

Show **all** of your work on this assignment and answer each question fully in the given context.

**If you cannot submit your homework in the class, you can drop it at my office door in 3220 Snedecore Hall by Thursday at 03:30 PM.**

Please staple your assignment and write your name !

1. [Ch. 5.1, Exercise 9, pg. 244] Transmission line interruptions in a telecommunication network occur at an average rate of one per day.

(a) Use a Poisson distribution as a model for

$X$  = the number of interruptions in the next five-day work week

Now, precisely specify the probability distribution.[5pts]

(b) Find  $P[X = 0]$ [5pts]

(c) Now consider the random variable

$Y$  = the number of work weeks in the next four in which there are no interruptions

What is a reasonable probability model for  $Y$ ?[5pts]

i.e. precisely specify the probability distribution.

(d) Find  $P[Y = 2]$ . [5pts]

2. Suppose a standup comedian plans to give a total of  $n = 5$  jokes in an entire 2-hour performance. Call a joke a success if at least one audience member laughs. If no audience member laughs, the joke is a failure.

Assume that all the jokes are equally funny, with  $p = P(\text{success}) = 0.2$ . Let  $X$  be the random variable associated with the number of successful jokes out of the total 5.

(a) Precisely state the distribution of  $X$ , giving the values of any parameters necessary.[2pts]

(b) Calculate the probability that the whole night is a failure. i.e. find the  $P(\text{no success})$ . [5pts]

(c) Calculate the probability that the comedian tells at least 4 successful jokes.[5pts]

(d) Calculate the expected number of successful jokes.[5pts]

(e) Calculate the standard deviation of the successful jokes.[5pts]

3. The number of computer shutdowns during any month has a Poisson distribution, averaging 0.25 shutdowns per month.

(a) What is the probability of at least 2 computer shutdowns during the next year?[5pts]

(b) What is the probability of at most 2 computer shutdowns during the next 6 month? [5pts]

(c) What is the variance of the number of computer shutdowns during the next year? [2pts]

4. Suppose that  $X$  is a random variable with probability density function

$$f(x) = \begin{cases} c \cdot x^2 & -2 \leq x \leq 2 \\ 0 & \text{o.w.} \end{cases}$$

- (a) Find the value of  $c$  that makes  $f(x)$  a valid probability density function.[5 pts]
  - (b) Find the CDF of the random variable  $X$ . [5 pts]
  - (c) What is  $P(|X| \geq -1)$  [5 pts].
  - (d) Find the expected value of  $X$ . [ 5 pts]
5. Consider a continuously distributed random variable,  $W$ , with a probability density function given by

$$f(w) = \begin{cases} \frac{1}{5(1-e^{-2})} e^{-w/5} & 0 \leq w \leq 10 \\ 0 & \text{otherwise} \end{cases}$$

- (a) Show that the function  $f(w)$  is a valid probability density function (i.e., show that (i)  $f(w)$  is non-negative and (ii)  $\int_{-\infty}^{\infty} f(w)dw = 1$ ). [5 pts]
  - (b) Find  $P(W \leq 2)$  [5 pts]
  - (c) Find  $P(2 \leq W \leq 10)$  [5 pts]
  - (d) Find  $E(W)$  [5 pts]
6. The mileage to first failure for a model of military personnel carrier can be modeled as exponential with mean 1,000 miles.
- (a) Find the probability that a vehicle of this type gives less than 500 miles of service before first failure.[5 pts]
  - (b) Find the probability that a vehicle of this type gives less than 2000 miles of service before first failure. [5 pts]
7. (Ch. 5.2, Exercise 2, pg. 263) Suppose that  $Z$  is a standard normal random variable. Evaluate the following probabilities involving  $Z$ :
- (a)  $P[Z < -.62]$  [3 pts]
  - (b)  $P[Z > 1.06]$  [3 pts]
  - (c)  $P[-.37 < Z < .51]$  [3 pts]
  - (d)  $P[|Z| \leq .47]$  [3 pts]
  - (e)  $P[|Z| > .93]$  [3 pts]
  - (f)  $P[-3.0 < Z < 3.0]$  [3 pts]

Now find numbers  $\#$  such that the following statements involving  $Z$  are true:

- (a)  $P[Z \leq \#] = .90$  [3 pts]
- (b)  $P[|Z| \leq \#] = .90$  [3 pts]
- (c)  $P[|Z| > \#] = .03$  [3 pts]

Total: 77 pts