

Show **all** of your work on this assignment and answer each question fully in the given context.

Please staple your assignment!

1. Ch. 5.1, Exercise 1, pg. 243 A discrete random variable  $X$  can be described using the probability function,  $f(x)$ :

x	2	3	4	5	6
$f(x)$	0.1	0.2	0.3	0.3	0.1

- (a) Plot  $F(x)$ , the cumulative probability function for  $X$ .[5 pts]
  - (b) Find the mean and standard deviation of  $X$ .[10 pts]
2. Ch. 5, Exercise 1, pg. 322: Suppose 90% of all students taking a beginning programming class fail to get their first program to run on first submission. Use a binomial distribution and assign probabilities to the possibilities that among a group of six such students,
- (a) all fail on their first submissions[5 pts]
  - (b) at least four fail on their first submissions[5 pts]
  - (c) less than four fail on their first submissions [5 pts]
- Continuing to use this binomial model,
- (d) what is the mean number who will fail?[5 pts]
  - (e) what are the variance and standard deviation of the number who will fail?[5 pts]
3. Ch. 5, Exercise 2, pg. 322: Suppose that for single launches of a space shuttle, there is a constant probability of O-ring failure (say .15), Consider ten future launches, and let  $X$  be the number of those involving an O-ring failure. Use an appropriate probability model and evaluate all of the following:
- (a) Precisely state the distribution of  $X$ , giving the values of any parameters necessary.[5 pts]
  - (b)  $P[X = 2]$ [5 pts]
  - (c)  $P[X \geq 1]$ [5 pts]
  - (d)  $EX$ [5 pts]
  - (e)  $\text{Var}X$ [5 pts]
  - (f) the standard deviation of  $X$ [5 pts]
4. Ch. 5.1, Exercise 6, pg. 244: Suppose that an eddy current nondestructive evaluation technique for identifying cracks in critical metal parts has a probability of about .20 of detecting a single crack of length .003in. in a certain material. Let  $Y$  be the number of specimens inspected in order to obtain the first crack detection. Use an appropriate probability model and evaluate all of the following:
- (a) Precisely state the distribution of  $X$ , giving the values of any parameters necessary.[5 pts]
  - (b)  $P[Y = 5]$ [5 pts]

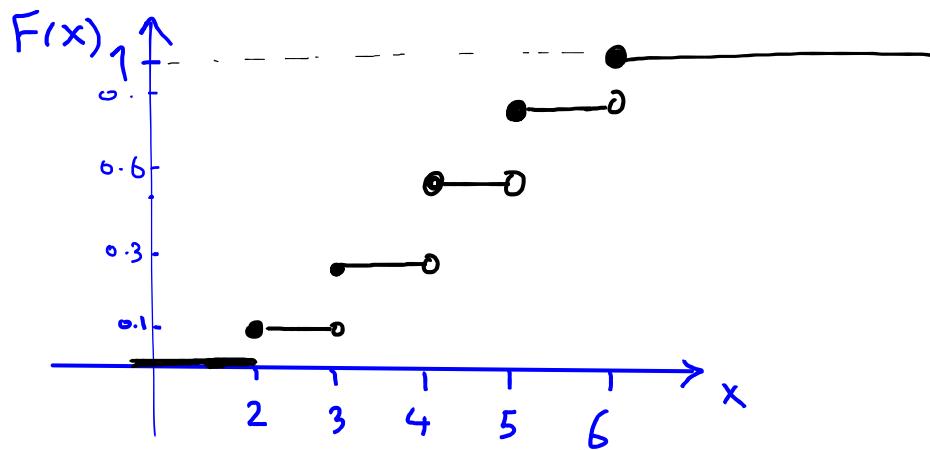
- (c)  $P[Y \leq 4]$ [5 pts]
- (d)  $EY$ [5 pts]
- (e)  $\text{Var}Y$ [5 pts]
- (f)  $\text{SD}(Y)$ [5 pts]

