6.11. Types of Thermodynamic Cycles

Though there are many types of thermodynamic cycles, yet the following are important from the subject point of view :

1. Carnot cycle, 2. Stirling cycle, 3. Ericsson cycle, 4. Joule cycle, 5. Otto cycle, 6. Diesel cycle, and 7. Dual combustion cycle.

The above mentioned cycles will be discussed, in detail, in the following pages.

6.12. Carnot Cycle

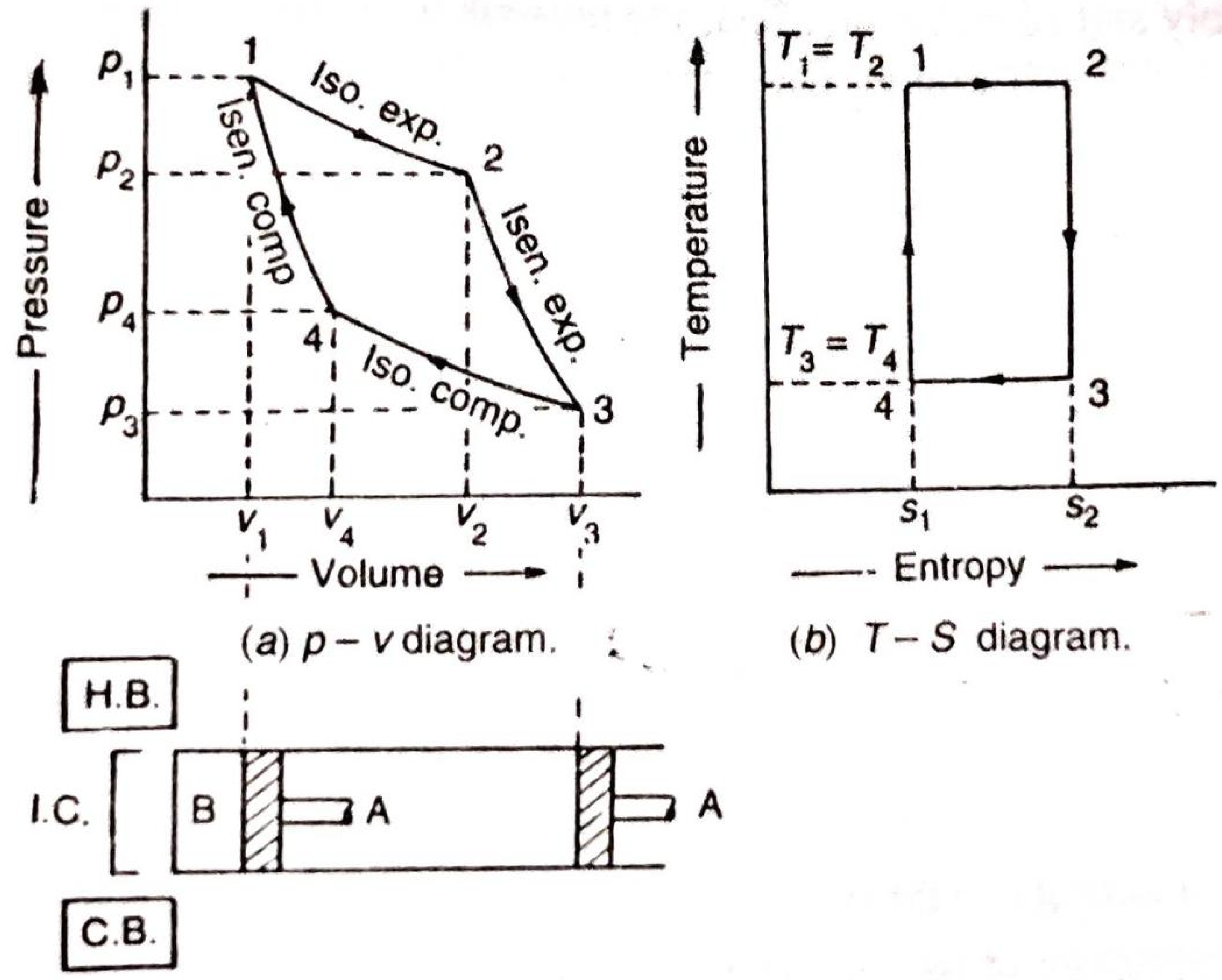

Fig. 6.4. Carnot cycle.

