STAT 305: Beyond Chapter 4

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Model Diagnostics

(Optional Reading)

Model Diagnostics

Model Assumptions

Model Diagnostics

Model Assumptions

- There are some assumptions in fitting a linear regression (either simple or multiple) to determine any possible relationship between response variable(s) and explanatory (experimental) variable(s). Some of them will be discussed in future chapters, and in this sub-section, we will discuss some assumptions relatd to residuals.

- The **Residals** are the difference between the observed data point and the fitted prediction:

$$e_i = y_i - \hat{y}_i$$

- **ROPe**: **R**esiduals = **O**bserved **-** **P**redicted (using symbol e_i)

- Obviously, we would like our residuals to be small compared to the size of response values.

Assumptions in Linear Regression

If a linear model makes sense, the residuals will

- have a constant (homogeneous) variance
- be approximately normally distributed (with a mean of zero), and
- be independent of one another.

The most useful graph for analyzing residuals is a **residual by predicted plot**. This is a graph of each residual value plotted against the corresponding predicted value.

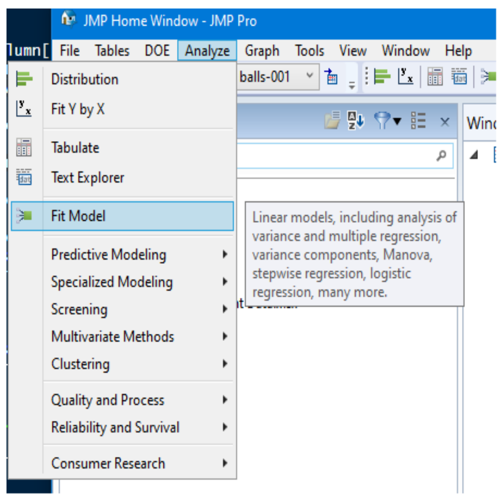
- If the assumptions are met, the residuals will be randomly scattered around the center line of zero, with no obvious pattern. The residuals will look like an unstructured cloud of points, centered at zero
- This checks the constant (homogeneous) variance and independence of residuals.

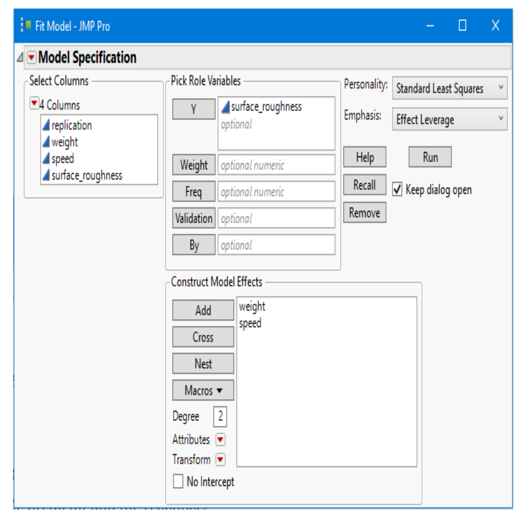
Assumptions in Linear Regression

Residual VS. predicted plot

> JMP: Analyze > Fit Model

then choose your response and explanatory variables and Run the model





After fiting a model, click on the red down arrow next to the model.