Total: 100 pts

$k = f(x)$

1. (a)

$x$	2	3	4	5	6
$P(x)$	0.1	0.2	0.3	0.3	0.1
$F(x)$	0.1	0.3	0.6	0.9	1



(b)

$E(x) = ?$

$x$	2	3	4	5	6
$P(x)$	0.1	0.2	0.3	0.3	0.1
$xP(x)$	0.2	0.6	1.2	1.5	0.6
$x^2 P(x)$	0.4	1.8	4.8	7.5	3.6

$$E(x) = \sum xP(x) = 0.2 + 0.6 + 1.2 + 1.5 + 0.6 = 4.1$$

$$E(x^2) = \sum x^2 P(x) = 0.4 + 1.8 + 4.8 + 7.5 + 3.6 = 18.1$$

$$\text{var}(x) = \sum (x - E(x))^2 P(x) = E(x^2) - (E(x))^2$$

$$= 18.1 - (4.1)^2 = 1.29$$

So, the standard deviation is:

$$SD(x) = \sqrt{\text{var}x} = \sqrt{1.29} = 1.13$$

problem 2)

Let  $x$  be the random variable connected to the numbers of students who fail to get their first program to run. Then

$$x \sim \text{Binomial}(n=6, p=0.9)$$

(a)  $P(\text{"all students fail to run their first program"})$

$$\begin{aligned} &= P(x=6) = \frac{6!}{6!(6-6)!} (0.9)^6 (1-0.9)^0 \\ &= (0.9)^6 = 0.531441 \end{aligned}$$

(b)  $P(\text{"at least 4 students fail"}) = P(x \geq 4)$

$$= P(x=4) + P(x=5) + P(x=6)$$

$$= \underbrace{\frac{6!}{4!(6-4)!}}_{2!} \cdot (0.9)^4 (1-0.9)^{6-4} + \underbrace{\frac{6!}{5!(6-5)!}}_{1!} (0.9)^5 (1-0.9)^{6-5}$$

(This shows how to cancel

$$+ \cancel{\frac{6!}{5!(6-6)!}} (0.9)^6 (1-0.9)^0$$

The Factorial terms)

$$= \frac{3 \cancel{6 \times 5 \times 4 \times 3 \times 2 \times 1}}{(4 \times 3 \times 2 \times 1) \times (2 \times 1)} (.9)^4 (.1)^2 + \frac{6 \cancel{5 \times 4 \times 3 \times 2 \times 1}}{(5 \times 4 \times 3 \times 2 \times 1) \times 1} (.9)^5 (.1)^1$$

$$+ (.9)^6$$

$$\begin{aligned}
 &= (3 \times 5) (0.9)^4 (0.1)^2 + 6 (0.9)^5 (0.1)^1 + (0.9)^6 \\
 &= 0.9841
 \end{aligned}$$

(c)  $P(\text{"less than 4 students fail"})$

$$\begin{aligned}
 &= P(X < 4) = P(X \leq 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3) \\
 &= \dots
 \end{aligned}$$

But, you can use the complement trick as you found the  $P(X \geq 4)$  in part (b).

$$P(X < 4) = 1 - P(X \geq 4) = 1 - 0.9841 = \boxed{0.0159}$$

(d)  $Ex$  = the mean (expected) number of fails.

$$X \sim \text{Binomial}(n=6, p=0.9)$$

$$\Rightarrow Ex = n \cdot p = 6(0.9) = \boxed{5.4}$$

It is expected that 5.4 student fail on their first programming.

