Assignment - 2
UNIT - II (Infinite Series)

1. Examine the following sequences for convergence:
   (a) $a_n = \dfrac{n^2 - 2n}{3n^2 + n}$
   (b) $a_n = 2^n$
   (c) $a_n = 3 + (-1)^n$

2. Examine the following series for convergence (Using Definition):
   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{3^{n-1}}$
   (b) $\displaystyle\sum_{n=1}^{\infty} 2^n$
   (c) $\displaystyle\sum_{n=1}^{\infty} (-1)^n$

3. Examine the following series for convergence (Using Geometric and p-series):
   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{5^n}$
   (b) $1 + \dfrac{1}{2} + \dfrac{1}{4} + \dfrac{1}{8} + \cdots$
   (c) $1 - \dfrac{1}{3} + \dfrac{1}{3^2} - \dfrac{1}{3^3} + \dfrac{1}{3^4} - \cdots$
   (d) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$
   (e) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$
   (f) $\displaystyle\sum_{n=1}^{\infty} n^2$

4. Test the following series for convergence:
   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{1.3.5...(2n-1)}{2.4.6...(2n)}$
   (b) $\dfrac{1}{1 \cdot 3} + \dfrac{2}{3 \cdot 5} + \dfrac{3}{5 \cdot 7} + \cdots\cdots$
   (c) $\dfrac{1}{1 \cdot 3 \cdot 5} + \dfrac{2}{3 \cdot 5 \cdot 7} + \dfrac{3}{5 \cdot 7 \cdot 9} + \cdots\cdots$
   (d) $\dfrac{1}{1^2} + \dfrac{1+2}{1^2+2^2} + \dfrac{1+2+3}{1^2+2^2+3^2} + \cdots\cdots$
   (e) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2 + 1}$

(f) $\displaystyle\sum_{n=1}^{\infty}\left(\sqrt{n^2+1}-n\right)$

(g) $\displaystyle\sum_{n=1}^{\infty}\left(\sqrt[3]{n^3+1}-n\right)$

(h) $\displaystyle\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}\sin\left(\frac{1}{n}\right)$

(i) $\displaystyle\frac{2^q}{1^p}+\frac{3^q}{2^p}+\frac{4^q}{3^p}+\cdots\cdots$

(j) $\displaystyle 1+\frac{x}{2}+\frac{x^2}{5}+\frac{x^3}{10}+\cdots\cdots,\ x>0$

(k) $\displaystyle\sum_{n=1}^{\infty}\frac{n!3^n}{n^n}$

(l) $\displaystyle\sum_{n=1}^{\infty}\frac{n}{1+2^n}$

(m) $\displaystyle\sum_{n=1}^{\infty}\frac{100^n}{n!}$

(n) $\displaystyle\sum_{n=1}^{\infty}\sqrt{\frac{n}{n^2+1}}x^n,\ x>0$

(o) $\displaystyle\sum_{n=1}^{\infty}\left(\frac{1+nx}{n}\right)^n,\ x>0$

(p) $\displaystyle\sum_{n=1}^{\infty}(\sqrt{n^2+1}-n)x^{2n},\ x>0$

(q) $\displaystyle\sum_{n=1}^{\infty}\frac{1}{(1+\frac{1}{n})^{n^2}}$

(r) $\displaystyle\sum_{n=1}^{\infty}\left(\frac{n+2}{n+3}\right)^n x^n,\ x>0$

(s) $\displaystyle\frac{3}{4}x+\left(\frac{4}{5}\right)^2x^2+\left(\frac{5}{6}\right)^3x^3+\cdots\cdots,\ x>0$

(t) $\displaystyle 1+\frac{1}{2}\frac{x^3}{3}+\frac{1}{2}\frac{3}{4}\frac{x^5}{5}+\frac{1}{2}\frac{3}{4}\frac{5}{6}\frac{x^7}{7}+\cdots\cdots,\ x>0$

(u) $\displaystyle 1+\frac{3}{7}x+\frac{3\cdot6}{7\cdot10}x^2+\frac{3\cdot6\cdot9}{7\cdot10\cdot13}x^3+\cdots\cdots,\ x>0$

(v) $\displaystyle\sum_{n=2}^{\infty}\frac{1}{n(\log n)^p},\quad$ where $p>0$.

5. Discuss the convergence of the following series:

(a) $\displaystyle\sum_{n=1}^{\infty}(-1)^{n-1}\frac{n}{n^2+1}$

(b) $\displaystyle\sum_{n=1}^{\infty}(-1)^{n-1}\frac{n}{5^n}$

(c) $\frac{1}{6} - \frac{2}{11} + \frac{3}{16} - \frac{4}{21} + \cdots\cdots$

(d) $\left(\frac{1}{2} - \frac{1}{\log 2}\right) - \left(\frac{1}{2} - \frac{1}{\log 3}\right) + \left(\frac{1}{2} - \frac{1}{\log 4}\right) - \left(\frac{1}{2} - \frac{1}{\log 5}\right) + \cdots\cdots$

(e) $\frac{1}{\sqrt{2}+1} - \frac{1}{\sqrt{3}+1} + \frac{1}{\sqrt{4}+1} - \frac{1}{\sqrt{5}+1} + \cdots\cdots$