KHi wave damping - Paper 1

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April 27, 2023

1 Introduction

Observations show that the solar atmosphere is filled with highly structured plasma forming loops and threads of material tracing the magnetic field. A number of different phenomena lead to oscillations of these structures in the direction transverse to the field. A few examples are the flare-generated transverse coronal loop oscillations (Aschwanden et al., 1999; Nakariakov et al., 1999); the more prevalent small-scale disturbances in active region loops (McIntosh et al., 2011); Doppler-shift disturbances in extended regions of the solar corona (Tomczyk et al., 2007); propagating transverse waves in prominences (Lin et al., 2007; Schmieder et al., 2013); or the occurrence of transverse waves generated from colliding plasma flows (Antolin et al., 2018). A common interpretation has been given to these oscillations, in terms of propagating or standing transverse magnetohydrodynamic (MHD) kink waves (see e.g., Ruderman and Erdélyi, 2009; Goossens et al., 2011; Nakariakov et al., 2021, for comprehensive reviews).

The usual paradigm under which theoretical studies on MHD kink waves have been carried out is that of waves modelled as linear perturbations to an initial background state (Roberts, 1983, 2000). Theoretical models for the damping of kink waves have also focused mainly on the linear regime (Goossens et al., 1992, 2002; Ruderman and Roberts, 2002; Goossens et al., 2006). However, as evidenced by the catalogues compiled by Goddard and Nakariakov (2016) and Nechaeva et al. (2019), many of the observed kink oscillations have amplitudes that are large and their damping seems to depend on the oscillation amplitude. For at least some of the observed oscillations, the nonlinearities associated with the perturbations to the system are non-negligible. This can lead to damping of the oscillations through nonlinearities (e.g. Chen and Schuck, 2007; Van Doorsselaere et al., 2021; Arregui, 2021).

A well-established result from analytical analysis and numerical simulation is that plasma motions in a flux tube undergoing a nonlinear kink oscillation can lead to the development of the Kelvin-Helmholtz instability (KHi) (see e.g. Terradas et al., 2008; Antolin et al., 2014, 2015; Magyar and Van Doorsselaere, 2016) and the subsequent turbulence the instability can induce (e.g. Hillier and Arregui, 2019; Hillier et al., 2020). In this case, the Kelvin-Helmholtz instability is a parasitic instability that grows on the shear-flow that exists on the flanks of the oscillating flux tube. If the instability can grow, it is then able to develop nonlinearities and if there is sufficient energy in the flow it can then develop turbulence (Hillier, 2019; Hillier and Arregui, 2019). This turbulence can extract energy from an oscillation, either providing a saturation mechanism for the amplitude of the wave for a driven oscillation (Hillier et al., 2020) or lead to damping of an impulsively excited mode. The turbulence excited by the Kelvin-Helmholtz instability is one that is fundamentally different from MHD wave turbulence (van Ballegooijen et al., 2011) (**Citations**) , where nonlinear MHD waves interact to create a daughter wave of higher frequency. The key difference being that even though both of these mechanisms use the large-scale oscillation as the energy source, in the wave turbulence model the large-scale oscillation is also involved in creating the energy cascade process. For Kelvin-Helmholtz turbulence this is not the case, with the energy cascade being related to the nonlinearities of an instability.

In this paper we perform a detailed analysis of the evolution of the Kelvin-Helmholtz instability on the surface of an oscillating flux tube, identifying how the turbulent dynamics results in two different phases of the evolution of the oscillation amplitude of the tube. We then develop an analytic model for the first stage of the evolution of the flux tube. Here we focus on the large scale response of the oscillating tube to the Kelvin-Helmholtz turbulence, in particular the damping of the oscillations and the heating this creates. The model for the second phase will be presented in a future work.

2 Simulations of kink-wave-driven KHi and related wave damping

To build a model of wave damping through Kelvin-Helmholtz turbulence, it is necessary to first identify the fundamental processes that are occurring. To do this we perform a 3D ideal MHD simulation of a nonlinear kink oscillation that goes unstable to the Kelvin-Helmholtz instability. Through this we look to identify what it means for the Kelvin-Helmholtz instability to damp a kink wave, and look at the fundamental processes involved in the damping.

2.1 Simulation setup

We perform this ideal MHD simulation of an impulsively excited MHD kink wave using the MHD routines of the (PIP) code (Hillier et al., 2016). We solve the evolution in 3D in a Cartesian reference frame using the non-dimensionalised ideal MHD equations in conservative form, namely:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0, \tag{1}$$

$$\frac{\partial}{\partial t}(\rho \mathbf{v}) + \nabla \cdot \left(\rho \mathbf{v}\mathbf{v} + P\mathbf{I} - \mathbf{B}\mathbf{B} + \frac{\mathbf{B}^2}{2}\mathbf{I} \right) = 0, \tag{2}$$

$$\frac{\partial}{\partial t}\left(e + \frac{B^2}{2} \right) + \nabla \cdot [\mathbf{v}(e+P) - (\mathbf{v} \times \mathbf{B}) \times \mathbf{B}] = 0, \tag{3}$$

$$\frac{\partial \mathbf{B}}{\partial t} - \nabla \times (\mathbf{v} \times \mathbf{B}) = 0, \tag{4}$$

$$\nabla \cdot \mathbf{B} = 0, \tag{5}$$

$$e \equiv \frac{P}{\gamma - 1} + \frac{1}{2}\rho v^2. \tag{6}$$

which allows us to calculate the evolution of the primitive variables, i.e the density (ρ), velocity field ($\mathbf{v}$), pressure (P) and magnetic field ($\mathbf{B}$), through the evolution of the relevant conserved quantities. Note that in our normalisation we have taken the magnetic permeability of a vacuum inside $\mathbf{B}$ meaning that the local Alfvén speed is given by $V_\mathrm{A} = |\mathbf{B}|/\sqrt{\rho}$. These equations are solved using a fourth-order central difference approximation for spatial derivatives (calculated on a uniform mesh) with a four-step Runge-Kutta integration.

