Show **all** of your work on this assignment and answer each question fully in the given context. You have 20 minutes. Each problem is designed to take 10 minutes. All answers in a topic must be correct for any credit for that topic. You may attempt multiple topics. You may use a calculator on this competency quiz.

1. **Competency Topic: Discrete Random Variables**

   Let X be a random variable following a binomial distribution with probability function

$$f(x) = \frac{4!}{x!(4-x)!}(0.6)^x(0.4)^{4-x}$$

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   a. Complete the probability table for X.

x	$P(X = x)$
0	$1 \cdot (0.4)^4 \cdot (0.6)^0 = 256/10000$
1	$4 \cdot (0.6)^1 \cdot (0.4)^3 = 1536/10000$
2	$6 \cdot (0.6)^2 \cdot (0.4)^2 = 3456/10000$
3	$4 \cdot (0.6)^3 \cdot (0.4)^1 = 3456/10000$
4	$1 \cdot (0.4)^0 \cdot (0.6)^4 = 1296/10000$

   b. Find the value of $E(X)$.

We can start with the definition:

$$\begin{aligned}
E(X) &= \sum_x x \cdot f(x) \\
&= 0 \cdot f(0) + 1 \cdot f(1) + 2 \cdot f(2) + 3 \cdot f(3) + 4 \cdot f(4) \\
&= 0 \cdot \frac{256}{10000} + 1 \cdot \frac{1536}{10000} + 2 \cdot \frac{3456}{10000} + 3 \cdot \frac{3456}{10000} + 4 \cdot \frac{1296}{10000} \\
&= \frac{1536}{10000} + \frac{6912}{10000} + \frac{10368}{10000} + \frac{5184}{10000} \\
&= 2.4
\end{aligned}$$

Since X, you could also use that to get your answer: $E(X) = n \cdot p = 4 \cdot 0.6 = 2.4$.

   c. Sketch the cumulative probability function, $F(x)$.

2. **Competency Topic: Continuous Random Variables**

Let X be a random variable with the following probability density function:

$$f(x) = \begin{cases} k \cdot x^3 & 0 \leq x \leq 2 \\ 0 & o.w. \end{cases}$$

for some constant k.

a. Find the value of k that makes $f(x)$ a valid pdf.

We need $\int_{-\infty}^{\infty} f(x)dx = 1$ for a valid pdf:

$$\begin{aligned} \int_{-\infty}^{\infty} f(x)dx &= \int_0^2 kx^3 dx \\ &= \frac{k}{4}x^4 \Big|_0^2 \\ &= \frac{k}{4}2^4 \\ &= 4k \end{aligned}$$

Which implies that $k = 1/4$.

b. Sketch the probability density function and illustrate the region corresponding to the value of $P(1 \leq X \leq 2)$.

c. Find the value of $E(X)$.

Starting wiht the definition,

$$\begin{aligned} E(X) &= \int_{-\infty}^{\infty} xf(x)dx \\ &= \int_0^2 x \cdot \frac{1}{4}x^3 dx \\ &= \int_0^2 \frac{1}{4}x^4 dx \\ &= \frac{1}{20}x^5 \Big|_0^2 \\ &= \frac{1}{20}2^5 \\ &= \frac{8}{5} \end{aligned}$$