Inference for matched pairs and two-sample data

An important type of application of confidence interval estimation and significance testing is when we either have *paired data* or *two-sample* data.

Recall: Matched pairs

Paired data is bivariate responses that consists of several determinations of basically the same characteristics

> **Example:**
>
> - Practice SAT scores *before* and *after* a preperation course
> - Severity of a disease *before* and *after* a treatment
> - Fuel economy of cars *before* and *after* testing new formulations of gasoline

Inference for matched pairs and two-sample data

One simple method of investigating the possibility of a consistent difference between paired data is to

> 1. Reduce the measurements on each object to a single difference between them
> 2. Methods of confidence interval estimation and significance testing applied to differences (using Normal or t distributions when appropriate)

Example:[Fuel economy]

Twelve cars were equipped with radial tires and driven over a test course. Then **the same twelve cars** (with the same drivers) were equipped with regular belted tires and driven over the same course.

After each run, the cars gas economy (in km/l) was measured. Using significance level $\alpha = 0.05$ and the method of critical values, test for a difference in fuel economy between the radial tires and belted tires.

Construct a 95% confidence interval for true mean difference due to tire type. (i.e μ_d)

car	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0	11.0	12.0
radial	4.2	4.7	6.6	7.0	6.7	4.5	5.7	6.0	7.4	4.9	6.1	5.2
belted	4.1	4.9	6.2	6.9	6.8	4.4	5.7	5.8	6.9	4.7	6.0	4.9

Hypothesis Testing

Null

Alternative

P-value

CI method

Matched Pairs

Two-sample

Example:[Fuel economy]

car	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0	11.0	12.0
radial	4.2	4.7	6.6	7.0	6.7	4.5	5.7	6.0	7.4	4.9	6.1	5.2
belted	4.1	4.9	6.2	6.9	6.8	4.4	5.7	5.8	6.9	4.7	6.0	4.9
d	0.1	-0.2	0.4	0.1	-0.1	0.1	0.0	0.2	0.5	0.2	0.1	0.3

Since we have paired data, the first thing to do is to find the differences of the paired data. ($d = d_1 - d_2$, where d_1 is associated with radial and d_2 is associated with belted tires.)

Then writing down the information available:

$$n = 12, \quad \bar{d} = 0.142, \quad s_d = 0.198$$

$$\bar{d} = \frac{1}{n}\sum_{i=1}^{n} d_i, \quad s_d^2 = \frac{1}{n-1}\sum_{i=1}^{n}(d_i - \bar{d})^2$$

Then we just need to apply steps of hypothesis testing. Note that the null hypothesis here is that **there is no difference** between the gas economy recorded of the two different tires.(i.e $\mu_d = 0$)

Example:[Fuel economy]

1- $H_0 : \boxed{\mu_d = 0} \, vs. \quad H_1 : \ \mu_d \neq 0$

2- $\alpha = 0.05$

3- I will use the test statistics $K = \frac{\bar{d}-0}{s_d/\sqrt{n}}$ which has a t_{n-1} distribution assuming that

- H_0 is true and
- $d_1, d_2, \cdots, d_{12}$ are iid $N(\mu_d, \sigma_d^2)$

critical value

Example:[Breaking strength of wire, cont'd]

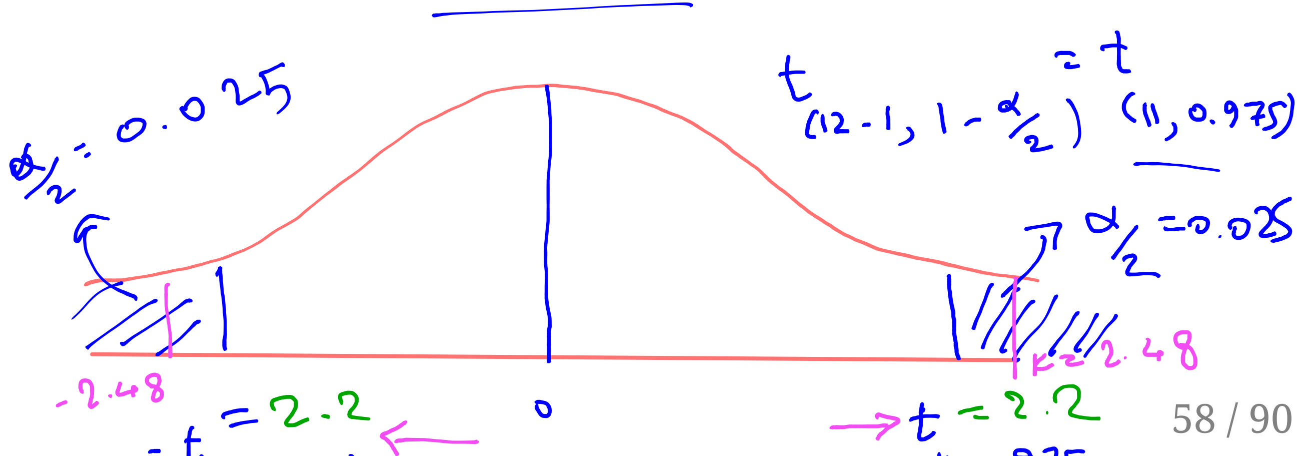
4- $K = \frac{0.421}{0.198/\sqrt{12}} = 2.48 \sim t_{(11,0.975)}$.

$$\begin{aligned} p-value &= P(|T|>K) = P(|T|>2.48) \\ &= P(T>2.48)+P(T<-2.48) \\ &= 1-P(T<2.48)+P(T<-2.48) \\ \text{(by software)} &= 1-0.9847+0.9694 = 0.03 \end{aligned}$$

5- Since p-value < 0.05, we **reject** H_0.

