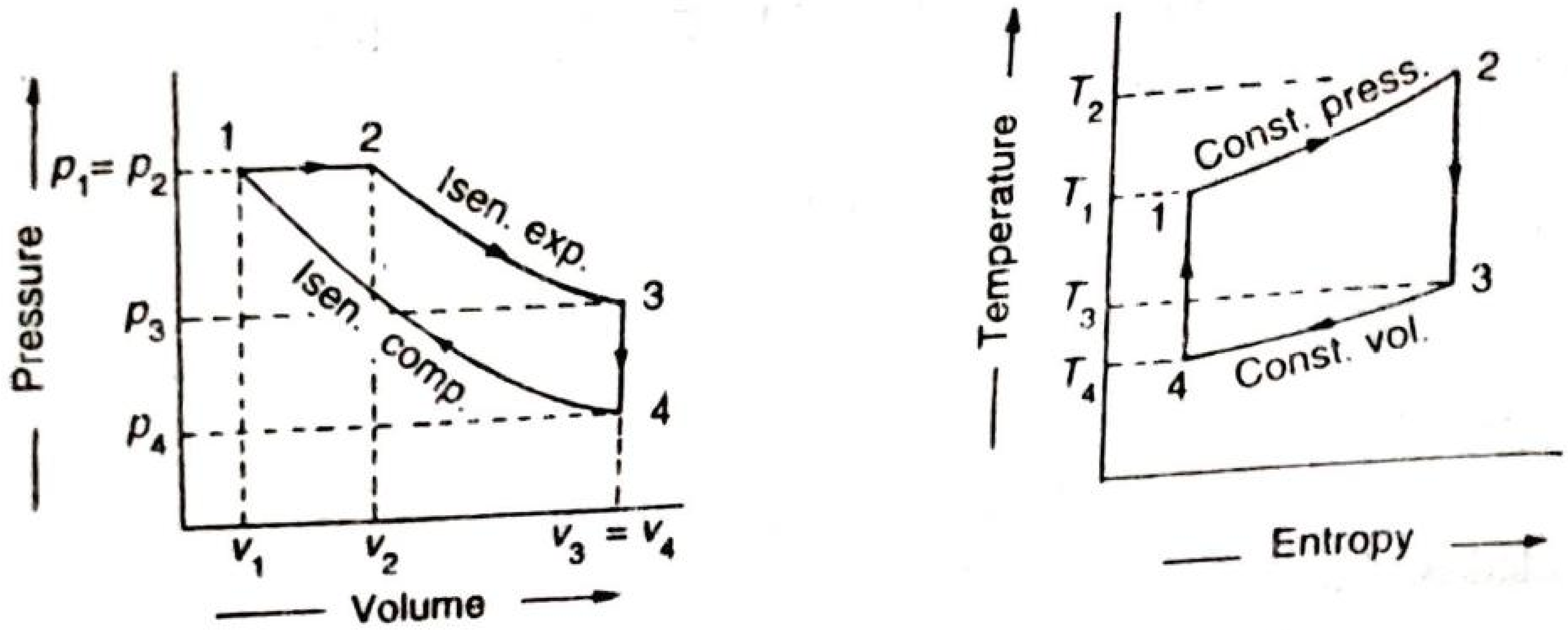
6.17. Diesel Cycle

This cycle was devised by Dr. Rudolph Diesel in 1893, with an idea to attain a higher thermal efficiency, with a high compression ratio. This is an important cycle on which all the diesel engines work. It is also known as *constant pressure cycle* as heat is received at a constant pressure.

The engine imagined by Diesel has air enclosed in the cylinder, whose walls are perfectly non-conductor of heat, but bottom is a perfect conductor of *heat. Again, there is a hot body, cold body and an insulating cap, which are alternately brought in contact with the cylinder.

The ideal diesel cycle consists of two reversible adiabatic or isentropic, a constant pressure and a constant volume processes. These processes are represented on p-v and T-S diagrams as shown in Fig. 6.10 (a) and (b).



(a) p-v diagram. (b) T-S diagram.

Fig. 6.10. Diesel cycle.

Let the engine cylinder contain m kg of air at point 1. At this point let, p_1, T_1 and v_1 be the pressure, temperature and volume of the air. Following are four stages of an ideal diesel cycle.

1. *First stage (Constant pressure heating).* The air is heated at constant pressure from initial temperature T_1 to a temperature T_2 represented by the curve 1-2 in Fig. 6.10 (a) and (b).

$\therefore$ Heat supplied to the air,

$$Q_{1-2} = m\, c_p\, (T_2 - T_1)$$

Note : Since the supply of heat is cut off at point 2, therefore it is known as cut-off point.

2. *Second stage (Reversible adiabatic or isentropic expansion).* The air is expanded reversibly and adiabatically from temperature T_2 to a temperature T_3 as shown by the curve 2-3 in Fig. 6.10 (a) and (b). In this process, no heat is absorbed or rejected by the air.

3. *Third stage (Constant volume cooling).* The air is now cooled at constant volume from temperature T_3 to a temperature T_4 as shown by the curve 3-4 in Fig. 6.10 (a) and (b).

$\therefore$ Heat rejected by the air,

$$Q_{3-4} = m\, c_v\, (T_3 - T_4)$$

4. *Fourth stage (Reversible adiabatic or isentropic compression).* The air is compressed reversibly and adiabatically from temperature T_4 to a temperature T_1 represented by the curve 4-1 in Fig. 6.10 (a) and (b). In this process, no heat is absorbed or rejected by the air.

