Quiz 1 (Section : 06)
MAT120 : Integral Calculus & Differential Equations
BRAC University

Date: 08/02/2023 Time: 35 minutes Total Mark: 15

Name:

ID:

1. Find the net signed area and the total area between the curve $y = 16 - x^2$ and the x-axis on the interval $[-5, 5]$. [5]

2. Evaluate [5+5]

   (a) $\int 12z^{-2}e^{4+z^{-1}}dz$

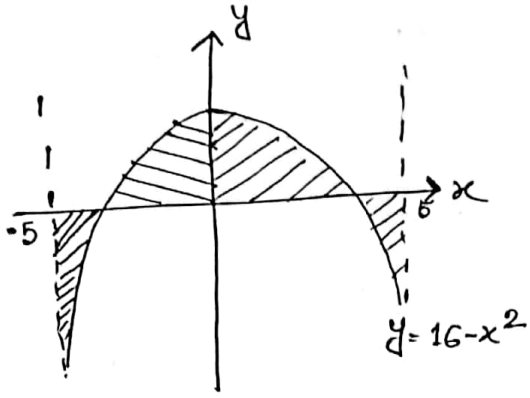
   (b) $\int -3\sin^{-1}(10x)dx$

[*Please start writing from here*]

Quiz 01 (Sec 06)

1. The given function is

$$y = 16 - x^2$$



Net signed Area

$$= \int_{-5}^{5} (16 - x^2)\, dx$$

$$= \left[16x - \frac{x^3}{3}\right]_{-5}^{5}$$

$$= \frac{230}{3} \text{ units}^2$$

The net signed area represents that the positive portion is more than the negative portion on that interval.

Total Area $= \int_{-5}^{5} |f(x)|\, dx$

$$= \int_{-5}^{5} |16 - x^2|\, dx$$

Here,

$$|16 - x^2| = \begin{cases} -(16-x^2), & -5 \le x \le -4 \\ 16 - x^2, & -4 \le x < 4 \\ -(16-x^2), & 4 \le x \le 5 \end{cases}$$

$$= \int_{-5}^{-4} -(16-x^2)\, dx + \int_{-4}^{4} (16-x^2)\, dx + \int_{4}^{5} -(16-x^2)\, dx$$

$$= \left[-16x + \frac{x^3}{3}\right]_{-5}^{-4} + \left[16x - \frac{x^3}{3}\right]_{-4}^{4} + \left[-16x + \frac{x^3}{3}\right]_{4}^{5} = 94 \text{ units}^2.$$

2. a) $\int 12z^{-2}e^{4+z^{-1}}dz$

Let,

$u = e^{4+z^{-1}}$

$= -\int 12\,du$

$\Rightarrow du = -z^{-2}e^{4+z^{-1}}dz$

$= -12\left[u\right] + c = -12e^{4+z^{-1}} + c$

Ans.

2. b) $\int -3\sin^{-1}(10x)\,dx$

$= \sin^{-1}(10x)\int(-3)dx - \int\left\{\frac{d}{dx}(\sin^{-1}10x)\int(-3)dx\right\}dx$

$= -3x\sin^{-1}(10x) - \int \frac{-3x\cdot 10}{\sqrt{1-100x^2}}\,dx$

Let,

$= -3x\sin^{-1}(10x) + 30\int\frac{x}{\sqrt{1-100x^2}}\,dx$

$u = 1-100x^2$

$\Rightarrow du = -200x\,dx$

$= -3x\sin^{-1}(10x) + \frac{30}{-200}\int\frac{du}{\sqrt{u}}$

$\Rightarrow \frac{du}{-200} = x\,dx$

$= -3x\sin^{-1}(10x) - \frac{30}{200}\left[\frac{u^{-1/2+1}}{-1/2+1}\right] + c$

$= -3x\sin^{-1}(10x) - \frac{30}{100}u^{\frac{1}{2}} + c$

$= -3x\sin^{-1}(10x) - \frac{30}{100}\sqrt{1-100x^2} + c$

Ans