Background

Terms and Use

Common Dists

Uniform

Exponential

Normal

Std. Normal

Example: Baby food

J. Fisher, in his article Computer Assisted Net Weight Control (**Quality Progress**, June 1983), discusses the filling of food containers with strained plums and tapioca by weight. The mean of the values portrayed is about 137.2g, the standard deviation is about 1.6g, and data look bell-shaped. Let

$$W = \text{the next fill weight}.$$

Let $W \sim N(137.2, 1.6^2)$. Find the probability that the next jar contains less food by mass than it's supposed to (declared weight = 135.05g).

$$P(W < 135.05) = P\left(\frac{W - 137.2}{1.6} \le \frac{135.05 - 137.2}{1.6}\right)$$

$Z \sim N(0,1)$

$$= P(Z \le -1.34)$$

$$= \Phi(-1.34) = 0.901$$

This means a 9% chance that the next

jar contains less food.

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Using the standard normal table, calculate the following:

- $P(X > 7), X \sim \text{Normal}(6, 9)$ → $\sigma^2 = 9$

standardization: $P\left(\frac{X-6}{3} > \frac{7-6}{3}\right) = P(Z > 1/3) = 1 - P(Z \le 0.33)$ Z

$= 1 - \Phi(0.33) = 1 - 0.6293$

- → $P(|X - 1| > 0.5), X \sim \text{Normal}(2, 4)$ → $\sigma^2 = 4$

→ $P(|X-1| > 0.5) = P(X-1 > 0.5$ or $X-1 < -0.5)$

$= P(X-1 > 0.5) + P(X-1 < -0.5)$

$= P(X > 1+0.5) + P(X < 1-0.5)$

$$\text{Standardization} = P\left(\frac{X-2}{\sqrt{4}} > \frac{1+0.5-2}{\sqrt{4}}\right) \quad -0.25$$

$$+\, P\left(\frac{X-2}{\sqrt{4}} < \frac{1-0.5-2}{\sqrt{4}}\right) \quad -0.75$$

$$= P(Z > -0.25) + P(Z < -0.75)$$

$$= 1 - P(Z \le 0.25) + P(Z \le 0.75)$$

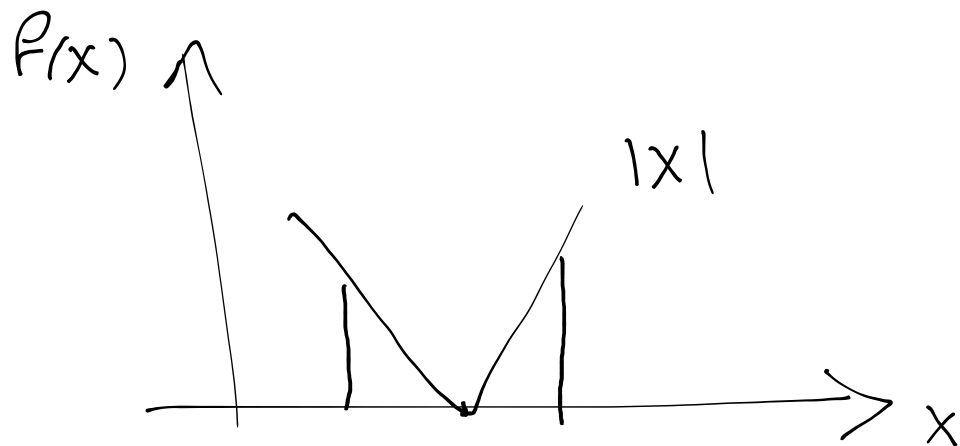
tables $= 1 - 0.4013 + 0.2266$

$$= 0.8253$$

Facts :

1. $P(|X| \leq c) = P(-c < X < c)$

2. $P(|X| > d) = P(X > d \text{ or } X < -d)$



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Find c such that

$$P(|X-2|>c)=0.01$$

where $X \sim \text{Normal}(\underbrace{2,4}) \Rightarrow \sigma^2 = 4$

$0.01 = P(|X-2|>c) = P(X-2>c \;\boxed{\text{or}}\; X-2<-c)$

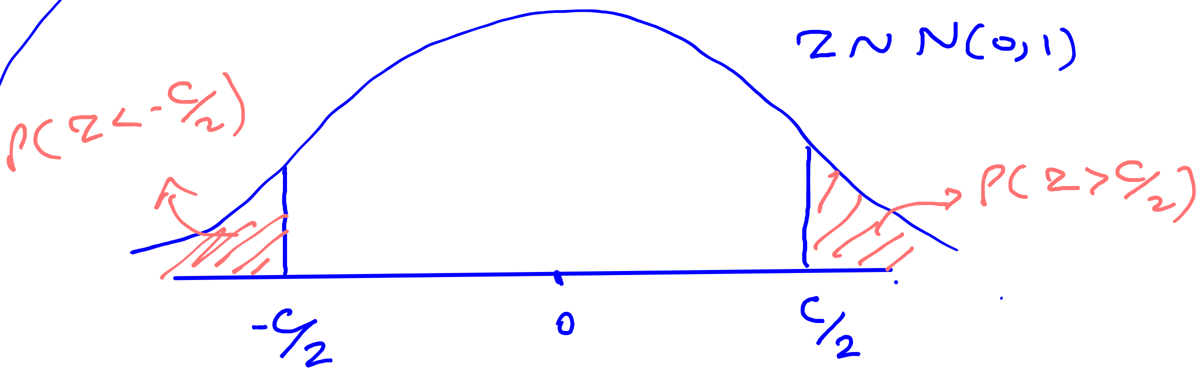
$= P(X-2>c) + P(X-2<-c)$

standardization $= P\left(\frac{X-2}{2} > \frac{c}{2}\right) + P\left(\frac{X-2}{2} < -\frac{c}{2}\right)$

$Z \sim N(0,1)$ $= P(Z > c/2) + P(Z < -c/2)$

$0.01 = 2P(Z < -c/2)$

$\Rightarrow \Phi(-c/2) = \frac{0.01}{2} = 0.005$



always in std. Normal dist.: $P(Z > a) = P(Z < -a)$

$$\Rightarrow -c/2 = -2.575 \Rightarrow c = 2(2.575)$$

- Hw 7 due Thursday Oct. 31st (in-class)

- Quiz II & Solution posted on the page.