

# **Search for production of a Higgs boson and a single top quark in $\mu\mu$ final states in proton collisions from LHC**



**Hiram Ernesto Damián**

Departamento de investigación en física  
Universidad de Sonora

Para obtener el grado de:  
**Maestría en Ciencias (Física)**

Hermosillo, Sonora

October 2019

I would like to dedicate this thesis to my loving mother . . .

## **Acknowledgements**

And I would like to acknowledge ...

## Abstract

I present a study on the production of tH in same sign dimuon channel  $\mu \pm \mu \pm$ . A likelihood scan is performed to show the probability of observation of the signal. By studying this process, I explore a Higgs production mechanism, which it has not been observed yet experimentally. Simulation is used to predict sensitivity for the LHC high luminosity phase. This could help to obtain better limits for the production of tH. I also include the model  $k_t = -1$ , where  $k_t$  is the top higgs coupling parameter, for compare it with the standard model.

# Table of contents

<b>List of figures</b>	<b>vii</b>
<b>List of tables</b>	<b>ix</b>
<b>1 Introduction</b>	<b>1</b>
1.1 General objective and motivations . . . . .	1
1.2 The Standard Model . . . . .	2
1.3 Higgs Boson . . . . .	6
1.4 Top quark . . . . .	7
1.5 Higgs mechanism . . . . .	8
1.6 Higgs production mechanisms . . . . .	12
1.7 Higgs decay rates . . . . .	14
1.8 $tH$ production mechanism . . . . .	15
<b>2 The LHC and CMS</b>	<b>17</b>
2.1 The Large Hadron Collider . . . . .	17
2.2 The Compact Muon Solenoid . . . . .	20
2.2.1 Silicon Tracker . . . . .	21
2.2.2 Electromagnetic calorimeter . . . . .	22
2.2.3 Hadron calorimeter . . . . .	24
2.2.4 Muon Detector . . . . .	25
<b>3 Event reconstruction and selection</b>	<b>27</b>
3.1 Signal Event Topology . . . . .	27
3.2 Backgrounds . . . . .	28
3.3 Event Selection . . . . .	31
3.4 Systematic uncertainties . . . . .	31
3.5 Multivariable discriminant . . . . .	32
<b>4 Statistical Analysis</b>	<b>34</b>
4.1 Likelihood and fit procedure . . . . .	34

---

4.2	Limit calculation . . . . .	37
4.3	Extrapolation for higher luminosities . . . . .	37
<b>5</b>	<b>Conclusion and outlook</b>	<b>42</b>
	<b>References</b>	<b>43</b>
	<b>Appendix A Scattering and cross section</b>	<b>45</b>

# List of figures

1.1	Standard Model to date <sup>1</sup> . . . . .	3
1.2	Particle interactions in Standard model . . . . .	4
1.3	Higgs potential $V(\Phi^\dagger \Phi)$ for $\mu^2 < 0$ and $\lambda > 0$ also called the mexican hat potential. . . . .	10
1.4	Different Higgs production mechanism, from the most likely to least likely	12
1.5	$tHW$ mechanism . . . . .	12
1.6	Cross section for different processes generated in a pp collision . . . . .	14
1.7	tH mechanism. Higgs radiated from a top quark (left). Higgs radiated from a W boson (right) [1] . . . . .	16
2.1	Large hadron collider from CERN . . . . .	18
2.2	LHC schedule, including future plans for increase the center of mass energy and luminosity. The periods labeled LS are the maintenance periods [2]. .	19
2.3	View of Compact muon solenoid (CMS) . . . . .	20
2.4	The Tracker layout distributed in terms of $\eta$ . . . . .	22
2.5	Total reconstruction efficiency for muons . . . . .	22
2.6	View of the ECAL . . . . .	23
2.7	View of the CMS HCAL showing the different layers and tower regions. In the endcap, the towers are defined with two longitudinal segments as shown by the colors . . . . .	24
2.8	Cross section of a quadrant of the CMS with axis parallel to the beam (z axis) horizontally and the radius of the detector in terms of the $\eta$ . . . . .	25
2.9	Muon reconstruction efficiency in terms of $\eta$ and $p_t$ [3] . . . . .	26
3.1	Topology of $tH$ process which generates two same sign muons , a forward jet and a b-jet. . . . .	28
3.2	Distribution of BDT discriminant for signal and background in the case of SM (left) and inverted coupling scenario (right) . . . . .	33
4.1	Post fit signal and background yields for tH process for SM (Left) and $k_t = -1$ (Right). . . . .	36

4.2	Illustration of the $\chi^2$ distribution with number of degrees of freedom of 7 used for the limit estimation . . . . .	39
4.3	Likelihood scan for SM (up) and $k_t=-1$ (Down). . . . .	39
4.4	Fit and likelihood scan for $150 \text{ fb}^{-1}$ , $300 \text{ fb}^{-1}$ and $3000 \text{ fb}^{-1}$ in SM model. . . . .	40
4.5	Fit and likelihood scan for $150 \text{ fb}^{-1}$ , $300 \text{ fb}^{-1}$ and $3000 \text{ fb}^{-1}$ in $k_t=-1$ model. . . . .	41
A.1	Diagram of scattering for a particle in a potential field[4] . . . . .	46
A.2	Drawing of an idealized scattering process showing the differential solid angle $\Delta\Omega$ and the scattering angle $\theta$ [5] . . . . .	47



# List of tables

1.1	Table of particles in SM[6]. Stable means no decay, or lifetime almost infinite. Particles speeds at 0.998c . . . . .	5
1.2	Higgs boson production cross sections in pp collisions for $\sqrt{s} = 13\text{TeV}$ (in pico barn) and number of events for an integrated luminosity of 35.9 $\text{fb}^{-1}$ for Run 2 [6] . . . . .	13
1.3	SM Higgs boson branching ratios for $M_H = 125\text{ GeV}$ [6] . . . . .	15
2.1	Accelerator operation energies . . . . .	18
2.2	Characteristic of the CMS superconducting solenoid [3] . . . . .	21
3.1	Expected number of events for different $tH$ decay chains assuming integrated luminosity of 35.9 $\text{fb}^{-1}$ . $l$ represents $\mu^\pm, e^\pm, \tau^\pm$ . . . . .	30
3.2	Main backgrounds and their same sign $\mu\mu$ decay process . . . . .	30
3.3	Event yields for signal and backgrounds after the event selection for a integrated luminosity of 35.9 $\text{fb}^{-1}$ . The uncertainties of yields include statistical and systematic . . . . .	31
4.1	Postfit yields for the fit to the Asimov data corresponding to 35.9 $\text{fb}^{-1}$ . The uncertainty given is the combined statistical plus systematic. . . . .	36
4.2	$\alpha$ and $\mu$ values for the fit to Asimov data corresponding to 35.9 $\text{fb}^{-1}$ . . . . .	36
4.3	Results of $\mu$ and upper limits for Asimov extrapolations for SM and $k_t = -1$ models. . . . .	38

# Chapter 1

## Introduction

### 1.1 General objective and motivations

Particle physics is a field of study of physics that studies the particles that create matter and its interactions to very small scale, even smaller than atoms. The particles analyzed in particle physics are the fundamental particles that compose the matter and the responsables for all forces of nature. All thoses particles and their properties are grouped in a theory called the Standard Model. Standard Model (SM) is a theory that describes the fundamental forces (except gravity) and classify the elementary particles in bosons (entire spin) and fermions (half spin). The SM , created almost 50 years ago, has been the most successul theory in explain the phenomena in the subatomic world.

One of the particles of the SM, the Higgs Boson , was recently discovered in 2012 in the ATLAS and CMS experiments at CERN. The discovery of Higgs boson in proton proton collisions marked a new era in the particle physics due to its characteristic of giving the matter the property of mass, according to previous theories and predictions . Since then, people around the world are working in the Large Hadron Collider at CERN in order to detect the Higgs bosons in higher energies and differents Higgs production processes. Also the impact of Higgs is also important in cosmology, where theoretical investigations relate the Higgs boson and the cosmic inflation.

In the recent years, there have been many studies in the Higgs production processes at CERN in pp collisions , such as gluon fusion (ggF), VBF, WH, ZH and  $t\bar{t}H$  and people have obtained consistent results in the search of Higgs in those processes. In this work, we will analyze the production of Higgs boson in association with a single top quark (tH) in proton-proton collisions with the CMS experiment of the LHC. The motivation of the work

is this mechanism of production of the Higgs boson has not been observed before by any experiment. The exploration of Higgs production on the tH channel is subject relatively new.

Many channels are yet out of reach experimentally. At the same time, many theoretical proposals are yet to be put on test. It is possible that studies on the Higgs boson give evidence of small deviations from the SM and test new physics that are beyond SM such as String Theory and Supersymmetry.

## 1.2 The Standard Model

The SM, created in the decade of seventies, is a successful theory that explains the structure of matter and the interactions of three of the four fundamental forces which are electromagnetic, strong and weak forces. The gravitational force is yet to be included due to the scale of interaction with respect to the other 3. All the matter is composed of three kinds of particles: leptons, quarks and bosons, also called mediators.

For the leptons we have electrons ( $e$ ), muons ( $\mu$ ), and taus ( $\tau$ ) with charge -1 and their neutrinos  $\nu_e, \nu_\mu, \nu_\tau$ , are chargeless particles. Leptons have their respective antiparticle; same structure with opposite charge. All of them have a spin (intrinsic angular momentum) of  $\frac{1}{2}$ . [7].

Quarks combine to form composite particles called hadrons. Quarks cannot be found in isolation, they always exist in form of hadrons that can be mesons or baryons. Mesons, are bosons made up of a quark-antiquark pair, such as pion ( $\pi$ ) and kaon ( $K$ ). Mesons were theorized by Yukawa in 1937 and discovered in 1947 by Powell. [7] Baryons are fermions made of 3 quarks. Examples of baryons are protons and neutrons. Both particles can interact via strong force.

Quarks were first theorized by Murray Gell-Mann and George Zweig in 1964, were introduced as parts of an ordering scheme for hadrons, and there was little evidence for their physical existence until scattering experiments at the Stanford Linear Accelerator Center in 1968.[7] There are 6 of them: up (1968), down (1968), strange (1968), charm (theorized in 1970, discovered 1974), bottom (theorized 1973, discovered 1977), top (theorized 1973, discovered 1995).

Quarks have characteristics such as electric charge, mass, color charge, and spin. The quarks up, charm and top have in common a charge of  $\frac{2}{3}$ , while the down, strange and

bottom quarks have charge of  $-\frac{1}{3}$ . Among them, the top quark is the most massive of all, while the up and down are the least massive. The lepton and quarks have their respective antiparticle, which have the same mass, but inverted charge.

