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State and Parameter Estimation Using Measurements with Unknown Time Delay

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***Abstract*— Standard Kalman filtering does not handle time-delayed measurements, and if the delay is significant, large estimation errors may accumulate over time. Furthermore, the delay value is typically unknown and variable in many real applications. To fuse measurements with unknown time delays, this study incorporates a parameter estimation technique into state estimation. In the combined parameter-state estimator, we directly estimate the delay value as an additional state and simultaneously obtain refined state estimates in the modified Kalman filter that compensates for delayed measurements. Since estimated delay value has some constraints, the estimator requires both interpolation and the truncation of the probability density function. Monte Carlo simulation results of this study show that this approach is more reliable than existing approaches for state estimation using measurements with unknown time delays.**

I. Introduction

The most widely used algorithm for estimating the state of a dynamic system is the Extended Kalman Filter (EKF) [1], [2]. However, its design does not inherently allow for significant sensor-related delays in computation. Fig. 1 shows that the delay is the time difference between an instant when a measurement is taken by a sensor and another instant when the measurement is available in the filter. As an example of a delay, some complex sensors such as vision processors for navigation often require extensive computations to obtain higher-level information from raw sensor data. Furthermore, a closed-loop system including control logics may be an overall computational burden to a single processing center. Delays resulting from heavy computation may distort the quality of state estimation since a current measurement corresponds to the past states of a system model.

In a number of applications, a vital problem for combining data from various sensors is the fusion of delayed observations, and if the computational delay is crucial, fusing the data in a Kalman filter is challenging. During the last 20 years, the sensor time-delay problem have been solved by a number of methods, most of which modify the Kalman filter so that it handles delay in the sensor fusion algorithm. Alexander [3] derived a method of calculating a correction term and then added it to filter estimates when lagged measurements arrive. However, because the uncertainty of measurements is often an unknown quantity until the data are processed, applying the method in time-varying systems is impossible. To overcome the drawback of Alexander's method, Larsen et al. [4] extrapolated a measurement to a current time using the past and present estimates of the Kalman filter and calculated an optimal gain for this extrapolated measurement. However, Larsen's approach is exact for only linear systems, but if the system dynamics and measurement equations are significantly nonlinear, it can be highly inaccurate. For optimally fusing lagged sensor data in a general nonlinear system, Van Der Merwe et al. [5] introduced a new technique called "sample-state augmentation," based on a sigma-point Kalman filter. Section II-B provides detailed background information about the new technique. Lastly, Gopalakrishnan et al. [6] provided a survey of all previously noted methods.

All of the above methods assume that the amount of delay is known. However, situations in which a current, accurate time delay might not be known can arise in real applications. For example, when a local clock is not synchronized with the centralized clock, deviations between clocks will occur, and nondeterministic delays can be introduced in the firmware updates of subsystems. Moreover, if the delay time is a required additional quantity measured by another sensor (e.g., additional timer), delay values at each discrete-time can be corrupted by measurement noise. To deal with the unknown time delays, Julier and Uhlmann [7] introduced the covariance union algorithm, and Sinopoli et al. [8] modeled the arrival of intermittent observations as a random variable with a probability. In addition, Choi et al. [9] augmented a state vector with as many past states as the maximum number of delayed steps. This augmented state vector is very wide, and calculations with a large-size vector might require additional extensive computational effort. Recently, for the uncertainty of time delays in state estimation, Lee and Johnson [10] also suggested an approach combined with multiple-model adaptive estimation. However, because of imperfect information on a certain range of the delay value, this method might not be suitable if too many models are candidates with delay values. Instead, we directly estimate the time delay as an additional state since augmentation is a straightforward means of handling unknown delay. Nilsson et al. [11] investigated this idea using Taylor series expansion for small delays. However, delay values are typically larger than a time step, and the linearization in their approach does not hold for large delays. Li and Mourikis [12] also examined the state augmentation for estimating an unknown time offset between the timestamps of two sensors. However, their approach is not optimal since it performs the measurement update of delayed sensor data without the cross-covariance term during the delay period and constrained

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Kalman filtering.

State estimation theory can apply to estimation of not only the states of a system but also the unknown parameters of the system. Kopp et al. [13] first suggested this application, which this study refers to as "parameter estimation," a term coined by [14]. For example, Simon and Simon [15] focused on estimating the parameters of the health of aircraft engines for the purpose of maintenance scheduling. Likely, our study characterizes the time delay of measurements as an unknown system parameter and considers it an additional state variable. Then, the dynamic model of the parameter is a slowly varying bias; that is, a small artificial noise allows the Kalman filter to update its estimate of the delay. Unlike that in [10], this approach, which directly leads to a filtering concept, does not need large computer storage.