We see that the air has been brought back to its original conditions of pressure, volume and temperature, thus completing the cycle. We know that

* A similar type of cylinder was designed by Otto.

$$\text{Work done} = \text{Heat absorbed} - \text{Heat rejected}$$
$$= m\,c_p\,(T_2 - T_1) - m\,c_v\,(T_3 - T_4)$$

$\therefore$ Air standard efficiency,

$$\eta = \frac{\text{Work done}}{\text{Heat absorbed}} = \frac{m\,c_p\,(T_2 - T_1) - m\,c_v\,(T_3 - T_4)}{m\,c_p\,(T_2 - T_1)}$$
$$= 1 - \frac{c_v}{c_p}\left(\frac{T_3 - T_4}{T_2 - T_1}\right) = 1 - \frac{1}{\gamma}\left(\frac{T_3 - T_4}{T_2 - T_1}\right) \qquad \ldots (i)$$

Now let compression ratio,

$$r = \frac{v_4}{v_1}$$

Cut-off ratio, $\rho = \dfrac{v_2}{v_1}$

Expansion ratio, $r_1 = \dfrac{v_3}{v_2} = \dfrac{v_4}{v_2} \qquad \ldots (\because\ v_3 = v_4)$

$$= \frac{v_4}{v_1} \times \frac{v_1}{v_2} = r \times \frac{1}{\rho} = \frac{r}{\rho}$$

We know for constant pressure heating process 1-2,

$$\frac{v_1}{T_1} = \frac{v_2}{T_2} \qquad \ldots \text{(Charles's law)}$$

$$\therefore \qquad T_2 = T_1 \times \frac{v_2}{v_1} = T_1 \times \rho \qquad \ldots (ii)$$

Similarly, for reversible adiabatic or isentropic expansion process 2-3,

$$\frac{T_3}{T_2} = \left(\frac{v_2}{v_3}\right)^{\gamma - 1} = \left(\frac{1}{r_1}\right)^{\gamma - 1} = \left(\frac{\rho}{r}\right)^{\gamma - 1}$$

$$\therefore \qquad T_3 = T_2\left(\frac{\rho}{r}\right)^{\gamma - 1} = T_1 \times \rho\left(\frac{\rho}{r}\right)^{\gamma - 1} \qquad \ldots (iii)$$

and for reversible adiabatic or isentropic compression process 4-1,

$$\frac{T_1}{T_4} = \left(\frac{v_4}{v_1}\right)^{\gamma - 1} = (r)^{\gamma - 1} \quad \text{or} \quad T_1 = T_4\,(r)^{\gamma - 1} \qquad \ldots (iv)$$

Substituting the value of T_1 in equations (*ii*) and (*iii*),

$$T_2 = T_4\,(r)^{\gamma - 1} \times \rho \qquad \ldots (v)$$

and

$$T_3 = T_4\,(r)^{\gamma - 1} \times \rho\left(\frac{\rho}{r}\right)^{\gamma - 1} = T_4\,\rho^{\gamma} \qquad \ldots (vi)$$

Now substituting the values of T_1, T_2 and T_3 in equation (i),

$$\eta = 1 - \frac{1}{\gamma}\left[\frac{(T_4\,\rho^{\gamma}) - T_4}{T_4\,(r)^{\gamma-1}\,\rho - T_4\,(r)^{\gamma-1}}\right]$$

$$= 1 - \frac{1}{(r)^{\gamma-1}}\left[\frac{\rho^{\gamma}-1}{\gamma(\rho-1)}\right] \qquad \ldots(vii)$$

Notes : 1. The efficiency of the ideal Diesel cycle is lower than that of Otto cycle, for the same compression ratio. This is due to the fact that the cut-off ratio (ρ) is always greater than unity and hence the term within the bracket of equation (*vii*) increases with the increase of cut-off ratio. Thus the negative term increases and the efficiency is reduced.

2. The Diesel cycle efficiency increases with decrease in cut-off ratio and approaches maximum (equal to Otto cycle efficiency) when the term within the bracket is unity.

Example 6.15. *In a diesel engine, the compression ratio is 13 : 1 and the fuel is cut-off at 8% of the stroke. Find the air standard efficiency of the engine. Take γ for air as 1.4.*

Solution. Given : $r = v_4 / v_1 = 13$; $\gamma = 1.4$

Since the cut-off takes place at 8% of the stroke, therefore volume at cut-off,

$$v_2 = v_1 + 8\% \text{ of stroke volume} = v_1 + 0.08\,(v_4 - v_1)$$

Let us assume that the clearance volume $(v_1) = 1\ \text{m}^3$.

$$\therefore \quad v_4 = 13\ \text{m}^3 \qquad \ldots(\because\ v_4/v_1 = 13)$$

and stroke volume, $v_4 - v_1 = 13 - 1 = 12\ \text{m}^3$

∴ Volume at cut-off

$$v_2 = v_1 + 0.08\,(v_4 - v_1) = 1 + 0.08 \times 12 = 1.96\ \text{m}^3$$

We know that cut-off ratio,

$$\rho = v_2 / v_1 = 1.96 / 1 = 1.96$$

∴ Air standard efficiency,

$$\eta = 1 - \frac{1}{(r)^{\gamma-1}}\left[\frac{\rho^{\gamma-1}}{\gamma(\rho-1)}\right] = 1 - \frac{1}{(13)^{1.4-1}}\left[\frac{(1.96)^{1.4}-1}{1.4\,(1.96-1)}\right]$$

$$= 1 - 0.417 = 0.583 \quad \text{or} \quad 58.3\%\ \textbf{Ans.}$$

Example 6.16. *In an ideal Diesel cycle, the temperatures at the beginning and end of compression are 57° C and 603° C respectively. The temperatures at the beginning and end of expansion are 1950° C and 870° C respectively. Determine the ideal efficiency of the cycle.* γ = 1.4.

If the compression ratio is 14 and the pressure at the beginning of the compression is 1 bar, calculate the maximum pressure in the cycle.

