←: BASIC ALGEBRA :→

→ Fundamental theorem, "Degree of polynomial = no of roots".

→ Linear equation:→

$$a_1x + b_1y + c_1 = 0$$
$$a_2x + b_2y + c_2 = 0$$

Unique solution	Infinite solution	No solution
$\boxed{\frac{a_1}{a_2} \neq \frac{b_1}{b_2}}$	$\boxed{\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}}$	$\boxed{\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}}$
→ Intersecting lines	→ coincident lines	→ parallel lines
→ Independent equation	→ Dependent equation	→ Inconsistent equation
→ consistent equation	→ consistent equation	(No solution)

(An equation is said to be consistent if it possesses at least one solution)

→ Quadratic equation:→

$$ax^2 + bx + c = 0$$

→ If α, β are two roots then $\boxed{\alpha + \beta = \frac{-b}{a}}$ & $\boxed{\alpha\beta = \frac{c}{a}}$

→ If roots (α, β) are given then to find out equation,

$$\boxed{x^2 - (\alpha+\beta)x + \alpha\beta = 0}$$

→ $$\boxed{x(\alpha, \beta) = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

If $b^2 - 4ac > 0$ then two distinct real roots

If $b^2 - 4ac = 0$ then two equal real roots

If $b^2 - 4ac < 0$ then No real roots.

→ If $x^2 \pm kx + 1 = 0$ then $\boxed{x + \frac{1}{x} = \mp k}$ (mind sign)

→ $y = +ax^2 \pm bx \pm c$, (or a greater than zero)

then, $\boxed{y_{min} = \pm\left(\frac{4ac - b^2}{4a}\right)}$

$y_{max} = \infty$, (can not determined)

→ $y = -ax^2 \pm bx \pm c$, (or $a < 0$)

then, $\boxed{y_{max} = \left(\frac{\pm 4ac + b^2}{4a}\right)}$

$y_{min} = \infty$, (can not be determined)

Identities :→

→ $(a+b)^2 = a^2 + 2ab + b^2$

or $= (a-b)^2 + 4ab$

→ $(a-b)^2 = a^2 - 2ab + b^2$

or $= (a+b)^2 - 4ab$

→ $(a+b)^2 + (a-b)^2 = 2(a^2+b^2)$

→ $\frac{(a+b)^2 + (a-b)^2}{2} = (a^2+b^2)$

→ $\frac{(a+b)^2 + (a-b)^2}{a^2+b^2} = 2$

→ $(a+b)^2 - (a-b)^2 = 4ab$

→ $\frac{(a+b)^2 - (a-b)^2}{4} = ab$

→ $\frac{(a+b)^2 - (a-b)^2}{ab} = 4$

Applications :→

→ $(a+1)^2 = a^2 + 2a + 1$

→ $(a-1)^2 = a^2 - 2a + 1$

→ $a^2 + 1 = (a+1)^2 - 2a$

or $= (a-1)^2 + 2a$

or $= \frac{(a+1)^2 + (a-1)^2}{2}$

→ $\left(a + \frac{1}{a}\right)^2 = a^2 + \frac{1}{a^2} + 2$

→ $\left(a - \frac{1}{a}\right)^2 = a^2 + \frac{1}{a^2} - 2$

→ $a^2 + \frac{1}{a^2} = \left(a + \frac{1}{a}\right)^2 - 2$

or $= \left(a - \frac{1}{a}\right)^2 + 2$

or $= \frac{\left(a+\frac{1}{a}\right)^2 + \left(a-\frac{1}{a}\right)^2}{2}$

→ $\left(a + \frac{1}{a}\right)^2 - \left(a - \frac{1}{a}\right)^2 = 4$

$\rightarrow (a-b)^2 - (a+b)^2 = -4ab$

$\rightarrow a^2 - b^2 = (a+b)(a-b)$

$\rightarrow a^2 + b^2 = (a+b)^2 - 2ab$

or $= (a-b)^2 + 2ab$

or $= \dfrac{(a+b)^2 + (a-b)^2}{2}$

$\rightarrow \left(a+\frac{1}{a}\right)^2 + \left(a-\frac{1}{a}\right)^2 = 2\left(a^2+\frac{1}{a^2}\right)$

