Quiz II

STAT 305, Section 3
FALL 2019

Instructions

- The quiz is scheduled for 80 minutes, from 09:30 to 10:50 AM. At 10:50 AM the exam will end.
- Total points for the exam is 60. Points for individual questions are given at the beginning of each problem. Show all your calculations clearly to get full credit. Put final answers in the box at the right (except for the diagrams!).
- A forumula sheet is attached to the end of the exam. Feel free to tear it off.
- Normal quantile table is attached to the end of the exam. Feel free to tear it off.
- You may use a calculator during this exam.
- Answer the questions in the space provided. If you run out of room, continue on the back of the page.
- If you have any questions about, or need clarification on the meaning of an item on this exam, please ask your instructor. No other form of external help is permitted attempting to receive help or provide help to others will be considered cheating.
- **Do not cheat on this exam.** Academic integrity demands an honest and fair testing environment. Cheating will not be tolerated and will result in an immediate score of 0 on the exam and an incident report will be submitted to the dean's office.

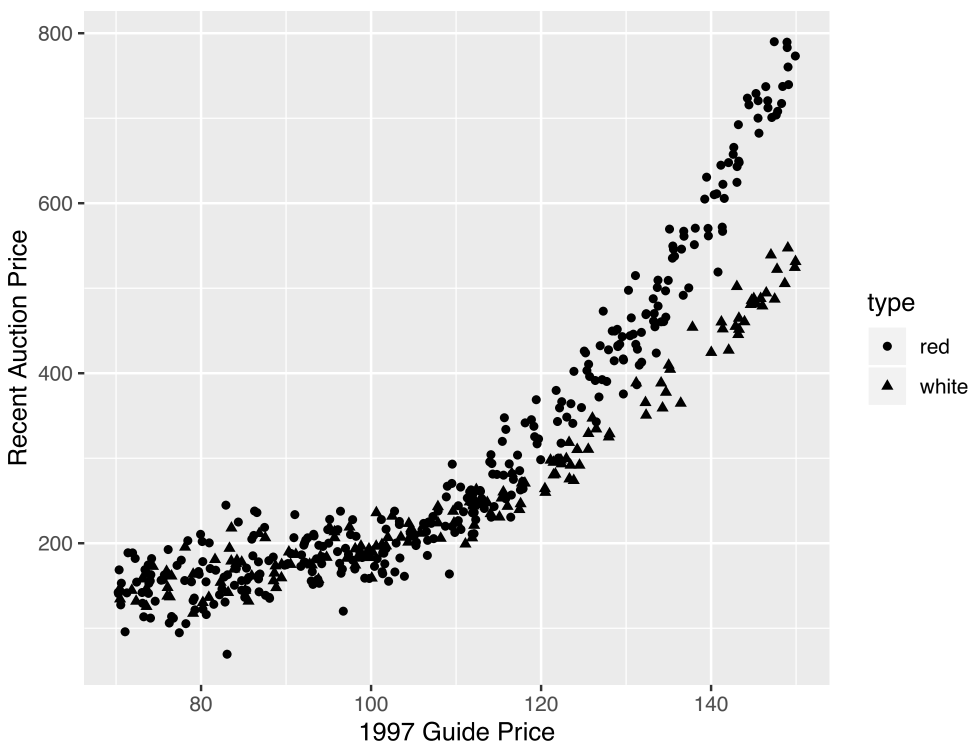
Name: Key

Student ID: ______________________

1. **Champions Red Wine**

   Oenophiles, or connoisseurs of fine wines, have been benefitted by sort of open information sharing we have in the days of online auctions. By monitoring the prices for which sought after bottles actually sell for, wine enthusiasts are able to both determine whether or not they are getting a fair price and whether or not they might make a nice profit by selling off a few of the bottles they have in storage. Before this era of easy and free data though, there was one source of information that stood above all the others: *Le Champions de vin rouge*, a guide to the world of fine wines published in 1997. While the information contained it in is largely still relevant in general, the price index has become somewhat outdated.

   Or has it? Is it possible that the prices from the guide in 1997 could help us understand the market today? A certain statistician decided to do a deep analysis of the relationship between the cost of wine bottles in the 1997 book and the actual prices the same bottles fetched on open auction sites recently. The dataset he created consists of the last 500 bottles with publically available auction prices that were also listed in *Le Champions de vin rouge* price index.



