1 NUMBER SYSTEM

Importance : Being a basic concept of mathematics : 1 and 2 questions on number system are regularly asked in different competitive exams. Its knowledge is also essential to solve other questions.

Scope of questions : Different type of questions like based on fractions, even/odd/whole/divisible/prime/coprime/rational/irrational/numbers and related to divisibility, order, ascending, descending, addition, multiplication, inverse numbers may be asked.

Way to success : These questions are solved by different methods. Maximum practice and rechecking is the way to success for this chapter.

Natural Numbers : Set of counting numbers is callled natural numbers. It is denoted by N. where,

N = {1, 2, 3, ...∞}

Even Numbers : The set of all natural numbers which are divisible by 2 are called even numbers. It is denoted by E.

Where, E = {2, 4, 6, 8, 10, ...∞}

Odd Numbers : The set of all natural numbers which are not divisible by 2 are called odd numbers. In other words, the natural numbers which are not even numbers, are odd numbers. i.e.,

O = {1, 3, 5, 7, ...∞}

Whole Numbers : When zero is included in the set of natural numbers, then it forms set of whole numbers. It is denoted by W. where,

W = {0, 1, 2, 3, ...∞}

Integers : When in the set of whole numbers, natural numbers with negative sign are included, then it becomes set of integers. It is denoted by I or Z.

I : [– ∞, ... –4, –3, –2, –1, 0, 1, 2, 3, 4, ...∞]

Integers can further be classified into negative or positive Integers. Negative Integers are denoted by Z^- and positive Integers are denoted by Z^+.

Z^- = {– ∞, ... –3, – 2, –1} and

Z^+ = {1, 2, 3, ... ∞}

Further 0 is neither negative nor positive integer.

Prime Numbers: The natural numbers which have no factors other than 1 and itself are called prime numbers.

Note that, (i) In other words they can be divided only by themselves or 1 only. As, 2, 3, 5, 7, 11 etc.

(ii) All prime numbers other than 2 are odd numbers but all odd numbers are not prime numbers.

2 is the only one even Prime number.

Co-Prime Numbers : Two numbers which have no common factor except 1, are called Co–Prime numbers. Such as, 9 and 16, 4 and 17, 80 and 81 etc.

It is not necessary that two co–prime numbers are prime always. They may or may not be prime numbers.

Divisible numbers/composite numbers : The whole numbers which are divisible by numbers other than itself and 1 are called divisible numbers or we can say the numbers which are not prime numbers are composite or divisible numbers. As, 4, 6, 9, 15, ...

Note : 1 is neither Prime number nor composite number. Composite numbers may be even or odd.

Rational Numbers : The numbers which can be expressed in the form of $\frac{p}{q}$ where p and q are integers and coprime and $q \neq 0$ are called rational numbers. It is denoted by Q. These may be positive, or negative.

e.g. $\frac{4}{5}, \frac{5}{1}, -\frac{1}{2}$ etc are rational numbers.

Irrational Numbers : The numbers which are not rational numbers, are called irrational numbers. Such as

$\sqrt{2}$ = 1.414213562..........

π = 3.141592653 ....

Real Numbers: Set of all rational numbers as well as irrational numbers is called Real numbers. The square of all of them is positive.

Cyclic Numbers : Cyclic numbers are those numbers of n digits which when multiplied by any other number upto n gives same digits in a different order. They are in the same line. As 142857

2 × 142857 = 285714 : 3 × 142857 = 428571

4 × 142857 = 571428 : 5 × 142857 = 714285

Perfect Numbers : If the sum of all divisors of a number N (except N) is equal to the number N itself then the number is called perfect number. Such as, 6, 28, 496. 8128 etc.

The factor of 6 are 1, 2 and 3

Since, 6 : 1 + 2 + 3 = 6

28 : 1 + 2 + 4 + 7 + 14 = 28

496 : 1 + 2 + 4 + 8 + 16 + 31 + 62 + 124 + 248 = 496

8128 : 1 + 2 + 4 + 8 + 16 + 32 + 64 + 127 + 254 + 508 + 1016 + 2032 + 4064 = 8128. etc.

Note : In a perfect number, the sum of inverse of all of its factors including itself is 2 always.

