Lecture 06

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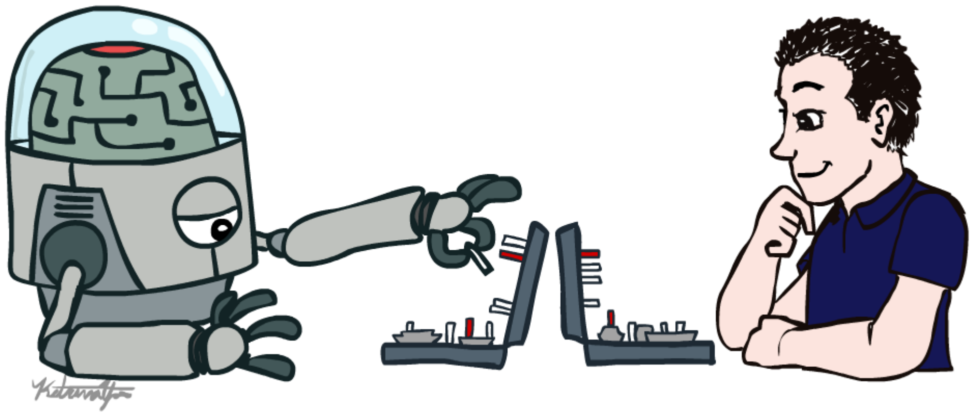

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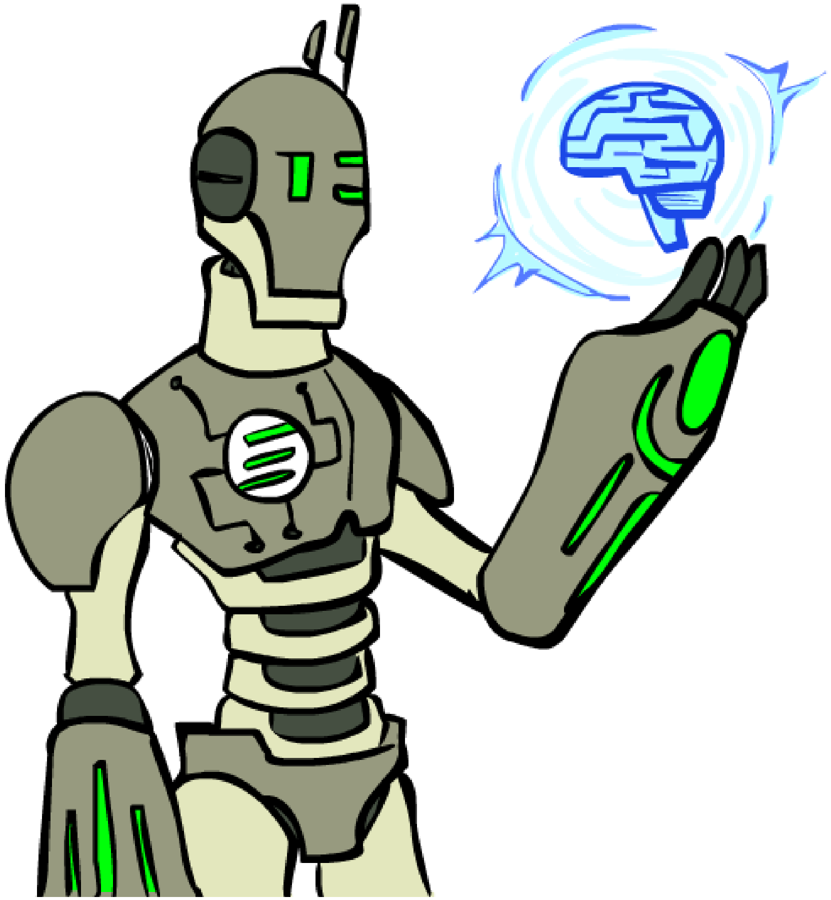
Today



- Propositional logic
- First order logic
- Quantifier

Logical agents

- Human acts based on his **logical sense**.
- A knowledge base agent use a process of reasoning over an internal representation of knowledge to decide what action to take.
- In our previous classes, we saw some problem-solving agents with limited knowledge.
- They know what actions are available and what the result of performing a special action from a specific state will be.
- However, they don't know **general facts**.

