Show **all** of your work on this assignment and answer each question fully in the given context.

Please write as legible as possible because if the TA cannot read your handwriting, it may cause confusion and ends up in deduction.

If you are going to type your answers, submit a pdf. If you take a photo/scan your handwritten answers, submit them in a single file.
You can download scanning apps for iOS or via Microsoft Office Lens|PDF Scan App or for Android

1. The mileage to first failure for a model of military personnel carrier can be modeled as exponential with mean $1,000$ miles.

   (a) Find the probability that a vehicle of this type gives less than 500 miles of service before first failure.[5 pts]

   (b) Find the probability that a vehicle of this type gives less than 2000 miles of service before first failure. [5 pts]

2. (Ch. 5.2, Exercise 2, pg. 263) Suppose that Z is a standard normal random variable. Evaluate the following probabilities involving Z:

   (a) $P[Z < -.62]$ [3 pts]
   (b) $P[Z > 1.06]$ [3 pts]
   (c) $P[-.37 < Z < .51]$ [3 pts]
   (d) $P[|Z| \leq .47]$ [3 pts]
   (e) $P[|Z| > .93]$ [3 pts]
   (f) $P[-3.0 < Z < 3.0]$ [3 pts]

   Now find numbers # such that the following statements involving Z are true:

   (a) $P[Z \leq \#] = .90$ [3 pts]
   (b) $P[|Z| \leq \#] = .90$ [3 pts]
   (c) $P[|Z| > \#] = .03$ [3 pts]

3. **Ch. 5, Exercise 7, pg. 323:** In a grinding operation, there is an upper specification of 3.150 in. on a dimensions of a certain part after grinding. Suppose that the standard deviation of this normally distributed dimension for parts of this type ground to any particular mean dimension μ is $\sigma = 0.002$ in. Suppose further that you desire to have no more than 3% of the parts fail to meet specifications. What is the maximum (minimum machining cost) μ that can be used if this 3% requirement is to be met?[10 pts]
   Hint: the question is giving you information on $P(X > 3.15) \leq 0.03$.

4. **Ch 5, Exercise 42, pg. 332:** Suppose that engineering specifications on the shelf depth of a certain slug to be turned on a CNC lathe are from 0.0275 in. to 0.0278 in. and that values of this dimension produced on the lathe can be described using a normal distribution with mean μ and standard deviation σ.

   (a) If $\mu = 0.0276$ and $\sigma = 0.0001$, about what fraction of shelf depths are in specifications?[10 pts]

(b) What machine precision (as measured by σ) would be required in order to produce about 98% of shelf depths within engineering specifications (assuming that μ is at the midpoint of the specifications)?[10 pts]
Hint: you are looking for the value of σ in this question.

5. **Ch 5.4, Exercise 2, pg. 300:** Quality audit records are kept on numbers of major and minor failures of circuit packs during burn-in of large electronic switching devices. They indicate that for a device of this type, the random variables

$$X = \text{ the number of major failures}$$

and

$$Y = \text{ the number of minor failures}$$

can be described at least approximately by the accompanying joint distribution.

Y | X	**0**	**1**	**2**
0	0.15	0.05	0.01
1	0.1	0.08	0.01
2	0.1	0.14	0.02
3	0.1	0.08	0.03
4	0.05	0.05	0.03

(a) Find the marginal probability functions for both X and Y ($f_x(x)$ and $f_y(y)$, respectively).[10 pts]

(b) Are X and Y independent? Explain.[5pts]

(c) Find the mean and variance of X (EX and Var$X$)[10 pts]

(d) Find the mean and variance of Y (EY and Var$Y$)[10 pts]

(e) Find the conditional probability function for Y, given that $X = 0$ – i.e., that there are no major circuit pack failures ($f_{Y|X}(y|0)$). What is the mean of this conditional distribution?[10 pts]

6. **Ch. 5.2, Exercise 3, pg. 263:** Suppose that X is a normal random variable with mean 43 and standard deviation 3.6. Evaluate the following probabilities involving X:

(a) $P[X < 45.2]$ [5 pts]

(b) $P[|X - 43| \leq 2]$ [5 pts]

(c) $P[|X - 43| > 1.7]$ [5 pts]

Now find numbers # such that the following statements involving X are true:

(d) $P[X < \#] = .95$ [5 pts]

(e) $P[|X - 43| > \#] = .05$ [5 pts]

Total: 137 pts

1/ X: The random variable associated with the mileage until the first failure.

