

1. New York Air Quality Measurement: The daily air quality in New York was measured from May to September 1973. To see the effect of different variables on Ozone layer, linear regression was used. The JMP output shows the linear regression between wind, temprature, solar radiation and Ozone.

R^2

$P = 4$

$\begin{matrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \end{matrix}$

$\hat{\beta}_0 = b_0$
 $\hat{\beta}_1 = b_1$
 $\hat{\beta}_2 = b_2$
 $\hat{\beta}_3 = b_3$

$\sqrt{MSE} = \sqrt{S^2_{SF}}$

$MSE = S^2_{SF}$

$SE(b_i)$

Summary of Fit					
RSquare	*	0.500653			
RSquare Adj		0.490599			
Root Mean Square Error		0.063432			
Mean of Response		0.119673			
Observations (or Sum Wgts)		153			

Analysis of Variance				
Source	DF	Sum of Squares	Mean Square	F Ratio
Model	3	0.6010814	0.200360	49.7966
Error	149	0.5995125	0.004024	Prob > F
C. Total	152	1.2005939		<.0001*

Parameter Estimates				
Term	Estimate	Std Error	t Ratio	Prob> t
Intercept	-0.182964	0.055639	-3.29	0.0013*
Wind	-2.460579	0.548765	-4.48	<.0001*
Temp	1.4731095	0.21135	6.97	<.0001*
Solar.R	0.0518205	0.02024	2.56	0.0115*

Use the output to answer the questions.

- (a) Write out the model with the appropriate estimates.

$$\hat{y} = -0.1829 - 2.4605 \text{ wind} + 1.473 \text{ temp} + 0.0518 \text{ Solar.R}$$

- (b) For the linear relationship, find r , the sample correlation coefficient and R^2 , the coefficient of determination and interpret R^2

$r^2 = 50\%$: 50% of the variation among air quality can be explained by a multiple regression between wind, temperature, Solar radiation & air quality. / r : sample correlation = $\sqrt{R^2}$

$$= \sqrt{0.5} = 0.7$$

- (c) Provide an estimate for σ^2

$$MSE = (0.063432)^2 = 0.004024 = S^2_{SE}$$

- (d) Provide an estimate for the variance of the coefficient of wind.

$$\widehat{\text{var}}(b_1) = (SE(b_1))^2 = (0.5487)^2 = 0.00310$$

$\hat{\beta}_1 = b_1$

- (e) Calculate and interpret the 95% two-sided confidence interval for the coefficient of wind

$$\begin{aligned} & \hat{b}_1 \pm t_{(n-p, 1-\alpha/2)} \times SE(b_1) \\ & = -2.4605 \pm t_{(149, 0.975)} \times (0.5487) \end{aligned}$$

$= \infty$ on t-table \leftarrow

$$= -2.4605 \pm (1.96)(0.5487) = (\quad , \quad)$$

- (f) Conduct a formal hypothesis test at the $\alpha = 0.05$ significance level to determine if there is significance relationship between air quality (y) and solar radiation (x_3), holding depth constant.

$$\begin{aligned} \textcircled{1} & \left\{ \begin{array}{l} H_0: \beta_3 = 0 \\ H_a: \beta_3 \neq 0 \end{array} \right. \end{aligned}$$

$$\textcircled{2} \quad \alpha = 0.05$$

③ I will use the statistic

$$K = \frac{b_3 - 0}{SE(b_3)}$$

which has $t_{(n-p)} \equiv t_{(149)}$ distribution assuming

that

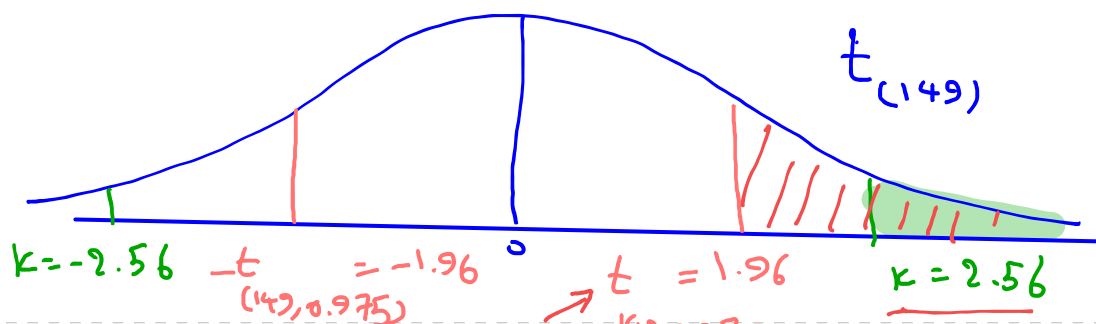
① H_0 is true

② the MLR is valid

$$(i.e. \hat{y} = \beta_0 + \beta_1 \text{Wind} + \beta_2 \text{temp} + \beta_3 \text{solar.R} + \epsilon)$$

④ calculate $K = \frac{b_3 - 0}{SE(b_3)} = \frac{0.05182 - 0}{0.02024} = 2.56$

P-value = $P(|T| > k) = P(|T| > 2.56)$
 $T \sim t_{(149)}$
 $t_{(n-p, 1-\alpha/2)} = t_{(149, 0.975)} = 1.96$



⑤ Since $p\text{-value} < \alpha$, we reject H_0 .

⑥ There is enough evidence to reject H_0 concluding that there is a significant relationship between the air quality & solar radiation, holding wind and temperature constant.