Ex: 8-puzzle agent doesn't know that two titles cannot occupy the same place.

Logic

- Our goal is developing **logic** as a general class of representations to support knowledge-based agents.
- A **logic** defines the **semantics** or meaning of sentences.
- The **semantics** defines the truth of each sentence with respect to each possible world.

- $X+Y = 4$ is a sentence.
- It is true, if $X = 2$ and $Y = 2$.
- It is false, if $X = 1$ and $Y = 1$.

- Every sentence must be either true or false.

Propositional Logic

- Assume that there are facts that either hold or do not hold (T or F)

- If the sun is shining, then I wear sunglass.
- The sun is shining.

- If P is true, then Q is true. Here, P and Q are proposition symbol.
- P is true.

- $Q \leftarrow P$ or we can write $P \rightarrow Q$
- P

Syntax and Semantics

The symbols of Propositional logic consist of

1. The truth symbol T and F
2. Proposition Symbols
3. The connectives ~ (not), Λ (and), V (or)
4. Implies → (Implications are also known as if-then statements)
5. ⇔ If and only if (bi-directional)
6. The symbol ")", "(", and ","

Proposition

A proposition is either a truth symbol,
a proposition symbol or
a formula formed
from the propositions P and Q in one of the following ways

1. $\sim P$
2. $P \vee Q$
3. $P \wedge Q$

Grammar of sentences with operator precedence

$$
\begin{array}{rcl}
Sentence & \rightarrow & AtomicSentence \mid ComplexSentence \\
AtomicSentence & \rightarrow & True \mid False \mid P \mid Q \mid R \mid \ldots \\
ComplexSentence & \rightarrow & (\ Sentence\ ) \mid [\ Sentence\ ] \\
& \mid & \neg\ Sentence \\
& \mid & Sentence \land Sentence \\
& \mid & Sentence \lor Sentence \\
& \mid & Sentence \ \Rightarrow\ Sentence \\
& \mid & Sentence \ \Leftrightarrow\ Sentence \\
\\
\textsc{Operator Precedence} & : & \neg, \land, \lor, \Rightarrow, \Leftrightarrow
\end{array}
$$

The "not" operator ($\neg$) has the highest precedence

Example

Suppose, we have a sentence,

$\neg A \land B$

Since not has high precedence, the sentence would be

$(\neg A) \land B$

rather than $\neg(A \land B)$.

Truth table for logical connectives

P	Q	$\neg P$	$P \wedge Q$	$P \vee Q$	$P \Rightarrow Q$	$P \Leftrightarrow Q$
false	*false*	*true*	*false*	*false*	*true*	*true*
false	*true*	*true*	*false*	*true*	*true*	*false*
true	*false*	*false*	*false*	*true*	*false*	*false*
true	*true*	*false*	*true*	*true*	*true*	*true*

P ⇔ Q is true whenever both P ➔ Q and Q➔P are true.

“Madison will eat the fruit if and only if it is an apple"
It is equivalent to saying that "Madison will eat the fruit if the fruit is an apple, and will eat no other fruit“.

A given fruit is an apple is both a *necessary* and a *sufficient* condition for Madison to eat the fruit.