$\rightarrow$ If $a+\frac{1}{a} = p$ then $a^2+\frac{1}{a^2} = p^2-2$

$\rightarrow a^2 - \frac{1}{a^2} = \left(a+\frac{1}{a}\right)\left(a-\frac{1}{a}\right)$

$\rightarrow (a+b+c)^2 = a^2+b^2+c^2 + 2(ab+bc+ac)$

$\rightarrow (a-b)^2 + (b-c)^2 + (c-a)^2 = 2a^2+2b^2+2c^2 - 2ab - 2bc - 2ac$

$= 2(a^2+b^2+c^2 - ab - bc - ac)$

Cubic form: $\rightarrow$

$\rightarrow (a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

or $= a^3 + b^3 + 3ab(a+b)$

$\rightarrow (a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

$= a^3 - b^3 - 3ab(a-b)$

$= a^3 - b^3 + 3ab(b-a)$

$\rightarrow (a^3+b^3) = (a+b)^3 - 3ab(a+b)$

$= (a+b)(a^2 - ab + b^2)$

$= (a+b)((a+b)^2 - 3ab)$

$\rightarrow \dfrac{a^3+b^3}{a^2-ab+b^2} = (a+b)$

$\rightarrow \dfrac{a^2-ab+b^2}{a^3+b^3} = \dfrac{1}{a+b}$

$\rightarrow (a^3-b^3) = (a-b)^3 + 3ab(a-b)$

$= (a-b)^3 - 3ab(b-a)$

$= (a-b)\{(a-b)^2 + 3ab\}$

$= (a-b)(a^2+ab+b^2)$

Applications: $\rightarrow$

$\rightarrow (a+1)^3 = a^3 + 1 + 3a^2 + 3a$

$= a^3 + 1 + 3a(a+1)$

$\rightarrow (a-1)^3 = a^3 - 3a^2 + 3a - 1$

$= a^3 - 1 - 3a(a-1)$

$= a^3 - 1 + 3a(1-a)$

$\rightarrow \left(a+\frac{1}{a}\right)^3 = a^3 + \frac{1}{a^3} + 3\left(a+\frac{1}{a}\right)$

$\rightarrow \left(a-\frac{1}{a}\right)^3 = a^3 - \frac{1}{a^3} - 3\left(a-\frac{1}{a}\right)$

$= a^3 - \frac{1}{a^3} + 3\left(\frac{1}{a} - a\right)$

$\rightarrow a^3 + \frac{1}{a^3} = \left(a+\frac{1}{a}\right)^3 - 3\left(a+\frac{1}{a}\right)$

$= \left(a+\frac{1}{a}\right)\left(a^2+\frac{1}{a^2} - 1\right)$

$= \left(a+\frac{1}{a}\right)\left(\left(a+\frac{1}{a}\right)^2 - 3\right)$

$\rightarrow a^3 - \frac{1}{a^3} = \left(a-\frac{1}{a}\right)^3 + 3\left(a-\frac{1}{a}\right)$

$= \left(a-\frac{1}{a}\right)\left(a^2+\frac{1}{a^2} + 1\right)$

$\rightarrow \dfrac{a^3-b^3}{a^2+ab+b^2} = (a-b)$

$\rightarrow \dfrac{a^2+ab+b^2}{a^3-b^3} = \dfrac{1}{(a-b)}$

$\rightarrow a^3 - \frac{1}{a^3} = \left(a-\frac{1}{a}\right)\left\{\left(a-\frac{1}{a}\right)^2+3\right\}$

