Show **all** of your work on this assignment and answer each question fully in the given context.

Please staple your assignment!

1. Chapter 4, Exercise 12, page 208(skip part d) [5 pts each part, 15 pts total]

(a) is the sum of the R^2 values from the two one-variable linear equations.

P208. 12. (a) The least squares equation is

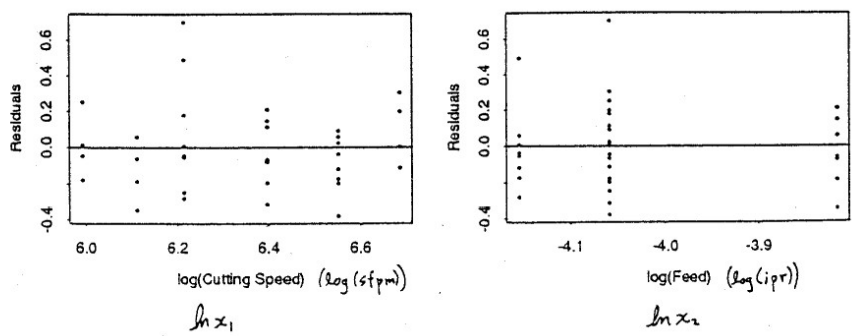
$$\widehat{\ln y} = 18.750 - 5.1209 \ln x_1 - 3.7379 \ln x_2,$$

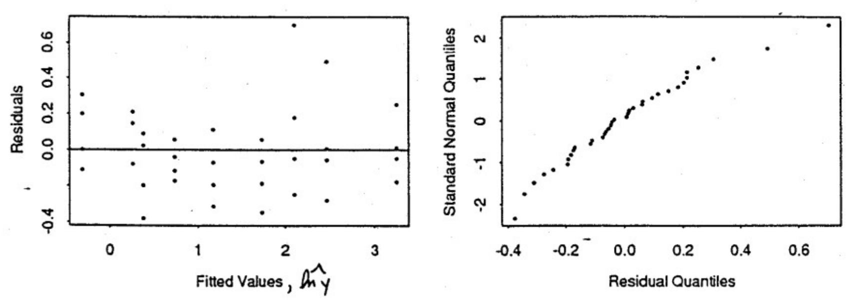
with an R^2 of .960. The relationship $y x_1^{\alpha_1} x_2^{\alpha_2} = C$ implies that

$$\ln y = \ln C - \alpha_1 \ln x_1 - \alpha_2 \ln x_2,$$

so $\hat{\alpha}_1 = -b_1 = 5.1209$, $\hat{\alpha}_2 = -b_2 = 3.7379$, and $\hat{C} = e^{b_0} = 1.39 \times 10^8$.

(b)





The plot of Residuals versus $\widehat{\ln y}$ shows a slight amount of curvature, but the pattern is not strong. The plot of Residuals versus $\ln x_1$ shows that there is more spread in the response when $\ln x_1 = 6.2146$ ($x_1 = 500$), and the plot of Residuals versus $\ln x_2$ shows that there is more spread in the response when $\ln x_2 = -4.05994$ ($x_2 = .01725$). The normal plot of residuals is fairly linear, indicating that the residuals are bell-shaped. Overall, the residual plots do not reveal any major problems with the fitted model.

(c) For $x_1 = 550$ and $x_2 = .01650$,

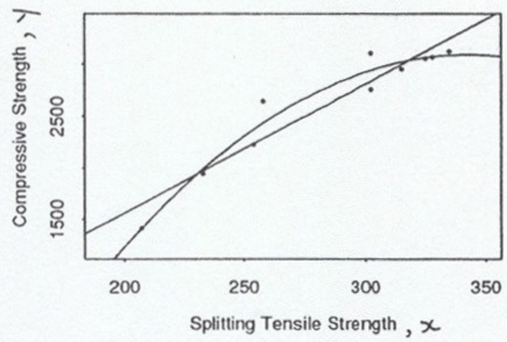
$$\widehat{\ln y} = \ln \hat{C} - \hat{\alpha}_1 \ln(550) - \hat{\alpha}_2 \ln(.01650) = 1.7789,$$

so $\hat{y} = e^{1.7789} = 5.92$ minutes.

(d)

2. Chapter 4, Exercise 16 [a-g][5 pts each part, 35 pts total]

16. (a)



The relationship is not quite linear.

(b) The calculations are given below:

i	x_i	x_i^2	y_i	y_i^2	x_iy_i
1	207	42849	1420	2016400	293940
2	233	54289	1950	3802500	454350
3	254	64516	2230	4972900	566420
4	328	107584	3070	9424900	1006960
5	325	105625	3060	9363600	994500
6	302	91204	3110	9672100	939220
7	258	66564	2650	7022500	683700
8	335	112225	3130	9796900	1048550
9	315	99225	2960	8761600	932400
10	302	91204	2760	7617600	833520
	2859	835285	26340	72451000	7753560

$$r = \frac{\sum x_iy_i - \frac{(\sum x_i)(\sum y_i)}{n}}{\sqrt{\left(\sum x_i^2 - \frac{(\sum x_i)^2}{n}\right)\left(\sum y_i^2 - \frac{(\sum y_i)^2}{n}\right)}}$$

$$= \frac{7753560 - \frac{(2859)(26340)}{10}}{\sqrt{\left(835285 - \frac{(2859)^2}{10}\right)\left(72451000 - \frac{(26340)^2}{10}\right)}} = .951.$$

This is close to 1, so there is a fairly strong positive linear relationship between y and x.

