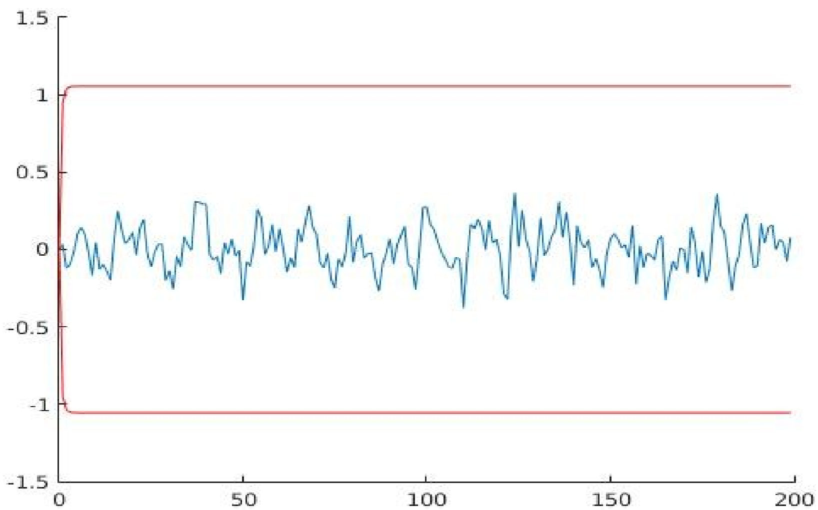
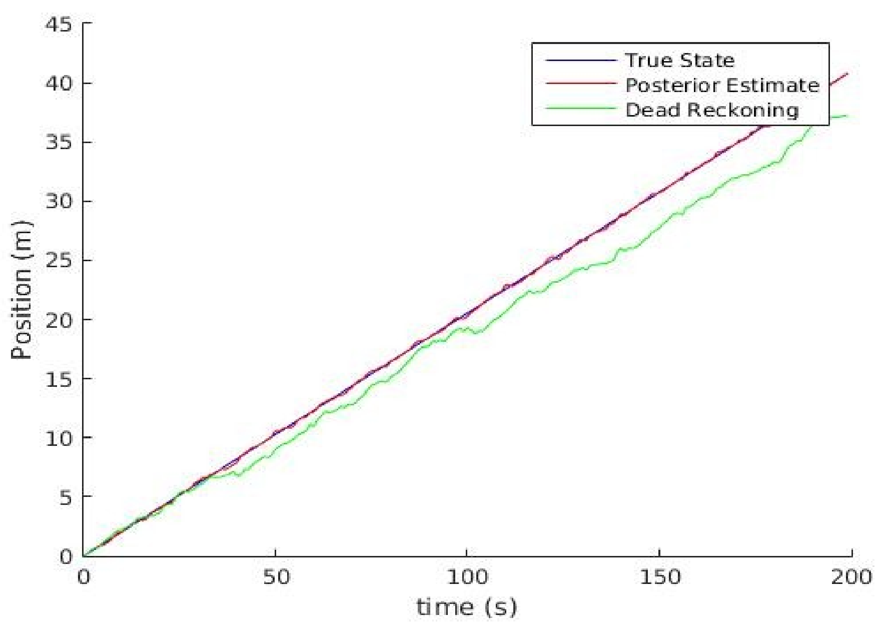
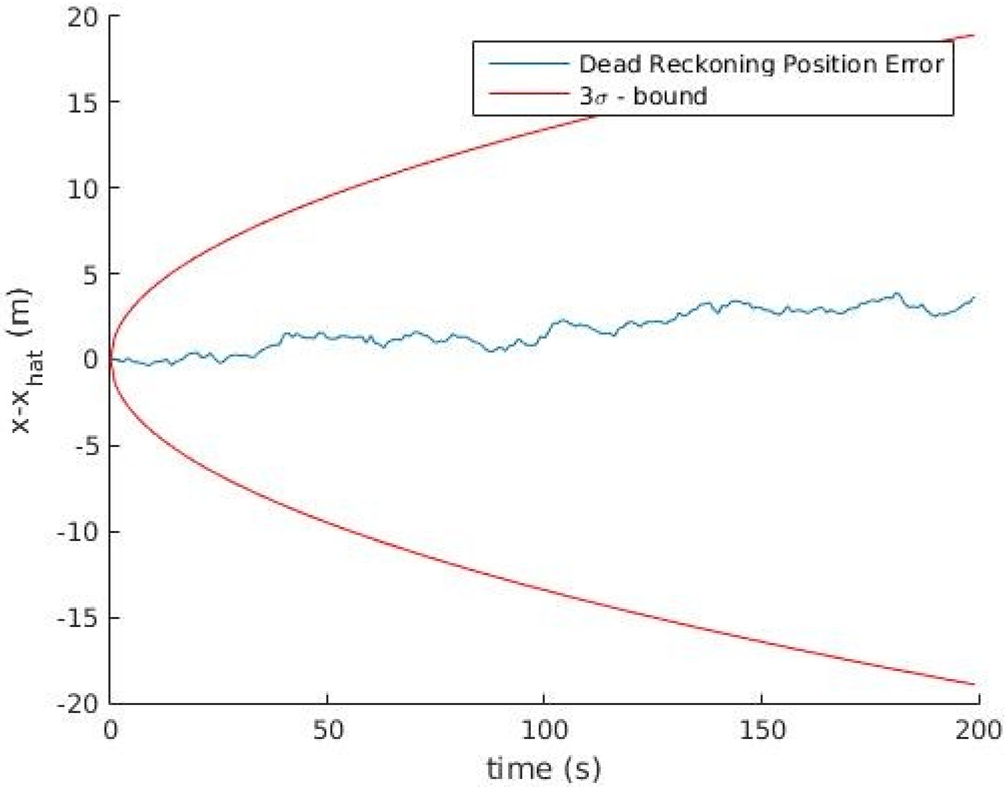
Q3 1-D Kalman Coding