$\rightarrow a^3+1 = (a+1)^3 - 3a(a+1)$

$= (a+1)(a^2+1-a)$

$\rightarrow a^3-1 = (a-1)^3 + 3a(a-1)$

$= (a-1)^3 - 3a(1-a)$

$= (a-1)(a^2+1+a)$

$\rightarrow x^3+y^3+z^3-3xyz = (x+y+z)(x^2+y^2+z^2-xy-yz-zx)$

or $= \dfrac{(x+y+z)}{2}\left\{(x-y)^2+(y-z)^2+(z-x)^2\right\}$

Application :→

→ If $x+y+z=0$ then $x^3+y^3+z^3 = 3xyz$

or $\dfrac{x^3+y^3+z^3}{xyz} = 3$

or $\dfrac{x^2}{yz} + \dfrac{y^2}{xz} + \dfrac{z^2}{xy} = 3.$

→ If $(x+y-z)=0$ then $x^3+y^3-z^3+3xyz=0$

→ If $(x-y-z)=0$ then $x^3-y^3-z^3-3xyz=0$

⊛→ Some major applications of square & cubic form:→

$\rightarrow (x+1) = (x^{1/3}+1)(x^{2/3}+1-x^{1/3})$

$\rightarrow (x-1) = (x^{1/3}-1)(x^{2/3}+1+x^{1/3})$

→ If $a+\frac{1}{a}=1$ then $a^3+1=0$ or $a^3=-1$

→ If $a+\frac{1}{a}=-1$ then $1-a^3=0$ or $a^3-1=0$ or $a^3=1$

→ If $\frac{a}{b}+\frac{b}{a}=1$ then $a^3+b^3=0$

→ If $\frac{a}{b}+\frac{b}{a}=-1$ then $a^3-b^3=0$

→ If $a+\frac{1}{a}=\sqrt{3}$ then $a^3+\frac{1}{a^3}=0$ (or) $a^6+1=0$ (or) $a^6=-1$

→ If $x+\frac{1}{x}=2$ then $x^n+\frac{1}{x^n}=2$

→ If $a^4+b^4=a^2b^2$ then $a^6+b^6=0$

→ If $x=\frac{2ab}{a+b}$ then $\frac{x+a}{x-a}+\frac{x+b}{x-b}=2$

→ If $x=\frac{4ab}{a+b}$ then $\frac{x+2a}{x-2a}+\frac{x+2b}{x-2b}=2$

→ If $x=\frac{6ab}{a+b}$ then $\frac{x+3a}{x-3a}+\frac{x+3b}{x-3b}=2$

→ If $\frac{\sqrt{a+x}+\sqrt{a-x}}{\sqrt{a+x}-\sqrt{a-x}}=b$ then $x=\frac{2ab}{b^2+1}$

→ $x^2+\frac{1}{x^2}\geqslant 2$ (Always)

→ $(x^3)^2+(y^3)^2=(x^6+y^6)=(x^3+y^3)^2-2x^3y^3$

(or) $=(x^3-y^3)^2+2x^3y^3$

→ $(x^2)^3+(y^2)^3=(x^6+y^6)=(x^2+y^2)^3-3x^2y^2(x^2+y^2)$

→ $(x^3)^2-(y^3)^2=(x^6-y^6)=(x^3-y^3)(x^3+y^3)$

→ $(x^2)^3-(y^2)^3=(x^6-y^6)=(x^2-y^2)^3+3x^2y^2(x^2-y^2)$

→ $a^2+b^2+ab=(a+b+\sqrt{ab})(a+b-\sqrt{ab})$

→ $a^4+b^4+a^2b^2=(a^2+b^2+ab)(a^2+b^2-ab)$

→ $\left(x^2+\frac{1}{x^2}+1\right)=\left(x+\frac{1}{x}+1\right)\left(x+\frac{1}{x}-1\right)$

→ $\left(x^4+\frac{1}{x^4}+1\right)=\left(x^2+\frac{1}{x^2}+1\right)\left(x^2+\frac{1}{x^2}-1\right)$

$\rightarrow \left(x^2+\frac{1}{x^2}\right)\left(x^3+\frac{1}{x^3}\right)=\left(x^5+\frac{1}{x^5}\right)+\left(x+\frac{1}{x}\right)$ $[2\times3=5+1]$

$\rightarrow \left(x^4+\frac{1}{x^4}\right)\left(x^3+\frac{1}{x^3}\right)=\left(x^7+\frac{1}{x^7}\right)+\left(x+\frac{1}{x}\right)$

$\rightarrow x^5+\frac{1}{x^5}=\left(x^2+\frac{1}{x^2}\right)\left(x^3+\frac{1}{x^3}\right)-\left(x+\frac{1}{x}\right)$ $[5=2\times3-1]$