Here we do not include any explicit resistive or viscous terms in the equations. Dissipation at some level is inherent in the simulation due to the finite grid size and the use of flux limiters to smooth sharp structures. As the code is written in conservative form, any extraction of energy from the flow or magnetic field results in a corresponding increase in internal energy (i.e. heating). This allows us to quantify the magnitude of any heating that occurs in the simulation whilst being able to run calculations in the least viscous, least resistive regime we can achieve for the resolution.

The initial conditions are of a dense tube, aligned with the direction of the magnetic field, which we then perturbed to excite the fundamental kink mode of the system. The initial density profile is given by

$$\rho(x,y,z) = \rho_e + \frac{\rho_e - \rho_i}{2}\left(1 - \tanh\left(2^6\left(\frac{r}{R} - 1\right)\right)\right), \tag{7}$$

with $r = \sqrt{x^2 + y^2}$, R the radius of the tube, and ρ_i and ρ_e the internal density of the tube and the external density. We take $R = 0.3$, $\rho_e = 1$ and $\rho_i = 3$. The initial pressure is uniform throughout the domain taking a value $P(x,y,z) = 1/\gamma$ (which gives an initial sound speed of $C_s = 1$) and a magnetic field of $\mathbf{B} = (B_x, B_y, B_z)^\mathrm{T} = \sqrt{2/(\gamma\beta)}(0,0,1)^\mathrm{T}$, with $\beta = 0.05$. The calculations are performed in a domain of $x \in [-L_x, L_x]$, $y \in [0, L_y]$ and $z \in [0, L_z]$ with $L_x = L_y = 1$ and $L_z = 10$. We use a grid size of $\Delta x = 0.005$, $\Delta y = 0.0025$ and $\Delta z = 0.1$. This tube is perturbed with a lateral velocity perturbation to the tube in the form

$$u(x,y,z) = V_0 \frac{\rho - \rho_e}{\rho_i - \rho_e} \sin\left(\frac{\pi z}{2L_z}\right) \tag{8}$$

to excite a fundamental kink mode of the system. We use a value of $V_0 = 0.2$. We also set the y- and z-components of the velocity to zero (i.e. $v = w = 0$).

We set the boundaries of our domain to be either periodic or possessing symmetries. The boundaries at $x = -L_x$ and $x = L_x$ are set to be periodic. The boundaries at $y = 0$ and $y = L_y$ are symmetric boundaries such that the magnetic field is imposed to be in the (x, z)-plane, as is the flow. This implies that $v = B_y = 0$ on the boundary and $\partial u/\partial y = \partial w/\partial y = \partial B_x/\partial y = \partial B_z/\partial y = \partial \rho/\partial y = \partial P/\partial y = 0$. The z-boundaries are symmetric with the magnetic field penetrating the boundary with the imposed symmetries on the magnetic and flow field being of those of a node ($z = 0$) and anti-node ($z = L_z$). **this needs more detail**

Figure 1: Contour plots of the density distribution in the $(x,\ y)$-plane taken at $z = L_z/2$ showing the time evolution of the KHi on the surface of the oscillating flux tube. do we need to add axes labels and lateral color bars with magnitude of density?, t is in units of Alfvén crossing time?

2.2 The evolution of the kink wave and their damping through Kelvin-Helmholtz induced turbulence

Having been subject to its initial kick, as explained in the previous subsection, the tube begins to oscillate. The wave excited is (or at least dominated by) a fundamental kink mode with frequency **give frequency** (period is 12.8), resulting in the dense tube oscillating back and forth. For our initial conditions we have set a sharp boundary in density between the tube and the external medium. Therefore, these oscillations are not subject to resonant damping, which requires a continuous non-uniform variation of density such that the fundamental kink mode has its frequency in the Alfvén continuum. However, as the natural state for these oscillations is for a strong shear flow to develop between the tube and the external medium (Sakurai et al., 1991; Goossens et al., 1992), the boundary of the oscillating tube becomes unstable to the Kelvin-Helmholtz instability (Terradas et al., 2008; Antolin et al., 2014, 2015; Magyar and Van Doorsselaere, 2016).

Figure 1 shows the density evolution of the tube cross-section at the wave apex. These snapshots are taken approximately at times when the displacement of the apex is at its greatest. As the oscillations proceed, we can clearly see the development of Kelvin-Helmholtz roll-ups on the top of the tube. After the first quarter period (top left panel) the instability has started to grow, though the scales associated with this (and with that the area of the cross-section of the tube that has been disturbed by this) is small. As the wave continues to oscillate, with the panels in the figure showing the situation at half-period intervals from the first panel, the KH vortices become larger, and with this the area of the cross section that has been disturbed by the turbulence keeps on increasing until the vast majority of the tube has become part of the turbulent layer.

Looking at this in ~~great~~ (more?) detail, we ~~can see~~ (show?) in Figure 2 the area of the cross section of the tube ~~cross section~~ at the apex (normalised by the initial value) that has a density $\rho \geq 2.9$ (shown by the black line) and for $\rho \geq 1.92$ (red line). The area shown by the black curve acts as a proxy for the area that has not been disturbed by the Kelvin Helmholtz induced turbulence. It is clear that once the KHi develops there is a clear evolution of material that is initially part of the kink oscillation of the tube but then joins the mixing layer (are we providing a movie?). Once the undisturbed area of the tube at the apex becomes about 20% of its initial area, we can see that a transition in the dynamics happens. The area of material with $\rho \geq 1.92$ stays approximately constant over time. This is an important result as it connects to the model of Hillier et al. (2019), as this is the density threshold that the model predicts is area conserving for the particular external and internal density values used in this study.

Looking back at Figure 1 we can see by eye that the total displacement of the tube, at least as measured by the displacement along the x axis, does not appear to be showing a strong decay with time. We can see this

Figure 2: Normalised area of the tube cross section at the apex that has $\rho > 2.9$ (red line) and $\rho = 1.64$ (black line) and $\rho = 1.1$ (blue line).

somewhat clearer in Figure 3 which shows the spatial evolution of v_x at $y = 0$ and $z = L_z$ with time. The blue and red colours show the positive and negative velocities respectively. Until a time of $t \approx 33$, the magnitude of the displacement and the magnitude and coherency of the velocity field do not show any drastic change. However, after that time, corresponding to where the change of area stops decreasing in Figure 2, there is a complete change in the dynamics with clear reductions in the lateral displacement of the tube and a velocity field which is less coherent and of smaller magnitude.

