

**ASSIGNMENT 1: Fourier Spectrum of Coherent Wavefield**

Checkout: Friday, January 25  
Due: Tuesday, January 29, 6 pm

Name: \_\_\_\_\_

**Objective:** The main objective of this homework assignment is to visualize the spectral distribution of 2-*D* coherent wavefield patterns. This leads you to

- (i) the generation of the coherent wavefield,
- (ii) observations of the spectral distribution of coherent wavefield,
- (iii) understanding of the phase-only concept, and
- (iv) observations of changes due to the variation of wavelength, sample spacing, and aperture size.

- (1) Consider a point source at the origin,  $(x_o, y_o) = (0, 0)$ . This centered point source produces a 2-*D* coherent wavefield pattern in the form of

$$h(x,y) = (j\lambda_o r)^{-1/2} \exp(j2\pi r/\lambda_o)$$

where  $r = (x^2 + y^2)^{1/2}$  and  $\lambda_o$  is the operating wavelength. Generate the resultant 2-*D* wave-field pattern for a region within the radius of  $30\lambda_o$ . For simplicity, sample the 2-*D* wave-field uniformly, in both directions, with the sample spacing

$$\Delta x = \Delta y = \lambda_o / 4.$$

Evaluate the 2-*D* Fourier spectrum of the coherent wave-field with a  $512 \times 512$  FFT, and plot the amplitude of the 2-*D* spectrum. (Remember to zero out the data points around  $r = 0$ .)

(2) Repeat Part (1) with 6 active point sources, located at  $(x_n, y_n)$ ,  $n = 1, 2, \dots, 6$ .

	<i>scatters</i>	<i>scatter locations</i>
1	$(x_1, y_1)$	$(0, 0)$
2	$(x_2, y_2)$	$(10 \lambda_o, 0)$
3	$(x_3, y_3)$	$(0, 10 \lambda_o)$
4	$(x_4, y_4)$	$(-10 \lambda_o, 0)$
5	$(x_5, y_5)$	$(-8 \lambda_o, -6 \lambda_o)$
6	$(x_6, y_6)$	$(8 \lambda_o, -6 \lambda_o)$

The resultant wavefield pattern over the aperture region is the superposition of 6 coherent waveform patterns.

(3) Now consider a different case that each of these 6 active sources is a generating independent coherent wavefield pattern, with a different operating wavelength  $\lambda_n$ , where

$$\lambda_n = n \lambda_o$$

	<i>scatters</i>	<i>scatter locations</i>	<i>wavelength</i>
1	$(x_1, y_1)$	$(0, 0)$	$\lambda_o$
2	$(x_2, y_2)$	$(10 \lambda_o, 0)$	$2 \lambda_o$
3	$(x_3, y_3)$	$(0, 10 \lambda_o)$	$3 \lambda_o$
4	$(x_4, y_4)$	$(-10 \lambda_o, 0)$	$4 \lambda_o$
5	$(x_5, y_5)$	$(-8 \lambda_o, -6 \lambda_o)$	$5 \lambda_o$
6	$(x_6, y_6)$	$(8 \lambda_o, -6 \lambda_o)$	$6 \lambda_o$

Again, the overall wavefield pattern over the aperture is the superposition of 6 waveform patterns. Sample the composite waveform in both directions, with the same sample spacing  $\Delta x = \Delta y = \lambda_o/4$ . Evaluate the 2-D Fourier spectrum of the composite waveform with a  $512 \times 512$  FFT, and plot the amplitude of the spectrum.

(4) Repeat the exercise by using a modified version of the Green's function

$$h'(x,y) = A \exp(j2\pi r/\lambda_o)$$

(This is to replace the amplitude portion of the Green's function with a constant  $A$ , which is known as the *phase-only* version.)

(5) The sample spacing was set to  $\lambda_o/4$  for simplicity. Examine the change of the spectral distribution as you vary the sample spacing of the array.

(6) The radius of the aperture was set to  $30\lambda_o$  for simplicity. Examine the change of the spectral distribution as you vary the aperture size.

Report format:

1. Cover page
2. Figures
3. Summary: (comments based on your observations)
4. Appendix: (computer code)