Quiz 4 (Section : 06)
MAT120 : Integral Calculus & Differential Equations
BRAC University

Date: 27/04/2024 Time: 35 minutes Total Mark: 15

Name:

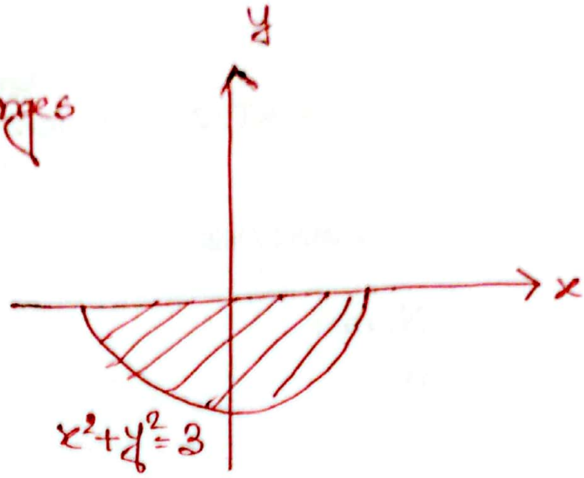
ID:

1. Evaluate $\int\int_{\mathcal{D}} \sqrt{1+2x^2+2y^2}\,dA$ where $\mathcal{D}$ is the bottom half of $x^2+y^2=3$. [5]
2. Determine whether the differential equation $(3x^2y+e^y)dx+(x^3+xe^y-2y)dy=0$ is exact or not. Then solve the equation by using the proper method. [5]
3. If a town's population, currently at 50000 experiences 5% growth every 10 years, what will its population be in 25 years? [5]

[*Please start writing from here*]

Quiz 04 (Sec 06)

Q1 In the bottom half, θ changes from π to 2π.



Now,

$$\iint_D \sqrt{1+2x^2+2y^2}\, dA$$

$$= \int_{\pi}^{2\pi}\int_0^{\sqrt{3}} \sqrt{1+2r^2}\; r\,dr\,d\theta$$

Let,

$u = 1+2r^2$

$\Rightarrow du = 4r\,dr$

$\Rightarrow \frac{du}{4} = r\,dr$

When $r=0$, $u=1$

When $r=\sqrt{3}$, $u=7$

$$= \frac{1}{4}\int_{\pi}^{2\pi}\int_1^7 \sqrt{u}\,du\,d\theta = \frac{1}{4}\int_{\pi}^{2\pi} \frac{2}{3}\left[u^{3/2}\right]_1^7 d\theta$$

$$= \frac{\pi}{6}\left(7^{3/2}-1\right) = 9.174$$

Ans.

Q2 Given that,

$$(3x^2y+e^y)\,dx + (x^3+xe^y-2y)\,dy = 0.$$

Here, $M(x,y) = 3x^2y + e^y$

$N(x,y) = x^3 + xe^y - 2y$

Since $\frac{\partial M}{\partial y} = 3x^2 + e^y = \frac{\partial N}{\partial x}$, the equation is exact

$$\frac{\partial f}{\partial x} = 3x^2 y + e^y$$

$$\Rightarrow \int \frac{\partial f}{\partial x}\, dx = \int (3x^2 y + e^y)\, dx$$

$$\Rightarrow f(x, y) = x^3 y + x e^y + g(y)$$

and,

$$\frac{\partial f}{\partial y} = x^3 + x e^y - 2y$$

$$\Rightarrow x^3 + x e^y + g'(y) = x^3 + x e^y - 2y$$

$$\Rightarrow g'(y) = -2y \Rightarrow g(y) = -y^2 + c$$

Therefore, $f(x, y) = x^3 y + x e^y - y^2 + c$

Ans.

Q3| Let $P(t)$ be the population at any time t.

At time $t = 0$, $P(0) = 50000$

At time $t = 10$, $P(10) = 50000 + 50000 \times \frac{15}{100}$

$$= 57500$$

According to the question we set the relation as

$$\frac{dP}{dt} \propto P(t)$$

$$\Rightarrow \frac{dP}{dt} = kP$$

$$\Rightarrow \int \frac{dP}{P} = \int k\,dt$$

$$\Rightarrow P(t) = c_1 e^{kt}$$

Here, $P(0) = 50000$ and $P(10) = 52500$

$\Rightarrow c_1 = 50000$

$$\Rightarrow c_1 e^{10k} = 52500$$

$$\Rightarrow k = \frac{1}{10} \ln \left| \frac{52500}{50000} \right|$$

$$\Rightarrow k = \cancel{0.01398}\ 4.87 \times 10^{-3}$$

Therefore, $P(t) = 50000\, e^{\cancel{0.01398t}\ 4.87 \times 10^{-3}}$

After 25 years, $P(25) = 50000\, e^{\cancel{0.01398} \times 25\ 4.87 \times 10^{-3}}$

$$\simeq \cancel{70917.9096}\ 56473.58$$

$$\simeq \cancel{70918}\ 56474 \text{ persons}$$

Ans.