Quiz IV

STAT 305, Section 7
SPRING 2020

Instructions

- The quiz is scheduled for 80 minutes, from 09:30 to 10:50 AM. At 10:50 AM the exam will end.
- Total points for the exam is 60. Points for individual questions are given at the beginning of each problem. Show all your calculations clearly to get full credit. Put final answers in the box at the right (except for the diagrams!).
- A forumula sheet is attached to the end of the exam. Feel free to tear it off.
- Normal quantile table is attached to the end of the exam. Feel free to tear it off.
- You may use a calculator during this exam.
- Answer the questions in the space provided. If you run out of room, continue on the back of the page.
- If you have any questions about, or need clarification on the meaning of an item on this exam, please ask your instructor. No other form of external help is permitted attempting to receive help or provide help to others will be considered cheating.
- **Do not cheat on this exam.** Academic integrity demands an honest and fair testing environment. Cheating will not be tolerated and will result in an immediate score of 0 on the exam and an incident report will be submitted to the dean's office.

Name: ______________________________

Student ID: Key

1. A small military subcontractor has secured a contract to develop drones capable of providing light air support for smaller naval vessels. Successfully fulfilling the contract requires that the drone be able to takeoff with a minimum runway of 10 meters. However, late changes in the prototype's weight distribution have led to concerns that they are no longer satisfying this requirement. Using 100 takeoffs, they found the average distance before takeoff was 9.92 meters with a standard deviation of 0.4 meters.

   (a) (4 points) Provide a two-sided 95% confidence interval for the true average distance before takeoff.

   $\boxed{(9.8416 \quad , 9.9984 \quad)}$

Large sample CI:

$\bar{x} = 9.92$

$n = 100$

$s = 0.4$

$$\bar{x} \pm z_{1-\alpha/2} \, s/\sqrt{n}$$

$$\Rightarrow 9.92 \pm z_{0.975} \cdot \frac{0.4}{\sqrt{100}}$$

$$\Rightarrow 9.92 \pm 1.96 \, \frac{0.4}{10}$$

$$\Rightarrow 9.92 \pm 0.0784$$

$$= (9.8416, 9.9984)$$

   (b) (4 points) Provide a 90% lower bound confidence interval for the true average distance before takeoff.

   $\boxed{(9.8688 \quad , \quad +\infty \quad)}$

$$\left(\bar{x} - z_{1-\alpha} \, s/\sqrt{n} \, , \, \infty\right)$$

$$= \left(9.92 - z_{0.9} \cdot \frac{0.4}{\sqrt{100}}, +\infty\right)$$

$$= \left(9.92 - 1.28 \, (0.04), +\infty\right)$$

$$= (9.8688, +\infty)$$

(c) (10 points) Conduct a hypothesis test at the $\boxed{\alpha = 0.01}$ significance level for μ, the true average distance before takeoff with the null hypothesis $\mu \geq 10$ against the alternative hypothesis of $\mu < 10$. Include the hypothesis statement, the test statistic, the p-value, and the conclusion.
Note: Write down all six steps for full credit.

1, $H_0: \mu \geq 10$ vs. $H_a: \mu < 10$

2, $\alpha = 0.01$

3, for $n \geq 25$ and under H_0 being true, I'll use the test statistic

$$K = \frac{\bar{X} - \mu_0}{S/\sqrt{n}} \sim N(0,1)$$

4, $K = \dfrac{9.92 - 10}{0.4/\sqrt{100}} = \dfrac{-0.08}{\frac{0.4}{10}} = -\dfrac{0.08}{0.04} = -2$

p-value $= P(Z < K) = P(Z < -2) = \Phi(-2)$

$= 0.0228$

5, Since p-value $> \alpha (= 0.01)$ we fail to reject H_0

6, There's not enough evidence to reject H_0,

So, we conclude that the average distance before takeoff is more than 10 meters.

2. An inspector examining the dependability of a certain gas pump fills 50 containers until the pump reads 1.00 gallons. If the pump is completely accurate, then each container should have 1.00 gallons of gasoline. However, since nothing is completely consistent, there will be differences from one container to the next.