q3.1

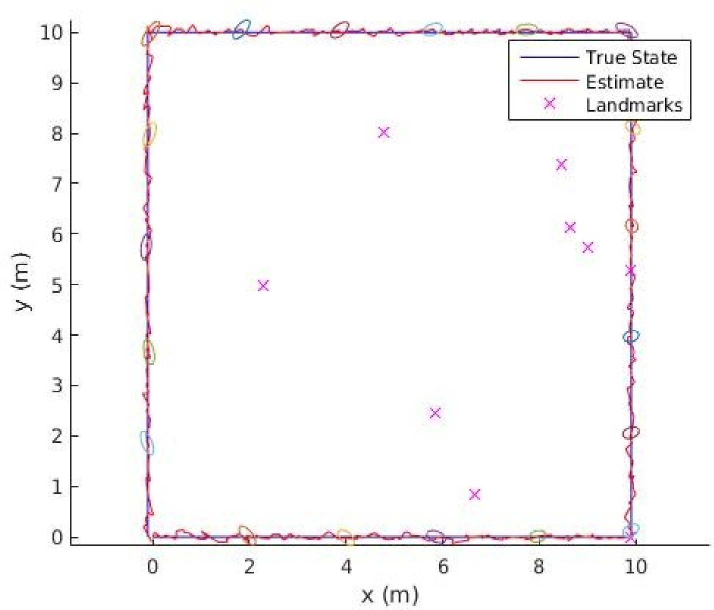
q3.3



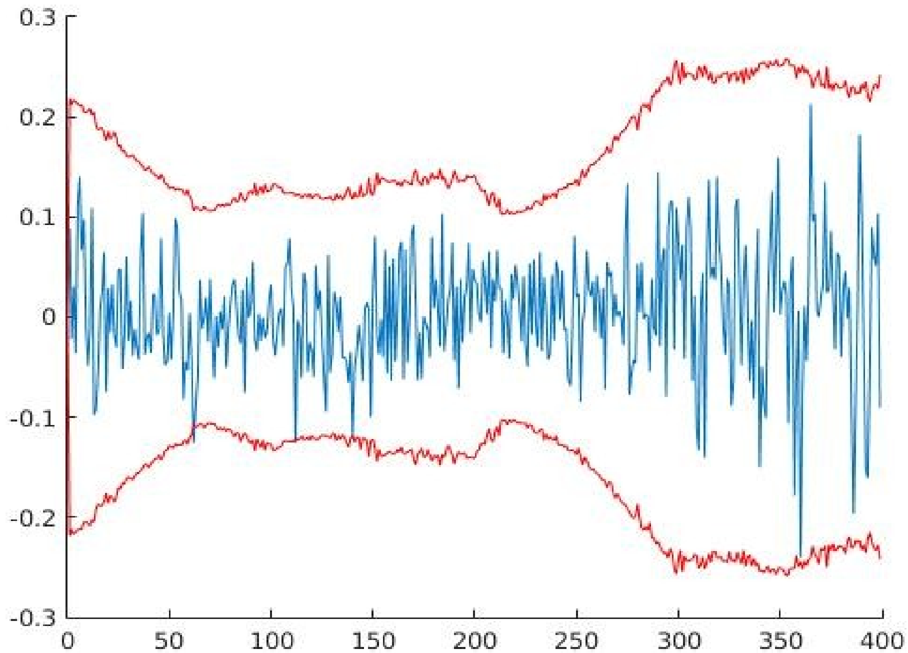



Dead Reckoning Position Error
3σ - bound
x-x_hat (m)
time (s)
20
15
10
5
0
-5
-10
-15
-20
0
50
100
150
200