This cycle was devised by *Carnot, who was the first scientist to analyse the problem of the efficiency of a heat engine, disregarding its mechanical details. He focussed his attention on the basic features of a heat engine. In a Carnot cycle, the working substance is subjected to a cyclic operation consisting of two isothermal and two reversible adiabatic or isentropic operations. The *p-v* and *T-S* diagrams of this cycle are shown in Fig. 6.4 (*a*) and (*b*).

* Nicolas Leonard Sadi Carnot was a French engineer. He devised this cycle in his early age.

The engine imagined by Carnot has air (which is supposed to behave like a perfect gas) as its working substance enclosed in a cylinder, in which a frictionless piston A moves. The walls of the cylinder and piston are perfect non-conductor of heat. However, the bottom B of the cylinder can be covered, at will, by an insulating cap (I.C.). The engine is assumed to work between two sources of infinite heat capacity, one at a higher temperature and the other at a lower temperature.

Now, let us consider the four stages of the Carnot's cycle. Let the engine cylinder contain m kg of air at its original condition represented by point 1 on the p-v and T-S diagrams. At this point, let p_1, T_1 and v_1 be the pressure, temperature and volume of the air, respectively.

1. *First stage (Isothermal expansion).* The source (hot body, H.B.) at a higher temperature is brought in contact with the bottom B of the cylinder. The air expands, practically at constant temperature T_1, from v_1 to v_2. It means that the temperature T_2 at point 2 is equal to the temperature T_1. This isothermal expansion is represented by curve 1-2 on p-v and T-S diagrams in Fig. 6.4 (*a*) and (*b*). It may be noted that the heat supplied by the hot body is fully absorbed by the air, and is utilised in doing external work.

$\therefore$ Heat supplied = *Work done by the air during isothermal expansion

or
$$Q_{1-2} = p_1 v_1 \log_e \left(\frac{v_2}{v_1}\right) = m\,R\,T_1 \log_e \left(\frac{v_2}{v_1}\right) \qquad \ldots (\because\ p_1 v_1 = mRT_1)$$
$$= 2.3\, m\,R\,T_1 \log r$$

where r = Expansion ratio $= v_2/v_1$.

2. *Second stage (Reversible adiabatic or isentropic expansion).* The hot body is removed from the bottom of the cylinder B and the insulating cap I.C. is brought in contact. The air is now allowed to expand reversibly and adiabatically. Thus the reversible adiabatic expansion is represented by the curve 2-3 on p-v and T-S diagrams. The temperature of the air falls from T_2 to T_3. Since no heat is absorbed or rejected by the air, therefore

Decrease in internal energy = Workdone by the air during adiabatic expansion
$$= \frac{p_2 v_2 - p_3 v_3}{\gamma - 1} = \frac{m\,R\,T_2 - m\,R\,T_3}{\gamma - 1} \qquad \ldots (\because\ p\,v = m\,R\,T)$$
$$= \frac{m\,R\,(T_1 - T_3)}{\gamma - 1} \qquad \ldots (\because\ T_1 = T_2)$$

3. *Third stage (Isothermal compression).* Now remove the insulating cap I.C. from the bottom of the cylinder and bring the cold body C.B. in its contact. The air is compressed practically at a constant temperature T_3 from v_3 to v_4. It means that the temperature T_4 (at point 4) is equal to the temperature T_3. This isothermal compression is represented by the curve 3-4 on p-v and T-S diagrams. It would be seen that during this process, the heat is rejected to the cold body and is equal to the work done on the air.

$\therefore$ Heat rejected = Work done on the air during isothermal compression
$$Q_{3-4} = p_3 v_3 \log_e \left(\frac{v_3}{v_4}\right) = m\,R\,T_3 \log_e \left(\frac{v_3}{v_4}\right) \qquad \ldots (\because\ p\,v = m\,R\,T)$$
$$= 2.3\, m\,R\,T_3 \log r$$

