Practice-set-I

1. Find the open intervals on which the following functions are increasing and decreasing. Also find the intervals, where such functions have concavity up and down. (i) $f(x) = x^2\sqrt{8 - x^3}$

   (ii) $g(x) = \dfrac{x^3}{3x^2 + 1}$

2. Find the open intervals on which the following functions are increasing and decreasing. Also find the intervals, where such functions have concavity up and down. (i) $f(x) = x^{1/3}(8 + x^2)$

   (ii) $g(x) = \dfrac{x^2 - 3}{(x + 1)^2}$

3. Find the open intervals on which the following functions are increasing and decreasing. Also find the intervals, where such functions have concavity up and down. (i) $f(x) = x^{2/3}(x^2 - 3)$

   (ii) $g(x) = \dfrac{x^3 + x}{x^2 + 9}$

4.

The strength of a rectangular beam is proportional to the breadth and square of the depth. Find the shape of the largest beam that can be cut from a log of given size.

5. A Tour bus has 80 seats. Experience shows that when a tour costs Rs.28000, all seats on the bus will be sold. For each additional Rs.1000 charged, however, 2 fewer seats will be sold. Find the largest possible revenue.

6. Find the critical points of the function $f(x)= x^4-4x^3+10$. Identify the intervals in which the function is monotonic. Determine the concavity of the function. Find the functions local extreme values.

For the given function $f(x) = 2x^3 - 9x^2 - 108x + 2$, determine the

7 intervals on which the function is concave upward and concave downward. 8.

The height of a body moving vertically is given by $s = \frac{-1}{2}gt^2 + v_o t + s_o$,

$g > 0$, with s in meters and t in seconds. Find the body's maximum height. 9.

(a) By exploiting Rolle's theorem, show that for no value of $k \in \mathbb{R}$, $f(x) = x^3 - 3x + k$ has two distinct zeros in $(0, 1)$. (5 Marks)

(b) It takes 12 hours to drain a storage tank by opening the valve at the bottom. The depth y of fluid in the tank after t hours the valve is opened is given by the formula $y = 6(1 - \frac{t}{12})^2$m. (5 Marks)

   (i) Find the rate at which the tank is draining at time t.

   (ii) When is the fluid level in the tank falling fastest? Slowest? What are the values of $\frac{dy}{dt}$ at these times?

10.

(a) Find $\dfrac{dy}{dt}$ at $x = 2$ if $y = x^2 + 7x - 5$ and $\dfrac{dx}{dt} = \dfrac{1}{3}$. [5]

(b) Determine where in the interval $[-1, 20]$ the function $f(x) = \ln(x^4 + 20x^3 + 100)$ is increasing and decreasing. [5]

11.

(a) The velocity of a particle is modeled as $v(t) = t^3 - 2t^2 + 22t - 3$, $t \in [0, 126]$. Then find the absolute maxima and minima of acceleration of the particle. [5]

(b) Find a positive number such that the sum of the number and its reciprocal is minimum. [5]