Some of the particles in the SM are not stable, so they decay and form particles with lower masses. Other particles do not decay or their lifetimes are very long. The final states can be represented with a Feynman diagram according to the SM vertices. [8]

## Standard Model of Elementary Particles

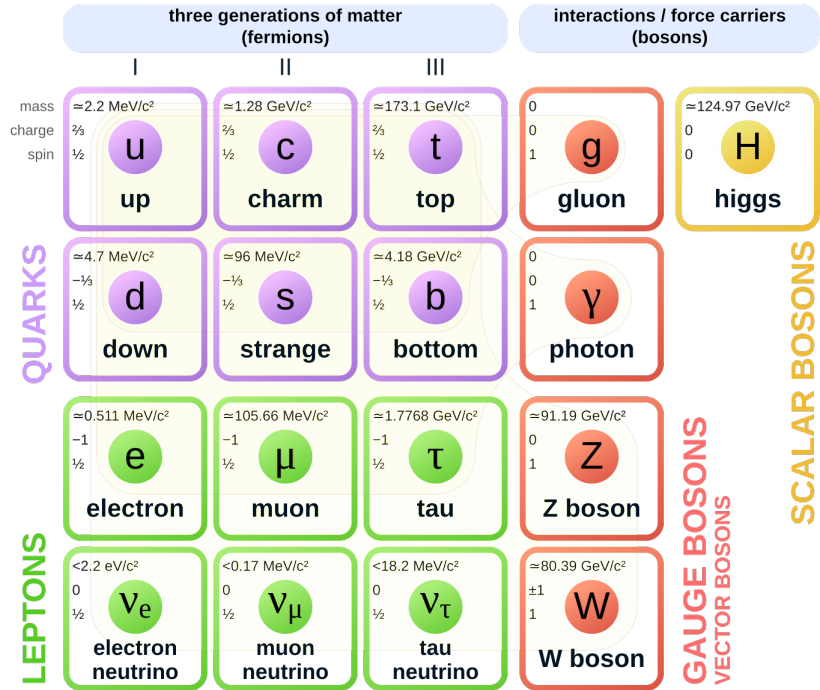


Fig. 1.1 Standard Model to date<sup>1</sup>

67

The last group of the family are the bosons, which have a spin 1 and 0. There are 5 bosons:  $W^\pm$ ,  $Z$  with neutral charge are responsible for weak force; photon, a massless particle, is responsible for electromagnetic force; gluon which is also massless, is related to strong force and Higgs boson, recently discovered, is in charge of giving the property of mass to particles via Higgs mechanism. Bosons let other particles to interact, even themselves, which is shown in figure 1.2.

The  $W$ ,  $Z$  and Higgs bosons, can interact with most of the particles, while the gluon can only interact with quarks and itself. Those bosons act as mediators in decays and creation of complex particles, as mesons and baryons.

77

<sup>1</sup>Taken from Wikimedia. [https://upload.wikimedia.org/wikipedia/commons/0/00/Standard\\_Model\\_of\\_Elementary\\_Particles.svg](https://upload.wikimedia.org/wikipedia/commons/0/00/Standard_Model_of_Elementary_Particles.svg)

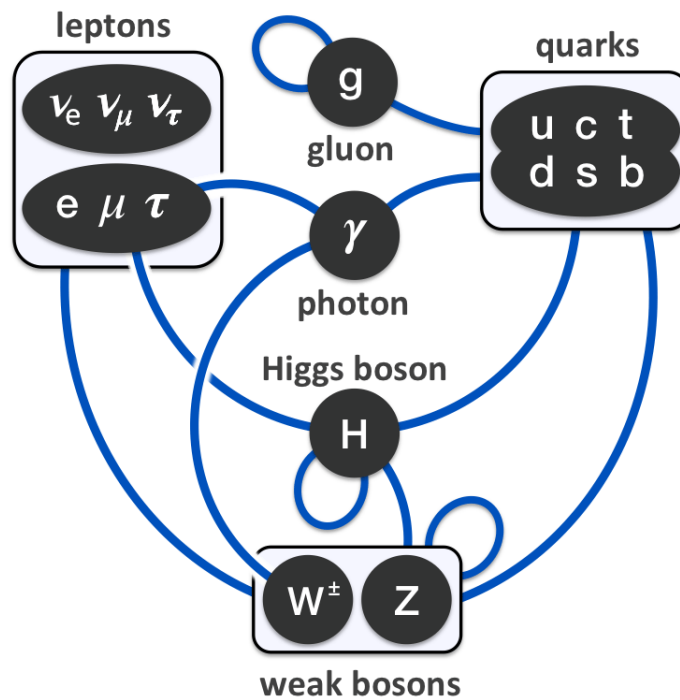


Fig. 1.2 Particle interactions in Standard model

78

79 There are experiments that detect particles around the world, such as the large hadron  
 80 collider (LHC) in Europe, the Fermilab in United States of America, KEK in Japan, etc.  
 81 More information about the LHC will be described in the next chapters.

82 Since quarks and gluons only exist in bound states, it is impossible to observe isolated  
 83 quarks except the top quark, which decays before it has time to form a bound state. As two  
 84 quarks are separated, the strong force between them increases which is equivalent to the  
 85 potential energy increasing. This energy transforms into quark-anti-quark pairs.

86

87 The table 1.1 contains the characteristics of each particle in the SM. Note the distance  
 88 column refers to particles moving with speeds near light speed. This column visualizes the  
 89 distance traveled in the predicted lifetime of the particle. And also give us a perspective of  
 90 how hard is the detection of the particles due to lifetime. Stable lifetime means that the  
 91 particles do not decay or can exist until they can interact with detectors.

92

Table 1.1 Table of particles in SM[6]. Stable means no decay, or lifetime almost infinite. Particles speeds at 0.998c

Particle	Mass (MeV/ $c^2$ )	Charge	spin	Lifetime (s)	Distance in lifetime (meters)
Up ( $u$ )	2.2	$\frac{2}{3}$	$\frac{1}{2}$	stable	-
Charm ( $c$ )	1280	$\frac{2}{3}$	$\frac{1}{2}$	$1.1 \times 10^{-12}$	$5.21 \times 10^{-3}$
Top ( $t$ )	173100	$\frac{2}{3}$	$\frac{1}{2}$	$5 \times 10^{-25}$	$2.37 \times 10^{-15}$
Down ( $d$ )	4.6	$-\frac{1}{3}$	$\frac{1}{2}$	Stable	-
Strange ( $s$ )	96	$-\frac{1}{3}$	$\frac{1}{2}$	$1.24 \times 10^{-8}$	58.7
Bottom ( $b$ )	4180	$-\frac{1}{3}$	$\frac{1}{2}$	$1.3 \times 10^{-12}$	$6.16 \times 10^{-3}$
$W$	80379	$\pm 1$	1	$3 \times 10^{-25}$	$1.42 \times 10^{-15}$
$Z$	91187.6	0	1	$3 \times 10^{-25}$	$1.42 \times 10^{-15}$
Photon ( $\gamma$ )	0	0	1	Stable	-
Gluon ( $g$ )	0	0	1	Stable	-
Higgs ( $H$ )	125.18	0	0	$1.56 \times 10^{-22}$	$7.39 \times 10^{-13}$
Electron ( $e$ )	0.511	-1	$\frac{1}{2}$	Stable	-
Muon ( $\mu$ )	105.7	-1	$\frac{1}{2}$	$2.2 \times 10^{-6}$	10419.85
$\tau$	1776.86	-1	$\frac{1}{2}$	$2.9 \times 10^{-13}$	$1.37 \times 10^{-3}$
$\nu_e \nu_\mu \nu_\tau$	0	0	$\frac{1}{2}$	Stable	-

### 1.3 Higgs Boson

The Higgs is a boson with spin 0 and chargeless. First theorized in 1964 by a group of theoretical physicist composed by Peter Higgs, François Englert, Robert Brout, Gerald Guralnik, C. Richard Hagen, and Tom Kibble in separated groups. They formulated the Higgs mechanism that explain the generation of mass for the particles .

98

The theory also explains Higgs bosons give the mass to the particles via Higgs Field and the mass of Higgs boson itself. The Higgs boson was discovered recently on year 2012 in the experiments ATLAS and CMS at CERN. The experiment was a proton proton collision with center of mass energy of 7 and 8 TeV ,where investigators worked on the data and detecting various decay modes that generates final states such as  $WW$ ,  $\gamma\gamma$ ,  $ZZ$ ,  $b\bar{b}$  and  $\tau\tau$  [9].

The mass of this boson has been measured and its production and decay rates are consistent with the SM prediction. The mass of the Higgs boson measured is approximately 125 MeV. The Higgs boson lifetime is  $1.56 \times 10^{-22}$ . That is why the Higgs boson was very difficult to detect in the particle detectors.

109

## 1.4 Top quark

The top quark is the most massive particle with a charge of  $2/3$  with spin of  $1/2$ . First proposed by M. Kobayashi and T. Maskawa in 1973 to explain observed CP violations in Kaon decay[7]. On 1995, in the Tevatron at Fermilab in Illinois, the top quark was discovered via proton antiproton collisions with center of mass energy between 1.8 to 1.96 TeV in the CDF and D0 experiments. The top quark production process was via ggF, which generated a pair of  $t\bar{t}$ [10]. Since then, the properties of the top quark have been studied at the Tevatron Collider at Fermilab, and by the ATLAS and CMS experiments at the Large Hadron Collider (LHC) at CERN.

However, the top quark often receives special attention in new physics models because its mass requires near unity coupling to the Higgs boson [10]. As shown in table 1.1, the top quark lifetime is so small that they decay before they can hadronise, that is to say, the process to create hadrons from quarks and gluons.

One of interesting the properties of top quark is its mass. The top quark mass is a free parameter in the SM and was extracted using indirect measurements and relying on SM calculations. The measured value of top quark mass is of 173 GeV[6]. The top quark mass is a very important parameter that puts a constraint in the Higgs boson mass and also takes an important role in electroweak symmetry breaking[10].

The production of top quark can be generated in different ways. In Tevatron and LHC, the production of  $t\bar{t}$  comes from ggF (gluon gluon fusion) via strong force. Even it is possible to create a single top quark in the LHC via electroweak interactions (with a W boson) [11]. The top quark, due to its big mass, always decays into a W boson and a b quark. But since the W boson have various decays, the top quark can generate many possible particles, according to W boson decays. Some possible decays of top quark is the decay to a lepton such as muon, electron or tau and their neutrinos .



## 1.5 Higgs mechanism

In the SM, mathematically the particles are described in form of fields. The SM for electroweak interactions is described as a gauge theory with a symmetry group  $SU(2) \otimes U(1)$  that describes the weak and electromagnetic interactions due to the exchange of spin 1 gauge fields ( $W^\pm, Z$ ) for the weak part and a photon for the electromagnetic part. The Higgs mechanism give bosons and fermions their mass.