Although the combined parameter-state estimator appears to be workable despite the delay, this paper tackles two limitations: First, since the delay represents the number of delayed samples in discrete-time systems, the delay value in each underlying function is quantized as an integer. However, the estimated result of the delay may not be an integer. Hence, interpolation [16] can construct a new approximated data point between two known adjacent estimates. Second, the delay is nonnegative; that is, it is bounded by zero. For example, if the delay value is zero, no delay occurs during the processing of measurements, but if the estimated delay is negative, obtaining measurements from the future is impossible. Hence, although the imposed inequality constraint does not easily fit into the structure of the Kalman filter, we must handle it to maintain the global optimality of the filter. An approach of constrained Kalman filtering in [17] and [18] truncates the probability density function (pdf) of the Kalman filter estimates at the enforced constraint and then computes the constrained estimates and their corresponding covariance by means of truncated pdf.

The rest of this paper is organized as follows. The next section demonstrates the formulation of the problem and the outline of our approach. Section III presents a novel combination of the modified EKF using lagged measurements with the parameter estimation technique. The last section describes the simulation environment and presents results using 100 Monte Carlo trials.

II. Preliminaries

A. Models and Setup

The system equations with continuous-time dynamics and two types of discrete-time measurements are as follows:

$$\dot{x}(t) = f(x(t), u(t), t) + w(t) \tag{1}$$
$$y_{1k} = c(x_k) + v_{1k} \tag{2}$$
$$y_{2k} = h(x_{k-N}) + v_{2k}, \tag{3}$$

where $x \in \mathbb{R}^n$ is the state and u is a control input. The first type of sensor, y_1, is an instantaneous non-lagged measurement, and the second type of sensor, y_2, is an infrequent and delayed measurement (Fig. 1). N is the number of delayed samples, and M_s denotes the number of intervals between two successive y_2 measurement samplings. f is the nonlinear dynamic function, and both c and h are nonlinear measurement functions. To clarify, subscript 1 represents the non-delayed sensor, subscript 2 the delayed sensor, and subscript k (or $k-N$) the k-th (or $(k-N)$-th) time step. Moreover, let's assume that both propagation and measurements are corrupted by additive zero-mean white Gaussian noise; that is, $w(t) \sim \mathcal{N}(0, Q_c(t))$, $v_{1k} \sim \mathcal{N}(0, R_{1k})$, and $v_{2k} \sim \mathcal{N}(0, R_{2k})$.

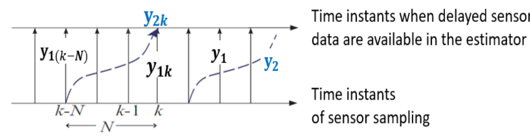

Fig. 1: Sensor fusion of non-delayed and delayed measurements

To estimate the state variables of the system, we design a hybrid EKF in the following steps. In the propagation step, state estimate $\hat{x}$ and its error-covariance P are integrated from time $(k-1)^+$ to time k^- with respect to variable t

$$\dot{\hat{x}}(t) = f(\hat{x}(t), u(t), t) \tag{4}$$
$$\dot{P}(t) = A(t)P(t) + P(t)A^T(t) + Q_c(t), \tag{5}$$

where hat "ˆ" denotes an estimate and $A(t) = \frac{\partial f}{\partial x}\big|_{\hat{x}(t)}$. Superscript $-$ and $+$ denote a priori and a posteriori estimates, respectively. At every time k,

$$K_{1k} = P_k^- C_k^T (C_k P_k^- C_k^T + R_{1k})^{-1} \tag{6}$$
$$\hat{x}_{1k}^+ = \hat{x}_k^- + K_{1k} (y_{1k} - c(\hat{x}_k^-)) \tag{7}$$
$$P_{1k}^+ = (I - K_{1k} C_k) P_k^-, \tag{8}$$

where $C_k = \frac{\partial c}{\partial x}\big|_{\hat{x}_k^-}$. During the period at which the second sensor does not arrive yet, the EKF performs the only measurement update of the first sensor (6)-(8) and then assigns the resulting estimate and covariance to the final estimate and covariance (i.e., $\hat{x}_k^+ \leftarrow \hat{x}_{1k}^+$ and $P_k^+ \leftarrow P_{1k}^+$), respectively.

