Exam 3

STAT 305, Section
Spring 2019

Instructions

- The exam is scheduled for 120 minutes, from 12:00 to 2:00pm. At 2:00pm the exam will end.
- A forumula sheet is attached to the end of the exam. Feel free to tear it off.
- You are allowed to use a self-produced one-page (front and back) formula sheet during this exam.
- You may use a calculator during this exam.
- Answer the questions in the space provided. If you run out of room, continue on the back of the page.
- If you have any questions about, or need clarification on the meaning of an item on this exam, please ask your instructor. No other form of external help is permitted attempting to receive help or provide help to others will be considered cheating.
- **Do not cheat on this exam.** Academic integrity demands an honest and fair testing environment. Cheating will not be tolerated and will result in an immediate score of 0 on the exam and an incident report will be submitted to the office of the dean.

Name: ______________________________

Student ID: ____________________________

1. (2 points) A random sample of 1000 students' Statistics exam scores was drawn from the population of all possible comparable Stat exam scores (an unknown population/distribution). The sample mean, once computed, has the exact value of the distribution/population mean.

   A. True B. False

2. (2 points) While trying to figure out the probability that the sample mean for a data of size 10 would exceed a value, we can apply the central limit theorem.

   A. True B. False

3. Two ancient scientists were promoting methods they had developed for determining the parts of gold in a given coin. While testing the methods, they were given the same coin (known to be 70 parts gold) on five different occasions. Their measurements on this coin are recorded below:

   - Scientist 1: $70.3, 70.3, 69.7, 69.7, 69.8$

   - Scientist 2: $34.6, 26.8, 20.1, 30.3, 19.4$

   (a) (2 points) Which scientist had the more accurate method? A. Scientist 1 B. Scientist 2

   (b) (2 points) Which scientist had the more precise method? A. Scientist 1 B. Scientist 2

4. The weights of the five remaining South Asian Black Rhinocerii are recorded below (in thousands of pounds):

   $$3,\ 2.8,\ 3.2,\ 3.2,\ 0.3$$

   Using these values, report the following:

   (a) (2 points) The mean weight of South Asian Black Rhinocerii

   (b) (2 points) The median weight of South Asian Black Rhinocerii

   (c) (3 points) The standard deviation of the weight of South Asian Black Rhinocerii

5. An agriculturist is attempting to determine which of three species of corn (A, B, and C) yield the most grain per acre. Since the yield may depend on the fertilizer used, the researcher intends to use fertilizers with different concentrations of Nitrogen as well - low Nitrogen, medium-low Nitrogen, medium-high Nitrogen, and high Nitrogen. There are 8 fields (scattered around Iowa) available to perform this expiriment. Each field is divided into 24 single acre plots and the combinations of species and fertilizer are randomly assigned so that within each field every combination is used exactly twice. Since the size of the plants may impact their growth when placed close by each other, it was decided that all species would be planted in a grid with each plant exactly four feet from its nearest neighbor. The agriculturalist also decided not to use any pest control system during growth. At harvest time, the weight of grain each plot yields is recorded and the combination of corn species and fertilizer that gives the highest average yield is chosen.

   (a) (2 points) Explain why this is an experiment and not an observational study.

   (b) Identify each of the following and describe them as numeric (in which case, identify whether it is continuous or discrete) or categorcial (in which case list the possible levels).

