

$$y_0(t) = S^{t_0}\{H\{x(t)\}\}$$

$$y_0(t) = S^{t_0} H\{x(t)\}$$

To be time invariant, $HS^{t_0} = S^{t_0} H$ i.e. H and S^{t_0} must commute.

Examples of time invariant (Orange khata class work) Last page.

Page 119-68

2.45) i) $y(n) = \cos[x(n)]$

$$y(n) = T[x(n)] = \cos[x(n)] \quad \text{—— (i)}$$

Introduce time delay in input,

$$x_1[n] = x(n-n_0)$$

$$y_1(n) = \cos[x_1(n)] = \cos[x(n-n_0)] \quad \text{—— (ii)}$$

Now introduce time delay in output,

$$y(n-n_0) = \cos[x(n-n_0)] \quad \text{—— (iii)}$$

Comparing (ii) and (iii) we get

$$y_1(n) = y(n-n_0)$$

$\therefore$ System is time invariant.

ii) $y(n) = \ln[x(n)]$

$$y(n) = T[x(n)] = \ln[x(n)] \quad \text{—— (i)}$$

Introduce time delay in input,

$$x_1[n] = x(n-n_0)$$

$$\Rightarrow y_1(n) = \ln[x(n-n_0)] \quad \text{—— (ii)}$$

Then introduce time delay n_0 in the output

$$y(n-n_0) = \ln[x(n-n_0)] \quad \text{—— (iii)}$$

Comparing (ii) an (iii) we get,

$$y_1(n) = y(n-n_0)$$

$\therefore$ system is time invariant

iii) $y(n) = x(n) - x(n-1)$

Introduce time delay in input

~~$x_1(n) = x(n-n_0)$~~

~~$x_1(n) = x(n-n_0) - x(n-n_0-1)$~~

~~$y_1(n) = x_1(n)$~~

$x_1(n) - x_1(n-1) = x(n-n_0) - x(n-n_0-1)$ — (i)

$y_1(n) = x_1(n) - x_1(n-1)$

$\Rightarrow y_1(n) = x(n-n_0) - x(n-n_0-1)$ — (ii)

Introduce time delay in output

$y(n-n_0) = x(n-n_0) - x(n-n_0-1)$ — (iii)

comparing (ii) and (iii)

$y_1(n) = y(n-n_0)$

∴ system is time invariant

iv) $y(n) = x(n)\, x(n+1)$

Introduce time delay n_0 in inp

$x_1(n)\, x_1(n+1) = x(n-n_0)\, x(n-n_0+1)$

$y_1(n) = x_1(n)\, x_1(n+1)$

$\Rightarrow y_1(n) = x(n-n_0)\, x(n-n_0+1)$ — (

Introduce time delay n_0 in output,

$y(n-n_0) = x(n-n_0)\, x(n-n_0+1)$ —

comparing (ii) and (iii)

$y_1(n) = y(n-n_0)$

∴ system is time invariant

Linear & Non-linear

2.47) $y(t) = tx(t)$

Let us define the input signal $x_1(t)$ whose response $y_1(t)$ is given by

$y_1(t) = \cancel{t_1 x(t)}\ t x_1(t)$ — (i)

Similarly, let us define another input signal $x_2(t)$ whose respons $y_2(t)$ is given by

$y_2(t) = t x_2(t)$ — (ii)

The two defined signals are related by $x_3(t) = ax_1(t) + bx_2(t)$

Linear and Non-linear check.

$T[ax_1(t)+bx_2(t)] = ay_1(t) + by_2(t)$

Then the output $y_3(t)$ is defined as

$$y_3(t) = t\,x_3(t)$$

$$\Rightarrow y_3(t) = t[ax_1(t)+bx_2(t)]$$

$$\Rightarrow y_3(t) = at\,x_1(t) + bt\,x_2(t)$$

$$\Rightarrow y_3(t) = ay_1(t) + by_2(t)$$

$\therefore$ system is linear

2.48) $y(t) = t\,x(t) + k$

Let us define the input signal $x_1(t)$ which results in an output $y_1(t)$

$$y_1 = t\,x_1(t) + k.$$

Another signal $x_2(t)$ which results in an output $y_2(t)$

$$y_2 = t\,x_2(t) + k$$

The above signals $x_1(t)$ and $x_2(t)$ are related as

$$x_3(t) = ax_1(t) + bx_2(t) \quad \text{here, } a, b \text{ are constants}$$

Output $y_3(t) = t\,x_3(t) + k$

$$= t[ax_1(t) + bx_2(t)] + k$$

$$= at\,x_1(t) + bt\,x_2(t) + k$$

$$\Rightarrow y_3(t) \neq ay_1(t) + by_2(t)$$

$\therefore$ non-linear

2.49) $y(n) = n^2 x(n-1)$

- Let input signal $x_1(n-1)$ which results in an output $y_1(n)$

$$y_1(n) = n^2 x_1(n-1)$$

Another input signal $x_2(n-1)$ and corresponding output $y_2(n)$

$$y_2(n) = n^2 x_2(n-1)$$

Now,

$$x_3(n) = a x_1(n) + b x_2(n)$$

output $y_3(n) = n^2 x_3(n-1)$

$$\cancel{= n^2[a x_1(n) + b x_2(n)]} = n^2[a x_1(n-1) + b x_2(n-1)]$$

$$\cancel{= an^2} = a n^2 x_1(n-1) + b n^2 x_2(n-1)$$

$$= a y_1(n) + b y_2(n)$$

$\therefore$ Linear

2.50) $y(n) = x^2(n)$

Let x_1 be any input which corresponding output is $y_1(n)$

$$y_1(n) = x_1^2(n)$$

Similarly $y_2(n) = x_2^2(n)$

~~Output $y_3(n)$~~ Now, $x_3(n) = a x_1(n) + b x_2(n)$

$y_3(n) = x_3^2(n)$

$= [a x_1(n) + b x_2(n)]^2$

$= a^2 x_1^2(n) + 2a x_1(n)\, b x_2(n) + b^2 x_2^2(n)$

$y_3(n) \neq a y_1(n) + b y_2(n)$

2. b)

i) $y(t) = x(\sin t)$

~~Let us define~~ $x_1(t)$ and $y_2(t)$ as

~~$x_1(t) =$~~ $y_1(t)$

Let input signal $x_1(n-1)$ which results in an output

$y_1(t) = x_1(\sin t)$ —— (i)