When the data from lagged y_{2k} are available in the filter, fusing the two-style measurements using the sequential Kalman filter [14] becomes simpler. Before the correction step of y_{2k}, let's assign the a posteriori estimate and covariance of the first non-delayed measurement to the a priori estimate and covariance of the second delayed measurement, respectively: $\hat{x}_{2k}^- \leftarrow \hat{x}_{1k}^+$, $P_{2k}^- \leftarrow P_{1k.}^+$. Then, the EKF processes the measurement update of the second sensor based on steps in the following section.

B. State Estimation for Delayed Measurements

Among the various fusing time-delayed observations techniques discussed in the Introduction, the sample-state augmentation-based method [5] is applicable to nonlinear functions as well as the case of delays that vary over time. In the method, at time k, when the actual readings of the second measurement become available in the filter,

$$K_{2k} = P^{\mathrm{cr}}_{2k^-|(k-N)^+} H^T_{k-N} (H_{k-N} P^+_{k-N} H^T_{k-N} + R_{2k})^{-1} \tag{9}$$

$$\hat{x}_k^+ = \hat{x}_{2k}^- + K_{2k}\left(\, y_{2k} - h(\hat{x}_{k-N}^+)\,\right) \tag{10}$$
$$P_k^+ = P_{2k}^- - K_{2k} H_{k-N}\, {P_{2k^-\,|\,(k-N)^+}^{\mathrm{cr}}}^T, \tag{11}$$

where $H_{k-N} = \frac{\partial h}{\partial x}\big|_{\hat{x}_{k-N}^+}$, and $P_{2k^-\,|\,(k-N)^+}^{\mathrm{cr}}$ is the relevant cross-covariance term during the delay period. This term, which fuses a current prediction of the state with an observation related to the lagged state of the system, is used for formulating the modified Kalman gain matrix (9). Eq. (11), like (8), is still Jeseph's form [19] of the covariance measurement update. This form preserves the symmetry of the updated covariance and ensures its the positive definiteness.

At time $(k-N)$, when the delayed-sensor signals are received for processing, the cross-covariance matrix is initialized with the current covariance matrix of the system; that is, $P_{(k-N)^+\,|\,(k-N)^+}^{\mathrm{cr}} \leftarrow P_{k-N}^+ \leftarrow P_{1\,(k-N)}^+$ after the EKF computes the step (8). From time $(k-N+1)$ to time k, no second measurement is fused during the delay period, and the time update and measurement update of the cross-covariance term are computed by either the Schmidt-Kalman filter [20] or stochastic cloning [21] as follows:

for $\;i = (k-N+1)$ to k **do**

$$\dot{P}^{\mathrm{cr}}(t) = A(t)\, P^{\mathrm{cr}}(t)$$
$$P_{(i)^-\,|\,(k-N)^+}^{\mathrm{cr}} = P_{(i-1)^+\,|\,(k-N)^+}^{\mathrm{cr}} + \int_{(i-1)^+}^{(i)^-} \dot{P}^{\mathrm{cr}}(t)\, dt$$
$$P_{1(i)^+\,|\,(k-N)^+}^{\mathrm{cr}} = (I - K_{1\,(i)}\, C_{(i)})\, P_{(i)^-\,|\,(k-N)^+}^{\mathrm{cr}}$$
$$P_{(i)^+\,|\,(k-N)^+}^{\mathrm{cr}} \leftarrow P_{1(i)^+\,|\,(k-N)^+}^{\mathrm{cr}} \qquad (\text{except } i = k)$$

end for

The recursive calculation of the correction terms is performed across time during the delayed period. At time k, before measurement update step of the second sensor, the EKF finally prepares the cross-covariance matrix, $P_{2k^-\,|\,(k-N)^+}^{\mathrm{cr}}$, for Eqs. (9)-(11); that is, when $i = k$ in the for-loop,

$$P_{2k^-\,|\,(k-N)^+}^{\mathrm{cr}} \leftarrow P_{1k^+\,|\,(k-N)^+}^{\mathrm{cr}} = (I - K_{1k} C_k)\, P_{k^-\,|\,(k-N)^+}^{\mathrm{cr}}.$$

The fusion problem with delayed measurements ends up being solved by the modified EKF introduced here. For more details, see [5], [21]. However, this approach works with only known delays. Typically, knowing the information of each delay value in real applications might be impossible. Hence, to solve the estimation problem with unknown time delays, we need another novel method that will be described in the next section.