6- There is enough evidence to conclude that fuel economy differs between radial and belted tires.

$\alpha = 0.05$



(11,.975)

p-value < α

Example:[Breaking strength of wire, cont'd]

A two-sided 95% confidence interval for the true mean fuel economy **difference** is

$$\left(\bar{d} - t_{(n-1,1-\alpha/2)}\frac{s_d}{\sqrt{n}}\ ,\ \bar{d} + t_{(n-1,1-\alpha/2)}\frac{s_d}{\sqrt{n}}\right)$$

$$= \left(0.142 - t_{(11,0.975)}\frac{0.198}{\sqrt{12}}\ ,\ 0.142 + t_{(11,0.975)}\frac{0.198}{\sqrt{12}}\right)$$

$$= \left(0.142 - \boxed{2.2}\frac{0.198}{\sqrt{12}}\ ,\ 0.142 + 2.2\frac{0.198}{\sqrt{12}}\right)$$

$$= (0.0164\ ,\ 0.2764)$$

Note that $d = d_1 - d_2$, so the interpretation will be:

We are 95% confident that the radial tires get between 0.0166 km/l and 0.2674 km/l more in fuel economy than belted tires on average

Hang on for a Second

Let's review slide 58 again

Example:[Breaking strength of wire, cont'd]

$$
\begin{aligned}
p - value &= P(|T| > K) = P(|T| > 2.48) \\
&= P(T > 2.48) + P(T < -2.48) \\
&= 1 - P(T < 2.48) + P(T < -2.48) \\
(\text{by software}) &= 1 - 0.9847 + 0.9694 = 0.03
\end{aligned}
$$

We have seen t-student table

How do we get that p-value usin software!!!

What is happening?

Hypothesis Testing

Null

Alternative

P-value

CI method

Matched Pairs

Two-sample

Unlike *standard Normal distribution* table which gives us probability under the standard Normal curve, t tables are quantile tables.

i.e We use the t table (Table B.4 in Vardeman and Jobe) to calculate quantiles.

To have exact probabilities, we need software.

Table B.4
t Distribution Quantiles

ν	$Q(.9)$	$Q(.95)$	$Q(.975)$	$Q(.99)$	$Q(.995)$	$Q(.999)$	$Q(.9995)$
1	3.078	6.314	12.706	31.821	63.657	318.317	636.607
2	1.886	2.920	4.303	6.965	9.925	22.327	31.598
3	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	1.476	2.015	2.571	3.365	4.032	5.893	6.869

e.g. we use these quantiles to make confidence intervals.

$$\bar{x} \pm t_{(n-1,\, 1-\alpha/2)} \cdot s/\sqrt{n}$$

a quantile, not probability.

The approach in calculating p-value when t distribution is involved

Two important points:

> P-value and α are $\boxed{\text{both probabilities.}}$ (so $\in [0, 1]$).
>
> They are areas under the curve in tails under *null hypothesis*.

H_0

Hypothesis Testing

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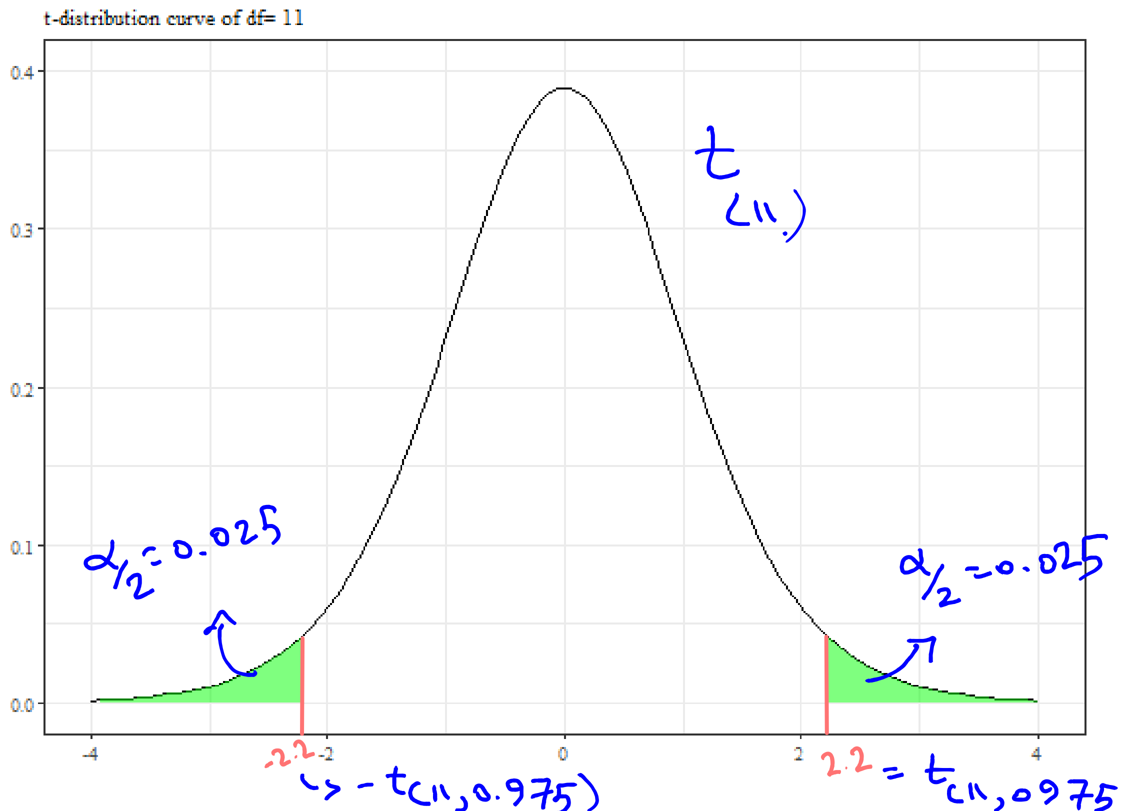
from example [breaking strength]