Solution. Given : $T_4 = 57°\text{C} = 57 + 273 = 330\ \text{K}$; $T_1 = 603°\text{C} = 603 + 273 = 876\ \text{K}$; $T_2 = 1950°\text{C} = 1950 + 273 = 2223\ \text{K}$; $T_3 = 870°\text{C} = 870 + 273 = 1143\ \text{K}$; $\gamma = 1.4$; $r = v_4/v_1 = 14$; $p_4 = 1$ bar

Ideal efficiency of the cycle

We know that ideal efficiency of the cycle,

$$\eta = 1 - \frac{1}{\gamma}\left(\frac{T_3 - T_4}{T_2 - T_1}\right) = 1 - \frac{1}{1.4}\left(\frac{1143 - 330}{2223 - 876}\right)$$

$$= 1 - 0.431 = 0.569 \quad \text{or} \quad 56.9\%\ \textbf{Ans.}$$

Maximum pressure in the cycle

Let p_1 = Maximum pressure in the cycle.

We know that for reversible adiabatic compression,

$$p_1 v_1^\gamma = p_4 v_4^\gamma \text{ or } p_1 = p_4\left(\frac{v_4}{v_1}\right)^\gamma = 1\,(14)^{1.4} = 40.23 \text{ bar} \textbf{ Ans.}$$

Example 6.17. *An ideal Diesel engine has a diameter 150 mm and stroke 200 mm. The clearance volume is 10 per cent of the swept volume. Determine the compression ratio and the air standard efficiency of the engine if the cut-off takes place at 6 per cent of the stroke.*

Solution. Given : d = 150 mm = 0.15 m ; l = 200 mm = 0.2 m ; v_c = 10% of $v_s = 0.1\ v_s$

Compression ratio

We know that stroke volume,

$$v_s = \frac{\pi}{4}\times d^2 \times l = \frac{\pi}{4}(0.15)^2\, 0.2 = 3.53\times 10^{-3}\ \text{m}^3$$

$$\therefore \quad v_c = 0.1\ v_s = 0.1\times 3.53\times 10^{-3} = 0.353\times 10^{-3}\ \text{m}^3$$

We know that compression ratio,

$$r = \frac{\text{Total volume}}{\text{Clearance volume}} = \frac{v_c + v_s}{v_c} = \frac{0.353\times 10^{-3} + 3.53\times 10^{-3}}{0.353\times 10^{-3}}$$

$$= 11 \textbf{ Ans.}$$

Air standard efficiency

Since the cut-off takes place at 6% of the stroke, therefore volume at cut-off,

$$v_2 = v_1 + 0.06\ v_s = v_c + 0.06\ v_s \qquad \ldots(\because v_1 = v_c)$$

$$= 0.353\times 10^{-3} + 0.06\times 3.53\times 10^{-3} = 0.565\times 10^{-3}\ \text{m}^3$$

$$\therefore \text{Cut-off ratio,} \quad \rho = \frac{v_2}{v_1} = \frac{v_2}{v_c} = \frac{0.565\times 10^{-3}}{0.353\times 10^{-3}} = 1.6$$

We know that air standard efficiency,

$$\eta = 1 - \frac{1}{(r)^{\gamma-1}}\left[\frac{\rho^\gamma - 1}{\gamma(\rho - 1)}\right] = 1 - \frac{1}{(11)^{1.4-1}}\left[\frac{(1.6)^{1.4} - 1}{1.4\,(1.6-1)}\right]$$

$$= 1 - \frac{1}{2.61}\times 1.11 = 1 - 0.4246 = 0.5753 \text{ or } 57.53\% \textbf{ Ans.}$$

Example 6.18. *The compression ratio of an ideal air standard Diesel cycle is 15. The heat transfer is 1465 kJ/kg of air. Find the pressure and temperature at the end of each process and determine the cycle efficiency.*

What is the mean effective pressure of the cycle, if the inlet conditions are 300 K and 1 bar.

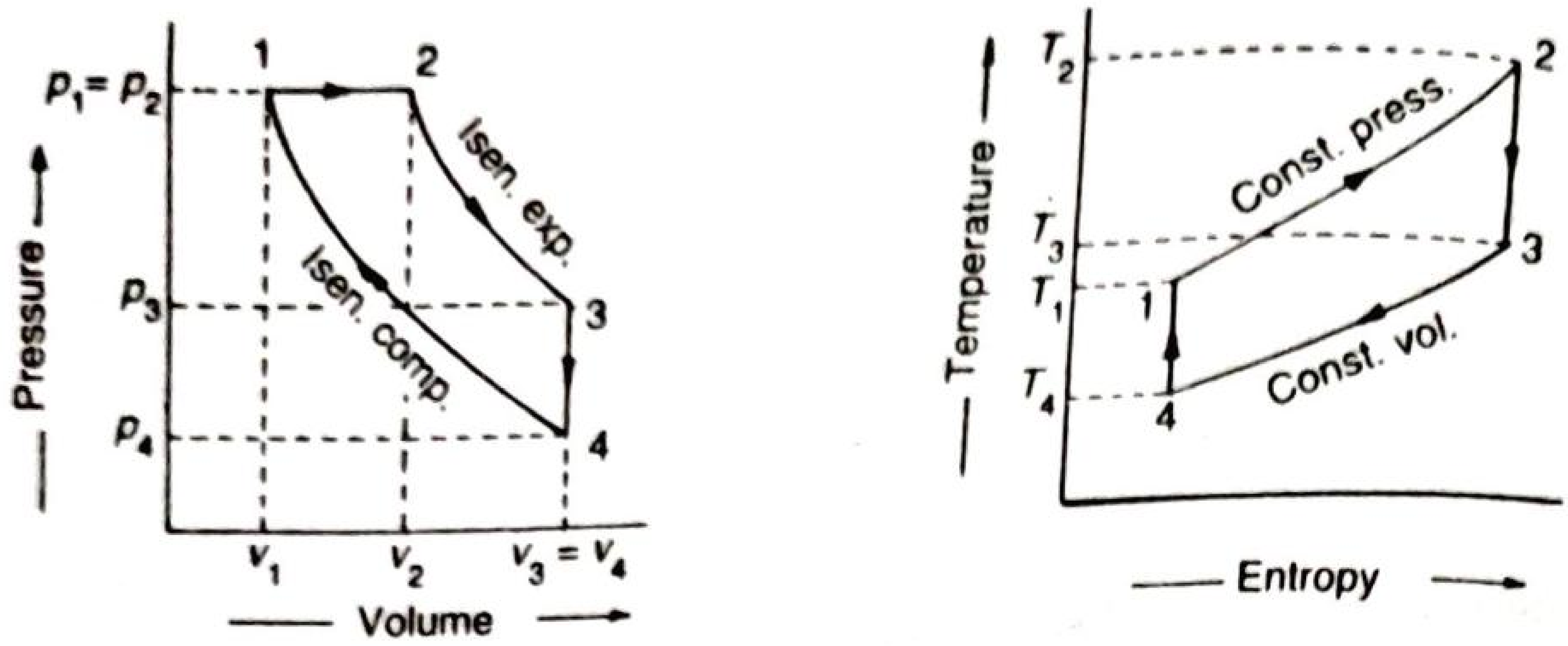
Solution. Given : $r = v_4 / v_1 = 15$; Q_{1-2} = 1465 kJ/kg ; T_4 = 300 K ; p_4 = 1 bar = 0.1×10^6 N/m²

