HW● Solutions

P344. 2

(a) You can use equation (6.9), since this is a large sample. The appropriate z for 90% confidence is 1.645. The interval is

$$142.7 \pm 1.645\left(\frac{98.2}{\sqrt{26}}\right) = 142.7 \pm 31.68 = [111.02, 174.38].$$

(b) Now $z = 1.96$, and the interval is

$$142.7 \pm 1.96\left(\frac{98.2}{\sqrt{26}}\right) = 142.7 \pm 37.75 = [104.95, 180.45].$$

This interval is wider than the one from (a). In order to have more confidence of containing the mean, the interval must be wider.

(c) To make a 90% one-sided confidence interval, construct a 80% two-sided confidence interval, and use the upper endpoint. The appropriate z for a 80% two-sided confidence interval is 1.28, so the 90% one-sided confidence interval is

$$142.7 + 1.28\left(\frac{98.2}{\sqrt{26}}\right) = 142.7 + 24.65 = 167.35.$$

This value is smaller than the upper endpoint from part (a). Setting the lower endpoint equal to $-\infty$ requires you to move the upper endpoint in so that the confidence remains at 90%.

(d) To make a 95% one-sided confidence interval, construct a 90% two-sided confidence interval, and use the upper endpoint. This was done in part (a), so the 90% one-sided confidence interval is 174.38. This is larger than the answer to (c); in order to achieve higher confidence, you must make the interval "wider".

(e) [111.02, 174.38] ppm is a set of plausible values for the mean aluminum content of samples of recycled PET plastic from the recycling pilot plant at Rutgers University. The method used to construct this interval correctly contains means in 90% of repeated applications. This particular interval either contains the mean or it doesn't (there is no probability involved). However, because the *method* is correct 90% of the time, we might say that we have 90% confidence that it was correct this time.

P44. 4. (a) $\bar{x} = 4.6858$ and $s = .02900317$.

(b) Since this is a large sample, you can use equation (6.9), with $z = 2.33$ for 98% confidence. The two-sided confidence interval is

$$\begin{aligned} 4.6858 \pm 2.33\left(\frac{.02900317}{\sqrt{50}}\right) &= 4.6858 \pm .009556884 \\ &= [4.676, 4.695] \text{ mm.} \end{aligned}$$

(c) $z = 2.58$ for 98% confidence. The two-sided confidence interval is

$$\begin{aligned} 4.6858 \pm 2.58\left(\frac{.02900317}{\sqrt{50}}\right) &= 4.6858 \pm .0105823 \\ &= [4.675, 4.696] \text{ mm.} \end{aligned}$$

This interval is wider than the one in (b). To increase the confidence that μ is in the interval, you need to make the interval wider.

(d) To make a 98% one-sided interval, construct a 96% two-sided interval and use the lower endpoint. For a 96% two-sided interval, the appropriate z is $Q_{SN}(.98) = 2.05$. The resulting 98% one-sided interval is

$$\begin{aligned} 4.6858 - 2.05\left(\frac{.02900317}{\sqrt{50}}\right) &= 4.6858 - .008408418 \\ &= 4.677 \text{ mm.} \end{aligned}$$

This is larger than the lower endpoint of the interval in (b). Since the upper endpoint here is set to ∞, the lower endpoint must be increased to keep the confidence level the same.

(e) To make a 99% one-sided interval, construct a 98% two-sided interval and use the lower endpoint. This was done in part (a), and the resulting lower bound is 4.676. This is smaller than the value in (d); to increase the confidence, the interval must be made "wider".

(f) [4.676, 4.695] ppm is a set of plausible values for the mean diameter of this type of screw as measured by this student with these calipers. The method used to construct this interval correctly contains means in 98% of repeated applications. This particular interval either contains the mean or it doesn't (there is no probability involved). However, because the *method* is correct 98% of the time, we might say that we have 98% confidence that it was correct this time.

P361. 1. Since the natural logarithms of the data are more bell-shaped than the raw data (see P113, 2), it would be better to test the null hypothesis that the mean of the logs is equal to ln(200) versus the alternative that the mean of the logs is greater than ln(200). However, since this is a large sample, using the raw data poses no major problem.

1. H_0: $\mu = 200$ ppm.
2. H_a: $\mu > 200$ ppm.
3. The test statistic is

$$Z = \frac{\bar{x} - 200}{\frac{s}{\sqrt{26}}}$$

and the reference distribution is the standard normal distribution. Observed values of Z far above zero will be considered as evidence against H_0.

4. The sample gives

$$z = -2.98.$$

5. The observed level of significance is

$$\begin{aligned} & P(\text{a standard normal random variable} > -2.98) \\ = \; & P(\text{a standard normal random variable} < 2.98) \end{aligned}$$

which is equal to .9986, according to Table B-3. There is no evidence that the mean aluminum content for samples of recycled plastic is greater than 200 ppm.

P361. 4.

1. H_0: $\mu = 4.70$ mm.
2. H_a: $\mu \neq 4.70$ mm.
3. The test statistic is

$$Z = \frac{\bar{x} - 4.70}{\frac{s}{\sqrt{50}}}$$

and the reference distribution is the standard normal distribution. Observed values of Z far above or below zero will be considered as evidence against H_0.

4. The sample gives

$$z = -3.46.$$

5. The observed level of significance is

$$2P(\text{a standard normal random variable} < -3.46)$$

which is equal to $2(.0003) = .0006$, according to Table B-3. There is very strong evidence that the mean measured diameter differs from nominal.

P427. 1. (a) The two-sided 95% confidence interval is given by equation (6-20). The required t is $Q(.975)$ of the t_9 distribution, since (by symmetry) there must be probability .025 in each tail. From Table B-4, $t = Q_9(.975) = 2.262$. From the data, $n = 10$, $\bar{x} = 9082.2$, and $s = 841.87$, so the confidence interval is

$$\begin{aligned} 9082.2 \pm 2.262\left(\frac{841.87}{\sqrt{10}}\right) &= 9082.2 \pm 602.19 \\ &= [8480.0, 9684.4] \text{ g.} \end{aligned}$$

To make the 95% one-sided confidence interval, construct a 90% two-sided confidence interval and use the lower endpoint. The appropriate t for a 90% two-sided confidence interval is $t = Q_9(.95) = 1.833$, and so the 95% one sided interval is

$$\begin{aligned} 9082.2 - 1.833\left(\frac{841.87}{\sqrt{10}}\right) &= 9082.2 - 487.99 \\ &= 8594.2 \text{ g.} \end{aligned}$$

(b) Use equation (6-78). The two-sided 95% prediction interval is

$$\begin{aligned} 9082.2 \pm 2.262(841.87)\sqrt{1+\frac{1}{10}} &= 9082.2 \pm 1997.25 \\ &= [7084.9, 11079.5] \text{ g.} \end{aligned}$$

To make the 95% one-sided prediction interval, construct a 90% two-sided prediction interval and use the lower endpoint.

$$\begin{aligned} 9082.2 - 1.833(841.87)\sqrt{1+\frac{1}{10}} &= 9082.2 - 1618.5 \\ &= 7463.7 \text{ g.} \end{aligned}$$