Common Distributions

Working with Off The Shelf Random Variables

Common Distributions

Why Are Some Distributions Worth Naming?

Even though you may create a random variable in a unique scenario, the way that it's probability distribution behaves (mathematically) may have a lot in common with other random variables in other scenarios. For instance,

> I roll a die until I see a 6 appear and then stop. I call X the number of times I have to roll the die in total.
>
> I flip a coin until I see heads appear and then stop. I call Y the number of times I have to flip the coin in total.
>
> I apply for home loans until I get accepted and then I stop. I call Z the number of times I have to apply for a loan in total.

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Why Are Some Distributions Worth Naming? (cont)

In each ot the above cases, we count the number of times we have to do some action until we see some specific result. The only thing that really changes from the random variables perspective is the likelihood that we see the specific result each time we try.

Mathematically, that's not a lot of difference. And if we can really understand the probability behavior of one of these scenarios then we can move our understanding to the different scenario pretty easily.

By recognizing the commonality between these scenarios, we have been able to identify many random variables that behave very similarly. We describe the similarity in the way the random variables behave by saying that they have a common/shared distribution.

We study the most useful ones by themselves.

The Bernoulli Distribution

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The Bernoulli Distribution

Origin: A random experiment is performed that results in one of two possible outcomes: success or failure. The probability of a successful outcome is p.

Definition: X takes the value 1 if the outcome is a success. X takes the value 0 if the outcome is a failure.

probability function: probability of success

success

$$f(x) = \begin{cases} p & x = 1, \\ 1-p & x = 0, \\ 0 & o.w. \end{cases}$$

Prob. of Failure. Failure

which can also be written as

$$f(x) = \begin{cases} p^x(1-p)^{1-x} & \boxed{x = 0, 1} \\ 0 & o.w. \end{cases}$$

i.e. for success ; $X=1 \rightarrow f(\text{success}) \equiv f(1) = P(X=1) = p^1(1-p)^{1-1} = p$

for failure ; $X=0 \rightarrow f(\text{failure}) = f(0) = P(X=0) = p^0(1-p)^{1-0} = 1-p$

Bernoulli Distribution

Expected Value and Variance

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Expected value: $E(X) = p$

proof: (optional reading):

$$E(X) = \sum_{x=0}^{1} x f(x) = 0 f(0) + (1) f(1)$$

$$= 0 + (1) p$$

$$= p$$

Recall: in Bernoulli: $f(0) = 1 - p$

$f(1) = p$

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Variance: $Var(X) = (1-p) \cdot p$

Proof: (optional reading):

First find Ex^2. Then use the formula

$$Var(X) = E(X^2) - [E(X)]^2$$

$$E(X^2) = \sum_{x=0}^{1} x^2 f(x) = 0^2 f(0) + 1^2 f(1)$$

$$= 0 + f(1)$$

$$= p$$

$$\Rightarrow Var(X) = E(X^2) - [E(X)]^2$$

$$= p - p^2 = p(1-p) \quad //$$

The Bernoulli Distribution

A few useful notes:

- In order to say that " X has a bernoulli distribution with success probability p " we write $X \sim Bernoulli(p)$
- Trials which results in which the only possible outcomes are "success" or "failure" are called **Bernoulli Trials**
- The value p is the Bernoulli distribution's **parameter**. We don't treat parameters like random values - they are fixed, related to the real process we are studying.
- "Success" does not mean something we would perceive as "good" has happened. It just means the outcome we were watching for was the outcome we got.
- Please note: we have two outcomes, but the probability for each outcome is **not** the same (duh!).

outcomes
success $\longrightarrow$ with prob. p
failure $\longrightarrow$ with prob. $(1-p)$

The Binomial Distribution

The Binomial Distribution → Repeat Bernoulli trial "n" times.

Origin: A series of n independent random experiments, or trials, are performed. Each trial results in one of two possible outcomes: success[redacted] or failure. The probability of a successful outcome, p, is the same across all trials.

Definition: For n trials, X is the number of trials with a successful outcome. X can take values $0, 1, \ldots, n$.

probability function:

With $0 < p < 1$,

$$P(X=x) = f(x) = \begin{cases} \dfrac{n!}{x!(n-x)!} p^x (1-p)^{n-x} & x = 0, 1, \ldots, n \\ 0 & o.w. \end{cases}$$

other wise

where $n! = n \cdot (n-1) \cdot (n-2) \cdot \ldots \cdot 1$ and $0! = 1$.

If we have only $n=1$ trial $\Rightarrow f(x) = \underbrace{\dfrac{1!}{x!\,(1-x)!}}_{=1} p^x (1-p)^{1-x} = p^x (1-p)^{1-x} \sim \text{Bernoulli}(p)$