$\rightarrow x^7+\frac{1}{x^7}=\left(x^3+\frac{1}{x^3}\right)\left(x^4+\frac{1}{x^4}\right)-\left(x+\frac{1}{x}\right)$

$\rightarrow x^9+\frac{1}{x^9}=\left(x^4+\frac{1}{x^4}\right)\left(x^5+\frac{1}{x^5}\right)-\left(x+\frac{1}{x}\right)$

Series: $\rightarrow$

Arithmatic progression (A.P): $\rightarrow$

Series $\rightarrow$ $(a, a+d, a+2d, a+3d \cdots\cdots)$

$\rightarrow$ If three numbers are in AP then you have to take $a-d,\ a,\ a+d$.

$\rightarrow$ $\boxed{n^{th} \text{ term}, \ t_n \text{ (or) } l = a+(n-1)d}$

$\Rightarrow \boxed{n = \frac{l-a}{d}+1}$

common difference

$$\boxed{\begin{aligned} d &= a_2-a_1 \\ &= a_3-a_2 \\ &= a_4-a_3 \\ &= a_n-a_{n-1} \end{aligned}}$$

$\rightarrow$ Sum of n terms,

$$\boxed{S_n = \frac{n}{2}[2a+(n-1)d]}$$

$$\boxed{S_n = \frac{n}{2}(a+l)}$$

$$\boxed{S_n = \frac{(l+a)(l-a+d)}{2d}}$$

$\rightarrow a_1 = S_1$

$a_2 = S_2 - S_1$

$a_3 = S_3 - S_2$

$\vdots \quad \vdots \quad \vdots$

$a_{10} = S_{10} - S_9$

$\vdots \quad \vdots \quad \vdots$

$$\boxed{a_n = S_n - S_{n-1}}$$

Arithmatic mean :→

→ If a, b & c are in A.P then $(b-a)=(c-b)$

So $\boxed{b = \frac{a+c}{2}}$, called arithmatic mean

Common term :→

→ If there are two series,

$a_1, a_2, a_3 \cdots\cdots$ (difference = d_1)

$b_1, b_2, b_3 \cdots\cdots$ (difference = d_2)

If a = first common term of two series.

d = lcm of d_1 & d_2

then $\boxed{t_n = a + (n-1)d}$

Application :→

→ $x, A_1, A_2, A_3, \cdots\cdots A_n, y$ are in A.P, then

$$\boxed{A_1 + A_2 + A_3 + \cdots\cdots + A_n = \frac{n(x+y)}{2}}$$

→ in A.P, If sum of n consecutive numbers is known then mid number $= \frac{sum}{n}$ (n should be always odd number).

Geometric progression (G.P): →

Series → $(a, ar, ar^2, ar^3 \ldots\ldots)$

→ If three numbers are in GP then you have to take

$$\left(\frac{a}{r}, a, ar\right)$$

→ n^{th} term, $\boxed{t_n = ar^{n-1}}$

→ Sum of n terms,

$$S_n = \frac{a(1-r^n)}{(1-r)}, \quad |r| < 1$$

$$S_n = \frac{a(r^n - 1)}{r-1}, \quad |r| > 1$$

$$S_\infty = \frac{a}{1-r}, \quad |r| < 1$$

→ $\boxed{r = \frac{a_2}{a_1} = \frac{a_3}{a_2} = \frac{a_{n+1}}{a_n}}$, called common ratio.

Geometric mean: →

→ If a, b, c are in G.P then $\frac{b}{a} = \frac{c}{b}$

$\Rightarrow \boxed{b^2 = ac}$ or $\boxed{b = \sqrt{ac}}$, called geometric mean.

Application: →

→ $x + xx + xxx + \ldots\ldots$ upto n terms $= \frac{x}{9}\left[\frac{10}{9}(10^n - 1) - n\right]$

→ Total apple $= (\text{Remaining apple} \times 2^n) + (2^n - 1)$

Harmonic progression :→

Series → $\frac{1}{3}, \frac{1}{6}, \frac{1}{9}, \frac{1}{12}, \frac{1}{15} \cdots\cdots$

