KHi wave damping - Paper 1

Andrew Hillier, Iñigo Arregui & Takeshi Matsumoto

March 3, 2023

1 Introduction

Observations show that the solar atmosphere is filled with highly structured plasma forming loops and threads of material tracing the magnetic field. A number of different phenomena lead to oscillations of these structures in the direction transverse to the field. A few examples are the flare-generated transverse coronal loop oscillations (Aschwanden et al., 1999; Nakariakov et al., 1999); the more prevalent small-scale disturbances in active region loops (McIntosh et al., 2011); Doppler-shift disturbances in extended regions of the solar corona (Tomczyk et al., 2007); propagating transverse waves in prominences (Lin et al., 2007; Schmieder et al., 2013); or the occurrence of transverse waves generated from colliding plasma flows (Antolin et al., 2018). A common interpretation has been given to these oscillations, in terms of propagating or standing transverse magnetohydrodynamic (MHD) kink waves (see e.g., Ruderman and Erdélyi, 2009; Goossens et al., 2011; Nakariakov et al., 2021, for comprehensive reviews).

The usual paradigm under which theoretical studies on MHD kink waves have been carried out is that of waves modelled as linear perturbations to an initial background state (Roberts, 1983, 2000). Theoretical models for the damping of kink waves have also focused mainly on the linear regime (Goossens et al., 1992, 2002; Ruderman and Roberts, 2002; Goossens et al., 2006). However, as evidenced by the catalogs compiled by Goddard and Nakariakov (2016) and Nechaeva et al. (2019), many of the observed kink oscillations have amplitudes that are large and their damping seems to depend on the oscillation amplitude. For at least some of the observed oscillations, the nonlinearities associated with the perturbations to the system are non-negligible. This can lead to damping of the oscillations through nonlinearities (e.g. Chen and Schuck, 2007; Van Doorsselaere et al., 2021; Arregui, 2021).

A well-established result from analytical analysis and numerical simulation is that plasma motions in a flux tube undergoing a nonlinear kink oscillation can lead to the development of the Kelvin-Helmholtz instability (KHi) (see e.g. Terradas et al., 2008; Antolin et al., 2014, 2015; Magyar and Van Doorsselaere, 2016) and the subsequent turbulence the instability can induce (e.g. Hillier and Arregui, 2019; Hillier et al., 2020). In this case, the Kelvin-Helmholtz instability is a parasitic instability that grows on the shear-flow that exists on the flanks of the oscillating flux tube. If the instability can grow, it is then able to develop nonlinearities and if there is sufficient energy in the flow it can then develop turbulence (Hillier, 2019; Hillier and Arregui, 2019). This turbulence can extract energy from an oscillation, either providing a saturation mechanism for the amplitude of the wave for a driven oscillation (Hillier et al., 2020) or lead to damping of an impulsively excited mode. The turbulence excited by the Kelvin-Helmholtz instability is one that is fundamentally different from MHD wave turbulence (van Ballegooijen et al., 2011) (**Citations**) IA: are we talking about the MHD wave turbulence models by e.g., Cranmer & Van Ballegooijen?, where nonlinear MHD waves interact to create a daughter wave of higher frequency. The key difference being that even though both of these mechanisms use the large-scale oscillation is the energy source, in the wave turbulence model the large-scale oscillation is also involved in creating the energy cascade process. For Kelvin-Helmholtz turbulence this is not the case, with the energy cascade being related to the nonlinearities of an instability.

In this paper we perform a detailed analysis of the evolution of the Kelvin-Helmholtz instability on the surface of an oscillating flux tube, identifying how the turbulent dynamics results in two different phases of the evolution of the oscillation amplitude of the tube. We then develop an analytic model for the first stage of the evolution of the flux tube. Here we focus on the large scale response of the oscillating tube to the Kelvin-Helmholtz turbulence, in particular the damping of the oscillations and the heating this creates. The model for the second phase will be presented in a future work.

2 Simulations of kink-wave-driven KHi and related wave damping

IA: I have removed the "the" in front of "related" in the title of this section

To build a model of wave damping through Kelvin-Helmholtz turbulence, it is necessary to first identify the fundamental processes that are occurring. To do this we perform a 3D ideal MHD simulation of a nonlinear kink oscillation that goes unstable to the Kelvin-Helmholtz instability. Through this we look to identify what it means for the Kelvin-Helmholtz instability to damp a kink wave, and look at the fundamental processes involved in the damping.

2.1 Simulation setup

We perform this ideal MHD simulation of an impulsively excited MHD kink wave using the MHD routines of the (PIP) code (Hillier et al., 2016). We solve the evolution in 3D in a Cartesian reference frame using the non-dimensionalised ideal MHD equations in conservative form, namely:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0, \tag{1}$$

$$\frac{\partial}{\partial t}(\rho \mathbf{v}) + \nabla \cdot \left(\rho \mathbf{v}\mathbf{v} + P\mathbf{I} - \mathbf{B}\mathbf{B} + \frac{\mathbf{B}^2}{2}\mathbf{I} \right) = 0, \tag{2}$$

$$\frac{\partial}{\partial t}\left(e + \frac{B^2}{2} \right) + \nabla \cdot [\mathbf{v}(e+P) - (\mathbf{v} \times \mathbf{B}) \times \mathbf{B}] = 0, \tag{3}$$

$$\frac{\partial \mathbf{B}}{\partial t} - \nabla \times (\mathbf{v} \times \mathbf{B}) = 0, \tag{4}$$

$$\nabla \cdot \mathbf{B} = 0, \tag{5}$$

$$e \equiv \frac{P}{\gamma - 1} + \frac{1}{2}\rho v^2. \tag{6}$$

which allows us to calculate the evolution of the primitive variables, i.e the density (ρ), velocity field ($\mathbf{v}$), pressure (P) and magnetic field ($\mathbf{B}$), through the evolution of the relevant conserved quantities. Note that in our normalisation we have taken the magnetic permeability of a vacuum inside $\mathbf{B}$ meaning that the local Alfvén speed is given by $V_\mathrm{A} = |\mathbf{B}|/\sqrt{\rho}$. These equations are solved using a fourth-order central difference approximation for spatial derivatives (calculated on a uniform mesh) with a four-step Runge-Kutta integration.

Here we do not include an explicit resistive or viscous terms in the equations. Dissipation at some level is inherent in the simulation due to the finite grid size and the use of flux limiters to smooth sharp structures. As the code is written in conservative form, any extraction of energy from the flow or magnetic field results in a corresponding increase in internal energy (i.e. heating). This allows us to quantify the magnitude of any heating that occurs in the simulation whilst being able to run calculations in the least viscous, least resistive regime we can achieve for the resolution.