(c)

$$b_1 = \frac{\sum x_iy_i - \frac{(\sum x_i)(\sum y_i)}{n}}{\sum x_i^2 - \frac{(\sum x_i)^2}{n}} = \frac{7753560 - \frac{(2859)(26340)}{10}}{835285 - \frac{(2859)^2}{10}} = 12.45769$$

$$b_0 = \bar{y} - b_1\bar{x} = \frac{26340}{10} - (12.45769)\frac{2859}{10} = -927.6531$$

So the least squares equation is

$$\hat{y} = -927.6531 + 12.45769x.$$

(d) There is an approximate $b_1 = 12.45769$ psi increase in compressive strength that accompanies a 1 psi increase in splitting tensile strength.

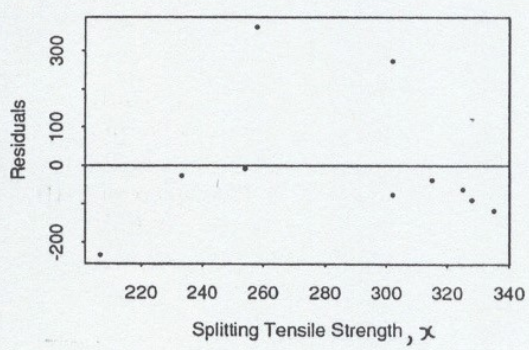
(e) $R^2 = r^2 = (.951)^2 = .904$.

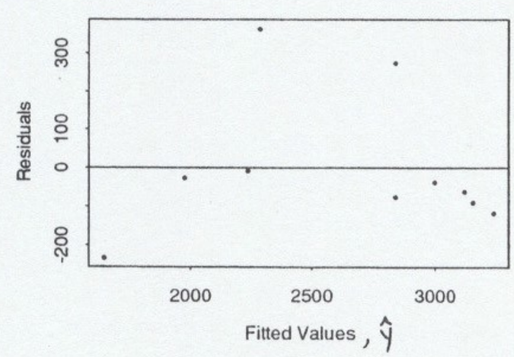
(f) For $x = 245$,

$$\hat{y} = -927.6531 + 12.45769(245) \approx 2124 \text{ psi}.$$

(g) The residuals e_i are computed below.

i	x_i	y_i	$\hat{y}_i = -927.6531 + 12.45769x_i$	$e_i = y_i - \hat{y}$
1	207	1420	1651.088	−231.088401
2	233	1950	1974.988	−24.988294
3	254	2230	2236.600	−6.599746
4	328	3070	3158.469	−88.468673
5	325	3060	3121.096	−61.095609
6	302	3110	2834.569	275.431220
7	258	2650	2286.430	363.569501
8	335	3130	3245.672	−115.672491
9	315	2960	2996.519	−36.518727
10	302	2760	2834.569	−74.568780





3. This is the rest of the problem 5 in HW 4.
   The major cause of axel failure in freight trucks is when shippers exceed the recommended weight limits that can be handled by the axels. Issues resulting from these failures have been becoming more frequent as shippers try to cut corners, leading members of the state's Department of Transportation to ask one of their civil engineers to look into the available data and better advise them on the relationship between excessive weight and axel failure.

   A company manufacturing axels provides the engineer with data gathered from conducting experiments loading axels with excessive weight and simulating traveling conditions. The data consists of two columns, **excessive weight (in tonnes)** is the amount of weight over the limit that was placed on the axel, and **distance to failure (in tens of thousands of miles)** is the simulated distance to the axel's failure.

Here are some summaries of the data:

$$\sum_{i=1}^{50} x_i = 64 \qquad \sum_{i=1}^{50} x_i^2 = 107$$

$$\sum_{i=1}^{50} y_i = 2025 \qquad \sum_{i=1}^{50} y_i^2 = 95247$$

$$\sum_{i=1}^{50} x_i y_i = 2028$$

The JMP output below comes from fitting a quadratic model using x and x^2.

Response Distance to Failure

Summary of Fit

RSquare	REDACTED
RSquare Adj	REDACTED
Root Mean Square Error	5.281589
Mean of Response	0.16
Observations (or Sum Wgts)	50

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	2	13229.647	6614.82	237.1314
Error	47	1311.073	27.90	**Prob > F**
C. Total	49	14540.720		<.0001*

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	16.27602	2.333507	6.97	<.0001*
Weight Exceeding Limit	4.6604349	4.221593	1.10	0.2752
(Weight Exceeding Limit)^2	-10.2775	1.604983	-6.40	<.0001*

(a) Write the equation of the fitted quadratic relationship. [5 pts]

$$\hat{y} = 16.27602 + 4.6604349x - 10.2775x^2$$

(b) Find and interpret the value of R^2 for the fitted quadratic relationship.[5 pts]

$$R^2 = 1 - SSE/SSTO = 1 - (1311.073/14540.720) = 0.909834382341452$$

In other words, 90.98% of the variability in travel distance to failure can be explained by the linear relationship with weight exceeding guidelines.

(c) Using the fitted quadratic relationship, provide a predicted value of travel distance to failure when the weight exceeding the guidelines is 3.4 tonnes.[5 pts]

$$\hat{y} = 16.27602 + 4.6604349(3.4) - 10.2775(3.4)^2 = -86.68640134$$

Total: 65 pts