And $x_2(n-1)$ is another input signal which results in an output

$y_2(t) = x_2(\sin t)$ —— (ii)

Now, $x_3(t) = a x_1(t) + b x_2(t)$

output $y_3(t) = x_3(\sin t) = a x_1(\sin t) + b x_2(\sin t)$

$= \cancel{a x_1(t) + b x_2(t)} = a y_1(t) + b y_2(t)$

$= \cancel{a}$

$\therefore$ system is Linear

ii) $y(t) = \sin[x(t)]$

Let input signal $x_1(t)$ result in an output

$$y_1(t) = \sin[x_1(t)] \quad \text{— (i)}$$

And another input signal $x_2(t)$ results in another input

$$y_2(t) = \sin[x_2(t)] \quad \text{— (ii)}$$

$$x_3(t) = a x_1(t) + b x_2(t)$$

output $y_3(t) = \sin[x_3(t)]$

$$= \sin[a x_1(t) + b x_2(t)]$$

$$\cancel{= a \sin[x_1(t)] + b \sin[x_2(t)]}$$

$$\cancel{= a y_1(t) + b y_2(t)}$$

$$= \sin a x_1(t) \cos b x_2(t) + \cos a x_1(t) \sin b x_2(t)$$

$$\therefore y_3(t) \neq a y_1(t) + b y_2(t)$$

iii) $y(t) = t^2 x(t-1)$

Let input signal x_1 results in an output

$$y_1(t) = t^2 x_1(t-1)$$

and $y_2(t) = t^2 x_2(t-1)$

$\cancel{x_3 = ax_1 + bx_2}$ $x_3(t) = ax_1(t) + bx_2(t)$

Output $y_3(t) = t^2 x_3(t-1)$

$$= t^2 [ax_1(t-1) + bx_2(t-1)]$$

$$= at^2 x_1(t-1) + bt^2 x_2(t-1)$$

$$\therefore y_3(t) = a y_1(t) + b y_2(t)$$

$\therefore$ Linear

iv) $y(t) = e^{x(t)}$

Let us define input signal x_1 that results in an output

$$y_1(t) = e^{x_1(t)}$$

Another input x_2

$$y_2(t) = e^{x_2(t)}$$

$$x_3(t) = ax_1(t) + bx_2(t)$$

Output becomes, $y_3(t) = e^{x_3(t)}$

$$= e^{ax_1(t) + bx_2(t)} \neq a y_1(t) + b y_2(t)$$

$\therefore$ Non-linear

2.52) i) $y(n) = m[x(n)]$

define input $x_1(n)$ as

$$y_1(n) = m[x_1(n)]$$

and $y_2(n) = m[x_2(n)]$

$$x_3(n) = a x_1(n) + b x_2(n)$$

and output $y_3(n) = m[x_3(n)]$

$$= m[a x_1(n) + b x_2(n)] \neq a y_1(n) + b y_2(n)$$

$$= \cancel{m}$$

Non-linear.

ii) $y(n) = x(n) - x(n-1)$

$\cancel{x_1(n) =}$ $y_1(n) = x_1(n) - x_1(n-1)$

$$y_2(n) = x_2(n) - x_2(n-1)$$

$$x_3(n) = a x_1(n) + b x_2(n)$$

output $y_3(n) = x_3(n) - x_3(n-1)$

$$= a x_1(n) + b x_2(n) - [a x_1(n-1) + b x_2(n-1)]$$

$$= a x_1(n) + b x_2(n) - a x_1(n-1) - b x_2(n-1)$$

$$= a x_1(n) - a x_1(n-1) + b x_2(n) - b x_2(n-1)$$

$$= a[x_1(n) - x_1(n-1)] + b[x_2(n) - x_2(n-1)]$$

$$= a y_1(n) + b y_2(n)$$ linear.

iii) $y(n) = x^2(n) + x^2(n-1)$

$y_1(n) = x_1^2(n) + x_1^2(n-1)$

$y_2(n) = x_2^2(n) + x_2^2(n-1)$

~~$x_3(n) = x_3^2(n) + x_3^2(n-1)$~~

$x_3(n) = a x_1(n) + b x_2(n)$

Output,

$y_3(n) = x_3^2(n) + x_3^2(n-1)$

$= [a x_1(n) + b x_2(n)]^2 + [a x_1(n-1) + b x_2(n-1)]^2$

~~$= a x_1(n) + a x_2(n-1)$~~

$= [a^2 x_1^2(n) + b^2 x_2^2(n) + 2a x_1(n) b x_2(n)] + [a^2 x_1^2(n-1) + b^2 x_2^2(n-1) + 2a x_1(n-1) b x_2(n-1)]$

$= a^2 [x_1^2(n) + x_1^2(n-1)] + b^2 [x_2^2(n) + x_2^2(n-1)] + 2ab [x_1(n) x_2(n) + x_1(n-1) x_2(n-1)]$

$\therefore y_3(n) \neq a y_1(n) + b y_2(n)$

$\therefore$ Non-Linear

iv) $y(n) = 2x(n) + 4$

$y_1(n) = 2x_1(n) + 4 \quad - x_1(n)$

$y_2(n) = 2x_2(n) + 4 \quad - x_2(n)$

$x_3(n) = a x_1(n) + b x_2(n)$

$y_3(n) = 2x_3(n) + 4$

$= 2[a x_1(n) + b x_2(n)] + 4$

$= 2[a\{2x_1(n) + 4\} + b\{2x_2(n) + 4\}] + 4$

$\neq a y_1 + b y_2 \quad \therefore$ Non-Linear

~~$= 2a[2x_1(n) + 4] + b[2x_2(n) + 4] + 4$~~

~~$= 4a x_1(n) + 8 + 2b x_2(n) +$~~

Examples (causal), Non-causal class book khata te kiya, page have already marked.

LINEAR TIME INVARIANT SYSTEM:

The main objective is to study the relationships betw input and output of LTI system in time domain.

IMPULSE RESPONSE:

Out of a system due to an impulse input applied at $t=0$ or $n=0$.