The initial conditions are of a dense tube, aligned with the direction of the magnetic field, which we then perturbed to excite the fundamental kink mode of the system. The initial density profile is given by

$$\rho(x,y,z) = \rho_e + \frac{\rho_e - \rho_i}{2}\left(1 - \tanh\left(2^6\left(\frac{r}{R} - 1\right)\right)\right), \tag{7}$$

with $r = \sqrt{x^2 + y^2}$, R the radius of the tube, and ρ_i and ρ_e the internal density of the tube and the external density. We take $R = 0.3$, $\rho_e = 1$ and $\rho_i = 3$. The initial pressure is uniform throughout the domain taking a value $P(x,y,z) = 1/\gamma$ (which gives an initial sound speed of $C_s = 1$) and a magnetic field of $\mathbf{B} = (B_x, B_y, B_z)^\mathrm{T} = \sqrt{2/(\gamma\beta)}(0,0,1)^\mathrm{T}$ with $\beta = 0.05$ (I have introduced a value for β copied from the mixing paper. It is OK?). The calculations are performed in a domain of $x \in [-L_x, L_x]$, $y \in [0, L_y]$ and $z \in [0, L_z]$ with $L_x = L_y = 1$ and $L_z = 10$. We use a grid size of $\Delta x = 0.005$, $\Delta y = 0.0025$ and $\Delta z = 0.1$. This tube is perturbed with a lateral velocity perturbation to the tube in the form

$$u(x,y,z) = V_0 \frac{\rho - \rho_e}{\rho_i - \rho_e} \sin\left(\frac{\pi z}{2L_z}\right) \tag{8}$$

to excite a fundamental kink mode of the system. We use a value of $V_0 = 0.2$. We also set $v = w = 0$ I am unable to find where w was defined.

~~For~~ We set the boundaries of our domain to be either periodic or possessing symmetries. The boundaries at $x = -L_x$ and $x = L_x$ are set to be periodic. The boundaries at $y = 0$ and $y = L_y$ are symmetric boundaries such that the magnetic field is imposed to be in the (x, z)-plane, as is the flow. This implies that $v = B_y = 0$ on the boundary and $\partial u/\partial y = \partial w/\partial y = \partial B_x/\partial y = \partial B_z/\partial y = \partial \rho/\partial y = \partial P/\partial y = 0$. The z-boundaries are symmetric with the magnetic field penetrating the boundary with the imposed symmetries on the magnetic and flow field being of those of a node ($z = 0$) and anti-node ($z = L_z$). **this needs more detail**

Figure 1: Contour plots of the density distribution in the $(x,\ y)$- plane taken at $z = L_z/2$ showing the time evolution of the KHi on the surface of the oscillating flux tube. do we need to add axes labels and lateral color bars with magnitude of density?, t is in units of Alfvén crossing time?

2.2 The evolution of the kink wave and their damping through Kelvin-Helmholtz turbulence

Would "Kelvin-Helmholtz induced turbulence" be more acurate than "Kelvin-Helmholtz turbulence"?

Having been subject to its initial kick, as explained in the previous subsection, the tube begins to oscillate. The wave excited is (or at least dominated by) a fundamental kink mode with frequency **give frequency** (period is 12.8), resulting in the dense tube oscillating back and forth. For our initial conditions we have set a sharp boundary in density between the tube and the external medium. Therefore, these oscillations are not subject to resonant damping, which requires a continuous non-uniform variation of density such that the fundamental kink mode has its frequency in the Alfvén continuum. However, as the natural state for these oscillations is for a strong shear flow to develop between the tube and the external medium (Sakurai et al., 1991; Goossens et al., 1992), the boundary of the oscillating tube becomes unstable to the Kelvin-Helmholtz instability (Terradas et al., 2008; Antolin et al., 2014, 2015; Magyar and Van Doorsselaere, 2016).

Figure 1 shows the density evolution of the tube cross-section at the wave apex. These snapshots are taken approximately at times when the displacement of the apex is at its greatest. As the oscillations proceed, we can clearly see the development of Kelvin-Helmholtz roll-ups on the top of the tube. After the first quarter period (top left panel) the instability has started to grow, though the scales associated with this (and with that the area of the cross-section of the tube that has been disturbed by this) is small. As the wave continues to oscillate, with the panels in the figure showing the situation at half-period intervals from the first panel, the KH vortices become larger, and with this the area of the cross section that has been disturbed by the turbulence keeps on increasing until the vast majority of the tube has become part of the turbulent layer.

Looking at this in ~~great~~ (more?) detail, we ~~can see~~ (show?) in Figure 2 the area of the cross section of the tube ~~cross section~~ at the apex (normalised by the initial value) that has a density $\rho \geq 2.9$ (shown by the black line) and for $\rho \geq 1.92$ (red line). The area shown by the black curve acts as a proxy for the area that has not been disturbed by the Kelvin Helmholtz induced turbulence. It is clear that once the KHi develops there is a clear evolution of material that is initially part of the kink oscillation of the tube but then joins the mixing layer (are we providing a movie?). Once the undisturbed area of the tube at the apex becomes about 20% of its initial area, we can see that a transition in the dynamics happens. The area of material with $\rho \geq 1.92$ stays approximately constant over time. This is an important result as it connects to the model of Hillier et al. (2019), as this is the density threshold that the model predicts is area conserving for the particular external and internal density values used in this study.

Looking back at Figure 1 we can see by eye that the total displacement of the tube, at least as measured by

Figure 2: Normalised area of the tube cross section at the apex that has $\rho > 2.9$ (black line) and $\rho = 1.92$ (red line) **NEED TO SHOW THAT AT THE OTHER END THERE IS NO EVOLUTION** need to explain blue curve

the displacement along the x axis, does not appear to be showing a strong decay with time. We can see this somewhat clearer in Figure 3 which shows the spatial evolution of v_x at $y = 0$ and $z = L_z$ with time. The blue and red colours show the positive and negative velocities respectively. Until a time of $t \approx 33$, the magnitude of the displacement and the magnitude and coherency of the velocity field do not show any drastic change. However, after that time, corresponding to where the change of area stops decreasing in Figure 2, there is a complete change in the dynamics with clear reductions in the lateral displacement of the tube and a velocity field which is less coherent and of smaller magnitude.

To detail the evolution of the energy sources for the development of the Kelvin-Helmholtz turbulence, i.e. the energy of the oscillation, we can look at how the axis of the tube is displaced. One way to investigate this is to look at the temporal evolution of the velocity field

look at velocity field and talk about damping

Hasegawa et al. (2006) to talk about the local speed-ups.

Clearly the way we measure the bulk motion is determining how we view the change in the oscillation amplitude with time. We can see this very aspect in Figure 5 where the centre-of mass displacement in the x direction has been calculated in two slightly different ways resulting in very different displacements measured. For the solid black line, the displacement in the 2D plane at the apex for all the material above a density threshold of $\rho \geq 1.64$ is displayed. The solid red line is a similar calculation, but for the centre of mass displacement at the apex for the same density threshold but calculated along the 1D line at $y = 0$. We can see that initially the amplitude of the oscillation, as given by the maximum and minimum displacements, decreases monotonically for the case shown in black. However, for the case shown in red, it is only once a time of $t \approx 33$ has been reached that the amplitude starts to decay. Once we get into this later regime both amplitudes are decaying with time, though this is accompanied by a period drift.