* Since the temperature is constant, therefore there is no change in internal energy of the air, *i.e.* $dU = 0$. According to the first law of thermodynamics,
$$Q_{1-2} = dU + W_{1-2} \quad \text{or} \quad Q_{1-2} = W_{1-2}$$

where r = *Compression ratio $= v_3/v_4$

4. *Fourth stage (Reversible adiabatic or isentropic compression).* Now again the insulated cap I.C. is brought in contact with the bottom of the cylinder B, and the air is allowed to be compressed reversibly and adiabatically. The reversible adiabatic compression is represented by the curve 4-1 on p-v and T-S diagrams. The temperature of the air increases from T_4 to T_1. Since no heat is absorbed or rejected by the air, therefore

Increase in internal energy = Work done on the air during adiabatic compression

$$= \frac{p_1 v_1 - p_4 v_4}{\gamma - 1} = \frac{m R T_1 - m R T_4}{\gamma - 1} \qquad \ldots (\because pv = mRT)$$

$$= \frac{m R (T_1 - T_3)}{\gamma - 1} \qquad \ldots (\because T_3 = T_4)$$

We see from the above discussion that the decrease in internal energy during reversible adiabatic expansion 2-3 is equal to the increase in internal energy during reversible adiabatic compression 4-1. Hence their net effect during the whole cycle is zero. We know that

Work done, $\quad W$ = Heat supplied − Heat rejected

$$= 2.3\, m R T_1 \log r - 2.3\, m R T_3 \log r = 2.3\, m R \log r (T_1 - T_3)$$

and efficiency

$$^{**}\eta = \frac{\text{Work done}}{\text{Heat supplied}} = \frac{2.3\, m R \log r (T_1 - T_3)}{2.3\, m R T_1 \log r}$$

$$= \frac{T_1 - T_3}{T_1} = 1 - \frac{T_3}{T_1}$$

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* The expansion and compression ratios (r) must be equal, otherwise the cycle would not close.

We know that for reversible adiabatic or isentropic expansion process 2-3,

$$\frac{T_2}{T_3} = \left(\frac{v_3}{v_2}\right)^{\gamma - 1} \quad \text{or} \quad \frac{v_3}{v_2} = \left(\frac{T_2}{T_3}\right)^{\frac{1}{\gamma - 1}} \qquad \ldots (i)$$

Similarly, for reversible adiabatic or isentropic compression process 4-1,

$$\frac{T_1}{T_4} = \left(\frac{v_4}{v_1}\right)^{\gamma - 1} \quad \text{or} \quad \frac{v_4}{v_1} = \left(\frac{T_1}{T_4}\right)^{\frac{1}{\gamma - 1}} \qquad \ldots (ii)$$

Since $T_1 = T_2$ and $T_3 = T_4$, therefore

$$\frac{v_3}{v_2} = \frac{v_4}{v_1} \quad \text{or} \quad r = \frac{v_2}{v_1} = \frac{v_3}{v_4}$$

** **Alternative Proof.**

Heat supplied during isothermal expansion 1-2,

$$Q_{1-2} = T_1 (S_2 - S_1) \qquad \ldots (\because \delta Q = T\,dS)$$

and heat rejected during isothermal compression 3-4,

$$Q_{3-4} = T_4 (S_2 - S_1) = T_3 (S_2 - S_1) \qquad \ldots (\because T_4 = T_3)$$

We know that work done = Heat supplied − Heat rejected

$$= T_1 (S_2 - S_1) - T_3 (S_2 - S_1) = (T_1 - T_3)(S_2 - S_1)$$

$\therefore$ Efficiency,

$$\eta = \frac{\text{Work done}}{\text{Heat supplied}} = \frac{(T_1 - T_3)(S_2 - S_1)}{T_1 (S_2 - S_1)} = \frac{T_1 - T_3}{T_1} = 1 - \frac{T_3}{T_1}$$

The expression for the efficiency of a Carnot cycle may also be written as discussed below.