→ If a, H, b are in H.P then $\boxed{H = \frac{2ab}{a+b}}$, called Harmonic mean

→ $\boxed{A.M \times H.M = (G.M)^2}$

Some useful applications of AP & GP :→

→ Ⓐ $\boxed{1+2+3+4+\cdots\cdots + n = \frac{n(n+1)}{2}}$

Ⓑ Sum of all consecutive numbers from 'a' to 'b' $= \frac{(b+a)(b-a+1)}{2}$

Ⓒ Sum of all even/odd numbers from 'a' to 'b' $= \frac{(b+a)(b-a+2)}{4}$

{where, a → first even/odd number
b → last even/odd number}

ⓓ Sum of first n multiples of $x = \frac{x \times n(n+1)}{2}$

→ Sum of first even natural numbers $= \frac{n}{2}\left(\frac{n}{2}+1\right)$
(must start with 2 & n is the last term)

→ Sum of first 'n' even natural numbers $= n(n+1)$

→ Average of 'n' even natural numbers $= (n+1)$

→ Sum of first odd numbers $= \left(\frac{n+1}{2}\right)^2$
(must start with 1 & n is the last term)

→ Sum of first 'n' odd natural numbers $= n^2$

→ Average of 'n' odd natural numbers $= n$.

$$1^2+2^2+3^2+\cdots\cdots+n^2 = \frac{n(n+1)(2n+1)}{6}.$$

$$1^2+3^2+5^2+\cdots\cdots+n^2\text{(odd)} = \frac{n(n+1)(n+2)}{6}.$$

$$2^2+4^2+6^2+8^2+\cdots\cdots+n^2\text{(even)} = \frac{n(n+1)(n+2)}{6}.$$

$$1^3+2^3+3^3+\cdots\cdots+n^3 = \left[\frac{n(n+1)}{2}\right]^2.$$

Sum of first n terms of following series
1, 3, 6, 10, 15, 21, ……… is $\frac{n(n+1)(n+2)}{6}$

$$\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\cdots\cdots\left(1-\frac{1}{n}\right) = \frac{1}{n} \quad \text{(A)}$$

$$\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\left(1+\frac{1}{4}\right)\cdots\cdots\left(1+\frac{1}{n}\right) = \frac{(n+1)}{2} \quad \text{(B)}$$

$$\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\left(1-\frac{1}{4^2}\right)\cdots\cdots\left(1-\frac{1}{n^2}\right) = \frac{(n+1)}{2n} \leftarrow (A\times B)$$

$$\frac{1}{1\times2}+\frac{1}{2\times3}+\frac{1}{3\times4}+\cdots\cdots+\frac{1}{(n-1)n} = \frac{n-1}{n}, <1 \left\{\begin{matrix}\text{may in}\\ \text{option}\end{matrix}\right\}$$

$$\frac{1}{1\times2}+\frac{1}{2\times3}+\frac{1}{3\times4}+\cdots\cdots\frac{1}{n(n+1)} = \frac{n}{n+1}, <1 \text{ (MIO)}$$

$$\frac{1}{1\times3}+\frac{1}{3\times5}+\frac{1}{5\times7}+\cdots\cdots+\frac{1}{(2n-1)(2n+1)} = \frac{1}{2}\left(\frac{2n}{2n+1}\right)$$

$$\frac{1}{1\times2}+\frac{1}{2\times3}+\cdots\cdots+\frac{1}{n(n+1)} = \frac{1}{d}\left\{\frac{1}{\text{first term}}-\frac{1}{\text{last term}}\right\}$$

$$\boxed{\left(1-\frac{1}{n}\right)+\left(1-\frac{2}{n}\right)+\left(1-\frac{3}{n}\right) \cdots\cdots n \text{ terms} = \frac{n-1}{2}}$$

$$\rightarrow \frac{1}{\sqrt{x}+\sqrt{y}} + \frac{1}{\sqrt{y}+\sqrt{z}} + \frac{1}{\sqrt{z}+\sqrt{p}} + \cdots\cdots\cdots + \frac{1}{\sqrt{A}+\sqrt{B}}$$

Answer: →

① If difference is 1 then (last term − first term) i.e. $(\sqrt{B}-\sqrt{x})$

difference ↓ $(x-y)$ or $(y-z)$ or $(z-p)$ ... etc

② If difference is 2 then, $\frac{1}{2}$ (last term − first term)