MORE ON CHANGE IN DYNAMICS

Ultimately this leads us to a fundamental question: How do we quantify damping in a turbulent loop? The standing kink wave is a coherent oscillation of a dense tube aligned with magnetic field. The damping of these oscillations, when thought about through the lens of normal-mode linear wave theory (**careful here as kink wave is quasi-mode when resonant absorption is there**), should be the same wherever it is measured, not heavily dependent on how the tube is being measured. **need to say something more here!**

What we see in Figures 4 and 5 is that if we measure the amplitude of the oscillation by only looking at the motions in the dense core, there is a long time period where the wave does not display any change in amplitude, **then has changes**. However, if we look at the cross-section as a composite of the dense material and the turbulent mixing layer then this shows changes in amplitude from the start, with the rate at which the amplitude changes being larger during the period the dense core is not displaying a change in amplitude.

even the period of the wave measured will change

Figure 3:

Figure 4:

Figure 5:

That is to say the bulk motions will coherently reduce in scale. However, the turbulent motions that we are studying here that have been excited by a nonlinear kink wave behave differently. The turbulent layer grows, and naturally "forgets" its initially conditions (i.e. that it is part of a kink wave). As part of this process, the material in the turbulent mixing layer starts to oscillate at a different frequency to the original kink wave.

For the simulation we present here, we can at least determine two regimes in the system. These are , i.e. the magnitude of the shear flow is independent of the

However, for the scenario investigated here, we see that the damping created by the nonlinear development of the Kelvin-Helmholtz instability is one where the coherent bulk oscillations of a loop structure are progressively turned into the random, chaotic motions of turbulence.

list key characteristic including needed two different regimes.

To summarise the different phases of the dynamics. In the first phase we observe the transition from the oscillatory motions of a coherent tube to a turbulent cross-section

with the kinetic energy associated with the

The period of the coherent oscillations of the turbulent layer is (XX%) shorter than the period of the original kink oscillations. During this period the kinetic energy of the turbulent flow (i.e. the 0

When the kinetic energy associated with the oscillations of the turbulent layer becomes comparable with the oscillations of the core of the tube, a new phase in the dynamics begins. In this phase, both the amplitude of the oscillations of the core and the turbulent envelope both decrease with time.

3 Modelling the development of a turbulent oscillating tube

In this section we develop a philosophy a formulation? to describe the evolution of the kink oscillation into a turbulent tube, i.e. the first stage of the damping as described in the previous section. A model for the second stage will be presented in a future paper.

The first stage of the damping process happens as the turbulent layer on the boundary grows. This process exchanges momentum from inside and outside of the tube, manifesting as the originally coherent oscillations of the tube developing a more incoherently moving upper layer, with a coherently moving inner region. This can be seen in Figure 5 where the oscillations of the centre-of-mass for the material with density greater than 1.56 has an amplitude decaying from the beginning of the motion, but the centre-of-mass motions along the x-axis does not show any decay until $t \approx 30$.

In this regime, the velocity jump that is driving the turbulent motions does not change as the amplitude of the centre of mass motions decreases. For a simpler flow, i.e. a hydrodynamic shear layer, it is well known that the turbulent layer grows as

$$h = \beta_{\mathrm{MIX}} \Delta V t, \tag{9}$$

where β_{MIX} is a constant for a given mixing layer but depends on the density contrast of the layer (Baltzer and Livescu, 2020). Using a dimensional analysis of the model proposed in Hillier and Arregui (2019) and Hillier, Snow, and Arregui (2023) (add bibtex entry when available) gives a layer that grows as

$$h = C_1 \sqrt{0.5} \frac{(\rho_i \rho_e)^{1/4}}{\sqrt{\rho_i} + \sqrt{\rho_e}} \Delta V t, \tag{10}$$

where C_1 is a constant of order 1. Through comparison with mixing in hydrodynamic shear flows, the value of C_1 is expected to be in the range of $C_1 = 0.3$ (Baltzer and Livescu, 2020) to 0.5 (Brown and Roshko, 1974) as discussed in Hillier, Snow, and Arregui (2023). To determine an appropriate value for the mixing constant C_1 for this problem we will look more at the mixing in this section. By doing this we can then use this model of an expanding shear layer, combined with information on the structure of the shear layer taken from Hillier and Arregui (2019), to calculate the rate at which mass and momentum are transferred into or out from the oscillating structure and with that how its amplitude damps.

Before it is possible to make these comparisons, it is necessary to measure the magnitude of the shear flow driving the Kelvin-Helmholtz instability. Figure 6 shows the shear flow at the surface of the tube measured at $x = 0$ at the apex at $t =$?????? and $t =$?????. The velocity reformulated such that the value is positive at $y = 0$ for easy comparison. Two dashed horizontal lines are added at 0.15 and 0.05 to show the approximate values of the flow inside the dense tube and outside the tube. This implies the magnitude of the shear flow is $\Delta V \approx 0.2$. IA: I am here

AH: Note to self, need to move from ΔV to $\overline{\Delta V}$ notation as the shear has a time dependence and we look at the time average (we don't use the spatial dependence but could as all this does is change the value of C_1)

3.1 Mass evolution

To study the evolution of the tube oscillations in Section 2.2 we looked at the centre of mass motions of the system based on given density thresholds (e.g. Figure 5). However, these different thresholds result in changes in the mass over time, so it is first important step is to determine how the mass above a given threshold evolves with time. Following the self-similar argument, we can predict the evolution of the total mass as

$$\frac{dm_{\rho>\rho_T}}{dt} = \frac{d}{dt}(\Delta m_{\rho>\rho_T} h 2R) \tag{11}$$

where $m_{\rho>\rho_T}$ is the mass above a given density threshold and $\Delta m_{\rho>\rho_T}$ is the change in mass above that density threshold for a mixing layer of unit width as predicted by the model of Hillier et al. (2019). As for this model, the actual detailed wave dynamics are not so important for the construction of the model we have decided for simplicity of the model to take a rectangular cross-section of length $2R$ and initial height $H = \pi/4$ to model the tube to simplify the calculation of the evolution of the dynamics in the analytical model. The simple justification of this is that the shear layer develops on the top and bottom of the tube and not all the way around.

Based on the model predictions of Hillier and Arregui (2019), $\Delta m_{\rho>\rho_T}$ is a constant in time, therefore our model becomes

$$\frac{dm_{\rho>\rho_T}}{dt} = \Delta m_{\rho>\rho_T} 2RC_1 \sqrt{0.5} \frac{(\rho_i \rho_e)^{1/4}}{\sqrt{\rho_i} + \sqrt{\rho_e}} \Delta V. \tag{12}$$

Using the model of Hillier and Arregui (2019), the value of $\Delta m_{\rho>\rho_T}$ is calculated by first calculating the initial mass of the dense region by multiplying the density by the fraction of a unit length that would have been

Figure 6:

occupied by the dense phase pre-mixing**explain calculation better**. Then we integrate the density component of the model of Hillier and Arregui (2019) over a unit length from the density threshold value to the maximum density value (in this case 3). This is plotted in Figure **give figure** for different thresholds for the initial density contrast of our simulation. We can see for a small enough threshold the mass will be growing in time, but for larger thresholds there is a loss of mass. The value where the mass evolution is roughly neutral is $\rho_T = 1.64$. **MAKE PLOT OF THIS FOR the density contrast of the simulation**.

Equation 12 can be integrated to give

$$m_{\rho>\rho_T}(t) = m_{\rho>\rho_T}(0) + \Delta m_{\rho>\rho_T} 2RC_1\sqrt{0.5}\frac{(\rho_i\rho_e)^{1/4}}{\sqrt{\rho_i}+\sqrt{\rho_e}}\Delta V t. \tag{13}$$

Here m_0 is the initial mass, and ΔV is the velocity difference averaged along z and over a half period giving $\Delta V = 0.17 \times 0.5$. Note that the RMS value has been used here as the motions driving the mixing are calculated as the RMS of the turbulent velocity fluctuations in the layer.

