Data Compression

- introduction
- basic coding schemes
- an application
- entropy
- LZW codes

References:

Algorithms 2nd edition, Chapter 22

http://www.cs.princeton.edu/introalgsds/65compression

- introduction
- basic coding schemes
- an application
- entropy
- LZW codes

Data Compression

Compression reduces the size of a file:

- To save space when storing it.
- To save time when transmitting it.
- Most files have lots of redundancy.

Who needs compression?

- Moore's law: # transistors on a chip doubles every 18-24 months.
- Parkinson's law: data expands to fill space available.
- Text, images, sound, video, …

> All of the books in the world contain no more information than is broadcast as video in a single large American city in a single year. Not all bits have equal value. -Carl Sagan

Basic concepts ancient (1950s), best technology recently developed.

Applications

Generic file compression.

- Files: GZIP, BZIP, BOA.
- Archivers: PKZIP.
- File systems: NTFS.

Multimedia.

- Images: GIF, JPEG.
- Sound: MP3.
- Video: MPEG, DivX™, HDTV.

Communication.

- ITU-T T4 Group 3 Fax.
- V.42bis modem.

Databases. Google.

Encoding and decoding

Message. Binary data M we want to compress.

Encode. Generate a "compressed" representation C(M). uses fewer bits (you hope)

Decode. Reconstruct original message or some approximation M'.

$$M \longrightarrow \boxed{\text{Encoder}} \longrightarrow C(M) \longrightarrow \boxed{\text{Decoder}} \longrightarrow M'$$

Compression ratio. Bits in C(M) / bits in M.

Lossless. M = M', 50-75% or lower.
Ex. Natural language, source code, executables.

this lecture

Lossy. $M \approx M'$, 10% or lower.
Ex. Images, sound, video.

"Poetry is the art of lossy data compression."

Food for thought

Data compression has been omnipresent since antiquity,

- Number systems.
- Natural languages.
- Mathematical notation.

has played a central role in communications technology,

- Braille.
- Morse code.
- Telephone system.

and is part of modern life.

- zip.
- MP3.
- MPEG.

What role will it play in the future?

Ex: If memory is to be cheap and ubiquitous, why are we doing lossy compression for music and movies??

- introduction
- **basic coding schemes**
- an application
- entropy
- LZW codes

Fixed length encoding

- Use same number of bits for each symbol.
- k-bit code supports 2^k different symbols

Ex. 7-bit ASCII

char	decimal	code
NUL	0	0
...	...	
a	97	1100001
b	98	1100010
c	99	1100011
d	100	1100100
...	...	
~	126	1111110
	127	1111111

this lecture:
special code for
end-of-message

a	b	r	a	c	a	d	a	b	r	a	!
1100001	1100010	1110010	1100001	1100011	1100001	1100100	1100001	1100010	1110010	1100001	1111111

12 symbols × 7 bits per symbol = 84 bits in code

Fixed length encoding

- Use same number of bits for each symbol.
- k-bit code supports 2^k different symbols

Ex. 3-bit custom code

char	code
a	000
b	001
c	010
d	011
r	100
!	111

a	b	r	a	c	a	d	a	b	r	a	!
000	001	100	000	010	000	011	000	001	100	000	111

12 symbols × 3 bits
36 bits in code

Important detail: decoder needs to know the code!

Fixed length encoding: general scheme

- count number of different symbols.
- $\lceil \lg M \rceil$ bits suffice to support M different symbols

Ex. genomic sequences

- 4 different codons
- 2 bits suffice

char	code
a	00
c	01
t	10
g	11

a	c	t	a	c	a	g	a	t	g	a
00	01	10	00	01	00	11	00	10	11	00

2N bits to encode genome with N codons

- Amazing but true: initial databases in 1990s did not use such a code!

Decoder needs to know the code

- can amortize over large number of files with the same code
- in general, can encode an N-char file with $N \lceil \lg M \rceil + 16 \lceil \lg M \rceil$ bits

Variable-length encoding

Use different number of bits to encode different characters.

Ex. Morse code.

Issue: ambiguity.

• • • _ _ _ • • •

SOS ?

IAMIE ?

EEWNI ?

V7O ?

Letters		Numbers	
A	•—	1	• — — — —
B	—•••	2	• • — — —
C	—•—•	3	• • • — —
D	—••	4	• • • • —
E	•	5	• • • • •
F	••—•	6	— • • • •
G	— —•	7	— — • • •
H	••••	8	— — — • •
I	••	9	— — — — •
J	•— — —	0	— — — — —
K	—•—		
L	• — • •		
M	— —		
N	— •		
O	— — —		
P	• — — •		
Q	— — • —		
R	• — •		
S	• • •		
T	—		
U	• • —		
V	• • • —		
W	• — —		
X	— • • —		
Y	— • — —		
Z	— — • •		

Variable-length encoding

Use different number of bits to encode different characters.

Q. How do we avoid ambiguity?

A1. Append special stop symbol to each codeword.

A2. Ensure that no encoding is a prefix of another.

S	• • •	← prefix of V
E	•	← prefix of I, S
I	• •	← prefix of S
V	• • • -	

Ex. custom prefix-free code

char	code
a	0
b	111
c	1010
d	100
r	110
!	1011

28 bits in code

a	b			r			a	c				a	d			a	b			r			a	!			
0	1	1	1	1	1	0	0	1	0	1	0	0	1	0	0	0	1	1	1	1	1	0	0	1	0	1	1

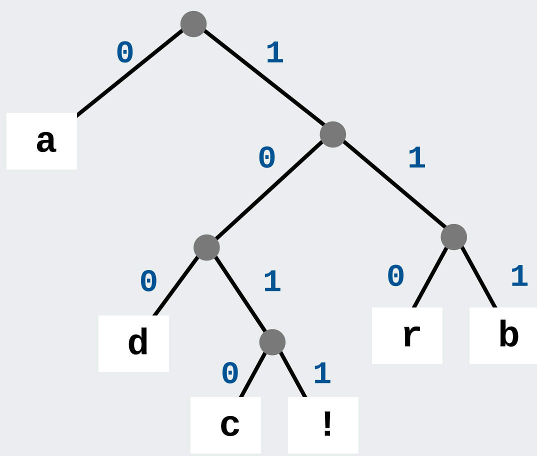
Note 1: fixed-length codes are prefix-free

Note 2: can amortize cost of including the code over similar messages

Prefix-free code: Encoding and Decoding

How to represent? Use a binary trie.

