Quiz 2 (Section : 06)

MAT120 : Integral Calculus & Differential Equations

BRAC University

Date: 24/02/2023 | Time: 35 minutes | Total Mark: 15

Name:

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1. Find the surface area generated by the curve $12xy = 4x^4 + 3$ over the interval $\cancel{\frac{7}{12} \leq y \leq \frac{67}{24}}$ with respect to x-axis. [5]
   $1 \leq x \leq 2$
2. Evaluate the following integrals: [5+5]

   (a) $\int_2^\infty \frac{9}{(1-3z)^4} dz$

   (b) $\int \frac{\sqrt{x^2 - 15}}{x^3} dx$

[***Please start writing from here***]

Quiz 02 (Sec 06)

(1) Given that,

$$12xy = 4x^4 + 3$$

$$\Rightarrow y = \frac{4x^4+3}{12x}$$

$$\Rightarrow y = \frac{1}{3}x^3 + \frac{1}{4}x^{-1}.$$

$$\frac{dy}{dx} = x^2 - \frac{1}{4}x^{-2}$$

Arc Length, $L = \int_1^2 \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$

$$= \int_1^2 \sqrt{1+\left(x^2-\frac{1}{4}x^{-2}\right)^2}\, dx$$

$$= \int_1^2 \sqrt{1+x^4-2.x^2\frac{1}{4}x^{-2}+\frac{1}{16}x^{-4}}\, dx$$

$$= \int_1^2 \sqrt{x^4+\frac{1}{2}+\frac{1}{16}x^{-4}}\, dx$$

$$= \int_1^2 \sqrt{(x^2)^2+2.x^2.\frac{1}{4}x^{-2}+\left(\frac{1}{4}x^{-2}\right)^2}\, dx$$

$$= \int_1^2 \sqrt{\left(x^2+\frac{1}{4}x^{-2}\right)^2}\, dx = \int_1^2 \left(x^2+\frac{1}{4}x^{-2}\right) dx = \frac{59}{24} \text{ units.}$$

$$= 2.46 \text{ units.}$$

Ans

2. (a) $\displaystyle\int_2^\infty \frac{9}{(1-3z)^4}\,dz$.

Since the upper limit of the integral is infinity, it is an improper integral.

Now $\displaystyle\int_2^\infty \frac{9}{(1-3z)^4}\,dz = \lim_{K\to\infty} \int_2^K \frac{9}{(1-3z)^4}\,dz$

$$= \lim_{K\to\infty} \left[\frac{9}{-3}\,\frac{(1-3z)^{-3}}{-3}\right]_2^K$$

$$= \lim_{K\to\infty} \left[\frac{1}{9(1-3z)^{+3}}\right]_2^K$$

$$= \lim_{K\to\infty} \left[\frac{1}{9(1-3K)^3} - \frac{1}{9(1-6)^3}\right]$$

$$= \frac{1}{125}$$ Ans.

$$\int \frac{\sqrt{x^2-15}}{x^3}\,dx$$

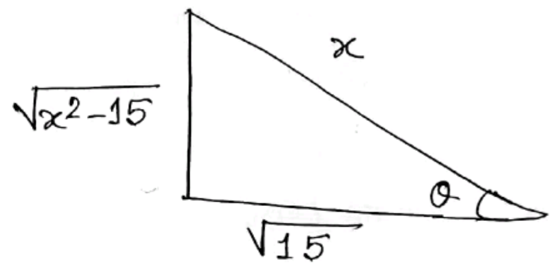
Let,

$$x = \sqrt{15}\sec\theta$$

$$\Rightarrow dx = \sqrt{15}\sec\theta\tan\theta\,d\theta$$

$$= \int \frac{\sqrt{15\sec^2\theta - 15}}{(\sqrt{15})^3\sec^3\theta}\sqrt{15}\sec\theta\tan\theta\,d\theta \,.$$

$$= \int \frac{\sqrt{15}\sqrt{\sec^2\theta-1}}{15\sec^2\theta}\tan\theta\,d\theta$$



$$= \int \frac{1}{\sqrt{15}}\,\frac{\tan^2\theta}{\sec^2\theta}\,d\theta$$

$$= \int \frac{1}{\sqrt{15}}\,\frac{\sin^2\theta}{\cos^2\theta} \times \cos^2\theta\,d\theta \,.$$

$$\sin\theta = \frac{\sqrt{x^2-15}}{x}$$

$$\cos\theta = \frac{\sqrt{15}}{x}$$

$$= \frac{1}{\sqrt{15}}\int \sin^2\theta\,d\theta$$

$$= \frac{1}{2\sqrt{15}}\int 2\sin^2\theta\,d\theta$$

$$= \frac{1}{2\sqrt{15}}\int (1-\cos 2\theta)\,d\theta$$

$$= \frac{1}{2\sqrt{15}}\left[\theta - \frac{\sin 2\theta}{2}\right] + c$$

$$= \frac{1}{2\sqrt{15}}\left[\theta - \sin\theta\cos\theta\right] + c = \frac{1}{2\sqrt{15}}\left[\sec^{-1}\frac{x}{\sqrt{15}} - \frac{\sqrt{15}\sqrt{x^2-15}}{x^2}\right] + c$$