To confirm that Equation 13 gives the expected evolution of the mass we compare the evolution of the mass for three different density thresholds from the simulation with comparisons to the predicted evolution given by Equation 13. Figure 7 shows the evolution of the mass at the apex for three density thresholds: $\rho_T = 1.1$ (blue line), $\rho_T = 1.64$ (black line) and $\rho_T = 2.7$ (red line). The dashed lines are the predicted evolution curves for these three thresholds (during this initial regime of the evolution) using $C_1 = 0.3$, which was determined by fitting by eye to the $\rho_T = 1.1$ lines. The value of C_1 found here ($C_1 = 0.3$) is in the range of the values found in hydrodynamic simulations/experiments (**CITATIONS**).

Overall the simulated curves follow the predicted linear trend. It is clear that though there is some evolution in the total mass for the density threshold of $\rho_T = 1.64$, this is a sufficiently accurate threshold to use to maintain an approximately constant mass in a centre-of-mass velocity calculations. The other two curves show the large variation in mass that can occur when different thresholds are employed. The reason the $\rho_T = 1.64$ threshold shows some evolution could be to do with errors in applying this Cartesian model for the mixing to this more complex geometry, but other, well-established reasons, unrelated to the mixing process like the ponderamotive force driving mass accumulation at the loop apex (**citations)**, exist to explain this. It is worth noting that the density Threshold of $\rho_T = 1.92$, which is predicted to be area conserving by the model of Hillier et al. (2019), is approximately area conserving for the duration of the simulation (as evidenced in Figure 2)

One consequence of having determined the value for C_1 is it allows an upper bound for the time this model holds to be derived. Using this we can calculate when the time when the mixing layer has become large enough

Figure 7: Evolution of the mass of material found above three different density thresholds. With $\rho_T = 1.1$ shown in blue, $\rho_T = 1.64$ shown in black, and $\rho_T = 2.7$ shown in red. The dashed lines are the predicted evolution value using $C_1 = 0.3$ with the colours corresponding to the appropriate density threshold.

that all of the dense tube has joined the mixing. Simply put (using the asymmetry of the mixing layer predicted by Hillier et al. (2019)) is when

$$\frac{\sqrt{\rho_e}}{\sqrt{\rho_i}+\sqrt{\rho_e}}h(t) = H \tag{14}$$

This implies that at a time of

$$t = \frac{\sqrt{2}H}{0.3\Delta V}\frac{(\sqrt{\rho_i}+\sqrt{\rho_e})^2}{\rho_i^{1/4}\rho_e^{3/4}} = 44.5, \tag{15}$$

CHECK NUMBER!! the whole tube would have become turbulent and, therefore, the model of a growing turbulent layer is invalid implying a second stage to the dynamics must have been reached by this time.

3.2 Momentum Evolution

Following on from the model of the mass evolution, we can develop a similar (though slightly more complex) model for the evolution of the momentum. To do this we split the momentum above a given density threshold into two components: 1) the momentum held in the material still corresponding to the original tube ($M_{\rm core}$), and 2) the momentum injected into the mixing layer ($M_{\rm mix}$. This is the same concept as has been used for the modelling of the mass evolution, but with the added complexity that the momentum naturally changes sign as the system undergoes its natural oscillations. The combination of a natural oscillation frequency with the momentum extraction/injection process acting like an oscillatory forcing on the system makes a system that is analogous to a forced linear oscillator

Firstly, to model the evolution of the momentum in the core, we need to look at the case where the forcing (in terms of the extraction of momentum through mixing) occurs at the same frequency (i.e. resonant frequency) as the natural oscillations of the system (the kink frequency of the oscillation). This can be stated in equation form as

$$\frac{dM_{\rm core}}{dt} = -\omega_{\rm KINK}^2 I_{\rm core} - 2\dot{F}_{\rm core}\cos(\omega_{\rm KINK}t), \tag{16}$$

i.e. a forced linear oscillator where the system is being forced at a resonant frequency with a forcing given by

$$\dot{F} = \frac{M_0}{H}\frac{0.3}{\sqrt{2}}\frac{\rho_i^{1/4}\rho_e^{3/4}}{(\sqrt{\rho_i}+\sqrt{\rho_e})^2}\Delta V \tag{17}$$

with M_0 the initial momentum given by

$$M_0 = \rho_i V_i \pi R^2 = 3 \times 0.15 \times \pi 0.3^2, \tag{18}$$

where V_i is the speed of the dense material, in this case $V_i = 0.15$ as can be seen in Figure 6.

This have the solution

$$I_{\text{core}}(t) = A\sin(\omega_{\text{KINK}}t) + B\cos(\omega_{\text{KINK}}t) - \frac{\dot{F}}{\omega_{\text{KINK}}} t\sin(\omega_{\text{KINK}}t). \tag{19}$$

If we follow our initial assumption that the forcing term is in phase with the oscillations, this implies that $B = -\dot{F}/\omega_{\text{KINK}}^2$, which leads to

$$I_{\text{core}}(t) = \frac{M_0}{\omega_{\text{KINK}}}\sin(\omega_{\text{KINK}}t) - \frac{\dot{F}}{\omega_{\text{KINK}}^2}\cos(\omega_{\text{KINK}}t) - \frac{\dot{F}}{\omega_{\text{KINK}}} t\sin(\omega_{\text{KINK}}t). \tag{20}$$

Mathematically wrong, must fix This leads to the momentum evolution of the core following

$$M_{\text{core}} = M_0\cos(\omega_{\text{KINK}}t)\left(1 - \frac{0.3}{\sqrt{2}H}\frac{\rho_i^{1/4}\rho_e^{3/4}}{(\sqrt{\rho_i}+\sqrt{\rho_e})^2}\Delta V t\right). \tag{21}$$

To model the momentum injected into the mixing layer requires more subtlety. Unlike the mass evolution, the momentum is not a positive definite quantity, it oscillates around zero, so the sign of the momentum injected into the mixing layer changes periodically. On top of this, the local field lines in the mixing layer have their own characteristic frequency of oscillation (i.e. Alfvén frequencies), which can be different from that of the frequency with which momentum is being injected.

The simple model we put forward to explain how the momentum in the mixing layer develops is based on again on a forced linear oscillator, but this time the characteristic frequency of the oscillation of the layer and the . **EXPLAIN KEY IDEAS** We consider a single, representative Alfvén frequency for the mixing layer (a composite Alfvén frequency or kink frequency of the layer) based on the turbulent motions coupling all the different regions of the layer allowing it to develop oscillations at a single representative frequency. As the model of Hillier et al. (2019) proposes the representative density of the layer is $\sqrt{\rho_i\rho_e}$, for our non-dimensional frequency this gives $\omega_{\text{A}} = B/(\rho_i\rho_e)^{1/4}$.

and model the whole layer as a single forced nonlinear oscillator with momentum $M_{1\text{L}}$. Mathematically, the evolution of the momentum would be modelled as:

$$\frac{dM_{1\text{L}}}{dt} = -\omega_{\text{A}}^2 I_{1\text{L}} - 2\dot{F}\cos(\omega_{\text{KINK}}t), \tag{22}$$

where we define I such that $dI_{1\text{L}}/dt = M_{1\text{L}}$ and $\dot{F}$ is the forcing term relating to the rate at which momentum is added into the mixing layer from the external regions. The basic concept is that in the layer, momentum is added through the turbulent mixing of the momenta of the internal and external layers, effectively forcing the mixing layer to respond. This driving naturally happens at the kink frequency (ω_{KINK}).