Model Diagnostics

Model Assumptions

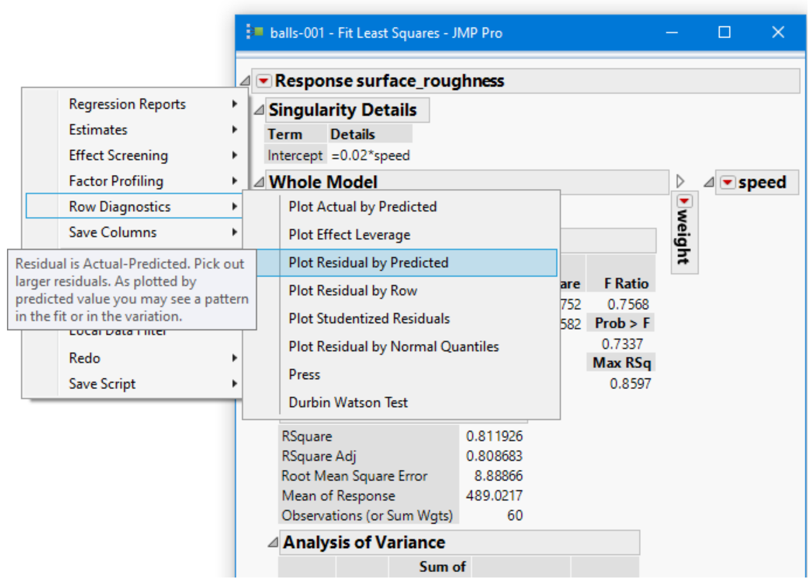
Residual plot

Assumptions in Linear Regression

Residual VS. predicted plot

After fiting a model, click on the red down arrow next to the model

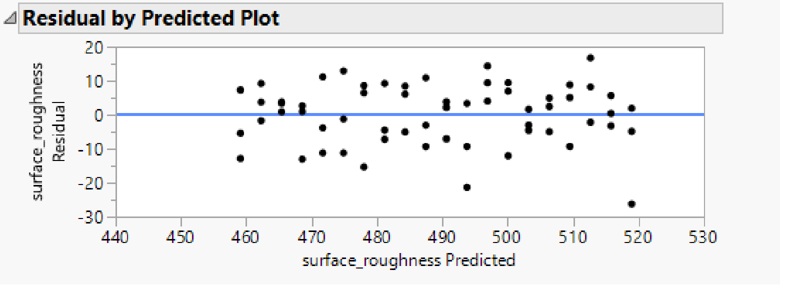
> Row diagnostics> Plot residuals by predicated



Assumptions in Linear Regression

Residual VS. predicted plot

Then you have **residuals** (e_i) on y axis and **predicted values** $(\hat{y})$ on x axis

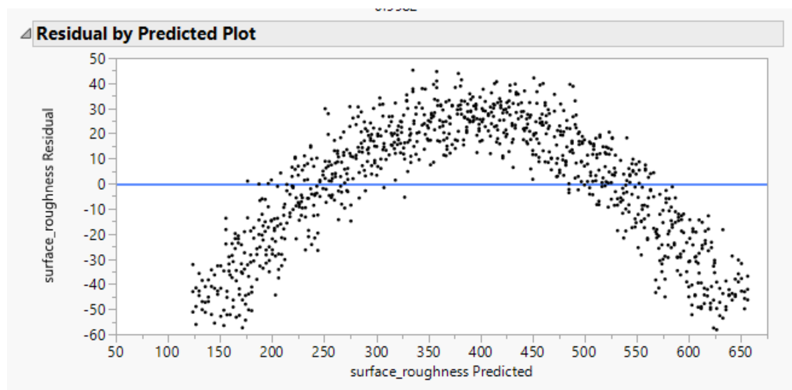


If there is a non-random pattern, the nature of the pattern can pinpoint potential issues with the model.

Assumptions in Linear Regression

Residual VS. predicted plot

For example, if curvature is present in the residuals, then it is likely that there is curvature in the relationship between the response and the predictor that is not explained by our model. A linear model does not adequately describe the relationship between the predictor and the response.



