Taxi demand prediction in New York City

```
In [1]:
```

```
#Importing Libraries
# pip3 install graphviz
#pip3 install dask
#pip3 install toolz
#pip3 install cloudpickle
# https://www.youtube.com/watch?v=ieW3G7ZzRZ0
# https://github.com/dask/dask-tutorial
# please do go through this python notebook: https://github.com/dask/dask-
tutorial/blob/master/07 dataframe.ipynb
import dask.dataframe as dd#similar to pandas
import pandas as pd#pandas to create small dataframes
#pip3 install folium
# if this doesnt work refere install folium.JPG in drive
import folium #open street map
# unix time: https://www.unixtimestamp.com/
import datetime #Convert to unix time
import time #Convert to unix time
# if numpy is not installed already : pip3 install numpy
import numpy as np#Do aritmetic operations on arrays
# matplotlib: used to plot graphs
import matplotlib
# matplotlib.use('nbagg') : matplotlib uses this protocall which makes plots more user intractive
like zoom in and zoom out
matplotlib.use('nbagg')
import matplotlib.pylab as plt
import seaborn as sns#Plots
from matplotlib import rcParams#Size of plots
# this lib is used while we calculate the stight line distance between two (lat,lon) pairs in mile
import gpxpy.geo #Get the haversine distance
from sklearn.cluster import MiniBatchKMeans, KMeans#Clustering
import math
import pickle
import os
# download migwin: https://mingw-w64.org/doku.php/download/mingw-builds
# install it in your system and keep the path, migw_path ='installed path'
mingw path = 'C:\\Program Files\\mingw-w64\\x86 64-5.3.0-posix-seh-rt v4-rev0\\mingw64\\bin'
os.environ['PATH'] = mingw_path + ';' + os.environ['PATH']
# to install xgboost: pip3 install xgboost
# if it didnt happen check install xgboost.JPG
import xgboost as xgb
# to install sklearn: pip install -U scikit-learn
from sklearn.ensemble import RandomForestRegressor
from sklearn.metrics import mean_squared_error
from sklearn.metrics import mean_absolute error
import warnings
warnings.filterwarnings("ignore")
/home/paperspace/anaconda3/lib/python3.7/site-packages/sklearn/ensemble/weight boosting.py:29: Dep
recationWarning: numpy.core.umath_tests is an internal NumPy module and should not be imported. It
will be removed in a future NumPy release.
  from numpy.core.umath_tests import inner1d
```

Dala IIIIOIIIIalioii

Ge the data from: http://www.nyc.gov/html/tlc/html/about/trip_record_data.shtml (2016 data) The data used in the attached datasets were collected and provided to the NYC Taxi and Limousine Commission (TLC)

Information on taxis:

Yellow Taxi: Yellow Medallion Taxicabs

These are the famous NYC yellow taxis that provide transportation exclusively through street-hails. The number of taxicabs is limited by a finite number of medallions issued by the TLC. You access this mode of transportation by standing in the street and hailing an available taxi with your hand. The pickups are not pre-arranged.

For Hire Vehicles (FHVs)

FHV transportation is accessed by a pre-arrangement with a dispatcher or limo company. These FHVs are not permitted to pick up passengers via street hails, as those rides are not considered pre-arranged.

Green Taxi: Street Hail Livery (SHL)

The SHL program will allow livery vehicle owners to license and outfit their vehicles with green borough taxi branding, meters, credit card machines, and ultimately the right to accept street hails in addition to pre-arranged rides.

Credits: Quora

Footnote:

In the given notebook we are considering only the yellow taxis for the time period between Jan - Mar 2015 & Jan - Mar 2016

Data Collection

We Have collected all yellow taxi trips data from jan-2015 to dec-2016(Will be using only 2015 data)

file name	file name size	number of records	number of features	
yellow_tripdata_2016-01	1. 59G	10906858	19	
yellow_tripdata_2016-02	1. 66G	11382049	19	
yellow_tripdata_2016-03	1. 78G	12210952	19	
yellow_tripdata_2016-04	1. 74G	11934338	19	
yellow_tripdata_2016-05	1. 73G	11836853	19	
yellow_tripdata_2016-06	1. 62G	11135470	19	
yellow_tripdata_2016-07	884Mb	10294080	17	
yellow_tripdata_2016-08	854Mb	9942263	17	
yellow_tripdata_2016-09	870Mb	10116018	17	
yellow_tripdata_2016-10	933Mb	10854626	17	
yellow_tripdata_2016-11	868Mb	10102128	17	
yellow_tripdata_2016-12	897Mb	10449408	17	
yellow_tripdata_2015-01	1.84Gb	12748986	19	
yellow_tripdata_2015-02	1.81Gb	12450521	19	
yellow_tripdata_2015-03	1.94Gb	13351609	19	
yellow_tripdata_2015-04	1.90Gb	13071789	19	
yellow_tripdata_2015-05	1.91Gb	13158262	19	
yellow_tripdata_2015-06	1.79Gb	12324935	19	
yellow_tripdata_2015-07	1.68Gb	11562783	19	
yellow_tripdata_2015-08	1.62Gb	11130304	19	
yellow_tripdata_2015-09	1.63Gb	11225063	19	

yellow_tripdata_2015-10	1.79Gb	12315488	19	
yellow_tripdata_2015-11	1.65Gb	11312676	19	
yellow_tripdata_2015-12	1.67Gb	11460573	19	

In [2]:

In [3]:

```
# However unlike Pandas, operations on dask.dataframes don't trigger immediate computation,
# instead they add key-value pairs to an underlying Dask graph. Recall that in the diagram below,
# circles are operations and rectangles are results.

# to see the visulaization you need to install graphviz
# pip3 install graphviz if this doesnt work please check the install_graphviz.jpg in the drive
#month.visualize()
```

In [4]:

```
#month.fare_amount.sum().visualize()
```

'improvement_surcharge', 'total_amount'],

Features in the dataset:

dtype='object')

Field Name	Description
VendorID	A code indicating the TPEP provider that provided the record. 1. Creative Mobile Technologies 2. VeriFone Inc.
tpep_pickup_datetime	The date and time when the meter was engaged.
tpep_dropoff_datetime	The date and time when the meter was disengaged.
Passenger_count	The number of passengers in the vehicle. This is a driver-entered value.
Trip_distance	The elapsed trip distance in miles reported by the taximeter.
Pickup_longitude	Longitude where the meter was engaged.
Pickup_latitude	Latitude where the meter was engaged.
RateCodeID	The final rate code in effect at the end of the trip. 1. Standard rate 2. JFK 3. Newark 4. Nassau or Westchester 5. Negotiated fare 6. Group ride
Store_and_fwd_flag	This flag indicates whether the trip record was held in vehicle memory before sending to the vendor, aka "store and forward," because the vehicle did not have a connection to the server. Y= store and forward trip N= not a store and forward trip
Dropoff_longitude	Longitude where the meter was disengaged.
Dropoff_ latitude	Latitude where the meter was disengaged.
	A numeric code signifying how the passenger paid for the trip. 1. Credit card

Payment_type	2. Cash3. No charge4. Dispute5. Unknown6. Voided trip
Fare_amount	The time-and-distance fare calculated by the meter.
Extra	Miscellaneous extras and surcharges. Currently, this only includes. the $0.50 and 1$ rush hour and overnight charges.
MTA_tax	0.50 MTA tax that is automatically triggered based on the metered rate in use.
Improvement_surcharge	0.30 improvement surcharge assessed trips at the flag drop. the improvement surcharge began being levied in 2015.
Tip_amount	Tip amount – This field is automatically populated for credit card tips.Cash tips are not included.
Tolls_amount	Total amount of all tolls paid in trip.
Total_amount	The total amount charged to passengers. Does not include cash tips.

ML Problem Formulation

Time-series forecasting and Regression

- To find number of pickups, given location cordinates(latitude and longitude) and time, in the query reigion and surrounding regions.

To solve the above we would be using data collected in Jan - Mar 2015 to predict the pickups in Jan - Mar 2016.

Performance metrics

- 1. Mean Absolute percentage error.
- 2. Mean Squared error.

Data Cleaning

In this section we will be doing univariate analysis and removing outlier/illegitimate values which may be caused due to some error

In [5]:

#table below shows few datapoints along with all our features month.head(5)

Out[5]:

	VendorID	tpep_pickup_datetime	tpep_dropoff_datetime	passenger_count	trip_distance	pickup_longitude	pickup_latit
0	2	2015-01-15 19:05:39	2015-01-15 19:23:42	1	1.59	-73.993896	40.750111
1	1	2015-01-10 20:33:38	2015-01-10 20:53:28	1	3.30	-74.001648	40.724243
2	1	2015-01-10 20:33:38	2015-01-10 20:43:41	1	1.80	-73.963341	40.802788
3	1	2015-01-10 20:33:39	2015-01-10 20:35:31	1	0.50	-74.009087	40.713818
4	1	2015-01-10 20:33:39	2015-01-10 20:52:58	1	3.00	-73.971176	40.762428
4	•						Þ

1. Pickup Latitude and Pickup Longitude

It is inferred from the source https://www.flickr.com/places/info/2459115 that New York is bounded by the location cordinates (lat,long) - (40.5774, -74.15) & (40.9176,-73.7004) so hence any cordinates not within these cordinates are not considered by us as we are only concerned with pickups which originate within New York.

In [6]:

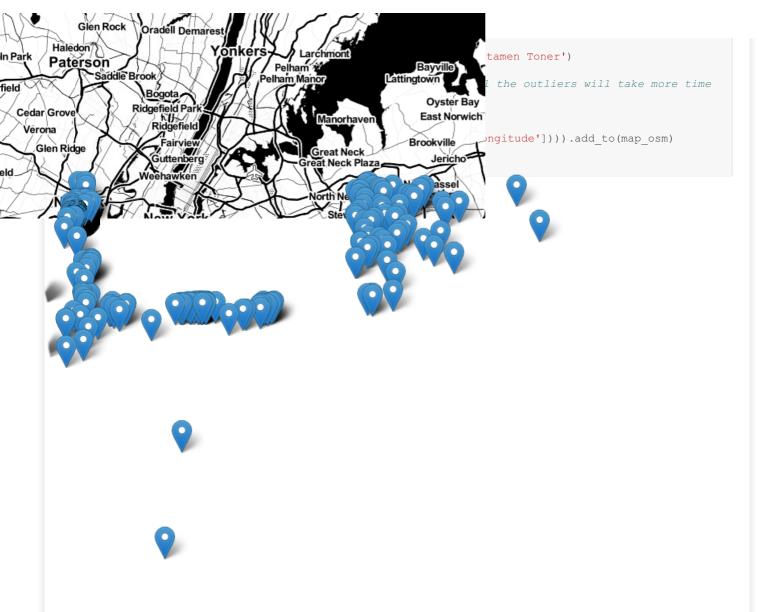
```
outlier_locations = month[((month.pickup_longitude <= -74.15) | (month.pickup_latitude <= 40.5774)|
                               (month.pickup_longitude >= -73.7004) | (month.pickup_latitude >= 40.9176))]
          # creating a map with the a base location
          # read more about the folium here: http://folium.readthedocs.io/en/latest/quickstart.html
                                                                               indeepth knowledge on these maps and
            Glen Rock
        Haledon"
                                                                               tamen Toner')
       Paterson
                                            Pelham *
                                                                    Bayville
               Saddle Brook
                                                               Lattingtown
                                                                                 the outliers will take more time
field
                                                                     Oyster Bay
  Cedar Grov
                                                                    East Norwich
   Verona
                                                                               _itude']))).add_to(map_osm)
                                                                   Brookville
     Glen Ridge
                                                   Great Neck
                                                                      Jericho*
                                                   eat Neck Plaza
                                                                      isbury
```

Observation:- As you can see above that there are some points just outside the boundary but there are a few that are in either South america. Mexico or Canada

2. Dropoff Latitude & Dropoff Longitude

It is inferred from the source https://www.flickr.com/places/info/2459115 that New York is bounded by the location cordinates (lat,long) - (40.5774, -74.15) & (40.9176,-73.7004) so hence any cordinates not within these cordinates are not considered by us as we are only concerned with dropoffs which are within New York.