$$\dot{F} = \overline{M}\frac{0.3}{\sqrt{2}H}\frac{(\rho_i\rho_e)^{1/4}}{\sqrt{\rho_i}+\sqrt{\rho_e}}\Delta V \tag{23}$$

with $\overline{M}$ as the momentum injection calculated from the model of Hillier et al. (2019) over a region of unit thickness. The precise calculation is given by

$$\overline{M} = \sqrt{\frac{1}{2}}\left(0.15 - \frac{\Delta V\sqrt{\rho_e}}{\sqrt{\rho_i}+\sqrt{\rho_e}}\right) \times \int_{\rho>\rho_{min}}^{\rho<\rho_{max}} \rho dy \tag{24}$$

This is

(calculated in the rest frame of the system frame **poorly worded so probably inaccurate in its current form**) of the region of the mixing layer above the given density threshold as calculated from Hillier et al. (2019). **need to explain that this is integrated for a partial region of a layer that has unit width. I will need to add an integral formula for $\overline{M}$ that shows the average in z if that is what I did. I think I calculated for an average in z for the density to calculate the Alfv'en frequency, but I can't recall any others. Maybe I did it for the momentum evolution and then rescaled for the apex (I seem to recall doing this and I have the gut feeling it would be more robust as it takes in the global response to the layer and not just the local response, but maybe this was not what I did).**.

Equation 22 leads to the well know solution (in the case that the characteristic Alfvén frequency is not the kink frequency of)

$$I_{1\text{L}} = A\sin(\omega_{\text{A}}t) + B\cos(\omega_{\text{A}}t) + \frac{2\dot{F}}{\omega_{\text{A}}^2 - \omega_{\text{KINK}}^2}\cos(\omega_{\text{KINK}}t). \tag{25}$$

Figure 8: Comparison of the model and the simulation for (a) the momentum evolution and (b) the kinetic energy associated with the bulk motions. The black lines show the distribution for material with $\rho < 2.7$ for the simulation (solid) and model (dashed). The red lines show the evolution for the quantities where $1.1 < \rho < 2.7$ for the simulation (solid) and model (dashed).

If we take that at $t = 0$ we have $I_{1\mathrm{L}} = 0$ this implies that

$$B = -\frac{2\dot{F}}{\omega_{\mathrm{A}}^2 - \omega_{\mathrm{KINK}}^2} \tag{26}$$

Equation 25 can be differentiated to give the momentum evolution in the mixing layer as

$$M_{1\mathrm{L}} = A\omega_{\mathrm{A}}\cos(\omega_{\mathrm{A}}t) + \frac{2\dot{F}\omega_{\mathrm{A}}}{\omega_{\mathrm{A}}^2 - \omega_{\mathrm{KINK}}^2}\sin(\omega_{\mathrm{A}}t) - \frac{2\dot{F}\omega_{\mathrm{KINK}}}{\omega_{\mathrm{A}}^2 - \omega_{\mathrm{KINK}}^2}\sin(\omega_{\mathrm{KINK}}t). \tag{27}$$

We then have $A = M_{1\mathrm{L}}(0)/\omega_{\mathrm{A}}$ giving

$$M_{1\mathrm{L}} = M_{1\mathrm{L}}(0)\cos(\omega_{\mathrm{A}}t) + \frac{2\dot{F}\omega_{\mathrm{A}}}{\omega_{\mathrm{A}}^2 - \omega_{\mathrm{KINK}}^2}\sin(\omega_{\mathrm{A}}t) - \frac{2\dot{F}\omega_{\mathrm{KINK}}}{\omega_{\mathrm{A}}^2 - \omega_{\mathrm{KINK}}^2}\sin(\omega_{\mathrm{KINK}}t). \tag{28}$$

We can take the

Figure 9 shows the comparison between the predicted momentum and kinetic evolution of the core of the tube and the mixing layer compared with the simulation results. What is presented here is not a fit, but the solution from an initial value problem compared with the results from the 3D simulations. This model clearly captures the key features of the evolution.

Going further, we can simplify Equation 28 by taking $M_{1\mathrm{L}}(0) = 0$ and looking at the case where $\omega_{\mathrm{A}} - \omega_{\mathrm{KINK}}$ is small. By setting $\omega_{\mathrm{D}} = (\omega_{\mathrm{A}} - \omega_{\mathrm{KINK}})/2$ and $\omega_{\mathrm{S}} = (\omega_{\mathrm{A}} + \omega_{\mathrm{KINK}})/2$. This means $\omega_{\mathrm{A}} = \omega_{\mathrm{S}} + \omega_{\mathrm{D}}$ and $\omega_{\mathrm{KINK}} = \omega_{\mathrm{S}} - \omega_{\mathrm{D}}$. Therefore we have

$$M_{1\mathrm{L}} \approx \frac{\dot{F}(\omega_{\mathrm{S}} + \omega_{\mathrm{D}})}{2\omega_{\mathrm{S}}\omega_{\mathrm{D}}}\sin((\omega_{\mathrm{S}} + \omega_{\mathrm{D}})t) - \frac{\dot{F}(\omega_{\mathrm{S}} - \omega_{\mathrm{D}})}{2\omega_{\mathrm{S}}\omega_{\mathrm{D}}}\sin((\omega_{\mathrm{S}} - \omega_{\mathrm{D}})t) \tag{29}$$

This can be expanded out through double angle formulas to give

$$M_{1\mathrm{L}} \approx \frac{\dot{F}}{2\omega_{\mathrm{S}}\omega_{\mathrm{D}}}\left((\omega_{\mathrm{S}} + \omega_{\mathrm{D}})(\sin(\omega_{\mathrm{S}}t)\cos(\omega_{\mathrm{D}}t) + \cos(\omega_{\mathrm{S}}t)\sin(\omega_{\mathrm{D}}t)) - (\omega_{\mathrm{S}} - \omega_{\mathrm{D}})(\sin(\omega_{\mathrm{S}}t)\cos(\omega_{\mathrm{D}}t) - \cos(\omega_{\mathrm{S}}t)\sin(\omega_{\mathrm{D}}t))\right). \tag{30}$$

Taking that $\omega_{\mathrm{D}} \ll \omega_{\mathrm{S}}$ this can again be simplified to

$$M_{1\mathrm{L}} \approx \frac{\dot{F}}{\omega_{\mathrm{D}}}\cos(\omega_{\mathrm{S}}t)\sin(\omega_{\mathrm{D}}t)). \tag{31}$$

This implies we expect our momentum in the mixing layer to oscillate with a frequency of ω_{S} inside an envelope that evolves as $\sin(\omega_{\mathrm{D}}t)$.

Our result above implies that we can refine our prediction for when this initial phase of the dynamics will end. The model we have put forward for this period of the dynamics is that of the oscillations in the core of the tube driving the motions. However, it is clear from panel (b) of Figure 9 that as time goes on, even assuming that the core of the tube holds the majority of the kinetic energy associated with coherent motions no longer holds. At a time of $t \approx 33$ the energy of the coherent motions of the mixing layer becomes greater than

Figure 9: Comparison of the model and the simulation for (a) the velocity amplitude and (b) the amplitude of the . The black lines show the distribution for material with $\rho < 2.7$ for the simulation (solid) and model (dashed). The red lines show the evolution for the quantities where $1.1 < \rho < 2.7$ for the simulation (solid) and model (dashed).