- Symbols are stored in leaves.
- Encoding is path to leaf.



Encoding.

- Method 1: start at leaf; follow path up to the root, and print bits in reverse order.
- Method 2: create ST of symbol-encoding pairs.

Decoding.

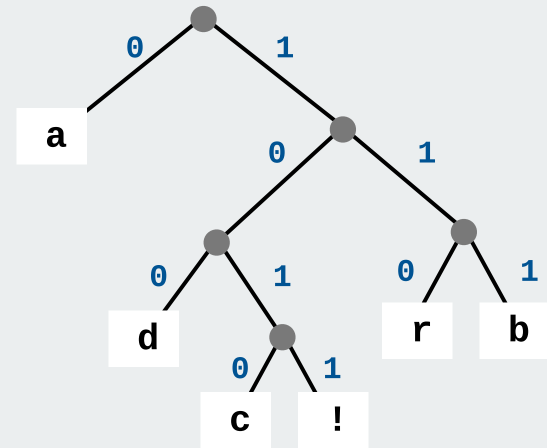
- Start at root of tree.
- Go left if bit is 0; go right if 1.
- If leaf node, print symbol and return to root.

char	encoding
a	0
b	111
c	1010
d	100
r	110
!	1011

Providing the code

How to transmit the trie?

- send preorder traversal of trie.
  - we use * as sentinel for internal nodes
  - [what if no sentinel is available?]
- send number of characters to decode.
- send bits (packed 8 to the byte).



preorder traversal	`*a**d*c!*rb`
# chars to decode	`12`
the message bits	`0111110010100100011111001011`

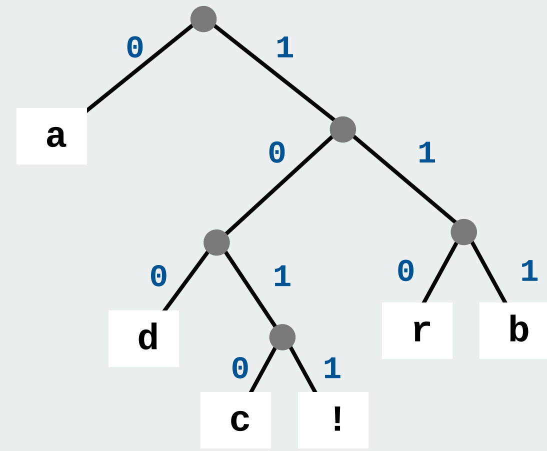
If message is long, overhead of transmitting trie is small.

a	b	r	a	c	a	d	a	b	r	a	!
0	1 1 1	1 1 0	0	1 0 1 0	0	1 0 0	0	1 1 1	1 1 0	0	1 0 1 1

char	encoding
a	0
b	111
c	1010
d	100
r	110
!	1011

Prefix-free decoding implementation

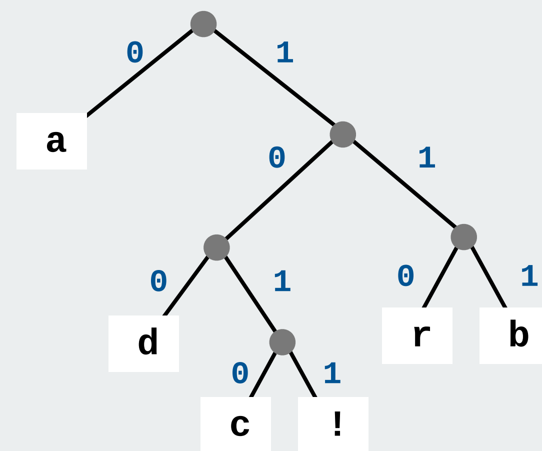
```
public class PrefixFreeDecoder
{
   private Node root = new Node();
   private class Node
   {
      char ch;
      Node left, right;
      Node()
      {
         ch = StdIn.readChar();
         if (ch == '*')
         {
            left  = new Node();
            right = new Node();
         }
      }

     boolean isInternal() { }

   }

   public void decode()
   {  /* See next slide. */   }

}
```



build tree from
preorder traversal

***a**d*c!*rb**

Prefix-free decoding iImplementation




```
public void decode()
{
   int N = StdIn.readInt();
   for (int i = 0; i < N; i++)
   {
      Node x = root;
      while (x.isInternal())
      {
         char bit = StdIn.readChar();
         if      (bit == '0') x = x.left;
         else if (bit == '1') x = x.right;
      }
      System.out.print(x.ch);
   }
}
```

use bits, not chars in actual applications

```
more code.txt
12
0111110010100100011111001011

% java PrefixFreeDecoder < code.txt
abacadabra!
```

Introduction to compression: summary

Variable-length codes can provide better compression than fixed-length

a	b	r	a	c	a	d	a	b	r	a	!
1100001	1100010	1110010	1100001	1100011	1100001	1100100	1100001	1100010	1110010	1100001	1111111

a	b	r	a	c	a	d	a	b	r	a	!
000	001	100	000	010	000	011	000	001	100	000	111

a	b	r	a	c	a	d	a	b	r	a	!
0	1 1 1	1 1 0	0	1 0 1 0	0	1 0 0	0	1 1 1	1 1 0	0	1 0 1 1

Every trie defines a variable-length code

Q. What is the best variable length code for a given message?

Huffman coding

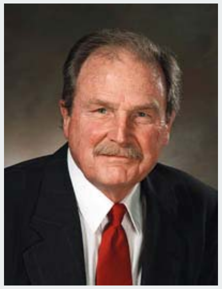
Q. What is the best variable length code for a given message?

