Show **all** of your work on this assignment and answer each question fully in the given context.

Please staple your assignment!

1. **Ch. 5, Exercise 7, pg. 323:** In a grinding operation, there is an upper specification of 3.150 in. on a dimensions of a certain part after grinding. Suppose that the standard deviation of this normally distributed dimension for parts of this type ground to any particular mean dimension μ is $\sigma = 0.002$ in. Suppose further that you desire to have no more than 3% of the parts fail to meet specifications. What is the maximum (minimum machining cost) μ that can be used if this 3% requirement is to be met?[10 pts]
   Hint: the question is giving you information on $P(X > 3.15) \leq 0.03$.

2. **Ch 5, Exercise 42, pg. 332:** Suppose that engineering specifications on the shelf depth of a certain slug to be turned on a CNC lathe are from 0.0275 in. to 0.0278 in. and that values of this dimension produced on the lathe can be described using a normal distribution with mean μ and standard deviation σ.

   (a) If $\mu = 0.0276$ and $\sigma = 0.0001$, about what fraction of shelf depths are in specifications?[10 pts]

   (b) What machine precision (as measured by σ) would be required in order to produce about 98% of shelf depths within engineering specifications (assuming that μ is at the midpoint of the specifications)?[10 pts]
   Hint: you are looking for the value of σ in this question.

3. **Ch 5.4, Exercise 2, pg. 300:** Quality audit records are kept on numbers of major and minor failures of circuit packs during burn-in of large electronic switching devices. They indicate that for a device of this type, the random variables

$$X = \text{ the number of major failures}$$

and

$$Y = \text{ the number of minor failures}$$

can be described at least approximately by the accompanying joint distribution.

Y | X	**0**	**1**	**2**
0	0.15	0.05	0.01
1	0.1	0.08	0.01
2	0.1	0.14	0.02
3	0.1	0.08	0.03
4	0.05	0.05	0.03

   (a) Find the marginal probability functions for both X and Y ($f_x(x)$ and $f_y(y)$, respectively).[10 pts]

   (b) Are X and Y independent? Explain.[5pts]

   (c) Find the mean and variance of X (EX and Var$X$)[10 pts]

   (d) Find the mean and variance of Y (EY and Var$Y$)[10 pts]

(e) Find the conditional probability function for Y, given that $X = 0$ – i.e., that there are no major circuit pack failures ($f_{Y|X}(y|0)$). What is the mean of this conditional distribution?[10 pts]

4. **Ch. 5.2, Exercise 3, pg. 263:** Suppose that X is a normal random variable with mean 43 and standard deviation 3.6. Evaluate the following probabilities involving X:

   (a) $P[X < 45.2]$ [5 pts]

   (b) $P[|X - 43| \leq 2]$ [5 pts]

   (c) $P[|X - 43| > 1.7]$ [5 pts]

   Now find numbers # such that the following statements involving X are true:

   (d) $P[X < \#] = .95$ [5 pts]

   (e) $P[|X - 43| > \#] = .05$ [5 pts]

   Total: 90 pts

Key :

1/

$$P(X > 3.15) \leq 0.03 \quad , \quad X \sim N(\mu, \sigma = 0.002)$$

$$\Rightarrow P\left(\frac{X-\mu}{\sigma} > \frac{3.15-\mu}{\sigma}\right) \leq 0.03$$

$$\Rightarrow P\left(\frac{X-\mu}{0.002} > \frac{3.15-\mu}{0.002}\right) \leq 0.03$$

$$\Rightarrow P\left(Z > \frac{3.15-\mu}{0.002}\right) \leq 0.03$$

$$\Rightarrow 1 - P\left(Z < \frac{3.15-\mu}{0.002}\right) \leq 0.03$$

$$\Rightarrow P\left(Z < \frac{3.15-\mu}{0.002}\right) \geq 1 - 0.03$$

$$\Rightarrow \Phi\left(\frac{3.15-\mu}{0.002}\right) \geq 0.97$$

by the table, $\dfrac{3.15-\mu}{0.002} \geq 1.88$

$$\Rightarrow \mu < 3.146$$

The maximum μ can be used

2, a) $X \sim N(\mu = 0.0276, \sigma = 0.0001)$

$P(0.0275 \leq X \leq 0.0278) = ?$

$$\Rightarrow P\left(\frac{0.0275 - 0.0276}{0.0001} \leq Z < \frac{0.0278 - 0.0276}{0.0001}\right)$$

$$= P(-1 \leq Z \leq 2) = P(Z \leq 2) - P(Z \leq -1)$$

$$= \Phi(2) - \Phi(1)$$

by the table

$$= 0.9773 - 0.1587$$

$$\boxed{= 0.8186}$$

(81.86% of the shelf depth are between 0.0275 in and 0.0278 in.

