Problem no: 01

Problem name:

Suppose we have a population of four measurements 2, 4, 6, 8. Draw a random sample of size 2 without replacement demonstrate that

(i) The sample mean is an unbiased estimate of population mean.

(ii) $V(\bar{y}) = \frac{\sigma^2}{n} \cdot \frac{N-n}{N-1}$

(iii) Veraify is s^2 an unbiased estimate of σ^2?

(iv) Find 95% confidence interval for population mean and population total.

(v) Answer the above questions for sampling with replacement.

Code:

(i)

```
data = c(2,4,6,8); data
sample.data = rbind(c(2,4), c(2,6), c(2,8), c(4,6),
                    c(6,8)); sample.data
pop.mean = mean(data); pop.mean
yibar = rowMeans(sample.data); yibar
E.ybar = sum(yibar*(1/6)); E.ybar
```

(ii)

```
E.yibar2 = sum((yibar^2)*(1/6)); E.yibar2
v.ybar = E.yibar - (E.ybar)^2; v.ybar
N = length(data); N
n = 2; n
var = var(data); var
sigma2 = var*(N-1)/(N-1)); R.H.S
```

(iii)

```
s2 = (sample.data[,1] - yibar)^2 + (sample.
    data[,2] - yibar)^2; s2
E.s2 = sum(s2*(1/6)); E.s2
```

(iv)

```
alpha = 0.05; alpha
ztab = qnorm(alpha/2, mean = 0, sd = 1); ztab
LCL = pop.mean - abs(ztab)*sqrt(var/N); LCL
```

UCL = pop.mean + (z tab) * sqrt (vartotal/N). ucl

total = pop.mean * N; total

var total = N^2 * sigma2 ; vartotal

LCLt = total - abs (z tab) * sqrt (vartotal/N);
lclt

UCLt = total + abs (z tab) * sqrt (vartotal/N)
uclt

Output:

(i) ~~dat~~

data = c(2,4,6,8); data

[1] 2. 4. 6 8

Sample.data

	[,1]	[,2]
[1,]	2	4
[2,]	2	6
[3,]	2	8
[4,]	4	6
[5,]	4	8
[6,]	6	8

```
Pop. mean
[1]  5
yibar = rowMeans(sample.data); yibar
[1] 3 4 5 5  6 7

E.ybar = sum(yibar*(1/6)); E.ybar
[1]  5
```

~~Coment~~
Comment: The sample mean is 5 and population mean is 5. Both are equal.

(ii)

```
E.yibar2 = sum((yibar^2)*(1/6)); E.yibar2
[1] 26.66667
V.ybar = E.yibar2 - (E.ybar)^2; V.ybar
[1] 1.66667
N = length(data); N
[1] 4
n = 2; n
[1] 2
var = var(data); var
[1] 6.66667
sigma2 = var*(N-1)/N; sigma2
[1]  5
```

>RHS = (sigma2/n)*((N-n)/N-1); R.H.S
[1] 1.66667

~~V.Y~~ Comment: V.ybar and ~~sig~~ RHS are equal. $V(\bar{y}) = \frac{\sigma^2}{n} \cdot \frac{N-n}{N-1}$ equation is varified.

(iii).

S2 = (sample.data[,1] - yibar)^2 + (sample.data[,2] - yibar)^2; S2

[1] 2 8 18 2 8 2

>E.S2 = sum(S2*(1/6)); E.S2
[1] 6.66667

~~Comm~~
Comment: From (ii) $\sigma^2 = 6.66667$ and (iii) $s^2 = 6.6667$. So s^2 is an unbiased estimate of σ^2. varrified.

(iv) ~~alph~~

>alpha = 0.05; alpa
[1] 0.05
>z tab = qnorm(alph/2, mean=0, sd=1); z tab
[1] -1.959964

```
>LCL = pop.mean-abs(ztab)*sqrt(var/N);LCL
[1] 2.469697
>UCL = pop.mean+abs(ztab)*sqrt(var/N);UCL
[1] 7.530303
>total = pop.mean*N; total
[1] 20
>vartotal = N^2*sigma2; vartotal
[1] 80
>LCLt = total-abs(ztab)*sqrt(vartotal/N);
LCLt
[1] 11.23477
>UCLt = total+abs(ztab)*sqrt(vartotal/N);
UCLt
[1] 28.76523
```

Proplem no: 02

Problem name:

let X and Y denote the strength of concrete beams and cylinders. The following data are obtained

X: 5.9, 7.2, 7.3, 6.3 .... 7.7, 11.6... 10.7

Y: 6.1, 5.8, 7.8, 7.1 .... 14.1, 12.6, 11.2

(i) show that $\bar{X} - \bar{Y}$ is an unbiased estimator of $\mu_1 - \mu_2$. calculate it for the given data.

