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## §1 Measures

### §1.1 Algebras and Sigma-Algebras

### §1.2 Measures

### §1.3 Outer Measures

### §1.4 Lebesgue Measure

### §1.5 Completeness and Regularity

### §1.6 Dynkin Classes

## §2 Functions and Integrals

### §2.1 Measurable Functions

#### §2.1.1 Question 1

Observe that all this question wants us to prove is that

$$\chi_{\limsup A_n} = \limsup \chi_{A_n}$$

This is easy and simply follows from the definition.

### §2.1.2 Question 2

Let  $Y$  be a subset of  $\mathbb{R}$  which is not Borel measurable. Then  $\chi_Y$  cannot be Borel measurable (See Example 2.1.2 (b) in the book). Observe that  $Y$  cannot be countable for otherwise we could write  $Y$  as union of its singleton members of  $Y$  and hence  $Y$  would be Borel set. Thus,  $Y$  is uncountable. Now, consider the set of functions  $J := \{\chi_{\{y\}} : y \in Y\}$ . It is easy to see that  $\chi_Y = \sup \{\chi_{\{y\}} : y \in Y\}$ . Clearly,  $J$  is set of Borel measurable functions whose supremum is not Borel measurable.

### §2.1.3 Question 3

Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  be a function which is differentiable everywhere. Consider the sequence of function  $\{f_n\}_{n \in \mathbb{N}}$  which is given by  $f_n(x) = \frac{f(x + \frac{1}{n}) - f(x)}{1/n}$  for every  $x \in \mathbb{R}$ . Clearly,  $f_n \rightarrow f$  pointwise on  $\mathbb{R}$  and each  $f_n$  is measurable. Since limit of Borel measurable functions is measurable, we have that  $f$  is Borel measurable.

### §2.1.4 Question 4