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Design of PID Controllers for 4-DOF Planar and Spatial Manipulators

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Abstract- Robotic manipulators are the most important components in the present manufacturing industry for handling materials and manipulation. It is important to note that these manipulators are used in the complex planar and spatial work spaces to perform the task. Further, it is also very difficult to manipulate and dynamically control the manipulators when the degrees of freedom (DOF) of the manipulator are increased. In the present manuscript, the dynamic analysis and design of PID controller for two different manipulator configurations, namely planar and spatial manipulators have been attempted. Moreover, both the manipulators considered in this study are provided with 4-DOF and dynamic analysis has been carried out using Lagrange-Euler (L-E) formulation. The main difference between the planar and spatial manipulator is that the former works in the two dimensional space and later works in a three dimensional space. In addition to the dynamic analysis, PID controllers have also been designed for the said configurations to follow a path between the given boundary conditions. Further, the developed controllers are tested in simulations, and found working satisfactorily.

Keywords -Planar manipulator, spatial manipulator, dynamic analysis, PID controller.

I. INTRODUCTION

Robotic manipulators are used in aerospace, manufacturing, automotive industry and medical applications. These manipulators are working in hazardous, unpredictable and inhospitable circumstances, which are difficult for the human operations to perform the task. It consists of three models, namely mechanical, electrical and control modules. In the mechanical point of view, robot manipulators are collection of serial or parallel links which are connected with the help of revolute and/or prismatic joints between the base and the end effector frame. Number of researchers had developed several methods to identify the factors influencing the performance and accuracy of the robotic manipulators [1].

The serial manipulator with 3-revolute joints that can operate in 2-D space was developed by soomro [2] and Fu et al. [3]. In both the cases, the authors had used forward kinematics approach to establish the mathematical model of the manipulator. Further, these mathematical models were used to determine mass moment of inertia, velocity and acceleration of various joints of the manipulator. Moreover, Hatem et al. [4] developed a kinematic and dynamic model of a 3-DOF planar and spatial manipulators to overcome the control and noise factors. In [5], a general purpose algorithm had been developed for accurate dynamic

modelling of a spatial RRR flexible manipulator that includes both link and joint flexibility. This algorithm was implemented by using the principle of virtual work, finite element method and recursive kinematic formulation. The benefit of the algorithm was that it uses less set of equations that define the dynamics of flexible manipulator, which was essential for the controller to reduce the computational cost. Moreover, the workspace of a robotic manipulator was determined after using an analytical method [6]. This method was based on analytical criteria for determining the singular behaviour of the mechanism. The authors attempted to compute the singularities after manipulating the Jacobian of the robot condition. These singularities were substituted in the wrist vector to acquire the range of motion of the robot wrist in 3D space.

During manipulation of the robot, many problems need to be tracked simultaneously. Among those problems, controlling of the robotic manipulator is a challenging task. The basic idea of a robot controller is to make the joints of a manipulator to track a desired trajectory to perform a specified task. The dynamic modelling of a simple 2-DOF robotic manipulator, using PD controller with online gravity compensation was developed by Dung et al. [7]. Further, Ohri et al. [8] designed PID controller and sliding mode controller for desired trajectory, in the presence of highly nonlinear coupled dynamics with high accuracy. It was observed that, when the payload was changed, the SMC produced better response compared to PID control strategy. In [9], the authors developed force-torque control strategy for two degrees of freedom robotic manipulator. Two types of control strategies were developed to control the slow part of system dynamics, i.e. impedance control and force/torque control with gravity compensation. Moreover, a classical PID controller [10] was implemented for a six-link, six-joint flexible robotic manipulator model. The flexibility was given to the joints of the system model.

