HW2_ISYE6414_ashish_dhiman

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```
library(ggplot2)
```

Part 1

Read Data and Summary

```
head -5 ./6414\-HW2\-taxes.csv
```

```
## "The following data are sale price, y ($10,000) and taxes, x ($10,000). ",
## Sale Price,Taxes
## 25.9,4.9176
## 29.5,5.0208
## 27.9,4.5429
```

Since the first line does not have data we should skip it, also both the data columns are in 10k USD scale.

```
df_tax = read.table(file ="./6414-HW2-taxes.csv",skip=1, sep=",",header=TRUE)
head(df_tax)
```

```
##   Sale.Price  Taxes
## 1       25.9 4.9176
## 2       29.5 5.0208
## 3       27.9 4.5429
## 4       25.9 4.5573
## 5       29.9 5.0597
## 6       29.9 3.8910
```

```
dim(df_tax)
```

```
## [1] 26  2
```

```
summary(df_tax)
```

```
##    Sale.Price        Taxes
##  Min.   :25.90   Min.   :3.891
##  1st Qu.:29.90   1st Qu.:5.057
```

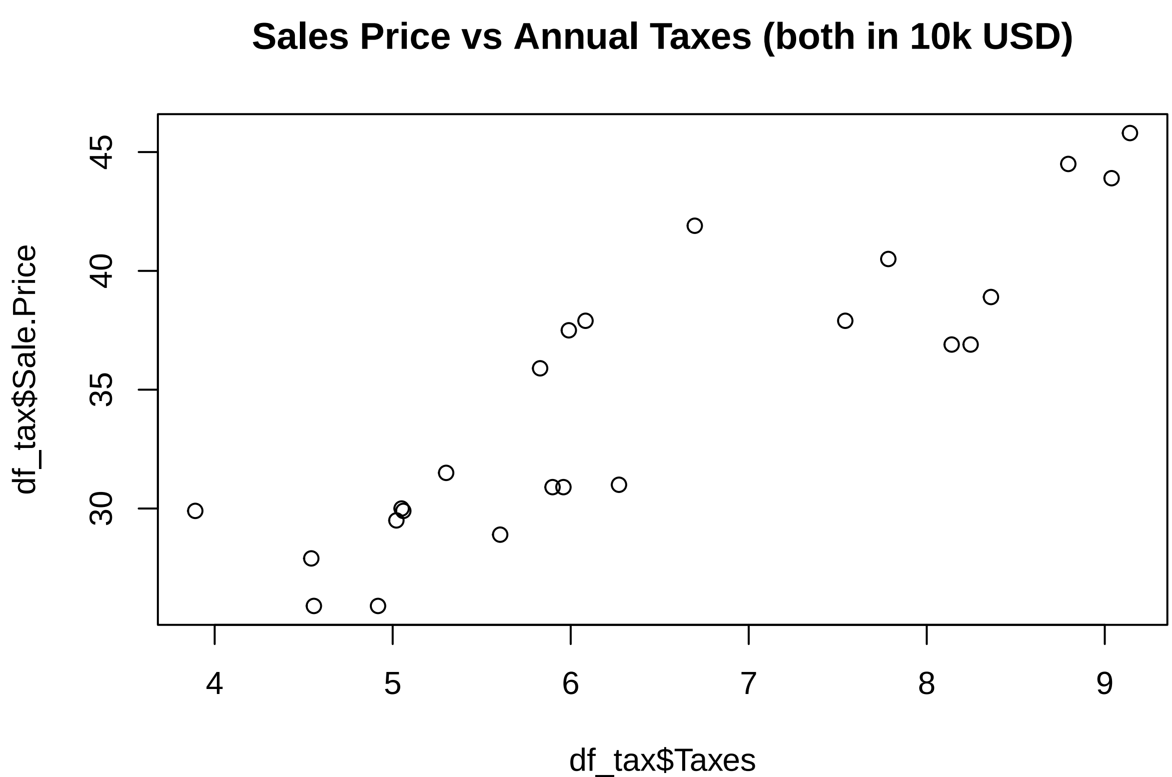
```
##  Median :33.70   Median :5.974
##  Mean   :34.61   Mean   :6.405
##  3rd Qu.:38.15   3rd Qu.:7.873
##  Max.   :45.80   Max.   :9.142
##  NA's   :2       NA's   :2
```

```
df_tax = df_tax[1:(nrow(df_tax)-2),] #Remove last two empty rows
dim(df_tax)
```

```
## [1] 24  2
```

Question 1: Scatter Plot

```
title_i = "Sales Price vs Annual Taxes (both in 10k USD)"
plot(x=df_tax$Taxes, y=df_tax$Sale.Price, type ="p",main = title_i)
```



From the above plot, a linear realationship between Sales Price and Taxes is apparent.

The strength of the linear realtionship can also be tested with corealtion between x and y.

