

Exam II

STAT 305, Section B Spring 2019

Instructions

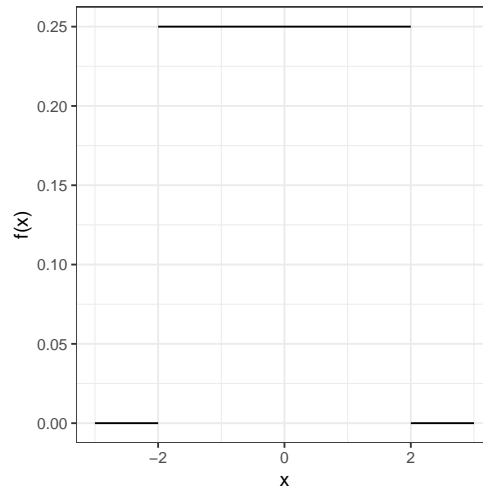
- The exam is scheduled for 80 minutes, from 3:40 to 5:00 PM. At 5:00 PM the exam will end.
- A formula sheet is attached to the end of the exam. Feel free to tear it off.
- You are allowed to use a self-produced one-page (front and back) formula sheet during this exam.
- You may use a calculator during this exam.
- Answer the questions in the space provided. If you run out of room, continue on the back of the page.
- If you have any questions about, or need clarification on the meaning of an item on this exam, please ask your instructor. No other form of external help is permitted attempting to receive help or provide help to others will be considered cheating.
- **Do not cheat on this exam.** Academic integrity demands an honest and fair testing environment. Cheating will not be tolerated and will result in an immediate score of 0 on the exam and an incident report will be submitted to the office of the dean.

Name: _____

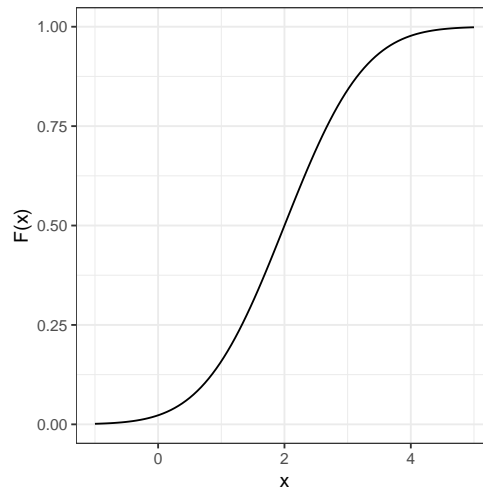
Student ID: _____

1. (2 points) For continuous distributions, the probability density function is _____ continuous and the cumulative density function is _____ continuous.
- A. **always** and **always** B. **always** and **sometimes** C. **sometimes** and **always** D. **sometimes** and **sometimes**

2. (3 points) Circle the name of the distribution which best matches the plot of the **probability density function**:

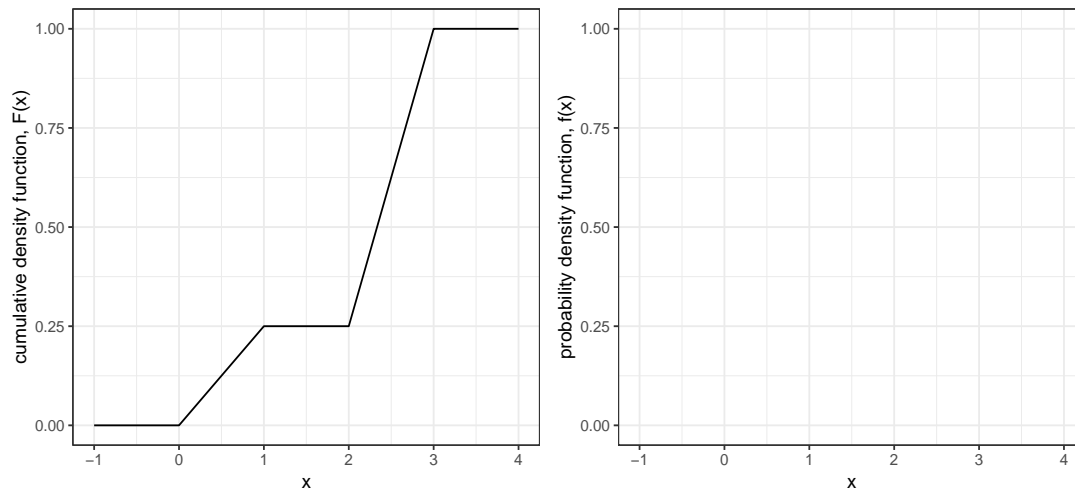


- A. standard normal B. normal C. exponential D. poisson E. uniform
3. (3 points) Circle the name of the distribution which best matches the plot of the **cumulative density function**:



- A. standard normal B. normal C. exponential D. binomial E. uniform

4. (4 points) The left plot depicts the **cumulative density function** of a random variable X . In the plot on the right, sketch the corresponding probability density function.