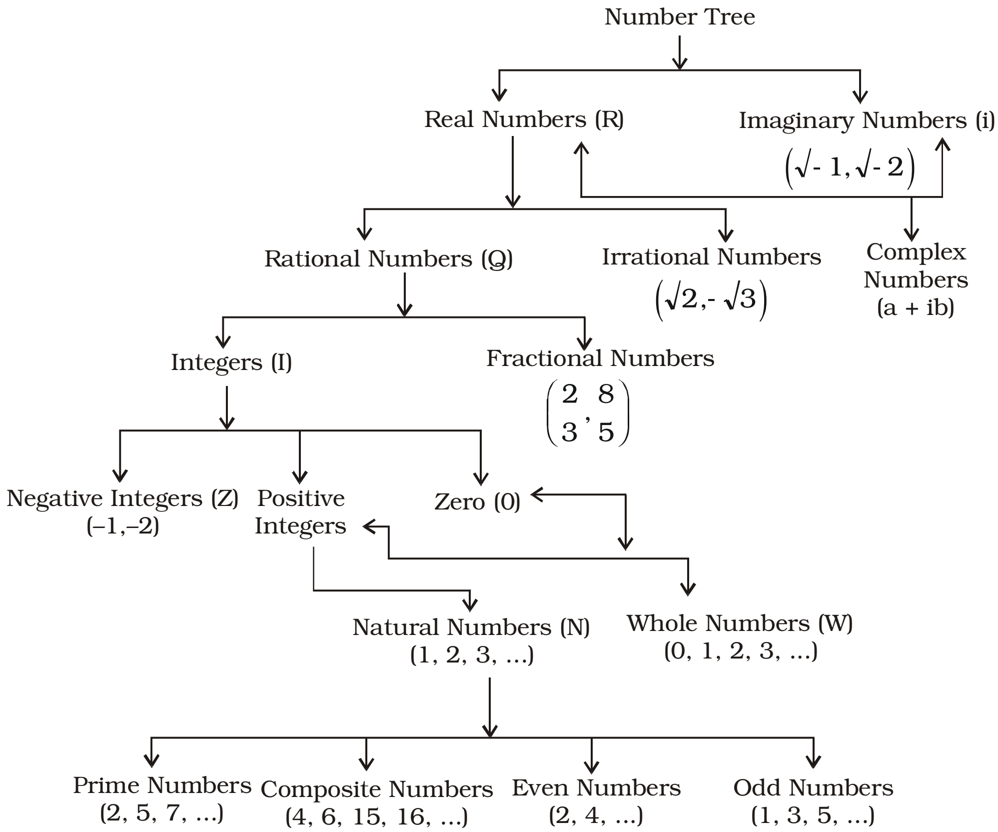
e.g. Factors of 28 are 1,2,4,7,14 are

$$= \frac{1}{1}+\frac{1}{2}+\frac{1}{4}+\frac{1}{7}+\frac{1}{14}+\frac{1}{28}=\frac{56}{28} = 2$$

Complex Numbers : Z = a + ib is called complex number, where a and b are real numbers, $b \neq 0$ and $i = \sqrt{-1}$.

Such as, $\sqrt{-2}$, $\sqrt{-3}$ etc.

So, a + ib or 4 + 5i are complex numbers.



Additive Identity : If a + 0 = a, then 0 (zero) is called additive identity.

Additive Inverse : If a + (–a) = 0, so 'a' and '–a' are called additive inverse to each other. As, 2 + (–2) = 0

Additive inverse of 2 is –2.

Multiplicative Identity : If a × 1 = a. then 1 is called multiplicative identity. e.g. 3 × 1 = 3 etc.

Multiplicative Inverse : If a × b = 1. then we can say that a and b are multiplicative invverse of each other. As

$$2 \times \frac{1}{2} = 1$$

So, multiplicative inverse of 2 is $\frac{1}{2}$

SOME IMPORTANT POINTS ON NUMBERS

(a) 2 is the only even prime number.

(b) Number 1 is neither divisible nor prime.

(c) Two consecutive odd prime numbers are called prime pair.

(d) All natural numbers are whole, rational, integer and real.

(e) All whole numbers are rational Integer and real.

(f) All whole numbers are rational and real.

(g) All whole numbers, rational and irrational numbers are real.

(h) Whole numbers and natural numbers can never be negative.

(i) Natural (including Prime, Composite, even or odd) numbers and whole numbers are never negative.

(j) Fractions are rational.

(k) All prime numbers except 2 are odd.

(l) 0 is neither negative nor positive number.

(m) If a is any number then, if a divides zero, result will be zero. If 0 divides a, then result will be infinite or not defined or undetermined i.e.

$$\frac{0}{a} = 0 \text{ but } \frac{a}{0} = \infty \text{ (infinite)}$$

where a is real number.

(n) Dividing 0 by any number gives zero e.g. $\frac{0}{a} = 0$

(o) The place or position of a digit in a number is called its place value such as
Place value of 2 in 5283 is 200.

(p) The real value of any digit in a certain number is called its face value. As, face value of 2 in 5283 is 2.

(q) The sum and the product of two rational numbers is always a rational number.

(r) The product or the sum of a rational number and irrational number is always an irrational number.

(s) π is an irrational number.

(t) There can be infinite number of rational or irrational numbers between two rational numbers or two irrational numbers.