Logical equivalences

$$
\begin{aligned}
(\alpha \land \beta) &\equiv (\beta \land \alpha) && \text{commutativity of } \land \\
(\alpha \lor \beta) &\equiv (\beta \lor \alpha) && \text{commutativity of } \lor \\
((\alpha \land \beta) \land \gamma) &\equiv (\alpha \land (\beta \land \gamma)) && \text{associativity of } \land \\
((\alpha \lor \beta) \lor \gamma) &\equiv (\alpha \lor (\beta \lor \gamma)) && \text{associativity of } \lor \\
\neg(\neg\alpha) &\equiv \alpha && \text{double-negation elimination} \\
(\alpha \Rightarrow \beta) &\equiv (\neg\beta \Rightarrow \neg\alpha) && \text{contraposition} \\
(\alpha \Rightarrow \beta) &\equiv (\neg\alpha \lor \beta) && \text{implication elimination} \\
(\alpha \Leftrightarrow \beta) &\equiv ((\alpha \Rightarrow \beta) \land (\beta \Rightarrow \alpha)) && \text{biconditional elimination} \\
\neg(\alpha \land \beta) &\equiv (\neg\alpha \lor \neg\beta) && \text{De Morgan} \\
\neg(\alpha \lor \beta) &\equiv (\neg\alpha \land \neg\beta) && \text{De Morgan} \\
(\alpha \land (\beta \lor \gamma)) &\equiv ((\alpha \land \beta) \lor (\alpha \land \gamma)) && \text{distributivity of } \land \text{ over } \lor \\
(\alpha \lor (\beta \land \gamma)) &\equiv ((\alpha \lor \beta) \land (\alpha \lor \gamma)) && \text{distributivity of } \lor \text{ over } \land
\end{aligned}
$$

The statement ~(P∧Q), which we can interpret as meaning that it is not the case that both *P* and *Q* are true. If it is not the case that both *P* and *Q* are true, then at least one of *P* or *Q* is false, in which case (~P)∨(~Q) is true. Thus ~(P∧Q) means the same thing as (~P)∨(~Q)

Exercise

- **Let A be “It is sunny” and B be “it is cold”.**
- It is sunny and cold
  - $A \wedge B$
- It is either sunny or cold
  - $A \vee B$
- If it is sunny, then it is cold
  - $A \to B$
- It is not sunny
  - $\neg A$

Quiz

- **P: It is raining**
- **Q: It is sunny.**
- **R: There is a rainbow.**

- It is raining, but there is no rainbow.
- If there is no rainbow, then it is not raining.
- If it is raining and sunny, then there is a rainbow.
- There is a rainbow if and only if there is rain and sunny.

Interpretation

An Interpretation or truth assignment (I) is an assignment of the truth values T and F to each of the propositional symbol.

Each propositional symbol represents a proposition (T or F).

Formula of propositional logic are simply formal strings of symbols.
Ex: $(P \lor \neg Q \land R)$.

A literal is a propositional symbol P or its negation ~p.

First Order Logic

- Assumes that there are objects with relations among them that do or do not hold.

- FOL allows to describe much more complicated situations and is more powerful.

- Ex: All kings are person.

Quantifier

- Once we have a logic that allows objects, it is natural to express properties of entire collections of objects.
- A quantifier is "an operator that limits the variables of a proposition".
- First order logic allows two standard quantifiers,
  - i) universal,
  - ii) existential.

Ex: All kings are person.
For all x, if x is a king, then x is a person.
Ex: x is greater than 0 and less than 1.
There exists a natural number x which is greater than 0 and less than 1

Universal quantifiers 1

- Represented by an upside-down A: $\forall$
  - It means "for all"
  - Let $P(x) = x+1 > x$
- We can state the following:
  - $\forall x\ P(x)$
  - English translation: "for all values of x, P(x) is true"
  - English translation: "for all values of x, x+1>x is true"

Universal quantifiers 2

- But is that always true?
  - $\forall x\ P(x)$
- Let x = the character ‘a’
  - Is ‘a’+1 > ‘a’?
- Let x = the state of Virginia
  - Is Virginia+1 > Virginia?
- You need to specify your universe!
  - What values x can represent
  - Called the “domain” or “universe of discourse” by the textbook