b)

μ: the midpoint of the specification

$$\Rightarrow \mu = \frac{0.0275 + 0.0278}{2} = 0.02765$$

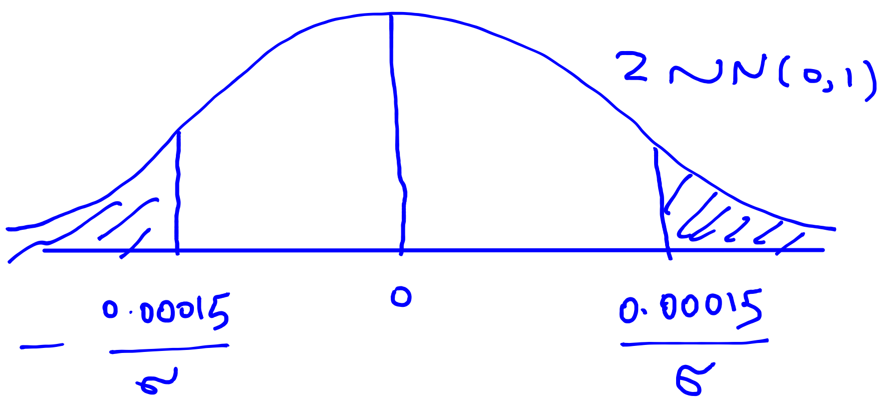
$$\Rightarrow X \sim N(\mu = 0.02765, \sigma)$$

$$\Rightarrow P(0.0275 \le X \le 0.0278) = 0.98$$

$$\Rightarrow P\left(\frac{0.0275-0.02765}{\sigma} \le Z \le \frac{0.0278-0.02765}{\sigma}\right) = 0.98$$

$$\Rightarrow P\left(-\frac{0.00015}{\sigma} \le Z \le \frac{0.00015}{\sigma}\right) = 0.98$$

$$\Rightarrow P\left(Z \le \frac{0.00015}{\sigma}\right) - P\left(Z \le -\frac{0.00015}{\sigma}\right) = 0.98$$



$$\Rightarrow 1 - 2P\left(Z \le -\frac{0.00015}{\sigma}\right) = 0.98$$

$$\Phi\left(-\frac{0.00015}{\sigma}\right) = 0.01$$

by the table : $\frac{-0.00015}{\sigma} \approx -2.32 \Rightarrow \boxed{\sigma \approx 0.000064}$

3,

(a)

y	$f_y(y)$
0	0.21
1	0.19
2	0.26
3	0.21
4	0.13

X	$f_X(x)$
0	0.5
1	0.4
2	0.1

(b) No, for example

$f_X(0) = 0.5$

$f_Y(0) = 0.21$

$f_{X,Y}(x=0, y=0) = 0.15$

$\Rightarrow f_{X,Y}(0,0) = 0.15 \neq f_X(0)\, f_Y(0) = (0.5)(0.21)$

(c) $EX = \sum_{x=0}^{2} x f_X(x) = 0(0.5) + 1(0.4) + 2(0.1) = \boxed{0.6}$

$Ex^2 = \sum_{x=0}^{2} x^2 f_X(x) = 0^2(0.5) + 1^2(0.4) + 2^2(0.1) = \boxed{0.8}$

$\text{Var}\, x = Ex^2 - (Ex)^2 = 0.8 - (0.6)^2 = 0.764$

(d) $EY = \sum_{y=0}^{4} y\, f_Y(y) = 0(0.21) + 1(0.19) + 2(0.26) + 3(0.21) + 4(0.13) = \boxed{1.86}$

$EY^2 = \sum_{y=0}^{4} y^2 f_Y(y) = 0^2(0.21) + 1^2(0.19) + 2^2(0.26) + 3^2(0.21) + 4^2(0.13) = \boxed{5.2}$