A. Huffman code. [David Huffman, 1950]

To compute Huffman code:

- count frequency p_s for each symbol s in message.
- start with one node corresponding to each symbol s (with weight p_s).
- repeat until single trie formed:
  - select two tries with min weight p_1 and p_2
  - merge into single trie with weight $p_1 + p_2$

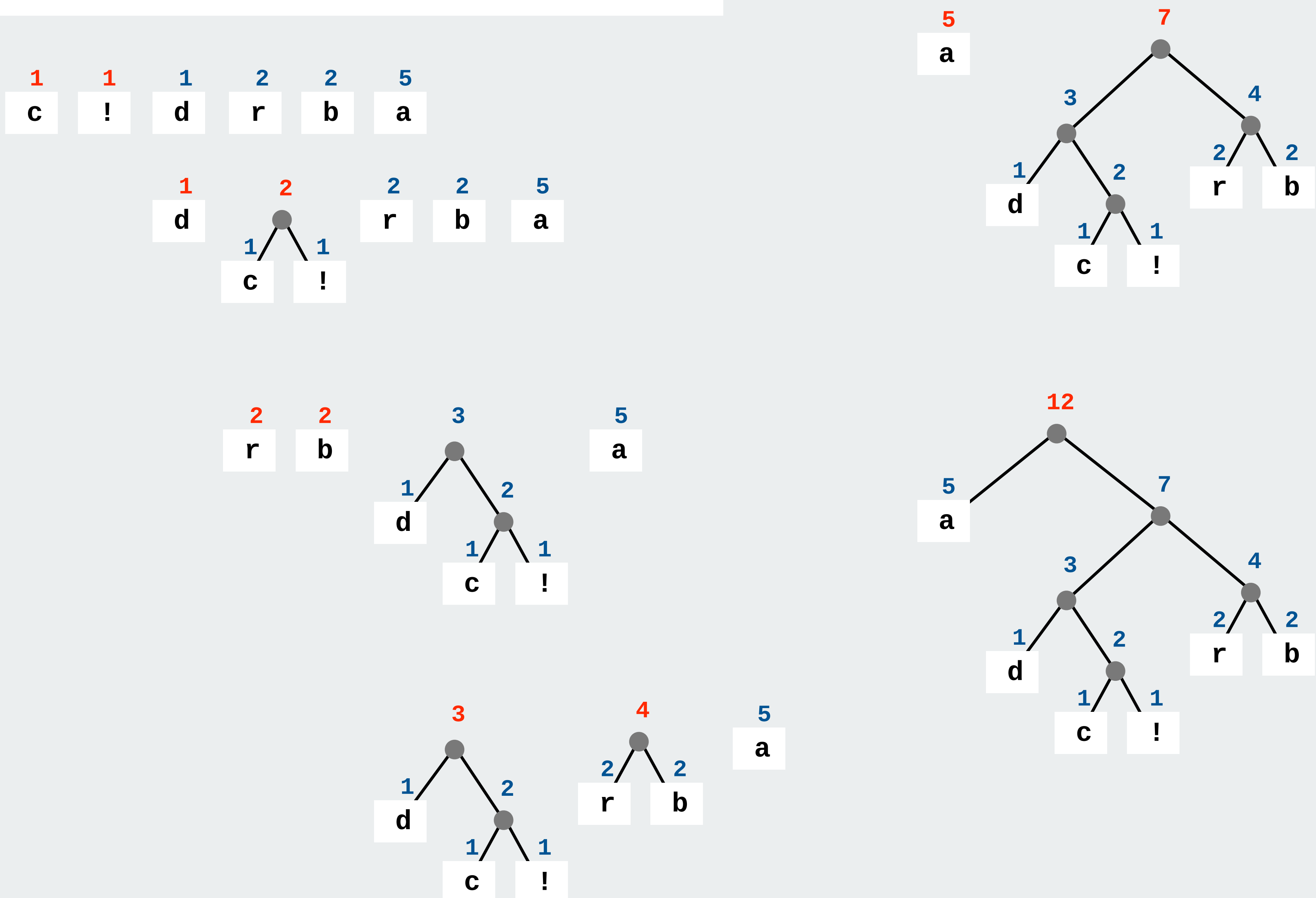
Applications. JPEG, MP3, MPEG, PKZIP, GZIP, …



David Huffman

Huffman coding example

a b r a c a d a b r a !



Huffman trie construction code

```
int[] freq = new int[128];
for (int i = 0; i < input.length(); i++)
{  freq[input.charAt(i)]++;  }
```

tabulate frequencies

```
MinPQ<Node> pq = new MinPQ<Node>();
for (int i = 0; i < 128; i++)
   if (freq[i] > 0)
      pq.insert(new Node((char) i, freq[i], null, null));
```

initialize PQ

```
while (pq.size() > 1)
{
   Node x = pq.delMin();
   Node y = pq.delMin();
   Node parent = new Node('*', x.freq + y.freq, x, y);
   pq.insert(parent);
}
root = pq.delMin();
```

merge trees

two subtrees

internal node marker

total frequency

Huffman encoding summary

Theorem. Huffman coding is an optimal prefix-free code.

no prefix-free code uses fewer bits

Implementation.

- pass 1: tabulate symbol frequencies and build trie
- pass 2: encode file by traversing trie or lookup table.

Running time. Use binary heap $\Rightarrow$ $O(\underset{\text{output bits}}{M} + \underset{\text{distinct symbols}}{N} \log N)$.

Can we do better? [stay tuned]

