Show **all** of your work on this assignment and answer each question fully in the given context. You have 20 minutes. Each problem is designed to take 10 minutes. All answers in a topic must be correct for any credit for that topic. You may attempt multiple topics. You may use a calculator on this competency quiz.

3. **Competency Topic: Joint Distributions**

   Let $X$ be uniform on $(0, 2)$, i.e., $X$ has pdf

$$f_X(x) = \begin{cases} 0.5 & 0 \leq x \leq 2 \\ 0 & o.w. \end{cases}$$

   Also, suppose that $Y$ has a uniform distribution that depends on $X$ so that

$$f_{Y|X}(y|x) = \begin{cases} \frac{1}{2x} & -x \leq y \leq x \\ 0 & o.w. \end{cases}$$

   a. Sketch the pdf of $Y$ given that $X = 2$.

   b. Find the joint probability density function of $X$ and $Y$, $f_{XY}(x, y)$

4. **Competency Topic: Functions of Random Variables**

   Suppose that $X$ has an exponential distribution with mean $\alpha = 3$. In other words, $f_X(x) = \frac{1}{3}\exp\left(\frac{x}{3}\right)$ for $x \geq 0$ and is 0 everywhere else. Let $Y$ be a random variable defined by $Y = X^2$.

   a. Find the probability that $Y \leq 4$. (hint: you may want the cdf of $X$)

   b. Find the probability density function of $Y$.