To detail the evolution of the energy sources for the development of the Kelvin-Helmholtz turbulence, i.e. the energy of the oscillation, we can look at how the axis of the tube is displaced. One way to investigate this is to look at the temporal evolution of the velocity field

look at velocity field and talk about damping

Hasegawa et al. (2006) to talk about the local speed-ups.

Clearly the way we measure the bulk motion is determining how we view the change in the oscillation amplitude with time. We can see this very aspect in Figure 5 where the centre-of mass displacement in the x direction has been calculated in two slightly different ways resulting in very different displacements measured. For the solid black line, the displacement in the 2D plane at the apex for all the material above a density threshold of $\rho \geq 1.64$ is displayed. The solid red line is a similar calculation, but for the centre of mass displacement at the apex for the same density threshold but calculated along the 1D line at $y = 0$. We can see that initially the amplitude of the oscillation, as given by the maximum and minimum displacements, decreases monotonically for the case shown in black. However, for the case shown in red, it is only once a time of $t \approx 33$ has been reached that the amplitude starts to decay. Once we get into this later regime both amplitudes are decaying with time, though this is accompanied by a period drift.

MORE ON CHANGE IN DYNAMICS

Ultimately this leads us to a fundamental question: How do we quantify damping in a turbulent loop? The standing kink wave is a coherent oscillation of a dense tube aligned with magnetic field. The damping of these oscillations, when thought about through the lens of normal-mode linear wave theory (**careful here as kink wave is quasi-mode when resonant absorption is there**), should be the same wherever it is measured, not heavily dependent on how the tube is being measured. **need to say something more here!**

What we see in Figures 4 and 5 is that if we measure the amplitude of the oscillation by only looking at the motions in the dense core, there is a long time period where the wave does not display any change in amplitude, **then has changes**. However, if we look at the cross-section as a composite of the dense material and the turbulent mixing layer then this shows changes in amplitude from the start, with the rate at which the amplitude changes being larger during the period the dense core is not displaying a change in amplitude.

even the period of the wave measured will change

That is to say the bulk motions will coherently reduce in scale. However, the turbulent motions that we are studying here that have been excited by a nonlinear kink wave behave differently. The turbulent layer grows,

Figure 3:

Figure 4:

Figure 5:

and naturally "forgets" its initially conditions (i.e. that it is part of a kink wave). As part of this process, the material in the turbulent mixing layer starts to oscillate at a different frequency to the original kink wave.

For the simulation we present here, we can at least determine two regimes in the system. These are , i.e. the magnitude of the shear flow is independent of the

However, for the scenario investigated here, we see that the damping created by the nonlinear development of the Kelvin-Helmholtz instability is one where the coherent bulk oscillations of a loop structure are progressively turned into the random, chaotic motions of turbulence.

list key characteristic including needed two different regimes.

To summarise the different phases of the dynamics. In the first phase we observe the transition from the oscillatory motions of a coherent tube to a turbulent cross-section

with the kinetic energy associated with the

The period of the coherent oscillations of the turbulent layer is (XX%) shorter than the period of the original kink oscillations. During this period the kinetic energy of the turbulent flow (i.e. the 0

When the kinetic energy associated with the oscillations of the turbulent layer becomes comparable with the oscillations of the core of the tube, a new phase in the dynamics begins. In this phase, both the amplitude of the oscillations of the core and the turbulent envelope both decrease with time.

3 Modelling the development of a turbulent oscillating tube

In this section we develop a formulation to describe the evolution of the kink oscillation into a turbulent tube, i.e. the first stage of the damping as described in the previous section. A model for the second stage will be presented in a future paper.

The first stage of the damping process happens as the turbulent layer on the boundary grows. This process exchanges momentum from inside and outside of the tube, manifesting as the originally coherent oscillations of the tube developing a more incoherently moving upper layer, with a coherently moving inner region. This can be seen in Figure 5 where the oscillations of the centre-of-mass for the material with density greater than 1.56 has an amplitude decaying from the beginning of the motion, but the centre-of-mass motions along the x-axis does not show any decay until $t \approx 30$.

In this regime, the velocity jump that is driving the turbulent motions does not change as the amplitude of the centre of mass motions decreases. For a simpler flow, i.e. a hydrodynamic shear layer, it is well known that the turbulent layer grows as

$$h = \beta_{\mathrm{MIX}} \Delta V t, \tag{9}$$

where β_{MIX} is a constant for a given mixing layer but depends on the density contrast of the layer (Baltzer and Livescu, 2020). Using a dimensional analysis of the model proposed in Hillier and Arregui (2019) and developed in Hillier et al. (2023) we propose that the mixing layer grows as

$$h = C_1 \sqrt{0.5} \frac{(\rho_i \rho_e)^{1/4}}{\sqrt{\rho_i} + \sqrt{\rho_e}} \Delta V t, \tag{10}$$

where C_1 is a constant of order 1. Through comparison with mixing in hydrodynamic shear flows, the value of C_1 is expected to be in the range of $C_1 = 0.3$ (Baltzer and Livescu, 2020) to 0.5 (Brown and Roshko, 1974) as discussed in Hillier et al. (2023). To determine an appropriate value for the mixing constant C_1 for this problem we will look more at the mixing in this section. By doing this we can then use this model of an expanding shear layer, combined with information on the structure of the shear layer taken from Hillier and Arregui (2019), to calculate the rate at which mass and momentum are transferred into or out from the oscillating structure and with that how its amplitude damps.

Before it is possible to make these comparisons, it is necessary to measure the magnitude of the shear flow driving the Kelvin-Helmholtz instability. Figure 6 shows the shear flow at the surface of the tube measured at $x = 0$ at the apex at $t = 11.6$ and $t = 18$. The velocity reformulated such that the value is positive at $y = 0$ for easy comparison. Two dashed horizontal lines are added at 0.15 and 0.05 to show the approximate values of the flow inside the dense tube and outside the tube. This implies the magnitude of the shear flow is $\Delta V \approx 0.2$.