Universal quantifiers 3

- Let the universe be the real numbers.
  - Then, $\forall x\, P(x)$ is true

- Let $P(x) = x/2 < x$
  - Not true for the negative numbers!
  - Thus, $\forall x\, P(x)$ is false
    - When the domain is all the real numbers

- In order to prove that a universal quantification is true, it must be shown for ALL cases
- In order to prove that a universal quantification is false, it must be shown to be false for only ONE case

Universal quantification 4

- Given some propositional function P(x)
- And values in the universe $x_1 .. x_n$
- The universal quantification $\forall x\, P(x)$ implies:

$$P(x_1) \wedge P(x_2) \wedge \ldots \wedge P(x_n)$$

Universal quantification 5

- Think of $\forall$ as a for loop:

- $\forall x\ P(x)$, where $1 \leq x \leq 10$

- … can be translated as …

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for ( x = 1; x <= 10; x++ )
     is P(x) true?
```

- If P(x) is true for all parts of the for loop, then $\forall x\ P(x)$
  - Consequently, if P(x) is false for any one value of the for loop, then $\forall x\ P(x)$ is false

Existential quantification 1

- Represented by an bacwards E: ∃
  - It means “there exists”
  - Let P(x) = x+1 > x

- We can state the following:
  - ∃x P(x)
  - English translation: “there exists (a value of) x such that P(x) is true”
  - English translation: “for at least one value of x, x+1>x is true”

Existential quantification 2

- Note that you still have to specify your universe
  - If the universe we are talking about is all the states in the US, then $\exists x\ P(x)$ is not true

- Let $P(x) = x+1 < x$
  - There is no numerical value x for which $x+1<x$
  - Thus, $\exists x\ P(x)$ is false

Existential quantification 3

- Let $P(x) = x+1 > x$
  - There is a numerical value for which $x+1>x$
    - In fact, it's true for all of the values of x!
  - Thus, $\exists x\ P(x)$ is true

- In order to show an existential quantification is true, you only have to find ONE value
- In order to show an existential quantification is false, you have to show it's false for ALL values

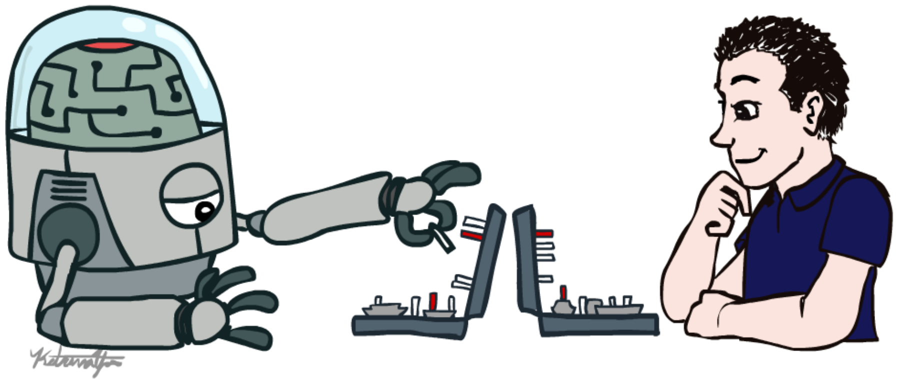
Existential quantification 4

- Given some propositional function $P(x)$
- And values in the universe $x_1 .. x_n$
- The existential quantification $\exists x\ P(x)$ implies:

$$P(x_1) \vee P(x_2) \vee \ldots \vee P(x_n)$$

A note on quantifiers

- Recall that P(x) is a propositional function
  - Let P(x) be "x == 0"
- Recall that a proposition is a statement that is either true or false
  - P(x) is not a proposition
- There are two ways to make a propositional function into a proposition:
  - Supply it with a value
    - For example, P(5) is false, P(0) is true
  - Provide a quantifiaction
    - For example, $\forall x\ P(x)$ is false and $\exists x\ P(x)$ is true
      - Let the universe of discourse be the real numbers



Thanks!