Pressure and temperature at the end of each process

The *p-v* and *T-S* diagram for the Diesel cycle is shown in Fig. 6.11.

Let p_1, p_2 and p_3 = Pressures at points 1, 2 and 3 respectively.

T_1, T_2 and T_3 = Temperature at points 1, 2 and 3 respectively.



(a) p–v diagram. (b) T–S diagram.

Fig. 6.11

First of all, consider the isentropic compression process 4-1. We know that

$$p_4 v_4^{\gamma} = p_1 v_1^{\gamma} \text{ or } p_1 = p_4 \left(\frac{v_4}{v_1}\right)^{\gamma} = 1\,(15)^{1.4} = 44.3 \text{ bar } \textbf{Ans.}$$

and

$$\frac{T_1}{T_4} = \left(\frac{v_4}{v_1}\right)^{\gamma-1} = (15)^{1.4-1} = (15)^{0.4} = 2.954$$

$$\therefore \quad T_1 = T_4 \times 2.954 = 300 \times 2.954 = 886.2 \text{ K } \textbf{Ans.}$$

Now consider the constant pressure process 1-2. We know that

$$p_2 = p_1 = 44.3 \text{ bar } \textbf{Ans.}$$

and heat supplied per kg of air during constant pressure process 1-2 (Q_{1-2}),

$$1465 = m\,c_p (T_2 - T_1) = 1 \times 1\,(T_2 - 886.2) = T_2 - 886.2$$

... (Taking c_p for air = 1 kJ/kg K)

$$\therefore \quad T_2 = 1465 + 886.2 = 2351.2 \text{ K } \textbf{Ans.}$$

Now consider the isentropic expansion process 2-3. First of all, let us find the volumes at points 2 and 3.

Let v_2 = Volume at point 2, and

v_3 = Volume at point 3.

We know that

$$p_4 v_4 = m\,R\,T_4 \text{ or } v_4 = \frac{m\,R\,T_4}{p_4} = \frac{1 \times 287 \times 300}{0.1 \times 10^6} = 0.861 \text{ m}^3$$

... ($\because$ R for air = 287 J/kg K)

$$\therefore \quad v_3 = 0.861 \text{ m}^3 \qquad \ldots (\because v_3 = v_4)$$

and

$$v_1 = v_4 / 15 = 0.861 / 15 = 0.0574 \text{ m}^3 \qquad \ldots (\because v_4 / v_1 = 15)$$

We also know that for the constant pressure process 1-2,

$$\frac{v_1}{T_1} = \frac{v_2}{T_2} \quad \text{or} \quad v_2 = v_1 \times \frac{T_2}{T_1} = 0.0574 \times \frac{2351.2}{886.2} = 0.1523 \text{ m}^3$$

Now
$$\frac{T_2}{T_3} = \left(\frac{v_3}{v_2}\right)^{\gamma-1} = \left(\frac{0.861}{0.1523}\right)^{1.4-1} = (5.65)^{0.4} = 2$$

$$\therefore \quad T_3 = T_2/2 = 2351.2/2 = 1175.6 \text{ K} \textbf{ Ans.}$$

We know that $p_2 v_2^\gamma = p_3 v_3^\gamma$ or $p_3 = p_2\left(\frac{v_2}{v_3}\right)^\gamma = 44.3\left(\frac{0.1523}{0.861}\right)^{1.4} = 3.92$ bar **Ans.**

Cycle efficiency

We know that heat rejected per kg of air during the constant volume process 3-4,

$$Q_{3-4} = m\,c_v\,(T_3 - T_4) = 1 \times 0.712\,(1175.6 - 300) = 623.4 \text{ kJ}$$

