MAT 120 Lab Final
Integral Calculus and Differential Equations
Time: **45 minutes** , Total Marks: **20**

Name: ____________________

Student ID: ____________________ Section: ____________________

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*Answer all the questions properly.

MCQ

Select the best option: [$8 \times 1 = 8$]

1. Which function is used to convert a string expression into a SymPy expression?
   [Ans. = c. eval()]

2. Which SymPy function is used to obtain integrated expression? (i.e., if input is $2x$, output is x^2)
   [Ans. = b. integrate()]

3.
```
from numpy.random import rand
print(round(3*rand()-1,4))
```
   Which of the following cannot be an output of the above code?
   (a) 2.3753 (b) 1.5691 (c) 0.4708 (d) -0.3896

   [Ans. = (a) 2.3753, because *rand()* $\in [0, 1]$, so the possible outputs are $\in [-1, 2]$]

4. Which Matplotlib function is used to show the label of different plots in a graph?
   [Ans. = d. pyplot.legend()]

5. Which SciPy submodule does contain functions for solving ODE?
   [Ans. = (a) integrate]

6. Suppose you need to generate a NumPy array $x = [x_1, x_2, ..., x_N]$ such that $x_i \in [1, 3]$, and $x_i - x_j = 0.125n$ (Here n is an integer and $i, j = 1, 2, .., N$). Which of the following statements generates this?
   [c. x= numpy.linspace(1,3,17)]

7.
```
from sympy import *
from sympy.utilities.lambdify import lambdify
x = symbols('x')
expr = diff((2/3)*x**(3/2))
f = lambdify(x, expr, modules='numpy')
print(round(f(2), 2))
```
   What is the output of this code?
   [Ans. = (d) 1.41]

8. The key characteristics of pseudo-random number generators are that they-
   i. Guarantee perfectly uniform distributions of numbers.
   ii. Exhibit no statistical patterns detectable by standard tests.
   iii. Generate sequences that eventually repeat with a period.
   Which of the following is correct?
   [Ans. = (b) ii]

Written Part

1. Write down the differential equation and the initial values which the following code tries to solve. [2]

```
import numpy as np
t = np.linspace(0, 1, 101)
y = 0*t
v = 0*t
y[0] = 1
%Euler
for i in range(1, 101):
    v[i] = v[i - 1] - (0.5*v[i-1]+2*y[i-1])) /100
    y[i] = y[i - 1] + v[i] /100
```

$$\frac{d^2y}{dt^2} + 0.5\frac{dy}{dt} + 2y = 0$$
$$y(0) = 1, y'(0) = 0$$

2. Consider the function $f(x) = 3x^2dx$. To numerically calculate $\int_0^1 f(x)dx$, four points have been taken on the curve: $(0, 0), (0.4, 0.48), (0.75, 1.6875), (1, 3)$. [6]
Now evaluate the area using the Trapezoidal method using these given points.

$$A_1 = \frac{0.4}{2}(0 + 0.48) = 0.096$$
$$A_2 = \frac{0.75 - 0.4}{2}(0.48 + 1.6875) = 0.3793125$$
$$A_3 = \frac{1 - 0.75}{2}(1.6875 + 3) = 0.5859375$$
$$A = A_1 + A_2 + A_3 = 1.06125$$

3. Consider the ODE: $(x + y)dy + (y - x)dx = 0, y(1) = 2$. [4]
Using step size $h = 0.125$, calculate $y(1.25)$ using the Euler method.

$$dy/dx = f(x, y) = (x - y)/(x + y), x_0 = 1, y_0 = 2, h = 0.125$$
$$y(1.125) = y_1 = y_0 + hf(x_0, y_0) = 47/24, x_1 = 1.125$$
$$y(1.25) = y_2 = y_1 + hf(x_1, y_1) \approx 1.92455$$