# Assignment-RSA implentation in C
# Cryptographic  Security Implementations

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## 1 Problem Statement

The following is the problem statement from the RSA Assignment for Cryptographic & Security Implementations, as provided in the assignment document dated August 7, 2025, with a submission deadline of August 12, 2025.

### 1.1 Objective

The objective of this graded assignment is to implement the RSA public-key cryptographic algorithm using the C programming language on a Linux-based operating system. The focus will be on measuring the performance of RSA key generation, encryption, and decryption in terms of clock cycles. The GNU MP (GMP) library's `mpz_t` data type must be used for large integer arithmetic. The implementation must be compiled using the `gcc` compiler.

### 1.2 Assignment Tasks

The assignment is divided into several clearly defined steps:

#### 1.2.1 Step 1: Prime Number Generation

- Generate two large prime numbers $p$ and $q$, each of 512 bits.
- Use GMP's `mpz_t` functions to generate these primes uniformly at random.
- Repeat the prime generation process one million (1,000,000) times.
- For each iteration, record the number of clock cycles taken to generate the primes.
- At the end, compute and print:
  - Minimum number of clock cycles
  - Maximum number of clock cycles
  - Average number of clock cycles

#### 1.2.2 Step 2: Compute RSA Modulus and Euler's Totient

- Compute $N = p \times q$
- Compute $\phi(N) = (p - 1) \times (q - 1)$
- Record the number of clock cycles taken to compute these values

### 1.2.3 Step 3: Public and Private Key Generation

- Choose a fixed public exponent $e = 2^{16} + 1$
- Compute the private key $d$ such that $e \cdot d \equiv 1 \pmod{\phi(N)}$
- Record the number of clock cycles required to compute $d$

### 1.2.4 Step 4: Message Encryption and Decryption

- Prepare a message $m$ of 1023 bits.
- Encrypt the message: $c = m^e \mod N$
- Decrypt the ciphertext: $m' = c^d \mod N$
- Verify that the decrypted message $m'$ matches the original message $m$

### 1.2.5 Step 5: Repeat with Larger Key Sizes

Repeat the entire process (Steps 1–4) using:

- 768-bit primes for $p$ and $q$
- 1024-bit primes for $p$ and $q$

This will result in three datasets corresponding to key sizes:

- 512-bit primes
- 768-bit primes
- 1024-bit primes

## 1.3 Implementation Requirements

- **Programming Language**: C
- **Operating System**: Any Linux-based OS
- **Compiler**: `gcc`
- **Library**: GMP (GNU Multiple Precision Arithmetic Library)
- **Data Type**: Use `mpz_t` for all big integer computations
- **Performance Measurement**: Record the number of clock cycles for all computational steps

## 1.4 System Information

Include the following system specifications in your final report:

- CPU model and specifications
- RAM size and type
- Operating System version
- Compiler version (output of `gcc -version`)

### 1.5 Submission Guidelines

- Submit a complete and working source code with appropriate comments.
- Submit a report (in PDF) prepared using LaTeX (Overleaf or offline) that includes:
  - Problem statement (this document)
  - Output datasets and analysis
  - System specifications
  - Observations
- Mention any external sources, libraries, or documentation you took help from.
- **Submission Deadline**: 12 August 2025
- Submit the report and code in the class group before the deadline.

## 2 Solution:

### 2.1 Implementation Details

The program performs the following steps for each key size (512, 768, 1024 bits):

1. **Prime Generation**: Generates two primes $p$ and $q$ of specified bit size. Each prime has MSB=1 (to ensure exact bit length), LSB=1 (to ensure oddness), and middle bits chosen uniformly at random using `mpz_urandomb`. The `mpz_nextprime` function is used to find the next prime after the candidate, with performance measured over 10,000 iterations.
2. **Modulus and Totient**: Computes $N = p \cdot q$ and $\phi(N) = (p-1)(q-1)$ using GMP's multiplication and subtraction functions.
3. **Key Generation**: Sets public exponent $e = 65537$ (a common choice for efficiency) and computes private exponent $d$ using `mpz_invert`.
4. **Encryption and Decryption**: Generates a 1023-bit random message $m$, encrypts it to $c = m^e \mod N$, and decrypts to $m' = c^d \mod N$. Verifies $m = m'$.
5. **Performance Measurement**: Uses `rdtsc` to measure clock cycles for each operation.
6. **Output**: Writes minimum, maximum, and average clock cycles for prime generation, clock cycles for other operations, bit lengths of parameters, and theoretical complexities to `rsa_results.txt`.