$X \sim Exp(1000)$

$$f_X(x) = \begin{cases} \frac{1}{1000} e^{-\frac{x}{1000}} & , x \geq 0 \\ 0 & o.w \end{cases}$$

(a) $P(X \leq 500) = ?$ (we know in Exponential distribution, $F_X(x) = 1 - e^{-x/\alpha}$)

So, $P(X \leq 500) = F(500) = 1 - e^{-\frac{500}{1000}} = 1 - e^{-0.5}$

$= 0.3934$

(b) $P(X \le 2000) = F(2000) = 1 - e^{-\frac{2000}{1000}}$

$= 1 - e^{-2}$

$= 0.8646$

2, $Z \sim N(0,1)$

by the table

(a) $P(Z < -0.63) = \Phi(-0.63) = 0.2643$

(b) $P(Z > 1.06) = ?$

method ①: $P(Z > 1.06) = 1 - P(Z \le 1.06)$

$= 1 - \Phi(1.06)$

by the table $= 1 - 0.8554$

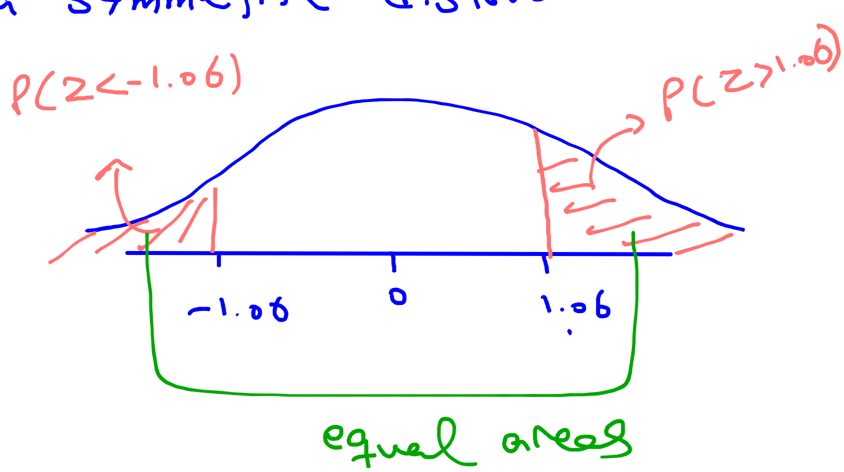
$= 0.1445$

method ②: Normal is a symmetric distribution.

