As always, this is not intended to be an complete list of problems. However, you can consider these to be a representative subset of of the population of problems.

Competency Topic: Discrete Random Variables

1. Suppose that U has a distribution so that, for some constant c, the probability function is given by $f(u) = c$ for $u = -2, -1, 0, 1, 2$ and 0 everywhere else.

   a. Find the value of c that makes $f(u)$ a valid probability function.
   b. Find the expected value of U.
   c. Find the variance of U.

2. Suppose that X has the following distribution:

$$f(x) = \begin{cases} \dfrac{3!}{x!(3-x)!}(0.5)^3 & x = 0, 1, 2, 3 \\ 0 & o.w. \end{cases}$$

   a. Show that $f(x)$ is a valid probability function.
   b. Find the probability that X is less than 2.
   c. Find the expected value of X.

3. Suppose that X has the following cumulative probability function:

$$F(x) = \begin{cases} 0 & x < -1.5 \\ 0.3 & -1.5 \le x < 2 \\ 1 & x \ge 2 \end{cases}$$

   a. Sketch the cumulative probability function.
   b. Find the probability function of X (hint: it may be easiest to write it as a piecewise function).
   c. Find the expected value of X.

Competency Topic: Continuous Random Variables

1. Suppose that X has a step-uniform distribution - that is, X has a probability density function given by

$$f(x) = \begin{cases} 0.2 & -1 \le x < 0 \\ 0.8 & 0 \le x \le 1 \\ 0 & otherwise \end{cases}$$

   a. Sketch the probability density function of X.
   b. Find the probability that X is less than 0.5.
   c. Find the expected value of X.

2. Suppose that X has a distribution so that, for some constant c, the cumulative probability density function is given by

$$F(X) = \begin{cases} 0 & x \le -1 \\ 0.5x + 0.5 & -1 < x < 1 \\ 1 & x \ge 1 \end{cases}$$

   a. Sketch the cumulative density function of X.
   b. Find the probability that X is less than 0.
   c. Find the probability density function of X.

3. Suppose that X has a gamma distribution with integer valued shape parameter $k \ge 1$ and rate parameter β, i.e.,

$$f(x) = \begin{cases} \dfrac{\beta^k}{(k-1)!} x^{k-1} \exp(-\beta x) & x \ge 0 \\ 0 & o.w. \end{cases}$$

   It can be shown that the expected value and the variance are based on the parameters, so that

$$E(X) = \frac{k}{\beta}$$

   and

$$Var(X) = \frac{k}{\beta^2}$$

   a. Suppose that X has a gamma distribution with shape $k = 4$ and variance 10. Find the value of β.
   b. Suppose that X has a gamma distribution with mean 0.25 and variance 0.125. Find the value of the shape parameter k and the rate parameter β.

Competency Topic: Joint Distributions

1. Suppose that X and Y are independent normal random variables with the same mean μ and variance σ^2. That means they both follow a distribution with pdf where

$$f(t) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{1}{2\sigma^2}(t-\mu)^2\right)$$

for any $-\infty < t < \infty$ (just replace t with x for X and y for Y).

   a. Find $f_{XY}(x, y)$, the joint probability density function of X and Y (it will include the parameters μ and σ).

   b. Suppose that a is a value such that

$$\int_{-\infty}^{a} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(\frac{1}{2\sigma^2}(x-\mu)^2\right) dt = 0.95$$

   Find the value of

$$\int_{-\infty}^{a} \int_{-\infty}^{a} f_{XY}(x, y) dx dy$$

   .

2. If α and β are both integers, then a random variable P following a *beta*(α, β) distribution has pdf

$$f_P(p) = \begin{cases} \frac{(\alpha+\beta-1)!}{(\alpha-1)!(\beta-1)!} p^{\alpha-1}(1-p)^{\beta-1} & 0 \leq p \leq 1 \\ 0 & otherwise \end{cases}$$

. Further suppose that, given P, X has the following conditional pdf:

$$f_{X|P}(x|p) = \begin{cases} \frac{5!}{x!(5-x)!} p^x (1-p)^{5-x} & x = 0, 1, 2, \ldots, 5 \\ 0 & o.w. \end{cases}$$

   a. Find the probability that $X = 0$ given that $P = 0.1$ (simplify as much as possible, but the answer will include the terms α and β)

   b. Find the joint probability function, $f_{XP}(x, p)$.

3. Suppose that P has a beta distribution with $\alpha = 4$ and $\beta = 3$. Then the probability density function of P can be written as

$$f_P(p) = \begin{cases} \frac{6!}{3!2!} p^3 (1-p)^2 & 0 \leq p \leq 1 \\ 0 & otherwise \end{cases}$$

. Suppose that P has a joint distribution with X so that

$$f_{XP}(x, p) = \begin{cases} \frac{6!}{x!(3-x)!2!} p^{x+3}(1-p)^{n+2-x} & 0 \leq p \leq 1, x = 0, 1, 2, 3 \\ 0 & o.w. \end{cases}$$

   a. Find the conditional probability function, $f_{X|P}(x|p)$.

   b. Find the probability that $X = 0$ given that $P = 0.1$.

Competency Topic: Functions of Random Variables

1. Suppose that X is uniform on the interval $[2,5]$. That is, $f_X(x) = 1/3$ for x in $[2,5]$ and it is 0 everywhere else. Let $Y = \sqrt{X}$.

   a. Find the cumulative density function of Y (it may help to have the cumulative density function of X).
   b. Find the probability density function of Y.

2. Suppose that $X \sim N(3,2)$ and $Y \sim N(-2,2)$ are independent random variables. If $U = 2 - 2X - 2Y$, find

   a. The expected value of U.
   b. The variance of U

3. Suppose that X has the probability function

$$f_X(x) = \begin{cases} \dfrac{4!}{x!(4-x)!}\dfrac{1}{2^4} & x = 0,1,2,3,4 \\ 0 & o.w. \end{cases}$$

   and that $U = X - 2$.

   a. Find the probability function for U.
   b. Find the expected value of U.