So, $\boxed{\text{If difference is } d\text{, then } \frac{1}{d}(\text{last term} - \text{first term})}$

$\{$mind sign, If it is '×' then $\frac{1}{d}\left(\frac{1}{F\cdot T} - \frac{1}{L\cdot T}\right)$

If it is '+' then $\frac{1}{d}(L\cdot T - F\cdot T)\}$

Surds

→ $\sqrt{a\sqrt{a\sqrt{a\sqrt{a}}}} \cdots\cdots \infty = a$

→ $\sqrt{a\sqrt{a\sqrt{a \cdots}}} \cdots$ n times $= a^{\frac{2^n-1}{2^n}}$

→ $\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}} + \frac{\sqrt{a}-\sqrt{b}}{\sqrt{a}+\sqrt{b}} = \frac{2(a+b)}{(a-b)}$

→ $\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}} - \frac{\sqrt{a}-\sqrt{b}}{\sqrt{a}+\sqrt{b}} = \frac{4\sqrt{ab}}{(a-b)}$

→ $\left(\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}}\right)^2 + \left(\frac{\sqrt{a}-\sqrt{b}}{\sqrt{a}+\sqrt{b}}\right)^2 = \left[\frac{2(a+b)}{a-b}\right]^2 - 2$

→ If $x = \frac{\sqrt{3}}{2}$ then $\boxed{\sqrt{1+x} = \frac{\sqrt{3}+1}{2}}$ and $\boxed{\sqrt{1-x} = \frac{\sqrt{3}-1}{2}}$

→ $\boxed{a+\sqrt{b} = \frac{1}{(a-\sqrt{b})}}$ e.g $\boxed{2+\sqrt{3} = \frac{1}{2-\sqrt{3}}}$ (or) $\boxed{2-\sqrt{3} = \frac{1}{2+\sqrt{3}}}$

Extra :→

1. in cubic equation, $ax^3+bx^2+cx+d=0$, If roots are α, β, γ

then, $\boxed{\alpha+\beta+\gamma = \frac{-b}{a}}$ $\boxed{(\alpha\beta+\beta\gamma+\alpha\gamma) = \frac{c}{a}}$ & $\boxed{\alpha\beta\gamma = \frac{-d}{a}}$.

2. $\boxed{(ax-by)^2+(ay+bx)^2 = (a^2+b^2)(x^2+y^2)}$

Ratio and proportion :→

→ If $a:b :: c:d$ then $bc = ad$

→ If $a:b :: b:c$ then $b^2 = ac$ (or) $b = \sqrt{ac}$

→ mean proportion between a & b is $\sqrt{ab}$

→ Duplicate ratio of $a:b$ is $a^2:b^2$

→ sub-duplicate ratio of $a:b$ is $(\sqrt{a}:\sqrt{b})$

→ Triplate ratio of $a:b$ is $a^3:b^3$

→ sub-triplate ratio of $a:b$ is $\sqrt[3]{a}:\sqrt[3]{b}$

→ If $x \propto y$ then $x = ky$ (k = constant)

→ If $x \propto \frac{1}{y}$ then $x = \frac{k}{y}$ (k = constant)

→ inverse ratio of $a:b$ is $b:a$ (

→ If $x:y = a:b$ and $y:z = m:n$

then $x:y:z = ma:mb:nb$

→ If $p:q:r = a:b:c$ and $r:s = m:n$ then

$p:q:r:s = ma:mb:mc:nc$

→ The compound ratios of $(a:b), (c:d), (e:f)$ is $(ace:bdf)$.

→ componendo : If $\frac{a}{b} = \frac{c}{d}$ then $\frac{a+b}{b} = \frac{c+d}{d}$

→ Dividendo : If $\frac{a}{b} = \frac{c}{d}$ then $\frac{a-b}{b} = \frac{c-d}{d}$

→ componendo & Dividendo :

If $\frac{a}{b} = \frac{c}{d}$ then $\boxed{\frac{a+b}{a-b} = \frac{c+d}{c-d}}$

→ If $\frac{x+y}{x-y} = \frac{a}{b}$ then $\boxed{\frac{x}{y} = \frac{a+b}{a-b}}$