Viva * একটি impulse এর against এ output at $=0$ time এ output বা result হবে unit impulse response

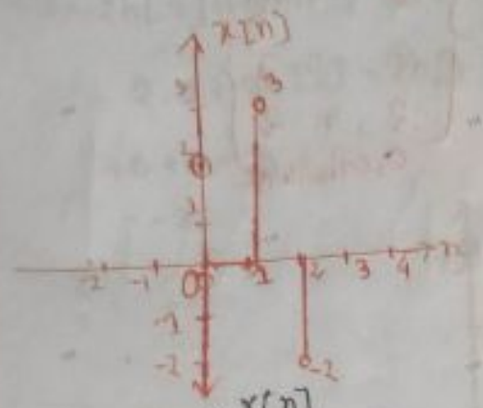
Type of convolution. (2 types)

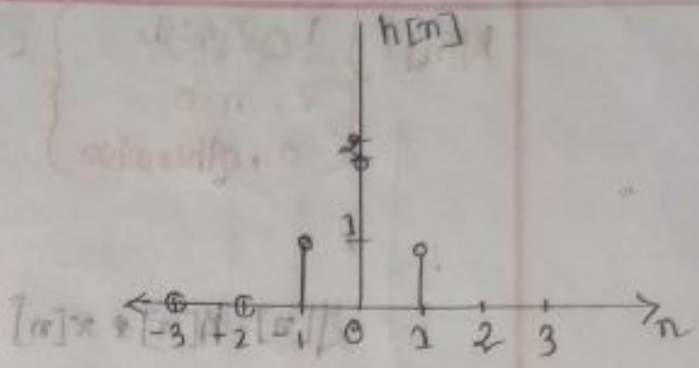
1. Sum → Discrete
2. Integral → Continuous

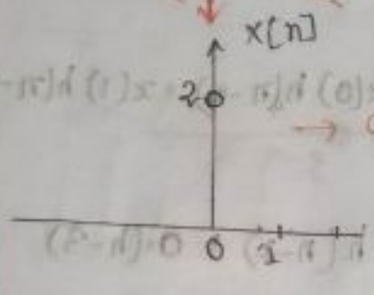
$H \rightarrow$ Impulse response

→ 1. Circular 2. Linear

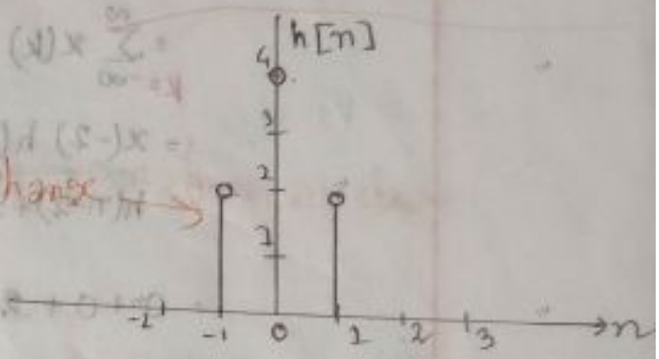
Graphical Representation of impulse function

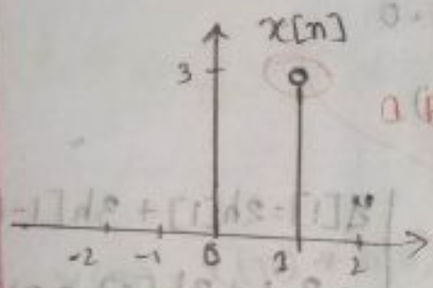




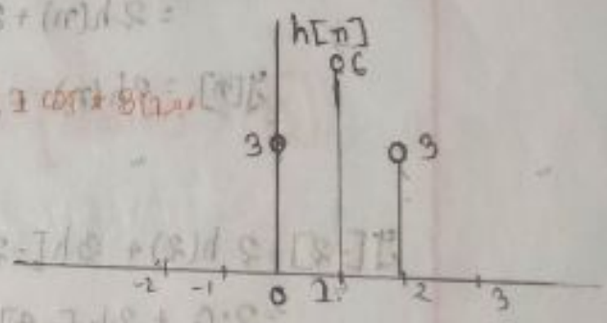


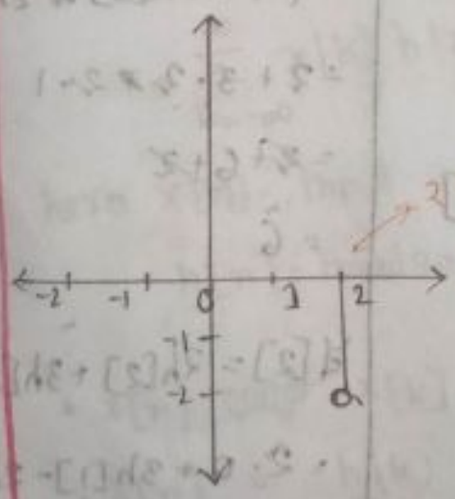
→ 0 position so no change



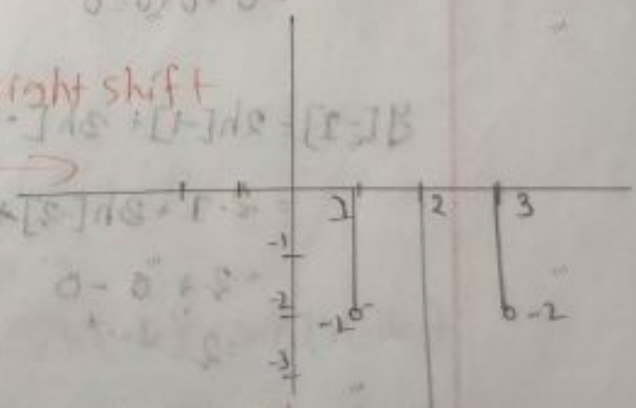


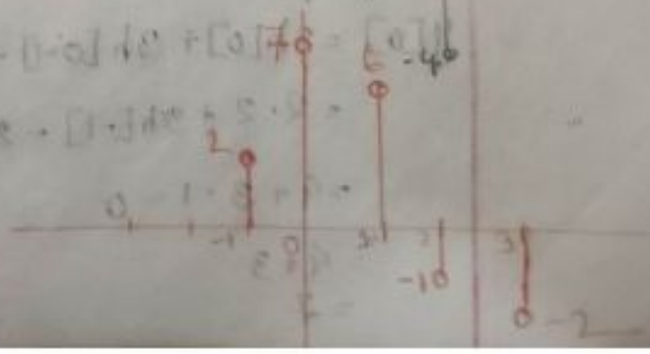
1 (position so right shift 1 and value × 2)