The lagrangian for the electroweak part of SM

$$\mathcal{L}_{EW} = \mathcal{L}_{\text{gauge}} + \mathcal{L}_f + \mathcal{L}_{\text{Higgs}} + \mathcal{L}_{\text{Yukawa}} \quad (1.1)$$

The first term of the lagrangian refers to gauge bosons

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4} [F^{\mu\nu} F_{\mu\nu}] - \frac{1}{4} \left[ \sum_i G^{i\mu\nu} G_{\mu\nu}^i \right] \quad (1.2)$$

where the F and G are

$$F^{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \quad (1.3)$$

$$G^{i\mu\nu} = \partial_\mu W_\nu^i - \partial_\nu W_\mu^i - g \epsilon^{ijk} W_\mu^j W_\nu^k \quad (1.4)$$

$W^i$  (i=1,2,3) are three  $SU(2)$  gauge bosons and B is a  $U(1)$  gauge boson.<sup>2</sup> In the end, those gauge bosons, after introduce the Higgs mechanism, become the  $W^\pm$ , Z and  $\gamma$  vector boson [12, 13].

The second term refers to kinematic energy of fermions

$$\mathcal{L}_f = \sum_{m=1}^3 (\bar{q}_L^m i \not{D}_L q_L^m + \bar{l}_L^m i \not{D}_L l_L^m + \bar{u}_R^m i \not{D}_R u_R^m + \bar{d}_R^m i \not{D}_R d_R^m + \bar{e}_R^m i \not{D}_R e_R^m) \quad (1.5)$$

---

<sup>2</sup>  $\epsilon^{ijk}$  represents the Levi-Civita tensor, where  $\epsilon^{ijk} = \begin{cases} +1 & \text{for even permutation} \\ -1 & \text{for odd permutation} \\ 0 & \text{i=j, or j=k, or k=i} \end{cases}$ .

where  $m$  is the family index, and L(R) refer to the left (right) handed particles.<sup>3</sup>. The gauge covariant derivatives are given by

$$D_L = \left( \partial_\mu + ig\sigma^i W_\mu^i + ig'YB_\mu \right) \quad (1.6)$$

$$D_R = \left( \partial_\mu + ig'YB_\mu \right) \quad (1.7)$$

where  $\sigma^i$  are the pauli matrices. L and R refers to left and right handed fields. The left handed fields for the first family are defined by the following doublets

$$q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \quad l_L = \begin{pmatrix} \nu_{eL} \\ e_L^- \end{pmatrix} \quad (1.8)$$

and the right handed fields

$$u_R, d_R, e_R^- \quad (1.9)$$

The hypercharges  $Y$  depend of the particle type. For left handed

$$Y(l_L) = -\frac{1}{2} \quad Y(q_L) = \frac{1}{6} \quad (1.10)$$

for right handed

$$Y(e_R) = -1 \quad Y(u_R) = \frac{2}{3} \quad Y(d_R) = -\frac{1}{3} \quad (1.11)$$

The lagrangian for Higgs field

$$\mathcal{L}_{\text{Higgs}} = (D_\mu \Phi)^\dagger (D^\mu \Phi) - V(\Phi^\dagger \Phi) \quad (1.12)$$

where  $D_\mu$  is a operator

$$D_\mu = \left( \partial_\mu + i\frac{1}{2}\sigma^i g W_\mu^i + i\frac{1}{2}g' B_\mu \right) \quad (1.13)$$

and the Higgs potential  $V(\Phi^\dagger \Phi)$

$$V(\Phi^\dagger \Phi) = \mu^2 \Phi^\dagger \Phi + \frac{1}{2}\lambda (\Phi^\dagger \Phi)^2 \quad (1.14)$$

---

<sup>3</sup>  $\bar{\psi}$  denotes the adjoint spinor  $\bar{\psi} = \psi^\dagger \gamma^0$ .  $\not{D} = \gamma^\mu D_\mu$ .  $\gamma$  are the Dirac matrices

147  $\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$  is a SU(2) doublet of scalar fields.  $V$  is symmetrical under rotations in  
 148  $\Phi$  space and parameters  $\lambda$  and  $\mu^2$  are parameters of the potential. The Higgs potential for  
 149  $\mu^2 < 0$  and  $\lambda > 0$  can be visualized in the figure 1.3

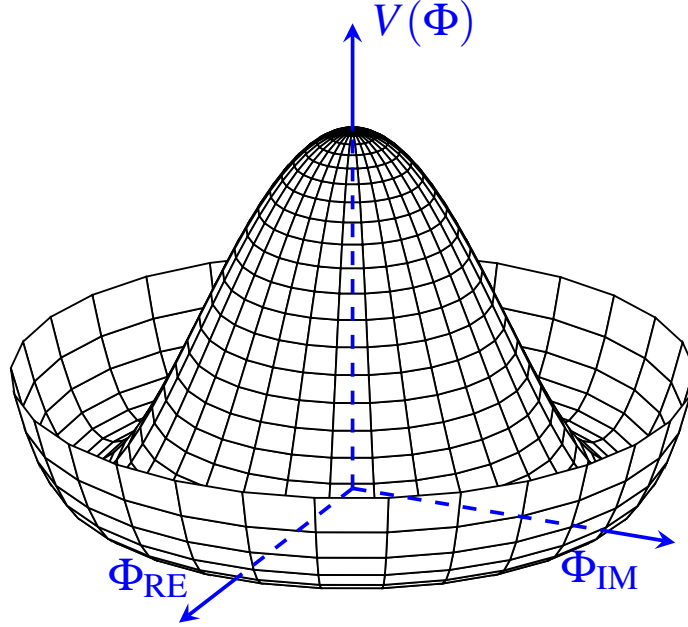


Fig. 1.3 Higgs potential  $V(\Phi^\dagger \Phi)$  for  $\mu^2 < 0$  and  $\lambda > 0$  also called the mexican hat potential.

Electroweak symmetry breaking refers to the choice of ground state

$$\Phi_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \quad (1.15)$$

150 where  $v = -\frac{\mu^2}{\sqrt{2}}$  is called the vacuum expectation value.

151

Taking 1.15 and substituting in  $\mathcal{L}_{\text{Higgs}}$ , we get

$$(D_\mu \Phi)^\dagger (D^\mu \Phi) = \frac{v^2}{8} \left( g^2 ((W_\mu^1)^2 + (W_\mu^2)^2) + (gW_\mu^3 - g'B_\mu)^2 \right) \quad (1.16)$$

we define the physical vector boson  $W_\mu^-$ ,  $W_\mu^+$  and  $Z$

$$W_\mu^\pm = \frac{1}{\sqrt{2}} (W_\mu^1 \mp iW_\mu^2) \quad (1.17)$$

$$Z_\mu = \frac{1}{\sqrt{g^2 + g'^2}} (gW_\mu^3 - g'B_\mu) \quad (1.18)$$

Now introducing 1.17 and 1.18 in 1.16,

$$(D_\mu \Phi)^\dagger (D^\mu \Phi) = \frac{v^2}{8} \left( 2g^2 W_\mu^+ W^{\mu-} + (g^2 + g'^2) Z_\mu Z^\mu \right) \quad (1.19)$$

from the coefficients, we get the  $W$  and  $Z$  mass

$$m_W = \frac{gv}{2} \quad m_Z = \frac{v}{2} \sqrt{g^2 + g'^2} \quad (1.20)$$

The last part of the lagrangian is the Yukawa lagrangian

$$\mathcal{L}_{\text{yukawa}} = \sum_{m,n}^3 \Gamma_{mn}^u \bar{q}_{m,L} \tilde{\Phi} u_{n,R} + \Gamma_{mn}^d \bar{q}_{m,L} \Phi d_{n,R} + \Gamma_{mn}^e \bar{l}_{m,L} \Phi e_{n,R} + h.c \quad (1.21)$$

152 The matrices  $\Gamma_{mn}$  describe the so called Yukawa couplings between Higgs doublet  $\Phi$  and  
153 the fermions.<sup>4</sup> The indices  $m$  and  $n$  mean sum over the families.

154

By using 1.15 it on  $\mathcal{L}_{\text{yukawa}}$ , it is obtained for  $u$  quark

$$\frac{\Gamma_{uu}^u v}{\sqrt{2}} (\bar{u}_L u_R + \bar{u}_R u_L)$$

from which the masses for the fermions shown to be proportional to the Yukawa couplings

$$m_u = -\frac{\Gamma_{uu}^u v}{\sqrt{2}}$$

---

<sup>4</sup>Here  $\tilde{\Phi} = i\sigma^2 \Phi^\dagger = \begin{pmatrix} \phi^{0\dagger} \\ -\phi^- \end{pmatrix}$  is necessary for the correct transformation in the case of up quarks.  
h.c refers to hermitian conjugate terms.

## 1.6 Higgs production mechanisms

During a particle collision, there are various ways that the Higgs boson can be created. The most common is the gluon gluon fusion process ( $ggF$ ) which is the process which involve two gluons mediated by the exchange of a virtual, heavy top quarks. Another process is vector boson fusion (VBF), where two fermions interact via a vector boson ( $W$  or  $Z$ ) and create a Higgs along with other particles. There is also the vector higgs process (VH), where the creation of particles using vector boson comes from a interaction between fermions and anti fermions[6]. These processes are shown in the figure 1.4

In the recent years, people have studied the top anti top Higgs ( $t\bar{t}H$ ) process, in order to detect a Higgs boson, via multilepton signals [11]. In this process, the creation of a Higgs boson comes from a intection via  $t\bar{t}$ . In the figure 1.4, the  $t\bar{t}H$  starts with 2 gluons which decay to two  $t\bar{t}$  pairs and after that , the interaction  $t\bar{t}$  generates a Higgs. The last process, subject of study in this thesis, is the top Higgs ( $tH$ ) process. This process is a rare expected process with a very small production rate[6]. In the figure 1.5, is another way to produce  $tH$ , in association with a  $W$ .

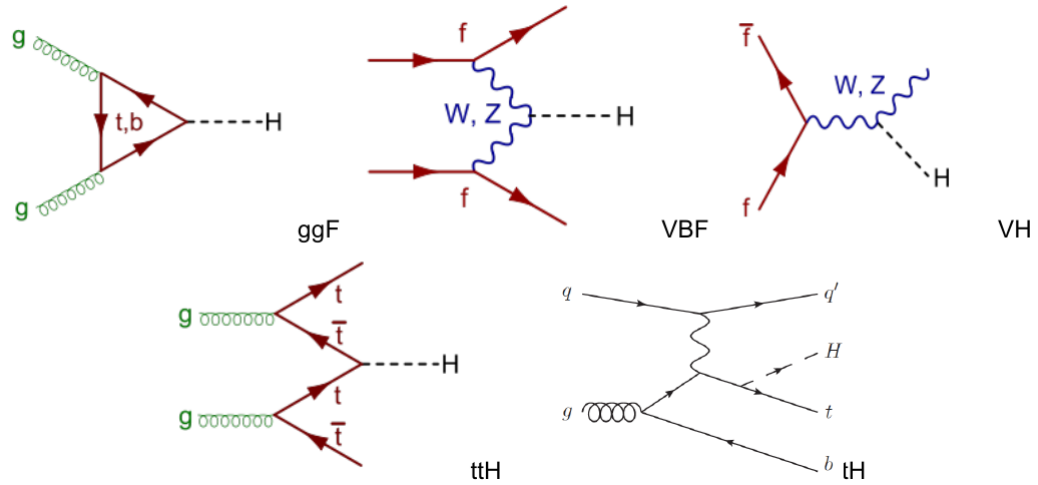


Fig. 1.4 Different Higgs production mechanism, from the most likely to least likely

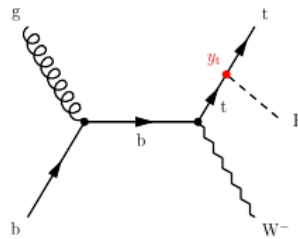


Fig. 1.5  $tHW$  mechanism

171 In tH process, Higgs boson is radiated from a single top quark as shown in 1.4. tH  
 172 process is a new field of study, given that there are few experiments and its production  
 173 rate (probability) is small, so the detection of Higgs bosons in this channel opens to new  
 174 discoveries.