- introduction
- basic coding schemes
- **an application**
- entropy
- LZW codes

An application: compress a bitmap

Typical black-and-white-scanned image

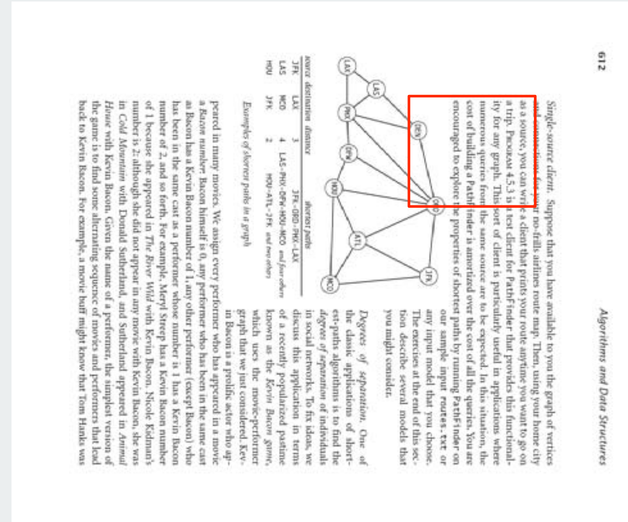
300 pixels/inch

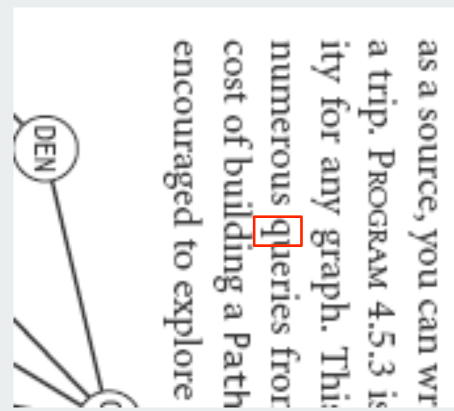
8.5 by 11 inches

300*8.5*300*11 = 8.415 million bits

Bits are mostly white

Typical amount of text on a page:
40 lines * 75 chars per line = 3000 chars





Natural encoding of a bitmap

one bit per pixel

```
000000000000000000000000000011111111111110000000000
000000000000000000000000001111111111111111100000000
000000000000000000000001111111111111111111111100000
000000000000000000000011111111111111111111111110000
000000000000000000001111111111111111111111111111110
000000000000000000011111110000000000000000001111111
000000000000000000011110000000000000000000000011111
000000000000000000011100000000000000000000000000111
000000000000000000011100000000000000000000000000111
000000000000000000011100000000000000000000000000111
000000000000000000011100000000000000000000000000111
000000000000000000001111000000000000000000000001110
000000000000000000000011100000000000000000000111000
011111111111111111111111111111111111111111111111111
011111111111111111111111111111111111111111111111111
011111111111111111111111111111111111111111111111111
011111111111111111111111111111111111111111111111111
011111111111111111111111111111111111111111111111111
011000000000000000000000000000000000000000000000011
```

19-by-51 raster of letter 'q' lying on its side

Run-length encoding of a bitmap

to encode number of bits per line

natural encoding. $(19 \times 51) + 6 = 975$ bits.

run-length encoding. $(63 \times 6) + 6 = 384$ bits.

63 6-bit run lengths

```
000000000000000000000000000011111111111111000000000
000000000000000000000000001111111111111111110000000
000000000000000000000001111111111111111111111110000
000000000000000000000011111111111111111111111111000
000000000000000000001111111111111111111111111111110
000000000000000000011111110000000000000000001111111
000000000000000000011111000000000000000000000011111
000000000000000000011100000000000000000000000000111
000000000000000000011100000000000000000000000000111
000000000000000000011100000000000000000000000000111
000000000000000000011100000000000000000000000000111
000000000000000000001111000000000000000000000001110
000000000000000000000011100000000000000000000111000
011111111111111111111111111111111111111111111111111
011111111111111111111111111111111111111111111111111
011111111111111111111111111111111111111111111111111
011111111111111111111111111111111111111111111111111
011111111111111111111111111111111111111111111111111
011000000000000000000000000000000000000000000000011
```

19-by-51 raster of letter 'q' lying on its side

```
51
28 14 9
26 18 7
23 24 4
22 26 3
20 30 1
19 7 18 7
19 5 22 5
19 3 26 3
19 3 26 3
19 3 26 3
19 3 26 3
20 4 23 3 1
22 3 20 3 3
1 50
1 50
1 50
1 50
1 50
1 2 46 2
```

RLE

Run-length encoding

- Exploit long runs of repeated characters.
- Bitmaps: runs alternate between 0 and 1; just output run lengths.
- Issue: how to encode run lengths (!)

10: 2
01: 1

`001001001001001`   `2121212121`   `10011001100110011001`

15 bits   20 bits

- Does not compress when runs are short.

Runs are long in typical applications (such as black-and-white bitmaps).

Run-length encoding and Huffman codes in the wild

ITU-T T4 Group 3 Fax for black-and-white bitmap images (~1980)

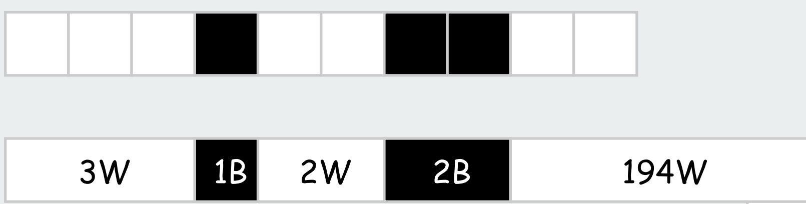
- up to 1728 pixels per line
- typically mostly white.

Step 1. Use run-length encoding.

Step 2. Encode run lengths using two Huffman codes.

one for white and one for black

194



...

1000 010 0111 11 010111 0111 ...

192 + 2

...

run	white	black
0	00110101	0000110111
1	000111	010
2	0111	11
3	1000	10
…	…	…
63	00110100	000001100111
64+	11011	0000001111
128+	10010	000011001000
…	…	…
1728+	010011011	0000001100101

Huffman codes built from frequencies in huge sample

BW bitmap compression: another approach

Fax machine (~1980)

- slow scanner produces lines in sequential order
- compress to save time (reduce number of bits to send)

Electronic documents (~2000)

- high-resolution scanners produce huge files
- compress to save space (reduce number of bits to save)

Idea:

- use OCR to get back to ASCII (!)
- use Huffman on ASCII string (!)

Ex. Typical page

- 40 lines, 75 chars/line ~ 3000 chars
- compress to ~ 2000 chars with Huffman code
- reduce file size by a factor of 500 (! ?)

Bottom line: Any extra information about file can yield dramatic gains

What data can be compressed?