→ If $\frac{a}{b} = \frac{c}{d}$ then $\boxed{\frac{a}{b} = \frac{c+am}{d+bm}}$

→ If $\frac{a}{b} = \frac{b}{c}$ then $\boxed{\frac{a}{c} = \frac{a^2}{b^2}}$

→ If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k$ then $\boxed{\frac{a+c+e}{b+d+f} = k}$

① Km $\xrightarrow[(10 \div)]{(\times 10)}$ Hecto meter → Deca meter (dam) → meter → deci meter → centi meter → mili meter

②

$100\ m^2 = 1$ are (a)	$1\ lt = 1000\ cm^3$
$10{,}000\ m^2 = 1$ hectare (ha)	$1000\ lt = 1\ m^3$

←: NUMBER SYSTEM :→

→ Let 'N' be a composite number & a, b, c, d ... be the prime factors and p, q, r, s ... be the powers or indices i.e N can be expressed as

$$\boxed{N = a^p \times b^q \times c^r \times d^s \cdots\cdots}, \text{ then}$$

1. Total prime factors $= p+q+r+s \cdots$
2. Total no of factors or divisors $= (p+1)(q+1)(r+1)(s+1)\cdots$
3. Sum of all factors or divisors $= \dfrac{(a^{p+1}-1)(b^{q+1}-1)(c^{r+1}-1)(d^{s+1}-1)}{(a-1)(b-1)(c-1)(d-1)}$
4. The no of ways of expressing composite number as a product of two factors $= \frac{1}{2} \times$ Total no of factors.
5. The product of all factors of composite number $N = N^{n/2}$ (where n is the total no of factors of N)
6. Total no of odd factors $= (p+1)(q+1)(r+1)(s+1)\cdots$ (where p, q, r, s ... are powers on odd numbers)
7. No of even factors = Total no of factors − Total no of odd factors

* UNIT DIGIT CONCEPT :→

→ 0, 1, 5, 6 — No change

→ $4^{even} = 6$ (unit digit), $9^{even} = 1$ (u.d)

$4^{odd} = 4$ (unit digit), $9^{odd} = 9$ (u.d)

→ $2^{4n} = 8^{4n} = 6$ (u.d)

$3^{4n} = 7^{4n} = 1$ (u.d)

→ Divide the power on them (number) by '4' and find out the remainder and if remainder is

1 → Number will be itself as ~~remainder~~ Unit digit (or) multiply that number (digit only) with 1.

2 → multiply that number (only digit) two times

3 → multiply that number (only digit) three times

0(4) → multiply that number (only digit) four times.

Between 1-100 ⇒ 25 prime numbers
100 - 200 ⇒ 21 prime numbers
200 - 300 ⇒ 16 prime numbers
300 - 400 ⇒ 16 prime numbers
400 - 500 ⇒ 17 prime numbers

* Remainder theorem :→

→ Divident = Divisor x quotient + Remainder

→ $\frac{(a-1)^n}{a}$, If n is even then remainder = 1
If n is odd then remainder = $(a-1)$ {Numerator}

(or) $\frac{a^n}{(a+1)}$, If n is even then remainder = 1
If n is odd then remainder = a {Numerator}

→ $\frac{(a+1)^n}{a}$, Remainder is always 1, wheather n is even or odd.

→ $x^n + y^n$ (n is positive integer, x, y are co-prime number)
⇒ Having no even factor
⇒ If n is odd then $(x+y)$ will divide it.

→ $x^n - y^n$ (n is positive integer
x, y are co-prime number)

↳ If n is odd then $x-y$ will divide it.

↳ If n is even then both $(x+y)$ & $(x-y)$ will divide it.

→ If a polynomial $f(x)$ is divided by $(x+a)$ then remainder is $f(-a)$.

→ If a polynomial $f(x)$ is divided by $(x-a)$ then $f(a)$ will be remainder.

→ If a polynomial $f(x)$ is divided by $(ax+b)$ then the remainder is $f\left(\frac{-b}{a}\right)$.

