Show **all** of your work on this assignment and answer each question fully in the given context. You have 20 minutes. Each problem is designed to take 10 minutes. All answers in a topic must be correct for any credit for that topic. You may attempt multiple topics. You may use a calculator on this competency quiz.

1. **Competency Topic: Discrete Random Variables**

   Suppose that X is a discrete random variable with a geometric distribution with probability of success p. That is, X has a probability function that is written as

$$f(x) = \begin{cases} (1-p)^{x-1}(p) & x = 1, 2, ... \\ 0 & o.w \end{cases}$$

   The mean and variance of a geometric random variable are based on the value of p so that $E(X) = \frac{1}{p}$ and $Var(X) = \frac{1-p}{p}$.

   a. Find the probability that $X \geq 2$ (hints: you do not need to do an infinite sum; the answer may include the term p).

   b. For any $x \geq 1$ and positive integer k, find an expression for the ratio $f(x+k)/f(x)$.

   c. Suppose that a particular geometric random variable has mean 10. Find the variance.

2. **Competency Topic: Continuous Random Variables**

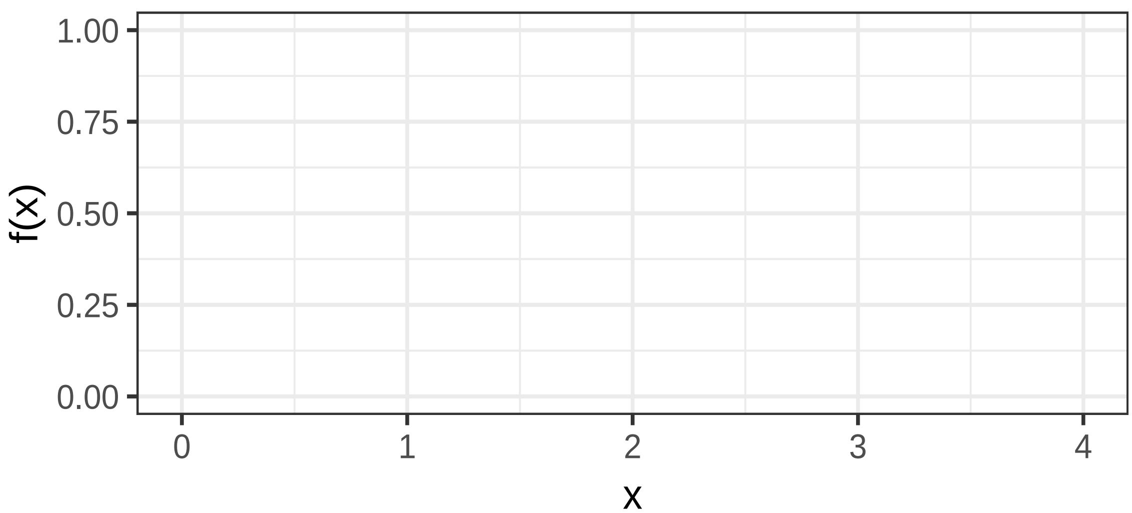
A gamma random variable depends on two parameters, a "rate" paramter k and a "scale" parameter θ. In the case where the shape parameter k is a positive integer, the probability density function can be written as:

$$f(x) = \begin{cases} \frac{1}{(k-1)!\theta^k} x^{k-1} \exp\left(-\frac{x}{\theta}\right) & x \geq 0 \\ 0 & o.w. \end{cases}$$

a. Using the plot below, provide a rough sketch of the following pdfs:

- (1) the pdf of a gamma random variable with $k = 3$ and $\theta = 1$ and
- (2) the pdf of a gamma random variable with $k = 1$ and $\theta = 1$

(for the sketch, it is enough to find and connect the values of the pdf at $x = 0, 1, 2, 3, 4$)



b. Based on your sketch, which of the two values of k will give the largest probability that $X \leq 1$?

3. **Competency Topic: Joint Distributions**

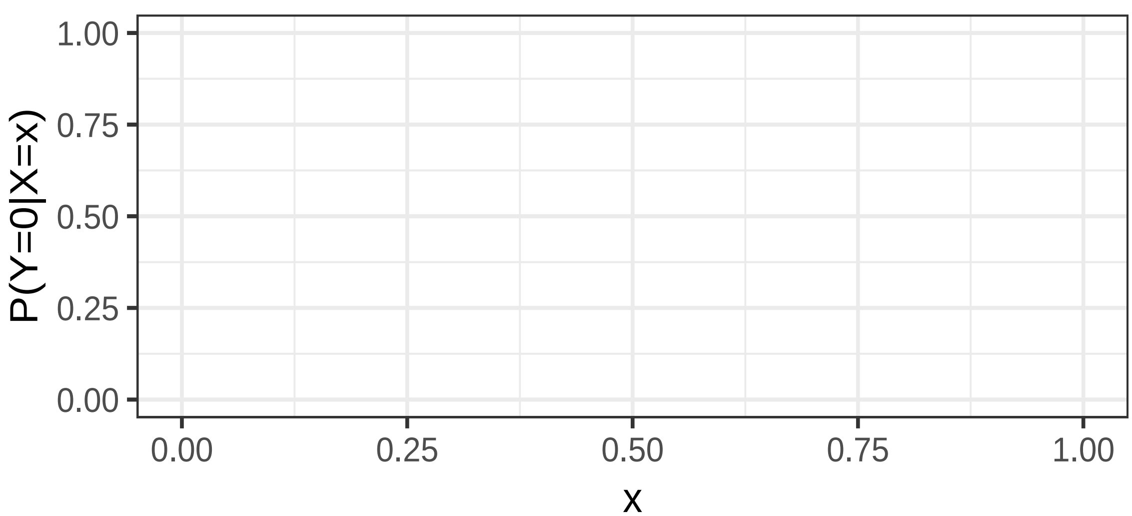
Suppose that X follows a uniform distribution on $[0, 1]$ - that is, X has a density function such that $f_X(x) = 1$ for $0 \leq x \leq 1$ and $f_X(x) = 0$ for any other value of x. Also suppose that the joint distribution of X and a second random variable Y can be written as:

$$f_{XY}(x, y) = \begin{cases} \frac{4!}{y!(4-y)!} x^y (1-x)^{4-y} & 0 \leq x \leq 1, y = 0, 1, 2, 3, 4 \\ 0 & otherwise \end{cases}$$

a. Find the conditional probability density function of Y given X, $f_{Y|X}(y|x)$.

b. Find the probability that $Y = 0$ given that $X = 0.1$.

c. Sketch the function $f_{Y|X}(0|x)$ as a function of x (in otherwords, a sketch of how the probability that $Y = 0$ changes as the value of X we are given changes)



4. **Competency Topic: Functions of Random Variables**

Suppose that X and Y are independent discrete random variables where, for $\lambda > 0$ and $\alpha > 0$, the joint probability function can be written as:

$$P(X = x, Y = y) = \begin{cases} \frac{\lambda^x}{x!} \exp\left(-\lambda\right) \frac{\alpha^y}{y!} \exp\left(-\alpha\right) & x = 0, 1, 2, ...; y = 0, 1, 2, ... \\ 0 & o.w. \end{cases}$$

It can be show that $E(X) = \lambda$ and $Var(X) = \lambda$ and that $E(Y) = \alpha$ and $Var(Y) = \alpha$.

Define $U = X - Y$.

a. Find $E(U)$ and $Var(U)$.

b. Find the probability that $U = 1$ (hints: consider the possible values of X and Y that could lead to $U = 0$; also, your answer will include λ and α)