Q4


True State
Estimate
Landmarks
y (m)
x (m)
10
9
8
7
6
5
4
3
2
1
0
0
2
4
6
8
10

Error in y with 3 sigma bound ('error in y' vs 'time step')



please find rest of the questions below

Q3 1. Propagation Step

$$\bar{\mu}_t = A_t \mu_{t-1} + B_t u_t \quad ; \quad \bar{\Sigma}_t = A_t \Sigma_{t-1} A_t^T + R_t$$

$$\bar{\mu}_t = 1 \times \mu_{t-1} + 1 \times u_t \quad ; \quad \bar{\Sigma}_t = 1 \times \Sigma_{t-1} \times 1 + R_t$$

~~Sigma_min = Sigma_plus~~

$\text{x\_hat\_min}_{(i)} = \text{x\_hat\_plus}_{(i-1)} + dt \times u_{(i)}$; $\text{Sigma\_min}_{(i)} = \text{Sigma\_plus}_{(i-1)} + \text{sigma\_u}$

Update step

$$K_t = \bar{\Sigma}_t C_t^T (C_t \bar{\Sigma}_t C_t^T + Q_t)^{-1}$$

$$= \bar{\Sigma}_t \times 1 \, (1 \times \bar{\Sigma}_t \times 1 + Q_t)^{-1}$$

$$= \text{Sigma\_min}_{(i)} (\text{Sigma\_min}_{(i)} + \text{sigma\_g})^{-1}$$

$$\mu_t = \bar{\mu}_t + K_t (z_t - C_t \bar{\mu}_t)$$

$$= \bar{\mu}_t + K_t (z_t - 1 \times \bar{\mu}_t)$$

$$\text{X\_hat\_plus}_{(i)} = \text{x\_hat\_min}_{(i)} + K_t (\text{z\_g}_{(i)} - \text{X\_hat\_min})$$

$$\Sigma_t = (I - K_t C_t) \bar{\Sigma}_t$$

$$= (1 - K_t) \times \bar{\Sigma}_t$$

$$\text{Sigma\_plus}_{(i)} = (1 - K_t) \, \text{Sigma\_min}$$

Q4 (3.) Update step

$$K_t = \bar{\Sigma}_t H_t^T (H_t \bar{\Sigma}_t H_t^T + Q_t)^{-1}$$

$$H_t = \begin{bmatrix} \partial h_1/\partial x_1 & \partial h_1/\partial x_2 \\ \partial h_2/\partial x_1 & \partial h_2/\partial x_2 \\ \vdots & \\ \partial h_{10}/\partial x_1 & \partial h_{10}/\partial x_2 \end{bmatrix} \qquad X_{state} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$m_{landmark} = \begin{bmatrix} m_1 \\ m_2 \\ \vdots \\ m_{10} \end{bmatrix}$$

$n \in (1,2 \ldots 10)$, $p \in (1,2)$

$$\frac{\partial h_n}{\partial x_m} = \frac{(\bar{\mu}_{t x_p} - m_n)}{\sqrt{(\bar{\mu}_{t x_1} - m_n)^2 + (\bar{\mu}_{t x_2} - m_n)^2}}$$

$$K_t = \underset{(i)}{\overset{(2\times 2)}{\text{Sigma\_min}}} * \underset{(i)}{\overset{(2\times 10)}{H_t^T}} \left(\underset{(i)}{\overset{(10\times 2)}{H_t}} * \underset{(i)}{\overset{(2\times 2)}{\text{Sigma\_min}}} * \underset{(i)}{\overset{(2\times 10)}{H^T}} + \text{sigma\_r}^2 * \text{eye}(10) \right)$$

$$\mu_t = \bar{\mu}_t + K_t (z_t - h(\bar{\mu}_t))$$

$$h(\bar{\mu}_t) = \begin{bmatrix} h_1(\bar{\mu}_t) \\ h_2(\bar{\mu}_t) \\ \vdots \\ h_{10}(\bar{\mu}_{10}) \end{bmatrix}$$

$$n \in (1,2 \ldots 10) \mid h_n(\bar{\mu}_t) = \sqrt{(\bar{\mu}_{tx_1} - m_n)^2 + (\bar{\mu}_{tx_2} - m_n)^2}$$