In [7]:



Observation:- The observations here are similar to those obtained while analysing pickup latitude and longitude

3. Trip Durations:

According to NYC Taxi & Limousine Commision Regulations the maximum allowed trip duration in a 24 hour interval is 12 hours.

```
In [8]:
```

```
#The timestamps are converted to unix so as to get duration(trip-time) & speed also pickup-times i
n unix are used while binning
# in out data we have time in the formate "YYYY-MM-DD HH:MM:SS" we convert thiss sting to python t
ime formate and then into unix time stamp
# https://stackoverflow.com/a/27914405
def convert_to_unix(s):
    return time.mktime(datetime.datetime.strptime(s, "%Y-%m-%d %H:%M:%S").timetuple())
# we return a data frame which contains the columns
# 1.'passenger_count' : self explanatory
# 2. 'trip distance' : self explanatory
 3.'pickup longitude' : self explanatory
# 4.'pickup latitude' : self explanatory
# 5.'dropoff longitude' : self explanatory
# 6.'dropoff_latitude' : self explanatory
 7.'total_amount' : total fair that was paid
 8. 'trip times' : duration of each trip
# 9.'pickup_times : pickup time converted into unix time
# 10.'Speed' : velocity of each trip
def return with trip times(month):
```

```
duration = month[['tpep_pickup_datetime','tpep_dropoff_datetime']].compute()
    #pickups and dropoffs to unix time
   duration pickup = [convert to unix(x) for x in duration['tpep pickup datetime'].values]
   duration drop = [convert to unix(x) for x in duration['tpep dropoff datetime'].values]
    #calculate duration of trips
   durations = (np.array(duration drop) - np.array(duration pickup))/float(60)
   #append durations of trips and speed in miles/hr to a new dataframe
   new frame =
month[['passenger count','trip distance','pickup longitude','pickup latitude','dropoff longitude',
'dropoff latitude', 'total amount']].compute()
   new_frame['trip_times'] = durations
   new frame['pickup times'] = duration pickup
   new frame['Speed'] = 60*(new frame['trip distance']/new frame['trip times'])
   return new frame
# print(frame_with durations.head())
# passenger count trip distance pickup longitude pickup latitude dropoff longitude
dropoff_latitude total_amount trip_times pickup_times Speed
                      1.59 -73.993896
                                                  40.750111
                                                                 -73.974785
         18.050000 1.421329e+09 5.285319
17.05
                    3.30 -74.001648
                                             40.724243 -73.994415
                                                                         40.759109
.80
      19.833333 1.420902e+09 9.983193
                             -73.963341
                                              40.802788
                                                             -73.951820
                    1.80
                                                                              40.824413
10.80
        10.050000 1.420902e+09 10.746269
# 1
                    0.50 -74.009087
                                              40.713818
                                                           -74.004326
                                                                             40.719986
4.80
       1.866667 1.420902e+09 16.071429
                    3.00 -73.971176
                                              40.762428
                                                           -74.004181
                                                                            40.742653
6.30 19.316667 1.420902e+09 9.318378
frame with durations = return_with_trip_times(month)
In [9]:
# the skewed box plot shows us the presence of outliers
sns.boxplot(y="trip times", data =frame with durations)
plt.show()
```

In [10]:

```
#calculating 0-100th percentile to find a the correct percentile value for removal of outliers
for i in range (0, 100, 10):
   var =frame_with_durations["trip_times"].values
   var = np.sort(var,axis = None)
    print("{} percentile value is {}".format(i,var[int(len(var)*(float(i)/100))]))
print ("100 percentile value is ",var[-1])
O percentile value is -1211.0166666666667
10 percentile value is 3.833333333333333
20 percentile value is 5.3833333333333334
30 percentile value is 6.816666666666666
40 percentile value is 8.3
50 percentile value is 9.95
60 percentile value is 11.86666666666667
70 percentile value is 14.283333333333333
80 percentile value is 17.633333333333333
90 percentile value is 23.45
100 percentile value is 548555.6333333333
```

In [11]:

```
#looking further from the 99th percecntile
for i in range(90,100):
   var =frame_with_durations["trip_times"].values
   var = np.sort(var,axis = None)
   print("{} percentile value is {}".format(i,var[int(len(var)*(float(i)/100))]))
print ("100 percentile value is ",var[-1])
90 percentile value is 23.45
```

```
91 percentile value is 24.35
00 noncontile malue to 0E 20222222222222
```

```
as betceurite natue is so.sosssssssss
93 percentile value is 26.55
94 percentile value is 27.933333333333334
95 percentile value is 29.583333333333332
96 percentile value is 31.683333333333333
97 percentile value is 34.4666666666667
98 percentile value is 38.71666666666667
99 percentile value is 46.75
100 percentile value is 548555.6333333333
In [12]:
#removing data based on our analysis and TLC regulations
frame with durations modified=frame with durations[(frame with durations.trip times>1) &
(frame with durations.trip times<720)]
In [13]:
#box-plot after removal of outliers
sns.boxplot(y="trip_times", data =frame_with_durations_modified)
plt.show()
In [14]:
#pdf of trip-times after removing the outliers
sns.FacetGrid(frame with durations modified, size=6) \
      .map(sns.kdeplot,"trip times") \
      .add legend();
plt.show();
In [15]:
#converting the values to log-values to chec for log-normal
frame with durations modified['log times'] = [math.log(i) for i in frame with durations modified['tri
p_times'].values]
In [16]:
#pdf of log-values
sns.FacetGrid(frame with durations modified,size=6) \
      .map(sns.kdeplot,"log times") \
      .add legend();
plt.show();
In [17]:
#Q-Q plot for checking if trip-times is log-normal
import scipy
scipy.stats.probplot(frame with durations modified['log times'].values, plot=plt)
plt.show()
4. Speed
In [18]:
# check for any outliers in the data after trip duration outliers removed
# box-plot for speeds with outliers
frame with durations modified['Speed'] =
```

60*(frame with durations modified['trip distance']/frame with durations modified['trip times'])

#sns.boxplot(y="Speed", data =frame with durations modified)

plt.show()

```
In [19]:
#calculating speed values at each percntile 0,10,20,30,40,50,60,70,80,90,100
for i in range (0, 100, 10):
    var =frame with durations modified["Speed"].values
   var = np.sort(var,axis = None)
    print("{} percentile value is {}".format(i,var[int(len(var)*(float(i)/100))]))
print("100 percentile value is ",var[-1])
0 percentile value is 0.0
10 percentile value is 6.409495548961425
20 percentile value is 7.80952380952381
30 percentile value is 8.929133858267717
40 percentile value is 9.98019801980198
50 percentile value is 11.06865671641791
60 percentile value is 12.286689419795222
70 percentile value is 13.796407185628745
80 percentile value is 15.963224893917962
90 percentile value is 20.186915887850468
100 percentile value is 192857142.85714284
In [20]:
#calculating speed values at each percntile 90,91,92,93,94,95,96,97,98,99,100
for i in range (90,100):
   var =frame with durations modified["Speed"].values
    var = np.sort(var,axis = None)
    print("{} percentile value is {}".format(i,var[int(len(var)*(float(i)/100))]))
print("100 percentile value is ",var[-1])
90 percentile value is 20.186915887850468
91 percentile value is 20.91645569620253
92 percentile value is 21.752988047808763
93 percentile value is 22.721893491124263
94 percentile value is 23.844155844155843
95 percentile value is 25.182552504038775
96 percentile value is 26.80851063829787
97 percentile value is 28.84304932735426
98 percentile value is 31.591128254580514
99 percentile value is 35.7513566847558
100 percentile value is 192857142.85714284
In [21]:
#calculating speed values at each percntile 99.0,99.1,99.2,99.3,99.4,99.5,99.6,99.7,99.8,99.9,100
for i in np.arange(0.0, 1.0, 0.1):
    var =frame with durations modified["Speed"].values
    var = np.sort(var,axis = None)
    print("{} percentile value is {}".format(99+i,var[int(len(var)*(float(99+i)/100))]))
print("100 percentile value is ",var[-1])
99.0 percentile value is 35.7513566847558
99.1 percentile value is 36.31084727468969
99.2 percentile value is 36.91470054446461
99.3 percentile value is 37.588235294117645
99.4 percentile value is 38.33035714285714
99.5 percentile value is 39.17580340264651
99.6 percentile value is 40.15384615384615
99.7 percentile value is 41.338301043219076
99.8 percentile value is 42.86631016042781
99.9 percentile value is 45.3107822410148
100 percentile value is 192857142.85714284
In [22]:
#removing further outliers based on the 99.9th percentile value
frame with durations modified=frame with durations[(frame with durations.Speed>0) \&
(frame_with_durations.Speed<45.31)]
```

In [23]:

```
sum(frame with durations modified["Speed"]) / float(len(frame with durations modified["Speed"]))
Out[23]:
12.450173996028015
The avg speed in Newyork speed is 12.45miles/hr, so a cab driver can travel2 miles per 10min on avg.
4. Trip Distance
In [24]:
# up to now we have removed the outliers based on trip durations and cab speeds
# lets try if there are any outliers in trip distances
# box-plot showing outliers in trip-distance values
sns.boxplot(y="trip distance", data =frame with durations modified)
plt.show()
In [25]:
#calculating trip distance values at each percntile 0,10,20,30,40,50,60,70,80,90,100
for i in range(0,100,10):
   var =frame with durations modified["trip distance"].values
    var = np.sort(var,axis = None)
    print("{} percentile value is {}".format(i,var[int(len(var)*(float(i)/100))]))
print("100 percentile value is ",var[-1])
0 percentile value is 0.01
10 percentile value is 0.66
20 percentile value is 0.9
30 percentile value is 1.1
40 percentile value is 1.39
50 percentile value is 1.69
60 percentile value is 2.07
70 percentile value is 2.6
80 percentile value is 3.6
90 percentile value is 5.97
100 percentile value is 258.9
In [26]:
#calculating trip distance values at each percntile 90,91,92,93,94,95,96,97,98,99,100
for i in range(90,100):
    var =frame with durations_modified["trip_distance"].values
    var = np.sort(var,axis = None)
    print("{} percentile value is {}".format(i,var[int(len(var)*(float(i)/100))]))
print("100 percentile value is ",var[-1])
90 percentile value is 5.97
91 percentile value is 6.45
92 percentile value is 7.07
93 percentile value is 7.85
94 percentile value is 8.72
95 percentile value is 9.6
96 percentile value is 10.6
97 percentile value is 12.1
98 percentile value is 16.03
99 percentile value is 18.17
100 percentile value is 258.9
In [27]:
#calculating trip distance values at each percntile
99.0,99.1,99.2,99.3,99.4,99.5,99.6,99.7,99.8,99.9,100
for i in np.arange(0.0, 1.0, 0.1):
    var =frame with durations modified["trip distance"].values
```

print("{} percentile value is {}".format(99+i,var[int(len(var)*(float(99+i)/100))]))

var = np.sort(var,axis = None)

```
|print("100 percentile value is ",var[-1])
99.0 percentile value is 18.17
99.1 percentile value is 18.37
99.2 percentile value is 18.6
99.3 percentile value is 18.83
99.4 percentile value is 19.13
99.5 percentile value is 19.5
99.6 percentile value is 19.96
99.7 percentile value is 20.5
99.8 percentile value is 21.22
99.9 percentile value is 22.57
100 percentile value is 258.9
In [28]:
#removing further outliers based on the 99.9th percentile value
frame_with_durations_modified=frame_with_durations[(frame_with_durations.trip_distance>0) &
(frame with durations.trip distance<23)]</pre>
In [29]:
#box-plot after removal of outliers
sns.boxplot(y="trip distance", data = frame with durations modified)
plt.show()
5. Total Fare
In [30]:
# up to now we have removed the outliers based on trip durations, cab speeds, and trip distances
# lets try if there are any outliers in based on the total amount
# box-plot showing outliers in fare
sns.boxplot(y="total amount", data =frame with durations modified)
plt.show()
In [31]:
#calculating total fare amount values at each percntile 0,10,20,30,40,50,60,70,80,90,100
for i in range(0,100,10):
    var = frame with durations modified["total amount"].values
    var = np.sort(var,axis = None)
    print("{} percentile value is {}".format(i,var[int(len(var)*(float(i)/100))]))
print("100 percentile value is ",var[-1])
O percentile value is -242.55
10 percentile value is 6.3
20 percentile value is 7.8
30 percentile value is 8.8
40 percentile value is 9.8
50 percentile value is 11.16
60 percentile value is 12.8
70 percentile value is 14.8
80 percentile value is 18.3
90 percentile value is 25.8
100 percentile value is 3950611.6
In [32]:
#calculating total fare amount values at each percntile 90,91,92,93,94,95,96,97,98,99,100
for i in range(90,100):
    var = frame_with_durations_modified["total amount"].values
    var = np.sort(var,axis = None)
    print("{} percentile value is {}".format(i,var[int(len(var)*(float(i)/100))]))
print("100 percentile value is ",var[-1])
90 percentile value is 25.8
```

```
91 percentile value is 27.3
92 percentile value is 29.3
93 percentile value is 31.8
94 percentile value is 34.8
95 percentile value is 38.53
96 percentile value is 42.6
97 percentile value is 48.13
98 percentile value is 58.13
99 percentile value is 66.13
100 percentile value is 3950611.6
In [331:
#calculating total fare amount values at each percntile
99.0,99.1,99.2,99.3,99.4,99.5,99.6,99.7,99.8,99.9,100
for i in np.arange(0.0, 1.0, 0.1):
    var = frame with durations modified["total amount"].values
    var = np.sort(var,axis = None)
    print("{} percentile value is {}".format(99+i,var[int(len(var)*(float(99+i)/100))]))
print("100 percentile value is ",var[-1])
99.0 percentile value is 66.13
99.1 percentile value is 68.13
99.2 percentile value is 69.6
99.3 percentile value is 69.6
99.4 percentile value is 69.73
99.5 percentile value is 69.75
99.6 percentile value is 69.76
99.7 percentile value is 72.58
99.8 percentile value is 75.35
99.9 percentile value is 88.28
100 percentile value is 3950611.6
```

