**Quiz 3 (Section : 06)**
**MAT120 : Integral Calculus & Differential Equations**
**BRAC University**

Date: 30/03/2024 Time: 40 minutes Total Mark: 15

---

**Name:**

**ID:**

---

1. Evaluate $\iint_{\mathcal{R}} \frac{y-4x}{y+4x} dA$ where $\mathcal{R}$ is the region enclosed by the lines $y = 4x$, $y = 4x + 2$, $y = 2 - 4x$, $y = 5 - 4x$. [6]

2. Evaluate the double integral $\iint_{\mathcal{R}} x \cos y dA$ where $\mathcal{R}$ is the triangular region enclosed by $y = x$, $y = 0$ and $x = \pi$. [5]

3. Find the Jacobean $\frac{\partial(x,y,z)}{\partial(\rho,\phi,\theta)}$ where the variables $(x, y, z)$ are defined as the co-ordinates of the spherical coordinates i.e. $x = \rho \sin\phi \cos\theta$, $y = \rho \sin\phi \sin\theta$ and $z = \rho \cos\phi$. [4]

---

[*Please start writing from here*]

# Quiz 03 (Sec 06)

**Q1** The region is enclosed by the lines

$y = 4x \Rightarrow 4x - y = 0$

$y = 4x + 2 \Rightarrow 4x - y = -2$

$y = 2 - 4x \Rightarrow 4x + y = 2$

$y = 5 - 4x \Rightarrow 4x + y = 5$

Let us consider the transformation

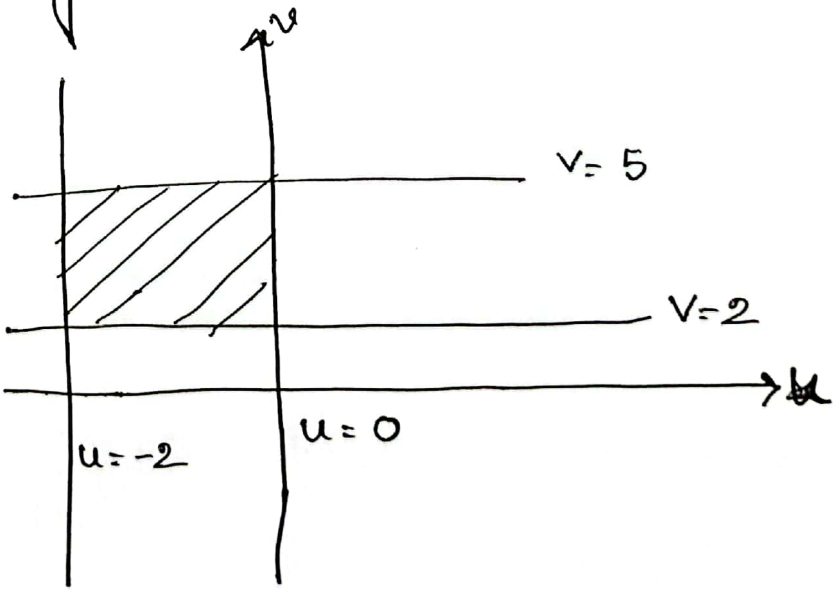
$4x - y = u, \quad 4x + y = v$

$\Rightarrow y - 4x = -u$

Then the transformed region looks like as follows :-

$u = 0, \quad u = -2$

$v = 2, \quad v = 5$




From the transformations,

$8x = u + v$

$\Rightarrow x = \dfrac{u+v}{8}$

and $-2y = u - v$

$$\Rightarrow y = \frac{v-u}{2}.$$

$$J = \frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{1}{8} & \frac{1}{8} \\ -\frac{1}{2} & \frac{1}{2} \end{vmatrix} = \frac{1}{8}$$

Thereforce,

$$\iint \frac{y-4x}{y+4x}\, dA = \int_{2}^{5}\int_{-2}^{0} \frac{-u}{v} \left|\frac{1}{8}\right| du\, dv$$