Although the researchers had developed various approaches for serial and parallel manipulators, those strategies may be divided into two categories, i.e. joint-space control and workspace control. The joint space controller considers the system as an independent SISO system to implement the developed joint space control strategy. Where, the workspace control is more attractive than joint space control because the tasks, such as painting, assembly etc., are performed by the end effectors in the robots workspace [11]. In addition to the above works, Su et al. [12] designed and explained a robust auto-disturbance rejection controller for 6-DOF parallel manipulator in joint space. Moreover, in [13], the authors discussed the

development of controller for robotic manipulators with complex nonlinear dynamics. Once the controllers are developed, the tuning of the gains of the controller plays an important role in determining the performance of the controller. There are different types of tuning methods developed by the researchers such as manual tuning, Ziegler and Nichols tuning and online tuning of the controllers. Ziegler and Nichols [14] proposed the first tuning method for proportional-integral-derivative (PID) controller. In [15], the authors designed a dynamic model and PID controller for a 2-DOF planar manipulator by using the manual tuning procedure. The present manuscript deals with the development of a PID controller for two robotic manipulators, namely 4-DOF planar and spatial robots, respectively.

The paper is organized as follows: section 2 explains the mathematical formulation of the problem, i.e. kinematics and dynamics of the robotic manipulator. Section 3 describes the design of a PID controller for both the 4-DOF planar and spatial manipulators. Section 4 shows the results of the simulation. Finally, section 5 concludes the paper.

II. MATHEMATICAL FORMULATION OF THE PROBLEM

The present section focused on the development of forward kinematics and dynamic modelling of two four-link robotic manipulators, such as planar (refer to Fig. 1) and spatial (refer to Fig. 2) manipulators. The links of the manipulators are considered to have masses and lengths equal to $\{m_1, m_2, m_3, m_4\}$ and $\{L_1, L_2, L_3, L_4\}$, respectively. Let the included angles made by the links (L_1 through L_4) of the manipulators are given as $\theta_1, \theta_2, \theta_3$ and θ_4 in order to define a position for the end effector. A systematic method known as D-H parameter setting has been used to assign the co-ordinate frames for the joints of the manipulator to derive the transformation matrix, that represent the forward kinematic model for both the manipulators. It is important to note that forward kinematic model helps in determining the position and orientation of end effector with respect to base. The D-H parameters obtained for 4-DOF planar and spatial manipulators are given in tables 1 and 2, respectively.

Table 1: The D-H parameters of the 4-DOF planar manipulator are as follows:

Link(i)	a_i	α_i	d_i	θ_i
1	L_1	0	0	θ_1
2	L_2	0	0	θ_2
3	L_3	0	0	θ_3
4	L_4	0	0	θ_4

$${}^0T_4 = \begin{bmatrix} C_{1234} & -S_{1234} & 0 & L_1C_1 + L_2C_{12} + L_3C_{123} + L_4C_{1234} \\ S_{1234} & C_{1234} & 0 & L_1S_1 + L_2S_{12} + L_3S_{123} + L_4S_{1234} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

This information is further used in determining the homogeneous transformation matrix (0T_4) that tells the relationship between end-effector and base, in terms of

length of the links and included angle between them. The homogeneous transformation matrices for 4-DOF planar and spatial manipulators are given by equations (1) and (2), respectively.

Table 2: The D-H parameters of the 4-DOF spatial manipulator are as follows:

Link (i)	a_i	α_i	d_i	θ_i
1	0	90°	0	θ_1
2	L_2	0	0	θ_2
3	L_3	0	0	θ_3
4	L_4	0	0	θ_4

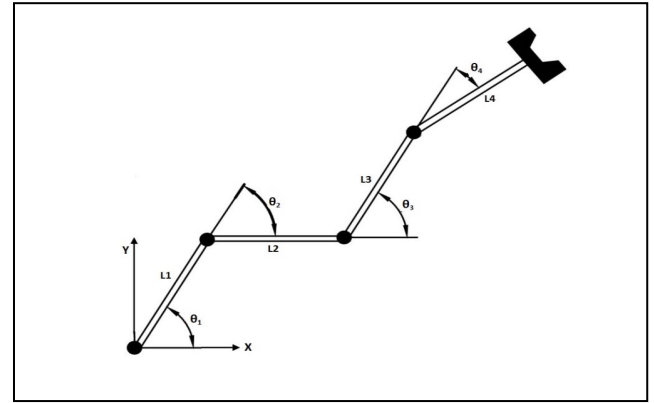


Fig 1: 4 DOF planar manipulator.