(u) Decimal indication of an irrational number is infinite coming. as $-\sqrt{3}, \sqrt{2}$

(v) The square of an even number is even and the square of an odd number is odd.

DECIMAL

(w) The decimal representation of a rational number is either finite or infinite recurring e.g. = $\frac{3}{4}$ = 0.75 (finite), $\frac{11}{3}$ = 3.666 ..... (infinite recurring)

(x) If decimal number 0, x and 0. xy are given, then they can be expressed in the form of $\frac{p}{q}$

As, $0.x = \frac{x}{10}$ and $0.xy = \frac{xy}{100}$

(y) If decimal recurring numbers $0.\overline{x}$ and $0.\overline{xy}$ are given, then they can be expressed in the form of $\frac{p}{q}$ As $0.\overline{x}$ $= \frac{x}{9}$ and $0.\overline{xy} = \frac{xy}{99}$

(z) The recurring decimal numbers of type $0.\overline{x}$ or $0.x\overline{yz}$ may be converted to rational form as $\frac{p}{q}$ follows.

$0.x\overline{y} = \frac{xy - x}{90}$ and $0.x\overline{yz} = \frac{xyz - x}{990}$

DIVISIBILITY

Importance : Divisibility questions, if not asked directly, still its knowledge is very essential to solve different questions in simplications.

Scope of questions : The study of this concept is very useful to increase speed in simplication and number system.

Way to success : The knowledge of divisibility rules (of ,2, 3, 4, 5, 6, 8, 9) and of osculaters for 7, 11, 13 etc & mental calculations increase our (speed) time management and accuracy.

Basic Formulae of Divisibility from 2 to 19:

1. Divisibility by 2 : If the last digit of a number is 0 or an even number then that number is diviisible by 2. Such as, 242, 540 etc.

2. Divisibility by 3 : If the sum of all digits of a number is divisible by 3, then that number will be divisible by 3. Such as.

432 : 4 + 3 + 2 = 9 which is divisible by 3.

So, 432 is divisible by 3.

3. Divisibility by 4 : If in any number last two digits are divisible by 4, then whole number will be divisible by 4. Such as,

48424. In this number 24 is divisible by 4. So, 48424 will be divisible by 4.

4. Divisibility by 5 : If last digit of a number is 5 or 0, then that number is divisible by 5. Such as 200, 225 etc.

5. Divisibility by 6 : If a number is divisible by both 2 and 3, then that number is divisible by 6 also, such as 216, 25614 etc.

6. Divisibility by 7 : Here concept of osculator should be applied. The meaning of negative osculator is – there increases or decreases 1 from the factor of 10 of the number. As, 21 : 2 × 10 + 1 = 21

49 : 5 × 10 – 1 = 50 – 1 = 49

To check the divisibility of 7, we use osculator '2', as , 112 : 11 – 2 × 2 = 7 which is divisible by 7

Again,

343 : 34 – 2 × 3 = 28 which is divisible by 7. Then 343 will be divisible by 7.

7. Divisibility by 8 : If in any number last three digits are divisible by 8, then whole number is divisible by 8, such as,

247864 since 864 is divisible by 8.

So, 247864 is divisible by 8.

Similarly, 289000 is divisible by 8.

8. Divisibility by 9 : If the sum of all digits of a number is divisible by 9, then that whole number will be divisible by 9. As, 243243 : 2 + 4 + 3 + 2 + 4 + 3 = 18 is divisible by 9.

So, 243243 is divisible by 9.

9. Divisibility by 10 : The number whose last digit is '0', is divisible by 10, such as, 10, 20, 200, 300 etc.

10. Divisibility by 11 : If the difference between "Sum of digits at even place" and "Sum of digits at odd place" is divisible by 11, then the whole number is divisible by 11 such as,

9 1 7 4

\+ 16   + 5

$\therefore$ (9 + 7) – (4 + 1) = 16 – 5 = 11 is divisible by 11.

So, 9174 will be divisible by 11.

11. Divisibility by 12 : If a number is divisible by 3 and 4 both. Then the number is divisible by 12. Such as, 19044 etc.

12. Divisibility by 13 : For 13 we use osculator 4, but our osculator is not negative here. It is one-more osculator (4).

143 : 14 + 3 × 4 = 26

and 26 is divisible by 13, So, 143 is divisible by 13.

Similarly for 325 : 32 + 5 × 4 = 52

52 is divisible by 13

Hence, 325 will also be divisible by 13.

13. Divisibility by 14 : If a number is divisible by 2 and 7 both then that number is divisible by 14 i.e. number is even and osculator 2 is applicable.

14. Divisibility by 15 : If a number is divisible by 3 and 5 both, then that number is divisible by 15.

15. Divisibility by 16 : If last 4 digits of a number are divisible by 16, then whole number is divisible by 16. Such as 34**1920**.