For a random variable with $\sim t_{(11,0.975)}$:

By the t table, the t quantile of $t_{(11,0.975)}$ is 2.2.



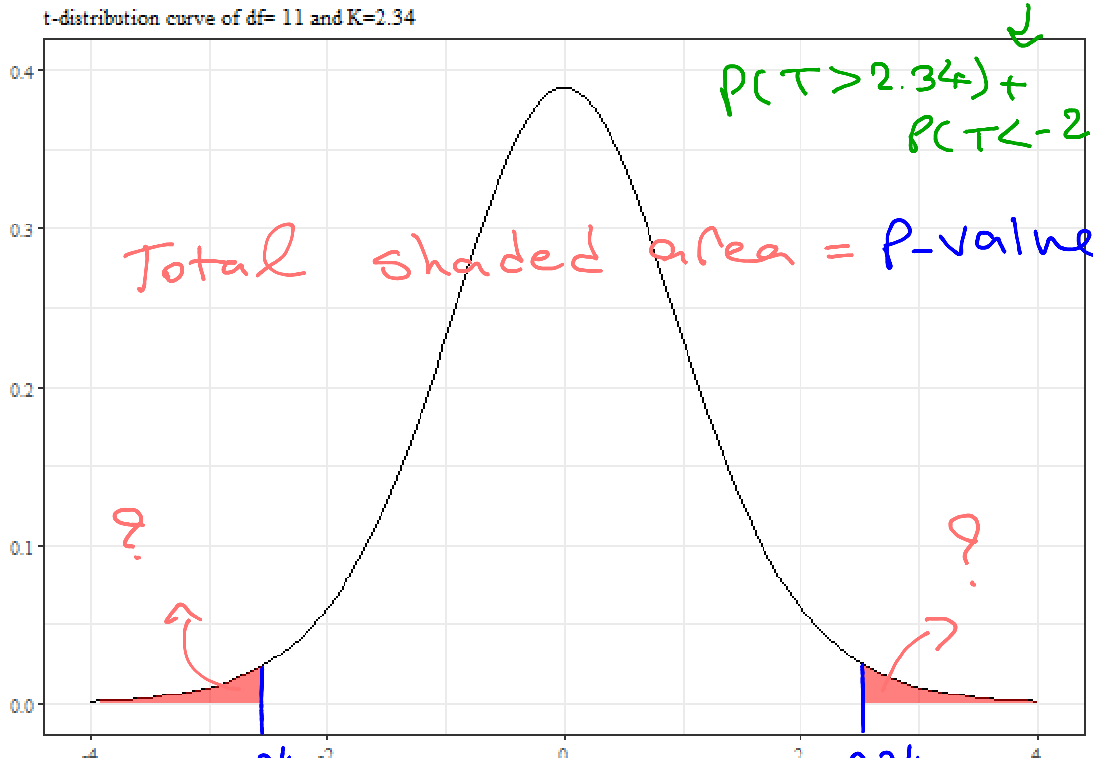
Total shaded area is $\alpha/2 + \alpha/2 = \alpha = 0.05$ (The significance level)

For the critical value we calculated under the null hypothesis:

The critical value calculated is $K = 2.34$

Recall: P-value $= P(|T| > K) = P(|T| > 2.34)$

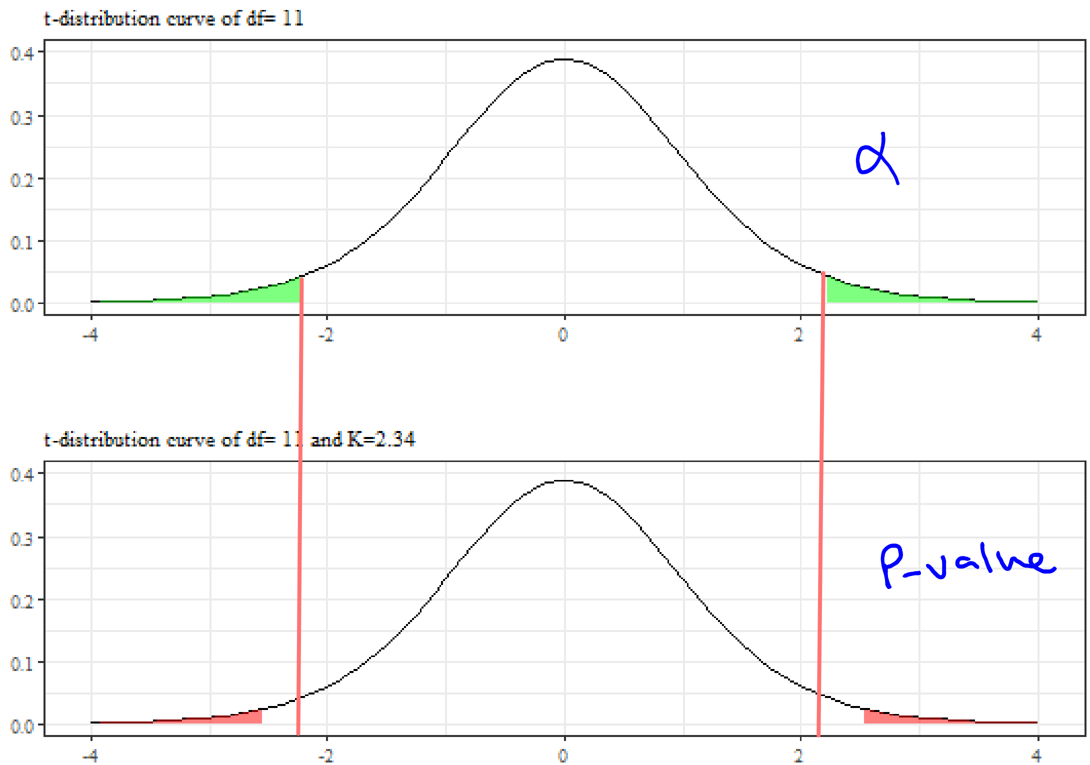
$P(T > 2.34) + P(T < -2.34)$



Here, we need software to find the total shaded area!

Both together

p-value $\leftarrow K \gtrless t_{(n-1,\, 1-\alpha/2)} \rightarrow \alpha$



We reject the null if p-value < α.

> Remember p-value and α are areas under the curve

p-value < α

* Total green area : $\alpha = 0.05$

* Total red area : $P(|T| > K)$ = P-value = ?

(we don't need to find ? just need

→ to know if ? $> \alpha$ or ? $< \alpha$)

① If the red area (p-value) $< \alpha$ (= 0.05 in this problem)

⇒ Reject H_0.

② If the red area (p-value) $> \alpha$ (= 0.05 in this problem)

⇒ Fail to Reject H_0

The steps for p-value :

calculate p-value using table on slide 39.

1 - If you use Z => use Normal table to find p-value

=> Then compare p-value & α (given in the problem)

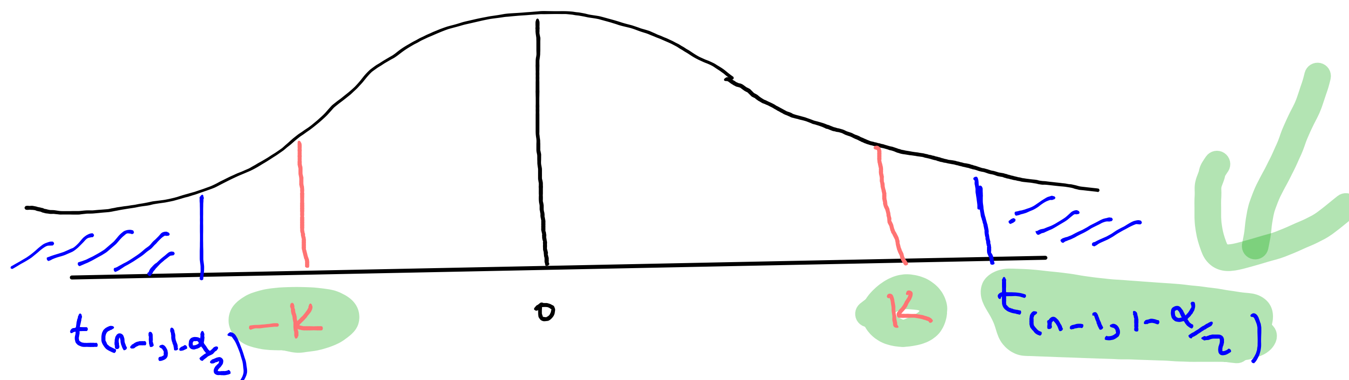
to reject or fail to reject H_0

2 - If you use T statistic, find the quantile

$\rightarrow t_{(n-1, 1-\alpha/2)}$ (or $t_{(n-1, 1-\alpha)}$ depending on the problem).

$\nearrow$

(I suggest quickly plot $t_{(n-1, 1-\alpha/2)}$ to understand better)



• write values of $\underline{K}$ (critical value) and the corresponding $t_{(n-1, 1-\alpha/2)}$

$$K = \frac{\bar{x} - \#}{s/\sqrt{n}}$$

$H_0: \mu = \#$

$H_a: \mu \neq \#$

• Note: area under the curve corresponding to $t_{(n-1, 1-\alpha/2)}$ is equal to α (e.g 0.05)

. Note: area under the curve corresponding to k (critical value) is p-value (which we don't need the exact value)

- Now compare the areas under the curve.