2 position, so right shift





batch 3(c)

Convolution Sum (Mathematical)

$$h[n] = \begin{cases} 1, & n=\pm 1 \\ 2, & n=0 \\ 0, & \text{otherwise} \end{cases}$$

$$x[n] = \begin{cases} 2, & n=0 \\ 3, & n=1 \\ -2, & n=2 \\ 0, & \text{otherwise} \end{cases}$$

$$y[n] = h[n] * x[n]$$

$$= \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

$$= x(-2)\, h(n+2) + x(-1)\, h(n+1) + x(0)\, h(n-0) + x(1)\, h(n-1) + x(2)\, h(n-2) + x(3)\, h(n-3)$$

$$= 0 + 0 + 2h(n) + 3h(n-1) + (-2)h(n-2) + 0\cdot(h-3)$$

$$= 2h(n) + 3h(n-1) - 2h(n-2) + 0$$

$$y[n] = 2h(n) + 3h(n-1) - 2h(n-2)$$

$$y[-2] = 2h(2) + 3h[-2-1] - 2h[-2-2]$$
$$= 2\cdot 0 + 3h[-3] - 2h[-4]$$
$$= 0 + 0 + 0 = 0$$

$$y[-1] = 2h[-1] + 3h[-1-1] - 2h[-1-2]$$
$$= 2\cdot 1 + 3h[-2] + 2h[-3]$$
$$= 2 + 0 - 0$$
$$= 2$$

$$y[0] = 2h[0] + 3h[0-1] - 2h[0-2]$$
$$= 2\cdot 2 + 3h[-1] - 2h[-2]$$
$$= 4 + 3\cdot 1 - 0$$
$$= 4 + 3$$
$$= 7$$

$$y[1] = 2h[1] + 3h[1-1] + 2h[1-$$
$$= 2\cdot 1 + 3h[0] + 2h[-1]$$
$$= 2 + 3\cdot 2 + 2\cdot 1$$
$$= \cancel{2} + 6 + \cancel{2}$$
$$= 6$$

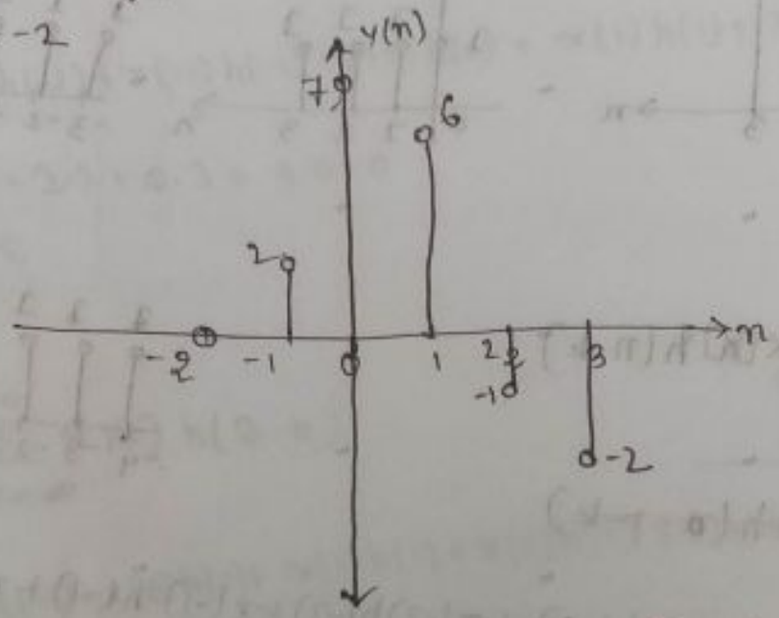
$$y[2] = 2h[2] + 3h[2-1] - 2h[$$
$$= 2\cdot 0 + 3h[1] - 2h[0]$$
$$= 0 + 3\cdot 1 - 2\cdot 2$$
$$= 3 - 4$$
$$= -1$$

$y[3] = 2h[3] + 3h[3-1] - 2h[3-2]$ $\qquad y[4] = 0$

$= 2.0 + 3h[2] - 2h[1]$

$= 0 + 0 - 2.1$

$= -2$



convolution sum [sliding way]

$$y(n) = x(n) * h(n)$$

$$= \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

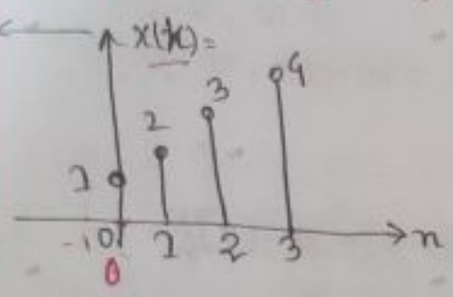
here $x[n]$ = Input

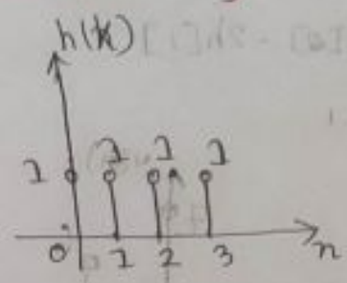
$h[n]$ = Impulse Response.