III. Parameter-State Estimator for Measurements with an Unknown Time Delay

A. Parameter Estimation

State estimation theory can be used to estimate not only the states but also the unknown parameters of the system [13]. Now delay N is an unknown parameter. From the system models given in (1)-(3), measurement data from only the second sensor depend on the unknown parameter. To estimate the unknown delay value, we first augment state x with the parameter to obtain augmented state vector x^{aug}:

$$x_k^{\mathrm{aug}} = \begin{bmatrix} x_k \\ N_k \end{bmatrix} \in \mathbb{R}^{n+1}$$

To estimate the delays that vary over time, we model the dynamics of the parameter as follows: $\dot{N}(t) = w_p(t)$, where $w_p(t)$ is a small artificial noise term that allows the EKF to change its estimate of N_k. Our augmented system model can be written as

$$\dot{x}^{\mathrm{aug}}(t) = \begin{bmatrix} f(x(t), u(t), t) + w(t) \\ 0 + w_p(t) \end{bmatrix}$$
$$= f^{\mathrm{aug}}(x^{\mathrm{aug}}(t), u(t), t) + w^{\mathrm{aug}}(t) \tag{12}$$
$$y_{1k} = c(x_k) + v_{1k} = c(x_k^{\mathrm{aug}}) + v_{1k} \tag{13}$$
$$y_{2k} = h(x_{k-N_k}) + v_{2k} = h^{\mathrm{aug}}(x_k^{\mathrm{aug}}) + v_{2k}, \tag{14}$$

where $w^{\mathrm{aug}}(t) = \begin{bmatrix} w(t) & w_p(t) \end{bmatrix}^T$. Note that $f^{\mathrm{aug}}(x^{\mathrm{aug}}(t), u(t), t)$ and $h^{\mathrm{aug}}(x_k^{\mathrm{aug}})$ are new nonlinear underlying functions of augmented state x^{aug}. Here let's call the estimation of the unknown parameter by the state augmentation "parameter estimation" [14]. Therefore, to reliably estimate x_k^{aug} and effectively compensate estimated delay $\hat{N}$, we incorporate this parameter estimation technique into the modified EKF introduced in the previous section.

B. Interpolation

Since the true N in discrete-time systems is defined as the number of delayed samples, N should be an integer. However, estimated delay value $\hat{N}_k$ might not be an integer after parameter estimation. Whenever time $k - \hat{N}_k$ is not an integer, we cannot directly access the values of either $\hat{x}_{k-\hat{N}_k}$ or its corresponding covariance $P_{k-\hat{N}_k}$, so interpolation is required instead. Mathematically, linear interpolation constructs a new

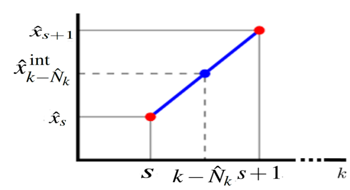

Fig. 2: A example of linear interpolation

data point within the range of two known adjacent data points by the same slope of two lines [16]. Let's take the nearest integer time step s, which is less than or equal to $k - \hat{N}_k$ (Fig. 2). With two data points, either $(s, \hat{x}_s)$ and $(s+1, \hat{x}_{s+1})$ or (s, P_s^{aug}) and $(s+1, P_{s+1}^{\mathrm{aug}})$, the interpolants at time $k - \hat{N}_k$ are given by

$$\hat{x}_{k-\hat{N}_k}^{\mathrm{int}} = \hat{x}_s + (\hat{x}_{s+1} - \hat{x}_s)\,(k - \hat{N}_k - s) \tag{15}$$
$$P_{k-\hat{N}_k}^{\mathrm{aug}} = P_s^{\mathrm{aug}} + \left(P_{s+1}^{\mathrm{aug}} - P_s^{\mathrm{aug}}\right)(k - \hat{N}_k - s). \tag{16}$$