... (Taking c_v for air = 0.712 kJ/kg K)

$$\therefore \text{ Cycle efficiency, } \eta = \frac{\text{Heat supplied} - \text{Heat rejected}}{\text{Heat supplied}} = \frac{1465 - 623.4}{1465}$$

$$= 0.5745 \text{ or } 57.45\% \textbf{ Ans.}$$

Mean effective pressure

We know that workdone per kg of air during the cycle

$$= \text{Heat supplied} - \text{Heat rejected} = 1465 - 623.4 = 841.6 \text{ kJ}$$

and stroke volume $= v_4 - v_1 = 0.861 - 0.0574 = 0.8036 \text{ m}^3$

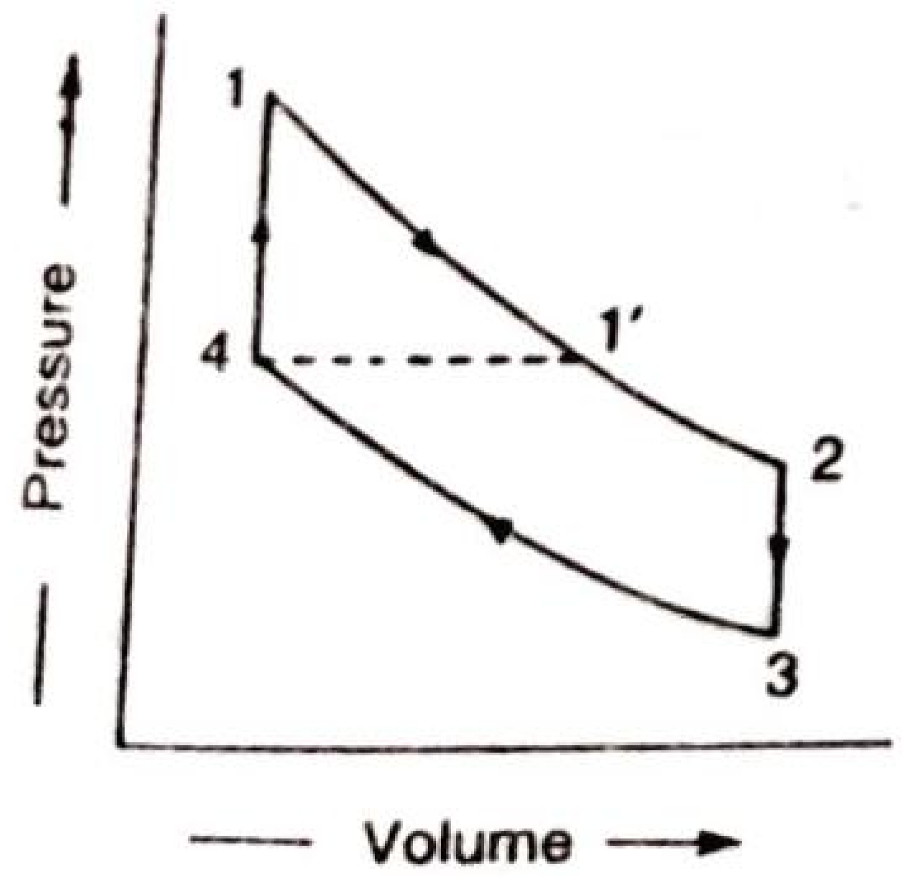
$\therefore$ Mean effective pressure

$$= \frac{\text{Workdone}}{\text{Stroke volume}} = \frac{841.6}{0.8036} = 1047.3 \text{ kN/m}^2$$

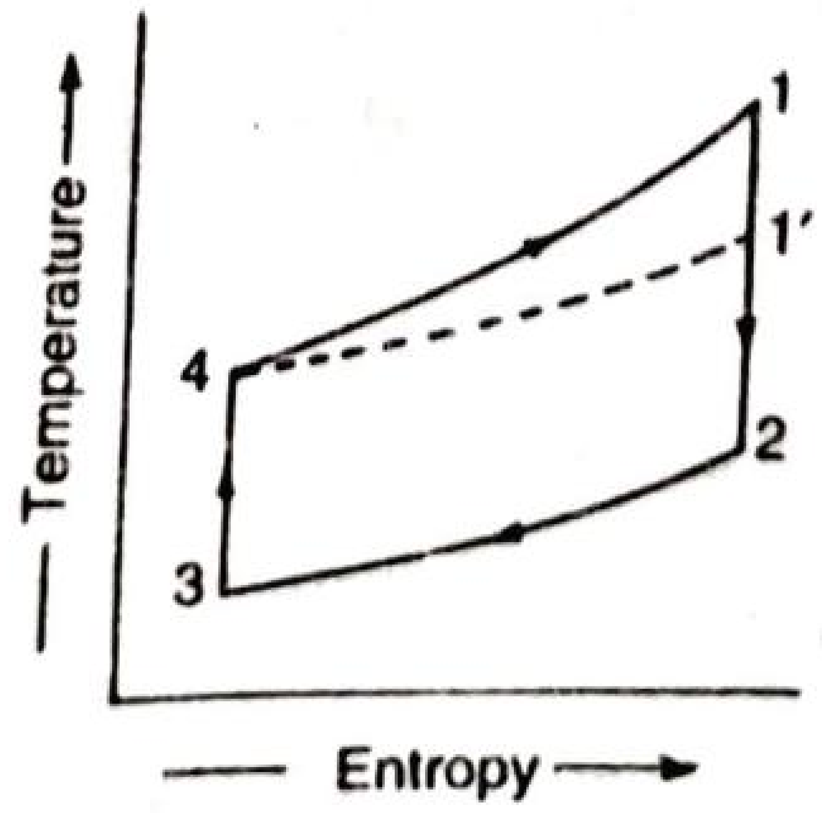
$$= 10.473 \text{ bar } \textbf{Ans.} \qquad \ldots(\because\ 1 \text{ bar} = 100 \text{ kN/m}^2)$$

Example 6.19. *Find the air standard efficiencies for the Otto and Diesel cycles on the basis of equal compression ratio of 10 and equal heat rejection of 840 kJ/kg. The suction conditions are 1 bar and 328 K.*

Solution. Given : $r = v_3 / v_4 = 10$; $Q_{2-3} = 840$ kJ/kg ; $p_3 = 1$ bar ; $T_3 = 328$ K



(a) p-v diagram.