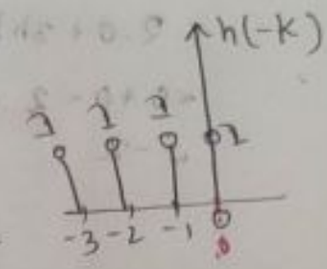
- $x[n]$ থেকে $x[k]$ বের করতে হবে।
- $h[n]$ " $h(k)$ " " " $\rightarrow h(-k) \rightarrow h(n-k)$

$x(n) = \{1, 2, 3, 4\}$, $h(n) = \{1, 1, 1, 1\}$

$x(n) = x(k)$





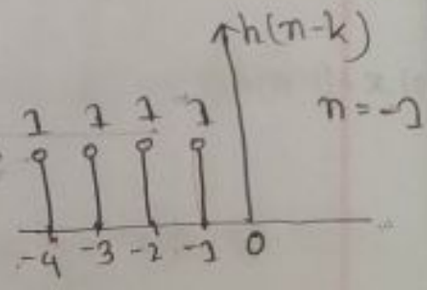


we know,

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

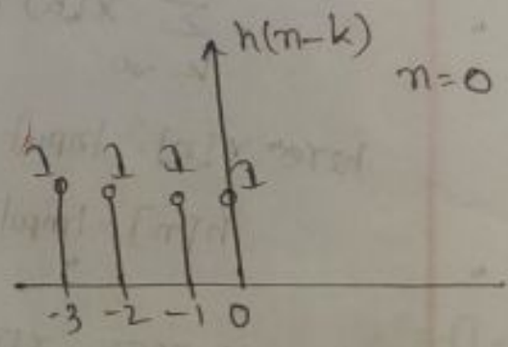
~~$y(-1) = \sum$~~

~~$y(-1) = \sum_{k=-\infty}^{\infty} x(k)\, h(-1-k)$~~



$$y(-1) = x(-4)h(-4) + x(-3)h(-3) + x(-2)h(-2) + x(-1)h(-1) + x(0)h(0) + x(1)h(1) + x(2)h(2) + x(3)h(3) + x(4)h(4)$$

$$= 0+0+0+0+0+0+0+0+0 = 0$$



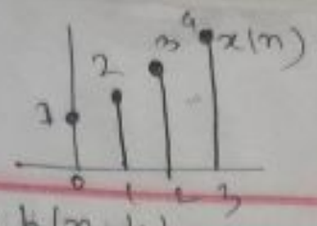
$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

~~$y(0) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-0)$~~

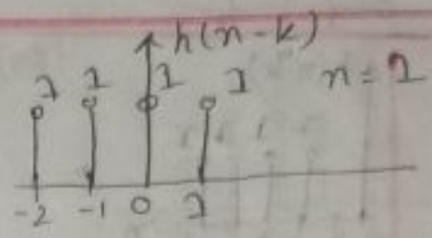
$$y(0) = x(-3)h(-3) + x(-2)h(-2) + x(-1)h(-1) + x(0)h(0) + x(1)h(1) + x(2)h(2) + x(3)h(3)$$

$$= 0+0+0+1\cdot 1 + 2\cdot 0 + 0 + 0$$

$$= 1$$



$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$



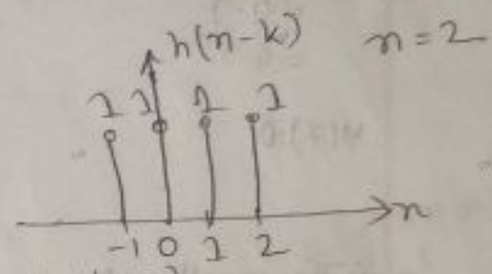
$$y(1) = \sum_{k=-\infty}^{\infty} x(k)\, h(1-k)$$

$$= x(-2)h(-2) + x(-1)h(-1) + x(0)h(0) + x(1)h(1) + x(2)h(2) + x(3)h(3)$$

$$= 0 + 0 + 1\cdot 1 + 2\cdot 1 + 3\cdot 0 + 0$$

$$= 1 + 2$$

$$= 3$$

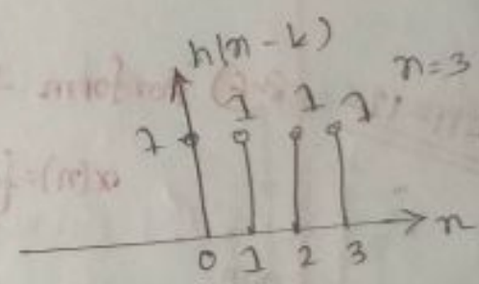


$$y(2) = \sum_{k=-\infty}^{\infty} x(k)\, h(2-k)$$

$$= x(-1)h(-1) + x(0)h(0) + x(1)h(1) + x(2)h(2) + x(3)h(3)$$

$$= 0 + 1\cdot 1 + 2\cdot 1 + 3\cdot 1 + 0$$

$$= 1 + 2 + 3 = 6$$

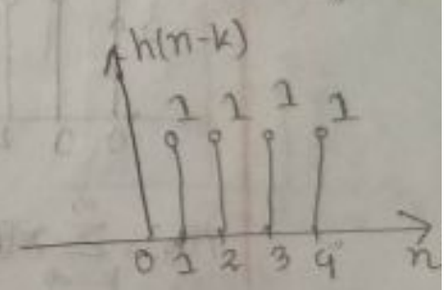


$$y(3) = \sum_{k=-\infty}^{\infty} x(k)\, h(3-k)$$

$$= x(0)h(0) + x(1)h(1) + x(2)h(2) + x(3)h(3)$$

$$= 1\cdot 1 + 2\cdot 1 + 3\cdot 1 + 4\cdot 1$$

$$= 1 + 2 + 3 + 4 = 10$$



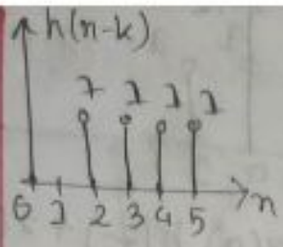
$$y(4) = \sum_{k=-\infty}^{\infty} x(k)\, h(4-k)$$

$$= x(0)h(0) + x(1)h(1) + x(2)h(2) + x(3)h(3) + x(4)h(4)$$

$$= 1\cdot 0 + 2\cdot 1 + 3\cdot 1 + 4\cdot 1 + 0$$

$$= 0 + 2 + 3 + 4 = 9$$

CamScanner

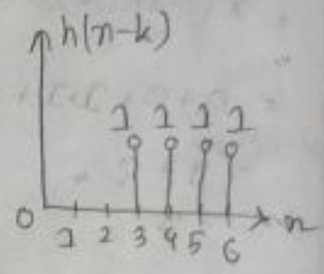


$y(5) = x(2)h(2) + x(3)h(3) + 0 + 0$

$= 3 \cdot 1 + 4 \cdot 1$

$= 3 + 4$

$= 7$



$y(6) = x(3)h(3)$

$= 4 \cdot 1$

$= 4$

$y(7) = 0$

$\therefore$ The result of convolution is

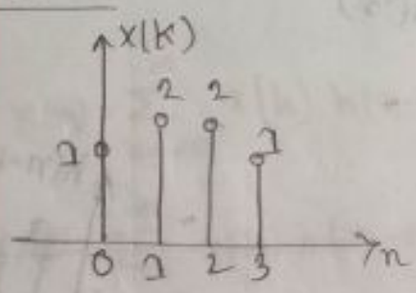
$$y(n) = x(n) * h(n) = \{1, 3, 6, 10, 9, 7, 4\}$$