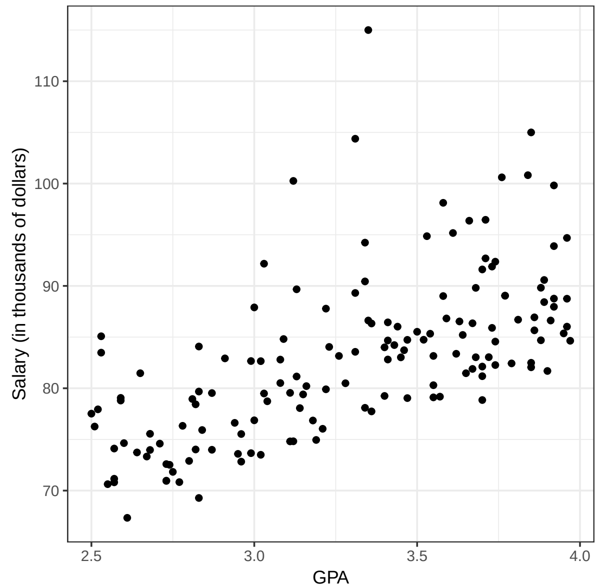
      i. (2 points) Identify the response variable(s):

      ii. (2 points) Identify the experimental variable(s):

      iii. (2 points) Blocking variable(s):

   (c) (2 points) Identify two controlled variables used in this process.

6. A survey given to members of a national engineering society who graduated five years prior is attempting to determine the relationship between salary and undergraduate GPA. The graph below displays 150 responses.



Here are some summaries of the data (with GPA as x and salary as y):

$$\sum_{i=1}^{150} x_i = 494 \qquad \sum_{i=1}^{150} x_i^2 = 1656$$

$$\sum_{i=1}^{150} y_i = 12455 \qquad \sum_{i=1}^{150} y_i^2 = 1043606$$

$$\sum_{i=1}^{150} x_i y_i = 41347$$

Using the summaries above, the survey workers fit a linear relationship between **GPA** (x) and **salary (in thousands of dollars)** (y).

(a) (3 points) Write the equation of the fitted linear relationship.

(b) (3 points) Using the fitted linear relationship, what do we predict will be the difference in salary of an engineer with an undergraduate GPA of 3.5 and an engineer with an undergraduate GPA of 3.0?

(c) (3 points) The actual income of one of the surveyed engineers who had an undergraduate GPA of 3.02 was 73.5 thousand dollars. What is the residual for this specific engineer using the linear relationship?

(d) (3 points) For the linear relationship, find r, the sample correlation coeffecient and R^2, the coeffecient of determination.

Discouraged by the relationship between salary and GPA, the surveyors remember that they know the address of each respondant and are able to determine the median income of the area in which the respondant lives. The JMP output below comes from fitting a linear relationship using for annual salary of the respondant ("`salary`") using both the undergraduate GPA ("`GPA`") and the median income of the area in which the respondant lives ("`med_salary_loc`") (in thousands of dollars).

Response salary

Effect Summary

Summary of Fit

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	2	8905.0931	4452.55	1278.142
Error	147	512.0904	3.48	**Prob > F**
C. Total	149	9417.1835		<.0001*

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	3.4351638	1.590643	2.16	0.0324*
GPA	10.143862	0.349311	29.04	<.0001*
med_salary_loc	1.3328123	0.034538	38.59	<.0001*

(a) (3 points) Write the equation of this fitted multivariate linear relationship.

(b) (3 points) Using this fitted multivariate linear relationship, what do we suppose the salary would be for an engineer with a undergraduate GPA of 3.02 living in a location with a median income of 30.41 thousand dollars?

(c) (3 points) The actual salary of one engineer surveyed with a undergraduate GPA of 3.02 living in a location with a median income of \$30,410 was \$73,500. What is the residual for this specific engineer's actual salary using the fitted multivariate linear relationship?

(d) (3 points) Find and interpret the value of R^2 for the fitted multivariate linear relationship.

(e) (2 points) One surveyor suggested adding an interaction term for GPA and the engineer's location's median income. What practical effect would such an interaction have?

(f) (3 points) Does it appear that a using the median income of the engineer's location improved the fit?