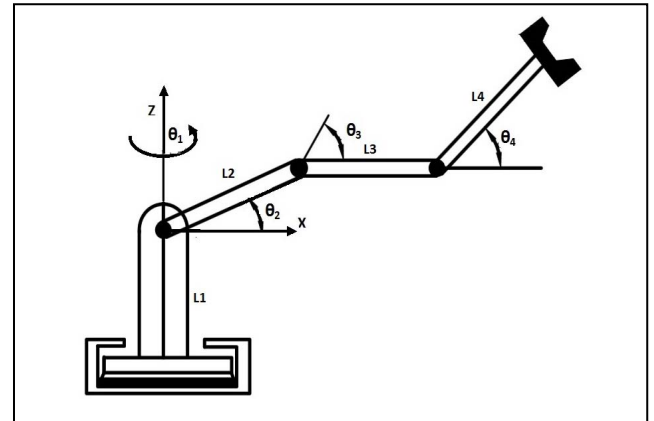


Fig 2: 4 DOF spatial manipulator.

$${}^0T_4 = \begin{bmatrix} C_1C_{234} & -C_1S_{234} & S_1 & C_1[L_2C_2 + L_3C_{23} + L_4C_{234}] \\ S_1C_{234} & -S_1S_{234} & -C_1 & S_1[L_2C_2 + L_3C_{23} + L_4C_{234}] \\ S_{234} & C_{234} & 0 & [L_2S_2 + L_3S_{23} + L_4S_{234}] \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2)$$

Where the notations are referring to $C_1=\cos(\theta_1)$, $S_1=\sin(\theta_1)$, $C_{12}=\cos(\theta_1+\theta_2)$, $S_{12}=\sin(\theta_1+\theta_2)$, $C_{123}=\cos(\theta_1+\theta_2+\theta_3)$, $S_{123}=\sin(\theta_1+\theta_2+\theta_3)$, $C_{1234}=\cos(\theta_1+\theta_2+\theta_3+\theta_4)$ and $S_{1234}=\sin(\theta_1+\theta_2+\theta_3+\theta_4)$.

Further, the dynamics of the robotic manipulators have been derived after considering Lagrange-Euler formulation, as the dynamics are going to play an important role in restricting the real time performance of the manipulator while executing a task. It is important to note that these restrictions are due to the velocity and acceleration of the masses that introduce the inertia of the moving masses into

the system. Therefore, it needs to be carefully modelled during the dynamic modelling of the manipulator. The dynamic equation of motion is considered as a combination of accelerations produced by inertia forces, velocity produced by centrifugal or coriolis components and gravity terms of the manipulator. The dynamic equation of motion, that is obtained from Lagrange-Euler formulation to calculate the torques required at different joints are given in equation (3).

$$\tau_i = \sum_{j=1}^n M_{ij}(q) \ddot{q}_j + \sum_{j=1}^n \sum_{k=1}^n h_{ijk} \dot{q}_j \dot{q}_k + G_i$$

$$i, j = 1, 2, \dots, n \quad \text{----- (3)}$$

Where τ_i = force applied at joint i due to motion of links

q = Displacement of the joint.

\dot{q}_j = Velocity of the joint j .

\ddot{q}_j = Acceleration of the joint j .

Where the inertia coefficient is M_{ij} is

$$M_{ij} = \sum_{p=\max(i,j)}^n \text{Tr}[d_{pj} I_p d_{pi}^T] \quad i, j = 1, 2, \dots, n$$

The Coriolis force coefficient is

$$h_{ijk} = \sum_{p=\max(i,j,k)}^n \text{Tr}\left[\frac{\partial(d_{pk})}{\partial q_p} I_p d_{pi}^T\right] \quad i, j, k = 1, 2, \dots, n$$

