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§2 Functions and Integrals

§2.1 Measurable Functions

§2.1.1 Question 1

Observe that all this question wants us to prove is that

$$\chi_{\limsup A_n} = \limsup \chi_{A_n}$$

This is easy and simply follows from the definition.

§2.1.2 Question 2

Let Y be a subset of $\mathbb{R}$ which is not Borel measurable. Then χ_Y cannot be Borel measurable (See Example 2.1.2 (b) in the book). Observe that Y cannot be countable for otherwise we could write Y as union of its singleton members of Y and hence Y would be Borel set. Thus, Y is uncountable. Now, consider the set of functions $J := \left\{\chi_{\{y\}} : y \in Y\right\}$. It is easy to see that $\chi_Y = \sup\left\{\chi_{\{y\}} : y \in Y\right\}$. Clearly, J is set of Borel measurable functions whose supremum is not Borel measurable.

§2.1.3 Question 3

Let $f : \mathbb{R} \to \mathbb{R}$ be a function which is differentiable everywhere. Consider the sequence of function $\{f_n\}_{n\in\mathbb{N}}$ which is given by $f_n(x) = \frac{f\left(x+\frac{1}{n}\right)-f(x)}{1/n}$ for every $x \in \mathbb{R}$. Clearly, $f_n \to f$ pointwise on $\mathbb{R}$ and each f_n is measurable. Since limit of Borel measurable functions is measurable, we have that f is Borel measurable.

§2.1.4 Question 4