Observation:- As even the 99.9th percentile value doesnt look like an outlier, as there is not much difference between the 99.8th percentile and 99.9th percentile, we move on to do graphical analyis

```
In [34]:
```

```
#below plot shows us the fare values(sorted) to find a sharp increase to remove those values as ou
tliers
# plot the fare amount excluding last two values in sorted data
plt.plot(var[:-2])
plt.show()
```

```
In [35]:
```

```
# a very sharp increase in fare values can be seen
# plotting last three total fare values, and we can observe there is share increase in the values
plt.plot(var[-3:])
plt.show()
```

```
In [36]:
```

```
#now looking at values not including the last two points we again find a drastic increase at aroun
d 1000 fare value
# we plot last 50 values excluding last two values
plt.plot(var[-50:-2])
plt.show()
```

Remove all outliers/erronous points.

```
In [37]:
```

```
#removing all outliers based on our univariate analysis above
def remove_outliers(new_frame):
```

```
a = new frame.shape[0]
           print ("Number of pickup records = ",a)
           \texttt{temp\_frame} = \texttt{new\_frame[((new\_frame.dropoff\_longitude}) = -74.15) \& (new\_frame.dropoff\_longitude) = -74.15) \& (new\_frame.dropoff\_longitude) = -74.15) \& (new\_frame.dropoff\_longitude) = -74.15) & (new\_frame.dropoff\_longitude) = -74.15) 
                                                            (new frame.dropoff latitude >= 40.5774) & (new frame.dropoff latitude <=
 40.9176)) & \
                                                             ((new frame.pickup longitude \geq -74.15) & (new frame.pickup latitude \geq
 40.5774)& \
                                                             (new frame.pickup longitude <= -73.7004) & (new frame.pickup latitude <=
 40.9176))]
          b = temp frame.shape[0]
          print ("Number of outlier coordinates lying outside NY boundaries:", (a-b))
          temp_frame = new_frame[(new_frame.trip_times > 0) & (new frame.trip times < 720)]</pre>
           c = temp frame.shape[0]
           print ("Number of outliers from trip times analysis:",(a-c))
           temp_frame = new_frame[(new_frame.trip_distance > 0) & (new_frame.trip_distance < 23)]</pre>
           d = temp frame.shape[0]
           print ("Number of outliers from trip distance analysis:", (a-d))
           temp_frame = new_frame[(new_frame.Speed <= 65) & (new_frame.Speed >= 0)]
           e = temp frame.shape[0]
           print ("Number of outliers from speed analysis:", (a-e))
           temp frame = new frame[(new frame.total amount <1000) & (new frame.total amount >0)]
           f = temp frame.shape[0]
           print ("Number of outliers from fare analysis:",(a-f))
           \verb|new_frame| = \verb|new_frame| [((\verb|new_frame.dropoff_longitude|) >= -74.15)| \& ((\verb|new_frame.dropoff_longitude|) <= -74.15| & ((\verb|new_frame.dropoff_longitude|) <= -74
 = -73.7004) & 
                                                             (new frame.dropoff latitude >= 40.5774) & (new frame.dropoff latitude <=
 40.9176)) & \
                                                            ((new frame.pickup longitude \geq -74.15) & (new frame.pickup latitude \geq
 40.5774)& \
                                                            (new frame.pickup longitude <= -73.7004) & (new frame.pickup latitude <=
 40.9176))]
          new frame = new frame[(new frame.trip times > 0) & (new frame.trip times < 720)]</pre>
          new frame = new frame[(new frame.trip distance > 0) & (new frame.trip distance < 23)]</pre>
          new_frame = new_frame[(new_frame.Speed < 45.31) & (new_frame.Speed > 0)]
          new frame = new frame[(new frame.total amount <1000) & (new frame.total amount >0)]
          print ("Total outliers removed",a - new_frame.shape[0])
          print ("---")
          return new_frame
 4
In [38]:
print ("Removing outliers in the month of Jan-2015")
 frame with durations outliers removed = remove outliers (frame with durations)
 print("fraction of data points that remain after removing outliers",
float(len(frame with durations outliers removed))/len(frame with durations))
Removing outliers in the month of Jan-2015
Number of pickup records = 12748986
Number of outlier coordinates lying outside NY boundaries: 293919
Number of outliers from trip times analysis: 23889
Number of outliers from trip distance analysis: 92597
Number of outliers from speed analysis: 24473
Number of outliers from fare analysis: 5275
Total outliers removed 377910
fraction of data points that remain after removing outliers 0.9703576425607495
```

Data-preperation

Clustering/Segmentation

```
In [39]:
#trying different cluster sizes to choose the right K in K-means
coords = frame with durations outliers removed[['pickup latitude', 'pickup longitude']].values
neighbours=[]
def find min distance (cluster centers, cluster len):
    nice points = 0
    wrong_points = 0
    less2 = []
    more2 = []
    min_dist=1000
    for i in range(0, cluster len):
        nice points = 0
        wrong_points = 0
        for j in range(0, cluster len):
            if j!=i:
                distance = gpxpy.geo.haversine_distance(cluster_centers[i][0], cluster centers[i][1
,cluster_centers[j][0], cluster_centers[j][1])
                min_dist = min(min_dist, distance/(1.60934*1000))
                if (distance/(1.60934*1000)) <= 2:</pre>
                    nice_points +=1
                else:
                    wrong points += 1
        less2.append(nice points)
        more2.append(wrong points)
    neighbours.append(less2)
    print ("On choosing a cluster size of ",cluster len,"\nAvg. Number of Clusters within the vici
nity (i.e. intercluster-distance < 2):", np.ceil(sum(less2)/len(less2)), "\nAvg. Number of
Clusters outside the vicinity (i.e. intercluster-distance > 2):", np.ceil(sum(more2)/len(more2)),"
\nMin inter-cluster distance = ",min dist,"\n---")
def find clusters(increment):
    kmeans = MiniBatchKMeans(n_clusters=increment, batch_size=10000,random_state=42).fit(coords)
    frame with durations outliers removed['pickup cluster'] =
kmeans.predict(frame with durations outliers removed[['pickup latitude', 'pickup longitude']])
    cluster centers = kmeans.cluster centers
    cluster len = len(cluster centers)
    return cluster_centers, cluster_len
# we need to choose number of clusters so that, there are more number of cluster regions
#that are close to any cluster center
# and make sure that the minimum inter cluster should not be very less
for increment in range(10, 100, 10):
    cluster_centers, cluster_len = find clusters(increment)
    find min distance (cluster centers, cluster len)
4
                                                                                                - N →
On choosing a cluster size of 10
Avg. Number of Clusters within the vicinity (i.e. intercluster-distance < 2): 2.0
Avg. Number of Clusters outside the vicinity (i.e. intercluster-distance > 2): 8.0
Min inter-cluster distance = 1.0945442325142543
On choosing a cluster size of 20
Avg. Number of Clusters within the vicinity (i.e. intercluster-distance < 2): 4.0
Avg. Number of Clusters outside the vicinity (i.e. intercluster-distance > 2): 16.0
Min inter-cluster distance = 0.7131298007387813
On choosing a cluster size of 30
Avg. Number of Clusters within the vicinity (i.e. intercluster-distance < 2): 8.0
Avg. Number of Clusters outside the vicinity (i.e. intercluster-distance > 2): 22.0
Min inter-cluster distance = 0.5185088176172206
On choosing a cluster size of 40
Avg. Number of Clusters within the vicinity (i.e. intercluster-distance < 2): 8.0
Avg. Number of Clusters outside the vicinity (i.e. intercluster-distance > 2): 32.0
Min inter-cluster distance = 0.5069768450363973
On choosing a cluster size of 50
Avg. Number of Clusters within the vicinity (i.e. intercluster-distance < 2): 12.0
Avg. Number of Clusters outside the vicinity (i.e. intercluster-distance > 2): 38.0
Min inter-cluster distance = 0.365363025983595
```

```
Un choosing a cluster size of 60
Avg. Number of Clusters within the vicinity (i.e. intercluster-distance < 2): 14.0
Avg. Number of Clusters outside the vicinity (i.e. intercluster-distance > 2): 46.0
Min inter-cluster distance = 0.34704283494187155
On choosing a cluster size of 70
Avg. Number of Clusters within the vicinity (i.e. intercluster-distance < 2): 16.0
Avg. Number of Clusters outside the vicinity (i.e. intercluster-distance > 2): 54.0
Min inter-cluster distance = 0.30502203163244707
On choosing a cluster size of 80
Avg. Number of Clusters within the vicinity (i.e. intercluster-distance < 2): 18.0
Avg. Number of Clusters outside the vicinity (i.e. intercluster-distance > 2): 62.0
Min inter-cluster distance = 0.29220324531738534
On choosing a cluster size of 90
Avg. Number of Clusters within the vicinity (i.e. intercluster-distance < 2): 21.0
Avg. Number of Clusters outside the vicinity (i.e. intercluster-distance > 2): 69.0
Min inter-cluster distance = 0.18257992857034985
```

Inference:

 The main objective was to find a optimal min. distance(Which roughly estimates to the radius of a cluster) between the clusters which we got was 40

```
In [40]:
```

```
# if check for the 50 clusters you can observe that there are two clusters with only 0.3 miles apa
rt from each other
# so we choose 40 clusters for solve the further problem

# Getting 40 clusters using the kmeans
kmeans = MiniBatchKMeans(n_clusters=40, batch_size=10000,random_state=0).fit(coords)
frame_with_durations_outliers_removed['pickup_cluster'] =
kmeans.predict(frame_with_durations_outliers_removed[['pickup_latitude', 'pickup_longitude']])
```

Plotting the cluster centers:

```
In [41]:
```

```
# Plotting the cluster centers on OSM
cluster_centers = kmeans.cluster_centers_
cluster_len = len(cluster_centers)
map_osm = folium.Map(location=[40.734695, -73.990372], tiles='Stamen Toner')
for i in range(cluster_len):
    folium.Marker(list((cluster_centers[i][0],cluster_centers[i][1])), popup=(str(cluster_centers[i][0])+str(cluster_centers[i][1]))).add_to(map_osm)
map_osm
```

Out[41]:

Plotting the clusters:

In [42]:

Time-binning

In [43]:

```
#Refer:https://www.unixtimestamp.com/
# 1420070400 : 2015-01-01 00:00:00
# 1422748800 : 2015-02-01 00:00:00
# 1425168000 : 2015-03-01 00:00:00
# 1427846400 : 2015-04-01 00:00:00
# 1430438400 : 2015-05-01 00:00:00
# 1433116800 : 2015-06-01 00:00:00
# 1451606400 : 2016-01-01 00:00:00
# 1454284800 : 2016-02-01 00:00:00
# 1456790400 : 2016-03-01 00:00:00
# 1459468800 : 2016-04-01 00:00:00
# 1462060800 : 2016-05-01 00:00:00
# 1464739200 : 2016-06-01 00:00:00
def add_pickup_bins(frame, month, year):
   unix_pickup_times=[i for i in frame['pickup_times'].values]
   unix times = [[1420070400,1422748800,1425168000,1427846400,1430438400,1433116800],\
                    [1451606400,1454284800,1456790400,1459468800,1462060800,1464739200]]
   start_pickup_unix=unix_times[year-2015][month-1]
    # https://www.timeanddate.com/time/zones/est
    \# (int((i-start_pickup_unix)/600)+33) : our unix time is in gmt to we are converting it to est
   tenminutewise_binned_unix_pickup_times=[(int((i-start_pickup_unix)/600)+33) for i in unix_picku
p times]
   frame['pickup bins'] = np.array(tenminutewise binned unix pickup times)
   return frame
```

In [44]:

```
# clustering, making pickup bins and grouping by pickup cluster and pickup bins
frame_with_durations_outliers_removed['pickup_cluster'] =
kmeans.predict(frame_with_durations_outliers_removed[['pickup_latitude', 'pickup_longitude']])
jan_2015_frame = add_pickup_bins(frame_with_durations_outliers_removed,1,2015)
jan_2015_groupby =
jan_2015_frame[['pickup_cluster','pickup_bins','trip_distance']].groupby(['pickup_cluster','pickup_bins']).count()
```