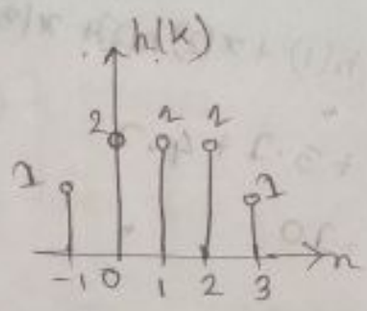
LT1-12

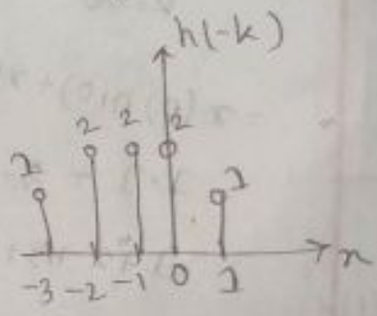
3.6) Perform the convolution of the given data sequences

$x(n) = \{\underset{\uparrow}{1}, 2, 2, 1\}$ $\quad h(n) = \{\underset{\uparrow}{1}, 2, 2, 2, 1\}$

Soln:







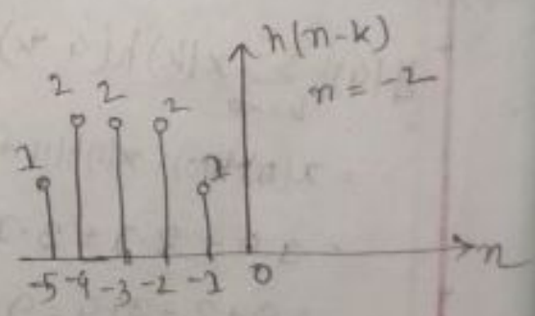
$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

$y(-2) = x(-5)h(-5) + x(-4)h(-4) + x(-3)h(-3)$

$+ x(-2)h(-2) + x(-1)h(-1) + x(0)h(0) + x(1)h(1)$

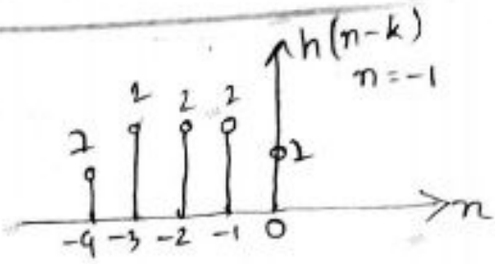
$+ x(2)h(2) + x(3)h(3)$

$= 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 = 0$



$y(-1) = x(-4)h(-4) + x(-3)h(-3) + x(-2)h(-2) + x(-1)h(-1) + x(0)h(0)$

$+ x(1)h(1) + x(2)h(2) + x(3)h(3)$

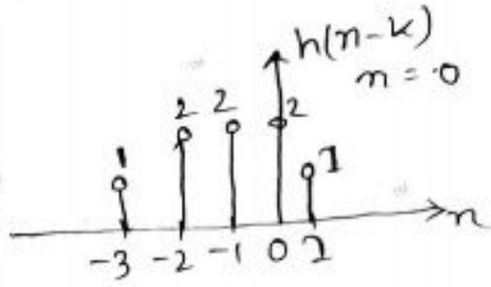


$= 0 + 0 + 0 + 1\cdot 1 + 2\cdot 0 + 0 + 0$

$= 1$

$y(0) = x(-3)h(-3) + x(-2)h(-2) + x(-1)h(-1) + x(0)h(0) + x(1)h(1)$

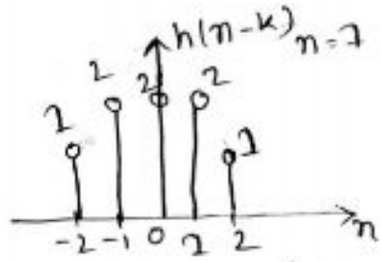
$+ x(2)h(2) + x(3)h(3)$



$= 0 + 0 + 0 + 1\cdot 2 + 2\cdot 1 + 0 + 0$

$= 2 + 2 = 4$

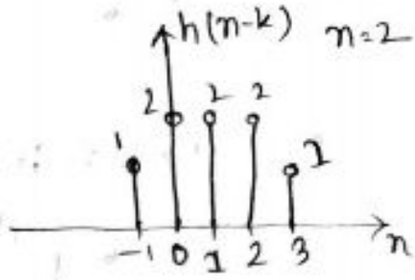
$y(1) = x(-2)h(-2) + x(-1)h(-1) + x(0)h(0) + x(1)h(1) + x(2)h(2) + x(3)h(3)$



$= 0 + 0 + 1\cdot 2 + 2\cdot 2 + 2\cdot 1 + 1\cdot 0$

$= 2 + 4 + 2 = 8$

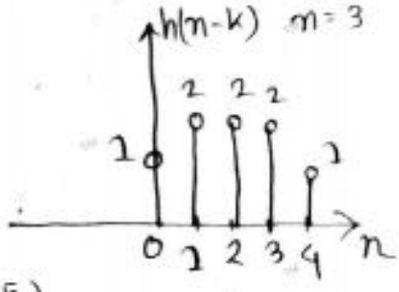
$y(2) = x(-1)h(-1) + x(0)h(0) + x(1)h(1) + x(2)h(2) + x(3)h(3)$