175  
 176 In particle physics, cross section ( $\sigma$ ) describes the likelihood of two particles interacting  
 177 under certain conditions. Cross sections are expressed in barns , where 1 barn= $10^{-34}$  cm<sup>2</sup> .  
 178 The cross section is important in the evaluation of events for specific processes. In order to  
 179 get the number of events for a specific process, the reaction rate  $N$  is determined by the  
 180 total cross section  $\sigma$  and the luminosity  $L$ .

181 The unit of measurement of luminosity is cm<sup>-2</sup>s<sup>-1</sup>. Therefore, The reaction rate is

$$N_R = \sigma L \quad (1.22)$$

182 In practice, the luminosity is measured by counting the number of events for a well known  
 183 process.

Table 1.2 Higgs boson production cross sections in pp collisions for  $\sqrt{s} = 13\text{TeV}$  (in pico barn) and number of events for an integrated luminosity of 35.9 fb<sup>-1</sup> for Run 2 [6]

Production mechanism	$\sigma$ (picobarns pb)	Number of events
$ggF$	48.93	1756587
VBF	3.78	135702
$WH$	1.35	48465
$ZH$	0.88	31592
$t\bar{t}H$	0.50	18255
$tH$	0.015	560.39

184 In the table 1.2 shows the Higgs production cross section for pp collision and the number  
 185 of events produced using a integrated luminosity of 35.9fb<sup>-1</sup> , that is the luminosity  
 186 measured for the Run 2 from LHC on 2016[6].

187 Here we can see ggF process have the biggest cross section of all Higgs mechanism  
 188 and generates the greatest number of events possibles. For the processes tH and  $t\bar{t}H$ , they  
 189 have the smallest cross section, so the probability for the processes is low and generates a  
 190 small number of events.

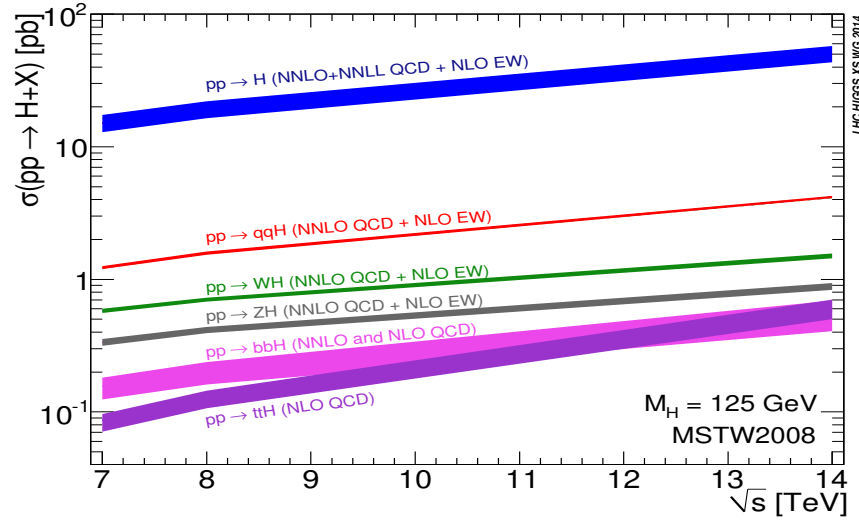


Fig. 1.6 Cross section for different processes generated in a pp collision

## 1.7 Higgs decay rates

In particle physics, two important properties are the lifetime and the decay rates of a particle. The lifetime is related to the total decay rate  $\Gamma$ , that is the probability per unit of time a particle decays,

$$dN = -\Gamma N dt \quad (1.23)$$

with  $N$  the original number of particles before the decay. From here it is evident that the number of particles left in a decay is

$$N(t) = N_0 e^{-\Gamma t} \quad (1.24)$$

The number of particles decreases over time exponentially and this can be measured. With this, it is possible to determine the mean lifetime [7]

$$\tau = \frac{1}{\Gamma} \quad (1.25)$$

In case of the decay rate to a specific process, it is necessary to get the branching fraction of the process. The branching ratio for a decay process is the ratio of the number of particles which decay via a specific decay mode with respect to the total number of particles which

decay via all decay modes.

$$BR_i = \frac{\Gamma_i}{\sum_i \Gamma_i} \quad (1.26)$$

Where  $\Gamma = \sum_i \Gamma_i$  is the total decay width (sum of all partial widths) of the particle and is related to lifetime of the particle:  $\Gamma = 1/\tau$ . Since the dimension of  $\Gamma$  is the inverse of time, in our system of natural units, it is measured in inverse seconds, it has the same dimension as mass (or energy)<sup>5</sup>. The lifetime of the Higgs boson is predicted to be  $1.56 \times 10^{-22}$  seconds and corresponds to  $\Gamma_H$  is about 4 MeV, but has not yet been measured due to the detector resolution [3].

The decay  $H \rightarrow b\bar{b}$  have a branching ratio a bit more than 50%, therefore is the most

Table 1.3 SM Higgs boson branching ratios for  $M_H = 125$  GeV [6]

Higgs decay	Branching ratio (BR)
$H \rightarrow b\bar{b}$	58.4%
$H \rightarrow W^+W^-$	21.4%
$H \rightarrow \tau^+\tau^-$	6.27%
$H \rightarrow ZZ$	2.62%
$H \rightarrow \gamma\gamma$	0.227%
$H \rightarrow Z\gamma$	0.153%
$H \rightarrow \mu^+\mu^-$	0.0218%

likely decay Higgs boson can decay. Due to this,  $H \rightarrow b\bar{b}$  is a very studied process and most likely to detect a Higgs boson in the data after reconstruction. The decay  $H \rightarrow \mu^+\mu^-$  has a very low branching ratio. This decay is very rare but not impossible to detect in the future. In this work the majority of the Higgs decays detected are from  $WW$ ,  $ZZ$ ,  $\tau\tau$

## 1.8 $tH$ production mechanism

The production of  $tH$ , where a Higgs boson can be radiated either from the top quark or from the exchanged  $W$  boson in the two dominant leading order diagrams in figure 1.7 provides a unique opportunity to study the relative sign of the coupling. In SM, the two diagrams interfere negatively and thereby suppress the production cross section. Any deviation from the SM can lead to a large enhancement of the event rate.  $\kappa_t$  and  $\kappa_W$  are the coupling parameters, where  $\kappa_t$  is for the top and  $\kappa_W$  for  $W$  boson.

<sup>5</sup>Cleaves H.J. (2011) Branching Ratio. In: Gargaud M. et al. (eds) Encyclopedia of Astrobiology. Springer, Berlin, Heidelberg



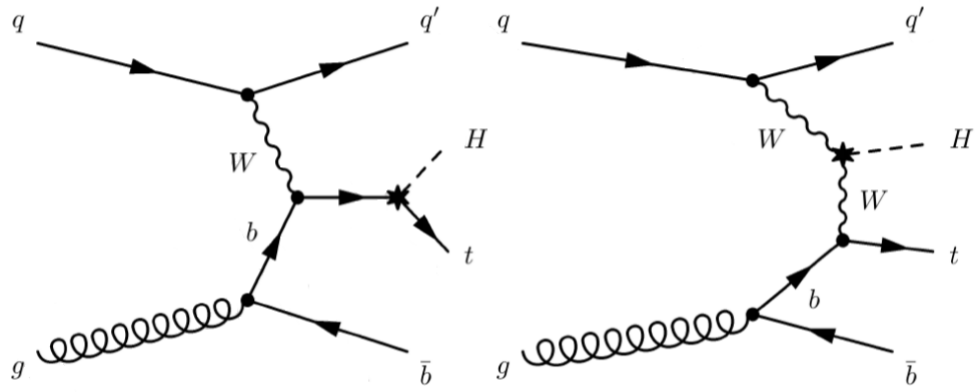


Fig. 1.7  $tH$  mechanism. Higgs radiated from a top quark (left). Higgs radiated from a  $W$  boson (right) [1]

# Chapter 2

## The LHC and CMS

### 2.1 The Large Hadron Collider

214 The Large Hadron Collider (LHC) is one the largest and most powerful particle accelerator  
215 in the world. It came on operation on 10 September 2008 and it is the most recent addition  
216 from the European Organization for Nuclear Research (CERN). The LHC is a 27 kilometer  
217 ring composed of superconducting magnets with accelerating structures to boost the energy  
218 of the particles.

219

220 The LHC is designed to accelerate particles to high energies and generate collisions.  
221 There are several experiments installed along the LHC ring. One of them is ATLAS,  
222 located on Point 1 between the two injection lines , which is a general purpose detector.  
223 At point 5 is another general purpose detector: the compact muon solenoid (CMS). A  
224 particle detector optimized for heavy ion physics, ALICE, is located at Point 2 and LHCb,  
225 a detector designed for B physics, is located at Point 7.[14, 15]

226

227 Inside the accelerator, two high energy particles beams travel at close to the speed of  
228 light before they collide. In order to make them collide, beams travel in opposite directions  
229 in separate beam pipes, two tubes kept at ultrahigh vacuum. By using of superconducting  
230 electromagnets that generates a powerful magnetic field , the beams are guided around  
231 the accelerator ring. The electromagnets are built from coils of special electric cable that  
232 operates in a superconducting state, efficiently conducting electricity without resistance  
233 or loss of energy. For that, it is required to have magnets to a temperature of -271.3 C. To  
234 reach such temperatures, a system of liquid helium is connected to the accelerator.[15]

