Show **all** of your work on this assignment and answer each question fully in the given context.

Please staple your assignment!

1. Ch. 5.1, Exercise 1, pg. 243 A discrete random variable X can be described using the probability function, $f(x)$:

x	2	3	4	5	6
f(x)	0.1	0.2	0.3	0.3	0.1

   (a) Plot $F(x)$, the cumulative probability function for X.[5 pts]
   (b) Find the mean and standard deviation of X.[10 pts]

2. Ch. 5, Exercise 1, pg. 322: Suppose 90% of all students taking a beginning programming class fail to get their first program to run on first submission. Use a binomial distribution and assign probabilities to the possibilities that among a group of six such students,

   (a) all fail on their first submissions[5 pts]
   (b) at least four fail on their first submissions[5 pts]
   (c) less than four fail on their first submissions [5 pts]
   Continuing to using this binomial model,
   (d) what is the mean number who will fail?[5 pts]
   (e) what are the variance and standard deviation of the number who will fail?[5 pts]

3. Ch. 5, Exercise 2, pg. 322: Suppose that for single launches of a space shuttle, there is a constant probability of O-ring failure (say .15), Consider ten future launches, and let X be the number of those involving an O-ring failure. Use an appropriate probability model and evaluate all of the following:

   (a) Precisely state the distribution of X, giving the values of any parameters necessary.[5 pts]
   (b) $P[X = 2]$[5 pts]
   (c) $P[X \geq 1]$[5 pts]
   (d) EX[5 pts]
   (e) Var$X$[5 pts]
   (f) the standard deviation of X[5 pts]

4. Ch. 5.1, Exercise 6, pg. 244: Suppose that an eddy current nondestructive evaluation technique for identifying cracks in critical metal parts has a probability of about .20 of detecting a single crack of length .003in. in a certain material. Let Y be the number of specimens inspected in order to obtain the first crack detection. Use an appropriate probability model and evaluate all of the following:

   (a) Precisely state the distribution of X, giving the values of any parameters necessary.[5 pts]
   (b) $P[Y = 5]$[5 pts]

(c) $P[Y \leq 4]$[5 pts]

(d) EY[5 pts]

(e) Var$Y$[5 pts]

(f) SD(Y)[5 pts]

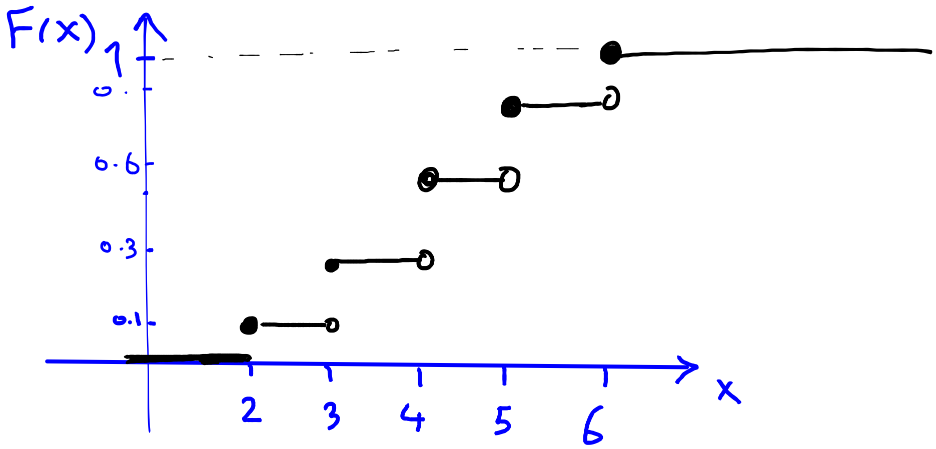
Total: 100 pts

Key:

7, (a)

x	2	3	4	5	6
$f(x)$	0.1	0.2	0.3	0.3	0.1
$F(x)$	0.1	0.3	0.6	0.9	$\boxed{1}$

CDF $\longrightarrow$ $F(x)$



(b) $Ex = ?$

x	2	3	4	5	6
$f(x)$	0.1	0.2	0.3	0.3	0.1
$xf(x)$	0.2	0.6	1.2	1.5	0.6
$x^2f(x)$	0.4	1.8	4.8	7.5	3.6

$$Ex = \sum x f(x) = 0.2 + 0.6 + 1.2 + 1.5 + 0.6 = \boxed{4.1}$$

$$Ex^2 = \sum x^2 f(x) = 0.4 + 1.8 + 4.8 + 7.5 + 3.6 = \boxed{18.1}$$

$$var(x) = \sum (x - Ex)^2 f(x) = Ex^2 - (Ex)^2$$

$$= 18.1 - (4.1)^2 = 1.29$$

So, the standard deviation is:

$$SD(X) = \sqrt{var\, x} = \sqrt{1.29} = \boxed{1.13}$$

problem 2)

Let x be the random variable connected to the number of students who fail to get their first program to run. Then

$$X \sim \text{Binomial}(n=6, p=0.9)$$

(a) P("all students fail to run their first program")

$$= P(X=6) = \frac{6!}{6!(6-6)!}(0.9)^6(1-0.9)^0$$

$$= (0.9)^6 = 0.531441$$

(b) P("at least 4 students fail") $= P(X \geq 4)$

$$= P(X=4) + P(X=5) + P(X=6)$$

$$= \frac{6!}{4!\underbrace{(6-4)!}_{2!}} \cdot (0.9)^4(1-0.9)^{6-4} + \frac{6!}{5!\underbrace{(6-5)!}_{1!}}(0.9)^5(1-0.9)^{6-5}$$

