HW2_ISYE6414_ashish_dhiman

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```
library(ggplot2)
```

Part 1

Read Data and Summary

```
head -5 ./6414\-HW2\-taxes.csv
```

```
## "The following data are sale price, y ($10,000) and taxes, x ($10,000). ",
## Sale Price,Taxes
## 25.9,4.9176
## 29.5,5.0208
## 27.9,4.5429
```

Since the first line does not have data we should skip it, also both the data columns are in 10k USD scale.

```
df_tax = read.table(file ="./6414-HW2-taxes.csv",skip=1, sep=",",header=TRUE)
head(df_tax)
```

```
##   Sale.Price  Taxes
## 1       25.9 4.9176
## 2       29.5 5.0208
## 3       27.9 4.5429
## 4       25.9 4.5573
## 5       29.9 5.0597
## 6       29.9 3.8910
```

```
dim(df_tax)
```

```
## [1] 26  2
```

```
summary(df_tax)
```

```
##    Sale.Price        Taxes
##  Min.   :25.90   Min.   :3.891
##  1st Qu.:29.90   1st Qu.:5.057
```

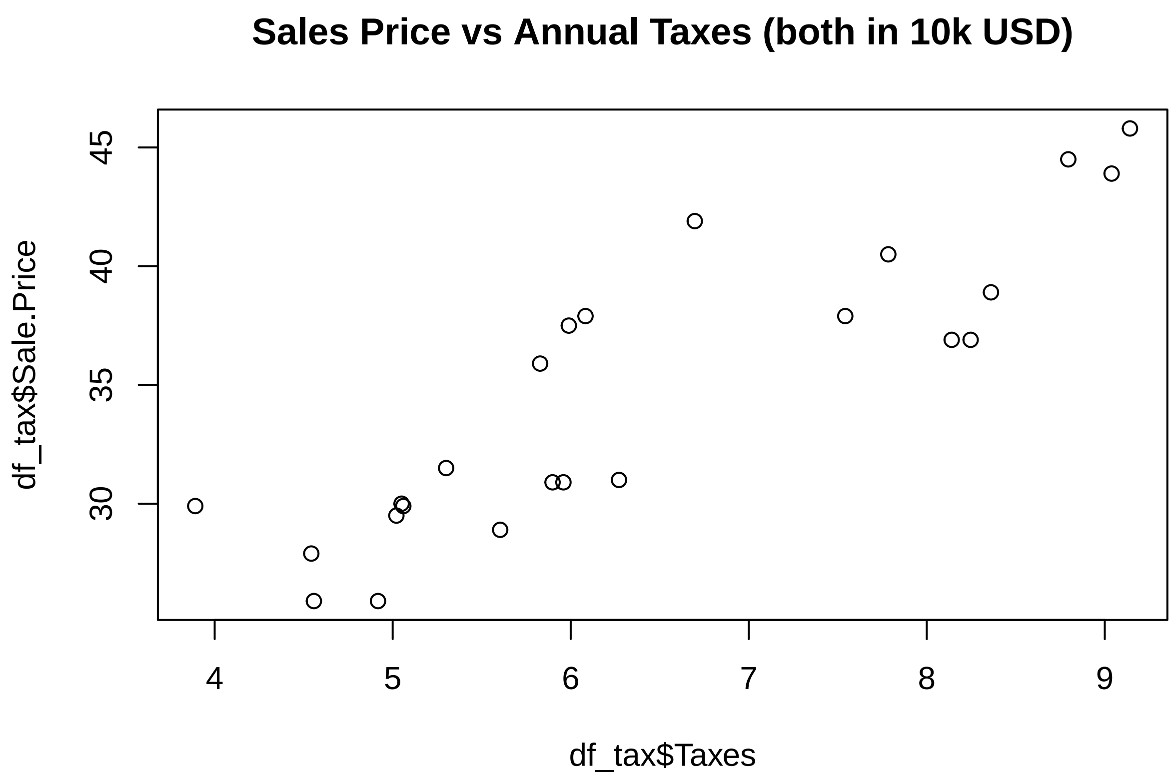
```
##  Median :33.70   Median :5.974
##  Mean   :34.61   Mean   :6.405
##  3rd Qu.:38.15   3rd Qu.:7.873
##  Max.   :45.80   Max.   :9.142
##  NA's   :2       NA's   :2
```

```
df_tax = df_tax[1:(nrow(df_tax)-2),] #Remove last two empty rows
dim(df_tax)
```

```
## [1] 24  2
```

Question 1: Scatter Plot

```
title_i = "Sales Price vs Annual Taxes (both in 10k USD)"
plot(x=df_tax$Taxes, y=df_tax$Sale.Price, type ="p",main = title_i)
```



From the above plot, a linear realationship between Sales Price and Taxes is apparent

The strength of the linear realtionship can also be tested with corealtion between x and y