(b) T-S diagram.

Fig. 6.12

In the p-v and T-S diagrams as shown in Fig. 6.12 (a) and (b), the cycle 1-2-3-4-1 represents Otto cycle and the cycle 4-1′-2-3-4 represents a Diesel cycle.

First of all, let us consider isentropic compression process 3-4, which is common for both the cycles.

Let T_4 and p_4 = Temperature and pressure at the end of isentropic compression.

We know that $$\frac{T_4}{T_3} = \left(\frac{v_3}{v_4}\right)^{\gamma-1} = (10)^{1.4-1} = (10)^{0.4} = 2.512$$

$$\therefore \quad T_4 = T_3 \times 2.512 = 328 \times 2.512 = 824 \text{ K}$$

We also know that

$$p_3 v_3^\gamma = p_4 v_4^\gamma \quad \text{or} \quad p_4 = p_3 \left(\frac{v_3}{v_4}\right)^\gamma = 1\,(10)^{1.4} = 25.12 \text{ bar}$$

Now consider the constant volume process 2-3 which is also common for both the cycles. We know that heat rejected per kg of gas during this process (Q_{2-3}),

$$840 = m\,c_v\,(T_2 - T_3) = 1 \times 0.712\,(T_2 - 328)$$

... (Taking c_v for air = 0.712 kJ/kg K)

$$\therefore \quad T_2 - 328 = 840/0.712 = 1180 \quad \text{or} \quad T_2 = 1180 + 328 = 1508 \text{ K}$$

and pressure at point 2, $$p_2 = p_3 \times \frac{T_2}{T_3} = 1 \times \frac{1508}{328} = 4.6 \text{ bar} \qquad \ldots\left(\because \frac{p_2}{T_2} = \frac{p_3}{T_3}\right)$$

Otto cycle

Consider the isentropic expansion process 1-2.

Let T_1 = Temperature at the beginning of isentropic expansion.

We know that $$\frac{T_1}{T_2} = \left(\frac{v_2}{v_1}\right)^{\gamma-1} = \left(\frac{v_3}{v_4}\right)^{\gamma-1} = (10)^{1.4-1} = 10^{0.4} = 2.512$$

... ($\because v_1 = v_4$ and $v_2 = v_3$)

$$\therefore \quad T_1 = T_2 \times 2.512 = 1508 \times 2.512 = 3788 \text{ K}$$

and heat supplied per kg of air during constant volume process 4-1,

$$Q_{4-1} = m\,c_v\,(T_1 - T_4) = 1 \times 0.712\,(3788 - 824) = 2110 \text{ kJ/kg}$$

$\therefore$ Efficiency of Otto cycle,

$$\eta_O = \frac{\text{Heat supplied} - \text{Heat rejected}}{\text{Heat supplied}} = \frac{2110 - 840}{2110} = 0.60 \text{ or } 60\% \quad \textbf{Ans.}$$

Diesel cycle

Consider the isentropic expansion process 1′–2.

Let T_1' = Temperature at the beginning of isentropic expansion.

We know that $$\frac{T_1'}{T_2} = \left(\frac{p_1'}{p_2}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{25.12}{4.58}\right)^{\frac{1.4-1}{1.4}} = (5.485)^{0.286} = 1.627$$

... ($\because p_1' = p_4 = 25.12$ bar)

$$\therefore \quad T_1' = T_2 \times 1.627 = 1508 \times 1.627 = 2453 \text{ K}$$

and heat supplied per kg of air during constant pressure process 4-1′,

$$Q_{4-1}{}' = m\,c_p\,(T_1{}' - T_4) = 1 \times 1\,(2453 - 824) = 1629 \text{ kJ/kg}$$

... (Taking c_p for air = 1 kg/kg K)

∴ Efficiency of the Diesel cycle,

$$\eta_D = \frac{\text{Heat supplied} - \text{Heat rejected}}{\text{Heat supplied}} = \frac{1629 - 840}{1629}$$

$$= 0.484 \quad \text{or} \quad 48.4\ \% \quad \textbf{Ans.}$$

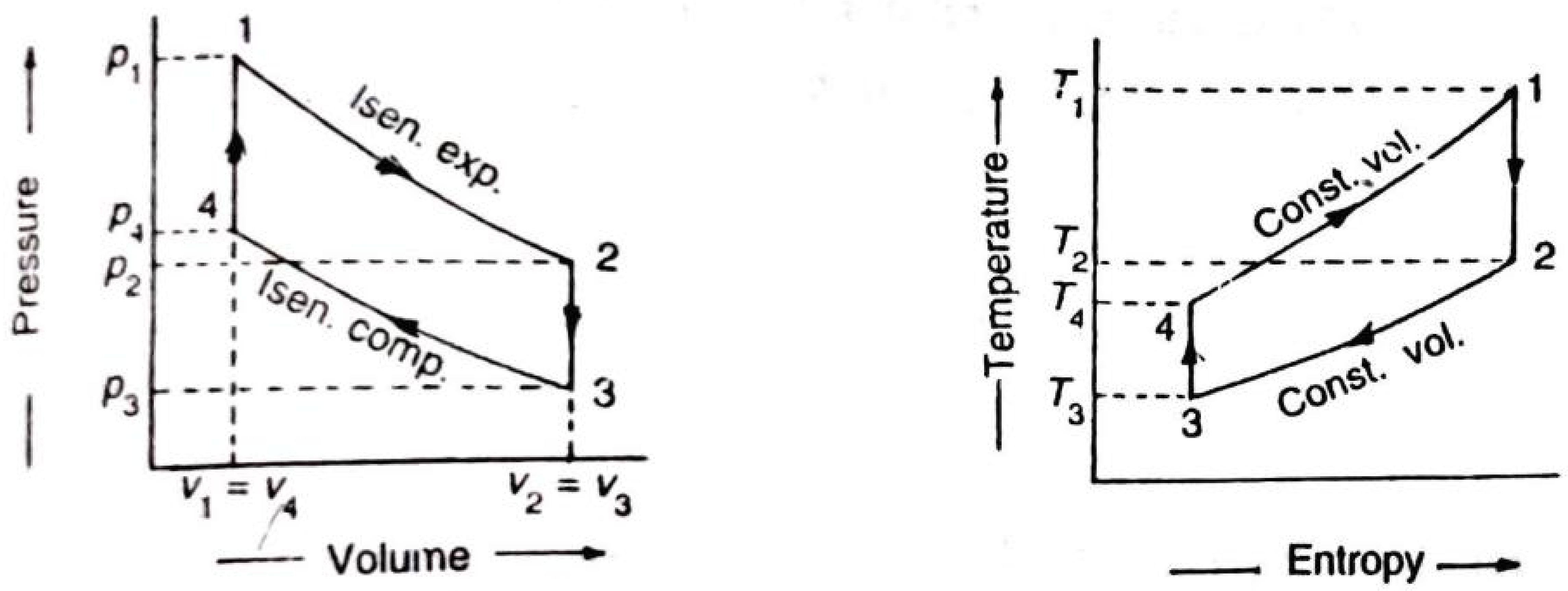
Example 6.20. *Two engines are to operate on Otto and Diesel cycles with the following data :*

