STAT 305 Exam II Reference Sheet

Basic Probability

Definitions

Random experiment	A series of actions that lead to an observable result. The result may change each time we perform the experiment.
Outcome	The result(s) of a random experiment.
Sample Space (S)	A set of all possible results of a random experiment.
Event (A)	Any subset of sample space.
Probability of an event ($P(A)$)	the likelihood that the observed outcome of a random experiment is one of the outcomes in the event.
A^C	The outcomes that are not in A.
$A \cap B$	The outcomes that are both in A and in B.
$A \cup B$	The outcomes that are either A or B.

General Rules

Probability A given B	$P(A\|B) = P(A \cap B)/P(B)$
Probability A and B	$P(A \cap B) = P(A\|B)P(B) = P(B\|A)P(A)$
Probability A or B	$P(A \text{ or } B) = P(A) + P(B) - P(A, B)$

Independence

Two events are called independent if $P(A, B) = P(A) \cdot P(B)$. Clever students will realize this also means that if A and B are independent then $P(A|B) = P(A)$ and $P(B|A) = P(B)$.

Joint Probability

Joint Probability	The probability an outcome is in event A and in event $B = P(A, B)$.
Marginal Probability	If $A \subseteq B \cup C$ then $P(A) = P(A \cap B) + P(A \cap C)$.
Conditional Probability	For events A and B, if $P(B) \neq 0$ then $P(A\|B) = P(A \cap B)/P(B)$.

Discrete Random Variables

General Rules

Probability function	$f_X(x) = P(X = x)$
Cumulative probability function	$F_X(x) = P(X \leq x)$
Expected Value	$\mu = E(X) = \sum_x x f_X(x)$
Variance	$\sigma^2 = Var(X) = \sum_x (x - \mu)^2 f_X(x)$
Standard Deviation	$\sigma = \sqrt{Var(X)}$

Joint Probability Functions

Joint Probability Function	$f_{XY}(x, y) = P[X = x, Y = y]$
Marginal Probability Function	$f_X(x) = \sum_y f_{XY}(x, y)$ $f_Y(y) = \sum_x f_{XY}(x, y)$
Conditional Probability Function	$f_{X\|Y}(x\|y) = f_{XY}(x, y)/f_Y(y)$ $f_{Y\|X}(y\|x) = f_{XY}(x, y)/f_X(x)$

Geometric Random Variables

X is the trial count upon which the first successful outcome is observed performing independent trials with probability of success p.

Possible Values	$x = 1, 2, 3, \ldots$
Probability function	$P[X = x] = f_X(x) = p^x(1 - p)^{x-1}$
Expected Value	$\mu = E(X) = \frac{1}{p}$
Variance	$\sigma^2 = Var(X) = \frac{1-p}{p^2}$

Binomial Random Variables

X is the number of successful outcomes observed in n independent trials with probability of success p.

Possible Values	$x = 0, 1, 2, \ldots, n$
Probability function	$P[X = x] = f_X(x) = \frac{n!}{(n-x)!x!} p^x(1 - p)^{n-x}$
Expected Value	$\mu = E(X) = np$
Variance	$\sigma^2 = Var(X) = np(1 - p)$

Poisson Random Variables

X is the number of times a rare event occurs over a predetermined interval (an area, an amount of time, etc.) where the number of events we expect is λ.

Possible Values	$x = 0, 1, 2, 3, \ldots$
Probability function	$P[X = x] = f_X(x) = \frac{e^{-\lambda}\lambda^x}{x!}$
Expected Value	$E(X) = \lambda$
Variance	$Var(X) = \lambda$

Continuous Random Variables

General Rules

Probability density function	$P[a \leq X \leq b] = \int_a^b f_X(x)dx$
Cumulative density function	$P[X \leq x] = F_X(x) = \int_{-\infty}^{x} f_X(t)dt$
Expected Value	$\mu = E(X) = \int_{-\infty}^{\infty} x f_X(x)dx$
Variance	$\sigma^2 = Var(X) = \int_{-\infty}^{\infty}(x-\mu)^2 f_X(x)dx$
Standard Deviation	$\sigma = \sqrt{Var(X)}$

Joint Probability Density Functions

Joint Probability Density Function	$f_{XY}(x,y)$ is the joint density of both X and Y.
	$P(a \leq X \leq b, c \leq Y \leq d) = \int_a^b \int_c^d f_{XY}(x,y)dydx$
Marginal Probability Density Function	$f_X(x) = \int_{-\infty}^{\infty} f_{XY}(x,y)dy$
	$f_Y(y) = \int_{-\infty}^{\infty} f_{XY}(x,y)dx$
Conditional Probability Density Function	$f_{X\|Y}(x\|y) = f_{XY}(x,y)/f_Y(y)$
	$f_{Y\|X}(y\|x) = f_{XY}(x,y)/f_X(x)$

Uniform Random Variables

Used when we believe an outcome could be anywhere between two values a and b but have no other beliefs.