   Suppose that it is known that the true standard deviation of the amount of gasoline the pump recognizes as 1.00 gallons is $\sigma = 0.2$ gallons.

   The average of the 50 gallon samples is $\bar{x} = 0.992$ gallons

   The following table may be useful:

Table 1: z's for use in **Two-sided** Large-n intervals for the mean

Desired Confidence	z
80%	1.28
90%	1.645
95%	1.96
98%	2.33
99%	2.58

(a) (4 points) Provide a two-sided 90% confidence interval for the mean volume of gasoline recognized by the pump to be 1.00 gallons. (0.9558 , 1.0282)

$n \geq 25 \to$ large sample with known variance / $\alpha = 0.1$

$$\bar{x} \pm z_{1-\alpha/2} \, \sigma/\sqrt{n}$$

$$= 0.992 \pm z_{0.9} \, 0.2/\sqrt{50}$$

$$= 0.992 \pm 0.0362 \longrightarrow (0.9558, 1.0282)$$

(b) (4 points) Provide a one-sided upper bound 95% confidence interval for the mean volume of gasoline recognized by the pump to be 1.00 gallons. ($-\infty$, 1.038528)

$\alpha = 0.05$

$$(-\infty, \bar{x} + z_{1-\alpha} \, \sigma/\sqrt{n})$$

$$= (-\infty, 0.992 + z_{0.95} \, 0.2/\sqrt{50})$$

$$= (-\infty, 0.992 + 1.645 \cdot \frac{0.2}{\sqrt{50}})$$

$$= (-\infty, 1.038528)$$

(c) (10 points) Conduct a hypothesis test at the $\alpha = 0.05$ significance level for μ, the true mean volume of gasoline recognized by the pump with the null hypothesis $\mu = 1$ against the alternative hypothesis of $\mu \neq 1$. Include the hypothesis statement, the test statistic, the p-value, and the conclusion. Note: Write down all six steps for full credit.

1, $H_0: \mu = 1$ vs $H_a: \mu \neq 1$

2, $\alpha = 0.05$

3, Since $n \geq 25$ and under H_0 with known σ, I'll use

$$K = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} \sim N(0,1)$$

4, $$K = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} = \frac{0.992 - 1}{0.2/\sqrt{50}} = \frac{-0.008}{\frac{0.2}{\sqrt{50}}}$$

$$= -0.2828$$

$$\text{P-value} = P(|Z| > |K|) = P(|Z| > |-0.2828|)$$

$$= P(|Z| > 0.2828) = 2\Phi(-0.2828)$$

$$= 2(0.3886) = 0.7773$$

5, P-value > α, so, we fail to reject H_0

6, There's not enough evidence to reject H_0, So we conclude that on average the true mean volume of casoline is equal to 1.

3. O-rings are elastomer loops designed to create a seal between the interface of two parts of a mechanical device. Because the elasticity of the material used to make them can be impacted by temperature (which can lead to the seal being broken) it is important to make sure that the O-ring is functional at the temperatures the part they are used in will be exposed to. Two composites (Composite X and Composite Y) are being tested in an O-ring that will be used in a part of a satellite that will be exposed to very low temperatures. A sample of 50 O-rings from each composite are placed in a chamber, where the temperature is gradually reduced until the seal is broken. Suppose that each composite has some mean failure temperature, μ_X for Composite X and μ_Y for Composite Y, and some variance in failure temperature, σ_X^2 for Composite X and σ_Y^2 for Composite Y. Before any observations are recorded, we can consider the sampled values from Composite X to be random variables $X_1, X_2, \ldots, X_{50}$ with $\mathbb{E}(X_i) = \mu_X$ and $Var(X_i) = \sigma_X^2$. We can also consider the sampled values from Composite Y to be random variables $Y_1, Y_2, \ldots, Y_{50}$ with $\mathbb{E}(Y_i) = \mu_Y$ and $Var(Y_i) = \sigma_Y^2$.

   Let $\bar{X} = \frac{1}{50}X_1 + \frac{1}{50}X_2 + \ldots + \frac{1}{50}X_{50}$ and let $\bar{Y} = \frac{1}{50}Y_1 + \frac{1}{50}Y_2 + \ldots + \frac{1}{50}Y_{50}$.

