Dynamic Linear Projection Model for Semantic Prompting

A Mathematical Framework for Semantic Structuring and Human-AI Dialogue

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Abstract

In modern information systems, natural language remains inherently ambiguous and unstructured, whereas AI systems require precise, structured input for optimal operation. To bridge this gap, the concept of **Semantic Prompting**—the progressive semantic structuring of natural language prompts into structured formats—has become increasingly significant. However, the mathematical foundation for such a process has been largely underdeveloped.

In this paper, we propose the **Dynamic Linear Projection Model (DLP-Model)**, a novel mathematical framework to formalize Semantic Prompting as a dynamic sequence of linear projections onto progressively evolving semantic subspaces, ultimately converging toward an Efficient Prompt Space characterized by structured representations such as JSON or XML.

We provide a theoretical framework for the iterative projection process, establish convergence conditions based on semantic shift magnitudes, and demonstrate the model's applicability through concrete examples. Furthermore, we discuss future extensions toward nonlinear projection models and semantic distance minimization, suggesting that the DLP-Model offers a foundational step toward more sophisticated human-AI dialogue and meaning co-creation processes.

1 Introduction

Natural language is widely utilized for information communication but inherently suffers from ambiguity and lack of structure. In contrast, AI systems demand highly structured and semantically coherent input to operate effectively.

Semantic Prompting refers to the process of transforming ambiguous natural language prompts into structured, machine-readable formats. Despite its practical importance, the mathematical modeling of Semantic Prompting remains underdeveloped.

Building upon the fundamental idea of linear projection, we propose the **Dynamic Linear Projection Model (DLP-Model)**. Our model extends static projection concepts into a dynamic and iterative formulation suitable for modeling the progressive semantic structuring required by Semantic Prompting.

This paper aims to establish a mathematical foundation for Semantic Prompting and position it within the broader context of human-AI meaning co-creation.

2 Related Work

Existing linear projection models treat information as vectors in a space V and structure it through projection onto a semantic subspace $W \subset V$. Properties such as idempotence ($P^2 = P$) and self-adjointness ($P = P^\top$) guarantee consistent mapping onto meaningful structures.

However, traditional models assume a one-shot static projection, insufficient for processes necessitating progressive semantic refinement.

Meanwhile, in the domain of prompt engineering, techniques such as converting natural language into structured formats like JSON and XML have emerged, but lack a rigorous mathematical framework.

Our study unifies these streams by proposing a dynamic, iterative extension of linear projection concepts tailored for Semantic Prompting.

3 Proposed Model: Dynamic Linear Projection Model (DLP-Model)

We define:

- V: the vector space representing unstructured prompts.
- $\{W_n\}$: a sequence of progressively refined semantic subspaces.
- $P_n : V \to W_n$: linear projections at each step.

The process is described by the iteration:

$$v_{n+1} = P_n(v_n) \tag{1}$$

where each v_n represents the state of the prompt after n refinements.

The ultimate goal is convergence to an Efficient Prompt Space $E \subset V$, characterized by machine-readable, structured formats.

4 Convergence Theory

We define the **semantic shift magnitude**:

$$\delta_n = \|v_{n+1} - v_n\| \tag{2}$$

We hypothesize exponential decay of semantic shift:

$$\delta_n \leq \delta_0 \rho^n \quad (0 < \rho < 1) \tag{3}$$

where:

- δ_0 is the initial semantic shift,
- ρ is the convergence rate.

We relax the requirement for strict convergence of subspaces. Instead, it suffices that the sequence $\{v_n\}$ converges in distance to E:

$$d(v_n, E) \to 0 \quad (n \to \infty) \tag{4}$$

where:

$$d(v, E) := \inf_{e \in E} \|v - e\| \tag{5}$$

5 Examples and Applications

5.1 Practical Example

Original Unstructured Prompt:

> Create a screen where users can search for products.

First refinement (Prompt Tuning): list screen elements.
Second refinement (Communication): convert into JSON.
Final structured prompt:

```
{
  "prompt_type": "screen_design",
  "elements": [
    {"type": "input", "name": "product_name"},
    {"type": "button", "label": "Search"}
  ]
}
```

5.2 Applications

- **Prompt Optimization**: Enhance AI consistency and response quality.
- **Semantic Space Design**: Tailor prompt structures for specific application domains.
- **Human-AI Dialogue Protocols**: Model meaning negotiation and alignment processes.

6 Discussion and Future Work

6.1 Strengths

- Dynamically models progressive semantic structuring.
- Provides mathematical criteria for iterative convergence.
- Bridges the gap between unstructured language and structured AI input.

6.2 Limitations

- Convergence is not guaranteed without appropriate projection design.
- Linearity assumptions limit the model's ability to capture complex semantic transformations.

6.3 Future Directions

- **Nonlinear Projection Models**: Incorporate complex and emergent semantic dynamics.
- **Semantic Distance Minimization**: Formulate prompt refinement as an energy minimization problem.
- **Human-AI Meaning Co-creation**: Model collaborative semantic evolution between humans and AI.

7 Conclusion

We have proposed the Dynamic Linear Projection Model (DLP-Model) to mathematically formalize Semantic Prompting as an iterative, dynamic process. Through structured convergence mechanisms, the DLP-Model connects natural language ambiguity and machine-readable clarity.

The DLP-Model not only advances Semantic Prompting, but also fundamentally redefines the pathway toward structured human-AI meaning negotiation and collaborative meaning construction in future information systems.

References

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