3.3 Predicted Evolution of Amplitude and Velocity Amplitude of the Wave motions

With a prediction for both the momentum evolution and the mass evolution above a given density threshold of the oscillation, this can then be turned into an estimate of the centre of mass velocity of the oscillation. This can be formulated mathematically as

$$V_{\mathrm{CoM},\rho\geq\rho_{\mathrm{T}}} = \frac{\int_{\rho\geq\rho_{\mathrm{T}}} \rho V dA}{\int_{\rho\geq\rho_{\mathrm{T}}} \rho dA} = \frac{M_{\rho\geq\rho_{\mathrm{T}}}}{m_{\rho\geq\rho_{\mathrm{T}}}}, \tag{32}$$

with ρ_{T} the threshold density used. That is the integral of the density weighted velocity divided by the integral of the density give a centre of mass velocity, but this is just the momentum divided by the mass. As both $m_{\rho\geq\rho_{\mathrm{T}}}$ and $M_{\rho\geq\rho_{\mathrm{T}}}$ have been directly predicted previosuly, we can use these to subsequently make a prediction of $V_{\mathrm{CoM},\rho\geq\rho_{\mathrm{T}}}$. Then by integrating $V_{\mathrm{CoM},\rho\geq\rho_{\mathrm{T}}}$ over time, we can make a subsequent prediction for $A_{\mathrm{CoM},\rho\geq\rho_{\mathrm{T}}}$.

3.3.1 Connecting with observed wave amplitudes

3.4 Energy evolution

An important question, especially in terms of how the results of this paper may impact our understanding of wave heating of the solar corona is: can we predict the heating rates in the simulation from the model of Kelvin-Helmholtz turbulence. To make a prediction of the heating rates, first we need to make an estimate of the level of the turbulence (both in the velocity and the magnetic fields). To do this we can make a model how the energy lost from the shear flow is distributed between turbulent fluctuations an heating. The predict how turbulence grows at the apex of the loop to compare with the simulations and then use this to infer the turbulence throughout the whole domain. Finally, this can be used to predict the heating rate for the whole domain which can be compared with the increase in thermal energy in the simulation domain.

Firstly, we need to propose a model for how the kinetic energy that enters the shear layer is divided between heating and turbulence. As we are assuming that the heating process comes from the turbulence, All the kinetic energy that is brought into the shear layer and forms part of the turbulent structures will ultimately become heat. However, this process takes time which allows the energy stored as turbulence to grow even though it is being extracted. To make the model, we can use the approximate self-similarity of the turbulent energy cascade which implies that the velocity fluctuations at a given length scale (l) as $\propto l^{1/3}$.

NEED TO BRING ENERGY SCALING PART HERE!!

This gives a ratio of the time scales of the large scale motions which bring in the kinetic energy and the cascade to dissipation we can see that

so at any given time the energy that has been brought into the mixing layer and has formed part of the turbulent fluctuations, 1/2.7 of the energy will have been dissipated leaving 1.7/2.7 as part of the turbulence.

The next step is to then make a prediction for how the turbulent component of the kinetic energy grows at the apex of the loop (based on the assumption of our model that compressibility is small in the Kelvin-Helmholtz dynamics which would imply that due to boundary conditions all the turbulent energy is held as kinetic energy). Following the work of Hillier and Arregui (2019) we expect that the average kinetic

A simple prediction for the upper bound of the heating rate can be made by adapting Equation 29 of Hillier et al. (2019). By recognising that the width of the layer is now given by h (instead of a nondimensional width of 2) and by integrating the flow along z (which reduces the kinetic energy by 1/2) and across the width of the turbulent layer ($2R$) we can determine the dimensions of the turbulent layer. As we would like to get the most extreme upper bound possible, we make the assumption that the turbulence extracts energy from both the oscillatory component of the flow and from the magnetic field (which increases our estimate by a factor of 2 as we do not need to average the kinetic energy over time).

With the above concept we should be able to make a fair estimate. However, one aspect that we have learnt from Section 3.2 is that the momentum (and with that energy of the coherent motions) that exists in the turbulent layer cannot be added efficiently. This means that compared to the model of Hillier et al. (2019) more energy is added to the turbulent component. To make sure that the true upper limit is presented, we assume that all the energy that could have been added to create coherent oscillations of the mixing layer if the forcing was near resonance is added to the turbulence. We can quantify this by introducing ΔV_{REF} as the difference in velocity between the the rest frame of the layer as can be calculated from Hillier et al. (2019) at the apex of the wave and the observer rest frame of the simulation. For this case we calculate a value of $\Delta V_{\text{REF}} = 0.08$. obviously our assumption that all the energy goes to turbulence is an over estimate, but this allows a simple estimate of the maximum amount of energy added into the turbulent velocity and magnetic fluctuations.

This all leads to an estimate of the the total free energy (i.e. both kinetic and free magnetic energy) that has been added to the turbulence of

$$\begin{aligned} E_{\text{turb}}(t) <& \frac{1}{2}\sqrt{\rho_i\rho_e}\left(\frac{(\rho_i\rho_e)^{1/2}}{(\sqrt{\rho_i}+\sqrt{\rho_e})^2}\Delta V^2 + \Delta V_{\text{REF}}^2\right) hL_zR \\ =& \frac{1}{2}C_1\sqrt{0.5}\frac{(\rho_i\rho_e)^{3/4}}{(\sqrt{\rho_i}+\sqrt{\rho_e})}\left(\frac{\sqrt{\rho_i\rho_e}}{(\sqrt{\rho_i}+\sqrt{\rho_e})^2}\Delta V^2 + \Delta V_{\text{REF}}^2\right) L_zR\Delta Vt. \end{aligned} \tag{33}$$

As this gives the total possible energy given to turbulence at any time in this initial phase of the evolution, the time derivative can be used as an upper limit on the average dissipation by turbulence in this time as the turbulence can only dissipate the energy it has so is limited by the amount of energy contained by the turbulence at any given time. Therefore our predicted upper bound for the heating rate is

$$\overline{\dot{E}}_{\text{turb}} < \frac{1}{2}C_1\sqrt{0.5}\frac{(\rho_i\rho_e)^{3/4}}{(\sqrt{\rho_i}+\sqrt{\rho_e})}\left(\frac{\sqrt{\rho_i\rho_e}}{(\sqrt{\rho_i}+\sqrt{\rho_e})^2}\Delta V^2 + \Delta V_{\text{REF}}^2\right) L_zR\Delta V. \tag{34}$$

With this prediction, we can now compare the actual evolution of the heating in the simulation, as shown by the growth in internal energy, against this upper limit to see how tight a bound it is providing. This is shown in figure 10 where the internal energy (normalised by the initial value) is compared with the slope for the predicted growth (again normalised in the same way). The predicted value (again using $C_1 = 0.5$), instead

Figure 10: Change in internal energy (solid red line) with predicted slope based on highly efficient turbulent heating (dashed black line) for the evolution of internal energy (normalised by the initial value) with time.Dot dashed black line gives the upper limit of the increase based on all of the initial kinetic energy becoming internal energy.

of being a strong upper limit actually matches the slope of the energy growth reasonably well. In fact it is only approximately twice the slope of the internal energy growth found in the simulations, implying that the turbulent heating is actually quite efficient. However, the small rise shown ($\sim 0.2\%$) highlights that kicks to the tube leading to velocity shears even as large as 0.17 of the ambient sound speed are not capable of providing strong heating. Even when we consider that the tube only occupies about a tenth of the simulation domain, the heating of the volume of the tube could only be 2% of the initial internal energy. Note that due to the dissipation being through numerical effects, it is not possible for use to quantify the heating more accurately than what is presented here.