The gravity coefficient is

$$G_i = - \sum_{p=i}^n m_p g d_{pi} \bar{r}_p \quad i, j = 1, 2, \dots, n$$

III. DESIGN OF PID CONTROLLER

The purpose of using PID controller in controlling the robotic manipulator is to position the end-effector at the desired location after following the boundary conditions. It is important to note that the design of PID controller is relatively simple, when the transmission mechanism includes gears. The PID control algorithm works on manipulation of the controller gains (K_p -gain of proportional controller, K_i -gain of integral controller, K_d -gain of derivative controller) obtained based on the magnitude of error signal. The proportional part of the PID controller is obtained by multiplying K_p with error signal and it will reduce large part of the overall error. Generally, PD or PID controllers are used in the closed loop system to locate the end effector at the exact location. The integral part of the controller gain is obtained by multiplying the integral value of the error with K_i time. Finally derivative part of the controller gain is obtained by multiplying the first derivative of the error with derivative constant K_d . It is to be noted that the importance of K_d is to counteract the influence of K_p and K_i terms when the output changes quickly. Further, the derivative term also helps in reducing the overshoot and ringing and it is not going to exhibit any effect on final error in position. The expression for joint-based PID controller implemented in the study is given in equation (4).

$$F = K_p e + K_d \dot{e} + K_i \int e dt \quad \text{----- (4)}$$

Where K_p = Proportional gain

K_d = Derivative gain

K_i = Integral gain

The expanded form of the equation (4) after including the meaning of e and \dot{e} are given in equation (5).

$$f_i = K_{pi}(\theta_{if} - \theta_i) - K_{di}\dot{\theta}_i + K_{ii} \int e(\theta_i) dt \quad \text{----- (5)}$$

$$x_i = \int e(\theta_i) dt \Rightarrow \dot{x}_i = \theta_{if} - \theta_i$$

Where the values of i is varying from 1 through n , which represent the number of links of the manipulator.

IV. RESULTS & DISCUSSIONS

The main aim of the present research is to design and develop a PID control strategy for the control of 4-DOF planar and spatial manipulators. Once the dynamic models and control strategies are developed, both the manipulator models are tested in simulations to check the controlling accuracy of the strategies. It is important to note that the controller gain values have been tuned using manual tuning method for a set of boundary conditions. The three gain values (K_p , K_d and K_i) that resulted in best control action for 4-DOF planar and spatial manipulators are given in tables 3 and 4, respectively. The boundary conditions utilized for the four joints of the planar and spatial manipulator are given in equation (6) and (7) respectively.

Table 3: The tuning parameters of the planar manipulator are as follows:

JOINTS	K_p	K_i	K_d
1	110	40	90
2	90	35	80
3	80	30	70
4	70	25	60

Table 4: The tuning parameters of the spatial manipulator are as follows:

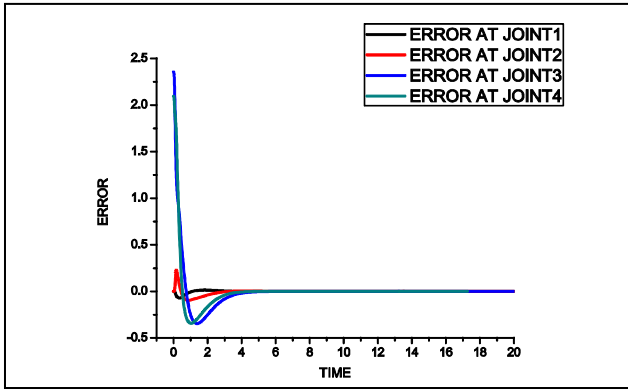
JOINTS	K_p	K_i	K_d
1	100	35	90
2	90	35	75
3	80	30	65
4	70	30	55

The boundary conditions for planar manipulator:

