

Contents

| | |
|--|----------|
| §1 Measures | 2 |
| §1.1 Algebras and Sigma-Algebras | 2 |
| §1.2 Measures | 2 |
| §1.3 Outer Measures | 2 |
| §1.4 Lebesgue Measure | 2 |
| §1.5 Completeness and Regularity | 2 |
| §1.6 Dynkin Classes | 2 |
| §2 Functions and Integrals | 3 |
| §2.1 Measurable Functions | 3 |
| §2.1.1 Question 1 | 3 |
| §2.1.2 Question 2 | 3 |
| §2.1.3 Question 3 | 3 |
| §2.1.4 Question 4 | 3 |

§1 Measures

§1.1 Algebras and Sigma-Algebras

§1.2 Measures

§1.3 Outer Measures

§1.4 Lebesgue Measure

§1.5 Completeness and Regularity

§1.6 Dynkin Classes

§2 Functions and Integrals

§2.1 Measurable Functions

§2.1.1 Question 1

Observe that all this question wants us to prove is that

$$\chi_{\limsup A_n} = \limsup \chi_{A_n}$$

This is easy and simply follows from the definition.

§2.1.2 Question 2

Let Y be a subset of \mathbb{R} which is not Borel measurable. Then χ_Y cannot be Borel measurable (See Example 2.1.2 (b) in the book). Observe that Y cannot be countable for otherwise we could write Y as union of its singleton members of Y and hence Y would be Borel set. Thus, Y is uncountable. Now, consider the set of functions $J := \{\chi_{\{y\}} : y \in Y\}$. It is easy to see that $\chi_Y = \sup \{\chi_{\{y\}} : y \in Y\}$. Clearly, J is set of Borel measurable functions whose supremum is not Borel measurable.

§2.1.3 Question 3

Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a function which is differentiable everywhere. Consider the sequence of function $\{f_n\}_{n \in \mathbb{N}}$ which is given by $f_n(x) = \frac{f(x + \frac{1}{n}) - f(x)}{1/n}$ for every $x \in \mathbb{R}$. Clearly, $f_n \rightarrow f$ pointwise on \mathbb{R} and each f_n is measurable. Since limit of Borel measurable functions is measurable, we have that f is Borel measurable.

§2.1.4 Question 4