Probability density function	$f_X(x) = \begin{cases} \frac{1}{b-a} & a \leq x \leq b \\ 0 & o.w. \end{cases}$
Cumulative density function	$F_X(x) = \begin{cases} 0 & x \leq a \\ \frac{1}{b-a}x - \frac{a}{b-a} & a \leq x \leq b \\ 1 & x > b \end{cases}$
Expected Value	$E(X) = \frac{1}{2}(b+a)$
Variance	$Var(X) = \frac{1}{12}(b-a)^2$

Exponential Random Variables

Used when we an outcome could be anything greater than 0 but the likelihood is concentrated on smaller values.

Probability density function	$f_X(x) = \begin{cases} \frac{1}{\alpha}\exp\left(-\frac{x}{\alpha}\right) & x \geq 0 \\ 0 & o.w. \end{cases}$
Cumulative density function	$F_X(x) = \begin{cases} 0 & x < 0 \\ 1 - \exp\left(-\frac{x}{\alpha}\right) & x \geq 0 \end{cases}$
Expected Value	$E(X) = \alpha$
Variance	$Var(X) = (\alpha)^2$

Normal Random Variables

Used when we believe an outcome could be above or below a certain value μ but we also believe it is more likely to be close to μ than it is to be far away.

Probability density function	$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$
Cumulative density function	There is no general formula.
Expected Value	$E(X) = \mu$
Variance	$Var(X) = \sigma^2$

Standard Normal Random Variables (Z)

A normal random variable with mean 0 and variance σ^2.

Probability density function	$f_Z(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}$
Cumulative density function	There is no general formula.
Expected Value	$E(Z) = 0$
Variance	$Var(Z) = 1$
Relationship with $X \sim N(\mu, \sigma^2)$	If X is normal(μ,σ^2) then $P[a \leq X \leq b] = P\left[\frac{a-\mu}{\sigma} \leq Z \leq \frac{b-\mu}{\sigma}\right]$

Functions of Random Variables

Linear Combinations of Independent Random Variables

For $X_1, X_2, \ldots, X_n$ independent random variables and $a_0, a_1, a_2, \ldots, a_n$ constants if $U = a_0 + a_1X_1 + \ldots + a_nX_n$:

- $E(U) = a_0 + a_1E(X_1) + a_2E(X_2) + \ldots + a_nE(X_n)$
- $Var(U) = a_1^2Var(X_1) + a_2^2Var(X_2) + \ldots + a_n^2Var(X_n)$

Standard Normal Probabilities

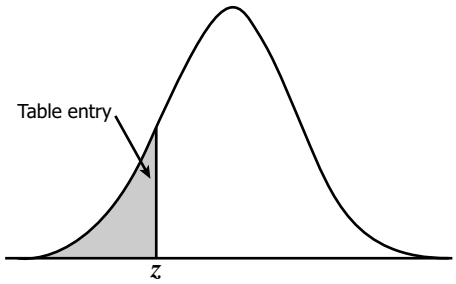

Table entry for z is the area under the standard normal curve to the left of z.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
–3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
–3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
–3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
–3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
–3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
–2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
–2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
–2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
–2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
–2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
–2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
–2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
–2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
–2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
–2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
–1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
–1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
–1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
–1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
–1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
–1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
–1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
–1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
–1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
–1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
–0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
–0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
–0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
–0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
–0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
–0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
–0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
–0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
–0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
–0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

Standard Normal Probabilities

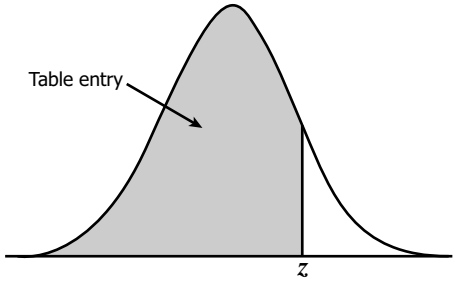

Table entry for z is the area under the standard normal curve to the left of z.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998