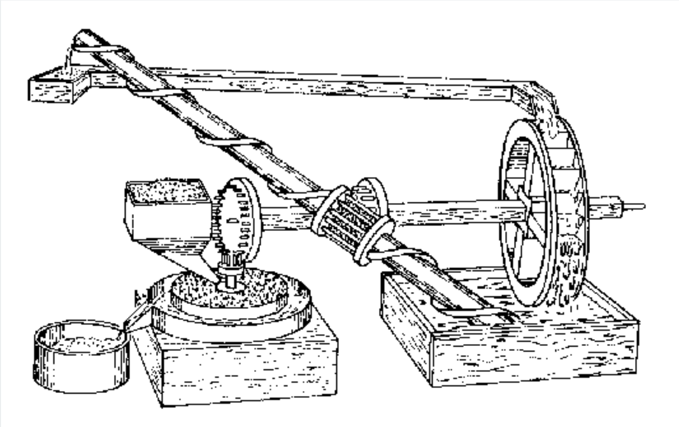
US Patent 5,533,051 on "Methods for Data Compression", which is capable of compression all files.

Slashdot reports of the Zero Space Tuner™ and BinaryAccelerator™.

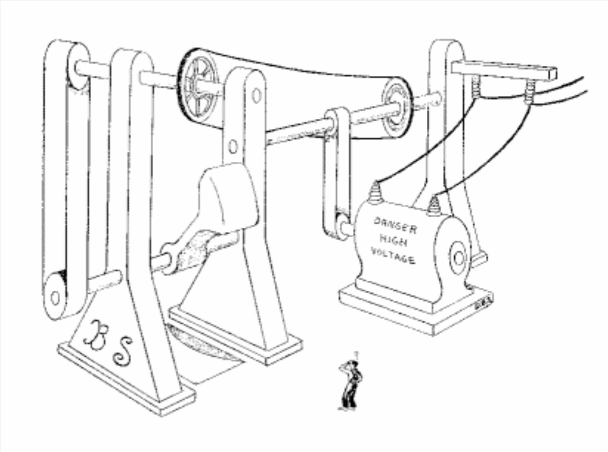
> "ZeoSync has announced a breakthrough in data compression that allows for 100:1 lossless compression of random data. If this is true, our bandwidth problems just got a lot smaller...."

Perpetual Motion Machines

Universal data compression algorithms are the analog of perpetual motion machines.



Closed-cycle mill by Robert Fludd, 1618



Gravity engine by Bob Schadewald

Reference: Museum of Unworkable Devices by Donald E. Simanek
http://www.lhup.edu/~dsimanek/museum/unwork.htm

What data can be compressed?

Theorem. Impossible to losslessly compress all files.

Pf 1.

- consider all 1,000 bit messages.
- 2^{1000} possible messages.
- only $2^{999} + 2^{998} + \ldots + 1$ can be encoded with ≤ 999 bits.
- only 1 in 2^{499} can be encoded with ≤ 500 bits!

Pf 2 (by contradiction).

- given a file M, compress it to get a smaller file M_1.
- compress that file to get a still smaller file M_2.
- continue until reaching file size 0.
- implication: all files can be compressed with 0 bits!

Practical test for any compression algorithm:

- given a file M, compress it to get a (smaller, you hope) file M_1
- compress that file to get a still smaller file M_2.
- continue until file size does not decrease

A difficult file to compress

One million pseudo-random characters (a – p)

```
fclkkacifobjofmkgdcoiicnfmcpcjfccabckjamolnihkbgobcjbngjiceeelpfgcjiihppenefllhglfemdemgahlbpi
ggmllmnefnhjelmgjncjcidlhkglhceninidmmgnobkeglpnadanfbecoonbiehglmpnhkkamdffpacjmgojmcaabpcjce
cplfbgamlidceklhfkkmioljdnoaagiheiapaimlcnlljniggpeanbmojgkccogpmkmoifioeikefjidbadgdcepnhdpfj
aeeapdjeofklpdeghidbgcaiemajllhnndigeihbebifemacfadnknhlbgincpmimdogimgeeomgeljfjgklkdgnhafoho
npjbmlkapddhmepdnckeajebmeknmeejnmenbmnnfefdbhpmigbbjknjmobimamjjaaaffhlhiggaljbaijnebidpaeigd
goghcihodnlhahllhhoojdfacnhadhgkfahmeaebccacgeojgikcoapknlomfignanedmajinlompjoaifiaejbcjcdibp
kofcbmjiobbpdhfilfajkhfmppcngdneeinpnfafaeladbhhifechinknpdnplamackphekokigpddmmjnbngklhibohdf
eaggmclllmdhafkldmimdbplggbbejkcmhlkjocjjlcngckfpfakmnpiaanffdjdlleiniilaenbnikgfnjfcophbgkhdg
mfpoehfmkbpiaignphogbkelphobonmfghpdgmkfedkfkchceeldkcofaldinljjcgafimaanelmfkokcjekefkbmegcgj
ifjcpjppnabldjoaafpbdafifgcoibbcmoffbbgigmngefpkmbhbghlbdjngenldhgnfbdlcmjdmoflhcogfjoldfjpaok
epndejmnbiealkaofifekdjkgedgdlgbioacflfjlafbcaemgpjlagbdgilhcfdcamhfmppfgohjphlmhegjechgdpkklj
pndphfcnnganmbmnggpphnckbieknjhilafkegboilajdppcodpeoddldjfcpialoalfeomjbphkmhnpdmcpgkgeaohfdm
cnegmibjkajcdcpjcpgjminhhakihfgiiachfepffnilcooiciepoapmdjniimfbolchkibkbmhbkgconimkdchahcnhap
fdkiapikencegcjapkjkfljgdlmgncpbakhjidapbldcgeekkjaoihbnbigmhboengpmedliofgioofdcphelapijcegej
gcldcfodikalehbccpbbcfakkblmoobdmdgdkafbbkjnidoikfakjclbchambcpaepfeinmenmpoodadoecbgbmfkkeabi
laoeoggghoekamaibhjibefmoppbhfbhffapjnodlofeihmjahmeipejlfhloefgmjhjnlomapjakhhjpncomippeanbik
khekpcfgbgkmklipfbiikdkdcbolofhelipbkbjmjfoempccneaebklibmcaddlmjdcajpmhhaeedbbfpjafcndianlfcj
mmbfncpdcccodeldhmnbdjmeajmboclkggojghlohlbhgjkhkmclohkgjamfmcchkchmiadjgjhjehflcbklfifackbecg
joggpbkhlcmfhipflhmnmifpjmcoldbeghpcekhgmnahijpabnomnokldjcpppbcpgcjofngmbdcpeeeiiiclmbbmfjkhl
anckidhmbeanmlabncnccpbhoafajjicnfeenppoekmlddholnbdjapbfcajblbooiaepfmmeoafedflmdcbaodgeahimc
gpcammjljoebpfmghogfckgmomecdipmodbcempidfnlcggpgbffoncajpncomalgoiikeolmigliikjkolgolfkdgiijj
iooiokdihjbbofiooibakadjnedlodeeiijkliicnioimablfdpjiafcfineecbafaamheiipegegibioocmlmhjekfikf
effmddhoakllnifdhckmbonbchfhhclecjamjildonjjdpifngbojianpljahpkindkdoanlldcbmlmhjfomifhmncikol
jjhebidjdphpdepibfgdonjljfgifimniipogockpidamnkcpipglafmlmoacjibognbplejnikdoefccdpfkomkimffgj
gielocdemnblimfmbkfbhkelkpfoheokfofochbmifleecbglmnfbnfncjmefnihdcoeiefllemnohlfdcmbdfebdmbeeb
balggfbajdamplphdgiimehglpikbipnkkecekhilchhhfaeafbbfdmcjojfhpponglkfdmhjpcieofcnjgkpibcbiblfp
njlejkcppbhopohdghljlcokhdoahfmlglbdkliajbmnkkfcoklhlelhjhoiginaimgcabcfebmjdnbfhohkjphnklcbhc
jpgbadakoecbkjcaebbanhnfhpnfkfbfpohmnkligpgfkjadomdjjnhlnfailfpcmnololdjekeolhdkebiffebajjpclg
hllmemegncknmkkeoogilijmmkomllbkkabelmodcohdhppdakbelmlejdnmbfmcjdebefnjihnejmnogeeafldabjcgfo
aehldcmkbnbafpciefhlopicifadbppgmfngecjhefnkbjmliodhelhicnfoongngemddepchkokdjafegnpgledakmbcp
cmkckhbffeihpkajginfhdolfnlgnadefamlfocdibhfkiaofeegppcjilndepleihkpkkgkphbnkggjiaolnolbjpobjd
cehglelckbhjilafccfipgebpc....
```