We know that for reversible adiabatic or isentropic expansion 2-3,

$$\frac{T_2}{T_3} = \left(\frac{v_3}{v_2}\right)^{\gamma-1} \quad \text{or} \quad \frac{T_1}{T_3} = \left(\frac{v_3}{v_2}\right)^{\gamma-1} \qquad \ldots (\because T_2 = T_1) \ldots (i)$$

Similarly, for reversible adiabatic or isentropic compression process 4-1,

$$\frac{T_1}{T_4} = \left(\frac{v_4}{v_1}\right)^{\gamma-1} \quad \text{or} \quad \frac{T_1}{T_3} = \left(\frac{v_4}{v_1}\right)^{\gamma-1} \qquad \ldots (\because T_4 = T_3) \ldots (ii)$$

From equations (*i*) and (*ii*),

$$\frac{v_3}{v_2} = \frac{v_4}{v_1} \quad \text{or} \quad \frac{v_2}{v_1} = \frac{v_3}{v_4} = r$$

where r = Ratio of expansion or compression.

$$\therefore \quad \frac{T_1}{T_3} = (r)^{\gamma-1}$$

We know that efficiency,

$$\eta = 1 - \frac{T_3}{T_1} = 1 - \left(\frac{1}{r}\right)^{\gamma-1} = 1 - \frac{1}{r^{\gamma-1}}$$

Notes : 1. From the above equation, we see that the efficiency of Carnot's cycle increases as T_1 is increased or T_3 is decreased. In other words, the heat should be taken in at as high a temperature as possible, and rejected at as low a temperature as possible. It may be noted that 100% efficiency can be achieved, only, if T_3 reaches absolute zero, though it is impossible to achieve in practice.

2. In the above theory, we have taken temperature at points 1, 2, 3 and 4 as T_1, T_2, T_3 and T_4 respectively in order to keep similarity between Carnot cycle and other cycles. But some authors take it T_1 (for points 1 and 2) and T_2 (for points 3 and 4). In that case, they obtain the relation for efficiency as,

$$\eta = \frac{T_1 - T_2}{T_1} = 1 - \frac{T_2}{T_1}$$

3. It may be noted that it is impossible to make an engine working on Carnot's cycle. The simple reason for the same is that the isothermal expansion 1-2 will have to be carried out extremely slow to ensure that the air is always at temperature T_1. Similarly, the isothermal compression 3-4 will have to be carried out extremely slow. But reversible adiabatic expansion 2-3 and reversible adiabatic compression 4-1 should be carried out as quickly as possible, in order to approach ideal adiabatic conditions. We know that sudden changes in the speed of an engine are not possible in actual practice. Moreover, it is impossible to completely eliminate friction between the various moving parts of the engine, and also heat losses due to conduction, radiation, etc. It is thus obvious, that it is impossible to realise Carnot's engine in actual practice. However, such an imaginary engine is used as the ultimate standard of comparison of all heat engines.

Example 6.1. *A Carnot engine, working between 650 K and 310 K, produces 150 kJ of work. Find thermal efficiency and heat added during the process.*

Solution. $T_1 = 650$ K ; $T_3 = 310$ K ; $W = 150$ kJ

Thermal efficiency

We know that thermal efficiency,

$$\eta = \frac{T_1 - T_3}{T_1} = \frac{650 - 310}{650} = 0.523 \quad \text{or} \quad 52.3\% \ \textbf{Ans.}$$

Heat added during the process

We know that heat added during the process,

$$Q_{1-2} = \frac{W}{\eta} = \frac{150}{0.523} = 286.8 \text{ kJ} \textbf{ Ans.}$$

Example 6.2. *A Carnot engine operates between two reservoirs at temperatures T_1 and T_3. The work output of the engine is 0.6 times the heat rejected. The difference in temperatures between the source and the sink is 200° C. Calculate the thermal efficiency, source temperature and the sink temperature.*

Solution. Given : $W = 0.6 \times \text{Heat rejected} = 0.6\, Q_{3-4}$; $T_1 - T_3 = 200°\text{ C}$

Thermal efficiency

We know that the thermal efficiency,

$$\eta = \frac{\text{Work done}}{\text{Heat supplied}} = \frac{\text{Work done}}{\text{Work done} + \text{Heat rejected}}$$

$$= \frac{0.6\, Q_{3-4}}{0.6\, Q_{3-4} + Q_{3-4}} = \frac{0.6}{1.6} = 0.375 \quad \text{or} \quad 37.5\% \textbf{ Ans.}$$

Source and sink temperatures

Let T_1 = Source temperature, and

T_3 = Sink temperature.