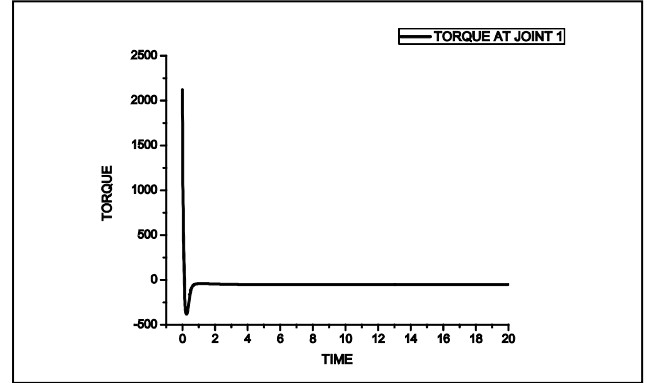
$$\begin{aligned} 45^\circ < \theta_1 < 90^\circ \\ 30^\circ < \theta_2 < 60^\circ \\ -45^\circ < \theta_3 < 90^\circ \\ -30^\circ < \theta_4 < 90^\circ \end{aligned} \quad \text{----- (6)}$$

The boundary conditions for spatial manipulator:

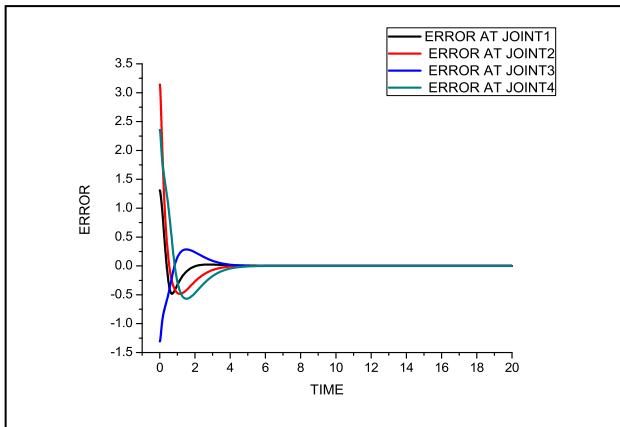
$$\begin{aligned} 0^\circ < \theta_1 < 180^\circ \\ 30^\circ < \theta_2 < -45^\circ \\ -45^\circ < \theta_3 < 90^\circ \\ -30^\circ < \theta_4 < 45^\circ \end{aligned} \quad \text{----- (7)}$$



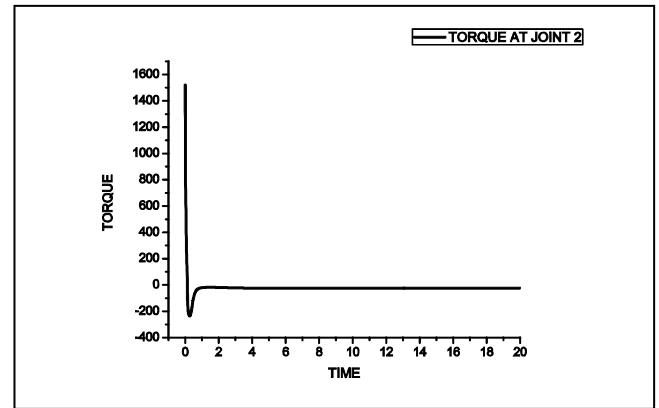
(a) Error on planar manipulator



(a)



(b) Error on spatial manipulator

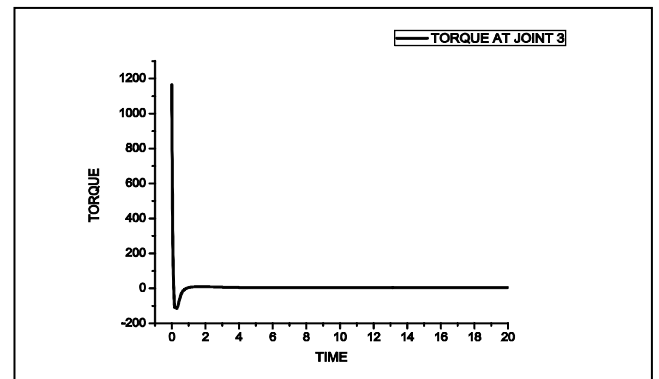


(b)