so $P(Z>1.06) = P(Z<-1.06)$

$= \Phi(-1.06)$

by the table $= 0.1445$



(c) $P(-0.37 < Z < 0.51) = P(Z < 0.51) - P(Z < -0.37)$

$$= \Phi(0.51) - \Phi(-0.37)$$

by the normal table $= 0.6949 - 0.3556$

$$= 0.3392$$

(d) $P(|Z| \leq 0.47) = P(-0.47 < Z < 0.47)$

$$= P(0.47) - P(Z < -0.47)$$

$$= \Phi(0.47) - \Phi(-0.47)$$

$$= 0.6808 - 0.3191$$

$$= 0.3616$$

(e) $P(|Z| > 0.93) = P(Z > 0.93 \text{ or } Z < -0.93)$

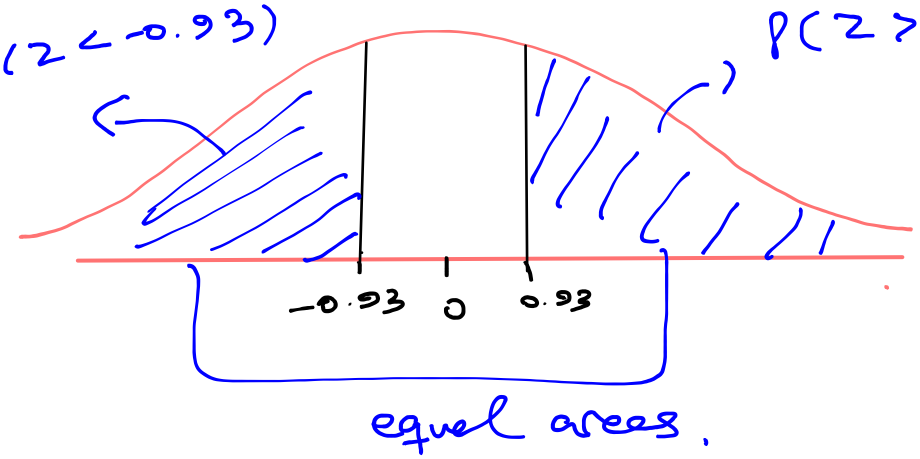
$$= P(Z > 0.93) + P(Z < -0.93)$$

method 1 : $= 1 - P(Z \leq 0.93) + P(Z < -0.93)$

$$= 1 - \Phi(0.93) + \Phi(-0.93)$$

$$= 1 - 0.8238 + 0.1761 = \boxed{0.3523}$$

method 2: $P(Z<-0.93)$ $P(Z>0.93)$



$P(Z>0.93) = P(Z<-0.93)$

$\Rightarrow P(Z>0.93) + P(Z<-0.93) = 2P(Z<0.93)$

$= 2(.1761)$

$= \boxed{0.3523}$

(f) $P(-3<Z<3) = P(Z<3) - P(Z<-3)$

$= \Phi(3) - \Phi(-3)$

$= 0.9986 - 0.0013$

$= 0.9973$

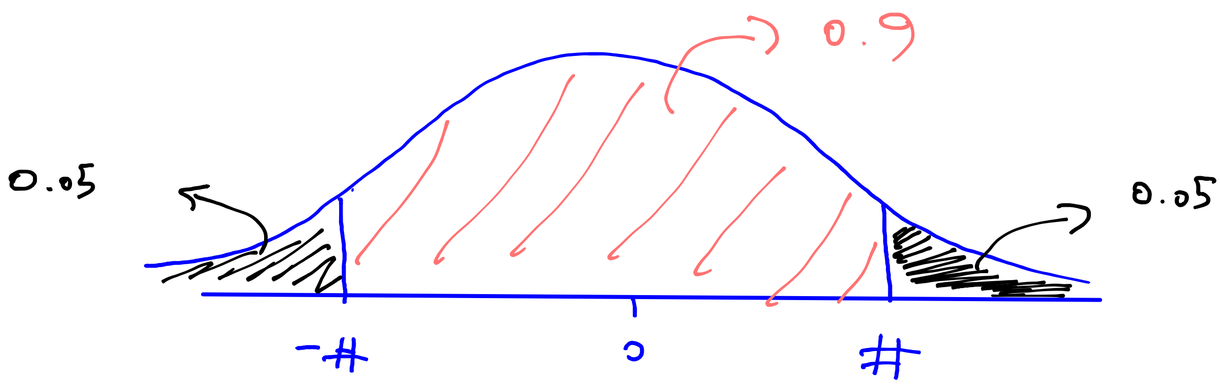
(a) $P(Z \leq \#) = 0.9$ $\longrightarrow$ by table, we should look for a value of Z which has probability 0.9

If we look it up in the standard normal

table, $\boxed{\# \approx 1.28}$

(b) $P(|Z| \leq \#) = 0.9$

$\hookrightarrow P(-\# < Z \leq \#) = 0.9$



because Normal is symmetric, $P(Z \leq -\#) = P(Z > \#) = 0.05$

So, by looking it up in the table,

$$-\# = -1.645$$

$$\boxed{\Rightarrow \# = 1.645}$$

The other method is:

$$P(-\# \leq Z \leq \#) = 0.9$$

by symmetry of Normal $\Rightarrow 1 - 2P(Z \leq -\#) = 0.9$

$\Rightarrow 2P(Z \le -\#) = 1 - 0.9$

$\Rightarrow P(Z \le -\#) = \frac{0.1}{2} = 0.05$

$\Rightarrow \Phi(-\#) = 0.05$

by the Normal table $\Rightarrow -\# = -1.645 \Rightarrow \# = 1.645$

(c) $P(|Z| > \#) = 0.03$

$\hookrightarrow P(Z > \#$ or $Z \le -\#) = 0.03$

$\Rightarrow \underbrace{P(Z > \#)}_{P(Z \le -\#)} + P(Z \le -\#) = 0.03$

$\Rightarrow 2P(Z \le -\#) = 0.03$

$\Rightarrow P(Z \le -\#) = \frac{0.03}{2} = 0.015$

$\Rightarrow \Phi(-\#) = 0.015$

by the Normal table : $-\# = -2.17 \Rightarrow \# = 2.17$

3,

$$P(X > 3.15) \leq 0.03 \quad , \quad X \sim N(\mu, \sigma = 0.002)$$

$$\Rightarrow P\left(\frac{X-\mu}{\sigma} > \frac{3.15-\mu}{\sigma}\right) \leq 0.03$$

$$\Rightarrow P\left(\frac{X-\mu}{0.002} > \frac{3.15-\mu}{0.002}\right) \leq 0.03$$