$= 0 + 1\cdot 2 + 2\cdot 2 + 2\cdot 2 + 1\cdot 1$

$= 2 + 4 + 4 + 1 = 11$

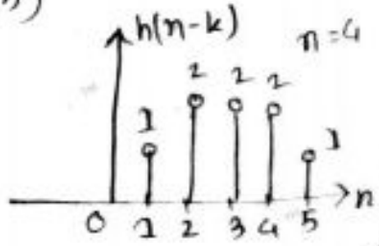
$y(3) = x(0)h(0) + x(1)h(1) + x(2)h(2) + x(3)h(3) + x(4)h(4)$



$= 1\cdot 1 + 2\cdot 2 + 2\cdot 2 + 1\cdot 2 + 0$

$= 1 + 4 + 4 + 2 = 11$

$y(4) = x(0)h(0) + x(1)h(1) + x(2)h(2) + x(3)h(3) + x(4)h(4) + x(5)h(5)$

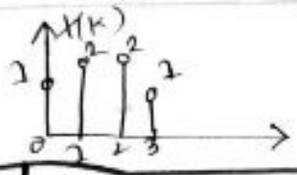


$= 1\cdot 0 + 2\cdot 1 + 2\cdot 2 + 1\cdot 2 + 0 + 0$

$= 2 + 4 + 2$

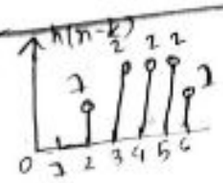
$= 8$

CamScanner



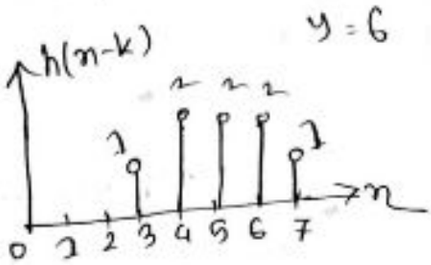
$y(5) = x(0)h(0) + x(1)h(1) + x(2)h(2) + x(3)h(3) + x(4)h(4)$
$+ x(5)h(5) + x(6)h(6)$

$= 0 + 0 + 2\cdot 1 + 1\cdot 2 + 0 + 0 = 2 + 2 = 4$



$y(6) = x(0)h(0) + x(1)h(1) + x(2)h(2) + x(3)h(3) + x(4)h(4)$
$+ x(5)h(5) + x(6)h(6) + x(7)h(7)$

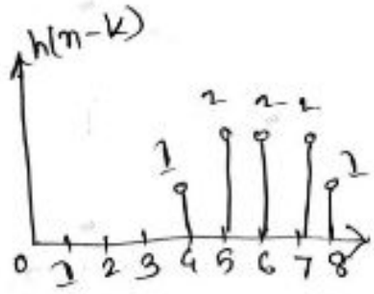
$= 0 + 0 + 0 + 1\cdot 1 + 0 + 0 + 0 + 0 = 1$



$y(7) = x(0)h(0) + x(1)h(1) + x(2)h(2) + x(3)h(3) + x(4)h(4)$
$+ x(5)h(5) + x(6)h(6) + x(7)h(7) + x(8)h(8)$

$= 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0$

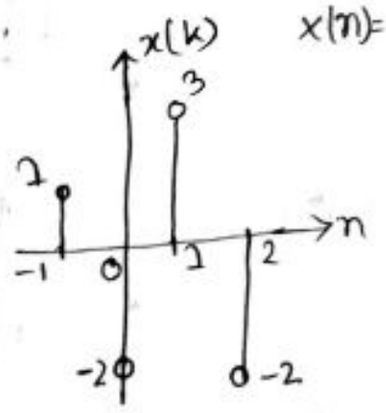
$= 0$

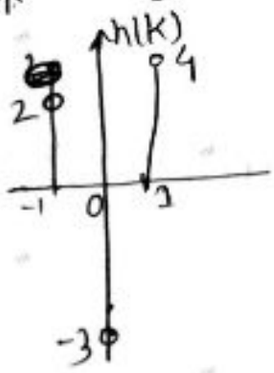


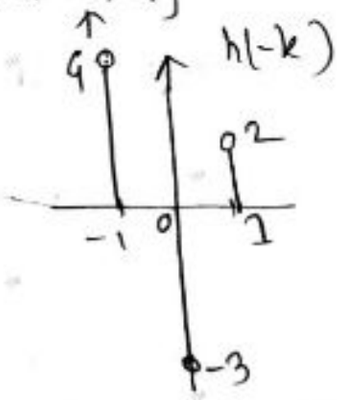
$\therefore$ The result of convolution is $y(n) = x(n) * h(n) = \{1, 4, 8, 10, 11, 8, 4, 1\}$

Ex-3.7) Perform the convolution of the given data sequences.

$x(n) = \{1, \underset{\uparrow}{-2}, 3, -2\}$; $h(n) = \{2, \underset{\uparrow}{-3}, 4\}$

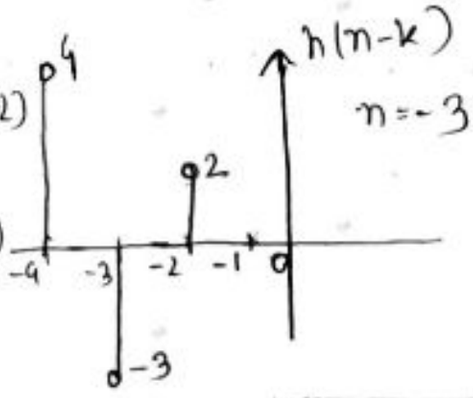


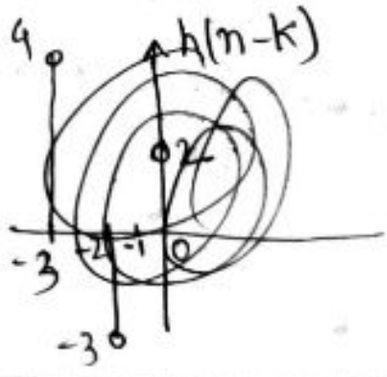




$y(-3) = x(-3)h(-3) + x(-2)h(-2)$
$+ x(-1)h(-1) + x(0)h(0)$
$+ x(1)h(1) + x(2)h(2)$

