Show **all** of your work on this assignment and answer each question fully in the given context. You have 20 minutes. Each problem is designed to take 10 minutes. All answers in a topic must be correct for any credit for that topic. You may attempt multiple topics. You may use a calculator on this competency quiz.

1. **Competency Topic: Discrete Random Variables**

   Suppose that X is a discrete random variable with the following probability function:

$$f(x) = \begin{cases} \dfrac{x^2}{c} & x = -3, -2, -1, 0, 1, 2, 3 \\ 0 & o.w \end{cases}$$

   where c is a constant.

   a. Find the value of c that makes $f(x)$ a valid probability function.

   b. Find $P(X \geq 2)$.

   c. Find $E(X)$.

2. **Competency Topic: Continuous Random Variables**

The mean-0 Laplace distribution is continuous distribution with the following probability density function:

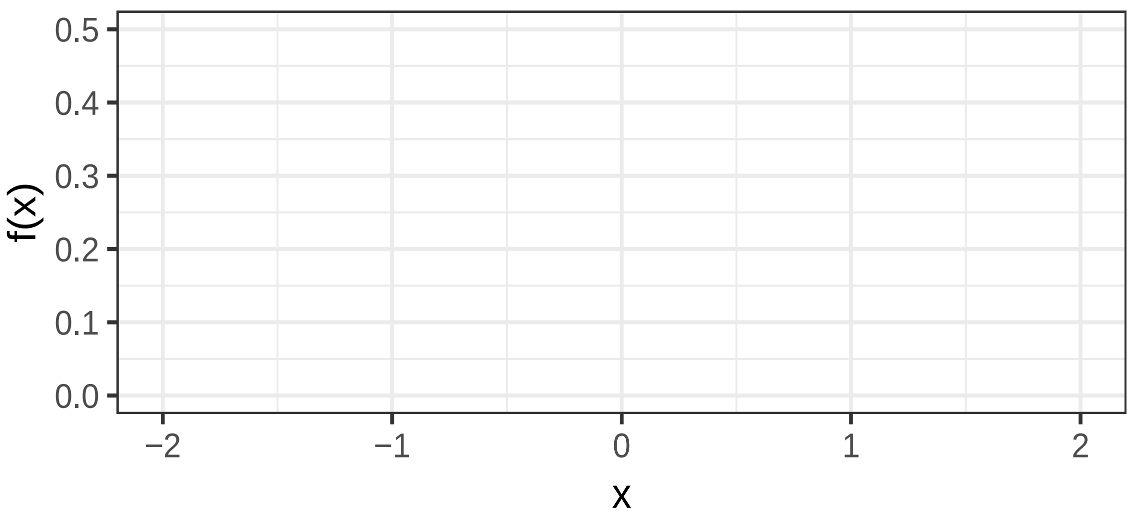
$$f(x) = \frac{1}{2\alpha} \exp\left(-\frac{|x|}{\alpha}\right) \quad -\infty < x < \infty$$

where α is a parameter which takes positive values (note: $|x|$ is the absolute value of x).

a. Show that regardless of the value of α the pdf above is symmetric (that is, show that $f(-x) = f(x)$).

b. Using the plot below, provide a rough sketch of the following pdfs:

- (1) the pdf of a variable with $\alpha = 0.5$.
- (2) the pdf of a variable with $\alpha = 1$. (for the sketch, show the values of the pdfs when $x = -2, -1, 0, 1, 2$)



c. Based on your sketch, would a random variable with $\alpha = 0.5$ have a larger or smaller variance than a random variable with $\alpha = 1$?

3. **Competency Topic: Joint Distributions**

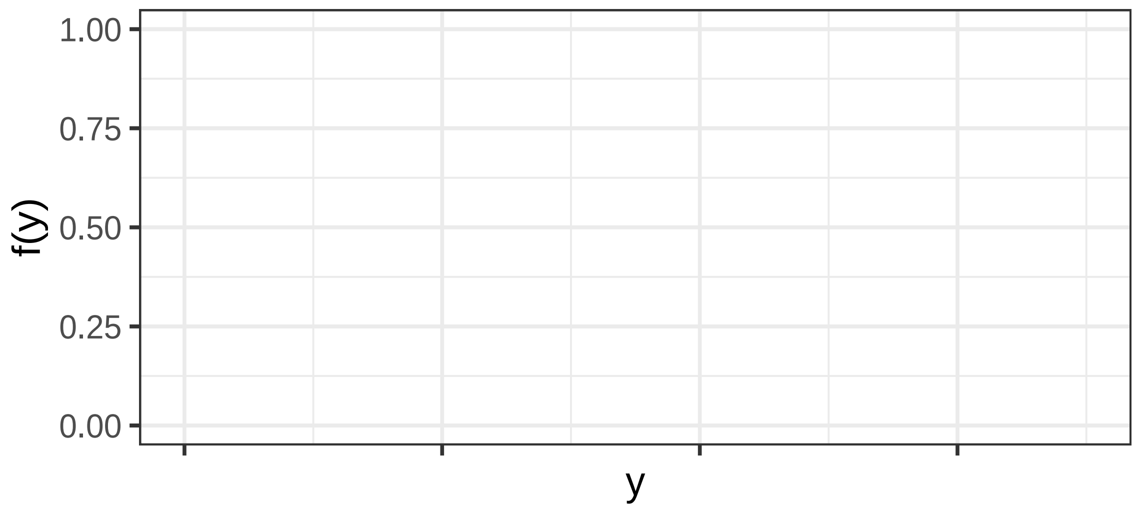
Suppose that X is a random variable with an exponential distribution with mean λ. That is

$$f_X(x) = \begin{cases} \frac{1}{\lambda}\exp\left(-\frac{x}{\lambda}\right) & x \geq 0 \\ 0 & otherwise \end{cases}$$

and suppose that a random variable Y follows an exponential distribution that depends on the value taken by X so that

$$f_Y|X(y|x) = \begin{cases} \frac{1}{x} & 0 \leq y \leq x \\ 0 & otherwise \end{cases}$$

a. Sketch the conditional probability density function of Y given that $X = 4$.



b. Find the joint probability function, $f_{XY}(x, y)$.

4. **Competency Topic: Functions of Random Variables**

   Suppose that $W_1, W_2, ...$ are independent random variables each with the same expected value μ and variance σ^2. We define a random variable U_n as a linear combination of n of these random variables using:

$$U_n = 2^{-1}W_1 + 2^{-2}W_2 + 2^{-3}W_3 + \ldots + 2^{-n}W_n$$

   note: if $r \geq 1$, the $r^{-1} + r^{-2} + \ldots r^{-n} = \sum_{k=1}^{n} r^{-k} = \dfrac{r^{n-1} - 1}{r^n - r^{n-1}}$

   a. Find an expression for $E(U_n)$ (hint: it will contain μ and n).

   b. Find an expression for $Var(U_n)$ (hint: it will contain σ^2 and n).