5. Suppose X is a discrete random variable with following probability function:

$$f(x) = \begin{cases} 0.1|x| + 0.08 & x = -2, -1, 0, 1, 2 \\ 0 & o.w. \end{cases}$$

- (a) (2 points) Find $E(X)$
- (b) (2 points) Find $P(|X| = 2)$
- (c) (2 points) Find $P(X \geq 0)$

6. Let X be a binomial random variable with $n = 6$ and $p = 0.2$.

(a) (2 points) Find $P(X = 3)$

(b) (2 points) Find $P(X > 4)$

(c) (2 points) Find $E(X)$

(d) (2 points) Find $Var(X)$

7. Suppose Y is an exponential random variable with $E(Y) = 2$.

(a) (2 points) Find $P(Y \leq 3)$

(b) (2 points) Find $P(1 \leq Y \leq 3)$

(c) (2 points) What is the cumulative density function of Y

(d) (2 points) Find $Var(Y)$

8. Let X be a normal random variable with a mean of 50 and a variance of 25 (i.e., $X \sim N(50, 25)$) and let Z be a random variable following a standard normal distribution. Find the following probabilities (note: the Standard Normal Probabilities table attached at to the exam will be helpful):

(a) (2 points) $P(Z > 0.1)$

(b) (2 points) $P(|Z| \leq 1.25)$

(c) (2 points) $P(1 < Z < 3)$

(d) (2 points) $P(X < 40)$

(e) (2 points) $P(|X - 50| \leq 12.5)$

(f) (5 points) Find the value a so that $P(X > a) = 0.85$

9. We are going to play a game involving four bags. The first bag contains 10 lettered wooden tiles (3 lettered A, 2 lettered B, and 5 lettered C). The other bags are labelled **Bag A**, **Bag B**, **Bag C** and contain the following:

- **Bag A**: 25 colored marbles: 16 Blue, 4 Red, 3 Green, and 2 Yellow.
- **Bag B**: 50 colored marbles: 10 Blue, 38 Red, 1 Green, and 1 Yellow.
- **Bag C**: 30 colored marbles: 1 Blue, 11 Red, 17 Green, and 1 Yellow.

Once you have learned the contents of the bags, we play the following game:

- First, I randomly select a tile from the bag of tiles (you do not see the tile).
- Next, I take the bag of marbles with the label matching the letter on my tile (you do not see which bag I take).
- Then, I randomly select a marble from the bag I took.
- Finally, I show you the marble.

You win the game by correctly guessing the letter on the tile I drew.

- (a) (2 points) What is the probability that I start the game by drawing a tile with the letter A?
- (b) (2 points) What is the probability that I draw a blue marble *given* that I drew a tile with the letter A?
- (c) (2 points) What is the probability that I start the game by drawing a tile with the letter A and then draw a blue marble?
- (d) (2 points) What is the probability that I start the game by drawing a tile with the letter C and then draw a blue marble?

- (e) (2 points) What is the probability that the marble I show you a blue marble?
- (f) (2 points) What is the probability that I drew a tile with the letter A *given* that I show you a blue marble?
- (g) (2 points) What is the probability that I drew a tile with the letter B *given* that I show you a green marble?

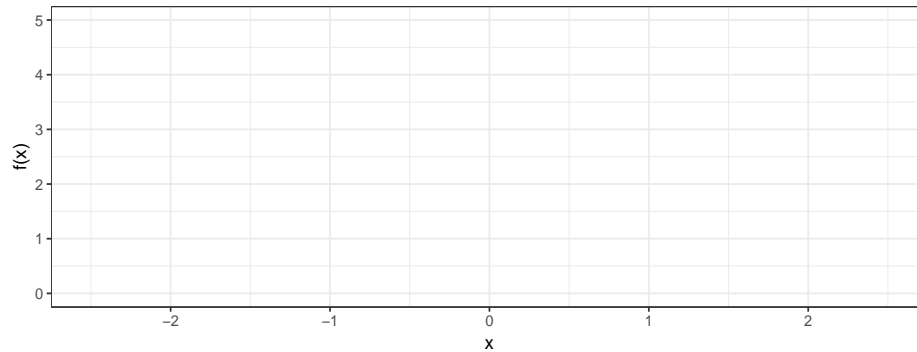
10. Suppose that X is a continuous random variable with probability density function (pdf):

$$f(x) = \begin{cases} c(4 - x^2) & -2 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

where c is a constant (not necessarily positive).

- (a) (2 points) What is the value of c if $f(x)$ is a valid probability density function?

- (b) (5 points) Sketch the probability density function using the grid below (including the points $(-2, f(2))$, $(0, f(0))$, and $(2, f(2))$).



- (c) (4 points) What is the cumulative density function, $F(x)$?
- (d) (2 points) What is the probability that X takes a value greater than 0.5?
- (e) (2 points) What is the probability that X takes a value between 0.5 and 1?