Show **all** of your work on this assignment and answer each question fully in the given context. You have 20 minutes. Each problem is designed to take 10 minutes. All answers in a topic must be correct for any credit for that topic. You may attempt multiple topics. You may use a calculator on this competency quiz.

1. **Competency Topic: Discrete Random Variables**

   Let X be a random variable with the following probability function:

$$f(x) = \begin{cases} 0.1x^2 & x = -2, -1, 0, 1, 2 \\ 0 & o.w. \end{cases}$$

   a. Find the probability that $X \geq 1$.

   b. Find the value of $E(X)$.

   c. Find the variance of X.

2. **Competency Topic: Continuous Random Variables**

   Let X be a random variable that follows an exponential distribution with mean $\alpha = 2$. In other words,

$$f(x) = \begin{cases} 0.5\exp(-0.5x) & x \geq 0 \\ 0 & otherwise \end{cases}$$

   a. Plot the probability density function, $f(x)$.

   b. Find the cumulative density function, $F(x)$.

   c. Find $P(1 \leq X \leq 3)$.

3. **Competency Topic: Joint Distributions**

Let X be has an exponential distribution with rate $\lambda > 0$ so that

$$f_X(x) = \begin{cases} \lambda \exp(-\lambda x) & x \geq 0 \\ 0 & o.w. \end{cases}$$

Additionally, suppose that the value of X acts as the rate of a second exponential random variable, Y. In other words,

$$f_{Y|X}(y|x) = \begin{cases} x \exp(-xy) & y \geq 0 \\ 0 & o.w. \end{cases}$$

a. Sketch the pdf of Y given that $X = 2$.

b. Find the joint probability density function of X and Y, $f_{XY}(x, y)$.

4. **Competency Topic: Functions of Random Variables**

Suppose that X_1, X_2, X_3, X_4, X_5 and Y_1, Y_2, Y_3, Y_4, Y_5 are all independent random variables where for any i

$$E(X_i) = \mu_X$$
$$Var(X_i) = \sigma_X^2$$
$$E(Y_i) = \mu_Y$$
$$Var(Y_i) = \sigma_Y^2$$

Suppose that we define a random variable U to help compare the values taken by X_is and the values taken by the Y_is by pairing the random variables like this:

$$U = \frac{1}{5}(X_1 - Y_1) + \frac{1}{5}(X_2 - Y_2) + \frac{1}{5}(X_3 - Y_3) + \frac{1}{5}(X_4 - Y_4) + \frac{1}{5}(X_5 - Y_5)$$

a. Find the mean of U (*hint: it will include* μ_X *and* μ_Y.)

b. Find the standard deviation of U (*hint: it will include* σ_X^2 *and* σ_Y^2)