Chapter 3 — Convergence in Measure

Last Updated: July 29, 2022

Contents

§1 Modes of Convergence

For simplicity, we will deal with $\mathbb{R}$-valued functions only. For further remarks, see the footnote in the book.

Definition §1.0.1. Let $(X, \mathscr{A}, \mu)$ be a measure space. Let $f, f_1, f_2, \ldots : X \to \mathbb{R}$ be a sequence of $\mathscr{A}$-measurable functions. We say that f_n converges to f *in measure* if for every $\varepsilon > 0$, we have that

$$\lim_n \mu\left(\left\{x \in X : \left|f_n(x) - f(x)\right| > \varepsilon\right\}\right) = 0$$

Remark §1.0.2. ⚠ General convergence in measure is neither implied nor implies convergence almost everywhere! As the following examples show:

Example §1.0.3.

1. Consider $(X, \mathscr{B}(\mathbb{R}), \lambda)$. Consider the sequence of functions $\{\chi_{[n,\infty)}\}$. Then $\chi_{[n,\infty)} \to 0$ function everywhere (hence, almost everywhere).But it does not converge in measure! To see this take $\varepsilon = \frac{1}{2}$.

2.