Here are some summaries of the dataset with the 1997 Guide Price as x and the Recent Auction Price as y.

$$\sum_{i=1}^{500} x_i = 54348 \qquad \sum_{i=1}^{500} x_i^2 = 6164828$$

$$\sum_{i=1}^{500} y_i = 146782 \qquad \sum_{i=1}^{500} y_i^2 = 55928348$$

$$\sum_{i=1}^{500} x_i y_i = 17573001$$

(a) Using the summaries, fit a linear relationship between **1997 Guide Price** (x) and **Recent Auction Price** (y).

i. (5 points) Write the equation of the fitted linear relationship.

To write down the fitted relationship, we need to find b_0, b_1. Then $\hat{y} = b_0 + b_1 X$.

$$b_1 = \frac{\sum x_i y_i - n\bar{x}\bar{y}}{\sum x_i^2 - n\bar{x}^2} = \frac{17573001 - 500\left(\frac{54348}{500}\right)\left(\frac{146782}{500}\right)}{\left(6164828 - 500\left(\frac{54348}{500}\right)^2\right)} = 6.28$$

$$b_0 = \bar{y} - b_1\bar{x} = \frac{146782}{500} - 6.28\left(\frac{54348}{500}\right) = -389.0469$$

Fitted relationship: $\hat{y} = -389.0469 + 6.28X$

ii. (5 points) Find and interpret the value of R^2 for the fitted linear relationship.

First find sample correlation (r) & then $R^2 = (r)^2$.

$$r = \frac{\sum x_i y_i - n\bar{x}\bar{y}}{\sqrt{\sum x_i^2 - n\bar{x}^2}\sqrt{\sum y_i^2 - n\bar{y}^2}} = \frac{17573001 - 500\left(\frac{54348}{500}\right)\left(\frac{146782}{500}\right)}{\sqrt{6164828 - 500\left(\frac{54348}{500}\right)^2}\sqrt{55928348 - 500\left(\frac{146782}{500}\right)^2}} = 0.89 \Rightarrow R^2 = (0.89)^2 = 0.79$$

79% of the variation in the response (auction price) can be explained by the linear relationship between the auction price and the 1997 Guide Price.

iii. (5 points) Using the fitted line, provide a predicted Recent Auction Price when the 1997 Guide Price was \$120.

$$\hat{y} = -389.0469 + 6.28(120)$$

$$= \$364.6$$

(b) The JMP output below comes from fitting a quadratic model using x and x^2 for *Red Wine Only* (left) and *White Wine Only* (right).

i. (5 points) Write the equation of the fitted quadratic relationship *for Red Wines* i.e the polynomial to degree two.

$$\hat{y} = 1165.806 - 24.00553(\text{Redwine}) + 0.142(\text{Red wine})^2$$

Response auction type=red

Effect Summary

Lack Of Fit

Summary of Fit

RSquare	0.966845
RSquare Adj	0.96665
Root Mean Square Error	32.17645
Mean of Response	307.4038
Observations (or Sum Wgts)	342

Analysis of Variance

	Source	DF	Sum of Squares	Mean Square	F Ratio
SSR	Model	2	10234985	5117493	4942.891
SSE	Error	339	350975	1035	Prob > F
SST	C. Total	341	10585960		<.0001*

Parameter Estimates

	Term	Estimate	Std Error	t Ratio	Prob>|t|
b_0	Intercept	1165.806	43.53581	26.78	<.0001*
(Red wine)^2	guide_sq	0.1421051	0.003729	38.11	<.0001*
(Red wine)	guide	-24.00553	0.81914	-29.31	<.0001*

() Red wine

Response auction type=white

Effect Summary

Lack Of Fit

Summary of Fit

RSquare	0.972108
RSquare Adj	0.971748
Root Mean Square Error	19.18487
Mean of Response	263.6046
Observations (or Sum Wgts)	158

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	2	1988322.8	994161	2701.091
Error	155	57049.2	368	Prob > F
C. Total	157	2045372.0		<.0001*

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	591.88351	36.65328	16.15	<.0001*
guide_sq	0.0713926	0.003075	23.22	<.0001*
guide	-11.13206	0.682587	-16.31	<.0001*

() White wine

Figure 1: JMP output of fitting a quadratic model using x and x^2 for *Red Wine Only* (left) and *White Wine Only* (right).

ii. (5 points) Write the equation of the fitted quadratic relationship *for White Wines* i.e the polynomial to degree two. .