→ $\boxed{R_3 = R_1 + R_2 - \text{Divisor}}$ // $\boxed{\text{Divisor} = R_1 + R_2 - R_3}$

→ Number = $\left(\begin{array}{l}\text{Lcm of} \\ \text{Divisors}\end{array} - \begin{array}{l}\text{Difference of Divisors} \\ \text{and remainders}\end{array}\right)$

→ If there is 'xyxyxy' type number then it is divisible by 7, 13 and 7×13 (both).

→ If there is 'xyoxy', 'xyzxyz' or 'xxxxxx' type number then it is divisible by 7, 11, & 13, and also by 1001 (7×11×13)

* HCF & LCM :→

$$a = H \times x \qquad b = H \times y$$

$\left\{\begin{array}{l} a, b \to \text{composite numbers} \\ x, y \to \text{prime numbers}\end{array}\right\}$

→ $H.C.f = H$

$L.C.M = H \times x \times y$

$= H.C.f \times (x \times y)$

→ $a \times b = LCM \times HCF$ {product of two numbers is equal to product of their HCF & LCM}

→ $a + b = H(x+y)$

→ $x + y = \frac{a+b}{H}$

→ $x \times y = \frac{a \times b}{H^2}$

→ $x \times y = \frac{LCM}{HCF}$

→ $\boxed{\frac{a+b}{LCM} = \frac{H(x+y)}{H(x \times y)} = \frac{(x+y)}{xy}}$

→ LCM of fractions $= \frac{\text{LCM of Numerator}}{\text{HCF of Denominator}}$

→ HCF of Fractions $= \frac{\text{HCF of Numerator}}{\text{LCM of Denominator}}$

→ Number $= \left(\text{LCM of Divisors} - \text{Difference of Divisors \& Remainder}\right)$

*→ Indices and Surds :→

→ $(xy)^m = x^m \times y^m$ (And viceversa)

→ $\left(\frac{x}{y}\right)^m = \frac{x^m}{y^m}$ (And viceversa)

→ $x^m \times x^n = x^{m+n}$ (And viceversa)

→ $\frac{x^m}{x^n} = x^{m-n}$ (And viceversa)

→ $(\sqrt[n]{x})^m = \sqrt[n]{x^m}$ (And viceversa)

→ $((x^m)^n)^p = x^{m \times n \times p}$ {Bracket must be there}

→ $x^{m^{n^p}} = x^{m^{n \times n \times n \cdots\cdots p \text{ times}}}$ $= x^{m \times m \times m \cdots\cdots n^p \text{ times}}$ (No Bracket)

→ $\boxed{\text{So} ((x^m)^n)^p \neq x^{m^{n^p}}}$

→ $x^0 = 1$ $(x \neq 0)$

→ If $x^m = y$ then $x = y^{1/m}$ (And viceversa)

→ If $x^{1/m} = y$ then $x = y^m$ (And viceversa)

→ If $a^x = b^y$ then It is possible when $x = y = 0$

→ If $a^x = b^y$ then $a = b^{y/x}$ (or) $b = a^{x/y}$

→ If $a^x = a^y$ then $x = y$

→ $a^n = \frac{1}{a^{-n}}$ (or) $a^{-n} = \frac{1}{a^n}$ (And viceversa)

→ If $(N_1)^p = (N_2)^q = (N_1 N_2)^{-z}$ then $\frac{1}{p} + \frac{1}{q} + \frac{1}{z} = 0$

(or) If $(N_1)^{-p} = (N_2)^{-q} = (N_1 N_2)^z$ then $\frac{1}{p} + \frac{1}{q} + \frac{1}{z} = 0$

→ If $xy = 0$ then $x = 0$ (or) $y = 0$

→ If $x^2 + y^2 = 0$ and x, y are real numbers then $x = 0$ (or) $y = 0$

→ If $x + y = K$ then maximum value of xy is when $x = y$

※ Surds :→

→ $\sqrt{a + \sqrt{a + \sqrt{a + \sqrt{a}}}} \cdots\cdots \infty = \frac{\sqrt{4a+1} + 1}{2}$ $(= x \text{ (say)})$

→ $\sqrt{a - \sqrt{a - \sqrt{a - \sqrt{a}}}} \cdots\cdots \infty = \frac{\sqrt{4a+1} - 1}{2}$ $\{= y \text{ (say)}\}$

{Relation between them is 1 i.e $(x - y) = 1$}