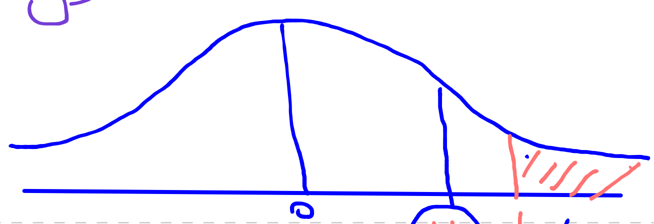
if p-value $< \alpha$ $\Rightarrow$ reject H_0

if p-value $> \alpha$ $\Rightarrow$ fail to reject H_0

Note Note Note:

* α is NOT always 0.05 (be careful)

* The test is NOT always two sided!



(k) $t_{(n-1,\, 1-\alpha)}$

Hypothesis Testing

Null

Alternative

P-value

CI method

Matched Pairs

Two-sample

Example:[End-cut router]

Consider the operation of an end-cut router in the manufacture of a company's wood product. Both a leading-edge and a trailing-edge measurement were made on each wooden piece to come off the router.

Paired data

Is the leading-edge measurement different from the trailing-edge measurement for a typical wood piece?

(Let's see if there's any difference between the measurement

Do a hypothesis test at $\alpha = 0.05$ to find out. Make a two-sided 95% confidence interval for the true mean of the difference between the measurements.

piece	1.000	2.000	3.000	4.000	5.000
leading_edge	0.168	0.170	0.165	0.165	0.170
trailing_edge	0.169	0.168	0.168	0.168	0.169
Difference: d_i	-.001	.002	-.003	-.003	.001

(-)

$n = 5$

$\bar{d} = \frac{1}{5}(-.001 + .002 - .003 - 0.003 + .001) = -8\times10^{-4}$

$S_d = \frac{1}{n-1}\sum_{i=1}^{5}(d_i - \bar{d})^2 = 0.0023$

Steps:

① $H_0: \mu_d = 0$ (There's no difference between measurements)

$H_a: \mu_d \neq 0$ (There's difference between measurements)

② $\alpha = 0.05$

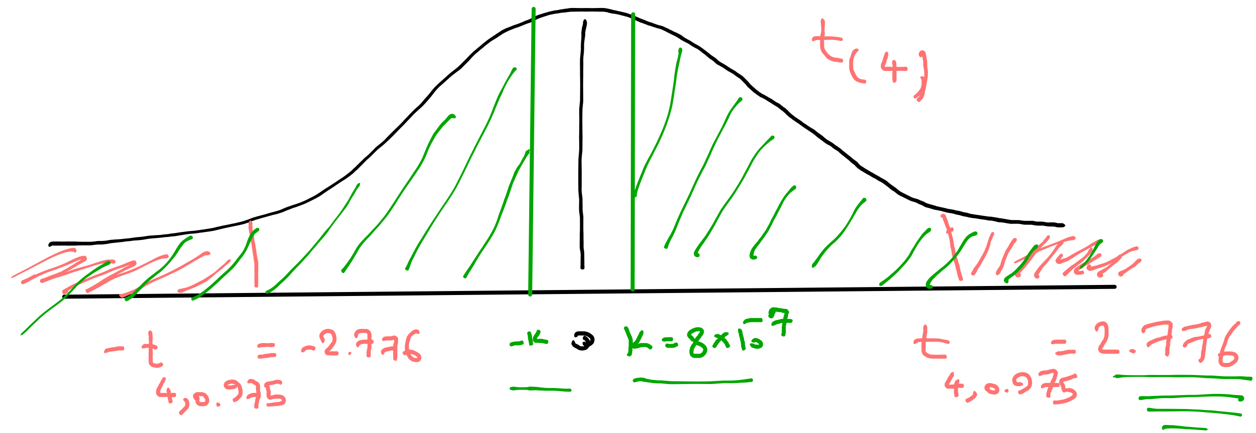
③ since σ_d is unknown & $n = 5 < 25$, I will use

④ $$K = \frac{\mu_d - 0}{\frac{s_d}{\sqrt{n}}} = \frac{-8\times10^{-4} - 0}{\frac{0.0023}{\sqrt{5}}} = \boxed{-8\times10^{-7}}$$

$$\text{p-value} = P(|T| > |K|) = P(|T| > |-8\times10^{-7}|)$$

$$= P(|T| > 8\times10^{-7})$$

by the table: $t_{(n-1, 1-\alpha/2)} = t_{(5-1, 0.975)} = 2.776$



(obviously area under the curve corresponding to p-value > area under the curve corresponding to $t_{4,0.975}$ $(= \alpha)$)

⑤ since p-value > α, we fail to reject H_0.

⑥ there is not enough evidence to conclude that

There's significant difference between the leading edge and trailing edge on average

Using 95% CI method:

$$\longrightarrow \bar{d} \pm \boxed{t_{(n-1,\, 1-\alpha/2)}} \, S_d/\sqrt{n}$$

$$\left\{ \begin{array}{l} H_0: \mu_d = 0 \\ H_a: \mu_d \neq 0 \end{array} \right.$$

$$= -8\times10^{-4} \pm t_{(4,\, 0.975)} \cdot \frac{0.0023}{\sqrt{5}}$$

$$= -8\times10^{-4} \pm 2.776\,(0.001)$$

$$\Rightarrow = (-0.00358,\ 0.00198)$$

Since the 95% CI contains zero, we fail to reject H_0. There's not enough evidence to conclude that the leading edge measurement is significantly different

from the trailing edge measurement.