Then, we substitute $\hat{x}_{k-\hat{N}_k}^{\mathrm{int}+}$ and its corresponding $P_{k-\hat{N}_k}^{\mathrm{aug}+}$ into the measurement-update step of the second sensor, (9)-(11). In other words, at time k, when the delayed measurement data are available in the filter,

$$K_{2k} = P^{\text{cr, aug}}_{2k^-|(k-\hat{N}_k)} H^{\text{aug}\ T}_{k-\hat{N}_k} \left(H^{\text{aug}}_{k-\hat{N}_k} P^{\text{aug}+}_{k-\hat{N}_k} H^{\text{aug}\ T}_{k-\hat{N}_k} + R_{2k} \right)^{-1} \tag{17}$$

$$\hat{x}^{\text{aug}+}_k = \hat{x}^{\text{aug}-}_{2k} + K_{2k} \left(y_{2k} - h(\hat{x}^{\text{int}+}_{k-\hat{N}_k}) \right) \tag{18}$$

$$P^{\text{aug}+}_k = P^{\text{aug}-}_{2k} - K_{2k}\ H^{\text{aug}}_{k-\hat{N}_k} P^{\text{cr, aug}\ T}_{2k^-|(k-\hat{N}_k)}, \tag{19}$$

where $H^{\text{aug}}_{k-\hat{N}_k} = \frac{\partial h^{\text{aug}}}{\partial x}\big|_{\hat{x}^{\text{aug}+}_{k-\hat{N}_k}}$. In particular, since estimated delay value $\hat{N}_k$ is unknown up to time k, we are not sure when the modified EKF begins computing cross-covariance $P^{\text{cr, aug}}_{2k^-|(k-\hat{N}_k)}$. Thus, instead of calculating the cross-covariance term recursively during the delay period, the filter computes $P^{\text{cr, aug}}_{2k^-|(k-\hat{N}_k)}$ once at time k with saved Jacobian A^{aug}, C, and gain K_1 during the delay period.

C. Constrained Kalman Filtering

To perform the combined parameter-state estimator with the interpolation, this section handles constrained Kalman filtering. The delay value must be nonnegative in the modified EKF. For example, if estimated delay $\hat{N}_k$ is negative (i.e., an exceeded index), estimation is impossible since this case is forecasting states or obtaining measurements from the future, so the delay has to be bounded by zero. If one neglects the delay whenever the estimated delay value in the modified EKF becomes negative (i.e., if one runs the modified EKF without the constraint and rounds negative values to zero for updating—called "Unconstrained"), the accuracy of the modified EKF estimate is significantly compromised, and it might be similar (or worse) to that of the standard EKF (Fig. 4). Thus, we cannot simply ignore any value of estimated delays, even negative values. Instead, to incorporate these constraints into the modified EKF equations, we employ the pdf truncation method [17]. In this approach, the pdf of estimates resulting from the modified EKF is truncated at the constraint edges. The constrained state estimates then become equal to the mean of the truncated pdf. Suppose that we have the following state constraints at time k.

$$0 = L_k \leq\ \phi_k^T x^{\text{aug}}_k = N_k\ \leq U_k,$$

where $\phi_k = \begin{bmatrix} 0 & \cdots & 0 & 1 \end{bmatrix}^T$, L is the lower limit of the delay, typically zero for the non-delay case, and U is the upper limit of the delay value. If we have only one-sided constraint L_k, then we can set the upper bound at infinity. Fortunately, we might heuristically expect a reasonable value of the upper bound by reading electric signals of the delayed second sensor. In other words, the pdf truncation approach works regardless of the existence of the upper bound.

With the requirement of constrained filtering in the estimation problem, the method truncates Gaussian pdf $\mathcal{N}(\hat{x}^{\text{aug}+}_k,\ P^{\text{aug}+}_k)$ at constraints and then computes constrained state estimate $\tilde{x}^{\text{aug}+}_k$ and its covariance $\tilde{P}^{\text{aug}+}_k$ of the truncated pdf. Providing more details about pdf truncation, Simon [14] derived the method, and Shimada et al. [17] and Simon et al. [18] proved its optimality and consistency for some class of systems. Based on their theoretical guarantees, our study simply applies the method to the estimation problem with delayed measurements. From the procedure in [14], first, computing the square root of $P^{\text{aug}+}_k$ is simpler by the following Jordan canonical decomposition $TWT^T = P^{\text{aug}+}_k$, where T is orthogonal and W is diagonal. Next, we use the Gram-Schmidt orthogonalization procedure [22] to find an orthogonal $\rho \in \mathbb{R}^{(n+1)\times(n+1)}$ matrix. In particular, the first row of the ρ, $\rho_1 = \frac{\phi_k^T T W^{1/2}}{(\phi_k^T P^{\text{aug}+}_k \phi_k)^{1/2}}$. Then, the pdf truncation problem is tractable by the following transformations:

$$z_k = \rho\, W^{-1/2}\, T^T \left(x^{\text{aug}}_k - \hat{x}^{\text{aug}+}_k\right)$$

$$\begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix} z_k\ \geq\ c_k = \frac{L_k - \hat{N}_k}{(\phi_k^T\, P^{\text{aug}+}_k\, \phi_k)^{1/2}}$$

$$\begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix} z_k\ \leq\ d_k = \frac{U_k - \hat{N}_k}{(\phi_k^T\, P^{\text{aug}+}_k\, \phi_k)^{1/2}},$$

where c_k and d_k define new two bounds. Since $z_k \sim \mathcal{N}(0, I)$, its elements are statistically independent. Only the first element of z_k is constrained, so the original pdf truncation problem is reduced to one-dimensional pdf truncation. Before constraint enforcement, the first element of z_k is distributed as $\mathcal{N}(0,1)$, but the new constraint shows that z_k must lie between c_k and d_k. We therefore remove the outside of the constraint and compute the area of the remaining portion of the Gaussian pdf. Then, we normalize the truncated pdf so that it has an area of one, and we find that the resulting truncated pdf (i.e., the constrained pdf of the first element of z_k) is given by $\text{pdf}(\zeta) = \begin{cases} \alpha\ \exp(-\zeta^2/2) & \zeta \in [c_k, d_k] \\ 0, & \text{otherwise} \end{cases}$ where $\alpha = \frac{\sqrt{2}}{\sqrt{\pi}\left[\ \text{erf}(d_k/\sqrt{2}) - \text{erf}(c_k/\sqrt{2})\ \right]}$ and $\text{erf}(\cdot)$ is the most commonly used error function. Furthermore, we define random variable z^{trun}_k, which has the same pdf as z_k, except that the pdf is truncated and normalized, so its pdf lies entirely between limits c_k and d_k; that is, $\text{pdf}\,(z^{\text{trun}}_k)$ = truncated $\text{pdf}\,(z_k)$. Thus, after enforcement of the constraint, the mean and the variance of the transformed state estimate are given as $\tilde{z}^{\text{trun}}_k = \begin{bmatrix} \mu & 0 & \cdots & 0 \end{bmatrix}^T$ and $\text{Cov}(\tilde{z}^{\text{trun}}_k) = \text{diag}(\sigma^2, 1, \cdots, 1)$,

$$\begin{aligned} \text{where}\quad \mu &= \mathbb{E}[z^{\text{trun}}_k] = \alpha\left[\exp(-c_k^2/2) - \exp(-d_k^2/2)\right] \\ \sigma^2 &= \mathbb{E}[(z^{\text{trun}}_k - \mu)^2] = \alpha\left[\exp\left(-c_k^2/2\right)(c_k - 2\mu)\ldots\right. \\ &\quad \left. - \exp\left(-d_k^2/2\right)(d_k - 2\mu)\right] + \mu^2 + 1 \end{aligned}$$

Finally, to find the mean and the variance of the state estimate after enforcement of the constraint, we take the inverse of the transformation.

$$\tilde{x}^{\text{aug}+}_k = TW^{1/2}\rho^T \tilde{z}^{\text{trun}}_k\ +\ \hat{x}^{\text{aug}+}_k \tag{20}$$

$$\tilde{P}^{\text{aug}+}_k = TW^{1/2}\rho^T\, \text{Cov}(\tilde{z}^{\text{trun}}_k)\, \rho W^{1/2} T^T. \tag{21}$$

These constrained quantities are the final state estimate and its corresponding covariance in the combined parameter-state filter. In sum, this paper presents everything to solve the problem, and Fig. 3 illustrates a flow chart of the overall process.

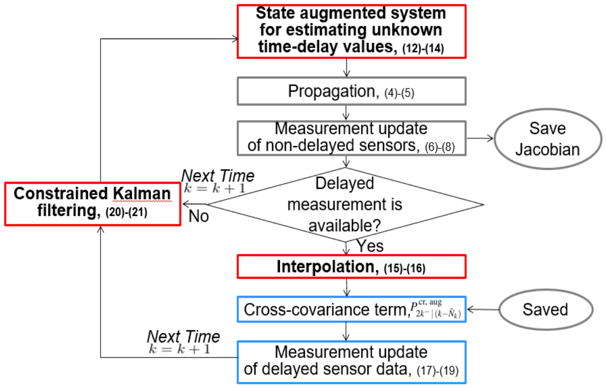

Fig. 3: A flow chart of the overall process of the combined parameter-state estimator