A difficult file to compress

```
public class Rand
{
   public static void main(String[] args)
   {
      for (int i = 0; i < 1000000; i++)
      {
         char c = 'a';
         c += (char) (Math.random() * 16);
         System.out.print(c);
      }
   }
}
```

231 bytes, but output is hard to compress
(assume random seed is fixed)

```
% javac Rand.java
% java Rand  > temp.txt
% compress -c  temp.txt > temp.Z
% gzip     -c  temp.txt > temp.gz
% bzip2    -c  temp.txt > temp.bz2
```

```
% ls -l
    231 Rand.java
1000000 temp.txt
 576861 temp.Z
 570872 temp.gz
 499329 temp.bz2
```

resulting file sizes (bytes)

Information theory

Intrinsic difficulty of compression.

- Short program generates large data file.
- Optimal compression algorithm has to discover program!
- Undecidable problem.

Q. How do we know if our algorithm is doing well?

A. Want lower bound on # bits required by any compression scheme.

Q. How do compression algorithms work?

A. They exploit statistical biases of input messages.

- ex: white patches occur in typical images.
- ex: ord `Princeton` occurs more frequently than `Yale`.

Basis of compression: probability.

- Formulate probabilistic model to predict symbols.
  - simple: character counts, repeated strings
  - complex: models of a human face
- Use model to encode message.
- Use same model to decode message.

Ex. Order 0 Markov model

- R symbols generated independently at random
- probability of occurrence of i th symbol: p_i (fixed).

Entropy

A measure of information. [Shannon, 1948]

$$H(M) = p_0/\lg p_0 + p_1/\lg p_1 + p_2/\lg p_2 + \ldots + p_{R-1}/\lg p_{R-1}$$

- information content of symbol s is proportional to $1/\lg_2 p(s)$.
- weighted average of information content over all symbols.
- interface between coding and model.


Claude Shannon

Ex. 4 binary models (R = 2)

	p_0	p_1	H(M)
1	1/2	1/2	1
2	0.900	0.100	0.469
3	0.990	0.010	0.0808
4	1	0	0

Ex. fair die (R = 6)

p(1)	p(2)	p(3)	p(4)	p(5)	p(6)	H(M)
1/6	1/6	1/6	1/6	1/6	1/6	2.585

Entropy and compression

Theorem. [Shannon, 1948] If data source is an order 0 Markov model, any compression scheme must use ≥ H(M) bits per symbol on average.

- Cornerstone result of information theory.
- Ex: to transmit results of fair die, need ≥ 2.58 bits per roll.

Theorem. [Huffman, 1952] If data source is an order 0 Markov model, Huffman code uses ≤ H(M) + 1 bits per symbol on average.

Q. Is there any hope of doing better than Huffman coding?

A1. Yes. Huffman wastes up to 1 bit per symbol.
  if H(M) is close to 0, this difference matters
  can do better with "arithmetic coding"

A2. Yes. Source may not be order 0 Markov model.

Entropy of the English Language

Q. How much redundancy is in the English language?

> "... randomising letters in the middle of words [has] little or no effect on the ability of skilled readers to understand the text. This is easy to denmtrasote. In a pubiltacion of New Scnieitst you could ramdinose all the letetrs, keipeng the first two and last two the same, and reibadailty would hadrly be aftcfeed. My ansaylis did not come to much beucase the thoery at the time was for shape and senqeuce retigcionon. Saberi's work sugsegts we may have some pofrweul palrlael prsooscers at work. The resaon for this is suerly that idnetiyfing coentnt by paarllel prseocsing speeds up regnicoiton. We only need the first and last two letetrs to spot chganes in meniang."

A. Quite a bit.

Entropy of the English Language

Q. How much information is in each character of the English language?

Q. How can we measure it?

model = English text

A. [Shannon's 1951 experiment]

- Asked subjects to predict next character given previous text.
- The number of guesses required for right answer:

# of guesses	1	2	3	4	5	≥ 6
Fraction	0.79	0.08	0.03	0.02	0.02	0.05

- Shannon's estimate: about 1 bit per char [0.6 - 1.3].