$$\Rightarrow P\left(Z > \frac{3.15-\mu}{0.002}\right) \leq 0.03$$

$$\Rightarrow 1 - P\left(Z < \frac{3.15-\mu}{0.002}\right) \leq 0.03$$

$$\Rightarrow P\left(Z < \frac{3.15-\mu}{0.002}\right) \geq 1 - 0.03$$

$$\Rightarrow \Phi\left(\frac{3.15-\mu}{0.002}\right) \geq 0.97$$

by the table, $\frac{3.15-\mu}{0.002} \geq 1.88$

$$\Rightarrow \mu < 3.146$$

The maximum μ can be used

4, a) $X \sim N(\mu = 0.0276, \sigma = 0.0001)$

$P(0.0275 \leq X \leq 0.0278) = ?$

$\Rightarrow P\left(\frac{0.0275 - 0.0276}{0.0001} \leq Z < \frac{0.0278 - 0.0276}{0.0001}\right)$

$= P(-1 \leq Z \leq 2) = P(Z \leq 2) - P(Z \leq -1)$

$= \Phi(2) - \Phi(1)$

by the table $= 0.9773 - 0.1587$

$\boxed{= 0.8186}$

(81.86% of the shelf depth are between 0.0275 in and 0.0278 in.

b) μ: the midpoint of the specification

$\Rightarrow \mu = \frac{0.0275 + 0.0278}{2} = 0.02765$

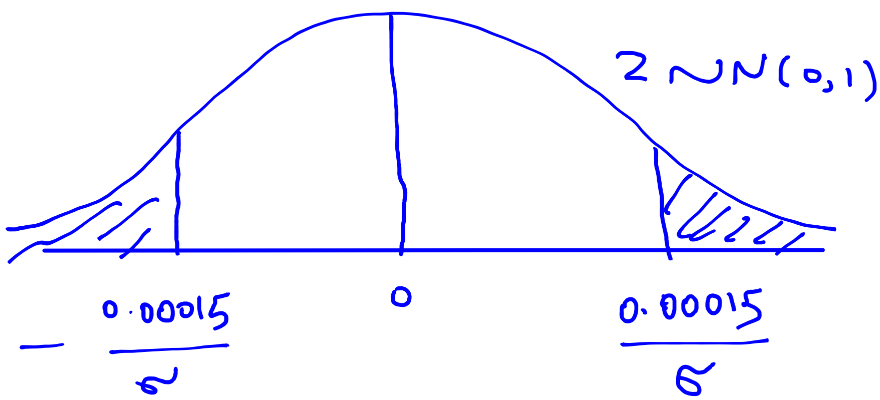
$\Rightarrow X \sim N(\mu = 0.02765, \sigma)$

$$\Rightarrow P(0.0275 \le X \le 0.0278) = 0.98$$

$$\Rightarrow P\left(\frac{0.0275-0.02765}{\sigma} \le Z \le \frac{0.0278-0.02765}{\sigma}\right) = 0.98$$

$$\Rightarrow P\left(-\frac{0.00015}{\sigma} \le Z \le \frac{0.00015}{\sigma}\right) = 0.98$$

$$\Rightarrow P\left(Z \le \frac{0.00015}{\sigma}\right) - P\left(Z \le \frac{-0.00015}{\sigma}\right) = 0.98$$



$$\Rightarrow 1 - 2P\left(Z \le -\frac{0.00015}{\sigma}\right) = 0.98$$

$$\Phi\left(\frac{-0.00015}{\sigma}\right) = 0.01$$

by the table: $\frac{-0.00015}{\sigma} \approx -2.32 \Rightarrow \boxed{\sigma \approx 0.000064}$

5,

(a)

y	$f_Y(y)$
0	0.21
1	0.19
2	0.26
3	0.21
4	0.13

X	$f_X(x)$
0	0.5
1	0.4
2	0.1

(b) No, for example

$$f_X(0) = 0.5$$

$$f_Y(0) = 0.21$$

$$f_{X,Y}(x=0, y=0) = 0.15$$

$$\Rightarrow f_{X,Y}(0,0) = 0.15 \neq f_X(0)\, f_Y(0) = (0.5)(0.21)$$

(c) $EX = \sum_{x=0}^{2} x f_X(x) = 0(0.5) + 1(0.4) + 2(0.1) = \boxed{0.6}$

$Ex^2 = \sum_{x=0}^{2} x^2 f_x(x) = 0^2(0.5) + 1^2(0.4) + 2^2(0.1) = \boxed{0.8}$