Two-Sample Data

Two-sample data

Paired differences provide inference methods of a special kind for comparison. Methods that can be used to compare two means where two different *unrelated* samples will be discussed next.

> SAT score of high school A vs. high school B
>
> Severity of a disease in men vs. women
>
> Height of Liverpool soccerr players vs. Man United soccer players
>
> Fuel economy of gas formula type A vs. formula type B

Hypothesis Testing

Null

Alternative

P-value

CI method

Matched Pairs

Two-sample

Two-sample data

Notations:

	Sample 1	Sample 2
Sample size	n_1	n_2
true means	μ_1	μ_2
Sample means	$\bar{x}_1$	$\bar{x}_2$
true variance	σ_1^2	σ_2^2
Sample variance	s_1^2	s_2^2

Large Samples

Hypothesis Testing

Null

Alternative

P-value

CI method

Matched Pairs

Two-sample

(want to compare two true means μ_1, μ_2)

Large samples $(n_1 \geq 25, n_2 \geq 25)$

$H_0: \mu_1 = \mu_2 \Rightarrow \boxed{\mu_1 - \mu_2 = 0}$ vs. $H_a: \boxed{\mu_1 - \mu_2 \neq 0}$

The difference in sample means $\overline{x}_1 - \overline{x}_2$ is a natural statistic to use in comparing μ_1 and μ_2.

$\mu_1 - \mu_2 < 10$

or $\mu_1 - \mu_2 > 15$

i.e

$$\mathrm{E}(\overline{X}_1) = \mu_1 \quad \mathrm{E}(\overline{X}_2) = \mu_2 \quad \mathrm{Var}(\overline{X}_1) = \frac{\boxed{\sigma_1^2}}{n_1} \quad \mathrm{Var}(\overline{X}_2) = \frac{\sigma_2^2}{n_2}$$

If σ_1 and σ_2 are **known**, then we have

$$\mathrm{E}(\overline{X}_1 - \overline{X}_2) = \mathrm{E}(\overline{X}_1) - \mathrm{E}(\overline{X}_2) = \mu_1 - \mu_2$$

$$\mathrm{Var}(\overline{X}_1 - \overline{X}_2) = \mathrm{Var}(\overline{X}_1) + \mathrm{Var}(\overline{X}_2) = \boxed{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$= \mathrm{Var}(\overline{X}_1) + (-1)^2 \mathrm{Var}(\overline{X}_2)$$

Large samples $(n_1 \geq 25, n_2 \geq 25)$

If, in addition, n_1 and n_2 are large,

$\overline{X}_1 \sim N(\mu_1, \frac{\sigma_1^2}{n_1})$ is independent of $\overline{X}_2 \sim N(\mu_2, \frac{\sigma_2^2}{n_2})$ (by CLT).

So that $\overline{X}_1 - \overline{X}_2$ is approximately Normal (trust me)

$$\rightarrow Z = \frac{\overline{X}_1 - \overline{X}_2 - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \sim N(0, 1)$$

Previously: $Z = \frac{\bar{X} - \mu}{\sqrt{\sigma^2 / n}} \sim N(0,1)$

(one sample)

Large samples $(n_1 \geq 25, n_2 \geq 25)$

So, if we want to test $\boxed{H_0 : \mu_1 - \mu_2 = \#}$ with some alternative hypothesis, σ_1 and σ_2 are known, and $n_1 \geq 25, n_2 \geq 25$, then we use the statistic

$$\boxed{K = \frac{\overline{X}_1 - \overline{X}_2 - (\#)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}}$$

which has a $N(0, 1)$ distribution if

> 1. H_0 is true
> 2. The sample 1 points are iid with mean μ_1 and variance σ_1^2, and the sample 2 points are iid with mean μ_2 and variance σ_2^2.
>
> 3.Sample I is independent of sample II

Large samples $(n_1 \geq 25, n_2 \geq 25)$

The confidence intervals (2-sided, 1-sided upper, and 1-sided lower, respectively) for $\mu_1 - \mu_2$ are:

- *Two-sided* $100(1-\alpha)\%$ confidence interval for $\mu_1 - \mu_2$

$$\underline{(\overline{x}_1 - \overline{x}_2)} \pm z_{1-\alpha/2} * \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

- *One-sided* $100(1-\alpha)\%$ confidence interval for $\mu_1 - \mu_2$ with a upper confidence bound

$$(-\infty\ ,\ (\overline{x}_1 - \overline{x}_2) \pm z_{1-\alpha} * \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}})$$

- *One-sided* $100(1-\alpha)\%$ confidence interval for μ with a lower confidence bound

$$((\overline{x}_1 - \overline{x}_2) \pm z_{1-\alpha} * \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}})\ ,\ \ +\infty)$$

Large samples $(n_1 \geq 25, n_2 \geq 25)$

If σ_1 and σ_2 are **unknown**, and $n_1 \geq 25, n_2 \geq 25$, then we use the statistic

replace σ_1^2 by s_1^2 and σ_2^2 by s_2^2

$$K = \frac{\overline{X}_1 - \overline{X}_2 - (\#)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

and confidence intervals (2-sided, 1-sided upper, and 1-sided lower, respectively) for $\mu_1 - \mu_2$:

- *Two-sided* $100(1-\alpha)\%$ confidence interval for $\mu_1 - \mu_2$

$$(\overline{x}_1 - \overline{x}_2) \pm z_{1-\alpha/2} * \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

Large samples $(n_1 \geq 25, n_2 \geq 25)$

- *One-sided* $100(1-\alpha)\%$ confidence interval for $\mu_1 - \mu_2$ with a upper confidence bound

$$\left(-\infty\ ,\ (\bar{x}_1 - \bar{x}_2) \pm z_{1-\alpha} * \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}\right)$$

- *One-sided* $100(1-\alpha)\%$ confidence interval for μ with a lower confidence bound

$$\left((\bar{x}_1 - \bar{x}_2) \pm z_{1-\alpha} * \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}\ ,\ +\infty\right)$$

Hypothesis Testing

Null

Alternative

P-value

CI method

Matched Pairs

Two-sample

Example:[Anchor bolts]

An experiment carried out to study various characteristics of anchor bolts resulted in 78 observations on shear strength (kip) of 3/8-in. diameter bolts and 88 observations on strength of 1/2-in. diameter bolts.

Let Sample 1 be the 1/2 in diameter bolts and Sample 2 be the 3/8 indiameter bolts.

Using a significance level of $\alpha = 0.01$, find out if the 1/2 in bolts are more than 2 kip stronger (in shear strength) than the 3/8 in bolts. Calculate and interpret the appropriate 99% confidence interval to support the analysis.

given info.

- $n_1 = 88, n_2 = 78$
- $\overline{x}_1 = 7.14, \overline{x}_2 = 4.25$
- $s_1 = 1.68, s_2 = 1.3$

① $H_0: \mu_1 - \mu_2 = 2$ vs. $H_a: \mu_1 - \mu_2 > 2$

② $\alpha = 0.01$

Hypothesis Testing

Null

Alternative

P-value

CI method

Matched Pairs

Two-sample

Example:[Anchor bolts]

- $n_1 = 88, n_2 = 78$
- $\overline{x}_1 = 7.14, \overline{x}_2 = 4.25$
- $s_1 = 1.68, s_2 = 1.3$

③ since $n_1, n_2 > 25$, I'll use

$$\boxed{K = \frac{(\bar{x}_1 - \bar{x}_2) - 2}{\sqrt{s_1^2/n_1 + s_2^2/n_2}}}$$

If we assume H_0 is true, sample I is iid with mean μ_1, variance σ_1^2 independent of sample II iid with mean μ_2 and variance σ_2^2, $K \approx N(0,1)$

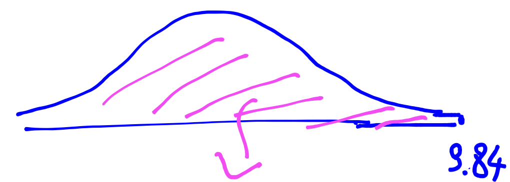
④ $k = \dfrac{(\bar{x}_1 - \bar{x}_2) - 2}{\sqrt{s_1^2/n_1 + s_2^2/n_2}} = \dfrac{(7.14 - 4.25) - 2}{\sqrt{\dfrac{1.68^2}{88} + \dfrac{1.3^2}{78}}} = \boxed{3.84}$

p-value $= P(Z > k) = P(Z > 3.84)$

$H_a: \mu_1 - \mu_2 > 2$

$= 1 - \Phi(3.84)$, $\Phi(3.84) \approx 1$

$\approx 1 - 1 \approx 0$



$\Phi(3.84)$

⑤ with a p-value $\approx 0 < \alpha = 0.01$, we reject H_0 in favor of H_a.

⑥ There's enough evidence that the $\frac{1}{2}$ in bolts are more than 2 kip stronger than $\frac{3}{8}$ in bolts on average.

99% lower bound CI (since $H_a: \mu_1 - \mu_2 > 2$)

$$\left(\bar{X}_1 - \bar{X}_2 - Z_{1-\alpha} \sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}} \, , +\infty \right)$$

$$= \left((7.14 - 4.25) - Z_{1-0.01} \sqrt{\frac{1.68^2}{88} + \frac{1.3^2}{78}} \, , +\infty \right)$$

$$Z_{.99}$$

$$= \left(2.89 - \boxed{2.33}(0.232) \, , +\infty \right)$$

$$= (2.35, +\infty)$$

we're 99% confident that the true mean strength of the ½ in bolts is at least 2.35 kip stronger than the true mean strength of the 3/8 in bolts.

Small Samples

Hypothesis Testing

Null

Alternative

P-value

CI method

Matched Pairs

Two-sample

Small samples

If $n_1 < 25$ or $n_2 < 25$, then we need some **other assumptions** to hold in order to complete inference on two-sample data.