In [45]:

```
# we add two more columns 'pickup_cluster'(to which cluster it belogns to)
# and 'pickup_bins' (to which 10min intravel the trip belongs to)
jan_2015_frame.head()
```

Out[45]:

	passenger_count	trip_distance	pickup_longitude	pickup_latitude	dropoff_longitude	dropoff_latitude	total_amount	tri
0	1	1.59	-73.993896	40.750111	-73.974785	40.750618	17.05	18
1	1	3.30	-74.001648	40.724243	-73.994415	40.759109	17.80	19
2	1	1.80	-73.963341	40.802788	-73.951820	40.824413	10.80	10
3	1	0.50	-74.009087	40.713818	-74.004326	40.719986	4.80	1.
4	1	3.00	-73.971176	40.762428	-74.004181	40.742653	16.30	19
4								F

In [46]:

```
# hear the trip_distance represents the number of pickups that are happend in that particular 10mi
n intravel
# this data frame has two indices
# primary index: pickup_cluster (cluster number)
# secondary index: pickup_bins (we devid whole months time into 10min intravels 24*31*60/10 =4464
bins)
jan_2015_groupby.head()
```

Out[46]:

		trip_distance
pickup_cluster	pickup_bins	
0	63	104
	64	200
	65	208
	66	141
	67	155

In [47]:

```
# upto now we cleaned data and prepared data for the month 2015,
# now do the same operations for months Jan, Feb, March of 2016
# 1. get the dataframe which inloudes only required colums
# 2. adding trip times, speed, unix time stamp of pickup_time
# 4. remove the outliers based on trip times, speed, trip duration, total amount
# 5. add pickup_cluster to each data point
# 6. add pickup bin (index of 10min intravel to which that trip belongs to)
# 7. group by data, based on 'pickup cluster' and 'pickuo bin'
# Data Preparation for the months of Jan, Feb and March 2016
def datapreparation(month, kmeans, month no, year no):
    print ("Return with trip times..")
    frame with durations = return with trip times(month)
    print ("Remove outliers..")
    frame with durations outliers removed = remove outliers (frame with durations)
    print ("Estimating clusters..")
    frame with durations outliers removed['pickup cluster'] =
kmeans.predict(frame_with_durations_outliers_removed[['pickup_latitude', 'pickup_longitude']])
    #frame with durations outliers removed 2016['pickup cluster'] =
kmeans.predict(frame with durations outliers removed 2016[['pickup latitude',
```

```
'pickup longitude']])
    print ("Final groupbying..")
    final updated frame = add pickup bins(frame with durations outliers removed, month no, year no)
    final_groupby_frame = final_updated_frame[['pickup_cluster','pickup_bins','trip_distance']].grc
upby(['pickup_cluster','pickup_bins']).count()
    return final updated frame, final groupby frame
month jan 2016 = dd.read csv('yellow tripdata 2016-01.csv')
month feb 2016 = dd.read csv('yellow tripdata 2016-02.csv')
month mar 2016 = dd.read csv('yellow tripdata 2016-03.csv')
jan_2016_frame,jan_2016_groupby = datapreparation(month_jan_2016,kmeans,1,2016)
feb_2016_frame,feb_2016_groupby = datapreparation(month_feb_2016,kmeans,2,2016)
mar_2016_frame,mar_2016_groupby = datapreparation(month_mar_2016,kmeans,3,2016)
Return with trip times..
Remove outliers..
Number of pickup records = 10906858
Number of outlier coordinates lying outside NY boundaries: 214677
Number of outliers from trip times analysis: 27190
Number of outliers from trip distance analysis: 79742
Number of outliers from speed analysis: 21047
Number of outliers from fare analysis: 4991
Total outliers removed 297784
Estimating clusters..
Final groupbying ...
Return with trip times..
Remove outliers..
Number of pickup records = 11382049
Number of outlier coordinates lying outside NY boundaries: 223161
Number of outliers from trip times analysis: 27670
Number of outliers from trip distance analysis: 81902
Number of outliers from speed analysis: 22437
Number of outliers from fare analysis: 5476
Total outliers removed 308177
Estimating clusters..
Final groupbying..
Return with trip times..
Remove outliers..
Number of pickup records = 12210952
Number of outlier coordinates lying outside NY boundaries: 232444
Number of outliers from trip times analysis: 30877
Number of outliers from trip distance analysis: 87318
Number of outliers from speed analysis: 23898
Number of outliers from fare analysis: 5859
Total outliers removed 324646
Estimating clusters..
Final groupbying..
```

Smoothing

```
In [48]:
```

```
# Gets the unique bins where pickup values are present for each each reigion

# for each cluster region we will collect all the indices of 10min intravels in which the pickups are happened

# we got an observation that there are some pickpbins that doesnt have any pickups

def return_unq_pickup_bins(frame):
    values = []
    for i in range(0,40):
        new = frame[frame['pickup_cluster'] == i]
        list_unq = list(set(new['pickup_bins']))
        list_unq.sort()
        values.append(list_unq)
    return values
```

In [49]:

```
# for every month we get all indices of 10min intravels in which atleast one pickup got happened
#jan
jan_2015_unique = return_unq_pickup_bins(jan_2015_frame)
jan_2016_unique = return_unq_pickup_bins(jan_2016_frame)

#feb
feb_2016_unique = return_unq_pickup_bins(feb_2016_frame)

#march
mar_2016_unique = return_unq_pickup_bins(mar_2016_frame)
```

```
In [50]:
# for each cluster number of 10min intravels with 0 pickups
for i in range (40):
  print("for the ",i,"th cluster number of 10min intavels with zero pickups: ",4464 -
len(set(jan 2015 unique[i])))
  print('-'*60)
for the 0 th cluster number of 10min intavels with zero pickups:
for the 1 th cluster number of 10min intavels with zero pickups:
_____
for the 2 th cluster number of 10min intavels with zero pickups:
for the 3 th cluster number of 10min intavels with zero pickups:
for the 4 th cluster number of 10min intavels with zero pickups:
for the 5 th cluster number of 10min intavels with zero pickups:
for the 6 th cluster number of 10min intavels with zero pickups:
_____
for the 7 th cluster number of 10min intavels with zero pickups:
for the 8 th cluster number of 10min intavels with zero pickups:
      _____
for the 9 th cluster number of 10min intavels with zero pickups:
   ._____
for the 10 th cluster number of 10min intavels with zero pickups:
for the 11 th cluster number of 10min intavels with zero pickups:
for the 12 th cluster number of 10min intavels with zero pickups:
_____
for the 13 th cluster number of 10min intavels with zero pickups:
      ______
for the 14 th cluster number of 10min intavels with zero pickups:
for the 15 th cluster number of 10min intavels with zero pickups:
for the 16 th cluster number of 10min intavels with zero pickups:
      _____
for the 17 th cluster number of 10min intavels with zero pickups:
______
for the 18 th cluster number of 10min intavels with zero pickups:
for the 19 th cluster number of 10min intavels with zero pickups:
      ______
for the 20 th cluster number of 10min intavels with zero pickups:
_____
```

for the 21 th cluster number of 10min intavels with zero pickups:

for the 22 th cluster number of 10min intavels with zero pickups:

for the 23 th cluster number of 10min intavels with zero pickups:

for the 24 th cluster number of 10min intavels with zero pickups:

for the 25 th cluster number of 10min intavels with zero pickups:

for the 26 th cluster number of 10min intavels with zero pickups:

```
for the 27 th cluster number of 10min intavels with zero pickups: 214
for the 28 th cluster number of 10min intavels with zero pickups:
for the 29 th cluster number of 10min intavels with zero pickups: 41
_____
for the 30 th cluster number of 10min intavels with zero pickups: 1180
     _____
for the 31 th cluster number of 10min intavels with zero pickups:
   _____
for the 32 th cluster number of 10min intavels with zero pickups:
for the 33 th cluster number of 10min intavels with zero pickups:
     _____
for the 34 th cluster number of 10min intavels with zero pickups:
______
for the \, 35 th cluster number of 10min intavels with zero pickups:
for the 36 th cluster number of 10min intavels with zero pickups:
      ______
for the 37 th cluster number of 10min intavels with zero pickups:
   _____
for the 38 th cluster number of 10min intavels with zero pickups: 36
  ______
for the 39 th cluster number of 10min intavels with zero pickups: 43
```

there are two ways to fill up these values

- Fill the missing value with 0's
- Fill the missing values with the avg values
 - Case 1:(values missing at the start)
 Ex1: _ _ \ x => ceil(x/4), ceil(x/4), ceil(x/4), ceil(x/4)
 Ex2: \ x => ceil(x/3), ceil(x/3), ceil(x/3)
 - Case 2:(values missing in middle)
 - Ex1: $x \setminus y = ceil((x+y)/4)$, ceil((x+y)/4), ceil((x+y)/4), ceil((x+y)/4)
 - Case 3:(values missing at the end)
 - Ex1: $x \setminus _ = \operatorname{ceil}(x/4)$, $\operatorname{ceil}(x/4)$, $\operatorname{ceil}(x/4)$, $\operatorname{ceil}(x/4)$ Ex2: $x \setminus = \operatorname{ceil}(x/2)$, $\operatorname{ceil}(x/2)$

In [51]:

```
# Fills a value of zero for every bin where no pickup data is present
# the count values: number pickps that are happened in each region for each 10min intravel
# there wont be any value if there are no picksups.
# values: number of unique bins
# for every 10min intravel(pickup bin) we will check it is there in our unique bin,
# if it is there we will add the count values[index] to smoothed data
# if not we add 0 to the smoothed data
 we finally return smoothed data
def fill missing(count values, values):
   smoothed_regions=[]
   ind=0
   for r in range (0,40):
       smoothed bins=[]
        for i in range (4464):
            if i in values[r]:
               smoothed bins.append(count values[ind])
            else:
                smoothed bins.append(0)
       smoothed regions.extend(smoothed bins)
   return smoothed_regions
```

In [52]:

```
# Fills a value of zero for every bin where no pickup data is present
# the count_values: number pickps that are happened in each region for each 10min intravel
# there wont be any value if there are no picksups.
# values: number of unique bins
```

```
# for every 10min intravel(pickup bin) we will check it is there in our unique bin,
# if it is there we will add the count_values[index] to smoothed data
# if not we add smoothed data (which is calculated based on the methods that are discussed in the
above markdown cell)
# we finally return smoothed data
def smoothing(count values, values):
   smoothed regions=[] # stores list of final smoothed values of each reigion
   ind=0
   repeat=0
   smoothed value=0
   for r in range (0,40):
        smoothed bins=[] #stores the final smoothed values
       repeat=0
       for i in range (4464):
            if repeat!=0: # prevents iteration for a value which is already visited/resolved
               repeat-=1
                continue
            if i in values[r]: #checks if the pickup-bin exists
               smoothed bins.append(count values[ind]) # appends the value of the pickup bin if it
exists
            else:
                if i!=0:
                    right hand limit=0
                    for j in range(i,4464):
                        if j not in values[r]: #searches for the left-limit or the pickup-bin
value which has a pickup value
                            continue
                        else:
                            right hand limit=j
                            break
                    if right hand limit==0:
                    #Case 1: When we have the last/last few values are found to be missing, hence we
have no right-limit here
                        smoothed value=count values[ind-1]*1.0/((4463-i)+2)*1.0
                        for j in range(i,4464):
                            smoothed bins.append(math.ceil(smoothed value))
                        smoothed_bins[i-1] = math.ceil(smoothed_value)
                        repeat=(4463-i)
                        ind-=1
                    else:
                    #Case 2: When we have the missing values between two known values
                        smoothed value=(count values[ind-1]+count values[ind])*1.0/((right hand lim
t-i)+2)*1.0
                        for j in range(i, right hand limit+1):
                            smoothed_bins.append(math.ceil(smoothed value))
                        smoothed bins[i-1] = math.ceil(smoothed value)
                        repeat=(right_hand_limit-i)
                else:
                    #Case 3: When we have the first/first few values are found to be missing, hence
we have no left-limit here
                    right hand limit=0
                    for j in range(i,4464):
                        if j not in values[r]:
                            continue
                        else:
                            right hand limit=j
                            break
                    smoothed value=count values[ind]*1.0/((right hand limit-i)+1)*1.0
                    for j in range(i, right hand limit+1):
                            smoothed bins.append(math.ceil(smoothed value))
                    repeat=(right_hand_limit-i)
            ind+=1
       smoothed regions.extend(smoothed bins)
   return smoothed regions
                                                                                                ▶
4
```

In [53]:

```
#Filling Missing values of Jan-2015 with 0
# here in jan_2015_groupby dataframe the trip_distance represents the number of pickups that are h
appened
jan_2015_fill = fill_missing(jan_2015_groupby['trip_distance'].values,jan_2015_unique)

#Smoothing Missing values of Jan-2015
jan_2015_smooth = smoothing(jan_2015_groupby['trip_distance'].values,jan_2015_unique)
```