$\hat{y} = 591.88351 - 11.13206$ (white wine) $+ 0.0713$ (white wine)^2

iii. (5 points) Interpret the value of R^2 for the two fitted quadratic relationships.

- 96% of the variation in the auction price (response) can be explained by the "quadratic" relationship of the response and red wine price.
- 97% of the variation in the auction price (response) can be explained by the "quadratic" relationship between the auction price & white wine price.

iv. (5 points) Using the fitted quadratic relationship, provide a predicted value of Recent Auction Price for a red wine with 1997 Guide Price of $120. 4cm]

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$\hat{y} = 1165.806 + 0.142(120) - 24(120)^2$

$= -345600$

Note that the result is a generated data set, and achieving a negative value for the recent auction price is acceptable in this case.

2. Suppose X is a discrete random variable with following probability function:

$$f(x) = \begin{cases} 0.1 & x = -2, 0, 2 \\ 0.35 & x = -1, 1 \\ 0 & o.w. \end{cases}$$

i. (2 points) Find $P(X = 0)$

$P(X=0) = 0.1$

ii. (2 points) Find $P(X \leq 0)$

$P(X \leq 0) = P(X = 0 \text{ or } X = -1 \text{ or } X = -2) = P(X=0) + P(X=-1) + P(X=-2)$
$= 0.1 + 0.35 + 0.1 = 0.55$

iii. (2 points) Find $P(|X| > 1)$

$P(|X| > 1) = P(X = -2 \text{ or } X = 2) = P(X=-2) + P(X=2)$
$= 0.1 + 0.1 = 0.2$

iv. (3 points) Find the CDF of X.

X	-2	-1	0	1	2
f(x)	0.1	0.35	0.1	0.35	0.1
F(x)	0.1	0.45	0.55	0.9	$\boxed{1}$

v. (3 points) Find the expected value of X.

$EX = \sum_{x \in \{-2,-1,0,1,2\}} x f(x) = -2f(-2) + (-1)f(-1) + (0)f(0) + (1)f(1) + (2)f(2)$
$= -2(.1) - 1(.35) + 0 + 0.35 + 2(.1)$
$= 0$

vi. (3 points) Find the variance of X.

you can use any formulas.

$Var(X) = \sum (x_i - E(X))^2 f(x) = (-2-0)^2 f(-2) + (-1-0)^2 f(-1) + (0-0)^2 f(0) + (1-0)^2 f(1) + (2-0)^2 f(2)$
$= 4(0.1) + (1)(0.35) + 0 + 1(0.35) + 4(0.1)$
$= 1.5$

3. Suppose a standup comedian plans to give a total of $n = 5$ jokes in an entire 2-hour performance. Call a joke a succes if at least one audience member laughs. If no audience member laughs, the joke is a failure. Assume that all the jokes are equally funny, with $p = p(success) = 0.2$. Let X be the random variable that denotes the number of jokes out of the total 5 were successes.

   i. (3 points) Precisely state the distribution of X, giving the values of any parameters necessary.

   $X \sim \text{binomial}(n=5, p=0.2)$

   ii. (3 points) Calculate the probability that the whole night is a failure: i.e., $P(no\ laughs)$

   $P(X=0) = \binom{5}{0}(0.2)^0(1-0.2)^{5-0}$

   $= (0.8)^5$

   iii. (3 points) Caculate the probability that the comedian tells at least 4 successful jokes.

   $P(X \geq 4) = P(X=4) + P(X=5)$

   $= \binom{5}{4}(0.2)^4(0.8)^1 + \binom{5}{5}(0.2)^5(0.8)^0$

   $= 0.0064 + 0.00032 = 0.00672$

   iv. (3 points) Calculate the expected number of successfull jokes.

   In Binomial distribution:

   $E(X) = n.p = 5(0.2) = 1$

   (This means it is expected that one of his jokes is successful

   v. (3 points) Calculate the standard deviation of X

in Binomial: $Var(X) = nP(1-P) = 5(0.2)(1-0.2) = 0.8$

$SD(X) = \sqrt{Var(X)} = \sqrt{0.8} = 0.89$