We know that thermal efficiency (η),

$$0.375 = \frac{T_1 - T_3}{T_1} = \frac{200}{T_1}$$

$$\therefore \quad T_1 = 200/0.375 = 533.3 \text{ K} = 260.3° \text{ C} \textbf{ Ans.}$$

and

$$T_3 = T_1 - 200 = 260.3 - 200 = 60.3° \text{ C} \textbf{ Ans.}$$

Example 6.3. *An engineer claims his engine to develop 3.75 kW. On testing, the engine consumes 0.44 kg of fuel per hour having a calorific value of 42 000 kJ/kg. The maximum temperature recorded in the cycle is 1400° C and minimum is 350° C. Find whether the engineer is justified in his claim.*

Solution. Give : $P = 3.75$ kW; Fuel consumed = 0.44 kg/h; Calorific value = 42 000 kJ/kg $T_1 = 1400°\text{ C} = 1400 + 273 = 1673$ K ; $T_3 = 350°\text{ C} = 350 + 273 = 623$ K

We know that the maximum efficiency, between two specified temperatures, is that of Carnot cycle.

$$\therefore \quad \eta_{carnot} = \frac{T_1 - T_3}{T_1} = \frac{1673 - 623}{1673} = 0.627 \quad \text{or} \quad 62.7\%$$

We also know that the heat supplied to the engine by the fuel

$$= \text{Fuel consumed} \times \text{Calorific value of fuel}$$

$$= 0.44 \times 42\,000 = 18\,480 \text{ kJ/h} = 5.13 \text{ kJ/s}$$

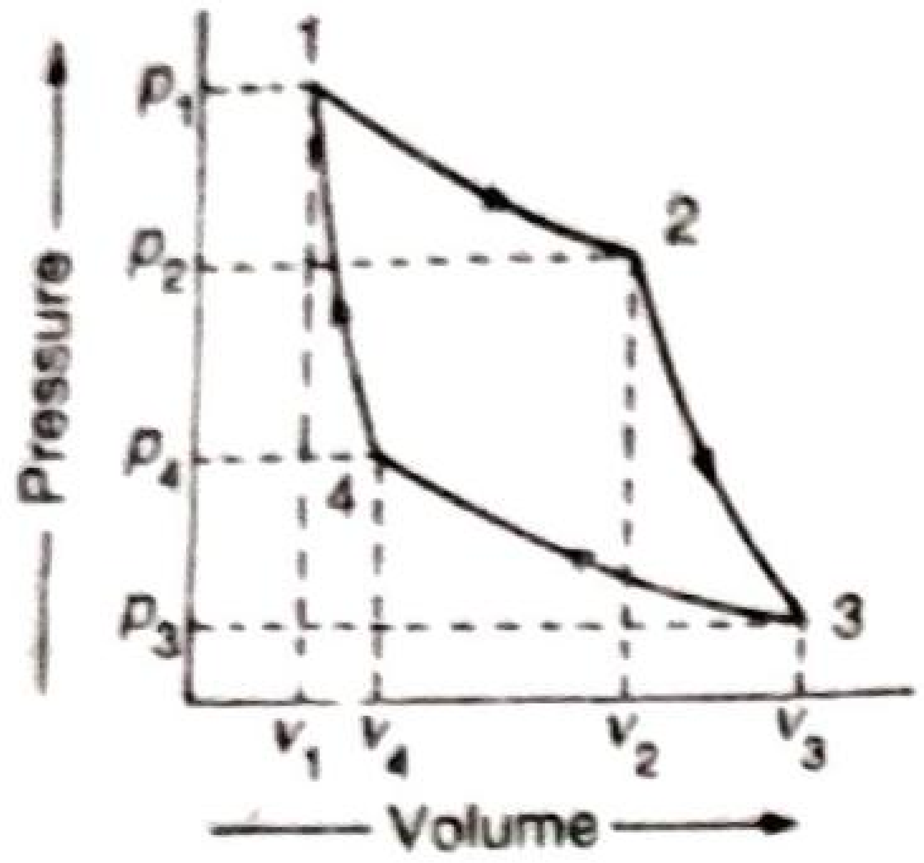
and workdone by the engine $= 3.75 \text{ kW} = 3.75 \text{ kJ/s}$

$$\therefore \text{Efficiency claimed} = \frac{\text{Workdone}}{\text{Heat supplied}} = \frac{3.75}{5.13} = 0.731 \quad \text{or} \quad 73.1\ \%$$