In [54]:

```
# number of 10min indices for jan 2015= 24*31*60/10 = 4464
# number of 10min indices for jan 2016 = 24*31*60/10 = 4464
# number of 10min indices for feb 2016 = 24*29*60/10 = 4176
# number of 10min indices for march 2016 = 24*30*60/10 = 4320
# for each cluster we will have 4464 values, therefore 40*4464 = 178560 (length of the jan_2015_fill)
print("number of 10min intravels among all the clusters ",len(jan_2015_fill))
```

number of 10min intravels among all the clusters 178560

In [55]:

```
# Smoothing vs Filling
# sample plot that shows two variations of filling missing values
# we have taken the number of pickups for cluster region 2
plt.figure(figsize=(10,5))
plt.plot(jan_2015_fill[4464:8920], label="zero filled values")
plt.plot(jan_2015_smooth[4464:8920], label="filled with avg values")
plt.legend()
plt.show()
```

In [56]:

```
# why we choose, these methods and which method is used for which data?

# Ans: consider we have data of some month in 2015 jan 1st, 10 _ _ _ 20, i.e there are 10 pickups that are happened in 1st
# 10st 10min intravel, 0 pickups happened in 2nd 10mins intravel, 0 pickups happened in 3rd 10min intravel
# and 20 pickups happened in 4th 10min intravel.
# in fill_missing method we replace these values like 10, 0, 0, 20
# where as in smoothing method we replace these values as 6,6,6,6,6 if you can check the number of pickups
# that are happened in the first 40min are same in both cases, but if you can observe that we look ing at the future values
# wheen you are using smoothing we are looking at the future number of pickups which might cause a data leakage.

# so we use smoothing for jan 2015th data since it acts as our training data
# and we use simple fill_misssing method for 2016th data.
```

In [57]:

```
# Jan-2015 data is smoothed, Jan, Feb & March 2016 data missing values are filled with zero
jan_2015_smooth = smoothing(jan_2015_groupby['trip_distance'].values,jan_2015_unique)
jan_2016_smooth = fill_missing(jan_2016_groupby['trip_distance'].values,jan_2016_unique)
feb_2016_smooth = fill_missing(feb_2016_groupby['trip_distance'].values,feb_2016_unique)
mar 2016 smooth = fill missing(mar 2016 groupby['trip distance'].values,mar 2016 unique)
# Making list of all the values of pickup data in every bin for a period of 3 months and storing t
hem region-wise
regions cum = []
\# a = [1, 2, 3]
# b = [2,3,4]
\# a+b = [1, 2, 3, 2, 3, 4]
# number of 10min indices for jan 2015= 24*31*60/10 = 4464
# number of 10min indices for jan 2016 = 24*31*60/10 = 4464
# number of 10min indices for feb 2016 = 24*29*60/10 = 4176
# number of 10min indices for march 2016 = 24*31*60/10 = 4464
# regions cum: it will contain 40 lists, each list will contain 4464+4176+4464 values which repres
ents the number of pickups
# that are happened for three months in 2016 data
for i in range (0,40):
    regions cum.append(jan 2016 smooth[4464*i:4464*(i+1)]+feb 2016 smooth[4176*i:4176*(i+1)]+mar 20
16 smooth [4464*i:4464*(i+1)])
```

```
# print(len(regions_cum))
# 40
# print(len(regions_cum[0]))
# 13104
```

Time series and Fourier Transforms

In [58]:

```
def uniqueish_color():
    """There're better ways to generate unique colors, but this isn't awful."""
    return plt.cm.gist_ncar(np.random.random())
first_x = list(range(0,4464))
second_x = list(range(4464,8640))
third_x = list(range(8640,13104))
for i in range(40):
    plt.figure(figsize=(10,4))
    plt.plot(first_x,regions_cum[i][:4464], color=uniqueish_color(), label='2016 Jan month data')
    plt.plot(second_x,regions_cum[i][4464:8640], color=uniqueish_color(), label='2016 feb month data')
    plt.plot(third_x,regions_cum[i][8640:], color=uniqueish_color(), label='2016 march month data')
    plt.legend()
    plt.show()
```

```
# getting peaks: https://blog.ytotech.com/2015/11/01/findpeaks-in-python/
# read more about fft function :
https://docs.scipy.org/doc/numpy/reference/generated/numpy.fft.fft.html
Y = np.fft.fft(np.array(jan_2016_smooth)[0:4460])
# read more about the fftfreq:
https://docs.scipy.org/doc/numpy/reference/generated/numpy.fft.fftfreq.html
freq = np.fft.fftfreq(4460, 1)
n = len(freq)
plt.figure()
plt.plot( freq[:int(n/2)], np.abs(Y)[:int(n/2)] )
plt.xlabel("Frequency")
plt.ylabel("Amplitude")
plt.show()
```

In [60]:

```
#Preparing the Dataframe only with x(i) values as jan-2015 data and y(i) values as jan-2016
ratios_jan = pd.DataFrame()
ratios_jan['Given']=jan_2015_smooth
ratios_jan['Prediction']=jan_2016_smooth
ratios_jan['Ratios']=ratios_jan['Prediction']*1.0/ratios_jan['Given']*1.0
```

Modelling: Baseline Models

Now we get into modelling in order to forecast the pickup densities for the months of Jan, Feb and March of 2016 for which we are using multiple models with two variations

- 1. Using Ratios of the 2016 data to the 2015 data i.e $R_t = P_t^{2016}/P_t^{2015}$
- 2. Using Previous known values of the 2016 data itself to predict the future values

Simple Moving Averages

The First Model used is the Moving Averages Model which uses the previous n values in order to predict the next value

Using Ratio Values - $R_t = (R_{t-1} + R_{t-2} + R_{t-3} \dots R_{t-n})/n$

```
In [61]:
```

```
def MA R Predictions(ratios, month):
    predicted ratio=(ratios['Ratios'].values)[0]
   error=[]
   predicted_values=[]
    window size=3
    predicted ratio values=[]
    for i in range(0,4464*40):
        if i%4464==0:
            predicted ratio values.append(0)
            predicted_values.append(0)
            error.append(0)
            continue
        predicted ratio values.append(predicted ratio)
        predicted values.append(int(((ratios['Given'].values)[i])*predicted ratio))
        error.append(abs((math.pow(int(((ratios['Given'].values)[i])*predicted ratio)-(ratios['Pred
iction'].values)[i],1))))
       if i+1>=window size:
            predicted_ratio=sum((ratios['Ratios'].values)[(i+1)-window_size:(i+1)])/window_size
            predicted ratio=sum((ratios['Ratios'].values)[0:(i+1)])/(i+1)
    ratios['MA R Predicted'] = predicted values
    ratios['MA R Error'] = error
   mape err = (sum(error)/len(error))/(sum(ratios['Prediction'].values)/len(ratios['Prediction'].values)/
alues))
   mse err = sum([e^{**2} for e in error])/len(error)
    return ratios, mape err, mse err
```

for getting the best results using Moving Averages using previous Ratio values therefore we get $R_t = (R_{t-1} + R_{t-2} + R_{t-3})/3$

Next we use the Moving averages of the 2016 values itself to predict the future value using $P_t = (P_{t-1} + P_{t-2} + P_{t-3}, \dots, P_{t-n})/n$

In [62]:

```
def MA P Predictions(ratios, month):
    predicted value=(ratios['Prediction'].values)[0]
    predicted values=[]
   window size=1
    predicted ratio values=[]
    for i in range(0,4464*40):
        predicted values.append(predicted value)
        error.append(abs((math.pow(predicted_value-(ratios['Prediction'].values)[i],1))))
        if i+1>=window size:
            predicted value=int(sum((ratios['Prediction'].values)[(i+1)-window size:
(i+1)])/window size)
        else:
            predicted value=int(sum((ratios['Prediction'].values)[0:(i+1)])/(i+1))
    ratios['MA P Predicted'] = predicted values
    ratios['MA P Error'] = error
    mape_err = (sum(error)/len(error))/(sum(ratios['Prediction'].values)/len(ratios['Prediction'].values)
alues))
   mse err = sum([e^{**2} for e in error])/len(error)
    return ratios,mape_err,mse_err
```

For the above the Hyperparameter is the window-size (n) which is tuned manually and it is found that the window-size of 1 is optimal for getting the best results using Moving Averages using previous 2016 values therefore we get $P_t = P_{t-1}$

Weighted Moving Averages

The Moving Avergaes Model used gave equal importance to all the values in the window used, but we know intuitively that the future is more likely to be similar to the latest values and less similar to the older values. Weighted Averages converts this analogy into a mathematical relationship giving the highest weight while computing the averages to the latest previous value and decreasing weights to the subsequent older ones

Weighted Moving Averages using Ratio Values - $R_t = (N * R_{t-1} + (N-1) * R_{t-2} + (N-2) * R_{t-3} + \dots 1 * R_{t-n})/(N * (N+1)/2)$

In [63]:

```
def WA R Predictions(ratios, month):
    predicted ratio=(ratios['Ratios'].values)[0]
   alpha=0.5
   error=[]
   predicted values=[]
    window_size=5
    predicted ratio values=[]
    for i in range (0,4464*40):
        if i%4464==0:
            predicted ratio values.append(0)
            predicted_values.append(0)
            error.append(0)
        predicted ratio values.append(predicted ratio)
        predicted values.append(int(((ratios['Given'].values)[i])*predicted ratio))
        error.append(abs((math.pow(int(((ratios['Given'].values)[i])*predicted ratio)-(ratios['Pred
iction'].values)[i],1))))
        if i+1>=window size:
            sum values=0
            sum of coeff=0
            for j in range(window size, 0, -1):
                sum_values += j*(ratios['Ratios'].values)[i-window_size+j]
                sum of coeff+=j
            predicted_ratio=sum_values/sum_of_coeff
        else:
            sum values=0
            sum of coeff=0
            for j in range(i+1,0,-1):
                sum values += j*(ratios['Ratios'].values)[j-1]
```

```
sum_of_coeff+=j
    predicted_ratio=sum_values/sum_of_coeff

ratios['WA_R_Predicted'] = predicted_values
    ratios['WA_R_Error'] = error
    mape_err = (sum(error))/len(error))/(sum(ratios['Prediction'].values)/len(ratios['Prediction'].v
alues))
    mse_err = sum([e**2 for e in error])/len(error)
    return ratios,mape_err,mse_err
```

For the above the Hyperparameter is the window-size (n) which is tuned manually and it is found that the window-size of 5 is optimal for getting the best results using Weighted Moving Averages using previous Ratio values therefore we get $R_t = (5*R_{t-1} + 4*R_{t-2} + 3*R_{t-3} + 2*R_{t-4} + R_{t-5})/15$

Weighted Moving Averages using Previous 2016 Values - $P_t = (N*P_{t-1} + (N-1)*P_{t-2} + (N-2)*P_{t-3}....1*P_{t-n})/(N*(N+1)/2)$

In [64]:

```
def WA P Predictions(ratios, month):
    predicted_value=(ratios['Prediction'].values)[0]
   error=[]
   predicted values=[]
    window size=2
    for i in range(0,4464*40):
        predicted values.append(predicted value)
        error.append(abs((math.pow(predicted value-(ratios['Prediction'].values)[i],1))))
        if i+1>=window size:
            sum values=0
            sum of coeff=0
            for j in range(window size, 0, -1):
                sum_values += j*(ratios['Prediction'].values)[i-window_size+j]
                sum of coeff+=j
            predicted value=int(sum values/sum of coeff)
        else:
            sum values=0
            sum of coeff=0
            for j in range (i+1,0,-1):
                sum_values += j*(ratios['Prediction'].values)[j-1]
                sum of coeff+=j
            predicted value=int(sum values/sum of coeff)
    ratios['WA P Predicted'] = predicted values
    ratios['WA P Error'] = error
    mape err = (sum(error)/len(error))/(sum(ratios['Prediction'].values)/len(ratios['Prediction'].values)
    mse err = sum([e**2 for e in error])/len(error)
    return ratios, mape err, mse err
```

For the above the Hyperparameter is the window-size (n) which is tuned manually and it is found that the window-size of 2 is optimal for getting the best results using Weighted Moving Averages using previous 2016 values therefore we get $P_t = (2 * P_{t-1} + P_{t-2})/3$

Exponential Weighted Moving Averages

https://en.wikipedia.org/wiki/Moving_average#Exponential_moving_average. Through weighted averaged we have satisfied the analogy of giving higher weights to the latest value and decreasing weights to the subsequent ones but we still do not know which is the correct weighting scheme as there are infinetly many possibilities in which we can assign weights in a non-increasing order and tune the hyperparameter window-size. To simplify this process we use Exponential Moving Averages which is a more logical way towards assigning weights and at the same time also using an optimal window-size.