7. (10 points) The data reported below are the masses of 10 squirrels (in kg) found on the campus at Iowa State University and 10 squirrels found at University of Iowa this weekend: % latex table generated in R 3.6.0 by xtable 1.8-4 package % Mon May 6 14:50:05 2019

	Squirrel Number									
	1	2	3	4	5	6	7	8	9	10
ISU	3.66	4.37	3.87	3.09	3.38	3.91	3.99	3.92	3.04	0.60
UI	5.90	5.15	4.45	3.29	2.94	4.91	5.70	2.53	2.12	8.50

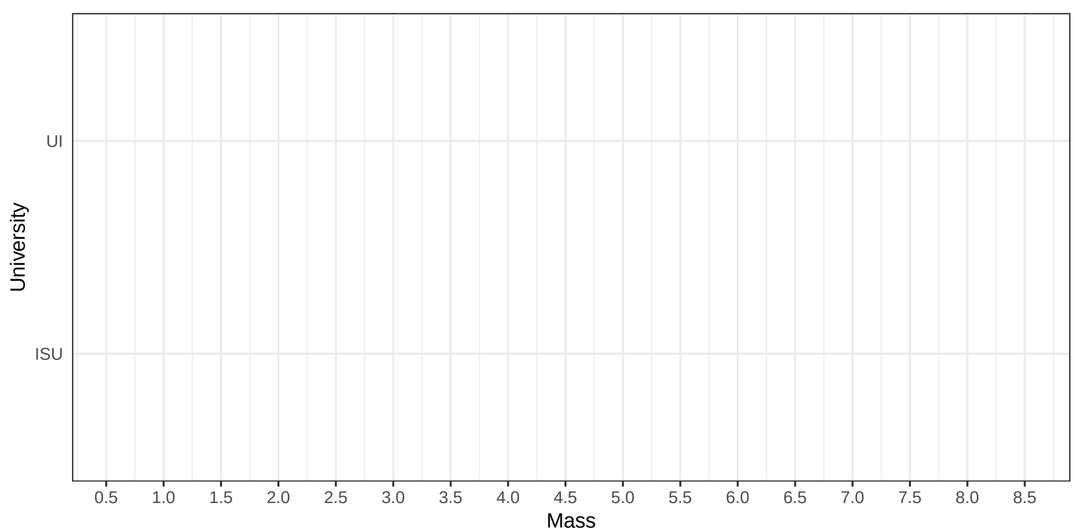
The third quartile of the Iowa State masses is 3.92 kg, the median of the masses is 3.77 kg and the interquartile range is 0.84 kg.

The first quartile of the University of Iowa masses is 2.94 kg, the median of the masses is 4.68 kg and the interquartile range is 2.76 kg.

Using the axes below, create a box plot for **time to failure** of each compound.

- Label the values of the boundaries of the boxes
- Label the values the ends of the upper and lower whiskers
- Draw a star over unusual observations.

For any partial credit, show all work in the additional space



8. Let X be a normal random variable with a mean of -2 and a varaince of 16 (i.e., $X \sim N(-2, 16)$) and let Z be a random variable following a standard normal distribution. Find the following probabilities (note: the attached standard normal probability table may be helpful):

   (a) (2 points) $P(Z \leq 1.0)$

   (b) (2 points) $P(|Z| \leq 2.5)$

   (c) (2 points) $P(-8 \leq X < 1)$

   (d) (3 points) $P(|X| \geq 4)$

9. Suppose that X is a discrete random variable with probability function:

$$f(x) = \begin{cases} 0.75(0.25)^{x-1} & x = 1, 2, 3, \ldots \\ 0 & otherwise \end{cases}$$

   (a) (2 points) What is the probability that $X = 2$?

   (b) (3 points) What is the probability that X takes a value less than 4?

   (c) (3 points) What is the probability that X takes a value greater than 2?