$\text{Var}\, X = Ex^2 - (Ex)^2 = 0.8 - (0.6)^2 = \underline{0.44}$

(d) $EY = \sum_{y=0}^{4} y f_Y(y) = 0(0.21) + 1(0.19) + 2(0.26) + 3(0.21) + 4(0.13) = \boxed{1.86}$

$EY^2 = \sum_{y=0}^{4} y^2 f_Y(y) = 0^2(0.21) + 1^2(0.19) + 2^2(0.26) + 3^2(0.21) + 4^2(0.13) = \boxed{5.2}$

$\text{var}\, Y = EY^2 - (EY)^2 = 5.2 - (1.86)^2 = \boxed{1.7404}$

(e)

$$f_{Y|X}(0|0) = \frac{f(0,0)}{f_X(0)} = \frac{0.15}{0.5} = 0.3$$

$$f_{Y|X}(1|0) = \frac{f(0,1)}{f_X(0)} = \frac{0.1}{0.5} = 0.2$$

$$f_{Y|X}(2|0) = \frac{f(0,2)}{f_X(0)} = \frac{0.1}{0.5} = 0.2$$

$$f_{Y|X}(3|0) = \frac{f(0,3)}{f_X(0)} = \frac{0.1}{0.5} = 0.2$$

$$f_{Y|X}(4|0) = \frac{f(0,4)}{f_X(0)} = \frac{0.05}{0.5} = 0.1$$

and in general we can write:

y	0	1	2	3	4
$f_{Y\|X}(y\|0)$	0.3	0.2	0.2	0.2	0.1

6/ $X \sim N(43, 3.6)$

a) $$P(X < 45.2) = P\left(Z < \frac{45.2 - 43}{3.6}\right)$$

$$= P\left(Z < \frac{2.2}{3.6}\right)$$

$$= \Phi(0.611)$$

$$= \underline{0.7294}$$

b) $$P(|X - 43| < 2) = P(-2 < X - 43 < 2)$$

$$= P\left(\frac{-2}{3.6} < \frac{X - 43}{3.6} < \frac{2}{3.6}\right)$$

$$= P(-0.555 < Z < 0.555)$$

$$= 2P(Z \leq -0.555)$$

$$= 2(0.2894)$$

$$= 0.5788$$

c) $$P(|X - 43| > 1.7) = P(X - 43 > 1.7 \text{ or } X - 43 < -1.7)$$

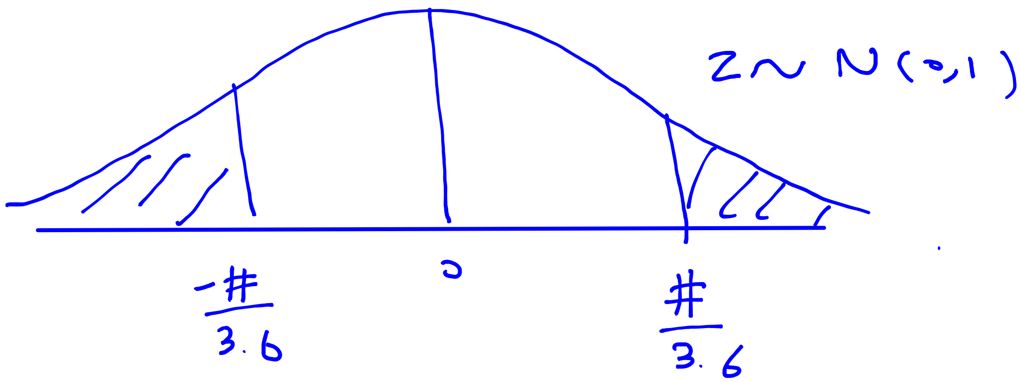
$$= P(X - 43 > 1.7) + P(X - 43 < -1.7)$$

(e) $0.05 = P(|X-43| > \#)$

$= P(X-43 > \#) + P(X-43 \leq -\#)$

$= P\left(\frac{X-43}{3.6} > \frac{\#}{3.6}\right) + P\left(\frac{X-43}{3.6} \leq \frac{-\#}{3.6}\right)$

$= P\left(Z > \frac{\#}{3.6}\right) + P\left(Z \leq \frac{-\#}{3.6}\right)$



$\Rightarrow 2P\left(Z < \frac{-\#}{3.6}\right) = 0.05$

$P\left(Z < \frac{-\#}{3.6}\right) = 0.025$

$\Rightarrow \frac{-\#}{3.6} = -1.96 \quad \Rightarrow \boxed{\# = 7.056}$