Model Diagnostics

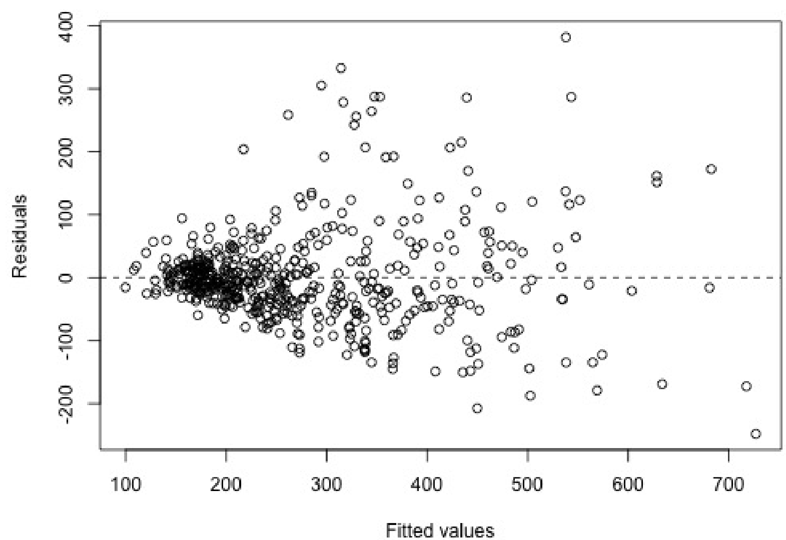
Model Assumptions

Residual plot

Assumptions in Linear Regression

Residual VS. predicted plot

Megaphone shaped pattern: variability of e_i increases or decreases as $\hat{y}_i$ increases.



This indicates non-constant (not homogeneous) variance.

Assumptions in Linear Regression

Normality of residuals

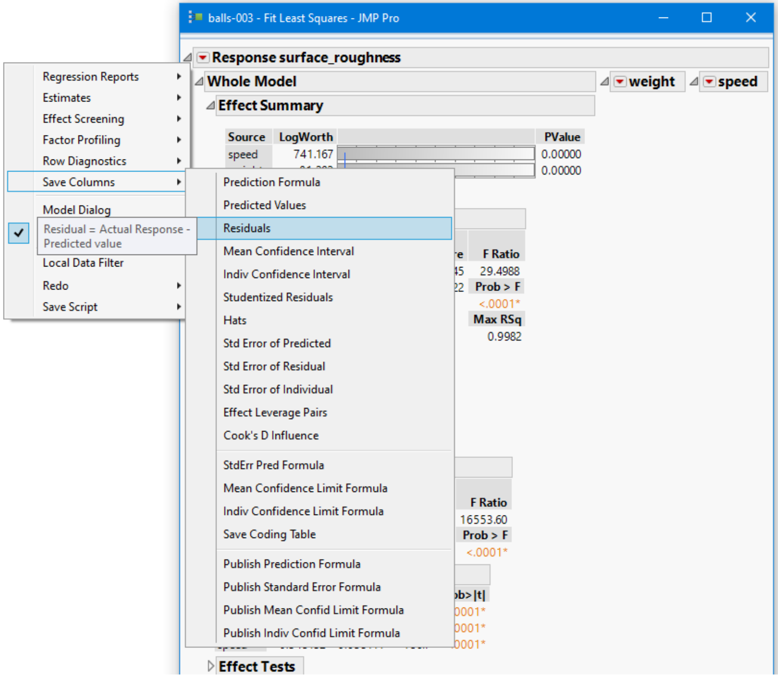
- In addition to the residual versus predicted plot, there are other residual plots we can use to check regression assumptions.
- A **histogram of residuals** and a **normal probability plot (QQ-plot)** of residuals can be used to evaluate whether our residuals are approximately normally distributed.
    - However, unless the residuals are far from normal or have an obvious pattern, we generally don't need to be overly concerned about normality.
- Note that we check the residuals for normality. We don't need to check for normality of the raw data. Our response and predictor variables do not need to be normally distributed in order to fit a linear regression model.

Assumptions in Linear Regression

Normality of residuals

To draw the histogram of the residuals, first save residuals of the model.