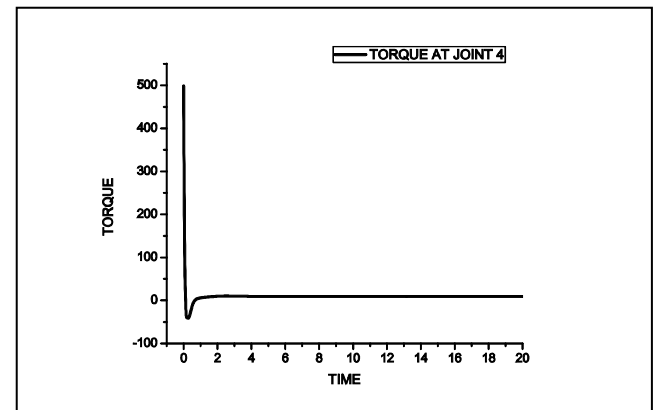
Fig 3: Schematic diagram showing the variation of error at joint 1, joint 2, joint 3, joint 4.

The variations in the value of error in angular position of all the four joints for both the planar and spatial manipulators are shown in Fig. 3(a) and (b), respectively. It has been observed that the value of error for joints 1 and 2 are seen to be low when compared with joints 3 and 4 for both the manipulators. It may be due to the fact that the angular distance covered by joints 1 and 2 are less than that of the joints 3 and 4 for the same interval of time in order to cover the segment of travel associated with that joint. It is interesting to note that all the joints are settled by 4 seconds and this result can be comparable with the existing literature [8, 15]. Further the variation of torques required at various joints, of the 4-DOF planar manipulator is shown in Fig. 4(a), (b), (c) and (d) respectively. From the above figures, it can be observed that the torque required at joint 1 is more than that of the joints 2, 3 and 4. This may be due to the reason, that the link 1 is driving the links 2, 3 and 4. Therefore, joint 1 requires more torque when compared to other joints. Similarly, the variation of torques for various joints of 4-DOF spatial manipulator is shown in Fig. (5). It is interesting to note that similar findings as discussed for planar manipulator are also true for the spatial manipulator.

Moreover, the coverage of work volume for 4-DOF planar and spatial manipulators for the given boundary conditions are shown in Fig. 6(a) and (b), respectively. It has been observed that both the manipulators have covered the work space smoothly after utilizing the developed PID controller.

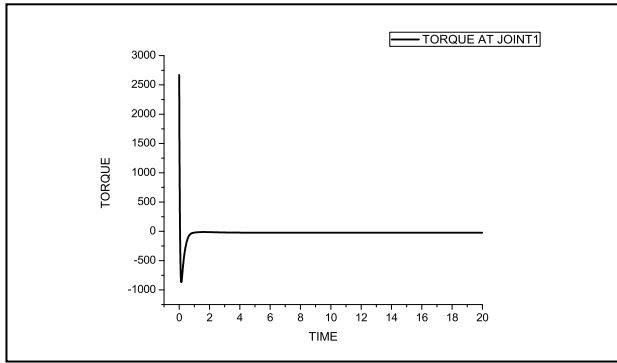


(c)

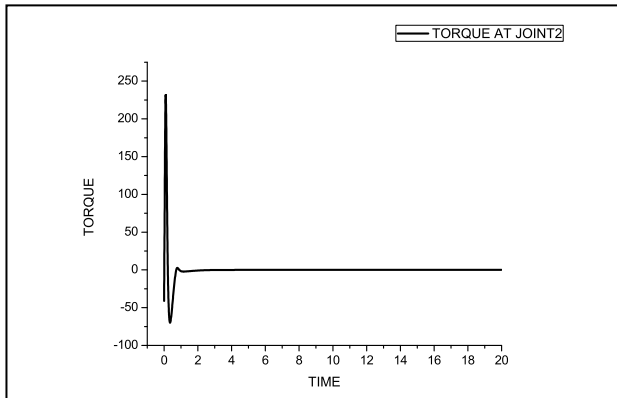


(d)