IV. Simulation Results

To validate the reliability of the proposed approach for estimating states and unknown delay values, we simulate a simple example system by 100 Monte Carlo trials. A augmented state vector of the system constructs the position and the velocity of a vehicle and the value of time delays of lagged measurements (i.e., $x^{\text{aug}}(t) = [p_X, p_Y, v_X, v_Y, N]^T$). The vehicle flies, given sinusoidal control inputs for a mission maneuver at a constant altitude with the following dynamics:

$$\begin{aligned}
\dot{x}^{\text{aug}}(t) &= \begin{bmatrix} v_X(t) & v_Y(t) & 0 & 0 & 0 \end{bmatrix}^T \ldots \\
&\quad + \begin{bmatrix} u_1(t) & u_2(t) & 0 & 0 & 0 \end{bmatrix}^T + w^{\text{aug}}(t), \\
\text{where } &\begin{cases} u_1(t) = -5\sin(t/2), \; u_2(t) = 5\sin(t/2) & t \geq 10s \\ u_1(t) = 0, \qquad\qquad\quad u_2(t) = 0 & t \geq 90s \end{cases} \\
w^{\text{aug}}(t) &\sim \mathcal{N}(0, \text{diag}(0, 0, 0.1^2, 0.1^2, \; 0)), \\
Q_c^{\text{aug}}(t) &= \text{diag}(0, 0, 0.1^2, 0.1^2, \; 0.5^2),
\end{aligned}$$

where the variance of $w^{\text{aug}}(t)$ differs from variance $Q_c^{\text{aug}}(t)$ since a small artificial noise, 0.5^2, is added to the last diagonal entity of $Q_c^{\text{aug}}(t)$. Additionally, inputs u are large excitations for easily displaying the responses of the state variables.

With the dynamic model, we introduce two types of sensors: measurements y_{1k} from a fast and non-lagged speed sensor and measurements y_{2k}, two delayed bearing angles measured from each location of two stations , $L_1 = [L_{1X}, L_{1Y}]^T = [100, 0]^T$ and $L_2 = [L_{2X}, L_{2Y}]^T = [0, 100]^T$. Measurements are generally obtained in discrete time, so y_{1k} samples at 20 (Hz), and sampling interval M_s of y_{2k} is 1 (s).

$$\begin{aligned}
y_{1k} &= \sqrt{v_{X,k}^2 + v_{Y,k}^2} \; + \; \nu_{1k}, \qquad\qquad \nu_{1k} \sim \mathcal{N}(0, I), \\
y_{2k} &= \begin{bmatrix} \tan^{-1}\left(\dfrac{p_{Y,(k-N)} \; - \; L_{1Y}}{p_{X,(k-N)} \; - \; L_{1X}}\right) \\ \tan^{-1}\left(\dfrac{p_{Y,(k-N)} \; - \; L_{2Y}}{p_{X,(k-N)} \; - \; L_{2X}}\right) \end{bmatrix} + \nu_{2k}, \;\; \nu_{2k} \sim \mathcal{N}(0, 10^{-4} I),
\end{aligned}$$

where the value of N, the number of delayed samples in the second measurements, is constant; that is, static N = 18 (0.9 s). The bearing sensor has finite delay N, which implies that the current reading actually corresponds to the position of the vehicle at some point (i.e., N time-prior) in the past.

Given the propagation and measurement models of the system, the combined parameter-state estimator initializes states and its covariance

$$\begin{aligned}
\tilde{x}_0^{\text{aug}+} &= \mathbb{E}(x_0^{\text{aug}}) = [0, 0, 1, 1, \text{random number}]^T \\
\tilde{P}_0^{\text{aug}+} &= \mathbb{E}\left[(x_0^{\text{aug}} - \tilde{x}_0^{\text{aug}+})(x_0^{\text{aug}} - \tilde{x}_0^{\text{aug}+})^T\right] = \text{diag}(I_{4\times4}, 5^2)
\end{aligned}$$

and follows the processes of cross covariance, interpolation, and constrained filtering (Fig. 3). The simplest solution to the estimation problem of the given system is to run a standard EKF that ignores any delay of the bearing sensor. However, the novel combined parameter-state estimator described in this paper compensates for delayed measurements at time when the data of the lagged second sensor are fused and estimates the delay value. This combined estimation is referred to as "delay-Param."

