Formulas associated with inference for simple linear regression (SLR):

Model: $y_i = \beta_0 + \beta_1 x_i + \epsilon_i$, $\epsilon_i \sim Normal(0, \sigma^2)$

Least Square Estimates for β_0, β_1: $b_1 = \frac{\sum(x_i-\bar{x})(y-\bar{y})}{\sum(x_i-\bar{x})^2} = \frac{\sum x_i y_i - n\bar{x}\bar{y}}{\sum x_i^2 - n\bar{x}\bar{y}}$, $\bar{y} = b_0 + b_1\bar{x}$

Estimate for σ^2: $s_{LF}^2 = \frac{1}{n-2}\sum(y_i - \hat{y}_i)^2$

Residuals: $e_i = y_i - \hat{y}_i$

Standardized Residuals: $e_i^* = \frac{e_i}{s_{LF}\sqrt{1-\frac{1}{n}-\frac{(x_i-\bar{x})^2}{\sum(x-\bar{x})^2}}}$

Inference for β_1:

C.I: $b_1 \pm t\frac{s_{LF}}{\sqrt{\sum(x-\bar{x})^2}}$, t is a t_{n-2} quantile. *Test for $H_0 : \beta_1 = \#$:* $T = \frac{b_1 - \#}{\frac{s_{LF}}{\sqrt{\sum(x-\bar{x})^2}}} \sim t_{n-2}$ under H_0.

. (if n large "$t_{n-2} \leftrightarrow Z$")

Inference for $\mu_{y|x} = \beta_0 + \beta_1\mathbf{x}$:

C.I: $\hat{y} \pm t.s_{LF}\sqrt{\frac{1}{n} + \frac{(\mathbf{x}-\bar{x})^2}{\sum(x-\bar{x})^2}}$, t is a t_{n-2} quantile.

Test for $H_0 : \mu_{y|x} = \#$: $T = \frac{\hat{y} - \#}{s_{LF}\sqrt{\frac{1}{n}+\frac{(\mathbf{x}-\bar{x})^2}{\sum(x-\bar{x})^2}}} \sim t_{n-2}$ under H_0.

Prediction interval: $\hat{y} \pm t.s_{LF}\sqrt{1 + \frac{1}{n} + \frac{(\mathbf{x}-\bar{x})^2}{\sum(x-\bar{x})^2}}$, t is a t_{n-2} quantile.

. (if n large "$t_{n-2} \leftrightarrow Z$")

Analysis of Variance (ANOVA):

$SSE = \sum(y - \hat{y})^2 = (n-2)s_{LF}^2$

$SSR = SSTot - SSE = \sum(\hat{y} - \bar{y})^2$

$R^2 = \frac{SSR}{SSTot} = 1 - \frac{SSE}{SSTot}$

. ($adj.R^2 = 1 - \frac{SSE/(n-k-1)}{SSTot/(n-1)}$, where for SLR, $k = 1$)

For testing $H_0 : \beta_1 = 0$ (or all $\beta_1 = \ldots = \beta_k = 0$, but for SLR, $k = 1$)

$F = \frac{SSR/1}{SSE/(n-2)} = MSR/MSE$ ($= t^2$) and under H_0, $F \sim F_{1,n-2}$ distribution.

. (in general, $F \sim F_{k,n-k-1}$ but in SLR, $k = 1$)