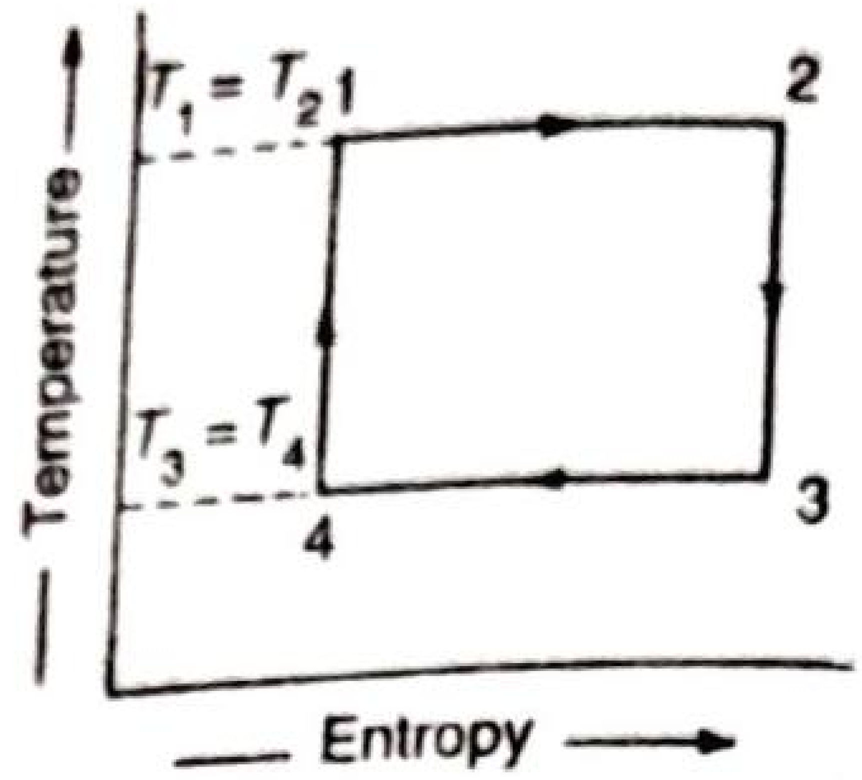
Since efficiency of the engine claimed (73.1%) is more than the maximum possible efficiency (62.7%), therefore the engineer is not justified in his claim. **Ans.**

Example 6.4. *A Carnot cycle works with isentropic compression ratio of 5 and isothermal expansion ratio of 2. The volume of air at the beginning of the isothermal expansion is 0.3 m³. If the maximum temperature and pressure is limited to 550 K and 21 bar, determine : 1. minimum temperature in the cycle ; 2. thermal efficiency of the cycle ; 3. pressure at all salient points ; 4. change of entropy during the isothermal expansion, 5. work done per cycle, and 6. mean effective pressure. Take ratio of specific heats as 1.4.*

Solution. Given : $v_4 / v_1 = 5$; $v_2 / v_1 = 2$; $v_1 = 0.3 \text{ m}^3$; $T_1 = 550$ K ; $p_1 = 21 \text{ bar} = 2.1 \times 10^6 \text{ N/m}^2$; $\gamma = 1.4$



(*a*) *p-v* diagram.



(*b*) *T-S* diagram.

Fig. 6.5

1. Minimum temperature in the cycle

Let T_4 (or T_3) = Minimum temperature in the cycle.

The cycle on *p-v* and *T-S* diagrams is shown in Fig. 6.5 (*a*) and (*b*) respectively. First of all, let us consider the isentropic compression process 4-1. We know that

$$\frac{T_1}{T_4} = \left(\frac{v_4}{v_1}\right)^{\gamma - 1} = (5)^{1.4-1} = (5)^{0.4} = 1.9036$$

$$\therefore \quad T_4 = T_1 / 1.9036 = 550 / 1.9036 = 289 \text{ K} = 16^\circ \text{C} \quad \textbf{Ans.}$$

2. Thermal efficiency of the cycle

We know that thermal efficiency of the cycle,

$$\eta = \frac{T_1 - T_3}{T_1} = \frac{550 - 289}{550} = 0.4745 \text{ or } 47.45\ \% \quad \textbf{Ans.}$$

$$\ldots (\because T_3 = T_4)$$

3. Pressure at all the salient points

Let p_2, p_3, p_4 = Pressures at points 2, 3 and 4 respectively.