In exponential moving averages we use a single hyperparameter alpha (a) which is a value between 0 & 1 and based on the value of the hyperparameter alpha the weights and the window sizes are configured.

For eg. If $\alpha=0.9$ then the number of days on which the value of the current iteration is based is~ $1/(1-\alpha)=10$ i.e. we consider values 10 days prior before we predict the value for the current iteration. Also the weights are assigned using 2/(N+1)=0.18, where N = number of prior values being considered, hence from this it is implied that the first or latest value is assigned a weight of 0.18 which keeps exponentially decreasing for the subsequent values.

$$R_{t}^{'} = \alpha * R_{t-1} + (1-\alpha) * R_{t-1}^{'}$$

```
In [65]:
```

```
def EA R1 Predictions(ratios, month):
   predicted ratio=(ratios['Ratios'].values)[0]
   alpha=0.6
   error=[]
   predicted values=[]
    predicted ratio values=[]
    for i in range (0,4464*40):
        if i%4464==0:
           predicted_ratio_values.append(0)
            predicted_values.append(0)
            error.append(0)
            continue
        predicted ratio values.append(predicted ratio)
       predicted values.append(int(((ratios['Given'].values)[i])*predicted ratio))
        error.append(abs((math.pow(int(((ratios['Given'].values)[i])*predicted ratio)-(ratios['Pred
iction'].values)[i],1))))
        predicted ratio = (alpha*predicted ratio) + (1-alpha)*((ratios['Ratios'].values)[i])
    ratios['EA R1 Predicted'] = predicted values
    ratios['EA R1 Error'] = error
   mape err = (sum(error)/len(error)) / (sum(ratios['Prediction'].values)/len(ratios['Prediction'].v
alues))
   mse err = sum([e^{**2} for e in error])/len(error)
   return ratios, mape err, mse err
```

```
P_{t}^{'} = \alpha * P_{t-1} + (1 - \alpha) * P_{t-1}^{'}
```

In [66]:

```
def EA P1_Predictions(ratios, month):
   predicted value= (ratios['Prediction'].values)[0]
    alpha=0.3
    error=[]
    predicted values=[]
    for i in range(0,4464*40):
       if i%4464==0:
            predicted values.append(0)
            error.append(0)
            continue
        predicted values.append(predicted value)
        error.append(abs((math.pow(predicted_value-(ratios['Prediction'].values)[i],1))))
        predicted_value =int((alpha*predicted_value) + (1-alpha)*((ratios['Prediction'].values)[i])
    ratios['EA P1 Predicted'] = predicted values
    ratios['EA P1 Error'] = error
    mape err = (sum(error))/len(error))/(sum(ratios['Prediction'].values)/len(ratios['Prediction'].values)
alues))
    mse err = sum([e^{**2} for e in error])/len(error)
    return ratios,mape_err,mse_err
```

In [92]:

```
mean_err=[0]*10
median_err=[0]*10
ratios_jan,mean_err[0],median_err[0]=MA_R_Predictions(ratios_jan,'jan')
ratios_jan,mean_err[1],median_err[1]=MA_P_Predictions(ratios_jan,'jan')
ratios_jan,mean_err[2],median_err[2]=WA_R_Predictions(ratios_jan,'jan')
ratios_jan,mean_err[3],median_err[3]=WA_P_Predictions(ratios_jan,'jan')
ratios_jan,mean_err[4],median_err[4]=EA_R1_Predictions(ratios_jan,'jan')
ratios_jan,mean_err[5],median_err[5]=EA_P1_Predictions(ratios_jan,'jan')
```

Comparison between baseline models

We have chosen our error metric for comparison between models as **MAPE** (**Mean Absolute Percentage Error**) so that we can know that on an average how good is our model with predictions and **MSE** (**Mean Squared Error**) is also used so that we have a clearer understanding as to how well our forecasting model performs with outliers so that we make sure that there is not much of a error margin between our prediction and the actual value

orror margin bottroom our production and the detadi raide

```
In [93]:
```

```
print ("Error Metric Matrix (Forecasting Methods) - MAPE & MSE")
print ("Moving Averages (Ratios) -
                                                    MAPE: ",mean err[0],"
                                                                          MSE: ",me
ian err[0])
                                                    MAPE: ",mean_err[1],"
print ("Moving Averages (2016 Values) -
                                                                          MSE: ", m
dian err[1])
print ("--
----")
print ("Weighted Moving Averages (Ratios) -
                                                    MAPE: ", mean err[2],"
                                                                         MSE: ",me
dian err[2])
print ("Weighted Moving Averages (2016 Values) -
                                                   MAPE: ",mean err[3],"
                                                                         MSE: ",me
dian err[3])
print ("----
print ("Exponential Moving Averages (Ratios) -
                                                 MAPE: ", mean err[4],"
                                                                       MSE: ", media
                                                MAPE: ",mean_err[5],"
print ("Exponential Moving Averages (2016 Values) -
                                                                       MSE: ", media
n err[5])
                                                                   Error Metric Matrix (Forecasting Methods) - MAPE & MSE
Moving Averages (Ratios) -
                                             MAPE: 0.26950909032781617
                                                                        MSE: 2480.
87830421147
                                             MAPE: 0.16378795278148583 MSE: 317.
Moving Averages (2016 Values) -
3727038530463
______
                                             MAPE: 0.27330740755134025
Weighted Moving Averages (Ratios) -
                                                                        MSE:
2057.5075100806453
                                            MAPE: 0.1556721390435653
Weighted Moving Averages (2016 Values) -
                                                                       MSE:
274.4154289874552
______
Exponential Moving Averages (Ratios) -
                                          MAPE: 0.2765065244870951 MSE:
2145.912539202509
Exponential Moving Averages (2016 Values) - MAPE: 0.15512682849413695 MSE: 271.4585
7974910393
4
```

Plese Note:- The above comparisons are made using Jan 2015 and Jan 2016 only

From the above matrix it is inferred that the best forecasting model for our prediction would be:- $P_{t}^{'} = \alpha * P_{t-1} + (1-\alpha) * P_{t-1}^{'}$ i.e Exponential Moving Averages using 2016 Values

Regression Models

Train-Test Split

Before we start predictions using the tree based regression models we take 3 months of 2016 pickup data and split it such that for every region we have 70% data in train and 30% in test, ordered date-wise for every region

Fourier Features:

```
In [94]:
```

```
# For each cluster from 0 to 39 i.e total clusters
# Fourier features dataframe - Stores fourier features for all clusters.
fourier_features = pd.DataFrame(['A1', 'A2', 'A3', 'A4', 'A5', 'F1', 'F2', 'F3', 'F4', 'F5'])
ans = []
for i in range(0,40):

# for each month calculate fft and get frequency
# regions cum hold data for each cluster in format jan,feb,mar. first 4464 values are for jan,
next 4176 values are for feb and rest are for march.
```

```
janiit_data = regions_cum[i][U:4464]
    febfft_data = regions_cum[i][4464:4464+4176]
    marfft data = regions cum[i][4464+4176: 4464+4176+4464]
    # calculate fft i.e Amplitude .....
    janfft_amp = np.fft.fft(janfft_data)
    janfft_freq = np.fft.fftfreq(4464, 1)
    febfft amp = np.fft.fft(febfft data)
    febfft_freq = np.fft.fftfreq(4176, 1)
    marfft amp = np.fft.fft(marfft data)
    marfft freq = np.fft.fftfreq(4464, 1)
    # Sort the amps and frequency and take only top 5 values..
    janfft amp = sorted(janfft amp, reverse = True)[:5]
    janfft freq = sorted(janfft freq, reverse = True)[:5]
    febfft amp = sorted(febfft amp, reverse = True)[:5]
    febfft freq = sorted(febfft freq, reverse = True)[:5]
    marfft amp = sorted(marfft amp, reverse = True)[:5]
   marfft_freq = sorted(marfft_freq, reverse = True)[:5]
    # Each Cluster contains 4464 values of jan , 4176 values of feb, 4464 values of march.
    # For eahc value of a month F1, A1 do not change sowe replicate these f1, a1 values as
follows;
   x = janfft amp
   y = febfft_amp
   z = marfft amp
    u = janfft freq
   v = febfft freq
    w = marfft freq
    for f in range (5):
        janfft amp[f] = [x[f]] * 4464
        febfft_amp[f] = [y[f]] * 4176
       marfft_amp[f] = [z[f]] * 4464
       janfft_freq[f] = [u[f]] * 4464
        febfft_freq[f] = [v[f]] * 4176
        marfft freq[f] = [w[f]] * 4464
    # Converting to numpy array and Transpose to get right dimension.
    janfft amp = np.array(janfft amp).T
    febfft_amp = np.array(febfft_amp).T
    marfft amp = np.array(marfft amp).T
    janfft freq = np.array(janfft freq).T
    febfft freq = np.array(febfft freq).T
    marfft_freq = np.array(marfft_freq).T
    # Joining amplitude and frequency of same month and combining different months together.
    jan clus = np.hstack((janfft amp, janfft freq))
    feb clus = np.hstack((febfft amp, febfft freq))
    mar_clus = np.hstack((marfft_amp, marfft_freq))
    clus = np.vstack((jan clus, feb clus))
    clus = np.vstack((clus, mar clus))
    #Cluster Frame stores the features for a single cluster
   cluster features = pd.DataFrame(clus, columns=['A1', 'A2', 'A3', 'A4', 'A5', 'F1', 'F2', 'F3',
'F4', 'F5'])
    cluster_features = cluster_features.astype(np.float)
    ans.append(cluster features)
# Combining 40 dataframes of fourier features belonging to each cluster into one dataframe
print(len(ans))
print(type(ans[0]))
fourier features = ans[0]
for i in range(1, len(ans)):
   fourier features = pd.concat([fourier features, ans[i]], ignore index=True)
fourier features = fourier features.fillna(0)
print("Shape of fourier transformed features for all points - ", fourier features.shape)
fourier features = fourier features.astype(np.float)
fourier features.tail(3)
```

```
40 <class 'pandas.core.frame.DataFrame'> Shape of fourier transformed features for all points - (524160, 10)
```

Out[94]:

	A1	A2	А3	A4	A5	F1	F2	F3	F4	F5
524157	277715.0	60810.898425	60810.898425	9679.827359	9679.827359	0.499776	0.499552	0.499328	0.499104	0.49888
524158	277715.0	60810.898425	60810.898425	9679.827359	9679.827359	0.499776	0.499552	0.499328	0.499104	0.49888
524159	277715.0	60810.898425	60810.898425	9679.827359	9679.827359	0.499776	0.499552	0.499328	0.499104	0.49888

In [95]:

```
# Preparing data to be split into train and test, The below prepares data in cumulative form which
will be later split into test and train
# number of 10min indices for jan 2015= 24*31*60/10 = 4464
# number of 10min indices for jan 2016 = 24*31*60/10 = 4464 # number of 10min indices for feb 2016 = 24*29*60/10 = 4176
# number of 10min indices for march 2016 = 24*31*60/10 = 4464
# regions cum: it will contain 40 lists, each list will contain 4464+4176+4464 values which repres
ents the number of pickups
# that are happened for three months in 2016 data
# print(len(regions_cum))
# 40
# print(len(regions cum[0]))
# 12960
# we take number of pickups that are happened in last 5 10min intravels
number of time stamps = 5
# output varaible
# it is list of lists
# it will contain number of pickups 13099 for each cluster
output = []
# tsne lat will contain 13104-5=13099 times lattitude of cluster center for every cluster
# Ex: [[cent lat 13099times],[cent lat 13099times], [cent lat 13099times].... 40 lists]
# it is list of lists
tsne_lat = []
# tsne lon will contain 13104-5=13099 times logitude of cluster center for every cluster
# Ex: [[cent long 13099times],[cent long 13099times], [cent long 13099times].... 40 lists]
# it is list of lists
tsne lon = []
# we will code each day
# sunday = 0, monday=1, tue = 2, wed=3, thur=4, fri=5, sat=6
# for every cluster we will be adding 13099 values, each value represent to which day of the week
that pickup bin belongs to
# it is list of lists
tsne weekday = []
# its an numbpy array, of shape (523960, 5)
# each row corresponds to an entry in out data
# for the first row we will have [f0,f1,f2,f3,f4] fi=number of pickups happened in i+1th 10min int
ravel (bin)
# the second row will have [f1,f2,f3,f4,f5]
# the third row will have [f2,f3,f4,f5,f6]
# and so on...
tsne feature = []
tsne feature = [0]*number of time stamps
for i in range (0,40):
    tsne_lat.append([kmeans.cluster_centers_[i][0]]*13099)
    tsne lon.append([kmeans.cluster centers [i][1]]*13099)
    \# jan 1st 2016 is thursday, so we start our day from 4: "(int(k/144))%7+4"
    \# our prediction start from 5th 10min intravel since we need to have number of pickups that ar
```

```
e happened in last 5 pickup bins
    tsne_weekday.append([int(((int(k/144))%7+4)%7) for k in range(5,4464+4176+4464)])
    # regions_cum is a list of lists [[x1,x2,x3..x13104], [x1,x2,x3..x13104], [x1,
```

