**Single Source Shortest Path on GPU**

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**Abstract**

This report presents analysis of some implementations for single source shortest path algorithm on GPU. First implementation tries to make bellman ford’s sequential algorithm, parallel and analyse runtime in various scenarios considering order of edges in the input and use of shared memory and segmented scan to improve efficiency. Second implementation is a work efficient variation of the first implementation which tries to decrease the workload at each iteration in bellman ford algorithm.

**1. List of graphs used for generating stats**

RoadNetCA: <http://snap.stanford.edu/data/roadNet-CA.html>

LiveJournal: <http://snap.stanford.edu/data/soc-LiveJournal1.html>

WebGoogle: http://snap.stanford.edu/data/web-Google.html

Amazon0312: <http://snap.stanford.edu/data/amazon0312.html>

msdoor: <http://www.cise.ufl.edu/research/sparse/matrices/INPRO/msdoor.html>

road-cal: <http://www.dis.uniroma1.it/challenge9/download.shtml> (CAL - California and Nevada - Distance Graph)

**2. Implementation using Bellman Ford**

The sequential bellman-ford algorithm checks all edges at every iteration and updates a node if the current estimate of its distance from the source node can be reduced. The number of iterations is at most the same as the number of vertices if no negative cycle exists. The complexity of the sequential bellman-ford algorithm is O(| V | · | E |).

In the parallel bellman-ford algorithm, we exploit the parallelism of edge processing at every iteration. Each of the input edges is checked at every iteration, we can distribute these edges evenly to different processors such that each processor is responsible for the same number of edges.

Following are the results for various configurations **1024x2 -**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Graph | incore src | incore  dest | outcore  src | outcore  dest | outcore  shared mem |
| amazon0312 | 156.2 | 164.8 | 439.3 | 463.9 | 1055.9 |
| msdoor | 2600.6 | 2609.5 | 6904.2 | 6874 | 27947 |
| roadnet-ca | 4391.1 | 4386.3 | 7011.4 | 7376.6 | 22461.9 |
| live journal | 1931.9 | 2224.4 | 4183.8 | 4203.2 | 9086.5 |
| road-cal | 28683.5 | 28369.5 | 48837.4 | 47809.5 | 144212 |
| web-google | 510.8 | 470.6 | 894.1 | 922.7 | 1736.6 |

Following are the **number of iterations** to complete the algorithm for various configurations **1024x2 -**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Graph | incore src | incore  dest | outcore  src | outcore  dest | outcore  shared mem |
| amazon0312 | 15 | 15 | 42 | 40 | 42 |
| msdoor | 165 | 168 | 439 | 439 | 439 |
| roadnet-ca | 363 | 363 | 555 | 555 | 555 |
| live journal | 7 | 8 | 15 | 15 | 15 |
| road-cal | 2620 | 2630 | 4165 | 4165 | 4165 |
| web-google | 19 | 18 | 33 | 33 | 33 |

Following are the results from varying block size -

**Incore - Src sorted -**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Graph | 256x8 | 384x5 | 512x4 | 768x2 | 1024x2 |
| amazon0312 | 151.3 | 160.4 | 157.3 | 145.2 | 156.2 |
| msdoor | 2669.5 | 2727.9 | 2687.9 | 2638.8 | 2600.6 |
| roadnet-ca | 4444.1 | 4511.5 | 4433.5 | 4302.1 | 4391.1 |
| live journal | 1940.7 | 2223.6 | 1932.4 | 1903.4 | 1931.9 |
| road-cal | 29232.9 | 30188.8 | 29022.9 | 28841.2 | 28683.5 |
| web-google | 515.3 | 486.4 | 541.1 | 505.7 | 510.8 |

Following are the results from varying block size -

**Incore - Dest sorted**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Graph | 256x8 | 384x5 | 512x4 | 768x2 | 1024x2 |
| amazon0312 | 164.8 | 167.8 | 165.2 | 163.7 | 164.8 |
| msdoor | 2723.6 | 2707.4 | 2684.6 | 2670.1 | 2609.5 |
| roadnet-ca | 4421.7 | 4497.9 | 4411.7 | 4341.6 | 4386.3 |
| live journal | 2188.9 | 2216.4 | 2210.8 | 2183.6 | 2224.4 |
| road-cal | 28869.7 | 30043.2 | 28622.8 | 28327.7 | 28369.5 |
| web-google | 444.9 | 527.4 | 500.5 | 495.2 | 470.6 |