> Save Columns> Residuals



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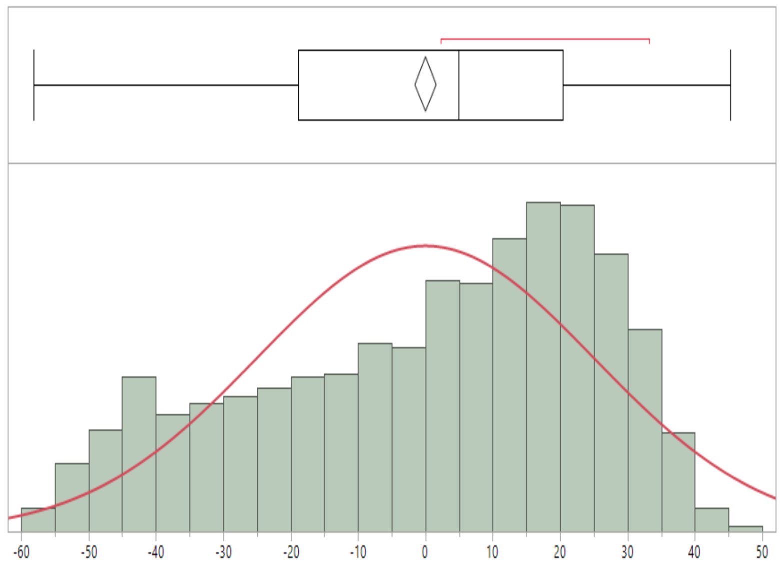
Residual plot

Normality

Assumptions in Linear Regression

Normality of residuals

Then draw a histogram of the residuals (review the JMP toturial for histograms)



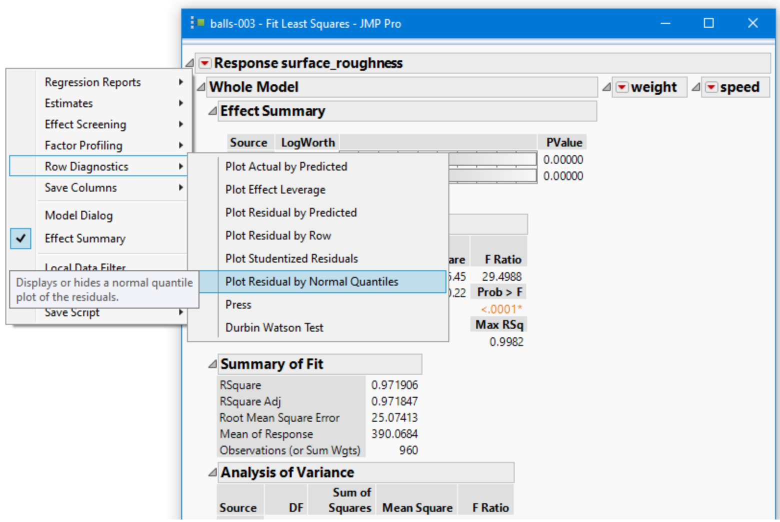
It seems the residuals are not normaly distributed in this example.

Assumptions in Linear Regression

Normality of residuals

As the instructions on the JMP toturials (and also HW #3), you can draw **Normal QQ-plot** to evaluate if the residuals meet the assumptions of normaly distributed.

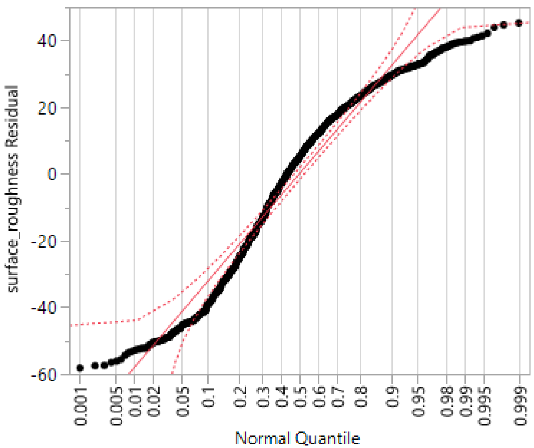
> Row Diagnostics> Plot Residual by Normal Quantile



Assumptions in Linear Regression

Normality of residuals

Plotting Normal QQ-plot of the same example



- Again, the QQ-plot also confirms that the assumption of Normal distribution of residuals is violated to some extend in this example.
- More examination is required to fix the issue or to find the problem.