COULD LOOK TO SEE WHAT RATES ARE NEEDED TO BALANCE RADIATIVE LOSSES OF A FLUX TUBE

The formulas in Equations 12 and 35 are somewhat complicated, making them difficult to apply directly to observations. However, a more approximate bound can be determined by realising that adding in the kinetic energy associated with $\Delta V_{\rm REF}$ has put the upper bound for the energy as being similar to that as if the velocity amplitude of the wave is used. Therefore, we can get a simpler approximate bound of

$$\overline{\dot{E}}_{\rm turb} \lesssim \frac{1}{2^{2.5}} \frac{(\rho_i \rho_e)^{3/4}}{(\sqrt{\rho_i} + \sqrt{\rho_e})} L_z R V_{\rm AMP}^3. \tag{35}$$

For the case presented in this paper, this approximate limit is 23% greater than the more detailed upper limit, but clearly provides a useful bound on the heating rate.

HERE

4 Modelling energy dissipation and wave damping in the second phase of the dynamics

Having developed a model to explain the evolution of the first phase, this now gives
to
In this section we develop a model to explain the

4.1 Heating through turbulent decay

In this phase, the energy associated with the turbulent fluctuations is already larger than the energy of the coherent oscillations. Therefore, the dissipation of energy can be understood without considering the energy contained in the oscillations. This makes the model we propose for understanding the rate at which the turbulence decays to be that of simple decaying turbulence, and independent of the oscillatory dynamics.

The model we use was first proposed by **Taylor** and **Kolmogorov**

$$\frac{dE_{\text{turb}}}{dt} = \mathbf{v} \cdot \nabla E_{\text{turb}} \tag{36}$$

We consider the simple situation where the characteristic lengthscale of the turbulent motions is approximately

$$\frac{dE_{\text{turb}}}{dt} = \mathbf{v} \cdot \nabla E_{\text{turb}} \approx C \frac{u_{\text{RMS}}}{L} E_{\text{turb}}. \tag{37}$$

The term on the righ-hand-side can be further simplified by realising that the RMS velocity is directly related to the turbulent energy by (following Hillier, 2019) $u_{\text{RMS}} = \sqrt{2E_{\text{turb}}/(A\sqrt{\rho_i \rho_e})}$ where A is the area of the turbulent region (which we take to be the)

$$\int_{E(t_0)}^{E(t)} E^{-3/2} dE = \frac{C}{L} \sqrt{\frac{2}{A\sqrt{\rho_e \rho_i}}} \int_{t_0}^{t} dt \tag{38}$$

Therefore,

$$E_{\text{turb}} = \frac{1}{(D(t - t_0) + E(t_0)^{-1/2})^2} \tag{39}$$

The simple implication from this is that the turbulent energy should decay at late times $\propto t^{-2}$.

4.2 Modelling the damping of the coherent oscillatory motions

Key details here. The amplitude of the dense core oscillates with a period of ~ 13. The kink frequency (using $\sqrt{3}$ as the characteristic external density) is predicted to be 12.8. The mixing layer (in the momentum) has a period of ~ 6.2, a bit under half of the core oscillation. It should be noted that the external region $\rho < 1.1$ has the same frequency at this time. Do kink waves naturally have a frequency doubling for the external dynamics (in the centre of mass velocity for example)? If not, why does this happen here as this is a way to explain the inner frequency. Otherwise some forcing dynamics my be at play? There is an interesting envelope developing around these late time oscillations in the layer momentum, which could be an example of a beat.

The key ingredients for understanding this phase of the damping come for the simple fact that the turbulent fluctuations are energetically most important. This has two consequences. Firstly, that the turbulence couples the core of the tube to the turbulent mixing region, meaning that a sensible prediction for the period would be the kink frequency based on the density of the tube ($\rho_{\text{tube}} = 3$) and the characteristic of the mixing layer ($\rho_{\text{layer}} = \sqrt{(3)}$). This is $\sim 9\%$ greater than the original kink period, so we are able to explain the period drift found in the simulations. Secondly, we can treat the key damping process of the oscillations to be the turbulent driven damping, but with the the key quality that the turbulence is decaying with time which means that the damping rate is a function of time.

Another key aspect of this period is that the evolution of the mass appears to evolve on slower timescales that the first phase of the dynamics (**refer to figure**). As such the mass can be treated as constant with time. Therefore, the model we propose for the evolution of the oscillations is

$$\frac{d^2 a}{dt^2} = -\omega_1^2 a - 2D(t)v, \tag{40}$$

with $\omega_1 = B/sqrt(3 + \sqrt{3})/2$ and $D(t) = 1/(At + B)$ (**calculate the constants in the damping rate from previous subsection**). As with the first stage of damping, this model will differ from the simple idea of damping given by an exponential decay as in this case the damping rate is not constant in time.

finish

Figure 11:

Figure 12:

5 Discussion

Question: Is the latter part of the wave damping important if the heating rate is completely unlrelated? What role might observations of this process have in understanding the corona?

5.1 Why is this model different from the other "KHi" damping model

The model presented by Van Doorsselaere et al. (2021) is labelled in their work as damping by the Kelvin-Helmholtz instability. However, the model presented in this paper and that model have vastly different formulations, with very different dependency on the density contrast (and in the case of this model two distinct regimes). This leads to the question: How can two different formulations be modelling the same physical phenomenon? The model presented in this paper is developed from direct analysis of the turbulence created by the Kelvin-Helmholtz instability and, as shown by the comparison in Hillier et al. (2020) and Section 2, gives results that are consistent with those found in simulations of loops that develop the Kelvin-Helmholtz instability.

The model presented in Van Doorsselaere et al. (2021) used the linear eigenfunction for a kink wave and then prescribed a nonlinear amplitude to the oscillation. By integrating the wave oscillation over one period they could calculate the energy extraction rate through these nonlinearities. As they only study the particular wave mode associated with the kink wave, there are no other modes that are excited in the system. This means that there is no perturbation to seed the linear growth of the Kelvin-Helmholtz instability, and as a consequence there can be no Kelvin-Helmholtz turbulence. That is to say, the model they calculated does not allow the growth of the Kelvin-Helmholtz instability so cannot have any damping through Kelvin-Helmholtz turbulence.

This leads to the question, what is the nature of the nonlinear damping investigated in Van Doorsselaere et al. (2021). The initial conditions of their model are not an exact solution of the nonlinear equations (unlike a linear Alfvén wave in incompressible nonlinear MHD). Therefore, some of the energy transfer may be as a result of unbalanced nonlinearities that work to redistribute the energy so the wave energy so it is closer to the nonlinear wave solution. Some of the energy may form part of a turbulent cascade, but the distribution of the energy between these two effects is not clear.