Following are the results from varying block size -

**Outcore - Src sorted**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Graph | 256x8 | 384x5 | 512x4 | 768x2 | 1024x2 |
| amazon0312 | 444.4 | 450.9 | 443.4 | 438.3 | 439.3 |
| msdoor | 7096.2 | 7352.3 | 7050.4 | 6944.9 | 6904.2 |
| roadnet-ca | 7196.1 | 7291.4 | 7167.5 | 6970.1 | 7011.4 |
| live journal | 4182.9 | 4153.1 | 4201.3 | 4106.5 | 4183.8 |
| road-cal | 50145.2 | 50741.2 | 49461.9 | 48852.2 | 48837.4 |
| web-google | 902.8 | 899.4 | 899.8 | 884.7 | 894.1 |

Following are the results from varying block size -

**Outcore - Dest sorted**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Graph | 256x8 | 384x5 | 512x4 | 768x2 | 1024x2 |
| amazon0312 | 465.9 | 466.7 | 468.9 | 462.5 | 463.9 |
| msdoor | 7063.5 | 7297.1 | 7001.3 | 6942.2 | 6874 |
| roadnet-ca | 7151.1 | 7252.8 | 7111.1 | 6960.7 | 7376.6 |
| live journal | 4132.1 | 4143.7 | 4170.2 | 4121.6 | 4203.2 |
| road-cal | 48523.2 | 49566.9 | 48217.1 | 47770 | 47809.5 |
| web-google | 926.2 | 925.3 | 926.4 | 916.6 | 922.7 |

Following are the results from varying block size -

**Outcore with shared mem - dest sorted**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Graph | 256x8 | 384x5 | 512x4 | 768x2 | 1024x2 |
| amazon0312 | 1225.6 | 968.2 | 981.4 | 1115.1 | 1055.9 |
| msdoor | 31563.7 | 27006.9 | 25056.2 | 29515.3 | 27947 |
| roadnet-ca | 25177.3 | 21713.9 | 21465.8 | 23482.7 | 22461.9 |
| live journal | 9721.7 | 8331.6 | 8501.4 | 9314.6 | 9086.5 |
| road-cal | 160607 | 136738 | 137129 | 149596 | 144212 |
| web-google | 1567.6 | 1560.8 | 1597.8 | 1772.6 | 1736.6 |

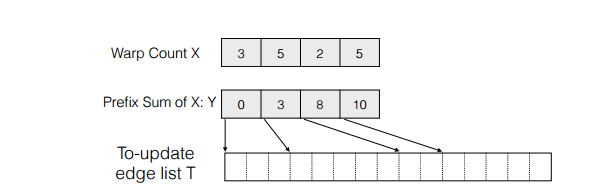
**2.1. Analyses and Observations**

1. Number of iterations for incore implementation is always less than that of the outcore version, probably because the parallel threads receive the updated vertex distances as soon as they are updated rather than wait for the next iteration.
2. When the input edges are sorted by destination node, the performance is worse compared to when they are sorted by source node in all most all cases. This can be because of many threads trying to update the same destination when edges are sorted in destination order. If they are sorted by source node there is a greater chance that parallel threads will be updating different destination vertices and thus will have less contention while writing data.
3. Shared memory version is performing really bad, I think this can be because of thread divergence in the segmentation step. Probably the cost of thread divergence here is much more than what we were trying to save while avoiding atomic operations.
4. For varying block size and block count the performance is not that much, but it can be observed that performance in case of 768 x 2 and 1024 x 2 is almost same in all cases, which is a bit surprising as the later one has more threads.

**3. Work Efficient implementation**

We can improve Implementation 1 by reducing the edges that need to be processed at each iteration. For any directed edge, if the starting point of the edge (the did not change, it will not affect the pointed-to end of the edge (the destination node on the other end). Thus, we can skip this edge when performing the comparison and updating “if (distance[u] + w < distance[v]) ...” computation.

Following image describes the steps involved in the filtering process -



In our implementation we have measured both processing time and filtering time and the stats are reported below -

Following are the **total-time** from various configurations **1024 x 2 -**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Graph | incore src | incore  dest | outcore  src | outcore  dest |
| amazon0312 | 322.9 | 768.3 | 329.2 | 867.8 |
| msdoor | 8722.6 | 10633.4 | 9761.2 | 11582.8 |
| roadnet-ca | 7525.9 | 11472.4 | 7878.4 | 11838.3 |
| live journal | 2022.2 | 6018 | 2780.7 | 7989.7 |
| road-cal | 52469.2 | 83708.4 | 59966.2 | 89994.1 |
| web-google | 421.4 | 1622.2 | 436.9 | 1789.3 |