(This shows how to cancel the factorial terms)

$$+ \frac{\cancel{6!}}{\cancel{6!}\underbrace{(6-6)!}_{0!}}(0.9)^6\underbrace{(1-0.9)^0}_{1}$$

$$= \frac{3\ 6\times5\times\cancel{4\times3\times2\times1}}{(\cancel{4\times3\times2\times1})\times(\cancel{2}\times1)}(.9)^4(.1)^2 + \frac{6\times\cancel{5\times4\times3\times2\times1}}{(\cancel{5\times4\times3\times2\times1})\times1}(.9)^5(.1)^1$$

$$+ (.9)^6$$

$= (3\times5)(0.9)^4(0.1)^2 + 6(0.9)^5(0.1)^1 + (0.9)^6$

$= 0.9841$

(c) $P(\text{"less than 4 students Fail"})$

$= P(X<4) = P(X\leq 3) = P(X=0)+P(X=1)+P(X=2)+P(X=3)$

$= \dots$

But, you can use the complement trick as you found the $P(X\geq 4)$ in part (b).

$P(X<4) = 1-P(X\geq 4) = 1-0.9841 = \boxed{0.0159}$

(d) EX = the mean (expected) number of fails.

$X \sim \text{Binomial}(n=6, p=0.9)$

$\Rightarrow EX = n.p = 6(0.9) = \boxed{5.4}$

It is expected that 5.4 student fail on their first programming.

(e) $Var(X) = ?$ in Binomial distribution,

$$Var(X) = np(1-p)$$

So, $Var(X) = 6(.9)(1-.9) = 6(.9)(.1)$

$$= \boxed{0.54}$$

and the standard variation is

$$SD(X) = \sqrt{Var(X)} = \sqrt{0.54} = \boxed{0.734}$$

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problem 3:

(a) $X \sim Binomial(n = 10, p = 0.15)$

(b) $P(X=2) = ?$

we know in Binomial, $P(X=x) = \dfrac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$

So,

$$P(X=2) = \frac{10!}{2!(10-2)!} \cdot (0.15)^2 (1-0.15)^{10-2}$$

$$= \frac{10!}{2!\,8!} (0.15)^2 (0.85)^8$$

$$= \frac{\cancelto{5}{10} \times 9 \times \cancel{8!}}{\cancel{2!}\ \cancel{8!}} (0.15)^2 (0.85)^8$$

$$= 5 \times 9 \times (0.15)^2 \times (0.85)^8$$

$$= \boxed{0.2758}$$

(c) $P(X \geq 1) = \sum_{x=1}^{10} P(X=x) = P(X=1) + P(X=2) + \cdots + P(X=10)$

(This is a very long way!)

So, use the compliment:

$$P(X \geq 1) = 1 - P(X<1) = 1 - P(X=0)$$

$$= 1 - \left[\frac{\cancel{10!}}{0!\ \cancel{(10-0)!}} \cdot \underbrace{(0.15)^0}_{1} \underbrace{(1-0.15)}_{0.85}{}^{10-0} \right]$$

$$= 1 - (0.85)^{10}$$

$$= \boxed{0.8031}$$

(d) $EX = ?$ Since we know $X \sim Binomial(10, 0.15)$, we can easily get EX.

$$EX = n.p = 6(0.15) = \boxed{0.9}$$

(e) $var(X) = ?$

$$Var(X) = n.p(1-p) = 6(0.15)(1-0.15)$$
$$= 6(0.15)(0.85)$$
$$= \boxed{0.765}$$

(f) standard deviation of X:

$$SD(X) = \sqrt{var\ X} = \sqrt{0.765} = \boxed{0.874}.$$

Problem 4)

(a) Y = The number of specimens inspected in order to see the first crack

$\Rightarrow Y \sim Geometric(p = 0.2)$

(b) we know that $Y \sim Geom(p)$,

$$f_Y(y) = p(1-p)^{y-1}, \quad y = 1, 2, 3, \ldots$$

$$P(Y=5) = p(1-p)^{5-1}$$

$$= (0.2)(1-0.2)^4$$

$$= (0.2)(0.8)^4$$

$$= 0.08192$$

(c) $P(Y \leq 4) = F_Y(4)$

Note: Geometric (unlike Binomial & Poisson) has a closed form CDF.

$$Y \sim Geom(p) \Rightarrow F_Y(y) = P(Y \leq y) = 1-(1-p)^y$$

$$P(Y \leq 4) = F_Y(4) = 1-(1-p)^4$$

$$= 1-(1-0.2)^4$$

$$= 1-(0.8)^4$$

$$= 0.5904$$

(d) we know in Geometric distribution,

$$EY = 1/p$$

$$\text{var } Y = \frac{1-p}{p^2}$$

So, $EY = \frac{1}{0.2} = 5$

(This means it is expected that we need 5 specimens inspected to observe the first crack)

(e) $\text{var}(Y) = \frac{1-p}{p^2} = \frac{1-0.2}{(0.2)^2} = \frac{0.8}{0.04} = \boxed{20}$

Then the $SD(Y) = \sqrt{\text{var}(Y)}$

$= \sqrt{20}$

$= 4.47$