```
print (paste("Corealtion on full data:",round(cor(df_tax$Taxes,df_tax$Sale.Price),2)))
```

```
## [1] "Corealtion on full data: 0.88"
```

A corelation of 0.88 is pretty significant and supports strong linear realtionship between x and y

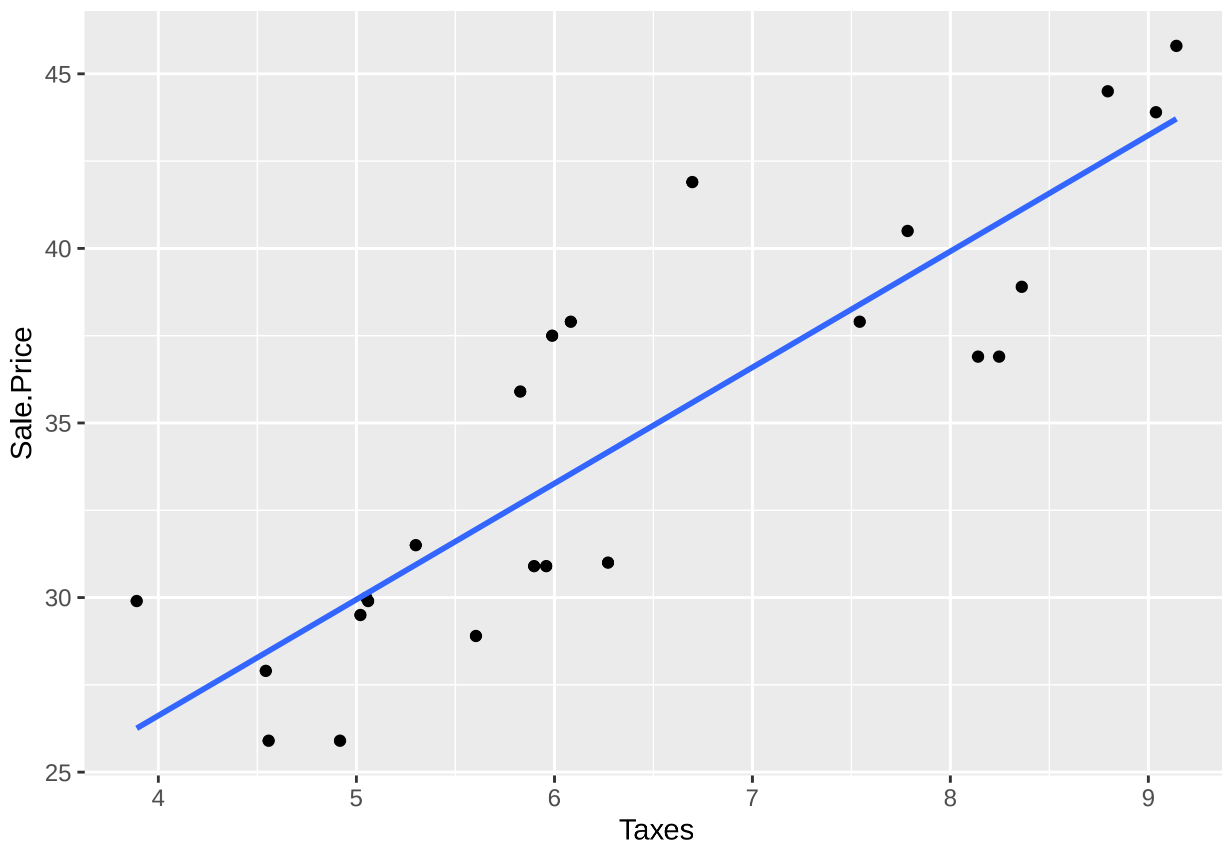
Question 2: Fit SLR

```
#Fit SLR
model0 <- lm(Sale.Price ~ Taxes, data = df_tax)
model0
```

```
##
## Call:
## lm(formula = Sale.Price ~ Taxes, data = df_tax)
##
## Coefficients:
## (Intercept)        Taxes
##      13.320        3.324
```

```
#Superpositioning regression line on
ggplot(df_tax, aes(Taxes, Sale.Price)) + #aes(x,y)
  geom_point() +
  stat_smooth(method = lm, se = FALSE)
```