$= 0$



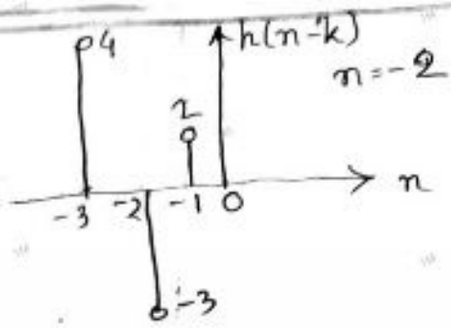


$y(-2) = x(-3)h(-3) + x(-2)h(-2) + x(-1)h(-1) + x(0)h(0) +$

$x(1)h(1) + x(2)h(2)$

$= 0 + 0 + 1 \cdot 2 + (-2) \cdot 0 + 0 + 0$

$= 2$

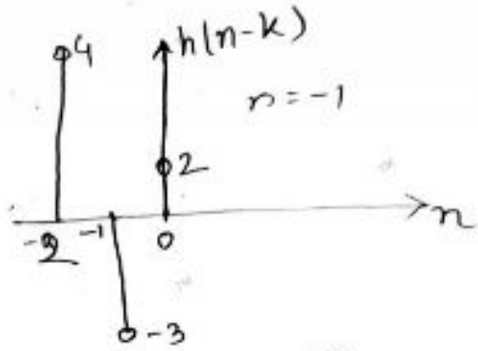


$y(-1) = x(-2)h(-2) + x(-1)h(-1) + x(0)h(0) + x(1)h(1) + x(2)h(2)$

$= 0 + 1 \cdot (-3) + (-2)\, 2 + 0 + 0$

$= -3 - 4$

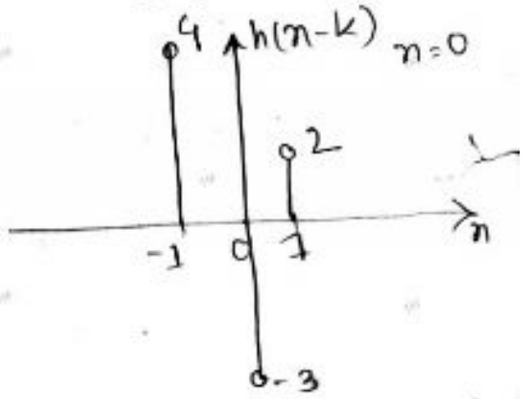
$= -7$



$y(0) = x(-1)h(-1) + x(0)h(0) + x(1)h(1) + x(2)h(2)$

$= 1 \cdot 4 + (-2)(-3) + 3 \cdot 2 + (-2) \cdot 0$

$= 4 + 6 + 6 = 16$

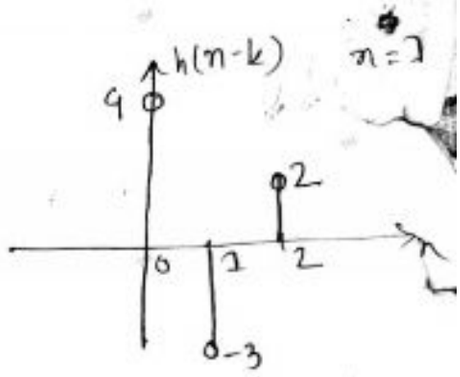


$y(1) = x(-1)h(-1) + x(0)h(0) + x(1)h(1) + x(2)h(2)$

$= 1 \cdot 0 + (-2)\, 4 + (-3)\, 3 + (-2)\, 2$

$= -8 - 9 - 4$

$= -21$

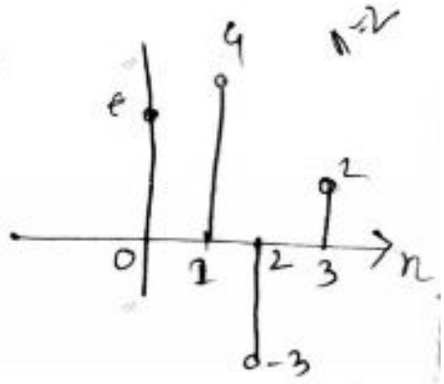


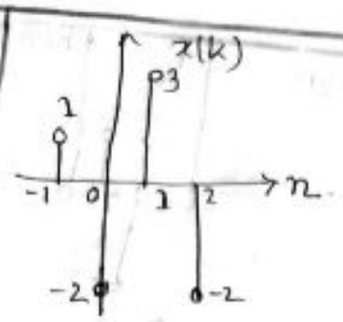
$y(2) = x(-1)h(-1) + x(0)h(0) + x(1)h(1) + x(2)h(2) + x(3)h(3)$

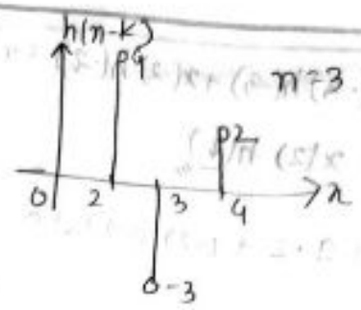
$= 1 \cdot 0 + 0 + 3 \cdot 4 + (-2)(-3) + 0 + 0$

$= 12 + 6$

$= 18$



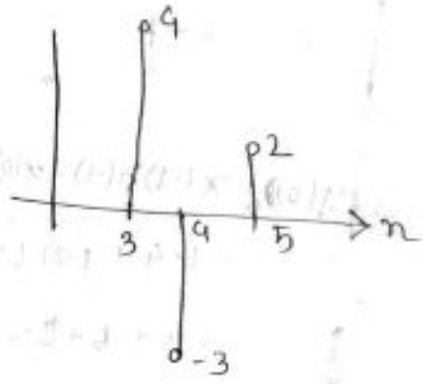




$Y(3) = x(-1)h(-1) + x(0)h(0) + x(1)h(1) + x(2)h(2) + x(3)h(3) + x(4)h(4)$

$= 1\cdot 0 + (-2)\cdot 0 + 0 + (-2)\cdot 4 + 0 + 0$

$= -8$



$Y(4) = x(-1)h(-1) + x(0)h(0) + x(1)h(1) + x(2)h(2)$

$+ x(3)h(3) + x(4)h(4) + x(5)h(5)$

$= 0 + 0 + 0 + 0 + 0 + 0$

$= 0$

$\therefore$ The result of the convolution is

$y(n) = x(n) * h(n)$

$= \{2, -7, 16, -21, 18, -8\}$

$= 0$

-3