Maximum temperature = 1500 K ; Exhaust temperature = 700 K ; Ambient conditions = 1 bar and 300 K.

Compare the compression ratios, maximum pressures, and efficiencies of the two engines.

Solution. Given : $T_1 = 1500$ K ; $T_2 = 700$ K ; $p_3 = 1$ bar ; $T_3 = 300$ K

First of all let us consider an Otto cycle as shown in Fig. 6.13.



(*a*) *p-v* diagram. (*b*) *T-S* diagram.

Fig. 6.13. Otto cycle.

Let $\quad r = \text{Compression ratio} = v_3 / v_4 = v_2 / v_1$, and

p_1 = Maximum pressure.

We know that for isentropic expansion process 1-2,

$$\frac{T_1}{T_2} = \left(\frac{v_2}{v_1}\right)^{\gamma-1} = r^{\gamma-1}$$

or

$$r = \left(\frac{T_1}{T_2}\right)^{\frac{1}{\gamma-1}} = \left(\frac{1500}{700}\right)^{\frac{1}{1.4-1}} = 6.72$$

Now for isentropic compression process 3-4,

$$\frac{T_4}{T_3} = \left(\frac{v_3}{v_4}\right)^{\gamma-1} = (r)^{\gamma-1} = (6.72)^{1.4-1} = 2.143$$

∴

$$T_4 = T_3 \times 2.143 = 300 \times 2.143 = 643 \text{ K}$$

and $$p_4 v_4^{\gamma} = p_3 v_3^{\gamma} \quad \text{or} \quad p_4 = p_3 \left(\frac{v_3}{v_4}\right)^{\gamma} = p_3 \times r^{\gamma} = 1\,(6.72)^{1.4} = 14.4 \text{ bar}$$

For constant volume process 4-1,

$$\frac{p_1}{T_1} = \frac{p_4}{T_4} \quad \text{or} \quad p_1 = p_4 \times \frac{T_1}{T_4} = 14.4 \times \frac{1500}{643} = 33.6 \text{ bar}$$

We know that efficiency of Otto cycle,

$$\eta_O = 1 - \frac{1}{(r)^{\gamma - 1}} = 1 - \frac{1}{(6.72)^{1.4-1}} = 1 - \frac{1}{2.143} = 1 - 0.467$$

$$= 0.533 \quad \text{or} \quad 53.3\% \quad \textbf{Ans.}$$

Note. The efficiency may also be calculated as follows :

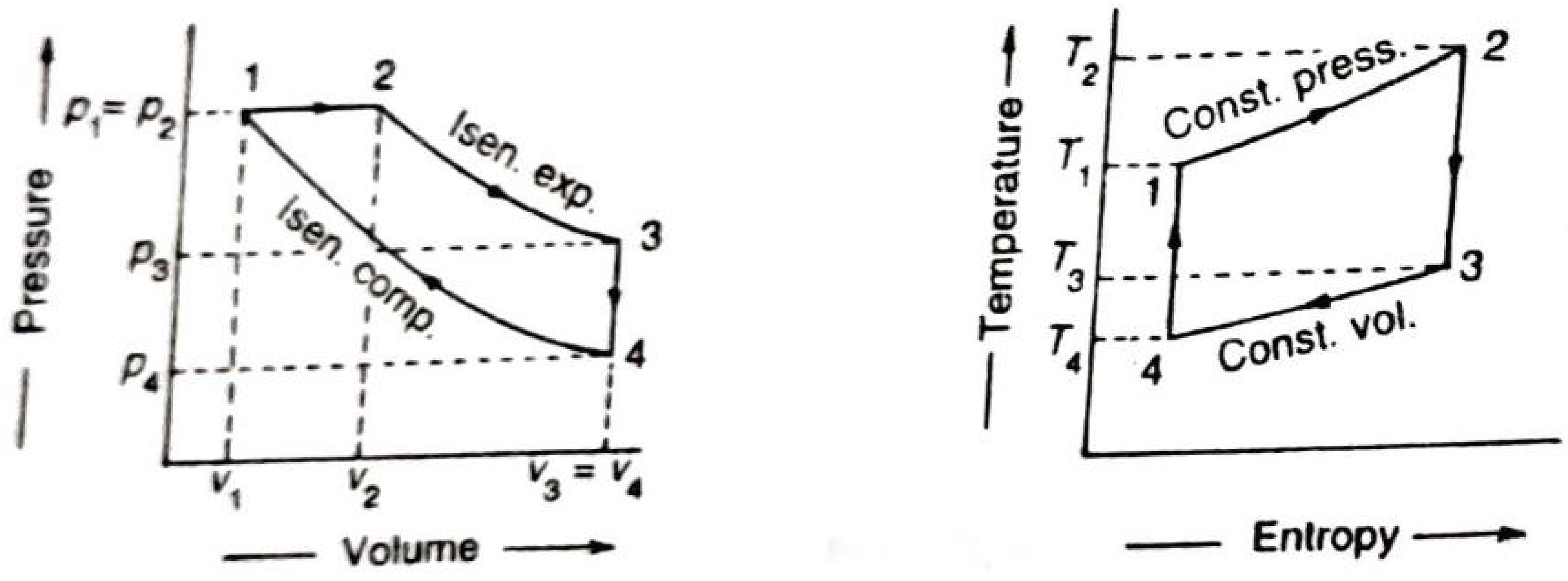
We know that $$\eta_O = 1 - \frac{T_2 - T_3}{T_1 - T_4} = 1 - \frac{700 - 300}{1500 - 643} = 1 - 0.467 = 0.533 \quad \text{or} \quad 53.3\%$$