Compression less than 1 bit/char for English ? If not, keep trying!

Statistical Methods

Static model. Same model for all texts.

- Fast.
- Not optimal: different texts have different statistical properties.
- Ex: ASCII, Morse code.

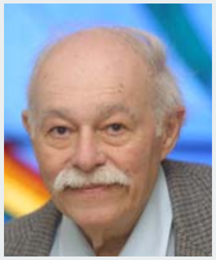
Dynamic model. Generate model based on text.

- Preliminary pass needed to generate model.
- Must transmit the model.
- Ex: Huffman code.

Adaptive model. Progressively learn and update model as you read text.

- More accurate modeling produces better compression.
- Decoding must start from beginning.
- Ex: LZW.

LZW Algorithm



Lempel-Ziv-Welch. [variant of LZ78]

- Create ST associating a fixed-length codeword with some previous substring.
- When input matches string in ST, output associated codeword.
- length of strings in ST grows, hence compression.

To send (encode) M.

- Find longest string s in ST that is a prefix of unsent part of M
- Send codeword associated with s.
- Add s · x to ST, where x is next char in M.

Ex. ST: **a**, **aa**, **ab**, **aba**, **abb**, **abaa**, **abaab**, **abaaa**,

- unsent part of M: **abaababbb**...
- s = **abaab**, x = **a**.
- Output integer associated with s; insert **abaaba** into ST.

LZW encoding example

input	code	add to ST
a	97	ab
b	98	br
r	114	ra
a	97	ac
c	99	ca
a	97	ad
d	100	da
a		
b	128	abr
r		
a	130	rac
c		
a	132	cad
d		
a	134	dab
b		
r	129	bra
a	97	
STOP	255	

input: 7-bit ASCII
output: 8-bit codewords

ASCII

key	value
	0
	...
a	97
b	98
c	99
d	100
	...
r	114
	...
	127

ST

key	value
ab	128
br	129
ra	130
ac	131
ca	132
ad	133
da	134
abr	135
rac	136
cad	137
dab	138
bra	139
	...
STOP	255

To send (encode) M.

- Find longest string s in ST that is a prefix of unsent part of M
- Send integer associated with s.
- Add s · x to ST, where x is next char in M.

LZW encoding example

input	code
a	97
b	98
r	114
a	97
c	99
a	97
d	100
a	
b	128
r	
a	130
c	
a	132
d	
a	134
b	
r	129
a	97
STOP	255

input: 7-bit ASCII
19 chars
133 bits

output: 8-bit codewords
14 chars
112 bits

Key point: no need to send ST (!)

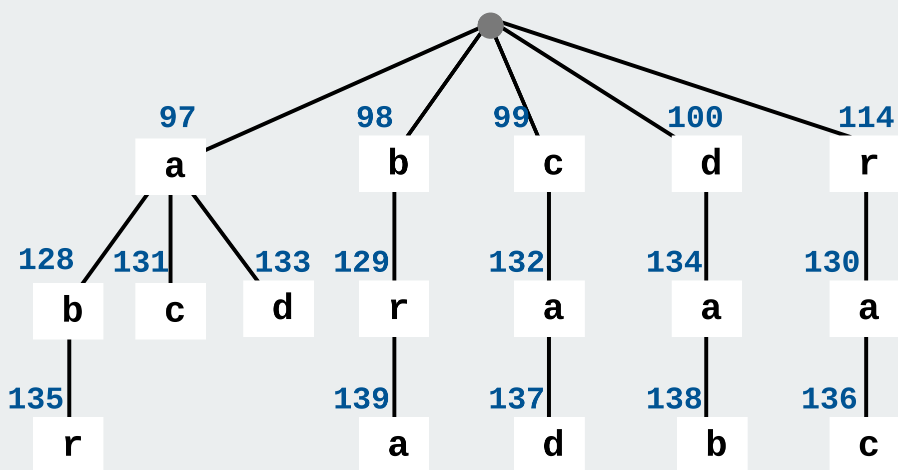
LZW encode ST implementation

Q. How to do longest prefix match?

A. Use a trie for the ST

Encode.

- lookup string suffix in trie.
- output ST index at bottom.
- add new node to bottom of trie.



ASCII

key	value
	0
	...
a	97
b	98
c	99
d	100
	...
r	114
	...
	127

ST

key	value
ab	128
br	129
ra	130
ac	131
ca	132
ad	133
da	134
abr	135
rac	136
cad	137
dab	138
bra	139
	...
STOP	255

Note that all substrings are in ST

LZW encoder: Java implementation

```
public class LZWEncoder
{
   public static void main(String[] args)
   {
      LookAheadIn in = new LookAheadIn();

      LZWst st = new LZWst();

      while (!in.isEmpty())
      {
         int codeword = st.getput(in);
         StdOut.println(codeword);
      }
   }
}
```

input stream with lookahead

specialized TST

encode text and build TST

postprocess to encode in binary

Use specialized TST

- initialized with ASCII chars and codes
- **`getput()`** method returns code of longest prefix s
  and adds s + next char to symbol table

Need input stream with backup [stay tuned]

LZW encoder: Java implementation (TST scaffolding)

```
public class LZWst
{
   private int i;                    // next codeword to assign
   private int codeword;             // codeword to return
   private Node[] roots;             // array of TSTs

   public LZWst()
   {
      roots = new Node[128];
      for (i = 0; i < 128; i++)
         roots[i] = new Node((char) i, i);
   }

   private class Node
   {
      Node(char c, int codeword)
      {  this.c = c; this.codeword = codeword;  }
      char c;
      Node left, mid, right;
      int codeword;
   }

   public int getput(LookAheadIn in)
   // See next slide.

}
```

next codeword to assign

codeword to return

array of TSTs

initialize with ASCII

standard node code

LZW encoder: Java implementation (TST search/insert)

caution: tricky recursive code

```
public int getput(LookAheadIn in)
{
   char c  = in.readChar();
   if (c == '!') return 255;
   roots[c] = getput(c, roots[c], in);
   in.backup();
   return codeword;
}
```