```
print (paste("Corealtion on full data:",round(cor(df_tax$Taxes,df_tax$Sale.Price),2)))
```

```
## [1] "Corealtion on full data: 0.88"
```

A corelation of 0.88 is pretty significant and supports strong linear realtionship between x and y.

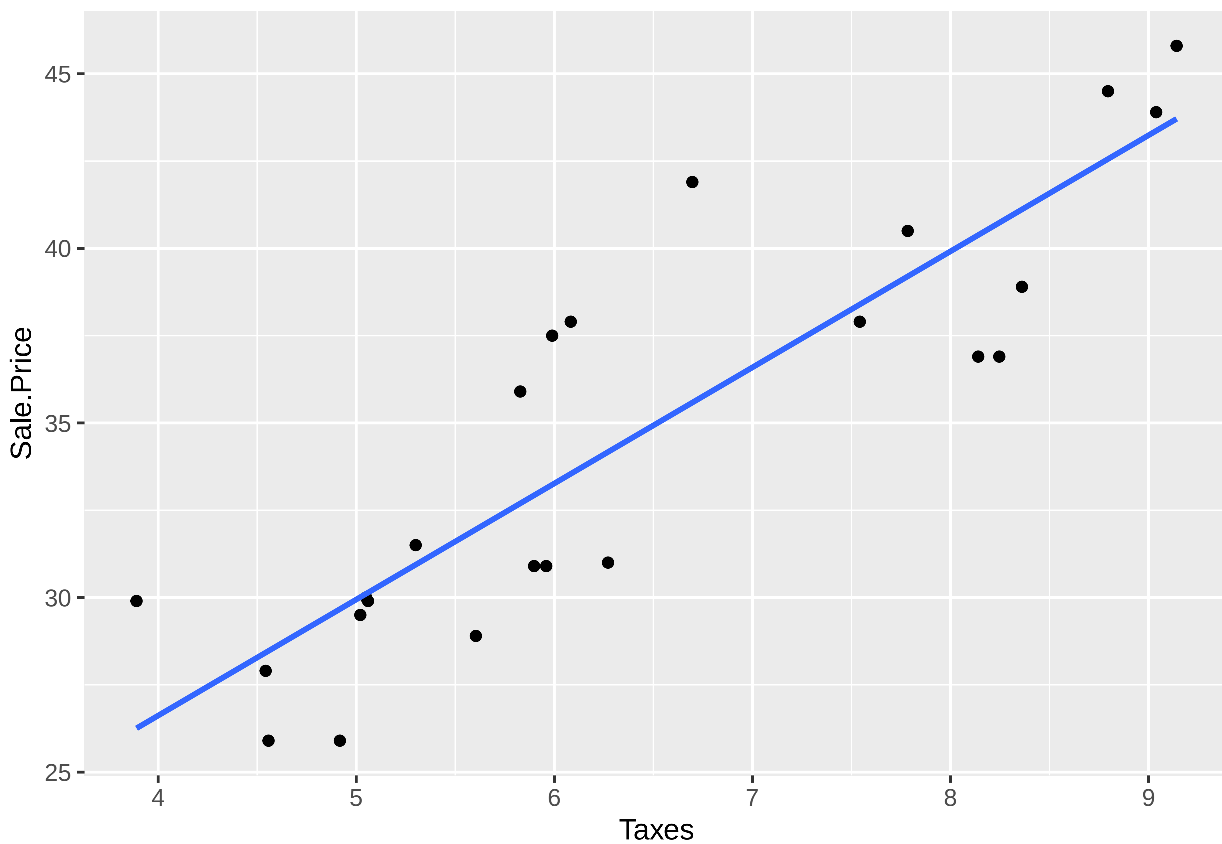
Question 2: Fit SLR