## CERN's Accelerator Complex

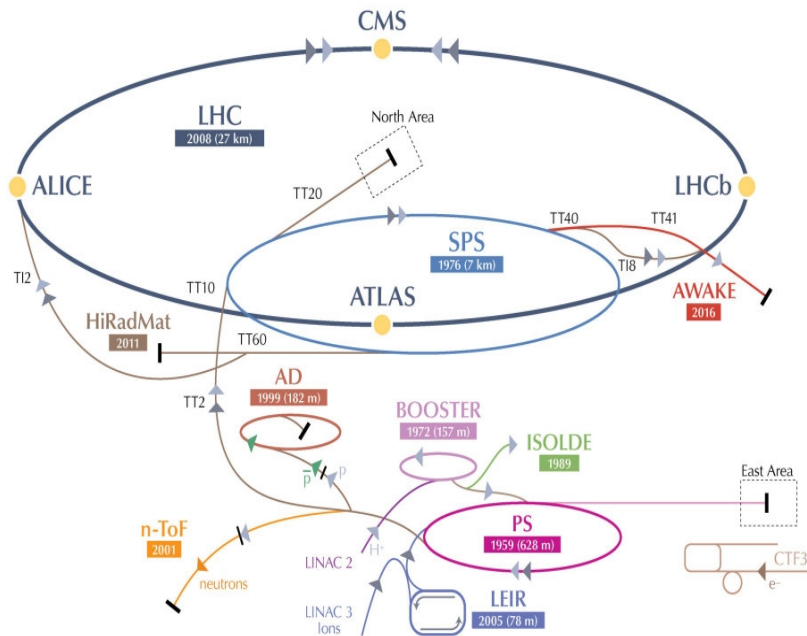


Fig. 2.1 Large hadron collider from CERN

Protons in the LHC start out as hydrogen atoms stripped of their electrons. The first accelerating stage the protons are subjected to is a linear accelerator, Linac2, which accelerates them up to 50 MeV . The protons are then sent into the Proton Synchrotron Booster which accelerates them up to 1.4 GeV before sending them to the Proton Synchrotron (PS). The protons leave the PS at 25 GeV before entering the Super Proton Synchrotron (SPS), which accelerates them to the LHC injection energy of 450 GeV. After this stage, the beam is ready to be injected into the LHC through one of two injection lines with 6.5 TeV.[2]

Table 2.1 Accelerator operation energies

Accelerator	Energy
Linac 2	50 MeV
PS Booster	1.4 GeV
Proton Scynchrotron (PS)	25 GeV
SPS	450 GeV
LHC	6.5 TeV

Inside the detectors, collided protons generates an amount of particles such as pion and kaons, which are the most commons, created by the jet particles that were created after the collision. Besides of the particles created by the quarks of the proton, gluons are also radiated and create new particles and photons are radiated too.

During the collision time, the main detectors of LHC capture the energy and save the data in supercomputers. Those high end computers starts to measure the signals in order to assign a type of events.

The main objctive of the LHC is to study the nature of electroweak symmetry breaking for the Higgs mechanism of the SM. Alternatives to the Standard Model that involves different symmetries are also put on test. With the exploration of theses theories, people hope for a discovery that guide them toward a unified theory, so the importance to accelerate particles to higher energy scales.

The LHC has a schedule where there is a period of experiments that last for three or four years. Ending the experimental time, the engineers of the LHC start the maintenance.

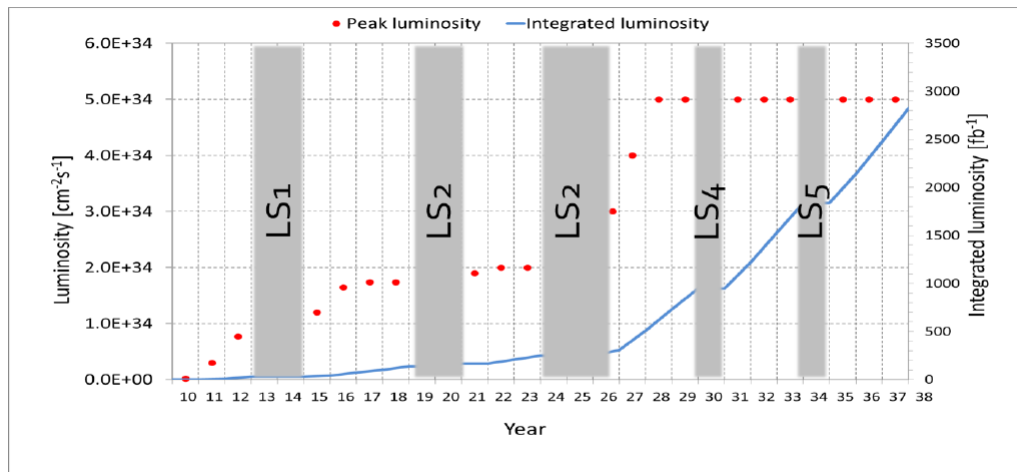


Fig. 2.2 LHC schedule, including future plans for increase the center of mass energy and luminosity. The periods labeled LS are the maintenance periods [2].

From the figure 2.2, shows the Large Hadron Collider forecast for the increase of luminosity for the next years. Red dots are peak luminosity expected to reach and blue line is integrated luminosity. In the year 2018, the maximum integrated luminosity is around  $150 \text{ fb}^{-1}$ [2]. From 2019 to 2021 the second phase of maintenance, where engineers increase the performance of the accelerator, give maintenance to the system and introduce new components to the complex. After that period, the LHC starts new collision period at even higher energies in order to explore new particle phenomena.

## 2.2 The Compact Muon Solenoid

The Compact Muon Solenoid (CMS) is a detector with multiple uses in the LHC and part of the main experiments at CERN. Located underground in the France- Switzerland border , in the city of Cessy, France. This detector was designed in the early 1990s, based on the mass limit of the Higgs boson, and put on operation in 2008, it has a big solenoid that generates a great magnetic field of 4 teslas with the objective to separate particles after a particle collision. The detector is 21 metres long, 15 meters wide, 15 meters high ,it has a diameter of 5.9 m with a weight of 12000 ton. The reason for such a strong magnetic field is to obtain a better momentum resolution . The magnetic flux is returned via a 1.5 m thick iron yoke instrumented with four stations of muon chambers.

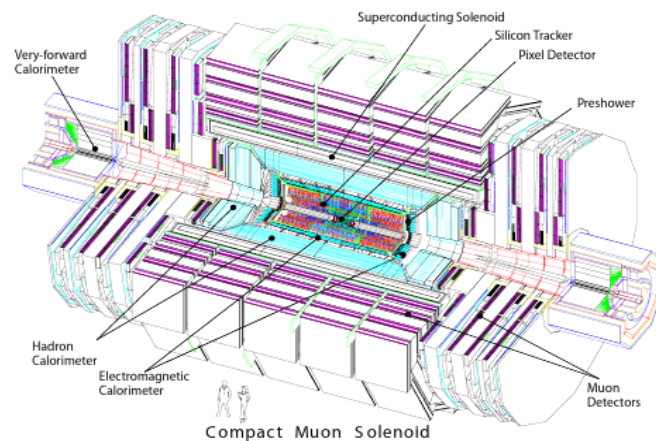


Fig. 2.3 View of Compact muon solenoid (CMS)

There are many layers of detectors to register different kinds of energy signals. But since muons go through materials very easy, the complex is buried 100 meters underground.

The CMS experiment is composed of several detector layers that allow identify and save different types of energy signals and it is saved in a powerful supercomputers that separate and classify the data according to certains variables. The general parts of the CMS are the Silicon Tracker , an Electromagnetic Calorimeter,a Hadron Calorimeter, the solenoid (superconducting magnet) and the Muon Detector.

The objectives of the CMS experiment are numerous. One of them is identification of muons by measuring the momenta and scattering angle. Muons can be produced in interesting events like Higgs,  $W^\pm$  and Z boson decays.This experiment along the ATLAS experiment discovered the Higgs boson in 2012.

Table 2.2 Characteristic of the CMS superconducting solenoid [3]

Field strength	4 T
Inner Bore	5.9 m
Length	12.9 m
Number of Turns	2168
Current	19.5 kA
Stored energy	2.7 GJ

### 2.2.1 Silicon Tracker

The Silicon Tracker is the first of the main subdetectors of CMS from inside to outside. The Tracker is composed of two sub components: Pixel and Strip detectors. The outer radius of the tracker is 110 cm, and its total length is 540 cm. The tracker coverage is  $|\eta| < 2.4^1$ .

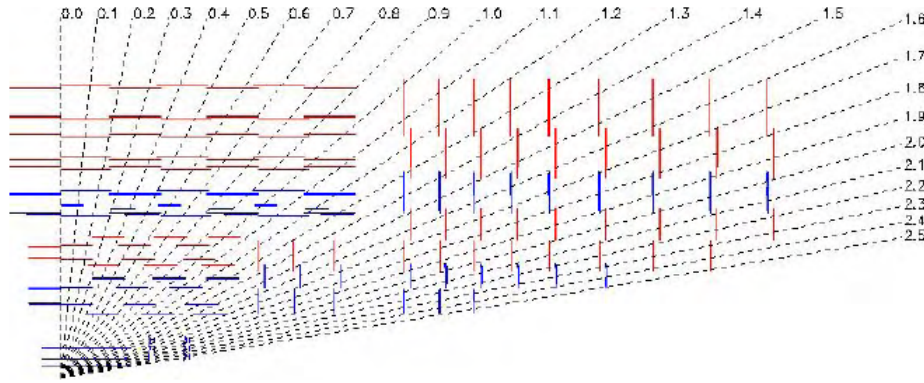
In the Pixel barrel section, there are 3 layers of pixel sensors with radii of 4.4, 7.3 and 10.2 cm with a length of 53 cm. Each pixel has an area of  $100 \times 150 \mu m^2$ . The barrel section has 768 pixel modules. The Pixel endcap has 2 disks on each side placed at 34.6 and 46.5 cm in the z axis. Each disk covers radii from 6 to 15 cm and is divided into 24 blades with 7 modules each blade. They are assembled in a turbine-like geometry that contains a total of 672 pixel modules.

The Strip Detector is divided in 4 sections: Tracker inner barrel (TIB), tracker outer barrel (TOB), tracker encap (TEC) and tracker inner disk (TID). The region that cover the barrel section is  $|z| < 65$  cm for TIB and  $|z| < 110$ cm for TOB. The first two are composed of silicon sensors layers of 4 and 6 layers respectively. The TEC has 9 disk extending in the region  $120 \text{ cm} > |z| > 280$  cm. The TID comprises 3 small disks that fill the gap between the TIB and the TEC.

The Strip Detector have almost 15 400 modules, mounted on carbon-fibre structures and housed inside by a temperature controlled outer support tube. The operating temperature must be around  $-20^\circ\text{C}$ . The total area of the pixel detector is around  $1 \text{ m}^2$  and the silicon strips is  $200 \text{ m}^2$ . The inner tracker comprises 66 million pixels and 9.6 million silicon strips [3]. A full coverage of the Tracker is shown in figure 2.4

During the particle collision, it generates a lot of different particles that pass first through the inner tracker, interacting with the sensors layers and registering the particle path. By reconstructing the path of the particles, it is possible to generate a track, which allows to measure the momentum of the particle by calculating its curvature. The tracker is

<sup>1</sup> $\eta$  , called pseudorapidity, is a coordinate where  $\eta = -\ln \tan \frac{\theta}{2}$

Fig. 2.4 The Tracker layout distributed in terms of  $\eta$ .

