Show **all** of your work on this assignment and answer each question fully in the given context.

If you cannot submit your homework in the class, you can drop it at my office door in 3220 Snedecore Hall by Thursday at 03:30 PM.

Please staple your assignment and write your name !

1. Ch. 5.1, Exercise 1, pg. 243 A discrete random variable X can be described using the probability function, $f(x)$:

x	2	3	4	5	6
f(x)	0.1	0.2	0.3	0.3	0.1

   (a) Plot $F(x)$, the cumulative probability function for X.[5 pts]
   (b) Find the mean and standard deviation of X.[10 pts]

2. Ch. 5, Exercise 1, pg. 322: Suppose 90% of all students taking a beginning programming class fail to get their first program to run on first submission. Use a binomial distribution and assign probabilities to the possibilities that among a group of six such students,

   (a) all fail on their first submissions[5 pts]
   (b) at least four fail on their first submissions[5 pts]
   (c) less than four fail on their first submissions [5 pts]
   Hint: You may think of using compliment of a probability.

   Continuing to using this binomial model,
   (d) what is the mean number who will fail?[5 pts]
   (e) what are the variance and standard deviation of the number who will fail?[5 pts]

3. Ch. 5, Exercise 2, pg. 322: Suppose that for single launches of a space shuttle, there is a constant probability of O-ring failure (say .15), Consider ten future launches, and let X be the number of those involving an O-ring failure. Use an appropriate probability model and evaluate all of the following:

   (a) Precisely state the distribution of X, giving the values of any parameters necessary.[5 pts]
   (b) $P[X = 2]$[5 pts]
   (c) $P[X \geq 1]$[5 pts]
   Hint: You may think of using compliment of a probability.
   (d) EX[5 pts]
   (e) Var$X$[5 pts]
   (f) the standard deviation of X[5 pts]

4. Ch. 5.1, Exercise 6, pg. 244: Suppose that an eddy current nondestructive evaluation technique for identifying cracks in critical metal parts has a probability of about .20 of detecting a single crack of length .003in. in a certain material. Let Y be the number of specimens inspected in order to obtain the first crack detection. Use an appropriate probability model and evaluate all of the following:

(a) Precisely state the distribution of X, giving the values of any parameters necessary.[5 pts]

(b) $P[Y = 5]$[5 pts]

(c) $P[Y \leq 4]$[5 pts]

(d) $\mathrm{E}Y$[5 pts]

(e) $\mathrm{Var}Y$[5 pts]

(f) $\mathrm{SD}(Y)$[5 pts]

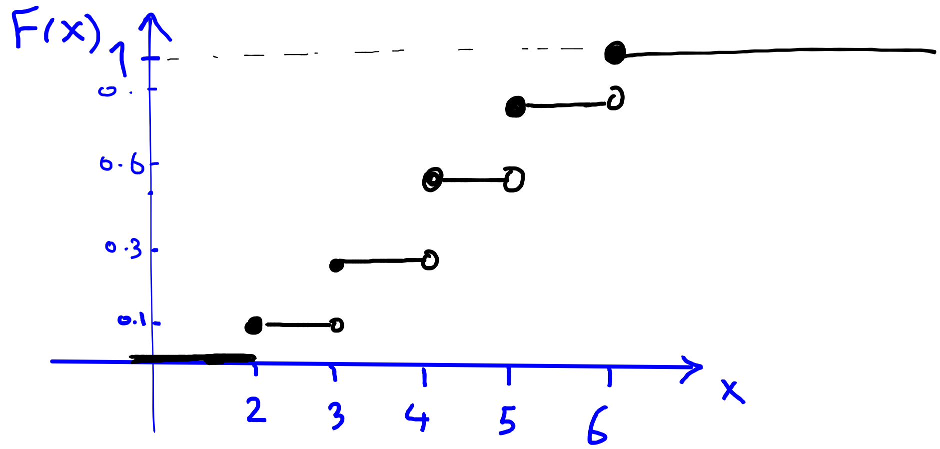
Total: 100 pts

Key:

7, (a)

x	2	3	4	5	6
$f(x)$	0.1	0.2	0.3	0.3	0.1
$F(x)$	0.1	0.3	0.6	0.9	$\boxed{1}$

CDF $\longrightarrow$ $F(x)$



(b) $Ex = ?$

x	2	3	4	5	6
$f(x)$	0.1	0.2	0.3	0.3	0.1
$xf(x)$	0.2	0.6	1.2	1.5	0.6
$x^2 f(x)$	0.4	1.8	4.8	7.5	3.6

$$Ex = \sum x f(x) = 0.2 + 0.6 + 1.2 + 1.5 + 0.6 = \boxed{4.1}$$

$$Ex^2 = \sum x^2 f(x) = 0.4 + 1.8 + 4.8 + 7.5 + 3.6 = \boxed{18.1}$$

$$var(x) = \sum (x - Ex)^2 f(x) = Ex^2 - (Ex)^2$$

$$= 18.1 - (4.1)^2 = 1.29$$

So, the standard deviation is;

$$SD(X) = \sqrt{var\,x} = \sqrt{1.29} = \boxed{1.13}$$

problem 2)

Let x be the random variable connected to the number of students who fail to get their first program to run. Then

$$X \sim \text{Binomial}(n=6, p=0.9)$$

(a) P("all students fail to run their first program")

$$= P(X=6) = \frac{6!}{6!(6-6)!}(0.9)^6(1-0.9)^0$$

$$=(0.9)^6 = 0.531441$$

(b) P("at least 4 students fail") = $P(X \geq 4)$

$$= P(X=4) + P(X=5) + P(X=6)$$

$$= \frac{6!}{4!\underbrace{(6-4)!}_{2!}} \cdot (0.9)^4(1-0.9)^{6-4} + \frac{6!}{5!\underbrace{(6-5)!}_{1!}}(0.9)^5(1-0.9)^{6-5}$$