   After running the O-ring experiment, the researchers found $\bar{x} = 50$ K and $\bar{y} = 53$ K. Suppose that $\sigma_X = 10$ and $\sigma_Y = 20$.

   (a) (4 points) Provide a two-sided 90% confidence interval for μ_X. (47.6736 , 52.3263)

   $\alpha = 0.1$

   $$\bar{x} \pm z_{1-\alpha/2} \, \sigma_X/\sqrt{n}$$

   $$= 50 \pm z_{0.95} \, 10/\sqrt{50}$$

   $$= 50 \pm 1.645 \cdot \frac{10}{\sqrt{50}} = (47.67362, 52.32638)$$

   (b) (4 points) Provide a one-sided lower bound 99% confidence interval for μ_X.
   (46.70488 , $+\infty$)

   $\alpha = 0.01$

   $$(\bar{x} - z_{1-\alpha} \cdot \sigma_X/\sqrt{n}, +\infty)$$

   $$= (50 - z_{.99} \, 10/\sqrt{50}, +\infty)$$

   $$= (50 - 2.33 \cdot 10/\sqrt{50}, +\infty) = (46.70488, +\infty)$$

   (c) (4 points) Provide a two-sided 95% confidence interval for μ_Y. (47.4562 , 58.54372)

   $\alpha = 0.05$

   $$\bar{y} \pm z_{1-\alpha/2} \, \sigma_Y/\sqrt{n}$$

   $$= 53 \pm z_{0.975} \, 20/\sqrt{50}$$

   $$= 53 \pm 1.96 \cdot \frac{20}{\sqrt{50}} = (47.4562, 58.54372)$$

(d) (6 points) Provide a two-sided 95% confidence interval for $\mu_X - \mu_Y$. Does this provide any evidence that one O-ring is better than the other? (-9.19 , 3.19)

large n two sample CI: $(\bar{X}-\bar{Y}) \pm Z_{1-\alpha/2}\sqrt{\frac{\sigma_X^2}{n}+\frac{\sigma_Y^2}{n}}$

$= (50-53) \pm 1.96\sqrt{\frac{100}{50}+\frac{400}{50}}$

$= (-3) \pm 1.96\sqrt{10}$

$= (-9.19 , 3.19)$. The CI contains zero, which means maybe $\mu_X = \mu_Y$

$n=50$
$\bar{X}=50$
$\bar{Y}=53$
$\sigma_X=10$
$\sigma_Y=20$

(e) (10 points) Conduct a hypothesis test at the $\alpha = 0.05$ significance level for the claim that the true difference between the true means of the two O-rings $(\mu_X - \mu_Y)$ are significantly different.
Note: Write down all six steps for full credit.

1, $H_0: \mu_X - \mu_Y = 0$ vs. $H_a: \mu_X - \mu_Y \neq 0$

2, $\alpha = 0.05$

3, for large sample $n_1 = n_2 = n = 50 \geq 25$, under H_0
if (1) $\bar{X} \sim N(\mu_X, \sigma_X^2/n)$ (2) independent from $\bar{Y} \sim N(\mu_Y, \sigma_Y^2/n)$ (3)

we'll use
$$K = \frac{\bar{X}-\bar{Y}-(\mu_X-\mu_Y)}{\sqrt{\sigma_X^2/n + \sigma_Y^2/n}} \sim N(0,1)$$

4, $K = \frac{\bar{X}-\bar{Y}-0}{\sqrt{\sigma_X^2/n+\sigma_Y^2/n}} = \frac{50-53}{\sqrt{\frac{100}{50}+\frac{400}{50}}} = \frac{-3}{\sqrt{10}}$

$= -0.948$

P-value $= P(|Z|>|K|) = P(|Z| > |-.948|)$
$= 2\Phi(-0.948) = 2(0.1736) = 0.3472$

5, P-value $> \alpha$, so we fail to reject H_0

6, There is not enough evidence against H_0 concluding that the difference between the true means of the two types of o-rings are on-average zero!