> We need two independent samples to be iid Normally distributed and $\sigma_1^2 \approx \sigma_2^2$

A test statistic to test $\mathrm{H}_0 : \mu_1 - \mu_2 = \#$ against some alternative is

$$K = \frac{\overline{X}_1 - \overline{X}_2 - (\#)}{S_p\sqrt{(\frac{1}{n_1} + \frac{1}{n_2})}}$$

where S_p^2 is called **pooled sample variance** and is defined as

$$S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}$$

Hypothesis Testing

Null

Alternative

P-value

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Matched Pairs

Two-sample

Small samples

Also assuming

- H_0 is true,
- The sample 1 points are iid $N(\mu_1, \sigma_1^2)$, the sample 2 points are iid $N(\mu_2, \sigma_2^2)$,
- and the sample 1 points are independent of the sample 2 points and $\sigma_1^2 \approx \sigma_2^2$.

Then

$$K = \frac{\overline{X}_1 - \overline{X}_2 - (\#)}{S_p\sqrt{(\frac{1}{n_1} + \frac{1}{n_2})}} \sim t_{(n_1+n_2-2)}$$

Small samples

$1-\alpha$ confidence intervals (2-sided, 1-sided upper, and 1-sided lower, respectively) for $\mu_1-\mu_2$ under these assumptions are of the form:

(let $\nu = n_1 + n_2 - 2$)

- *Two-sided* $100(1-\alpha)\%$ confidence interval for $\mu_1-\mu_2$

$$(\overline{x}_1-\overline{x}_2) \pm t_{(\nu,1-\alpha/2)} * S_p\sqrt{(\frac{1}{n_1}+\frac{1}{n_2})}$$

- *One-sided* $100(1-\alpha)\%$ confidence interval for $\mu_1-\mu_2$ with a upper confidence bound

$$(-\infty\ ,\ (\overline{x}_1-\overline{x}_2) + t_{(\nu,1-\alpha)} * S_p\sqrt{(\frac{1}{n_1}+\frac{1}{n_2})}$$

n_1+n_2-2

Small samples

- *One-sided* $100(1-\alpha)\%$ confidence interval for μ with a lower confidence bound

$$((\overline{x}_1 - \overline{x}_2) - t_{(\nu,1-\alpha)} * S_p\sqrt{(\frac{1}{n_1} + \frac{1}{n_2})}\ ,\ \ +\infty)$$

Hypothesis Testing

Null

Alternative

P-value

CI method

Matched Pairs

Two-sample

Small samples

Example:[Springs]

The data of W. Armstrong on spring lifetimes (appearing in the book by Cox and Oakes) not only concern spring longevity at a 950 N/ mm^2 stress level but also longevity at a 900 N/ mm^2 stress level.

Let sample 1 be the 900 N/ mm^2 stress group and sample 2 be the 950 N/ mm^2 stress group.

900 N/mm2 Stress	950 N/mm2 Stress
216, 162, 153, 216, 225, 216, 306, 225, 243, 189	225, 171, 198, 189, 189, 135, 162, 135, 117, 162

Hypothesis Testing

Null

Alternative

P-value

CI method

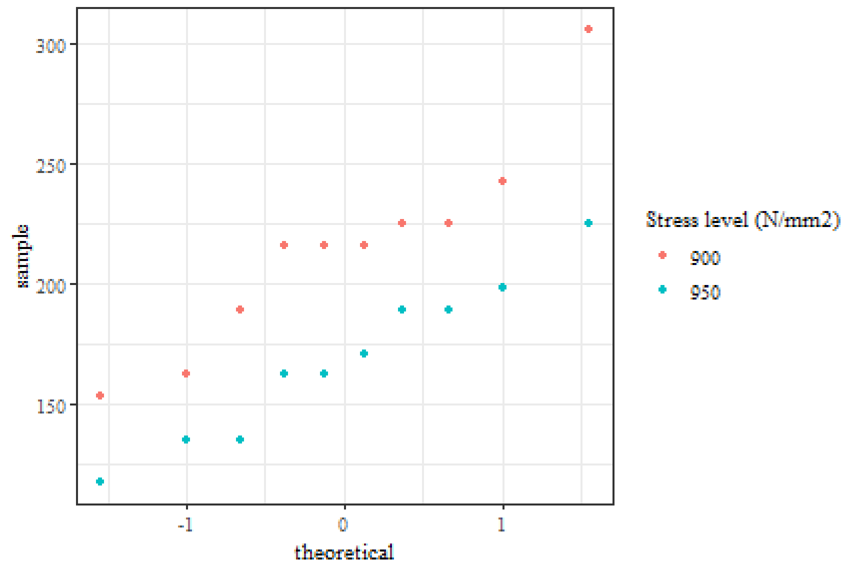
Matched Pairs

Two-sample

Small samples

Example:[Springs]

Normal plots of spring lifetime under two different stress level.



Let's do a hypothesis test to see if the sample 1 springs lasted significantly longer than the sample 2 springs.

Small samples

Example:[Stopping distance]

Suppose μ_1 and μ_2 are true mean stopping distances (in meters) at 50 mph for cars of a certain type equipped with two different types of breaking systems.

Suppose $n_1 = n_2 = 6, \overline{x}_1 = 115.7, \overline{x}_2 = 129.3, s_1 = 5.08$, and $s_2 = 5.38$.

Use significance level $\alpha = 0.01$ to test
$\mathrm{H}_0 : \mu_1 - \mu_2 = -10$ vs. $\mathrm{H}_A : \mu_1 - \mu_2 < -10$.

Construct a 2-sided 99% confidence interval for the true difference in stopping distances.