Algorithm - Secant Method

Given an equation f(x) = 0
Let the initial guesses be x_0 and x_1
Do

$$x_{i+1} = x_i - \frac{\left(f(x_i) * (x_i - x_{i-1})\right)}{f(x_i) - f(x_{i-1})}, \ \ where\ i = 1,2,3 \ldots \ldots$$

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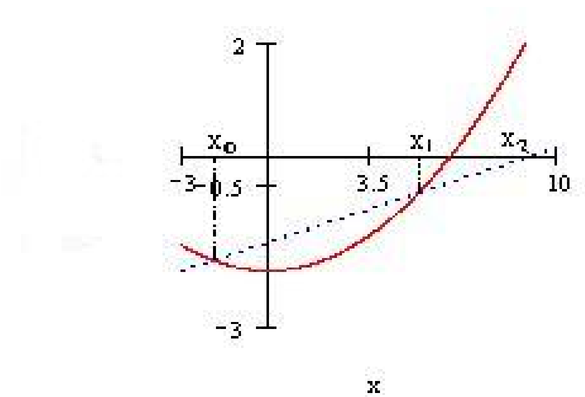
The Newton-Raphson algorithm requires the evaluation of two functions (the function and its derivative) per each iteration. If they are complicated expressions it will take considerable amount of effort to do hand calculations or large amount of CPU time for machine calculations. Hence it is desirable to have a method that converges (please see the section order of the numerical methods for theoretical details) as fast as Newton's method yet involves only the evaluation of the function. Let x0 and x1 are two initial approximations for the root 's' of f(x) = 0 and f(x0) & f(x1) respectively, are their function values. If x 2 is the point of intersection of x-axis and the line-joining the points (x0, f(x0)) and (x1, f(x1)) then x2 is closer to 's' than x0 and x1. The equation relating x0, x1 and x2 is found by considering the slope 'm'



$$m = \frac{f(x_1) - f(x_o)}{x_1 - x_0} = \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{0 - f(x_1)}{x_1 - x_0}$$

$$x_2 - x_1 = -\frac{\left(f(x_1) * (x_i - x_0)\right)}{f(x_1) - f(x_0)}$$

In general:

$$x_{i+1} = x_i - \frac{\left(f(x_i) * (x_i - x_{i-1})\right)}{f(x_i) - f(x_{i-1})}, \ \ where\ i = 1,2,3 \ldots \ldots$$

This formula is similar to Regula-falsi scheme of root bracketing methods but differs in the implementation. The Regula-falsi method begins with the two initial approximations 'a' and 'b' such that a < s < b where s is the root of f(x) = 0. It proceeds to the next iteration by calculating c(x2) using the above formula and then chooses one of the interval (a, c) or (c, h) depending on f(a) * f(c) < 0 or > 0 respectively. On the other hand, secant method starts with two initial approximation x0 and x1 (they may not bracket the root) and then calculates the x2 by the same formula as in Regula-falsi method but proceeds to the next iteration without bothering about any root bracketing.