Some consideration has to be given to the fact that this is an oscillatory flow and as such will not have the same magnitude of shear flow at any given time. We hypothesize that as the turbulent motions drive the growth in layer width, the Root-Mean-Squared (RMS) shear flow speed is the appropriate value to use, therefore we redefine h with a factor of $1/\sqrt{2}$,

$$h = C_1 \sqrt{0.5} \frac{(\rho_i \rho_e)^{1/4}}{\sqrt{\rho_i} + \sqrt{\rho_e}} \frac{\Delta V}{\sqrt{2}} t. \tag{11}$$

Note that this is not strictly necessary as the constant C_1 will have to be calculated and the value calculated will just scale up or down based on the numerical constants included in the model. To make the value of C_1 as close to unity as possible (i.e. including as many of the physical processes that determine its value as possible), we include this factor here.

3.1 Mass evolution

To study the evolution of the tube oscillations in Section 2.2 we looked at the centre of mass motions of the system based on given density thresholds (e.g. Figure 5). However, these different thresholds result in changes in the mass over time, so it is first important step is to determine how the mass above a given threshold evolves with time. Following the self-similar argument, we can predict the evolution of the total mass as

$$\frac{dm_{\rho>\rho_T}}{dt} = \frac{d}{dt}(\Delta m_{\rho>\rho_T} h 2R) \tag{12}$$

where $m_{\rho>\rho_T}$ is the mass above a given density threshold and $\Delta m_{\rho>\rho_T}$ is the change in mass above that density threshold for a mixing layer of unit width as predicted by the model of Hillier et al. (2019). As for this model, the actual detailed wave dynamics are not so important for the construction of the model we have decided for simplicity of the model to take a rectangular cross-section of length $2R$ and initial height $H = \pi/4$ to model the tube to simplify the calculation of the evolution of the dynamics in the analytical model. The simple justification of this is that the shear layer develops on the top and bottom of the tube and not all the way around.

Figure 6:

Based on the model predictions of Hillier and Arregui (2019), $\Delta m_{\rho>\rho_T}$ is a constant in time, therefore our model becomes

$$\frac{dm_{\rho>\rho_T}}{dt} = \Delta m_{\rho>\rho_T} 2RC_1\sqrt{0.5}\frac{(\rho_i\rho_e)^{1/4}}{\sqrt{\rho_i}+\sqrt{\rho_e}}\Delta V. \tag{13}$$

Using the model of Hillier and Arregui (2019), the value of $\Delta m_{\rho>\rho_T}$ is calculated by first calculating the initial mass of the dense region by multiplying the density of the high density region by the fraction of a unit length that would have been occupied by this dense phase pre-mixing. Then we integrate the density component of the model of Hillier and Arregui (2019) over a unit length from the density threshold value to the maximum density value (in this case 3). This is plotted in Figure **give figure** for different thresholds for the initial density contrast of our simulation. We can see for a small enough threshold the mass will be growing in time, but for larger thresholds there is a loss of mass. The value where the mass evolution is roughly neutral is $\rho_T = 1.64$.

Equation 13 can be integrated to give

$$m_{\rho>\rho_T}(t) = m_{\rho>\rho_T}(0) + \Delta m_{\rho>\rho_T} 2RC_1\sqrt{0.5}\frac{(\rho_i\rho_e)^{1/4}}{\sqrt{\rho_i}+\sqrt{\rho_e}}\Delta V t. \tag{14}$$

Here m_0 is the initial mass, and ΔV is the velocity difference averaged along z and over a half period giving $\Delta V = 0.17 \times 0.5$. Note that the RMS value has been used here as the motions driving the mixing are calculated as the RMS of the turbulent velocity fluctuations in the layer.

To confirm that Equation 14 gives the expected evolution of the mass we compare the evolution of the mass for three different density thresholds from the simulation with comparisons to the predicted evolution given by Equation 14. Figure 7 shows the evolution of the mass at the apex for three density thresholds: $\rho_T = 1.1$ (blue line), $\rho_T = 1.64$ (black line) and $\rho_T = 2.7$ (red line). The dashed lines are the predicted evolution curves for these three thresholds (during this initial regime of the evolution) using $C_1 = 0.3$, which was determined by fitting by eye to the $\rho_T = 1.1$ lines. The value of C_1 found here ($C_1 = 0.3$) is in the range of the values found in hydrodynamic simulations/experiments (Brown and Roshko, 1974; Baltzer and Livescu, 2020).

Overall the simulated curves follow the predicted linear trend. It is clear that though there is some evolution in the total mass for the density threshold of $\rho_T = 1.64$, this is a sufficiently accurate threshold to use to maintain an approximately constant mass in a centre-of-mass velocity calculations. The other two curves show the large variation in mass that can occur when different thresholds are employed. The reason the $\rho_T = 1.64$ threshold shows some evolution could be to do with errors in applying this Cartesian model for the mixing to this more complex geometry, but other, well-established reasons, unrelated to the mixing process like the

Figure 7: Evolution of the mass of material found above three different density thresholds. With $\rho_T = 1.1$ shown in blue, $\rho_T = 1.64$ shown in black, and $\rho_T = 2.7$ shown in red. The dashed lines are the predicted evolution value using $C_1 = 0.3$ with the colours corresponding to the appropriate density threshold.

ponderamotive force driving mass accumulation at the loop apex (**citations**), exist to explain this. It is worth noting that the density Threshold of $\rho_T = 1.92$, which is predicted to be area conserving by the model of Hillier et al. (2019), is approximately area conserving for the duration of the simulation (as evidenced in Figure 2)

One consequence of having determined the value for C_1 is it allows an upper bound for the time this model holds to be derived. Using this we can calculate when the time when the mixing layer has become large enough that all of the dense tube has joined the mixing. Simply put (using the asymmetry of the mixing layer predicted by Hillier et al. (2019)) is when

$$\frac{\sqrt{\rho_e}}{\sqrt{\rho_i}+\sqrt{\rho_e}}h(t) = H \tag{15}$$

This implies that at a time of

$$t = \frac{\sqrt{2}H}{0.3\Delta V}\frac{(\sqrt{\rho_i}+\sqrt{\rho_e})^2}{\rho_i^{1/4}\rho_e^{3/4}} = 44.5, \tag{16}$$

the whole tube would have become turbulent and, therefore, the model of a growing turbulent layer is invalid implying a second stage to the dynamics must have been reached by this time.