6 Summary

References

Antolin, P., Okamoto, T. J., De Pontieu, B., Uitenbroek, H., Van Doorsselaere, T., and Yokoyama, T. (2015). Resonant Absorption of Transverse Oscillations and Associated Heating in a Solar Prominence. II. Numerical Aspects. ApJ, 809(1):72.

Antolin, P., Pagano, P., De Moortel, I., and Nakariakov, V. M. (2018). In Situ Generation of Transverse Magnetohydrodynamic Waves from Colliding Flows in the Solar Corona. ApJ, 861(2):L15.

Antolin, P., Yokoyama, T., and Van Doorsselaere, T. (2014). Fine Strand-like Structure in the Solar Corona from Magnetohydrodynamic Transverse Oscillations. ApJ, 787(2):L22.

Arregui, I. (2021). Bayesian Evidence for a Nonlinear Damping Model for Coronal Loop Oscillations. ApJ, 915(1):L25.

Aschwanden, M. J., Fletcher, L., Schrijver, C. J., and Alexander, D. (1999). Coronal loop oscillations observed with theTransition region and coronal explorer. The Astrophysical Journal, 520(2):880–894.

Baltzer, J. R. and Livescu, D. (2020). Variable-density effects in incompressible non-buoyant shear-driven turbulent mixing layers. Journal of Fluid Mechanics, 900:A16.

Brown, G. L. and Roshko, A. (1974). On density effects and large structure in turbulent mixing layers. Journal of Fluid Mechanics, 64:775–816.

Chen, J. and Schuck, P. W. (2007). Damped Oscillations of Coronal Loops. Sol. Phys., 246(1):145–164.

Goddard, C. R. and Nakariakov, V. M. (2016). Dependence of kink oscillation damping on the amplitude. A&A, 590:L5.

Goossens, M., Andries, J., and Arregui, I. (2006). Damping of magnetohydrodynamic waves by resonant absorption in the solar atmosphere. Philosophical Transactions of the Royal Society of London Series A, 364(1839):433–446.

Goossens, M., Andries, J., and Aschwanden, M. J. (2002). Coronal loop oscillations. An interpretation in terms of resonant absorption of quasi-mode kink oscillations. A&A, 394:L39–L42.

Goossens, M., Erdélyi, R., and Ruderman, M. S. (2011). Resonant MHD Waves in the Solar Atmosphere. Space Sci. Rev., 158:289–338.

Goossens, M., Hollweg, J. V., and Sakurai, T. (1992). Resonant behaviour of MHD waves on magnetic flux tubes. III - Effect of equilibrium flow. Sol. Phys., 138:233–255.

Hasegawa, H., Fujimoto, M., Takagi, K., Saito, Y., Mukai, T., and RèMe, H. (2006). Single-spacecraft detection of rolled-up Kelvin-Helmholtz vortices at the flank magnetopause. Journal of Geophysical Research (Space Physics), 111(A9):A09203.

Hillier, A. (2019). Ion-neutral decoupling in the nonlinear Kelvin-Helmholtz instability: Case of field-aligned flow. Physics of Plasmas, 26(8):082902.

Hillier, A. and Arregui, I. (2019). Coronal Cooling as a Result of Mixing by the Nonlinear Kelvin-Helmholtz Instability. ApJ, 885(2):101.

Hillier, A., Barker, A., Arregui, I., and Latter, H. (2019). On Kelvin-Helmholtz and parametric instabilities driven by coronal waves. MNRAS, 482(1):1143–1153.

Hillier, A., Takasao, S., and Nakamura, N. (2016). The formation and evolution of reconnection-driven, slow-mode shocks in a partially ionised plasma. A&A, 591:A112.

Hillier, A., Van Doorsselaere, T., and Karampelas, K. (2020). Estimating the Energy Dissipation from Kelvin-Helmholtz Instability Induced Turbulence in Oscillating Coronal Loops. ApJ, 897(1):L13.

Lin, Y., Engvold, O., Rouppe van der Voort, L. H. M., and van Noort, M. (2007). Evidence of Traveling Waves in Filament Threads. Sol. Phys., 246:65–72.

Magyar, N. and Van Doorsselaere, T. (2016). Damping of nonlinear standing kink oscillations: a numerical study. A&A, 595:A81.

McIntosh, S. W., de Pontieu, B., Carlsson, M., Hansteen, V., Boerner, P., and Goossens, M. (2011). Alfvénic waves with sufficient energy to power the quiet solar corona and fast solar wind. Nature, 475:477–480.

Nakariakov, V. M., Anfinogentov, S. A., Antolin, P., Jain, R., Kolotkov, D. Y., Kupriyanova, E. G., Li, D., Magyar, N., Nisticò, G., Pascoe, D. J., Srivastava, A. K., Terradas, J., Vasheghani Farahani, S., Verth, G., Yuan, D., and Zimovets, I. V. (2021). Kink Oscillations of Coronal Loops. Space Sci. Rev., 217(6):73.

Nakariakov, V. M., Ofman, L., DeLuca, E. E., Roberts, B., and Davila, J. M. (1999). Trace observations of damped coronal loop oscillations: implications for coronal heating. Science, 285:862–864.

Nechaeva, A., Zimovets, I. V., Nakariakov, V. M., and Goddard, C. R. (2019). Catalog of Decaying Kink Oscillations of Coronal Loops in the 24th Solar Cycle. ApJS, 241(2):31.

Roberts, B. (1983). Wave propagation in intense flux tubes. Sol. Phys., 87:77–93.

Roberts, B. (2000). Waves and oscillations in the corona - (Invited Review). Sol. Phys., 193:139.

Ruderman, M. S. and Erdélyi, R. (2009). Transverse Oscillations of Coronal Loops. Space Sci. Rev., 149:199–228.

Ruderman, M. S. and Roberts, B. (2002). The damping of coronal loop oscillations. The Astrophysical Journal, 577(1):475–486.

Sakurai, T., Goossens, M., and Hollweg, J. V. (1991). Resonant Behaviour of Magnetohydrodynamic Waves on Magnetic Flux Tubes - Part One. Sol. Phys., 133(2):227–245.

Schmieder, B., Kucera, T. A., Knizhnik, K., Luna, M., Lopez-Ariste, A., and Toot, D. (2013). Propagating Waves Transverse to the Magnetic Field in a Solar Prominence. ApJ, 777:108.

Terradas, J., Andries, J., Goossens, M., Arregui, I., Oliver, R., and Ballester, J. L. (2008). Nonlinear Instability of Kink Oscillations due to Shear Motions. ApJ, 687(2):L115.

Tomczyk, S., McIntosh, S. W., Keil, S. L., Judge, P. G., Schad, T., Seeley, D. H., and Edmondson, J. (2007). Alfvén Waves in the Solar Corona. Science, 317:1192–.

van Ballegooijen, A. A., Asgari-Targhi, M., Cranmer, S. R., and DeLuca, E. E. (2011). Heating of the Solar Chromosphere and Corona by Alfvén Wave Turbulence. ApJ, 736(1):3.

Van Doorsselaere, T., Goossens, M., Magyar, N., Ruderman, M. S., and Ismayilli, R. (2021). Nonlinear Damping of Standing Kink Waves Computed With Elsässer Variables. ApJ, 910(1):58.