(ii) Find the varriance and standard deviation (standard error) of the estimator in part (i) and then compute the estimated standard error.

(iii) Calculate an estimate of the ratio $\frac{\sigma_1}{\sigma_2}$ of the two standard deviations.

(iv) Suppose a single beam X and a ~~sign~~ single cylinder Y are randomly selected. calculate an estimate of the varriance of the difference X-Y.

Code:

(i)
```
x = c(5.9,7.2,7.3 .......... 11.3,11.8,10.7); x
Y= c(6.1,5.8,7.8 .......... 14.1,12.6,11.2); Y
xbar = mean(x); xbar
Ybar = mean(Y); Ybar
dif = abs(xbar-Ybar); dif
```

(ii)
```
xvar=var(x); xvar
Yvar = var(Y); Yvar
n1 = length(x); n1
n2 = length(Y); n2
SE.dif = sqrt(xvar/n1)+(Yvar/n1)); SEdif
```

(iii)
```
ratio = sqrt(xvar)/sqrt(Yvar); ratio
```

(iv)
```
var.dif = xvar+Yvar; var.dif
```

Output:

(i)

```
>X = c(5.9,7.2,7.3 ............ 10.7); x
[27]   5.9, 7.2  7.3  ......  10.7

>Y = c(6.1,5.8,7.8,7.1,7.2 ......... 11.2); Y
[22]  6.1, 5.8, 7.8  7.1 ........ 11.2

>xbar = mean(x); xbar
[1]   8.140741

>Ybar = mean(Y); Ybar
[1] 8.575

> diff = abs(xbar - Ybar); dif
[1]   0.4342593

> dif' = abs( mean(x) - mean(x)
[1]   0.4342593
```

Comment: $\bar{X} - \bar{Y}$ is unbiased estimator of $\mu_X - \mu_Y$, we need to show that the expected value of the estimator is equal to the population paramn

$E(\bar{X} - \bar{Y}) = E(\bar{X}) - E(\bar{Y}) = \mu_1 - \mu_2$

(ii)

```
>Xvar = var(X); Xvar
[1] 2.754046
>Yvar = var(Y); Yvar
[1] 4.427237
>n1 = length(X); n1
[1] 27
>n2 = length(Y); n2
[1] 20
>SE.diff = sqrt((Xvar/n) + (Yvar/n2)); SE.diff
[1] 0.5686506
```

(iii)

```
>ratio = sqrt(Xvar)/sqrt(Yvar); ratio
[1] 0.7887133
```

(iv)

```
>var.diff = Xvar + Yvar ; var.diff
[1] 7.181282
```

CamScanner

Problem no: 03

Problem Name:

A farm grows grapes for jelly. The following data are measurement of suger in a grapes of a sample taken from each of 30 truckloads.

16.0, 15.2, 15.0, 16.9 .... 17.2, 14.7, 14.8

Assume that these observations of a random variable X that has mean μ and the standard deviation σ.

(i) Find point estimates of μ and σ^2

(ii) Construct an approximate 90%/ 95%/ 80% confidence interval for μ.

Code:

(i)

```
x = c(16.02, 15.2, 12.0 ..... 17.2, 14.7, 14.8); x
n = length(x); n
muhat = sum(x)/n; umhat
sigmahat = sqrt((sum(x^2) - n muhat^2)/n);
                                        sigmahat
```

(ii)

```
alpha = 0.10; alpha
ztab = qnorm(alpha/2, mean=0, sd=1);
                                        ztab
LCL = muhat - abs(ztab)*sqrt(sigmahat/n);
                                        LCL
UCL = muhat + abs(ztab)*sqrt(sigmahat/n)
                                        UCL
```

Output:

(i)

```
> x=c(16.0,15.2 ....... 14.7, 14.8); x
[1] 16.0, 15.02 ..... 15.5, 12.5
[14] 14.5, 14.9 ..... 15.1, 15.3
[27] 12.4, 17.2 .... 14.7, 14.8

> n=length(x): n
[1] 30
> muhat = sum(x)/n; muhat
[1] 14.72
> sigmahat = sqrt((sum(x^2)-n*mauhat^2)/n; sigmahat.
[1] 1.357547
```

(ii)

```
> alpha = 0.10; alpha
[1] 0.1
> ztab = qnorm(alpha/2, mean=0, sd=1); ztab
[1] -1.644854
> LCL = muhat - abs(ztab)*sqrt(sigmahat/n); LCL
[1] 14.3701
> UCL = muhat + abs(ztab)*sqrt(sigmahat/n); UCL
[1] 15.0699
```