3.2 Momentum Evolution

Following on from the model of the mass evolution, we can develop a similar (though slightly more complex) model for the evolution of the momentum. To do this we split the momentum above a given density threshold into two components: 1) the momentum held in the material still corresponding to the original tube (M_{core}), and 2) the momentum injected into the mixing layer (M_{mix}. This is the same concept as has been used for the modelling of the mass evolution, but with the added complexity that the momentum naturally changes sign as the system undergoes its natural oscillations. The combination of a natural oscillation frequency with the momentum extraction/injection process acting like an oscillatory forcing on the system makes a system that is analogous to a forced linear oscillator

Firstly, to model the evolution of the momentum in the core, we need to look at the case where the forcing (in terms of the extraction of momentum through mixing) occurs at the same frequency (i.e. resonant frequency) as the natural oscillations of the system (the kink frequency of the oscillation). This can be stated in equation form as

$$\frac{dM_{\text{core}}}{dt} = -\omega_{\text{KINK}}^2 I_{\text{core}} - 2\dot{F}_{\text{core}}\cos(\omega_{\text{KINK}}t), \tag{17}$$

i.e. a forced linear oscillator where the system is being forced at a resonant frequency with a forcing given by

$$\dot{F} = 2R\overline{M_0}\frac{dH}{dt}, \tag{18}$$

with $\overline{M_0}$ the initial momentum density given by

$$\overline{M_0} = \rho_i V_i, \tag{19}$$

where V_i is the speed of the dense material, in this case $V_i = 0.15$ as can be seen in Figure 6.

This have the solution

$$I_{\rm core}(t) = A\sin(\omega_{\rm KINK}t) + B\cos(\omega_{\rm KINK}t) - \frac{\dot{F}}{\omega_{\rm KINK}} t\sin(\omega_{\rm KINK}t). \tag{20}$$

If we follow our initial assumption that the forcing term is in phase with the oscillations, this implies that $B = -\dot{F}/\omega_{\rm KINK}^2$, which leads to

$$I_{\rm core}(t) = \frac{M_0}{\omega_{\rm KINK}}\sin(\omega_{\rm KINK}t) - \frac{\dot{F}}{\omega_{\rm KINK}^2}\cos(\omega_{\rm KINK}t) - \frac{\dot{F}}{\omega_{\rm KINK}} t\sin(\omega_{\rm KINK}t). \tag{21}$$

This leads to the momentum evolution of the core following

$$M_{\rm core} = M_0\cos(\omega_{\rm KINK}t)\left(1 - \frac{0.3}{\sqrt{2}H}\frac{\rho_i^{1/4}\rho_e^{3/4}}{(\sqrt{\rho_i}+\sqrt{\rho_e})^2}\Delta V t\right). \tag{22}$$

To model the momentum injected into the mixing layer requires more subtlety. Unlike the mass evolution, the momentum is not a positive definite quantity, it oscillates around zero, so the sign of the momentum injected into the mixing layer changes periodically. On top of this, the local field lines in the mixing layer have their own characteristic frequency of oscillation (i.e. Alfvén frequencies), which can be different from that of the frequency with which momentum is being injected.

The simple model we put forward to explain how the momentum in the mixing layer develops is based on again on a forced linear oscillator, but this time the characteristic frequency of the oscillation of the layer and that of the core are not assumed to be the same. We consider a single, representative Alfvén frequency for the mixing layer (a composite Alfvén frequency or kink frequency of the layer) based on the turbulent motions coupling all the different regions of the layer allowing it to develop oscillations at a single representative frequency. As the model of Hillier et al. (2019) proposes the representative density of the layer is $\sqrt{\rho_i\rho_e}$, for our non-dimensional frequency this gives $\omega_{\rm A} = B/(\rho_i\rho_e)^{1/4}$. Here we model the whole layer as a single forced nonlinear oscillator with momentum $M_{\rm 1L}$. Mathematically, the evolution of the momentum would be modelled as:

$$\frac{dM_{\rm 1L}}{dt} = -\omega_{\rm A}^2 I_{\rm 1L} - 2\dot{F}\cos(\omega_{\rm KINK}t), \tag{23}$$

where we define I such that $dI_{\rm 1L}/dt = M_{\rm 1L}$ and $\dot{F}$ is the forcing term relating to the rate at which momentum is added into the mixing layer from the external regions. The basic concept is that in the layer, momentum is added through the turbulent mixing of the momenta of the internal and external layers, effectively forcing the mixing layer to respond. This driving naturally happens at the kink frequency ($\omega_{\rm KINK}$).

$$\dot{F}_L = \overline{M}\frac{0.3}{\sqrt{2}H}\frac{(\rho_i\rho_e)^{1/4}}{\sqrt{\rho_i}+\sqrt{\rho_e}}\Delta V \tag{24}$$

with $\overline{M}$ as the momentum injection calculated from the model of Hillier et al. (2019) over a region of unit thickness. The precise calculation is given by

$$\overline{M} = \left(0.15 - \frac{\Delta V\sqrt{\rho_e}}{\sqrt{\rho_i}+\sqrt{\rho_e}}\right) \times \int_{y'(\rho_{min})}^{y'(\rho_{max})} \rho dy' \tag{25}$$

The term in the bracket gives the mean velocity of the mixing layer in the rest frame of the simulation. This is then multiplied by the integral of the density between the two density limits of interest across the predicted average density profile of the mixing layer model of Hillier et al. (2019) for a layer with width of unit length.