$$\underset{(i)}{\overset{(2\times 1)}{\text{x\_hat\_plus}}} = \underset{(i)}{\overset{(2\times 1)}{\text{x\_hat\_min}}} + \underset{(i)}{\overset{(2\times 10)}{K_t}} \left(\underset{(i)}{\overset{(10\times 1)}{\text{z\_g}}} - h\left(\underset{(i)}{\overset{(10\times 1)}{\text{x\_hat\_min}}} \right) \right)$$

$$\Sigma_t = (I - K_t H_t) \bar{\Sigma}_t$$

$$\underset{(i)}{\text{Sigma\_plus}} = \left(\overset{2\times 2}{I} - \overset{(2\times 10)}{K_t} \overset{(10\times 2)}{H_t} \right) * \overset{(2\times 2)}{\text{Sigma\_min}}$$

Q2 Part A

$$H(x) = -\int p(x)\log(p(x))\,dx \quad -①$$

$$\text{Gaussian 1-D} = p(x) = \frac{1}{\sigma\sqrt{2\pi}} \times e^{-\left(\frac{(x-\mu)^2}{2\sigma^2}\right)} \quad -②$$

Replacing ② in ①

$$H(x) = -\int \frac{1}{\sigma\sqrt{2\pi}} \times e^{\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)} \times \underbrace{\log\left(\frac{1}{\sigma\sqrt{2\pi}} \times e^{-\left(\frac{(x-\mu)^2}{2\sigma^2}\right)}\right)}_{\log(ab)} dx$$

$$\log(ab) = \log a + \log b$$

$$= \log\left(\frac{1}{\sigma\sqrt{2\pi}}\right) \times \left[\int \frac{-1}{\sigma\sqrt{2\pi}} \times e^{-\left(\frac{(x-\mu)^2}{2\sigma^2}\right)} dx\right] + \left[\int \frac{-1}{\sigma\sqrt{2\pi}} \times e^{-\left(\frac{(x-\mu)^2}{2\sigma^2}\right)} \times \log\left(e^{-\frac{(x-\mu)^2}{2\sigma^2}}\right) dx\right]$$

$$= \underbrace{\log\left(\frac{1}{\sigma\sqrt{2\pi}}\right)\left[\int \frac{-1}{\sigma\sqrt{2\pi}} \times e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx\right]}_{Ⓐ} + \underbrace{\left[\int \frac{+(x-\mu)^2}{2\sigma^2} \times \left(\frac{1}{\sigma\sqrt{2\pi}} \times e^{-\frac{(x-\mu)^2}{2\sigma^2}}\right) dx\right]}_{Ⓑ}$$

Ⓐ $= -\log\left(\frac{1}{\sigma\sqrt{2\pi}}\right) \times \int p(x)\,dx \rightarrow$ from ② above

We know that integration of probability over $-\infty$ to ∞ must be 1.

$$= -\log\left(\frac{1}{\sigma\sqrt{2\pi}}\right) \times 1$$

Ⓑ $= \frac{1}{2\sigma^2} \times \int (x-\mu)^2 \times p(x)\,dx$

We know that Variance of the Gaussian Random Variable $\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 p(x)\,dx$

$\therefore$

$$= \frac{1}{2\sigma^2} \times \sigma^2 = \frac{1}{2}$$

(A) + (B)

$$= -\log\left(\frac{1}{\sigma\sqrt{2\pi}}\right) + \frac{1}{2} = \frac{1}{2}\log\left((\sigma^2 2\pi)\right) + \frac{1}{2}$$

$$= \frac{1}{2}\left[\log\left(2\pi\sigma^2\right) + 1\right] = \frac{1}{2}\log\left(2\pi e\sigma^2\right) \Big] \star$$

Q2 Part C

Given

$$h(X) = \frac{1}{2}\log(2\pi\sigma^2) \quad -①$$

$$h(X,Y) = \frac{1}{2}\log\left((2\pi e)^2(\sigma_X^2\sigma_Y^2 - \sigma_{XY}^2)\right) \quad -②$$

$$\rho(X,Y) = \frac{\sigma_{XY}}{\sigma_X\sigma_Y} \quad -③$$

Also it is known that

$$I(X:Y) = H(X) + H(Y) - H(X,Y) \quad -④$$

∴ We can replace ① & ② in ③

$$I(X:Y) = \underbrace{\frac{1}{2}\log(2\pi\sigma_X^2) + \frac{1}{2}\log(2\pi e\sigma_Y^2)}_{\Downarrow} - \frac{1}{2}\log\left((2\pi e)^2(\sigma_X^2\sigma_Y^2 - \sigma_{XY}^2)\right)$$