(e)  $\text{Var}(X) = ?$  in Binomial distribution,

$$\text{Var}(X) = np(1-p)$$

$$\text{So, } \text{Var}(X) = 6(0.9)(1-0.9) = 6(0.9)(0.1)$$
$$= 0.54$$

and the standard variation is

$$\text{SD}(X) = \sqrt{\text{Var}(X)} = \sqrt{0.54} = 0.734$$

problem 3:

(a)  $X \sim \text{Binomial}(n=10, p=0.15)$

(b)  $P(X=2) = ?$

we know in Binomial,  $P(X=x) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$

So,

$$P(X=2) = \frac{10!}{2!(10-2)!} \cdot (0.15)^2 (1-0.15)^{10-2}$$

$$= \frac{10!}{2! 8!} (0.15)^2 (0.85)^8$$

$$\begin{aligned}
 &= \frac{\cancel{10} \times 9 \times 8!}{\cancel{2!} \cancel{8!}} (0.15)^2 (0.85)^8 \\
 &= 5 \times 9 \times (0.15)^2 \times (0.85)^8 \\
 &= \boxed{0.2758}
 \end{aligned}$$

(c)  $P(X \geq 1) = \sum_{x=1}^{10} P(X=x) = P(X=1) + P(X=2) + \dots + P(X=10)$

(This is a very long way!)

So, use the complement:

$$P(X \geq 1) = 1 - P(X < 1) = 1 - P(X=0)$$

$$\begin{aligned}
 &= 1 - \left[ \frac{\cancel{10!}}{\cancel{0!} \cancel{(10-0)!}} \cdot \underbrace{(0.15)^0}_{1} \underbrace{(1-0.15)^{10-0}}_{0.85} \right] \\
 &= 1 - (0.85)^{10} \\
 &= \boxed{0.8081}
 \end{aligned}$$

(d)  $E(X) = ?$  Since we know  $X \sim \text{Binomial}(10, 0.15)$ ,  
we can easily get  $E(X)$ .

$$E(X) = n \cdot p = 10(0.15) = \boxed{1.5}$$

(e)  $\text{Var}(X) = ?$

$$\begin{aligned}\text{Var}(X) &= n \cdot p(1-p) = 10(0.15)(1-0.15) \\ &= 10(0.15)(0.85) \\ &= \boxed{1.275}\end{aligned}$$

(f) standard deviation of  $X$ :

$$SD(X) = \sqrt{\text{Var } X} = \sqrt{1.275} = \boxed{1.129}.$$

Problem 4)

(a)  $Y =$  The number of specimens inspected in order  
to see the first crack

$$\Rightarrow Y \sim \text{Geometric}(p=0.2)$$

(b) we know that  $Y \sim \text{Geom}(p)$ ,

$$P_Y(y) = p(1-p)^{y-1}, y=1, 2, 3, \dots$$

$$\begin{aligned} P(Y=5) &= p(1-p)^{5-1} \\ &= (0.2)(1-0.2)^4 \\ &= (0.2)(0.8)^4 \\ &= 0.08192 \end{aligned}$$

(c)  $P(Y \leq 4) = F_Y(4)$

Note: Geometric (unlike Binomial & Poisson) has a closed form CDF.

$$Y \sim \text{Geom}(p) \Rightarrow F_Y(y) = P(Y \leq y) = 1 - (1-p)^y$$

$$\begin{aligned} P(Y \leq 4) &= F_Y(4) = 1 - (1-p)^4 \\ &= 1 - (1-0.2)^4 \\ &= 1 - (0.8)^4 \\ &= 0.5904 \end{aligned}$$

(d) we know in Geometric distribution,

$$EY = \frac{1}{p}$$

$$\text{Var } Y = \frac{1-p}{p^2}$$

So,  $E Y = \frac{1}{0.2} = 5$

(This means it is expected that we need 5 specimens inspected to observe the first crack)

(e)  $\text{Var}(Y) = \frac{1-p}{p^2} = \frac{1-0.2}{(0.2)^2} = \frac{0.8}{0.04} = 20$

Then the  $SD(Y) = \sqrt{\text{Var}(Y)}$

$$= \sqrt{20}$$

$$= 4.47$$