Equation 23 leads to the well know solution (in the case that the characteristic Alfvén frequency is not the kink frequency of)

$$I_{\rm 1L} = A\sin(\omega_{\rm A}t) + B\cos(\omega_{\rm A}t) + \frac{2\dot{F}_L}{\omega_{\rm A}^2 - \omega_{\rm KINK}^2}\cos(\omega_{\rm KINK}t). \tag{26}$$

If we take that at $t = 0$ we have $I_{\rm 1L} = 0$ this implies that

$$B = -\frac{2\dot{F}_L}{\omega_{\rm A}^2 - \omega_{\rm KINK}^2} \tag{27}$$

Figure 8: Comparison of the model and the simulation for (a) the momentum evolution and (b) the kinetic energy associated with the bulk motions. The black lines show the distribution for material with $\rho > 2.7$ for the simulation (solid) and model (dashed). The red lines show the evolution for the quantities where $1.1 < \rho < 2.7$ for the simulation (solid) and model (dashed).

Equation 26 can be differentiated to give the momentum evolution in the mixing layer as

$$M_{1\mathrm{L}} = A\omega_{\mathrm{A}} \cos(\omega_{\mathrm{A}} t) + \frac{2\dot{F}_L \omega_{\mathrm{A}}}{\omega_{\mathrm{A}}^2 - \omega_{\mathrm{KINK}}^2} \sin(\omega_{\mathrm{A}} t) - \frac{2\dot{F}_L \omega_{\mathrm{KINK}}}{\omega_{\mathrm{A}}^2 - \omega_{\mathrm{KINK}}^2} \sin(\omega_{\mathrm{KINK}} t). \tag{28}$$

We then have $A = M_{1\mathrm{L}}(0)/\omega_{\mathrm{A}}$ giving

$$M_{1\mathrm{L}} = M_{1\mathrm{L}}(0) \cos(\omega_{\mathrm{A}} t) + \frac{2\dot{F}_L \omega_{\mathrm{A}}}{\omega_{\mathrm{A}}^2 - \omega_{\mathrm{KINK}}^2} \sin(\omega_{\mathrm{A}} t) - \frac{2\dot{F}_L \omega_{\mathrm{KINK}}}{\omega_{\mathrm{A}}^2 - \omega_{\mathrm{KINK}}^2} \sin(\omega_{\mathrm{KINK}} t). \tag{29}$$

Figure 9 shows the comparison between the predicted momentum and kinetic evolution of the core of the tube (M_{core}) and the mixing layer $M_{1L}()$ compared with the simulation results. What is presented here is not a fit, but the solution from an initial value problem compared with the results from the 3D simulations. This model clearly captures the key features of the evolution.

Going further, we can simplify Equation 29 by taking $M_{1\mathrm{L}}(0) = 0$ and looking at the case where $\omega_{\mathrm{A}} - \omega_{\mathrm{KINK}}$ is small. By setting $\omega_{\mathrm{D}} = (\omega_{\mathrm{A}} - \omega_{\mathrm{KINK}})/2$ and $\omega_{\mathrm{S}} = (\omega_{\mathrm{A}} + \omega_{\mathrm{KINK}})/2$. This means $\omega_{\mathrm{A}} = \omega_{\mathrm{S}} + \omega_{\mathrm{D}}$ and $\omega_{\mathrm{KINK}} = \omega_{\mathrm{S}} - \omega_{\mathrm{D}}$. Therefore we have

$$M_{1\mathrm{L}} \approx \frac{\dot{F}(\omega_{\mathrm{S}} + \omega_{\mathrm{D}})}{2\omega_{\mathrm{S}}\omega_{\mathrm{D}}} \sin((\omega_{\mathrm{S}} + \omega_{\mathrm{D}})t) - \frac{\dot{F}(\omega_{\mathrm{S}} - \omega_{\mathrm{D}})}{2\omega_{\mathrm{S}}\omega_{\mathrm{D}}} \sin((\omega_{\mathrm{S}} - \omega_{\mathrm{D}})t) \tag{30}$$

This can be expanded out through double angle formulas to give

$$M_{1\mathrm{L}} \approx \frac{\dot{F}}{2\omega_{\mathrm{S}}\omega_{\mathrm{D}}} \left((\omega_{\mathrm{S}} + \omega_{\mathrm{D}})(\sin(\omega_{\mathrm{S}} t)\cos(\omega_{\mathrm{D}} t) + \cos(\omega_{\mathrm{S}} t)\sin(\omega_{\mathrm{D}} t)) - (\omega_{\mathrm{S}} - \omega_{\mathrm{D}})(\sin(\omega_{\mathrm{S}} t)\cos(\omega_{\mathrm{D}} t) - \cos(\omega_{\mathrm{S}} t)\sin(\omega_{\mathrm{D}} t))\right). \tag{31}$$

Taking that $\omega_{\mathrm{D}} \ll \omega_{\mathrm{S}}$ this can again be simplified to

$$M_{1\mathrm{L}} \approx \frac{\dot{F}}{\omega_{\mathrm{D}}} \cos(\omega_{\mathrm{S}} t)\sin(\omega_{\mathrm{D}} t)). \tag{32}$$

This implies we expect our momentum in the mixing layer to oscillate with a frequency of ω_{S} inside an envelope that evolves as $\sin(\omega_{\mathrm{D}} t)$.

Our result above implies that we can refine our prediction for when this initial phase of the dynamics will end. The model we have put forward for this period of the dynamics is that of the oscillations in the core of the tube driving the motions. However, it is clear from panel (b) of Figure 9 that as time goes on, even assuming that the core of the tube holds the majority of the kinetic energy associated with coherent motions no longer holds. At a time of $t \approx 33$ the energy of the coherent motions of the mixing layer becomes greater than that of the energy of the oscillations of the core. So even without including the energy of the turbulent motions we can see that our assumption that the core energetically dominates the dynamics. This switch from the core to the layer dominating the energetics might explain why the model is less accurate once we reach a time of $t \approx 40$.