In [96]:

Out[96]:

True

In [97]:

```
# Getting the predictions of exponential moving averages to be used as a feature in cumulative for
# upto now we computed 8 features for every data point that starts from 50th min of the day
# 1. cluster center lattitude
# 2. cluster center longitude
# 3. day of the week
# 4. f_t_1: number of pickups that are happened previous t-1th 10min intravel
# 5. f t 2: number of pickups that are happened previous t-2th 10min intravel
# 6. f t 3: number of pickups that are happened previous t-3th 10min intravel
# 7. f t 4: number of pickups that are happened previous t-4th 10min intravel
# 8. f t 5: number of pickups that are happened previous t-5th 10min intravel
# from the baseline models we said the exponential weighted moving avarage gives us the best error
# we will try to add the same exponential weighted moving avarage at t as a feature to our data
# exponential weighted moving avarage \Rightarrow p'(t) = alpha*p'(t-1) + (1-alpha)*P(t-1)
alpha=0.3
# it is a temporary array that store exponential weighted moving avarage for each 10min intravel,
# for each cluster it will get reset
# for every cluster it contains 13104 values
predicted values=[]
# it is similar like tsne lat
# it is list of lists
# predict list is a list of lists [[x5,x6,x7..x13104], [x5,x6,x7..x13104], [x5,x6,x7..x13104], [x5
,x6,x7..x13104], [x5,x6,x7..x13104], .. 40 lsits]
predict list = []
tsne_flat_exp_avg = []
for r in range (0,40):
    for i in range(0,13104):
        if i==0:
            predicted value= regions cum[r][0]
            predicted_values.append(0)
            continue
        predicted values.append(predicted value)
        predicted_value =int((alpha*predicted_value) + (1-alpha)*(regions_cum[r][i]))
    predict list.append(predicted values[5:])
    predicted values=[]
```

Holts Winter Triple exponential smoothing:

References - https://grisha.org/blog/2016/02/17/triple-exponential-smoothing-forecasting-part-iii/

In [98]:

```
def initial_trend(series, slen):
    sum = 0.0
    for i in range(slen):
        sum += float(series[i+slen] - series[i]) / slen
    return sum / slen
```

```
In [99]:
```

```
def initial_seasonal_components(series, slen):
    seasonals = {}
    season_averages = []
    n_seasons = int(len(series)/slen)
    # compute season averages
    for j in range(n_seasons):
        season_averages.append(sum(series[slen*j:slen*j+slen])/float(slen))
# compute initial values
for i in range(slen):
        sum_of_vals_over_avg = 0.0
        for j in range(n_seasons):
            sum_of_vals_over_avg += series[slen*j+i]-season_averages[j]
            seasonals[i] = sum_of_vals_over_avg/n_seasons
    return seasonals
```

In [100]:

```
def triple exponential smoothing(series, slen, alpha, beta, gamma, n preds):
    result = []
    seasonals = initial seasonal components(series, slen)
    for i in range(len(series)+n preds):
        if i == 0: # initial values
            smooth = series[0]
            trend = initial trend(series, slen)
            result.append(series[0])
            continue
        if i >= len(series): # we are forecasting
            m = i - len(series) + 1
            result.append((smooth + m*trend) + seasonals[i%slen])
        else:
            val = series[i]
            last smooth, smooth = smooth, alpha*(val-seasonals[i%slen]) + (1-alpha)*(smooth+trend)
            trend = beta * (smooth-last smooth) + (1-beta)*trend
            seasonals[i%slen] = gamma*(val-smooth) + (1-gamma)*seasonals[i%slen]
            result.append(smooth+trend+seasonals[i%slen])
    return result
```

In [101]:

```
alpha = 0.2
beta = 0.15
gamma = 0.2
season_len = 24

predict_values_2 =[]
predict_list_2 = []
tsne_flat_exp_avg_2 = []
for r in range(0,40):
    predict_values_2 = triple_exponential_smoothing(regions_cum[r][0:13104], season_len, alpha, bet
a, gamma, 0)
    predict_list_2.append(predict_values_2[5:])
```

In [102]:

```
# train, test split : 70% 30% split
# Before we start predictions using the tree based regression models we take 3 months of 2016 pick
up data
# and split it such that for every region we have 70% data in train and 30% in test,
# ordered date-wise for every region
print("size of train data :", int(13099*0.7))
print("size of test data :", int(13099*0.3))
size of train data : 9169
size of test data : 3929

In [103]:
```

```
# extracting first 9169 timestamp values i.e 70% of 13099 (total timestamps) for our training data train_features = [tsne\_feature[i*13099:(13099*i+9169)] for i in range(0,40)] # temp = [0]*(12955 - 9068)
```

```
test_features = [tsne_feature[(13099*(i))+9169:13099*(i+1)] for i in range(0,40)]
# Extracting the same for fourier features -->
fourier_features_train = pd.DataFrame(columns=['A1', 'A2', 'A3', 'A4', 'A5', 'F1', 'F2', 'F3', 'F4', 'F5'])
fourier_features_test = pd.DataFrame(columns=['A1', 'A2', 'A3', 'A4', 'A5', 'F1', 'F2', 'F3', 'F4', 'F5'])

for i in range(40):
    fourier_features_train = fourier_features_train.append(fourier_features[i*13099 : 13099*i + 916 9])

fourier_features_train.reset_index(inplace = True)

for i in range(40):
    fourier_features_test = fourier_features_test.append(fourier_features[i*13099 + 9169 : 13099*(i +1)])
fourier_features_test.reset_index(inplace = True)
```

In [104]:

```
print("Number of data clusters",len(train_features), "Number of data points in trian data",
len(train_features[0]), "Each data point contains", len(train_features[0][0]),"features")
print("Number of data clusters",len(train_features), "Number of data points in test data",
len(test_features[0]), "Each data point contains", len(test_features[0][0]),"features")
```

Number of data clusters 40 Number of data points in trian data 9169 Each data point contains 5 features

Number of data clusters 40 Number of data points in test data 3930 Each data point contains 5 feat

In [105]:

```
# extracting first 9169 timestamp values i.e 70% of 13099 (total timestamps) for our training data
tsne_train_flat_lat = [i[:9169] for i in tsne_lat]
tsne_train_flat_lon = [i[:9169] for i in tsne_lon]
tsne_train_flat_weekday = [i[:9169] for i in tsne_weekday]
tsne_train_flat_output = [i[:9169] for i in output]
tsne_train_flat_exp_avg = [i[:9169] for i in predict_list]
tsne_train_flat_triple_avg = [i[:9169] for i in predict_list_2]
```

In [106]:

```
# extracting the rest of the timestamp values i.e 30% of 12956 (total timestamps) for our test dat
a
tsne_test_flat_lat = [i[9169:] for i in tsne_lat]
tsne_test_flat_lon = [i[9169:] for i in tsne_lon]
tsne_test_flat_weekday = [i[9169:] for i in tsne_weekday]
tsne_test_flat_output = [i[9169:] for i in output]
tsne_test_flat_exp_avg = [i[9169:] for i in predict_list]
tsne_test_flat_triple_avg = [i[9169:] for i in predict_list_2]
```

In [107]:

```
len(predict_list_2[0])
```

Out[107]:

13099

In [108]:

```
# the above contains values in the form of list of lists (i.e. list of values of each region), her
e we make all of them in one list
train_new_features = []
for i in range(0,40):
    train_new_features.extend(train_features[i])
test_new_features = []
```

```
for i in range (0,40):
    test new features.extend(test features[i])
In [109]:
# converting lists of lists into sinle list i.e flatten
\# a = [[1,2,3,4],[4,6,7,8]]
# print(sum(a,[]))
# [1, 2, 3, 4, 4, 6, 7, 8]
tsne train lat = sum(tsne train flat lat, [])
tsne_train_lon = sum(tsne_train_flat_lon, [])
tsne_train_weekday = sum(tsne_train_flat_weekday, [])
tsne train output = sum(tsne train flat output, [])
tsne train exp avg = sum(tsne train flat exp avg,[])
tsne train triple avg = sum(tsne train flat triple avg,[])
In [110]:
# converting lists of lists into sinle list i.e flatten
\# a = [[1,2,3,4],[4,6,7,8]]
# print(sum(a,[]))
# [1, 2, 3, 4, 4, 6, 7, 8]
tsne_test_lat = sum(tsne_test_flat_lat, [])
tsne test lon = sum(tsne test flat lon, [])
tsne test weekday = sum(tsne test flat weekday, [])
tsne_test_output = sum(tsne_test_flat_output, [])
tsne_test_exp_avg = sum(tsne_test_flat_exp_avg,[])
tsne test triple avg = sum(tsne test flat triple avg,[])
In [111]:
# Preparing the data frame for our train data
columns = ['ft_5','ft_4','ft_3','ft_2','ft_1']
df train = pd.DataFrame(data=train new features, columns=columns)
df train['lat'] = tsne_train_lat
df_train['lon'] = tsne_train_lon
df_train['weekday'] = tsne_train_weekday
df_train['exp_avg'] = tsne_train_exp_avg
df_train['3EXP'] = tsne_train_triple_avg
print(df_train.shape)
(366760, 10)
In [112]:
# Preparing the data frame for our train data
df test = pd.DataFrame(data=test new features, columns=columns)
df test['lat'] = tsne_test_lat
df test['lon'] = tsne test lon
df_test['weekday'] = tsne_test_weekday
df_test['exp_avg'] = tsne_test_exp_avg
df_test['3EXP'] = tsne_test_triple_avg
print(df test.shape)
(157200, 10)
In [113]:
df test.head()
Out[113]:
```

ft 5 ft 4 ft 3 ft 2 ft 1 weekday 3EXP lat lon exp_avg 0 25 28 27 39 53 40.776228 -73.982119 47 44.684084 __ 140 770000 70 000440 4

1	∠8 ft 5	2/ ft 4	39 ft 3	ეკ ft 2	5/ ff 1	40.776228	-73.982119	4 weekday	54 exp. avg	05.385854 3FXP
2	27	39	53	57	84	40.776228	-73.982119	4	75	72.989870
3	39	53	57	84	77	40.776228	-73.982119	4	76	85.566655
4	53	57	84	77	89	40.776228	-73.982119	4	85	97.248740

Merging the fourier features:

In [114]:

```
df_train_2 = df_train
df_test_2 = df_test
df_train = pd.concat([df_train, fourier_features_train], axis = 1)
df_test = pd.concat([df_test, fourier_features_test], axis = 1)
```

In [115]:

```
print("Shape of Train Data Now - ", df_train.shape)
df_train.drop(['index'], axis = 1, inplace=True)
df_train.head()
```

Shape of Train Data Now - (366760, 21)

Out[115]:

	ft_5	ft_4	ft_3	ft_2	ft_1	lat	lon	weekday	exp_avg	3EXP	A1	A2	A3	
0	0	0	0	0	0	40.776228	- 73.982119	4	0	0.714487	363083.0	94443.16402	94443.16402	25408.
1	0	0	0	0	0	40.776228	- 73.982119	4	0	0.203261	363083.0	94443.16402	94443.16402	25408.
2	0	0	0	0	0	40.776228	- 73.982119	4	0	1.025893	363083.0	94443.16402	94443.16402	25408.
3	0	0	0	0	0	40.776228	- 73.982119	4	0	- 0.767218	363083.0	94443.16402	94443.16402	25408.
4	0	0	0	0	0	40.776228	- 73.982119	4	0	- 0.570430	363083.0	94443.16402	94443.16402	25408.
4														

In [116]:

```
print("Shape of Test Data Now - ", df_test.shape)
df_test.drop(['index'], axis = 1, inplace=True)
df_test.head()
```

Shape of Test Data Now - (157200, 21)

Out[116]:

	ft_5	ft_4	ft_3	ft_2	ft_1	lat	lon	weekday	exp_avg	3EXP	A1	A2	А3	
C	25	28	27	39	53	40.776228	- 73.982119	4	47	44.684084	385215.0	109221.086959	109221.086959	1
1	28	27	39	53	57	40.776228	- 73.982119	4	54	65.385854	385215.0	109221.086959	109221.086959	1
2	27	39	53	57	84	40.776228	- 73.982119	4	75	72.989870	385215.0	109221.086959	109221.086959	1
3	39	53	57	84	77	40.776228	- 73.982119	4	76	85.566655	385215.0	109221.086959	109221.086959	1
4	53	57	84	77	89	40.776228	- 73.982119	4	85	97.248740	385215.0	109221.086959	109221.086959	1