(This shows how to cancel the factorial terms)

$$+ \frac{\cancel{6!}}{\cancel{6!}\underbrace{(6-6)!}_{0!}}(0.9)^6 \underbrace{(1-0.9)^0}_{1}$$

$$= \frac{\overset{3}{\cancel{6}} \times 5 \times \cancel{4 \times 3 \times 2 \times 1}}{(\cancel{4 \times 3 \times 2 \times 1}) \times (\cancel{2} \times 1)}(.9)^4(.1)^2 + \frac{6 \times \cancel{5 \times 4 \times 3 \times 2 \times 1}}{(\cancel{5 \times 4 \times 3 \times 2 \times 1}) \times 1}(.9)^5(.1)^1$$

$$+ (.9)^6$$

$= (3\times 5)(0.9)^4(0.1)^2 + 6(0.9)^5(0.1)^1 + (0.9)^6$

$= 0.9841$

(c) P("less than 4 students fail")

$= P(X<4) = P(X\leq 3) = P(X=0)+P(X=1)+P(X=2)+P(X=3)$

$= \dots$

But, you can use the complement trick as you found the $P(X\geq 4)$ in part (b).

$P(X<4) = 1 - P(X\geq 4) = 1 - 0.9841 = \boxed{0.0159}$

(d) EX = the mean (expected) number of fails.

$X \sim \text{Binomial}(n=6, p=0.9)$

$\Rightarrow EX = n.p = 6(0.9) = \boxed{5.4}$

It is expected that 5.4 student fail on their first programming.

(e) $Var(X) = ?$ in Binomial distribution,

$$Var(X) = np(1-p)$$

So, $Var(X) = 6(.9)(1-.9) = 6(.9)(.1)$

$$= \boxed{0.54}$$

and the standard variation is

$$SD(X) = \sqrt{Var(X)} = \sqrt{0.54} = \boxed{0.734}$$

Problem 3:

(a) $X \sim Binomial(n = 10, p = 0.15)$

(b) $P(X=2) = ?$

we know in Binomial, $P(X=x) = \dfrac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$

So,

$$P(X=2) = \frac{10!}{2!(10-2)!} \cdot (0.15)^2 (1-0.15)^{10-2}$$

$$= \frac{10!}{2!\,8!} (0.15)^2 (0.85)^8$$

$$= \frac{\cancel{10}^{5} \times 9 \times \cancel{8!}}{\cancel{2!}\ \cancel{8!}} (0.15)^2 (0.85)^8$$

$$= 5 \times 9 \times (0.15)^2 \times (0.85)^8$$

$$= \boxed{0.2758}$$

(c) $P(X \geq 1) = \sum_{x=1}^{10} P(X = x) = P(X=1) + P(X=2) + \cdots + P(X=10)$

(This is a very long way!)

So, use the compliment:

$P(X \geq 1) = 1 - P(X < 1) = 1 - P(X=0)$

$$= 1 - \left[\frac{\cancel{10!}}{0!\ \cancel{(10-0)!}} \cdot \underbrace{(0.15)^0}_{1} \underbrace{(1-0.15)}_{0.85}{}^{10-0}\right]$$

$$= 1 - (0.85)^{10}$$

$$= \boxed{0.8031}$$

(d) $EX = ?$ Since we know $X \sim Binomial(10, 0.15)$, we can easily get EX.

$$EX = n.p = 10(0.15) = \boxed{1.5}$$

(e) $var(X) = ?$

$$Var(X) = n.p(1-p) = 10(0.15)(1-0.15)$$
$$= 10(0.15)(0.85)$$
$$= \boxed{1.275}$$

(f) standard deviation of X:

$$SD(X) = \sqrt{var\, X} = \sqrt{1.275} = \boxed{1.129}.$$

Problem 4)

(a) Y = The number of specimens inspected in order to see the first crack

$$\Rightarrow Y \sim Geometric(p = 0.2)$$

(b) we know that $Y \sim Geom(p)$,

$$f_Y(y) = p(1-p)^{y-1}, \quad y = 1, 2, 3, \dots$$

$$P(Y=5) = p(1-p)^{5-1}$$

$$= (0.2)(1-0.2)^4$$

$$= (0.2)(0.8)^4$$

$$= 0.08192$$

(c) $P(Y \leq 4) = F_Y(4)$

Note: Geometric (unlike Binomial & Poisson) has a closed form CDF.

$$Y \sim Geom(p) \Rightarrow F_Y(y) = P(Y \leq y) = 1-(1-p)^y$$

$$P(Y \leq 4) = F_Y(4) = 1-(1-p)^4$$

$$= 1-(1-0.2)^4$$

$$= 1-(0.8)^4$$

$$= 0.5904$$

(d) we know in Geometric distribution,

$$EY = 1/p$$

$$\text{var } Y = \frac{1-p}{p^2}$$

So, $EY = \frac{1}{0.2} = 5$

(This means it is expected that we need 5 specimens inspected to observe the first crack)

(e) $\text{var}(Y) = \frac{1-p}{p^2} = \frac{1-0.2}{(0.2)^2} = \frac{0.8}{0.04} = \boxed{20}$

Then the $SD(Y) = \sqrt{\text{var}(Y)}$

$$= \sqrt{20}$$

$$= 4.47$$