longest prefix codeword

```
public Node getput(char c, Node x, LookAheadIn in)
{
   if (x == null)
   {  x = new Node(c, i++); return x; }

   if      (c < x.c) x.left  = getput(c, x.left, in);
   else if (c > x.c) x.right = getput(c, x.right, in);
   else
                  {
                     char next  = in.readChar();
                     codeword = x.codeword;
                     x.mid =   getput(next, x.mid, in);
                  }
   return x;
}
```

recursive search and insert

check for codeword overflow omitted

LZW encoder: Java implementation (input stream with lookahead)

```
public class LookAheadIn
{
    In in = new In();
    char last;
    boolean backup = false;

    public void backup()
    { backup = true; }

    public char readChar()
    {
      if (!backup)
      {  last = in.readChar();  }
      backup = false;
      return last;
    }

    public boolean isEmpty()
    { return !backup && in.isEmpty();  }
}
```

Provides input stream with one-character lookahead.

`backup()` call means that last **`readChar()`** call was lookahead.

LZW Algorithm

Lempel-Ziv-Welch. [variant of LZ78]

- Create ST and associate an integer with each useful string.
- When input matches string in ST, output associated integer.
- length of strings in ST grows, hence compression.
- decode by rebuilding ST from code

To send (encode) M.

- Find longest string s in ST that is a prefix of unsent part of M
- Send integer associated with s.
- Add $s \cdot x$ to ST, where x is next char in M.

To decode received message to M.

- Let s be ST entry associated with received integer
- Add s to M.
- Add $p \cdot x$ to ST, where x is first char in s, p is previous value of s.

LZW decoding example

codeword	output	add to ST
97	a	
98	b	ab
114	r	br
97	a	ra
99	c	ac
97	a	ca
100	d	ad
128	a	
	b	da
130	r	
	a	abr
132	c	
	a	rac
134	d	
	a	cad
129	b	
	r	dab
97	a	bra
255	STOP	

role of keys and values switched

key	value
0	
...	
97	a
98	b
99	c
100	d
...	
114	r
...	
127	

key	value
128	ab
129	br
130	ra
131	ac
132	ca
133	ad
134	da
135	abr
136	rac
137	cad
138	dab
139	bra
...	
255	

Use an array to implement ST

To decode received message to M.

- Let s be ST entry associated with received integer
- Add s to M.
- Add p · x to ST, where x is first char in s, p is previous value of s.

LZW decoder: Java implementation

```
public class LZWDecoder
{
   public static void main(String[] args)
   {
      String[] st = new String[256];
      int i;
      for (i = 0; i < 128; i++)
      {  st[i] = Character.toString((char) i);  }
      st[255] = "!";

      String prev = "";
      while (!StdIn.isEmpty())
      {
         int codeword = StdIn.readInt();
         String s;
         if (codeword == i) // Tricky situation!
               s = prev + prev.charAt(0);
         else s = st[codeword];
         StdOut.print(s);
         if (prev.length() > 0)
         {  st[i++] = prev + s.charAt(0);  }
         prev = s;
      }
      StdOut.println();
   }
}
```

initialize ST with ASCII

decode text and build ST

preprocess to decode from binary

Ex: **ababababab**

LZW decoding example (tricky situation)

input	code	add to ST
a	97	ab
b	98	ba
a		
b	128	aba
a		
b		
a	130	abab
b		
STOP	255	

key	value
128	ab
129	ba
130	aba
131	abab
...	
255	

codeword	output	add to ST
97	a	
98	b	ab
128	a	
	b	ba
130	a	
	b	
	a	aba
98	b	
255	STOP	

needed before added to ST!

To send (encode) M.

- Find longest prefix
- Send integer associated with s.
- Add s · x to ST, where x is next char in M.

To decode received message to M.

- Let s be ST entry for integer
- Add s to M.
- Add p · x to ST where x is first char in s p is previous value of s.

LZW implementation details

How big to make ST?

- how long is message?
- whole message similar model?
- ...
- [many variations have been developed]

What to do when ST fills up?

- throw away and start over. GIF
- throw away when not effective. Unix compress
- ...
- [many other variations]

Why not put longer substrings in ST?

- ...
- [many variations have been developed]

LZW in the real world

Lempel-Ziv and friends.

- LZ77.
- LZ78.
- LZW.
- Deflate = LZ77 variant + Huffman.

LZ77 not patented ⇒ widely used in open source
LZW patent #4,558,302 expired in US on June 20, 2003
some versions copyrighted

PNG: LZ77.
Winzip, gzip, jar: deflate.
Unix compress: LZW.
Pkzip: LZW + Shannon-Fano.
GIF, TIFF, V.42bis modem: LZW.
Google: zlib which is based on deflate.

never expands a file

Lossless compression ratio benchmarks

Calgary corpus: standard data compression benchmark

Year	Scheme	Bits / char
1967	ASCII	7.00
1950	Huffman	4.70
1977	LZ77	3.94
1984	LZMW	3.32
1987	LZH	3.30
1987	Move-to-front	3.24
1987	LZB	3.18
1987	Gzip	2.71
1988	PPMC	2.48
1988	SAKDC	2.47
1994	PPM	2.34
1995	Burrows-Wheeler	2.29
1997	BOA	1.99
1999	RK	1.89

next assignment

Entropy	Bits/char
Char by char	4.5
8 chars at a time	2.4
Asymptotic	1.3

Data compression summary

Lossless compression.

- Represent fixed length symbols with variable length codes. [Huffman]
- Represent variable length symbols with fixed length codes. [LZW]

Lossy compression. [not covered in this course]

- JPEG, MPEG, MP3.
- FFT, wavelets, fractals, SVD, …

Limits on compression. Shannon entropy.

Theoretical limits closely match what we can achieve in practice.

Practical compression: Use extra knowledge whenever possible.

Butch: I don't mean to be a sore loser, but when it's done, if I'm dead, kill him.
Sundance: Love to.
Butch: No, no, not yet. Not until me and Harvey get the rules straightened out.
Harvey: Rules? In a knife fight? No rules.
Butch: Well, if there ain't going to be any rules, let's get the fight started...