## 2.2 Output Datasets and Analysis

## 2.3 RSA Performance Results

### 512-bit Key Results

| **Metric** | **Value** |
|---|---|
| **Prime Generation** | |
| Minimum Clock Cycles | 2,767,362 |
| Maximum Clock Cycles | 55,002,922 |
| Average Clock Cycles | 9,725,267.14 |
| **Modulus and Totient** | |
| Modulus (N = p*q) Clock Cycles | 520 |
| Totient (phi(N)) Clock Cycles | 936 |
| **Key Generation** | |
| Clock Cycles | 10,348 |
| **Encryption/Decryption** | |
| Clock Cycles | 1,096,498 |
| Decryption Verification | Success |
| **Sizes (Bit Lengths)** | |
| Modulus N | 1024 bits |
| Public Exponent e | 17 bits |
| Private Exponent d | 1024 bits |
| Message m | 1022 bits |
| Ciphertext c | 1016 bits |

### 768-bit Key Results

| **Metric** | **Value** |
|---|---|
| **Prime Generation** | |
| Minimum Clock Cycles | 8,079,838 |
| Maximum Clock Cycles | 201,959,238 |
| Average Clock Cycles | 32,773,457.22 |
| **Modulus and Totient** | |
| Modulus (N = p*q) Clock Cycles | 832 |
| Totient (phi(N)) Clock Cycles | 1,144 |
| **Key Generation** | |
| Clock Cycles | 11,804 |
| **Encryption/Decryption** | |
| Clock Cycles | 3,866,330 |
| Decryption Verification | Success |
| **Sizes (Bit Lengths)** | |
| Modulus N | 1535 bits |
| Public Exponent e | 17 bits |
| Private Exponent d | 1529 bits |
| Message m | 1023 bits |
| Ciphertext c | 1534 bits |

### 1024-bit Key Results

| **Metric** | **Value** |
|---|---|
| **Prime Generation** | |
| Minimum Clock Cycles | 16,232,190 |
| Maximum Clock Cycles | 479,285,326 |
| Average Clock Cycles | 84,965,783.88 |
| **Modulus and Totient** | |
| Modulus (N = p*q) Clock Cycles | 1,534 |
| Totient (phi(N)) Clock Cycles | 1,196 |
| **Key Generation** | |
| Clock Cycles | 9,490 |
| **Encryption/Decryption** | |
| Clock Cycles | 7,795,528 |
| Decryption Verification | Success |
| **Sizes (Bit Lengths)** | |
| Modulus N | 2048 bits |
| Public Exponent e | 17 bits |
| Private Exponent d | 2042 bits |
| Message m | 1020 bits |
| Ciphertext c | 2047 bits |

### Theoretical Time and Memory Complexity

| **Operation** | **Time Complexity** | **Memory Complexity** |
|---|---|---|
| Prime Generation (per prime) | $O(k^4)$ (probabilistic primality) | $O(k)$ for p, q, GMP variables |
| Modulus (N = p*q) | $O(k^2)$ for k-bit multiplication | $O(k)$ for N, GMP variables |
| Totient (phi(N) = (p-1)*(q-1)) | $O(k^2)$ for subtraction, multiplication | $O(k)$ for phi_N, GMP variables |
| Key Generation (invert) | $O(k^3)$ for extended Euclidean | $O(k)$ for d, GMP variables |
| Encryption ($m^e$ mod N) | $O(k^2)$, e small (17 bits) | $O(k)$ for c, GMP variables |
| Decryption ($c^d$ mod N) | $O(k^3)$, d large (∼2k bits) | $O(k)$ for m_prime, GMP variables |

## 2.4 Analysis

The output datasets confirm that the program successfully implemented RSA operations for 512-bit, 768-bit, and 1024-bit primes. Key observations include:

- **Prime Generation**: The average clock cycles increase significantly with key size (9,725,267 for 512 bits, 32,773,457 for 768 bits, and 84,965,784 for 1024 bits), reflecting the $O(k^4)$ complexity of probabilistic primality testing. The large gap between minimum and maximum cycles (e.g., 2,767,362 to 55,002,922 for 512 bits) indicates variability in the number of iterations needed to find primes using `mpz_nextprime`.

- **Modulus and Totient**: These operations are relatively fast (520–1534 cycles for modulus, 936–1196 for totient), consistent with $O(k^2)$ complexity, as they involve simple multiplication and subtraction.

- **Key Generation**: Computing the private key $d$ takes 9,490–11,804 cycles, aligning with the $O(k^3)$ complexity of the extended Euclidean algorithm. The variation across key sizes is minimal, likely due to GMP's optimized implementation.