3.3 Predicted Evolution of Amplitude and Velocity Amplitude of the Wave motions

With a prediction for both the momentum evolution and the mass evolution above a given density threshold of the oscillation, this can then be turned into an estimate of the centre of mass velocity of the oscillation. This can be formulated mathematically as

$$V_{\mathrm{CoM},\rho\geq\rho_{\mathrm{T}}} = \frac{\int_{\rho\geq\rho_{\mathrm{T}}} \rho V dA}{\int_{\rho\geq\rho_{\mathrm{T}}} \rho dA} = \frac{M_{\rho\geq\rho_{\mathrm{T}}}}{m_{\rho\geq\rho_{\mathrm{T}}}}, \tag{33}$$

with ρ_{T} the threshold density used. That is the integral of the density weighted velocity divided by the integral of the density give a centre of mass velocity, but this is just the momentum divided by the mass. As both $m_{\rho\geq\rho_{\mathrm{T}}}$ and $M_{\rho\geq\rho_{\mathrm{T}}}$ have been directly predicted previosuly, we can use these to subsequently make a prediction of $V_{\mathrm{CoM},\rho\geq\rho_{\mathrm{T}}}$.

Panels (a) and (c) of figure 9 show the temporal evolution of centre of mass velocity for just the core of the tube at its apex (panel a) and when the mixing layer is also included (panel c). Unsurprisingly, given the accuracy of the momentum and mass evolution up until a time of $t \approx 40$ these panels show the model is a good representation of the simulation results.

A measure of the evolution of the velocity of the loop apex over time can be used to make a prediction for the evolution of the position of the loop apex (i.e. the wave amplitude) over time. Then by integrating $V_{\mathrm{CoM},\rho\geq\rho_{\mathrm{T}}}$ over time, we can make a subsequent prediction for the amplitude evolution ($A_{\rho\geq\rho_{\mathrm{T}}}$) of

$$A_{\rho\geq\rho_{\mathrm{T}}} = \int_0^t \frac{\int_{\rho\geq\rho_{\mathrm{T}}} \rho V dA}{\int_{\rho\geq\rho_{\mathrm{T}}} \rho dA} dt = \int_0^t \frac{M_{\rho\geq\rho_{\mathrm{T}}}}{m_{\rho\geq\rho_{\mathrm{T}}}} dt. \tag{34}$$

Due to the nonlinear system we are studying, though dimensionally the same, the $A_{\rho\geq\rho_{\mathrm{T}}}$ defined above is different from the centre-of-mass amplitude which is defined as

$$A_{\mathrm{CoM},\rho\geq\rho_{\mathrm{T}}} = \frac{\int_{\rho\geq\rho_{\mathrm{T}}} \rho x dA}{\int_{\rho\geq\rho_{\mathrm{T}}} \rho dA}. \tag{35}$$

Here the position x connects as the time integral of the velocity. This difference in formulation is very important to consider when comparing results for a non-linear system, as differing formulations will lead to some differences in results. This is seen in panels (b) and (d) of Figure 9, where for looking only at the core of the tube (with its simpler evolution) the model prediction for the amplitude evolution ($A_{\rho\geq\rho_{\mathrm{T}}}$) is a reasonable representation of the centre-of-mass amplitude from the simulation ($A_{\mathrm{CoM},\rho\geq\rho_{\mathrm{T}}}$). However, in panel (d) there is a much greater decrease in the centre-of-mass amplitude (not seen in the centre of mass velocity) compared to the model.

Figure 9: Comparison of the model and the simulation for (a) the velocity amplitude and (b) the amplitude of the . The black lines show the distribution for material with $\rho < 2.7$ for the simulation (solid) and model (dashed). The red lines show the evolution for the quantities where $1.1 < \rho < 2.7$ for the simulation (solid) and model (dashed).

3.4 Bounding the heating rate in the first phase of the dynamics

An important question, especially in terms of how the results of this paper may impact our understanding of wave heating of the solar corona is: can we predict the heating rates in the simulation from the model of Kelvin-Helmholtz turbulence. To make a prediction of the heating rates, first we need to make an estimate of the level of the turbulence (both in the velocity and the magnetic fields). To do this we can make a model how the energy lost from the shear flow is distributed between turbulent fluctuations an heating. The predict how turbulence grows at the apex of the loop to compare with the simulations and then use this to infer the turbulence throughout the whole domain. Finally, this can be used to predict the heating rate for the whole domain which can be compared with the increase in thermal energy in the simulation domain.

Firstly, we need to propose a model for how the kinetic energy that enters the shear layer is divided between the kinetic and magnetic energy of turbulent fluctuations and heating. As we are assuming that the heating process comes from the turbulence, all the free kinetic energy and magnetic energy that are brought into the shear layer and forms part of the turbulent structures will ultimately become heat. However, this process takes time which allows the energy stored as turbulence to grow even though it is being extracted. To make the model, we can use the approximate self-similarity of the turbulent energy cascade which implies that the velocity fluctuations at a given length scale (l) as $\propto (l/l_0)^{1/3}$ where l_0 is the largest scale of the turbulent cascade (**CITE K41?**). Assuming that the turbulent cascade involves the standard nonlinear process where energy is passed from one scale to the scale of half that size and that the turbulence cascades to infinitesimal scales (the Reynolds and magnetic Reynolds number are tending to infinity) leads to the following estimate of the dissipation time τ_{DISS}

$$\tau_{\mathrm{DISS}} = \sum_{i=0}^{\infty} \frac{l_i}{v_i} = \frac{l_0}{v_0}\left(1 + \frac{1}{2^{2/3}} + \frac{1}{2^{4/3}} + ...\right) \approx 2.70\frac{l_0}{v_0} \tag{36}$$

where the subscript denotes the level in the cascade and $\tau_{\mathrm{EDDY}} = l_0/v_0$ is the large-scale vortex turnover time. This leads us to estimate that at any given time the energy that has been brought into the mixing layer and has formed part of the turbulent fluctuations, $\tau_{\mathrm{EDDY}}/\tau_{\mathrm{DISS}} = 1/2.7$ of the energy will have been dissipated leaving 1.7/2.7 as part of the turbulence.

