Show **all** of your work on this assignment and answer each question fully in the given context.

Please staple your assignment!

1. Suppose that X is a random variable with probability density function

$$f(x) = \begin{cases} c \cdot x^2 & -2 \le x \le 2 \\ 0 & o.w. \end{cases}$$

   (a) Find the value of c that makes $f(x)$ a valid probability density function.[5 pts]
   (b) Find the CDF of the random variable X.[5 pts]
   (c) What is $P(|X| \ge -1)$ [5 pts].
   (d) Find the expected value of X.[5 pts]

2. Consider a continuously distributed random variable, W, with a probability density function given by

$$f(w) = \begin{cases} \frac{1}{5(1-e^{-2})}e^{-w/5} & 0 \le w \le 10 \\ 0 & \text{otherwise} \end{cases}$$

   (a) Show that the function $f(w)$ is a valid probability density function (i.e., show that (i) $f(w)$ is non-negative and (ii) $\int_{-\infty}^{\infty} f(w)dw = 1$). [5 pts]
   (b) Find $P(W \le 2)$ [5 pts]
   (c) Find $P(2 \le W \le 10)$ [5 pts]
   (d) Find $E(X)$ [5 pts]

3. The mileage to first failure for a model of military personnel carrier can be modeled as exponential with mean $1,000$ miles.

   (a) Find the probability that a vehicle of this type gives less than 500 miles of service before first failure.[5 pts]
   (b) Find the probability that a vehicle of this type gives less than 2000 miles of service before first failure. [5 pts]

4. (Ch. 5.2, Exercise 2, pg. 263) Suppose that Z is a standard normal random variable. Evaluate the following probabilities involving Z:

   (a) $P[Z < -.62]$ [3 pts]
   (b) $P[Z > 1.06]$ [3 pts]
   (c) $P[-.37 < Z < .51]$ [3 pts]
   (d) $P[|Z| \le .47]$ [3 pts]
   (e) $P[|Z| > .93]$ [3 pts]
   (f) $P[-3.0 < Z < 3.0]$ [3 pts]

   Now find numbers # such that the following statements involving Z are true:

   (a) $P[Z \le \#] = .90$ [3 pts]
   (b) $P[|Z| \le \#] = .90$ [3 pts]
   (c) $P[|Z| > \#] = .03$ [3 pts]

Total: 77 pts