The simulation results from both approaches depict the influence of the delays and the effectiveness of delay-Param in the sensor fusion of the lagged measurements. Monte Carlo simulations produce the root mean square errors (RMSE) of both approaches (Fig. 4) and their averaged RMSE (Table I). Fig. 4 also shows the advantages of the constrained Kalman filtering in the delay-Param method by comparing with the Unconstrained approach. The EKF discards valuable information (i.e., time delays) in lagged measurements, and although delay-Param might increase the computational effort of the filter, it significantly improves the accuracy of estimation. In other words, when the value of the delay is large, the proposed estimator, delay-Param, outperforms the existing method, the EKF. Unlike the EKF,

TABLE I: RMS error comparison

Averaged RMS error	EKF	delay-Param
Position X | Y (ft)	3.4570 | 3.3927	0.9748 | 0.9849
Velocity X | Y (ft/s)	0.6761 | 0.6993	0.2509 | 0.2749

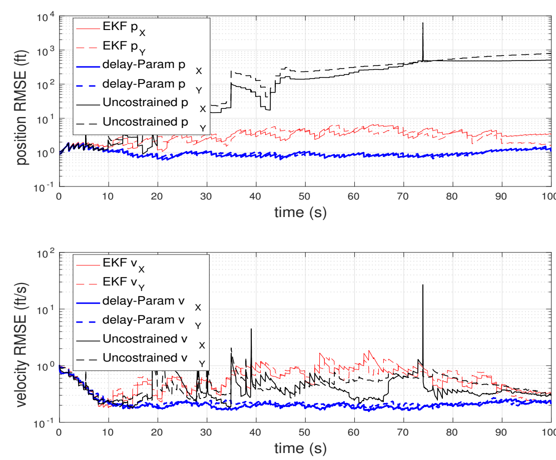

Fig. 4: RMS errors: EKF vs. delay-Param vs. Unconstrained

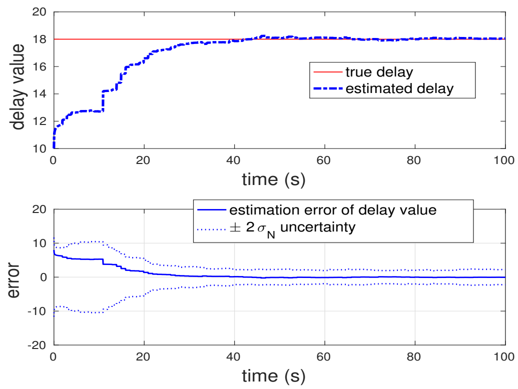

Fig. 5: Estimation of an unknown delay

delay-Param also estimates the values of time delays. Fig. 5 shows that the estimation error of N gradually decreases toward zero and that the estimation variance gradually decreases. In this example, we set the variance of artificial noise $w_p(t)$ to 0.5^2. The noise allows the delay-Param approach to more readily adjust its estimate of $\hat{N}_k$. The performance of the parameter estimation depends on the initial guess, so we set a uniformly distributed random number to the last element of the augmented vector at the starting point. Here, we set 50 (= 2.5s) to the upper bound of delays, but even if we assume the upper bound of the delay represents the total simulation time, 2,000 (= 100s), delay-Param works the same. Morever, we may wonder whether the delay-Param algorithm works when the delay is not static. See Fig. 6 for an answer. Although the values of unknown delays vary

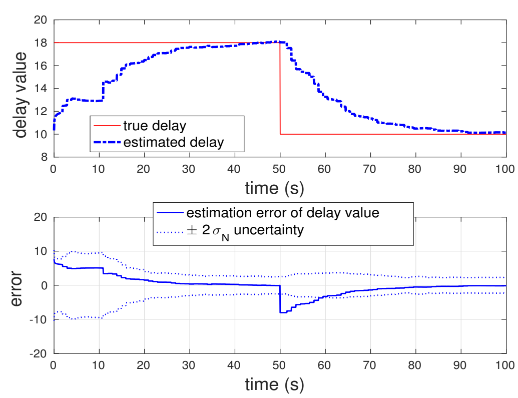

Fig. 6: Estimation of unknown time-varying delays

over time, estimation resulting from the delay-Param method converges to true delay values.

V. Conclusion

For compensating delayed measurements and estimating unknown delay values, this paper proposes a combined parameter-state estimator that includes state augmentation, interpolation, cross covariance, and constrained filtering. Results from Monte Carlo simulations show that the proposed approach can solve an estimation problem with unknown time delays.

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