Following are the **processing times** from various configurations **1024 x 2 -**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Graph | incore src | incore  dest | outcore  src | outcore  dest |
| amazon0312 | 44.7 | 50.7 | 52.1 | 50.2 |
| msdoor | 699.8 | 1018.9 | 1304.2 | 1467.8 |
| roadnet-ca | 763.2 | 771.2 | 1176.4 | 1184 |
| live journal | 694.9 | 826.1 | 725.8 | 745.9 |
| road-cal | 8721.6 | 8739.4 | 16135.2 | 14903.1 |
| web-google | 86.2 | 84.9 | 92.6 | 93.1 |

Following are the **number of iterations** to complete the algorithm from various configurations **1024x2** -

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Graph | incore src | incore  dest | outcore  src | outcore  dest |
| amazon0312 | 42 | 37 | 42 | 41 |
| msdoor | 417 | 418 | 439 | 439 |
| roadnet-ca | 555 | 555 | 555 | 555 |
| live journal | 11 | 11 | 15 | 15 |
| road-cal | 4176 | 4159 | 4165 | 4165 |
| web-google | 32 | 33 | 33 | 33 |

Following are only the **processing times**, for filtering times you can refer the corresponding .stats file generated by running the provided script (impl1.sh).

Following are the results from varying block size -

**Incore - Src sorted**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Graph | 256x8 | 384x5 | 512x4 | 768x2 | 1024x2 |
| amazon0312 | 45.3 | 36.7 | 1064.1 | 35.7 | 44.7 |
| msdoor | 729.1 | 725.5 | 738.2 | 751.3 | 699.8 |
| roadnet-ca | 755.8 | 766.2 | 806.3 | 806.2 | 763.2 |
| live journal | 690.9 | 699.6 | 699.5 | 683.9 | 694.9 |
| road-cal | 8722.1 | 8798.5 | 8792.2 | 8642.4 | 8721.6 |
| web-google | 77.8 | 78.8 | 85.8 | 76.7 | 86.2 |

Following are the results from varying block size -

**Incore - Dest sorted**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Graph | 256x8 | 384x5 | 512x4 | 768x2 | 1024x2 |
| amazon0312 | 148.9 | 167.2 | 49.2 | 50.9 | 50.7 |
| msdoor | 989.1 | 1079.8 | 1011.9 | 1030.9 | 1018.9 |
| roadnet-ca | 987.9 | 771.1 | 773.2 | 772.7 | 771.2 |
| live journal | 822.1 | 844.4 | 835.2 | 811 | 826.1 |
| road-cal | 8743.3 | 8657.3 | 8734.5 | 8637.8 | 8739.4 |
| web-google | 88.2 | 91.2 | 86.6 | 83.6 | 84.9 |

Following are the results from varying block size -

**Outcore - Src sorted**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Graph | 256x8 | 384x5 | 512x4 | 768x2 | 1024x2 |
| amazon0312 | 52.3 | 44.1 | 43.7 | 52 | 52.1 |
| msdoor | 1307.4 | 1335.3 | 1316.5 | 1343.5 | 1304.2 |
| roadnet-ca | 1177.4 | 1177.1 | 1214.9 | 1177.7 | 1176.4 |
| live journal | 724.1 | 735.5 | 730.2 | 713.9 | 725.8 |
| road-cal | 16269.2 | 16393.1 | 16312.1 | 16095.2 | 16135.2 |
| web-google | 85.8 | 88.9 | 92.3 | 91.1 | 92.6 |

Following are the results from varying block size -

**Outcore - Dest sorted**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Graph | 256x8 | 384x5 | 512x4 | 768x2 | 1024x2 |
| amazon0312 | 59.3 | 50.8 | 56 | 511.1 | 50.2 |
| msdoor | 1479.2 | 1503.6 | 1484.1 | 1504.3 | 1467.8 |
| roadnet-ca | 1417.2 | 1359.8 | 1184.7 | 1184.3 | 1184.0 |
| live journal | 740.5 | 751 | 747.3 | 734.1 | 745.9 |
| road-cal | 15068.8 | 15165.9 | 15068.1 | 14959.9 | 14903.1 |
| web-google | 97.8 | 96.7 | 95.6 | 94.3 | 93.1 |

**3.1. Analyses and Observations**

1. If we just take the processing times into account then they are way less than in the previous implementation, most of the time is spent in filtering step.
2. Very large amount of time goes in the filtering for some big graphs like live-journal and road-cal.
3. Number of iterations for incore implementation are much more than that in previous implementation - in some case twice. The iteration count is close to outcore version in previous implementation.
4. Number of iterations for incore implementation is always approximately same to that of outcore version.
5. When the input edges are sorted by destination node, the performance is worse compared to when they are sorted by source node in all most all cases. This can be because of many threads trying to update the same destination when edges are sorted in destination order. If they are sorted by source node there is a greater chance that parallel threads will be updating different destination vertices and thus will have less contention while writing data.

**4. Implementation using Priority Queue**

**Not Working!**

**3. References**

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