```r
#Fit SLR
model0 <- lm(Sale.Price ~ Taxes, data = df_tax)
model0
```

```
##
## Call:
## lm(formula = Sale.Price ~ Taxes, data = df_tax)
##
## Coefficients:
## (Intercept)        Taxes
##      13.320        3.324
```

```r
#Superpositioning regression line on
ggplot(df_tax, aes(Taxes, Sale.Price)) + #aes(x,y)
  geom_point() +
  stat_smooth(method = lm, se = FALSE)
```

```
## `geom_smooth()` using formula 'y ~ x'
```



$$From\ above, we\ have:$$
$$\hat{\beta}_0 := Intecept = 13.320,$$

$$\hat{\beta}_1 := Slope = 3.324, \ and$$
$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 * x = 13.320 + 3.324 * x$$

Question 3: Meaning of beta1 ($\hat{\beta}_1$):

$\hat{\beta}_1$ implies the slope of the SLR line we have fit to the data. In other words, it tells us, the change recorded in y (on average) for every one unit of change in x.

In this case, this implies for every 10k USD change in Taxes, the Price goies up by 3,324 USD (on average).

Question 4: Meaning of beta0 ($\hat{\beta}_0$):

$\hat{\beta}_0$ implies the predicted value of y given x is zero.

In this case, this implies for 0 USD in taxes (let's assume), the the expected prices is 13,320 USD

Question 5: value of s, s^2 and SSE:

s and s^2

```
s_squared = sum(sapply(model0["residuals"], function(x) x^2))/(nrow(df_tax)-2)
s_squared
```

```
## [1] 8.767753
```

```
s_squared^0.5
```

```
## [1] 2.961039
```

SSE from fitted values

$SSE = \sum_{i \in all \ data \ points}([y_i - \hat{y}_i]^2)$

```
y_hat = fitted(model0)
y_act = df_tax$Sale.Price

sse = sum((y_act - y_hat)^2)
print (paste("sse is",sse))
```

```
## [1] "sse is 192.89056494381"
```

```
sse/(nrow(df_tax)-2)
```

```
## [1] 8.767753
```

Part 2

Question 6: Least Square Estimate of beta0, beta1:

$$\hat{\beta}_1 = \frac{(\sum_i x_i y_i) - n\overline{xy}}{\sum_i {x_i}^2 - n\overline{x}^2},$$
$$we\ have:$$
$$\sum_i x_i y_i = 1697.8,$$
$$n\overline{xy} = n * \frac{\sum_i x_i}{n} * \frac{\sum_i y_i}{n},$$
$$\sum_i {x_i}^2 = 157.42,$$
$$n\overline{x}^2 = n * \frac{\sum_i x_i}{n} = \sum_i x_i = 14 * (43/14)^2$$

```
num = 1697.8 - (14 * (43/14) * (572/14))
num
```

```
## [1] -59.05714
```

```
denom = 157.42 - (14*(43/14)^2)
denom
```

```
## [1] 25.34857
```

```
beta1 = num/denom
print (paste("Intecept (or beta0)",beta1))
```

```
## [1] "Intecept (or beta0) -2.32980162308386"
```

$$\hat{\beta}_0 = \overline{y} - \hat{\beta}_1\overline{x}$$

```
beta0 = (572/14) - beta1*(43/14)
print (paste("Intecept (or beta0)",beta0))
```

```
## [1] "Intecept (or beta0) 48.0129621280433"
```

Question 7: Calculate SSE

$$SSE = SS_{yy} - \hat{\beta}_1 SS_{xy},$$
$$SS_{yy} = \sum_i (y_i - \overline{y})^2 = \sum_i {y_i}^2 - n\overline{y}^2$$
$$SS_{xy} = (\sum_i x_i y_i) - n\overline{xy}$$

```r
SS_yy = 23530 - 14 *((572/14))^2
SS_xy = num #from ques7
SSE = SS_yy - beta1 * SS_xy
sigma2 = SSE/(14-2)
print (paste("Sigma squared is",sigma2))
```

```
## [1] "Sigma squared is 1.84357153591864"
```

Question 8: y for x =3.7

```r
y_act = 46.1
y_hat = beta0 + beta1 * 3.7
residual = y_act - y_hat
print (paste("Predicted Y value is",y_hat))
```

```
## [1] "Predicted Y value is 39.392696122633"
```

```r
print (paste("Corresponding Residual is",y_hat))
```

```
## [1] "Corresponding Residual is 39.392696122633"
```