Now let us consider the Diesel cycle as shown in Fig. 6.14. In this case

$T_2 = 1500$ K ; $T_3 = 700$ K ; $T_4 = 300$ K ; $p_4 = 1$ bar

Let r = Compression ratio = v_4 / v_1, and

$p_1 = p_2$ = Maximum pressure.



(a) p-v diagram. (b) T-S diagram.

Fig. 6.14. Diesel cycle.

We know that for isentropic expansion process 2-3,

$$\frac{T_3}{T_2} = \left(\frac{v_2}{v_3}\right)^{\gamma - 1} \quad \text{or} \quad \frac{v_2}{v_3} = \left(\frac{T_3}{T_2}\right)^{\frac{1}{\gamma - 1}} = \left(\frac{700}{1500}\right)^{\frac{1}{1.4-1}} = 0.1488 \qquad \ldots (i)$$

Now for isentropic compression process 4-1,

$$\frac{T_1}{T_4} = \left(\frac{v_4}{v_1}\right)^{\gamma - 1}$$

or $$T_1 = T_4 \left(\frac{v_4}{v_1}\right)^{\gamma - 1} = 300 \left(\frac{v_4}{v_1}\right)^{1.4 - 1} = 300 \left(\frac{v_4}{v_1}\right)^{0.4} \qquad \ldots (ii)$$

and for constant pressure process 1-2,

$$\frac{v_1}{T_1} = \frac{v_2}{T_2} \quad \text{or} \quad T_1 = T_2 \times \frac{v_1}{v_2} = 1500 \times \frac{v_1}{v_2} \qquad \ldots (iii)$$

From equations (*ii*) and (*iii*),

$$300\left(\frac{v_4}{v_1}\right)^{0.4} = 1500 \times \frac{v_1}{v_2} \quad \text{or} \quad \left(\frac{v_4}{v_1}\right)^{0.4} \times \frac{v_2}{v_1} = \frac{1500}{300} \qquad \ldots (iv)$$

We know that

$$\frac{v_2}{v_1} = \frac{v_4}{v_1} \times \frac{v_2}{v_4} = \frac{v_4}{v_1} \times \frac{v_2}{v_3} \qquad \ldots (\because v_4 = v_3)$$

$$= \frac{v_4}{v_1} \times 0.1488 \qquad \ldots \text{[From equation } (i)]$$

Substituting the value of v_2/v_1 in equation (*iv*), we have

$$\left(\frac{v_4}{v_1}\right)^{0.4} \times \frac{v_4}{v_1} \times 0.1488 = \frac{1500}{300}$$

or

$$\left(\frac{v_4}{v_1}\right)^{1.4} = \frac{1500}{300} \times \frac{1}{0.1488} = 33.6$$

$$\therefore \quad \frac{v_4}{v_1} = (33.6)^{\frac{1}{1.4}} = 12.3 \quad \text{or} \quad r = 12.3$$

Now for isentropic compression process 4-1,

$$p_4 v_4^{\gamma} = p_1 v_1^{\gamma} \quad \text{or} \quad p_1 = p_4\left(\frac{v_4}{v_1}\right)^{\gamma} = 1\,(12.3)^{1.4} = 33.6 \text{ bar}$$

and from equation (*ii*), for isentropic compression process,

$$T_1 = 300\left(\frac{v_4}{v_1}\right)^{0.4} = 300\,(12.3)^{0.4} = 818.6 \text{ K}$$

We know that efficiency of Diesel cycle,

$$\eta_D = 1 - \frac{1}{\gamma}\left(\frac{T_3 - T_4}{T_3 - T_1}\right) = 1 - \frac{1}{1.4}\left(\frac{700 - 300}{1500 - 818.6}\right)$$

$$= 1 - 0.419 = 0.581 \quad \text{or} \quad 58.1\ \%$$

$\therefore$ Ratio of compression ratios,

$$\frac{r \text{ for Otto cycle}}{r \text{ for Diesel cycle}} = \frac{6.72}{12.3} = 0.546 \ \textbf{Ans.}$$

Ratio of maximum pressures,

$$\frac{p_1 \text{ for Otto cycle}}{p_1 \text{ for Diesel cycle}} = \frac{33.6}{33.6} = 1 \ \textbf{Ans.}$$

Ratio of efficiencies,

$$\frac{\eta \text{ for Otto cycle}}{\eta \text{ for Diesel cycle}} = \frac{0.533}{0.581} = 0.917 \ \textbf{Ans.}$$