317 used to reconstruct the path of charged particles (e.g. electrons, muons, pions). The most  
 318 important objects of study are the momentum and the vertex (origin of the track). The  
 319 efficiency of the tracker is estimated by using samples of muons and pions with  $p_t$  of 1,10  
 and 100 GeV. For muons the efficiency of the tracker is shown in 2.5

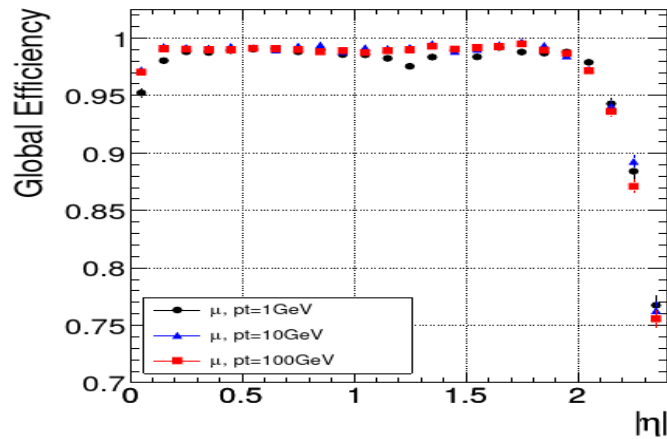


Fig. 2.5 Total reconstruction efficiency for muons

320

321 According to figure 2.5, the efficiency of the tracker is around 98 % for  $|\eta| < 2$ . The  
 322 resolution of the tracker for muons with  $p_t=100$  GeV is around 1-2 %.

323 Of course, this is a first stage of the measurement of particle momentum because the other  
 324 detectors are used to increase and improve the measurement and reconstruction of the  
 325 particle path.

### 326 2.2.2 Electromagnetic calorimeter

327 The electromagnetic calorimeter or ECAL is a calorimeter composed of 61200 lead  
 328 tungstate ( $\text{PbWO}_4$ ) crystals that act as scintillating crystals (emits light when particles  
 329 interact with the crystal). These calorimeters are mounted in the central barrel and closed

by 7324 crystals in the both endcaps. The barrel section of the ECAL covers the range  $|\eta| < 1.479$ . The crystals in the barrel have a pyramid shape and their cross section is  $22 \times 22 \text{ mm}^2$  at the front face and  $26 \times 26 \text{ mm}^2$  at rear face. The crystal length is 230 mm. The EB (barrel section of ECAL) has a inner radius of 129 cm[3]

The endcaps cover have the range  $1.479 < |\eta| < 3.0$ . The distance between the interaction point and the endcap is 3.144 m. The endcap has identically shaped crystals grouped in mechanical units of  $5 \times 5$  crystals. The crystal cross section for the rear face is  $30 \times 30 \text{ mm}^2$ , and for front face cross section is  $28.62 \times 28.62 \text{ mm}^2$  with length of 220 mm.

In the barrel, there photo detectors called avalanche photo diodes (APD), made of silicon

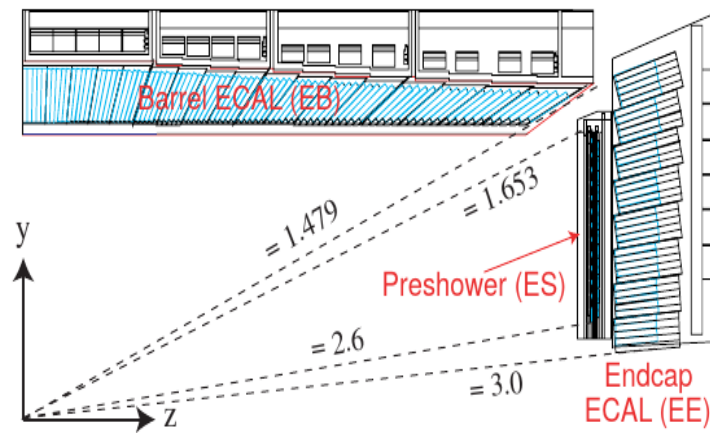


Fig. 2.6 View of the ECAL

with a active area of  $5 \times 5 \text{ mm}^2$  with 2 glued to the back of each crystal. At the endcap, the photo detectors are vacuum photo diodes (VPT), made of with an anode of copper mesh ( $10 \mu\text{m}$ ), allowing to operate in the magnetic field. VPT have a size of 25 mm in diameter, glued to the back of each crystal.

There is also a preshower detector, which principal function is to identify neutral pions in the endcaps within the region  $1.653 < |\eta| < 2.6$ . It also helps the identification of electrons against minimum ionizing particles, and improves the position determination of electrons and photons.

As its name says, the ECAL is mainly used for the electron detection, but also detects, photons, and neutral pions. The charged particles reach the crystals that provokes the photo diodes detect energy.

The resolution of the ECAL is estimated by using a test beam and registering the energy signals. The resolution for electrons with energies of 120 GeV, it has obtained a resolution of 0.5%.



### 2.2.3 Hadron calorimeter

The hadron calorimeter (HCAL) along the ECAL, form a complete calorimeter that allows to measure the jets and missing transverse energy. HCAL is located in the barrel and the endcaps, surrounding the ECAL and affected by the magnetic field generated by the solenoid. The Barrel section (HB) and endcap section (HE) cover the pseudorapidity  $0 < |\eta| < 1.3$  and  $1.3 < |\eta| < 3.0$  respectively.

The HB is an assembly of two half barrels, composed of 18 wedges. Each wedge has 17 active layers composed of scintillator tiles with 16 layers of absorber metal (brass and stainless steel). Each tile has a size of  $\Delta\eta\Delta\phi = 0.087 \times 0.087$ . Light of scintillator is collected by a single wave length shifting fiber (WLS) for each scintillator tile. The wedge has a inner radius of 1777 mm to an outer radius of 2876.5 mm.

HE is composed by brass absorber plates ,with the thickness of 78 mm, and a total of 19 scintillator layers with a thickness of 3.7 mm. HE is sectioned in 5 in  $\phi$  to match the barrel wedges. .

The outer barrel hadron calorimeter (HO) consists of layers of scintillator with thickness

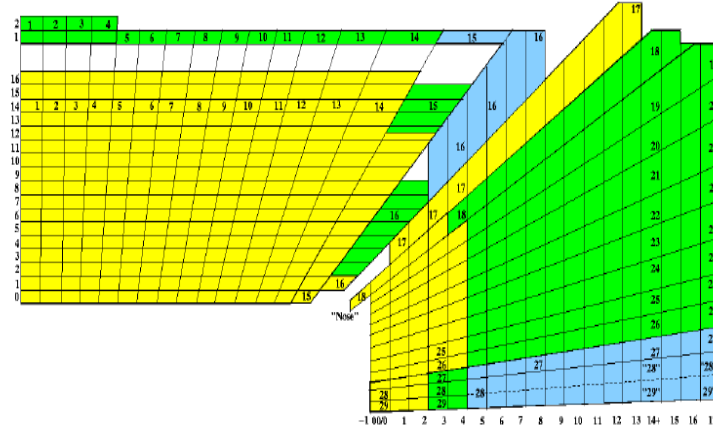


Fig. 2.7 View of the CMS HCAL showing the different layers and tower regions. In the endcap, the towers are defined with two longitudinal segments as shown by the colors

of 10 mm, located outside of the magnet coil that cover the region  $-1.26 < |\eta| < 1.26$ .

There is also a foward calorimeter (HF) located at 11.2 m from the interaction point, that cover the region of  $2.9 < |\eta| < 5$ . They are made of steel absorbers and embedded radiation hard quartz fibers, which collects Cherenkov light[3].

The tiles are arranged in a tower pattern in the  $\eta - \phi$  space, projective to the interaction point. In total there are 4176 towers. The towers are used as input to several jet reconstruction algorithms.

381 The HCAL measures the energy of hadrons, such as the pions and kaons. The energy  
 382 resolution of the HCAL was estimated using a test beam of pions. For 100 GeV pions, the  
 383 resolution obtained is 12%.

## 384 2.2.4 Muon Detector

385 The Muon Detector has three main subdetectors: the drift tubes chambers (DT), the cathode  
 386 strips chambers (CSC) and the resistive plate chamber (RPC). The DT's, CSC's and RPC's  
 387 are shown in the figure 2.8.

388  
 389 The DT chambers cover the barrel section  $|\eta| < 1.2$ , composed of rectangular gas filled  
 390 active cells. Those cells have a transverse size of  $42 \times 13 \text{ mm}^2$  with a  $50 \mu\text{m}$  diameter  
 391 anode wire at the center that operates at voltages of over 3600 V. The gas used in this cells  
 392 is a mix of Ar and  $\text{CO}_2$  with a proportion of 85 % and 15 %. There are 250 chambers in 4  
 393 layers inside the magnet return yoke, with radii of approximately 4.0, 4.9, 5.9 and 7.0 m  
 394 from the beam axis and 12 sections covering  $30^\circ$  in  $\phi$  each one.

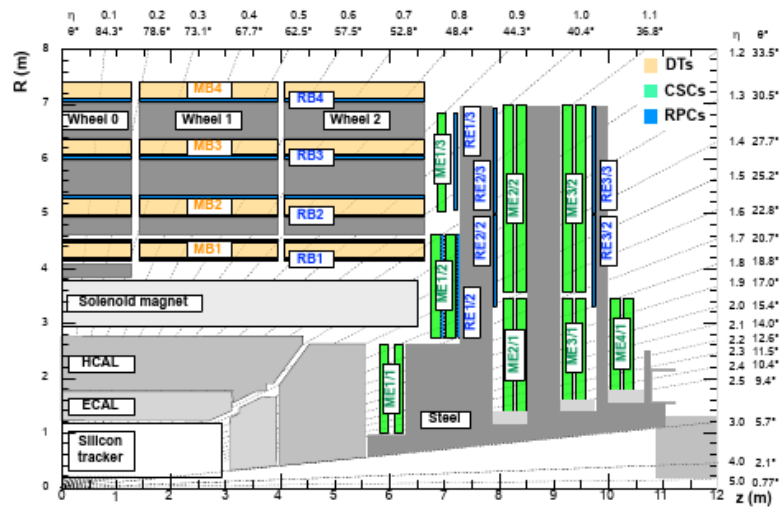


Fig. 2.8 Cross section of a quadrant of the CMS with axis parallel to the beam (z axis) horizontally and the radius of the detector in terms of the  $\eta$ .

395  
 396 The CSC's are located in the endcap regions  $0.9 < |\eta| < 2.4$ . The CSC's are installed  
 397 on the face of steel disks perpendicular to the beam. Each CSC has a trapezoidal shape  
 398 and consist of 6 gas gaps between 7 metal plates, where each gap contains copper cathode  
 399 strips and anode wires running almost perpendicularly to the strips, with a diameter of  $50 \mu\text{m}$   
 400 separated by 3.16 mm. The chambers use a gas mixture of 50%  $\text{CO}_2$ , 40 % Ar and  
 401 10%  $\text{CF}_4$ . The muon endcap system comprises 468 CSCs in the 2 endcaps [3, 16].