First of all, let us consider the isothermal expansion process 1-2. We know that in an isothermal expansion,

$$p_1 v_1 = p_2 v_2 \quad \text{or} \quad p_2 = p_1 \times \frac{v_1}{v_2} = 21 \times \frac{1}{2} = 10.5 \text{ bar} \quad \textbf{Ans.}$$

$$\ldots (\because v_2 / v_1 = 2)$$

Now consider the isentropic expansion process 2-3, we know that

$$p_2 v_2^\gamma = p_3 v_3^\gamma \quad \text{or} \quad p_3 = p_2 \left(\frac{v_2}{v_3}\right)^\gamma = 10.5 \left(\frac{1}{5}\right)^{1.4} = 10.5\,(0.2)^{1.4}$$

$$= 1.103 \text{ bar } \textbf{Ans.} \qquad \ldots \left(\because \frac{v_4}{v_1} = \frac{v_3}{v_2}\right)$$

Now consider the isentropic compression process 4-1. We know that

$$p_4 v_4^\gamma = p_1 v_1^\gamma \quad \text{or} \quad p_4 = p_1 \left(\frac{v_1}{v_4}\right)^\gamma = 21 \left(\frac{1}{5}\right)^{1.4} = 21\,(0.2)^{1.4}$$

$$= 2.206 \text{ bar } \textbf{Ans.}$$

4. *Change of entropy during the isothermal expansion*

We know that change of entropy during the isothermal expansion,

$$S_2 - S_1 = 2.3\, m\, R \log \left(\frac{v_2}{v_1}\right) = 2.3 \times \frac{p_1 v_1}{T_1} \log \left(\frac{v_2}{v_1}\right) \qquad \ldots (\because p_1 v_1 = m R T_1)$$

$$= 2.3 \times \frac{2.1 \times 10^6 \times 0.3}{550} \times \log 2 = 2.636 \times 10^3 \times 0.301 = 793 \text{ J/K}$$

$$= 0.793 \text{ kJ/K } \textbf{Ans.}$$

5. *Workdone per cycle*

We know that heat supplied during the cycle,

$$Q_{1-2} = T_1 (S_2 - S_1) = 550 \times 0.793 = 436 \text{ kJ}$$

and heat rejected during the cycle,

$$Q_{3-4} = T_3 (S_2 - S_1) = 289 \times 0.793 = 229 \text{ kJ}$$

$\therefore$ Workdone per cycle,

$$W = \text{Heat supplied} - \text{Heat rejected} = 436 - 229 = 207 \text{ kJ } \textbf{Ans.}$$

Note: The heat supplied and heat rejected may also be obtained as discussed below :

We know that heat supplied,

$$Q_{1-2} = 2.3\, p_1 v_1 \log \left(\frac{v_2}{v_1}\right) = 2.3 \times 2.1 \times 10^6 \times 0.3 \log 2 \text{ J}$$

$$= 1.449 \times 10^6 \times 0.301 = 436 \times 10^3 \text{ J} = 436 \text{ kJ}$$

and heat rejected,

$$Q_{3-4} = 2.3\, p_3 v_3 \log \left(\frac{v_3}{v_4}\right) = 2.3 \times 0.1103 \times 10^6 \times 3 \log 2$$

$$= 0.761 \times 10^6 \times 0.301 = 229 \times 10^3 \text{ J} = 229 \text{ kJ}$$

$$\ldots \left[\begin{array}{l} \because \dfrac{v_3}{v_4} = \dfrac{v_2}{v_1} = 2 \text{ and } \dfrac{v_4}{v_1} = 5 \\ \text{or } \dfrac{v_3}{v_4} \times \dfrac{v_4}{v_1} = \dfrac{v_3}{v_1} = 2 \times 5 = 10 \\ \therefore v_3 = 10\, v_1 = 10 \times 0.3 = 3 \text{ m}^3 \end{array}\right]$$

6. Mean effective pressure

We know that the stroke volume in a Carnot cycle

$$= v_3 - v_1 = 3.0 - 0.3 = 2.7 \text{ m}^3$$

$\therefore$ Mean effective pressure

$$= \frac{\text{Work done}}{\text{Stroke volume}} = \frac{207}{2.7} = 76.7 \text{ kN/m}^2 \quad \textbf{Ans.}$$