$\text{var}\,Y = EY^2 - (EY)^2 = 5.2 - (1.86)^2 = \boxed{1.7404}$

(e)

$$f_{Y|X}(0|0) = \frac{f(0,0)}{f_X(0)} = \frac{0.15}{0.5} = 0.3$$

$$f_{Y|X}(1|0) = \frac{f(0,1)}{f_X(0)} = \frac{0.1}{0.5} = 0.2$$

$$f_{Y|X}(2|0) = \frac{f(0,2)}{f_X(0)} = \frac{0.1}{0.5} = 0.2$$

$$f_{Y|X}(3|0) = \frac{f(0,3)}{f_X(0)} = \frac{0.1}{0.5} = 0.2$$

$$f_{Y|X}(4|0) = \frac{f(0,4)}{f_X(0)} = \frac{0.05}{0.5} = 0.1$$

and in general we can write:

y	0	1	2	3	4
$f_{Y\|X}(y\|0)$	0.3	0.2	0.2	0.2	0.1

4/ $X \sim N(43, 3.6)$

a) $P(X < 45.2) = P\left(Z < \frac{45.2 - 43}{3.6}\right)$

$= P\left(Z < \frac{2.2}{3.6}\right)$

$= \Phi(0.611)$

$= \underline{0.7294}$

b) $P(|X - 43| < 2) = P(-2 < X - 43 < 2)$

$= P\left(\frac{-2}{3.6} < \frac{X - 43}{3.6} < \frac{2}{3.6}\right)$

$= P(-0.555 < Z < 0.555)$

$= 2P(Z \leq -0.555)$

$= 2(0.2894)$

$= 0.5788$

c) $P(|X - 43| > 1.7) = P(X - 43 > 1.7$ or $X - 43 < -1.7)$

$= P(X - 43 > 1.7) + P(X - 43 < -1.7)$

$$= P\left(\frac{X-43}{3.6} > \frac{1.7}{3.6}\right) + P\left(\frac{X-43}{3.6} < \frac{-1.7}{3.6}\right)$$

$$= P(Z > 0.47) + P(Z < -0.47)$$

$$= 1 - P(Z < 0.47) + P(Z < -0.47)$$

$$= 1 - \Phi(0.47) + \Phi(-0.47)$$

$$= 1 - 0.6808 + 0.3191$$

$$= \boxed{0.6383}$$

(d) $P(X \leq \#) = 0.95$

$$\Rightarrow 0.95 = P\left(\frac{X-43}{3.6} \leq \frac{\#-43}{3.6}\right)$$

$$= P\left(Z \leq \frac{\#-43}{3.6}\right) = \Phi\left(\frac{\#-43}{3.6}\right)$$

by the table $\Rightarrow$

$$\frac{\#-43}{3.6} \simeq 1.645$$

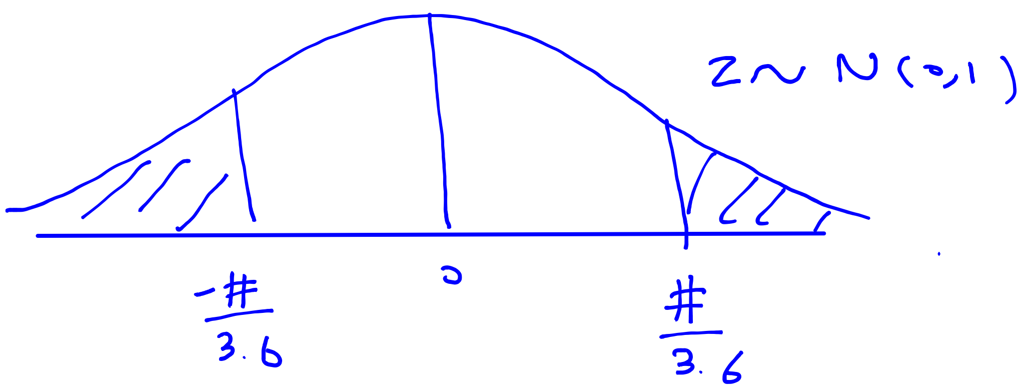
$$\Rightarrow \boxed{\# = 48.922}$$

(e) $0.05 = P(|X-43| > \#)$

$= P(X-43 > \#) + P(X-43 \leq -\#)$

$= P\left(\frac{X-43}{3.6} > \frac{\#}{3.6}\right) + P\left(\frac{X-43}{3.6} \leq \frac{-\#}{3.6}\right)$

$= P\left(Z > \frac{\#}{3.6}\right) + P\left(Z \leq \frac{-\#}{3.6}\right)$



$\Rightarrow 2P\left(Z < \frac{-\#}{3.6}\right) = 0.05$

$P\left(Z < \frac{-\#}{3.6}\right) = 0.025$

$\Rightarrow \frac{-\#}{3.6} = -1.96 \Rightarrow \# = 7.056$