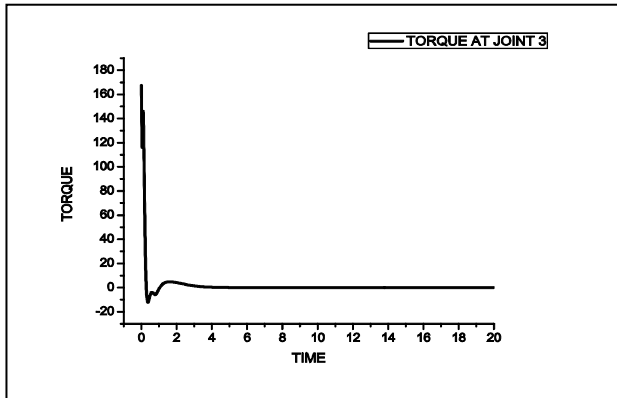
Fig 4: Schematic diagram showing the variation of torque in planar manipulator at (a) joint 1, (b) joint 2, (c) joint 3, (d) joint 4.



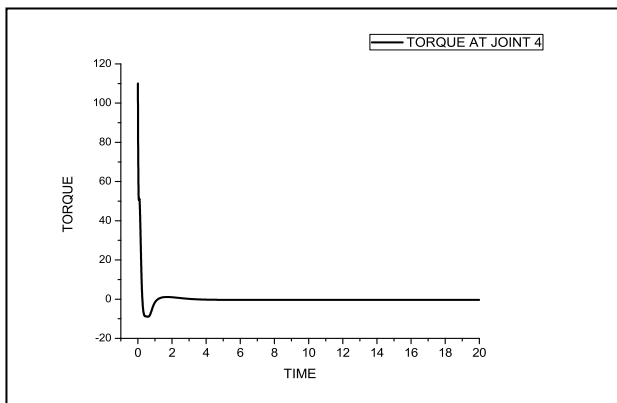
(a)



(b)

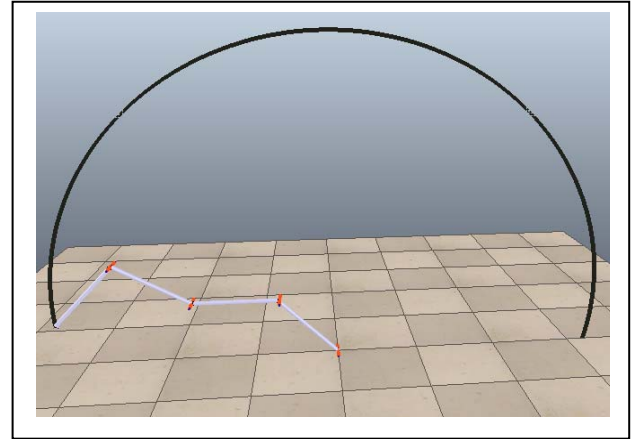


(c)

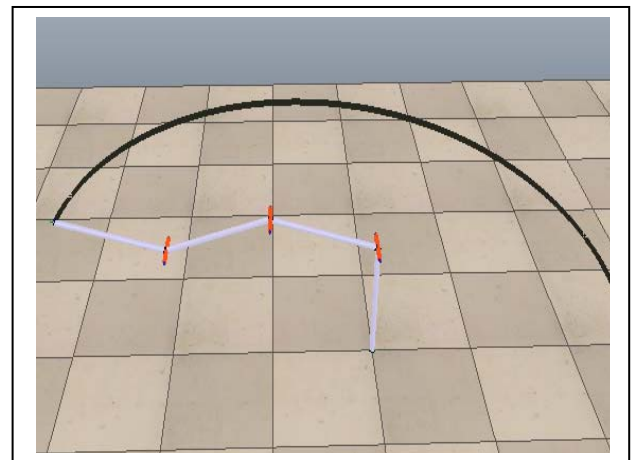


(d)

Fig 5: Schematic diagram showing the variation of torque in spatial manipulator at (a) joint 1, (b) joint 2, (c) joint 3, (d) joint 4.



(a)



(b)

Fig 6: Schematic diagram showing the variation of work volume (a) Planar manipulator, (b) Spatial manipulator.