$$= -\frac{1}{8}\int_{2}^{5}\left[\frac{u^2}{2v}\right]_{-2}^{0} dv$$

$$\cancel{= -\frac{1}{8}(-2)\left[v\right]_{2}^{5}}$$

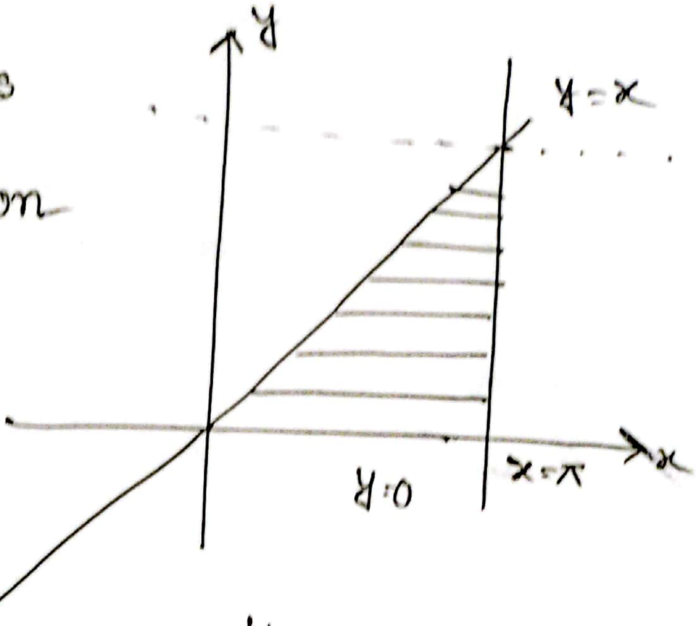
$$\cancel{= \frac{1}{4}(5-2) = \frac{3}{4}}$$

$$= -\frac{1}{8}\left(\frac{1}{2}\right)(-4)\int_{2}^{5}\frac{1}{v}\, dv.$$

$$= \frac{1}{4}\left[\ln|v|\right]_{2}^{5} = \frac{1}{4}\{\ln|5| - \ln|2|\} \quad \text{Ans.}$$

A We can view this as type II region. The region is bounded by

$y = 0, \quad y = x, \quad x = \pi.$




Therefore, as, $y = x$ and $x = \pi$ then $y = \pi$.

$$\therefore \iint_R x\cos y \, dA = \int_0^{\pi}\int_0^{x} x\cos y \, dy \, dx$$

$$= \int_0^{\pi} \Big[x\sin y\Big]_0^{x} dx = \int_0^{\pi} x\sin x \, dx$$

$\cancel{x}\int$

Then, $\int x\sin x \, dx$

$$= x\int \sin x \, dx - \int\left\{\frac{d}{dx}(x)\int \sin x \, dx\right\} dx$$

$$= -x\cos x + \int \cos x \, dx$$

$$= -x\cos x + \sin x + c$$

$$\therefore \int_0^{\pi} x\sin x \, dx = \Big[-x\cos x + \sin x\Big]_0^{\pi}$$

$$= -\pi(-1) = \pi$$

Ans

Given that,

$$x = \rho \sin\varphi \cos\theta$$

$$y = \rho \sin\varphi \sin\theta$$

$$z = \rho \cos\varphi$$

Jacobean, $\dfrac{\partial(x, y, z)}{\partial(\rho, \varphi, \theta)} = \begin{vmatrix} \frac{\partial x}{\partial \rho} & \frac{\partial x}{\partial \varphi} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial \rho} & \frac{\partial y}{\partial \varphi} & \frac{\partial y}{\partial \theta} \\ \frac{\partial z}{\partial \rho} & \frac{\partial z}{\partial \varphi} & \frac{\partial z}{\partial \theta} \end{vmatrix}$

$$= \begin{vmatrix} \sin\varphi\cos\theta & \rho\cos\varphi\cos\theta & -\rho\sin\varphi\sin\theta \\ \sin\varphi\sin\theta & \rho\cos\varphi\sin\theta & \rho\sin\varphi\cos\theta \\ \cos\varphi & -\rho\sin\varphi & 0 \end{vmatrix}$$

$$= \sin\varphi\cos\theta\left(\rho^2\sin^2\varphi\cos\theta\right)$$
$$- \rho\cos\varphi\cos\theta\left(-\rho\sin\varphi\cos\varphi\cos\theta\right)$$
$$- \rho\sin\varphi\sin\theta\left(-\rho\sin^2\varphi\sin\theta \cancel{+} \rho\cos^2\varphi\sin\theta\right.$$

$$= \rho^2\sin^3\varphi\cos^2\theta + \rho^2\sin\varphi\cos^2\varphi\cos^2\theta$$
$$+ \rho\sin^3\varphi\sin^2\theta + \rho^2\sin\varphi\cos^2\varphi\sin^2\theta$$

$$\rho^2\sin\varphi\left(\sin^2\varphi\cos^2\theta + \cos^2\varphi\cos^2\theta + \sin^2\varphi\sin^2\theta + \cos^2\varphi\sin^2\theta\right)$$

$$= \rho^2\sin\varphi\left\{\cos^2\theta\left(\sin^2\varphi + \cos^2\varphi\right) + \sin^2\theta\left(\sin^2\varphi + \cos^2\varphi\right)\right\}$$

$$= \rho^2\sin\varphi\left(\cos^2\theta + \sin^2\theta\right) = \rho^2\sin\varphi$$

Ans.