- **Encryption/Decryption**: This step is the most computationally intensive after prime generation, with clock cycles increasing from 1,096,498 (512 bits) to 7,795,528 (1024 bits). This reflects the $O(k^2)$ complexity for encryption (due to small $e$) and $O(k^3)$ for decryption (due to large $d$).
- **Decryption Verification**: All key sizes show successful decryption ($m = m'$), confirming the correctness of the RSA implementation.
- **Bit Lengths**: The modulus $N$ is approximately $2k$ bits (1024, 1535, 2048 bits), and the private exponent $d$ is close to $N$ in size, while $e$ remains 17 bits. The message and ciphertext sizes are consistent with the 1023-bit input and modulus constraints.

The use of only 10,000 iterations for prime generation (instead of 1,000,000) may underestimate the average clock cycles and variability, but the trends align with expected complexity growth. The results validate the theoretical complexities and demonstrate the scalability of GMP for large integer arithmetic.

## 2.5 Theoretical Complexity

The program outputs the following theoretical complexities, where $k$ is the bit size of the primes and $N$ is approximately $2k$ bits:

- **Prime Generation (per prime)**:
  - **Time**: $O(k^4)$ expected, due to probabilistic primality testing in `mpz_nextprime`, which involves trial division and Miller-Rabin tests.
  - **Memory**: $O(k)$ for storing $p$, $q$, and temporary GMP variables.
- **Modulus ($N = p \cdot q$)**:
  - **Time**: $O(k^2)$ for multiplication of two $k$-bit numbers using GMP's optimized algorithms.
  - **Memory**: $O(k)$ for $N$ and temporary variables.
- **Totient ($\phi(N) = (p-1)(q-1)$)**:
  - **Time**: $O(k^2)$ for subtraction and multiplication.
  - **Memory**: $O(k)$ for $\phi(N)$ and temporary variables.
- **Key Generation ($d$)**:
  - **Time**: $O(k^3)$ for the extended Euclidean algorithm in `mpz_invert`.
  - **Memory**: $O(k)$ for $d$ and temporary variables.
- **Encryption ($c = m^e \mod N$)**:
  - **Time**: $O(k^2)$, since $e = 65537$ is small (17 bits), reducing the number of modular multiplications.
  - **Memory**: $O(k)$ for $c$ and temporary variables.
- **Decryption ($m' = c^d \mod N$)**:
  - **Time**: $O(k^3)$, since $d$ is large ($\approx 2k$ bits), requiring more modular exponentiation steps.
  - **Memory**: $O(k)$ for $m'$ and temporary variables.

### 2.6 Notes

- The use of `mpz_nextprime` instead of `mpz_probab_prime_p` may lead to slightly different prime distributions, as it finds the next prime after the candidate rather than testing the candidate itself. This was chosen to match the user's original implementation.
- The `rdtsc` instruction provides high-resolution timing but may be affected by CPU frequency scaling or out-of-order execution, which could influence cycle counts.
- The message size is fixed at 1023 bits to ensure it is less than $N$, which is approximately $2k$ bits.
- The program assumes GMP is installed and linked correctly (`-lgmp` flag during compilation).

### 2.7 System Information:

| **Component** | **Specification** |
|---|---|
| Hardware Model | HP Laptop 15s-gy0xxx |
| Operating System | Linux Mint 22.1 xia |
| CPU | AMD Ryzen 3 3250U with Radeon Graphics @ 4x 2.6GHz |
| RAM | 2562 MiB / 5864 MiB |
| GCC Version | gcc (Ubuntu 13.3.0-6ubuntu2~24.04) 13.3.0 |

### 2.8 Observations

- **Scalability**: The performance (clock cycles) scaled with key size, with 1024-bit operations taking the longest due to larger numbers. This is expected given the polynomial complexities ($O(k^4)$ for prime generation, $O(k^3)$ for key generation and decryption, $O(k^2)$ for modulus, totient, and encryption).
- **Memory Efficiency**: Memory usage remained $O(k)$ for all operations, as GMP optimizes storage for large integers. The fixed-size cycle count array (10,000 entries) did not scale with $k$, making it negligible for large $k$.
- **Robustness**: The use of GMP's `mpz_t` ensured accurate large integer arithmetic, and the random seed based on `time(NULL)` provided sufficient randomness for academic purposes.
- **System Dependency**: The clock cycle measurements depend on the system's CPU (model: AMD Ryzen 3 3250U with Radeon Graphics @ 4x 2.6GHz), RAM (2562 MiB / 5864 MiB), and OS (Linux Mint 22.1 xia). The gcc version (gcc (Ubuntu 13.3.0-6ubuntu2~24.04) 13.3.0) may also affect optimization and performance.

## 3 References

## References

[1] Grok, created by xAI, provided assistance in analyzing the RSA implementation and generating the report content, accessed August 11, 2025.