V. CONCLUSIONS

In the present paper, the forward kinematic modelling, dynamic analysis and design of PID controllers for two 4-DOF planar and spatial robots are developed successfully. It is important to note that the manual method of tuning have the PID controllers been adopted to tune the proportional, integral and derivative gains of the controllers for both the robots. Once the controllers are developed, they are tested in simulations to verify the performance of the developed controllers. Both the PID controllers developed to control the planar and spatial manipulators are found to give satisfactory performance measures while reaching the specified boundary conditions with reasonable torque requirements at various joints.

VI. REFERENCES

- [1] Swevers, J., Verdonck, W., and De Schutter, J., "Dynamic Model Identification for Industrial Robots," IEEE Control Systems Magazine, Vol. 27, pp. 58-71 (2007).
- [2] Zulfikar Ali soomro., "Kinematic Modeling and Simulation of 2-D Link and 3-R Pendulum Serial Manipulator Robotic Arm," International Researcher, Vol.2, Issue No.4, December (2013).
- [3] Robotics: "Control, Sensing, Vision, and Intelligence," King Sun Fu, Rafael C. González, C. S. George Lee, McGraw-Hill, (1987).

- [4] Hatem A. Al-Dois., A.K Jha., and R.B Mishra., "Investigations into the Parameters Influencing the Dynamic Performance of 3-RRR Planar & Articulated Robot Manipulators," *Tamkang Journal of Science and Engineering*, Vol.14, No. 4, pp.313-322 (2011).
- [5] Rajesh K Moolam., Francesco Braghin., Federico Vicentini., "Dynamic Modeling and Simulation of Spatial Manipulator with Flexible Links and Joints," 11th International Conference on Vibration Problems Z. Dimitrovov'a et.al. (eds.) Lisbon, Portugal, 9–12 September(2013).
- [6] Khusheep Goyal, Davinder Sethi, "An Analytical Method to Find Workspace of a Robotic Manipulator," *Journal of mechanical engineering*, Vol. ME 41, No.1, June(2010).
- [7] Le Tien Dung., Hee-Jun Kang., and Young-shick Ro., "Robot Manipulator Modeling in Matlab Simmechanics with PD Control and Online Gravity Compensation," *IFOST 2010 Proceedings*, IEEE (2010).
- [8] Jyothi Ohri, Dhaval R. Vyas, Pretty Neelam Topno, "Comparison of Robustness of PID Control and Sliding Mode Control of Robotic Manipulator," *International symposium on devices MEMS, Intelligent systems & Communication(ISDMISC)*, (2011).
- [9] Victor Titov., I.Shardyko., Sergey Isaenko., "Force-Torque Control Implementation for 2 DoF Manipulator", 24th DAAAM International Symposium on Intelligent Manufacturing and Automation (2013).
- [10] Prasenjit Sarkhel., Nilotpal Banerjee., Nirmal Baran Hui., "Analysis and Control of a Six Link Serial Manipulator with Flexible Joints" 142-inacomm(2013).
- [11] Raouf Fareh, Mohamad Saad, and Maarouf Saad, "Workspace Tracking Trajectory for 7-DOF ANAT Robot Using a Hierarchical Control Strategy" 20th Mediterranean Conference on Control & Automation (MED) Barcelona, Spain (2012).
- [12] Y. X. Su, B. Y. Duan, C. H. Zheng, Y. F. Zhang, G. D. Chen, and J. W. Mi, "Disturbance-rejection High-precision Motion Control of a Stewart Platform," *IEEE Trans. Contr. Syst. Technol.*, vol. 12, no. 3, pp. 364-374, (2004).
- [13] Lewis, F.L. & et al. "Robotics Mechanical Engineer Handbook". Ed. Frank Kreith. Boca Raton: CRC Press LLC (1999).
- [14] J. G. Ziegler, and N.B. Nichols, "Optimum Setting for Automatic Controllers," *Trans. ASME*, vol. 64, pp. 759-768, (1942).
- [15] David I. Robles G., "PID Control Dynamics of a Robotic Arm Manipulator with Two Degrees of Freedom," August 17, (2012).