10. The electrical grid of the tiny, struggling republic of Freedonia's capital city depends on two independent (and sometimes functioning) generators. Each generator is functioning approximately 75% of the time, allowing us to describe the number of generators operating at any point in time, X, using a binomial distribution:

$$f_X(x) = \begin{cases} \dfrac{2!}{x!(2-x)!}(0.75)^x(0.25)^{2-x} & x = 0, 1, 2 \\ 0 & otherwise \end{cases}$$

Since the generators are down so frequently, the Freedonian Power Authority has difficulty maintaining power reserves. At any point in time, the amount of reserve power (Y, in GW) can be described using the conditional probability:

$$P(Y \geq y | X = x) = \begin{cases} \exp\left(-\frac{y}{x+1}\right) & y \geq 0 \\ 0 & otherwise \end{cases}$$

   (a) (2 points) At any point in time, what is the probability that both generators are working?

(b) (3 points) If we know that both generators are operating, what is probability that there is more than 1 GW in reserve?

(c) (3 points) Find the probability that at any point in time there is more than 1 GW in reserve *and* 2 generators are working.

(d) (3 points) What is the probability that there is more than 1 GW of power in reserve at any moment?

(e) (3 points) Suppose we know that we have more than 1 GW in reserve. Find the probability that both generators are operational.

11. In an effort to understand smartphone use, the American Association of Psychologists gained access to a single day's data on 625 smartphone users. They found that the smartphone users sampled spent an average of 270 minutes using their phone on that day with a standard deviation of phone use was 32.15 minutes.

    (a) (3 points) Provide a 95% confidence interval for true number of minutes smartphone users spend using thier phones on average.

    (b) (3 points) Provide a 99% confidence lower bound for the number of minutes smartphone users spend using thier phones on average.

    (c) (5 points) A similar study two years ago determined that smartphone users spend an average of 4 hours per day using their phones. Perform a hypothesis test at the $\alpha = 0.05$ significance level for the claim that this is no longer the case.

12. A team of engineers is studying the differences in camera systems on self-driving cars. Their primary concern is in the car's ability to avoid obstacles. Each system was installed in a test car and on 10 consecutive days the car was sent on a 15 hour drive through a closed obstacle course where the number, timing, location, and type of obstacle the car encounters is randomly determined. The proportion of obstacles avoided during the 15 hour drive is recorded below (along with relevant summary statistics):

- System 1: 0.79, 0.87, 0.86, 0.84, 0.86, 0.87, 0.85, 0.81, 0.84, 0.78 (with $\bar{x} = 0.84, s^2 = 0.0011$)
- System 2: 0.78, 0.77, 0.77, 0.83, 0.8, 0.81, 0.82, 0.75, 0.76, 0.75 (with $\bar{x} = 0.78, s^2 = 0.0008$)

(a) (3 points) Provide a 95% confidence interval for the mean proportion of obstacles avoided on the course using System 1.

(b) (3 points) Provide a 95% confidence interval for the mean proportion of obstacles avoided on the course using System 2.

(c) (3 points) Assuming that the proportion of obstacles avoided is roughly normally distributed and that the both systems have the same variance in proportion of obstacles avoided, provide a 95% confidence interval for the difference in the average proportion of obstacles avoided by two systems.

(d) (3 points) Does this previous confidence interval provide any clear evidence which system is best able to avoid obstacles on the course? Explain.

13. A small military subcontractor has secured a contract to develop drones capable of providing light air support for smaller naval vessels. Successfully fulfilling the contract requires that the drone be able to takeoff with a minimum runway of 10 meters. However, late changes in the prototype's weight distribution have led to concerns that they are no longer satisfying this requirement. Using 100 takeoffs, they found the average distance before takeoff was 9.92 meters with a standard deviation of 0.4 meters.

    (a) (10 points) Conduct a hypothesis test at the $\alpha = 0.01$ significance level for μ, the true average distance before takeoff with the null hypothesis $\mu \geq 10$ against the alternative hypothesis of $\mu < 10$. Include the hypothesis statement, the test statistic, the p-value, and the conclusion.

    (b) (3 points) Does the drone prototype meet the contract's runway requirements or will they need to redesign? Explain.