16. Divisibility by 17 : For 17, there is a negative 'osculator 5'. This process is same as the process of 7. As.

1904 : 190 – 5 × 4 = 170.

∵ 170 is divisible by 17. So 1904 will be divisible by 17.

17. Divisibility by 18 : If a number is divisible by 2 and 9 both, then that number is divisible by 18.

18. Divisibility by 19 : For 19, there is one–more (positive) osculator 2, which is same processed as 13. As,

361 = 36 + 1 × 2 = 38

∵ 38 is divisible by 19. So 361 is also divisible by 19.

Few more Important Points:

1. Out of a group of n consecutive integers one and only one number is divisible by n.

2. The product of n consecutive numbers is always divisible by n! or = $\underline{|n}$.

3. For any number n, $(n^p–h)$ is always divisible by P where P is a prime number, for e.g.,

if n = 2 and P = 5 then,

$(2^5 – 2) = (32 – 2) = 30$ which is divisible by 5.

4. The square of an odd number when divided by 8 always leaves a remainder 1, as

If we divide $7^2 = 49$ or $5^2 = 25$ by 8 then remainder will be 1.

5. For any natural number n, n^5 or n^{4k+1} is having same unit digit as n has, where k is a whole number, such as,

$3^5 = 243$ has 3 at its unit place.

6. Square of any natural number can be written in the form of 3n or 3n + 1 or 4n or (4n + 1).

e.g. square of 11 = 121 = 3 × 40 + 1

or 4 × 30 + 1

If N = $a^p\ b^q\ c^r$ .... where a, b and c are prime numbers and p, q and r are natural numbers, then

1. Number of factors of N is given by

F = (p + 1)(q + 1)(r + 1) ....

2. Number of ways to express the number as a product of two factors are $\frac{F}{2}$ F is even or $\frac{F+1}{2}$ if F is odd respectively.

3. Sum of all the factors of the number N.

$$S(F) = \frac{\left(a^{p+1}-1\right)}{(a-1)} \times \frac{\left(b^{q+1}-1\right)}{(b-1)} \times \frac{\left(c^{r+1}-1\right)}{(c-1)}$$

4. The number of ways in which a number N can be resolved into co–prime factors is 2^{k-1}, where k is the number of different Prime factors of the number N.

5. The number of co–primes to number N is given by

$$C(N) = n\left(1-\frac{1}{a}\right)\left(1-\frac{1}{b}\right)\left(1-\frac{1}{c}\right)$$

Special Rules :

Rule 1 : If the sum of digits of two digit number is 'a' and if the digits or the number are reversed, such that number reduces by 'b', then

$$\text{Original Number} = \frac{11a+b}{2}$$

For example : (For number 82) a = 8 + 2 = 10

and b = 82 – 28 = 54 is given then

$$\text{original number} = \frac{11\times10+54}{2} = \frac{164}{2} = 82$$

Rule 2 : If the sum of digits of two digit number is 'a' and if the digits of the number are reveresed, such that number increases by 'b', then,

$$\text{Original Number} = \frac{11a-b}{2}$$

e.g. (For number 47): a = 4 + 7 = 11

& b = 74 – 47 = 27 thus the

$$\text{original number} = \frac{11\times11-27}{2} = 47$$

Rule 3 : If the difference between a number and formed by number reversing digit is x, then the difference between both the digits of the number is $\frac{x}{9}$

eg. (for 63) x = 63 – 36 = 27

$$\Rightarrow \text{Required difference} = \frac{27}{9} = 3$$

Rule 4 : If the sum of a number and the number formed by reversing the digits is x, then the sum of digits of the number is $\frac{x}{11}$.

e.g. (For number 76) = x = 67 + 76 = 143 Required sum of numbers = 67 + 76 = 143

$$\text{Required sum} = \frac{143}{11} = 13$$

Dividend = (Divisor × Quotient) + Remainder

$$\text{Divisor} = \frac{\text{Dividend} - \text{Remainder}}{\text{Quotient}}$$

$$\text{Quotient} = \frac{\text{Dividend} - \text{Remainder}}{\text{Divisor}}$$

Remainder = Dividend – (Divisor × Quotient)

Special Rule for Remainder Calculation:

Rule 5 : If $\frac{a^n}{a-1}$ then remainder will always be 1, whether n is even or odd.

Rule 6 : If $\frac{a^{(\text{even number})}}{(a+1)}$, then remainder will be 1.

Rule 7 : If $\frac{a^{(\text{odd number})}}{(a+1)}$, then remainder will be a.

Rule 8 : If n is a single digit number, then in n^3, n will be at unit place. It is valid for the number 0, 1, 4, 5, 6 or 9 As, digit at unit place in (4^3) is 4.

Rule 9 : If n is a single digit number then in n^p, where p is any number (+ve), n will be at unit place. It is valid for 5 and 6.

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