The next step is to then make a prediction for how the turbulent component of the kinetic energy grows at the apex of the loop (based on the assumption of our model that compressibility is small in the Kelvin-Helmholtz dynamics which would imply that due to boundary conditions all the turbulent energy is held as kinetic energy). Following the work of Hillier and Arregui (2019) we expect that the predicted upper bound for TE (the turbulent energy - including both the kinetic and magnetic energy components of the turbulence) in the mixing layer at a given time is:

$$TE \approx \frac{(\rho_i\rho_e)}{8(\sqrt{\rho_i}+\sqrt{\rho_e})^2}\Delta V^2 2Rh = C_1\frac{(\rho_i\rho_e)^{5/4}}{8(\sqrt{\rho_i}+\sqrt{\rho_e})^3}\Delta V^3 Rt. \tag{37}$$

Here we have assumed that the magnitude of the turbulence is uniform all the way along the loop (though the magnitudes of the kinetic and magnetic energy components will vary), so have taken an average of the total energy available for the turbulence.

This, however, is not all the energy that exists for turbulence. Actually, the forced linear oscillator model used for the mixing layer, as a consequence of the layer not being in resonance with the frequency of the kink oscillations implies that not all the energy out into the layer

this doesn't take into account the extra turbulent energy that exists because the tube is oscillating back and forth. In Section **previous part of energy** that there is

Figure 10: Change in internal energy (solid black line) with predicted slope based on the model of turbulent heating as the Reynolds number tends to infinity (solid red line) and the predicted upper limit of the heating rate (dashed red line).

4 Modelling energy dissipation and wave damping in the second phase of the dynamics

Having developed a model to explain the evolution of the first phase, we now turn our attention to the second phase of the dynamics. The key characteristic in this regime that we use to guide the model is that the energy of the dynamics is dominated by the turbulent component (see Figure 11 after $t \approx 40$). It is also important to note that as the turbulent energy is dominant and no further energy is input into the system, that the turbulence will decay over time (again see Figure 11 after $t \approx 40$). Therefore, we focus on developing a model for the decay of the turbulence.

4.1 Heating through turbulent decay

In this phase, the energy associated with the turbulent fluctuations is already larger than the energy of the coherent oscillations. Therefore, the dissipation of energy can be understood without considering the energy contained in the oscillations. This makes the model we propose for understanding the rate at which the turbulence decays to be that of simple decaying turbulence, and independent of the oscillatory dynamics.

The model we use was first proposed by **Taylor** and **Kolmogorov** is based on the turbulent transport of the turbulent energy held at large scales to the dissipation scale through nonlinear turbulent transport through spatial scales:

$$\frac{dE_{\rm turb}}{dt} = \mathbf{v} \cdot \nabla E_{\rm turb} \tag{38}$$

We consider the simple situation where the characteristic lengthscale of the turbulent motions is approximately constant throughout the whole of the turbulent decay, this cane be approximated as

$$\frac{dE_{\rm turb}}{dt} = \mathbf{v} \cdot \nabla E_{\rm turb} \approx C \frac{u_{\rm RMS}}{L} E_{\rm turb}. \tag{39}$$

The term on the right-hand-side can be further simplified by realising that the RMS velocity is directly related to the turbulent energy by (following Hillier, 2019) $u_{\rm RMS} = \sqrt{2E_{\rm turb}/(A\sqrt{\rho_i \rho_e})}$ where A is the area of the turbulent region (which we take to be an annulus with outer radius of 0.45 and inner radius of 0.15) and L is the length-scale of the turbulent motions which we take to be the different between the inner and outer radius of 0.3. For dynamical consistency we take that the unknown constant C has the value $C_1/2.7$ where the 2.7 comes from the cascade time for Kolmogorov turbulence (see Section 3.4), i.e. $C = C_1/2.7 = 0.3/2.7$. Making

Figure 11: Kinetic energy of the x component of the velocity at the apex of the oscillation for the core of the tube (black line), the coherent motions of the mixing layer (red line), with the energy of everything left, i.e. the incoherent, turbulent motions, shown in blue.

Figure 12: Plot of the normalised turbulent energy against modified time. The black line shows the results of the simulation and the red line the predicted decay from the model.

the substitution for u_{RMS}, rearranging and then integrating leads to the following integral equation:

$$\int_{E(t_0)}^{E(t)} E^{-3/2} dE = \frac{C_1}{2.7L}\sqrt{\frac{2}{A\sqrt{\rho_e\rho_i}}}\int_{t_0}^{t} dt \tag{40}$$

Therefore,

$$E_{\mathrm{turb}} = \frac{1}{(\alpha(t-t_0)+E(t_0)^{-1/2})^2}. \tag{41}$$

with

$$\alpha = \frac{C_1}{2.7L}\sqrt{\frac{2}{A\sqrt{\rho_e\rho_i}}}. \tag{42}$$

The simple implication from this is that the turbulent energy should decay at late times $\propto t^{-2}$. This is a classic result for decaying turbulence with a fixed length-scale of the large scale motions (CITE GI TAYLOR). Figure 12 shows the comparison between the theoretically predicted decay and that shown for the kinetic energy in the simulation at the apex of the tube. Note that α has been multiplied by a factor of $\sqrt{4/3}$ because we predict that the total kinetic energy will be 4/3 greater than the kinetic energy in of the x-direction can be inferred from the model of Hillier and Arregui (2019). Overall the match between the model and simulation results is good, though this only over a relatively short range.

If the energy of the turbulence is decaying, the reason for this is that the turbulent fluctuations are being dissipated. Therefore, we can expect this to directly connect to heating. If the turbulent energy goes down, this should directly correspond to an increase in thermal energy. Assuming this decay (be it in turbulent kinetic energy or turbulent magnetic energy) is uniform along the flux tube, we can integrate $E(t_0) - E_{\mathrm{turb}}$ over the

Figure 13:

length of the tube and this gives the amount of energy dissipated. To take into account the velocity (and with that magnetic field) fluctuations in the y direction we multiply $E(t_0) - E_{\rm turb}$ by 4/3 and again take the same factor of $\sqrt{4/3}$ in α. The comparison between the relative increase in internal energy of the model and the simulation are shown in Figure 13 with the black line giving the simulation results and the red line that of the model (the model is only shown after a time of $t = 42$ for which it was derived). The model clearly provides a reasonable representation of the evolution implying that the key physical concepts that are needed have been included in the model.

5 Summary

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