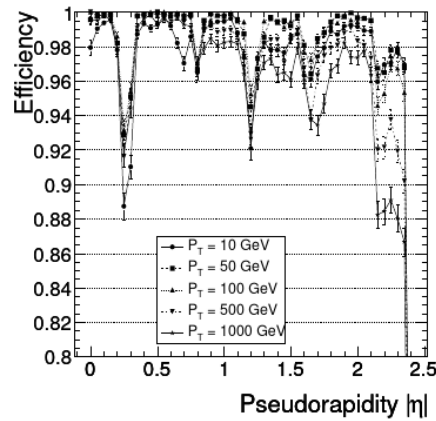


Fig. 2.9 Muon reconstruction efficiency in terms of  $\eta$  and  $p_t$  [3]

403 The RPC are located in the barrel and endcap regions that covers the range of  $|\eta| < 1.6$ .  
 404 The main purpose is to trigger events with muons. RPC's are structured with a double gas  
 405 filled gap and readout strips between the gaps .  
 406 The gas mix used in the RPC consist of 95.2% Freon ( $C_2H_2F_4$  ), 4.5% isobutane ( $C_4H_{10}$ ),  
 407 and 0.3% sulphur hexafluoride ( $SF_6$  ) [16].

408

409 The main purpose of the Muon System Detector is to identify muon tracks. The  
 410 efficiency of the muon system is estimated by using single muon samples simulated with  
 411  $p_t=10, 50, 100, 500$  and  $1000$  GeV. The results are shown in the figure 2.9. The graph  
 412 shows an efficiency of approximately 98% in the the fully instrumented regions. In the  
 413 intersection regions, there are drops in efficiency due to the separation of the chambers.

# Chapter 3

## Event reconstruction and selection

During the proton collision, many types of processes happen whose information are saved in the CMS data storage system. But the events are only saved if they fulfill certain conditions compatible with the signal. For the  $tH$  process, the search is based on the presence of a pair of muons with the same sign. The rest of the processes that also generates a pair of muons with the same sign will be considered backgrounds.

### 3.1 Signal Event Topology

In this search, top quark decays to  $Wb$  and from the  $W$  boson decays to a muon and a neutrino. The Higgs decays to a pair of opposite sign  $W$  bosons, where one of the bosons can decay to a  $\mu$  and its neutrino. This is the main process that generates events with two same sign muons. The  $tH$  process topology is shown in the figure 3.1.

Figure 3.1, also shows a b-jet that comes from the  $tH$  process. Finally there is a forward quark jet (highest  $\eta$  value) generated from the initial collision. Additional jets or leptons can be generated from the other  $W$  boson.

Due to the small cross section of the  $tH$  process, the amount of  $tH$  events is low compared to other Higgs production processes. Table 3.1 shows several processes that generate same sign muon events for  $tH$ . These numbers do not consider the detection efficiency.



Fig. 3.1 Topology of  $tH$  process which generates two same sign muons , a forward jet and a b-jet.

## 3.2 Backgrounds

Several processes contribute to the background in this search:

- $t\bar{t}W^\pm$  and  $t\bar{t}Z$  ( $t\bar{t}V$ ): One muons comes from a top and the other comes from the vector boson.
- $W^+Z$ : Diboson production with leptonic decays. One muon comes from  $W$  boson and the other from  $Z$  boson.
- $W^\pm W^\pm$ : A pair of same sign  $W$  bosons generate a muon each one.
- $tZq$ : Processes with single top quarks associated with a  $Z$  boson, where  $Z \rightarrow \mu^+ \mu^-$ , also contribute to the background.
- $t\bar{t}t\bar{t}$ : In these type of events, one  $t$  decays to  $Wb$  and  $W$  decays to a muon. The second muon comes from the other top decay.
- $W^+W^-Z$ ,  $ZZZ$  and  $W^+ZZ$  ( $VVV$ ): The leptonic decays of the  $W$  or  $Z$  bosons generate at least two same sign muons.
- $tZW^+$ : One muon comes from the  $Z$  while the other same sign muon can come from  $t$  or  $W$ .

- **$ZZ$** : Each muon comes from  $Z$  boson decays.
- **$t\bar{t}H$** : This Higgs production mechanism is considered a background in this analysis. The muons in this cases comes from one top and Higgs decays.
- ***Fakes***: This background refers to events where two muons come from  $b$  meson decays in jets.

Table 3.2 show the decay chains and production cross section for background. Due to small expected event yields, the processes  $W^\pm W^\pm, tZq, t\bar{t}t\bar{t} VVV, tZW^+$  and  $ZZ$  are grouped as one called Rares in the results below.

Table 3.1 Expected number of events for different  $tH$  decay chains assuming integrated luminosity of  $35.9 \text{ fb}^{-1}$ .  $l$  represents  $\mu^\pm, e^\pm, \tau^\pm$ .

Decay chain	BR	Events
$tH \rightarrow W^+ b W^+ W^- \rightarrow \mu^+ \nu_\mu b \mu^+ \nu_\mu q \bar{q}'$	$2.096 \times 10^{-3}$	1.173
$tH \rightarrow W^+ b W^+ W^- \rightarrow \mu^+ \nu_\mu b \mu^+ \nu_\mu l^- \bar{\nu}_l$	$3.37 \times 10^{-4}$	0.899
$tH \rightarrow W^+ b \tau^+ \tau^- \rightarrow \mu^+ \nu_\mu b \mu^+ \nu_\mu \bar{\nu}_\tau l^- \bar{\nu}_l \nu_\tau$	$3.637 \times 10^{-4}$	0.203
$tH \rightarrow W^+ b W^+ W^- \rightarrow \tau^+ \bar{\nu}_\tau b \mu^+ \nu_\mu q \bar{q} \rightarrow \mu^+ \nu_\mu \bar{\nu}_\tau \bar{\nu}_\tau b \mu^+ \nu_\mu q \bar{q}$	$1.890 \times 10^{-4}$	0.105
$tH \rightarrow W^+ b \tau^+ \tau^- \rightarrow \mu^+ \nu_\mu b \nu_\tau \mu^+ \nu_\mu \bar{\nu}_\tau q \bar{q}$	$1.681 \times 10^{-4}$	0.094
$tH \rightarrow W^+ b W^+ W^- \rightarrow \tau^+ \bar{\nu}_\tau b \mu^+ \nu_\mu l^- \bar{\nu}_l \rightarrow \mu^+ \nu_\mu \bar{\nu}_\tau \bar{\nu}_\tau b \mu^+ \nu_\mu l^- \bar{\nu}_l$	$3.045 \times 10^{-5}$	0.017
$tH \rightarrow W^+ b Z Z \rightarrow q \bar{q} b Z Z \rightarrow q \bar{q} b \mu^+ \mu^- \mu^+ \mu^-$	$1.966 \times 10^{-5}$	0.011
$tH \rightarrow W^+ b \tau^+ \tau^- \rightarrow \tau^+ \bar{\nu}_\tau b \mu^+ \nu_\mu \bar{\nu}_\tau q \bar{q}' \nu_\tau \rightarrow \mu^+ \nu_\mu \bar{\nu}_\tau \bar{\nu}_\tau b \mu^+ \nu_\mu \bar{\nu}_\tau q \bar{q}' \nu_\tau$	$1.549 \times 10^{-5}$	0.008

Table 3.2 Main backgrounds and their same sign  $\mu\mu$  decay process

Background	Decay process
$t\bar{t}W$	$t\bar{t}W \rightarrow W^+ b W^- \bar{b} \mu^+ \nu_\mu \rightarrow \mu^+ \nu_\mu b \mu^- \bar{\nu}_\mu \bar{b} \mu^+ \nu_\mu$
$t\bar{t}Z$	$t\bar{t}Z \rightarrow W^+ b W^- \bar{b} \mu^+ \mu^- \rightarrow \mu^+ \nu_\mu b \mu^- \bar{\nu}_\mu \bar{b} \mu^+ \mu^-$
$W^+ Z$	$W^+ Z \rightarrow \mu^+ \nu_\mu \mu^+ \mu^-$
$W^\pm W^\pm$	$W^+ W^+ \rightarrow \mu^+ \nu_\mu \mu^+ \nu_\mu$
$tZq$	$tZq \rightarrow W^+ b \mu^+ \mu^- q \rightarrow \mu^+ \nu_\mu b \mu^+ \mu^- q$
$t\bar{t}t\bar{t}$	$t\bar{t}t\bar{t} \rightarrow W^+ b W^- \bar{b} W^+ b W^- \bar{b} \rightarrow \mu^+ \nu_\mu b \mu^- \bar{\nu}_\mu \bar{b} \mu^+ \nu_\mu b \mu^- \bar{\nu}_\mu \bar{b}$
$W^+ W^- Z$	$W^+ W^- Z \rightarrow \mu^+ \nu_\mu \mu^- \bar{\nu}_\mu \mu^+ \mu^-$
$ZZZ$	$ZZZ \rightarrow \mu^+ \mu^- \mu^+ \mu^- l^+ l^-$
$W^+ ZZ$	$W^+ ZZ \rightarrow \mu^+ \nu_\mu \mu^+ \mu^- l^+ l^-$
$tZW^+$	$tZW^+ \rightarrow W^+ b \mu^+ \mu^- \mu^+ \nu_\mu \rightarrow \mu^+ \nu_\mu b \mu^+ \mu^- \mu^+ \nu_\mu$
$ZZ$	$ZZ \rightarrow \mu^+ \mu^- \mu^+ \mu^-$
$t\bar{t}H$	$t\bar{t}H \rightarrow W^+ b W^- \bar{b} W^+ W^- \rightarrow \mu^+ \nu_\mu b \mu^- \bar{\nu}_\mu \bar{b} \mu^+ \nu_\mu \mu^- \bar{\nu}_\mu$

### 3.3 Event Selection

In order to detect signal events and reject background, the following selections are applied.

- The events must contain two muons with the same sign.
- Transverse moment  $p_t > 25$  GeV for the highest  $p_t$  muon and  $p_t > 15$  GeV for the lowest  $p_t$  muon.
- A forward jet with  $p_t > 40$  GeV and  $|\eta| > 2.4$
- One or more b-tagged jets with  $|\eta| < 2.4$

The number of events that passed the event selection are shown in table 3.3 which corresponds to a integrated luminosity of  $35.9 \text{ fb}^{-1}$  [11]. The backgrounds with the most events are  $t\bar{t}W^\pm$ ,  $t\bar{t}Z$  and Fakes. For the SM signal  $tH$ , the number of expected events is 2.14, while for the inverted coupling scenario ( $k_t = -1$ ) the cross section is enhanced by a factor of approximately 10. This event yields will be used to estimate the signal sensitivity. The yields include statistical uncertainties due to the simulated samples using Monte Carlo (MC) samples and systematic uncertainties are described below.

