STAT 305: Chapter 5

Part II

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Discrete Random Variables

Meaning, Use, and Common Distributions

General Info About Discrete RVs

Reminder: What is a Random Variable?

Random Variables, we have already defined, take real-numbered ($\mathbb{R}$) values based on outcomes of a random experiment.

- If we know the outcome, we know the value of the random variable (so that isn't the random part).
- However, before we perform the experiment we do not know the outcome - we can only make statements about what the outcome is likely to be (i.e., we make "probabilistic" statements).
- In the same way, we do not know the value of the random variable before the experiment, but we can make probability statements about what value the RV might take.

General Info

Reminder: RVs

Discrete?

Terms & Notation

Common Terms and Notation for Discrete RVs

Of course, we can't introduce a *sort of* new concept without introducing a whole lot of new terminology.

We use capital letters to refer to discrete random variables: X, Y, Z, ...

We use lower case letters to refer to values the discrete RVs can take: x, x_1, y, z, ...

While we can use $P(X = x)$ to refer to the probability that the discrete random variable takes the value x, we usually use what we call the **probability function**:

- For a discrete random variable X, the probability function $f(x)$ takes the value $P(X = x)$
- In otherwords, we just write $f(x)$ instead of $P(X = x)$.

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Common Terms and Notation for Discrete RVs

We also have another function related to the probabilities, called the **cumulative probability function**.

For a discrete random variable X taking values $x_1, x_2, \ldots$ the CDF or **cumulative probability function** of X, $F(x)$, is defined as

$$F(x) = \sum_{z \leq x} f(z)$$

Which in other words means that for any value x,

$$f(x) = P(X = x)$$

and

$$F(x) = P(X \leq x)$$

Common Terms and Notation for Discrete RVs (cont)

The values that X can take and the probabilities attached to those values are called the **probability distribution** of X (since we are talking about how the total probability 1 gets spread out on (or distributed to) the values that X can take).

Example

Suppose that the we roll a die and let T be the number of dots facing up. Define the probability distribution of T. Find $f(3)$ and $F(6)$.

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Mean of a Discrete Random Variable

For a discrete random varable, X, which can take values $x_1, x_2, \ldots$ we define **the mean of X** (also known as **the expected value of X**) as:

$$E(X) = \sum_{i=1}^{n} x_i \cdot f(x_i)$$

We often use the symbol μ instead of $E(X)$.

Also, just to be confusing, you will often see EX instead of $E(X)$. Use context clues.

Example:

Suppose that the we roll a die and let T be the number of dots facing up. Find the expected value of T.

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Variance of a Discrete Random Variable

For a discrete random varable, X, which can take values $x_1, x_2, \ldots$ and has mean μ we define **the variance of X** as:

$$Var(X) = \sum_{i=1}^{n} (x_i - \mu)^2 \cdot f(x_i)$$

There are other usefule ways to write this, most importantly:

$$Var(X) = \sum_{i=1}^{n} x_i^2 \cdot f(x_i) - \mu^2$$

Example:

Suppose that the we roll a die and let T be the number of dots facing up. What is the variance of T?

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Summary

Discrete Random Variables

- Discrete RVs are RVs that will take one of a countable set of values.
- When working with a discrete random variable, it is common to need or use the RV's
  - probability distribution: the values the RV can take and their probabilities
  - probability function: a function where $f(x) = P(X = x)$
  - cumulative probability function: a function where $F(x) = P(X \leq x)$.
  - mean: a value for X defined by $EX = \sum_x x \cdot f(x)$
  - variance: a value for X defined by $VarX = \sum_x (x - \mu)^2 \cdot f(x)$

Common Distributions

Working with Off The Shelf Random Variables

General Info

Common Distributions

Background

Common Distributions

Why Are Some Distributions Worth Naming?

Even though you may create a random variable in a unique scenario, the way that it's probability distribution behaves (mathematically) may have a lot in common with other random variables in other scenarios. For instance,

> I roll a die until I see a 6 appear and then stop. I call X the number of times I have to roll the die in total.
>
> I flip a coin until I see heads appear and then stop. I call Y the number of times I have to flip the coin in total.
>
> I apply for home loans until I get accepted and then I stop. I call Z the number of times I have to apply for a loan in total.

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Why Are Some Distributions Worth Naming? (cont)

In each ot the above cases, we count the number of times we have to do some action until we see some specific result. The only thing that really changes from the random variables perspective is the likelyhood that we see the specific result each time we try.

Mathematically, that's not a lot of difference. And if we can really understand the probability behavior of one of these scenarios then we can move our understanding to the different scenario pretty easily.

By recognizing the commonality between these scenarios, we have been able to identify many random variables that behave very similarly. We describe the similarity in the way the random variables behave by saying that they have a common/shared distribution.

We study the most useful ones by themselves.

General Info

Common Distributions

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Bernoulli

The Bernoulli Distribution

Origin: A random experiment is performed that results in one of two possible outcomes: successful or failure. The probability of a successful outcome is p.

Definition: X takes the value 1 if the outcome is a success. X takes the value 0 if the outcome is a failure.

probability function:

$$f(x) = \begin{cases} p & x = 1, \\ 1-p & x = 0, \\ 0 & o.w. \end{cases}$$

which can also be written as

$$f(x) = \begin{cases} p^x(1-p)^{1-x} & x = 0, 1 \\ 0 & o.w. \end{cases}$$

The Bernoulli Distribution

Expected value: $E(X) = p$

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Bernoulli

The Bernoulli Distribution

Variance: $Var(X) = (1-p) \cdot p$