Using Linear Regression

```
In [117]
```

```
# find more about LinearRegression function here http://scikit-
learn.org/stable/modules/generated/sklearn.linear model.LinearRegression.html
# default paramters
# sklearn.linear model.LinearRegression(fit intercept=True, normalize=False, copy X=True, n jobs=1
# some of methods of LinearRegression()
# fit(X, y[, sample weight]) Fit linear model.
# get params([deep]) Get parameters for this estimator.
# predict(X) Predict using the linear model
# score(X, y[, sample weight]) Returns the coefficient of determination R^2 of the prediction.
# set params(**params) Set the parameters of this estimator.
# video link: https://www.appliedaicourse.com/course/applied-ai-course-online/lessons/geometric-in
tuition-1-2-copy-8/
from sklearn.linear_model import LinearRegression
from sklearn.model selection import GridSearchCV
from sklearn.preprocessing import MinMaxScaler
from sklearn.linear_model import SGDRegressor
#scaler = MinMaxScaler()
#df train = scaler.fit transform(df train)
#df test = scaler.transform(df test)
#lr_reg=LinearRegression().fit(df_train, tsne_train_output)
#y pred = lr reg.predict(df test)
#lr_test_predictions = [round(value) for value in y pred]
#y pred = lr reg.predict(df train)
#lr train predictions = [round(value) for value in y pred]
params = {'fit intercept':[True, False], 'normalize':[True, False]}
model = LinearRegression(n jobs = -1)
lr_reg = GridSearchCV(model, params, scoring = 'neg_mean_absolute_error', cv = 3)
lr_reg.fit(df_train, tsne_train_output)
y pred = lr reg.predict(df test)
lr test predictions = [round(value) for value in y pred]
y pred = lr reg.predict(df train)
lr train predictions = [round(value) for value in y pred]
```

Using Random Forest Regressor

In [118]:

```
# Training a hyper-parameter tuned random forest regressor on our train data
# find more about LinearRegression function here http://scikit-
learn.org/stable/modules/generated/sklearn.ensemble.RandomForestRegressor.html
# default paramters
# sklearn.ensemble.RandomForestRegressor(n_estimators=10, criterion='mse', max_depth=None, min_sam
ples split=2,
# min_samples_leaf=1, min_weight_fraction_leaf=0.0, max_features='auto', max_leaf_nodes=None, min_
impurity decrease=0.0,
# min impurity split=None, bootstrap=True, oob score=False, n jobs=1, random state=None,
verbose=0, warm start=False)
# some of methods of RandomForestRegressor()
# apply(X) Apply trees in the forest to X, return leaf indices.
# decision path(X) Return the decision path in the forest
\# fit(X, y[, sample weight]) Build a forest of trees from the training set (X, y).
# get params([deep]) Get parameters for this estimator.
# predict(X) Predict regression target for X.
# score(X, y[, sample weight]) Returns the coefficient of determination R^2 of the prediction.
# wides link1. https://www.appliedsicourse.com/course/applied.si_course_aplied/lossens/re
```

```
# video iinki: https://www.appiredatcourse.com/course/appired-ai-course-onithe/ressons/regression-
using-decision-trees-2/
# video link2: https://www.appliedaicourse.com/course/applied-ai-course-online/lessons/what-are-en
sembles/
\#regr1 =
RandomForestRegressor(max features='sqrt', min samples leaf=4, min samples split=3, n estimators=40,
n_{jobs=-1}
#regrl.fit(df train, tsne train output)
model = RandomForestRegressor(n jobs=-1)
params = {'max_depth' : [3, 4, 5], 'min_samples_split' : [2,3,5,7], 'max_features':['sqrt', 'log2'],
          'min samples leaf':[1, 10, 100]}
regr1 = GridSearchCV(model, params, scoring = 'neg mean absolute error', cv = None)
regr1.fit(df train, tsne train output)
Out[118]:
GridSearchCV(cv=None, error score='raise',
       estimator=RandomForestRegressor(bootstrap=True, criterion='mse', max depth=None,
           max features='auto', max leaf nodes=None,
           min_impurity_decrease=0.0, min_impurity_split=None,
           min_samples_leaf=1, min_samples_split=2,
           min weight fraction leaf=0.0, n estimators=10, n jobs=-1,
           oob_score=False, random_state=None, verbose=0, warm_start=False),
       fit_params=None, iid=True, n_jobs=1,
      param grid={'max depth': [3, 4, 5], 'min samples split': [2, 3, 5, 7], 'max features':
['sqrt', 'log2'], 'min_samples_leaf': [1, 10, 100]},
      pre dispatch='2*n jobs', refit=True, return train score='warn',
       scoring='neg mean absolute error', verbose=0)
In [119]:
# Predicting on test data using our trained random forest model
# the models regr1 is already hyper parameter tuned
# the parameters that we got above are found using grid search
y pred = regr1.predict(df test)
rndf_test_predictions = [round(value) for value in y pred]
y pred = regr1.predict(df train)
rndf train predictions = [round(value) for value in y pred]
```

Using XgBoost Regressor

In [120]:

```
# Training a hyper-parameter tuned Xg-Boost regressor on our train data
# find more about XGBRegressor function here
http://xgboost.readthedocs.io/en/latest/python/python api.html?#module-xgboost.sklearn
# default paramters
# xgboost.XGBRegressor(max depth=3, learning rate=0.1, n estimators=100, silent=True,
objective='reg:linear',
# booster='gbtree', n jobs=1, nthread=None, gamma=0, min child weight=1, max delta step=0, subsamp
le=1, colsample bytree=1,
# colsample_bylevel=1, reg_alpha=0, reg_lambda=1, scale_pos_weight=1, base_score=0.5,
random state=0, seed=None,
# missing=None, **kwargs)
# some of methods of RandomForestRegressor()
# fit(X, y, sample weight=None, eval set=None, eval metric=None, early stopping rounds=None, verbo
se=True, xgb model=None)
# get params([deep]) Get parameters for this estimator.
# predict(data, output_margin=False, ntree_limit=0) : Predict with data. NOTE: This function is no
t thread safe.
# get_score(importance_type='weight') -> get the feature importance
# video link1: https://www.appliedaicourse.com/course/applied-ai-course-online/lessons/regression-
using-decision-trees-2/
# video link2: https://www.appliedaicourse.com/course/applied-ai-course-online/lessons/what-are-en
```

```
sembles/
x \mod el = xgb.XGBRegressor(
 learning rate =0.1,
 n estimators=1000,
 max depth=3,
min child weight=3,
gamma=0,
 subsample=0.8,
 reg_alpha=200, reg_lambda=200,
 colsample bytree=0.8,nthread=4)
x_model.fit(df_train, tsne_train_output)
from xgboost import XGBRegressor
model = XGBRegressor(n_jobs = -1)
params = {
        'subsample':[0.7, 0.8, 0.9],
        'min child weight':[3, 5],
        'reg lambda':[200, 300, 400],
        'max depth': [3, 4, 5]
}
x_model = GridSearchCV(model, params, scoring = 'neg_mean_absolute_error', cv = None)
x_model.fit(df_train, tsne_train_output)
Out[120]:
GridSearchCV(cv=None, error_score='raise',
       estimator=XGBRegressor(base_score=0.5, booster='gbtree', colsample_bylevel=1,
       colsample_bytree=1, gamma=0, learning_rate=0.1, max_delta_step=0,
       max_depth=3, min_child_weight=1, missing=None, n_estimators=100,
       n jobs=-1, nthread=None, objective='reg:linear', random state=0,
       reg_alpha=0, reg_lambda=1, scale_pos_weight=1, seed=None,
       silent=True, subsample=1),
       fit params=None, iid=True, n jobs=1,
       param_grid={'subsample': [0.7, 0.8, 0.9], 'min_child_weight': [3, 5], 'reg_lambda': [200, 30]
0, 400], 'max depth': [3, 4, 5]},
      pre dispatch='2*n jobs', refit=True, return train score='warn',
       scoring='neg mean absolute error', verbose=0)
4
In [121]:
#predicting with our trained Xg-Boost regressor
# the models x model is already hyper parameter tuned
# the parameters that we got above are found using grid search
y pred = x model.predict(df test)
xgb_test_predictions = [round(value) for value in y_pred]
y pred = x model.predict(df train)
xgb train predictions = [round(value) for value in y pred]
In [122]:
#feature importances
#x_model.booster().get_score(importance_type='weight')
Calculating the error metric values for various models
```

```
In [123]:
```

```
columns = ['ft_5','ft_4','ft_3','ft_2','ft_1','lat','lon','weekday','exp_avg','A1','A2','A3','A4','
A5','F1','F2','F3','F4','F5']
df_train = pd.DataFrame(df_train, columns = columns)
df_test = pd.DataFrame(df_test, columns = columns)

train_mape=[]
test_mape=[]
train_mape.append((mean_absolute_error(tsne_train_output,df_train['ft_1'].values))/(sum(tsne_train_output)/len(tsne_train_output)))
```

```
train mape.append((mean absolute error(tsne train output,dr train['exp avg'].values))/(sum(tsne tra
in output)/len(tsne train output)))
train mape.append((mean absolute error(tsne train output, rndf train predictions))/(sum(tsne train c
utput)/len(tsne_train_output)))
train mape.append((mean absolute error(tsne train output,
xgb_train_predictions))/(sum(tsne_train_output)/len(tsne_train_output)))
train_mape.append((mean_absolute_error(tsne_train_output,
lr train predictions))/(sum(tsne train output)/len(tsne train output)))
\texttt{test\_mape.append((mean\_absolute\_error(tsne\_test\_output, \ df\_test['ft\_1'].values)))/(sum(tsne\_test\_output, \ df\_test['ft\_1'].values))/(sum(tsne\_test\_output, \ df\_test['ft\_1'].values)/(sum(tsne\_test\_output, \ df\_test['ft\_1'].values)/(sum(tsne\_test\_ou
put)/len(tsne test output)))
test mape.append((mean absolute error(tsne test output,
df test['exp_avg'].values))/(sum(tsne_test_output)/len(tsne_test_output)))
test mape.append((mean absolute error(tsne test output,
rndf test predictions))/(sum(tsne test output)/len(tsne test output)))
test mape.append((mean absolute error(tsne test output,
xgb test predictions))/(sum(tsne test output)/len(tsne test output)))
test_mape.append((mean_absolute_error(tsne_test_output,
lr test predictions))/(sum(tsne test output)/len(tsne test output)))
                                                                                                                                                                                                                                                                               Þ
```

Error Metric Matrix

```
In [125]:
```

```
print ("Error Metric Matrix (Tree Based Regression Methods) - MAPE")
----")
                                                                         Test: ",test_map
print ("Baseline Model -
                                              Train: ",train mape[0],"
print ("Exponential Averages Forecasting -
                                              Train: ",train_mape[1],"
                                                                         Test: ", test map
e[1])
                                              Train: ",train mape[4],"
print ("Linear Regression -
                                                                         Test: ", test mape
print ("Random Forest Regression -
                                              Train: ",train mape[2],"
                                                                         Test: ", test mape
                                              Train: ",train mape[3],"
print ("XgBoost Regression -
                                                                         Test: ", test map
[3])
print ("-----
Error Metric Matrix (Tree Based Regression Methods) - MAPE
                                       Train: 0.15219919303463372
                                                                     Test:
Baseline Model -
0.14527264149803146
Exponential Averages Forecasting -
                                      Train: 0.14453242288776613
                                                                     Test:
0.13742496628342404
Linear Regression -
                                       Train: 0.12331289993554674
                                                                     Test:
0.11103976486001399
                                       Train: 0.13726664415060033
Random Forest Regression -
                                                                     Test:
0.12730480669750005
XgBoost Regression -
                                       Train: 0.10748817782628778
                                                                      Test: 0.10221297417
59224
```

Results:

Model	Train Mape	Test Mape
BaseLine Model	15.21 %	14.52 %
Exponential Averages Forecasting	14.45 %	13.74 %
Linear Regression	12.33 %	11.10 %
Random Forest Regression	13.72 %	12.73 %
XgBoost Regression	10.74 %	10.22 %

Conclusions:

- 1. XgBoost has the least MAPE of all the models built.
- 2. Linear Regression despite being simpler than RandomForest performed better than it but not by much.
- 3. Holts Winter Features with vales of alpha, beta and gamma (0.2, 0.15, 0.2) helped to reduce the MAPE below 12 %.

References:

- 1. Holts Winter Triple Exponential https://grisha.org/blog/2016/02/17/triple-exponential-smoothing-forecasting-part-iii/
- 2. For FFT function https://docs.scipy.org/doc/numpy/reference/generated/numpy.fft.fft.html