

ASSIGNMENT 3: Multi-Frequency Backward Propagation

Checkout: Friday, February 8
Due: Tuesday, February 12, 6pm

Name: _____

Objective: The objective of this programming assignment is to experience image formation by multi-frequency backward propagation and to observe the effects on image resolution especially in the range direction.

The system configuration is the same as that in Assignment 2. The *receiver aperture* is a centered linear receiver array with a span of $60\lambda_0$ (from $x = -30\lambda_0$ to $x = +30\lambda_0$). This receiver array is located at the *plane* $y = y_o = -60\lambda_0$, with quarter-wavelength spacing ($\lambda_0/4$) spacing. For each coherent frequency, there are 241 wavefield samples in total over the $60\lambda_0$ -long aperture.

In assignment 2, one single wavelength is applied. As we move into the multi-frequency operating mode, we collect wavefield samples over a wide range of spectrum corresponding to 40 different wavelengths, in the form

$$\lambda_n = 40\lambda_0/(n+20)$$

where $n = 1, 2, \dots, 40$

Thus, this imaging modality operates with a sequence of wavelengths, from $0.67\lambda_0$ to $2\lambda_0$, corresponding to the spatial-frequency band from $0.5(1/\lambda_0)$ to $1.5(1/\lambda_0)$ resulting a bandwidth of $(1/\lambda_0)$. This is to perform coherent image formation procedure 40 times with different operating wavelengths, with the same receiver array configuration. This process produces 40 coherent sub-images, $\hat{s}_n(x, y)$, for $n = 1, 2, \dots, 40$. (It should be noted that the locations of the source distribution and receivers are independent of the change of operating wavelength.)

The goal is to perform multi-frequency image reconstruction of the $60\lambda_0 \times 60\lambda_0$ 2D source region with the 40 sets of coherent wavefield samples. The source region is an area centered at $(0, 0)$ and bounded by $x = \pm 30\lambda_0$ and $y = \pm 30\lambda_0$. For consistency, use quarter-wavelength ($\lambda_0/4$) spacing as the sample spacing in both directions.

- (A) By repeating the image formation procedure, produce the 40 coherent sub-images.
- (B) Observe the characteristics of the sub-images and compare to the results from Assignment 2.
- (C) Superimpose the 40 coherent sub-images *sequentially* and observe the convergence to the overall image,

$$\hat{S}_n(x, y) = \sum_{k=1}^n \hat{s}_k(x, y) \quad n = 1, 2, \dots, 40.$$

- (D) Plot the magnitude distribution of the 4 images $\hat{S}_{10}(x, y)$, $\hat{S}_{20}(x, y)$, $\hat{S}_{30}(x, y)$, and $\hat{S}_{40}(x, y)$.

Report format:

1. Cover page.
2. Magnitude distribution of the final images from Part (D).
3. Summary: (comments based on your observations)
4. Appendix: (computer code)