$$= \frac{1}{2}\log\left((2\pi e)^2\sigma_X^2\sigma_Y^2\right) - \frac{1}{2}\log\left[(2\pi e)^2(\sigma_X^2\sigma_Y^2 - \sigma_{XY}^2)\right]$$

$$= \frac{1}{2}\log\left[\frac{\sigma_X^2\sigma_Y^2}{\sigma_X^2\sigma_Y^2 - \sigma_{XY}^2}\right] = \frac{-1}{2}\log\left(1 - \frac{\sigma_{XY}^2}{\sigma_X^2\sigma_Y^2}\right) -⑤$$

Replacing ③ in ⑤

$$= \frac{-1}{2}\log\left(1 - \rho(X,Y)^2\right) \Big] \star$$

Q2 Part B

Given

$X = \begin{bmatrix} X \\ Y \end{bmatrix}$, $\Sigma = \begin{bmatrix} \sigma_X^2 & \sigma_{XY} \\ \sigma_{XY} & \sigma_Y^2 \end{bmatrix}$, $\underset{(2d)}{p(x)} = \frac{1}{2\pi|\Sigma|^{1/2}} \times e^{-1/2(x-\mu)^T \Sigma^{-1}(x-\mu)}$

To show $h(X,Y) = \frac{1}{2} \log\left[(2\pi e)^2(\sigma_X^2\sigma_Y^2 - \sigma_{XY}^2)\right]$

$$h(X,Y) = -\int p(x) \times \log(p(x))\,dx$$

$$= -\int p(x) \times \log\left[\frac{1}{2\pi|\Sigma|^{1/2}} \times e^{-1/2(x-\mu)^T\Sigma^{-1}(x-\mu)}\right] dx \quad \Big\}\ \log ab = \log a + \log b$$

$$= -\log\left(\frac{1}{2\pi|\Sigma|^{1/2}}\right) \times \underbrace{\int \frac{1}{2\pi|\Sigma|^{1/2}} \times e^{-\frac{1}{2}(x-\mu)^T\Sigma^{-1}(x-\mu)}\,dx}_{\text{again Probability Integration } -\infty \text{ to } \infty} + \int \frac{1}{2\pi|\Sigma|^{1/2}} \times e^{-1/2(x-\mu)^T\Sigma^{-1}(x-\mu)} \times \left(-\frac{1}{2}(x-\mu)^T\Sigma^{-1}(x-\mu)\right) dx$$

$$= -\log\left(\frac{1}{2\pi|\Sigma|^{1/2}}\right) \times 1 + \underbrace{\frac{1}{2}\int \frac{(x-\mu)^T\Sigma^{-1}(x-\mu) \times e^{-1/2(x-\mu)^T\Sigma^{-1}(x-\mu)}}{2\pi|\Sigma|^{1/2}}\,dx}_{B}$$

$$B = \frac{1}{2}\int (x-\mu)^T\Sigma^{-1}(x-\mu) \times \frac{1}{2\pi|\Sigma|^{1/2}} \times e^{-1/2(x-\mu)^T\Sigma^{-1}(x-\mu)}\,dx$$

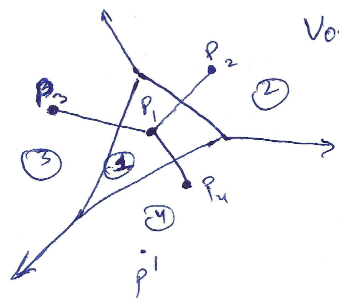
$$= \frac{1}{2} \times E\left((x-\mu)\Sigma^{-1}(x-\mu)\right) = \frac{1}{2}\Sigma^{-1} \underbrace{E\left((x-\mu)^T(x-\mu)\right)}_{\text{Covariance}}$$

$$= \frac{1}{2}\Sigma^{-1}\Sigma = \frac{1}{2}$$

$$= -\log\left(\frac{1}{2\pi|\Sigma|^{1/2}}\right) + \frac{1}{2}$$

$$= \frac{1}{2}\log\left[(2\pi e)^2(\sigma_X^2\sigma_Y^2 - \sigma_{XY}^2)\right]$$

Q1



In cell 4 it can be easily seen that
$P_4 P' <$ distance of P' from any other points outside cell 4

<u>To Proove</u>

~~Convex~~ Regions of voronoi cells are convex.

We know voronoi cells are Intersection of half planes as they are made from perpendicular bisectors. Also Intersection of Infinite number of half planes is convex. Hence Voronoi cells are convex.