```
## 'geom_smooth()' using formula 'y ~ x'
```




```
summary(model0)
```

```
##
## Call:
## lm(formula = Sale.Price ~ Taxes, data = df_tax)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -3.8343 -2.3157 -0.3669  1.9787  6.3168
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  13.3202     2.5717   5.179 3.42e-05 ***
## Taxes         3.3244     0.3903   8.518 2.05e-08 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.961 on 22 degrees of freedom
## Multiple R-squared:  0.7673, Adjusted R-squared:  0.7568
## F-statistic: 72.56 on 1 and 22 DF,  p-value: 2.051e-08
```

$$
\begin{gathered}
From\ above, we\ have:\\
\hat{\beta}_0 := Intecept = 13.320,\\
\hat{\beta}_1 := Slope = 3.324,\ and\\
\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 * x = 13.320 + 3.324 * x
\end{gathered}
$$

Question 3: Meaning of beta1 ($\hat{\beta}_1$):

$\hat{\beta}_1$ implies the slope of the SLR line we have fit to the data. In other words, it tells us, the change recorded in y (on average) for every one unit of change in x.

In this case, this implies for every 10k USD change in Taxes, the Price goies up by 3,324 USD *onaverage*.

Question 4: Meaning of beta0 ($\hat{\beta}_0$):

$\hat{\beta}_0$ implies the predicted value of y given x is zero.

In this case, this implies for 0 USD in taxes (let's assume), then the expected prices is 13,320 USD.

This case is improbable but still possible, and would imply that the state has relaxed the tax rate to 0%.

This could be done by the state to foster investment.

Question 5: value of s, s^2 and SSE:

s and s^2

```
s_squared = sum(sapply(model0["residuals"], function(x) x^2))/(nrow(df_tax)-2)
sse = s_squared * (24-2)
print (paste("Sigma is", s_squared^0.5))
```

```
## [1] "Sigma is 2.96103916758819"
```

```r
print (paste("Sigma^2 is", s_squared))
```

```
## [1] "Sigma^2 is 8.76775295199138"
```

```r
print (paste("sse is", sse))
```

```
## [1] "sse is 192.89056494381"
```

SSE from fitted values

$SSE = \sum_{i \in all\ data\ points}([y_i - \hat{y}_i]^2)$

```r
y_hat = fitted(model0)
y_act = df_tax$Sale.Price

sse = sum((y_act - y_hat)^2)
print (paste("sse is",sse))
```

```
## [1] "sse is 192.89056494381"
```

```r
print(paste("s^2 from fitted values",(sse/(nrow(df_tax)-2))))
```

```
## [1] "s^2 from fitted values 8.76775295199138"
```

Part 2

Question 6: Least Square Estimate of beta0, beta1:

$$\hat{\beta}_1 = \frac{(\sum_i x_i y_i) - n\overline{xy}}{\sum_i {x_i}^2 - n\overline{x}^2},$$
$$we\ have:$$
$$\sum_i x_i y_i = 1697.8,$$
$$n\overline{xy} = n * \frac{\sum_i x_i}{n} * \frac{\sum_i y_i}{n},$$
$$\sum_i {x_i}^2 = 157.42,$$
$$n\overline{x}^2 = n * \frac{\sum_i x_i}{n} = \sum_i x_i = 14 * (43/14)^2$$

```r
num = 1697.8 - (14 * (43/14) * (572/14))
num
```

```
## [1] -59.05714
```

```r
denom = 157.42 - (14*(43/14)^2)
denom
```

```
## [1] 25.34857
```

```r
beta1 = num/denom
print (paste("Slope (or beta1)",beta1))
```

```
## [1] "Slope (or beta1) -2.32980162308386"
```

$$\hat{\beta}_0 = \overline{y} - \hat{\beta}_1\overline{x}$$

```r
beta0 = (572/14) - beta1*(43/14)
print (paste("Intecept (or beta0)",beta0))
```

```
## [1] "Intecept (or beta0) 48.0129621280433"
```

Question 7: Calculate SSE

$$SSE = SS_{yy} - \hat{\beta}_1 SS_{xy},$$
$$SS_{yy} = \sum_i (y_i - \overline{y})^2 = \sum_i {y_i}^2 - n\overline{y}^2$$
$$SS_{xy} = (\sum_i x_i y_i) - n\overline{xy}$$

```r
SS_yy = 23530 - 14 *((572/14))^2
SS_xy = num #from ques7
SSE = SS_yy - beta1 * SS_xy
sigma2 = SSE/(14-2)
print (paste("SSE is",SSE))
```

```
## [1] "SSE is 22.1228584310236"
```

```r
print (paste("Sigma squared is",sigma2))
```

```
## [1] "Sigma squared is 1.84357153591864"
```

Question 8: y for x =3.7

```r
y_act = 46.1
y_hat = beta0 + beta1 * 3.7
residual = y_act - y_hat
print (paste("Predicted Y value is",y_hat))
```

```
## [1] "Predicted Y value is 39.392696122633"
```

```r
print (paste("Corresponding Residual is",residual))
```

```
## [1] "Corresponding Residual is 6.707303877367"
```