Table 3.3 Event yields for signal and backgrounds after the event selection for a integrated luminosity of  $35.9 \text{ fb}^{-1}$ . The uncertainties of yields include statistical and systematic

Process	Number of events
$t\bar{t}W$	$68 \pm 10$
$t\bar{t}Z$	$25.9 \pm 3.9$
$WZ$	$15.1 \pm 7.7$
Rares	$20.9 \pm 4.9$
Fakes	$80.9 \pm 9.4$
$t\bar{t}H$	$24.2 \pm 2.1$
$tH$ (SM)	$2.14 \pm 0.13$
$tH$ ( $k_t = -1$ )	$26.2 \pm 2.2$

### 3.4 Systematic uncertainties

The uncertainties on  $t\bar{t}W$  and  $t\bar{t}Z$  event yields are mainly due to the uncertainties of their production cross sections. The uncertainty on  $WZ$  background is estimated using real data events in a three lepton control region. In the Rare backgrounds, which are not measured, a 50% of uncertainty is assigned. The uncertainty on the Fakes background is estimated using real data in a control region, defined by the muon identification criteria. For the Higgs processes  $tH$  and  $t\bar{t}H$ , the uncertainty are due to the theoretical parameters used in that simulation.



### 3.5 Multivariable discriminant

Due to small signal to background ratio caused by the small cross section of  $tH$ , a multivariable discriminant, which separates signal from backgrounds, is necessary to optimize the signal sensitivity. The multivariable discriminant is a Boosted Decision Tree (BDT) that takes a set of input features and splits input data recursively based on those features. The features can be a mix of categorical and continuous data. The BDT training is performed using several event variables.

In this analysis, the BDT discriminant was trained to discriminate against  $t\bar{t}V$  background, because this background is one of the largest backgrounds. The variables used for the BDT were the following:

- Number of jets with  $p_T > 25$  GeV,  $|\eta| < 2.4$
- Maximum  $|\eta|$  of forward jet
- Sum of lepton charges
- Number of jets with  $|\eta| > 1.0$
- $\Delta\eta$  between forward jet and b-jet with highest  $p_t$
- $\Delta\eta$  between forward jet and b-jet with lower  $p_t$
- $\Delta\eta$  between forward jet and closest muon
- $\Delta\phi$  of highest  $p_t$  same-sign muon pair
- $\min \Delta R$  (muon pairs)<sup>1</sup>
- $p_t$  of muon with lower momentum

---

<sup>1</sup>  $\Delta R = \sqrt{\Delta\eta^2 + \Delta\phi^2}$ , where  $\Delta R$  is the distance in the  $\eta\phi$  plane

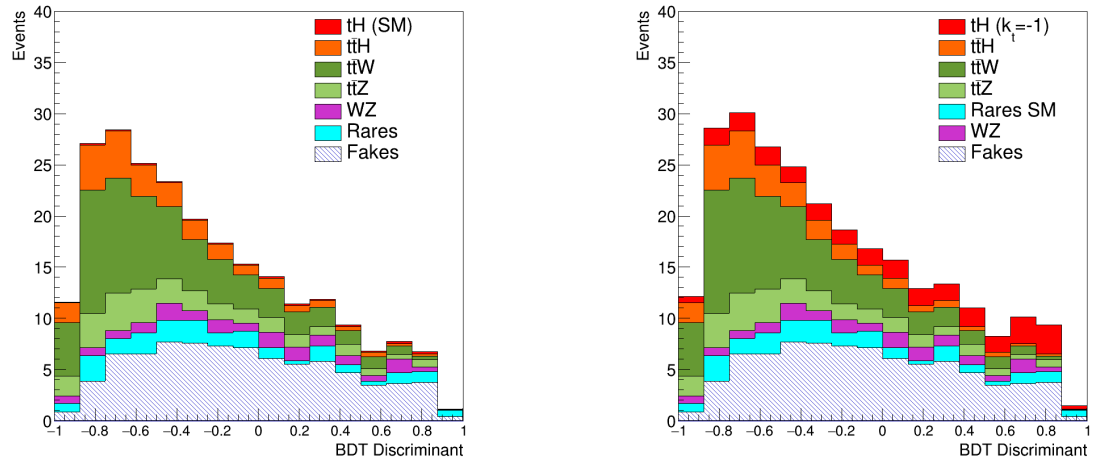


Fig. 3.2 Distribution of BDT discriminant for signal and background in the case of SM (left) and inverted coupling scenario (right)

# Chapter 4

## Statistical Analysis

414 To estimate the sensibility of the  $tH$  signal, we define an Asimov dataset, made by  
415 replacing the ensemble of simulated backgrounds and signal by a single one. The statistical  
416 uncertainty of the Asimov data is calculated as  $\sqrt{n}$ , with  $n$  the number of events. The  
417 uncertainty of the signal strength is estimated by applying a fit to the Asimov dataset,  
418 where the model is constructed from the sum of the individual backgrounds and signal. The  
419 fit is implemented using a Poisson likelihood and gaussian constraints for the systematical  
420 uncertainties in the model.

### 4.1 Likelihood and fit procedure

The likelihood function is the product of Poisson probabilities for all bins of the BDT distribution. The likelihood function has the form

$$L(\mu, \alpha) = \prod_{j=1}^N \frac{(\mu s_j + b_j)^{n_j}}{n_j!} e^{-(\mu s_j + b_j)} \prod_{k=1}^M e^{\frac{-\alpha_k^2}{2}} \quad (4.1)$$

where  $N$  is the total number of bins,  $n_j$  is the number of events in a bin  $j$ ,  $s_j$  is the number of signal events,  $\mu$  is a parameter that modifies the signal strength and  $b_j$  is the number of background events.  $b_j$  is the sum of different background processes  $k$

$$b_j = \sum_k^M b_j^k (1 + \alpha_k \sigma_k) \quad (4.2)$$

422  $\alpha_k$  is the parameter that modifies the expected background prediction and  $\sigma_k$  is the system-  
423 atic uncertainty of the associated background.  $\sigma_k$  for the backgrounds are shown in the  
424 Table 3.3. The fit is applied by minimizing the  $-\log L$  (NLL) with respect to the parameter

425  $\mu$  and  $\alpha_k$ . The minimization is performed by using the package ROOFIT [17]. In the  
426 figure 4.1 and Table 4.1 , shows the results of the model fitting using the Asimov data for  
427 both models.

428 As mentioned before, for the  $k_t=-1$  the number of events for signal is more than ten times  
429 compared to SM. This improves the sensitivity to the signal. Due to the small number of  
430 signal events the uncertainty for the SM is large compared to the  $k_t=-1$  scenario, where the  
431 uncertainty is around 50%. In this analysis only the signal is affected by the change  $k_t=1$   
432 (SM)  $\rightarrow k_t = -1$ , therefore the backgrounds remain the same.

433 In the Table 4.2, shows the  $\mu$  and  $\alpha$  parameters. For the pre-fit, values of  $\alpha$  are set to  
434 zero and  $\mu$  is set to 1. After the fit,  $\alpha$  parameters set to zero indicates that in the fit, the  
435 values of the backgrounds didn't changed for the SM and the  $\mu$  parameter set to 1 indicates  
436 that the signal strength also didn't changed.

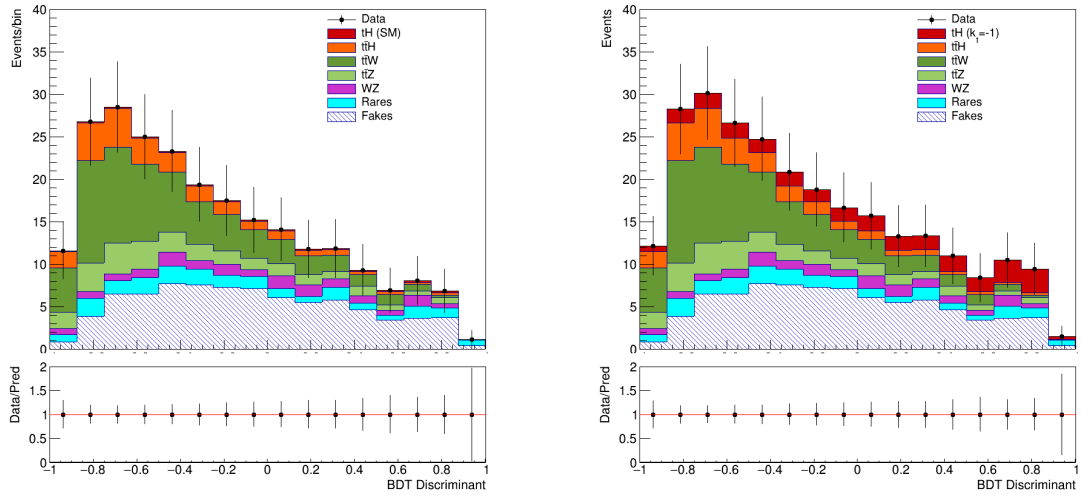


Fig. 4.1 Post fit signal and background yields for  $t\bar{t}H$  process for SM (Left) and  $k_t = -1$  (Right).

Table 4.1 Postfit yields for the fit to the Asimov data corresponding to  $35.9 \text{ fb}^{-1}$ . The uncertainty given is the combined statistical plus systematic.

Process	SM	$k_t = -1$
$t\bar{t}W$	$68 \pm 8.9$	$68 \pm 8.9$
$t\bar{t}Z$	$25.9 \pm 3.8$	$25.9 \pm 3.8$
$WZ$	$15.1 \pm 7.4$	$15.1 \pm 7.4$
Rares	$20.8 \pm 4.8$	$20.8 \pm 4.8$
Fakes	$80.9 \pm 9.0$	$80.9 \pm 8.9$
$t\bar{t}H$	$24.2 \pm 2.0$	$24.2 \pm 2.0$
$tH$	$2.1 \pm 16.5$	$26.2 \pm 13.1$

Table 4.2  $\alpha$  and  $\mu$  values for the fit to Asimov data corresponding to  $35.9 \text{ fb}^{-1}$

Parameter	SM	$k_t = -1$
$\mu$	$1.00 \pm 7.74$	$1.0 \pm 0.5$
$\alpha_{t\bar{t}W}$	$0.00 \pm 0.89$	$0.00 \pm 0.89$
$\alpha_{t\bar{t}Z}$	$0.00 \pm 0.98$	$0.00 \pm 0.98$
$\alpha_{WZ}$	$0.00 \pm 0.97$	$0.00 \pm 0.97$
$\alpha_{Rares}$	$0.00 \pm 0.95$	$0.00 \pm 0.98$
$\alpha_{Fakes}$	$0.00 \pm 0.96$	$0.00 \pm 0.95$
$\alpha_{t\bar{t}H}$	$0.00 \pm 0.98$	$0.00 \pm 0.98$

## 4.2 Limit calculation

Due to the large background, the signal strength for the Asimov data with  $35.9 \text{ fb}^{-1}$  is consistent with zero. Therefore, we estimate an upper limit on the signal strength at 95% confidence level.

We can define the likelihood ratio

$$\lambda(\mu, \alpha) = \frac{L(\mu, \alpha)}{L(\hat{\mu}, \hat{\alpha})} \quad (4.3)$$

Where  $\hat{\alpha}$  and  $\hat{\mu}$  are the parameters obtained in the previous section which correspond to the minimal of the NLL. To determine an upper limit on the strength parameter  $\mu$ , we use the following statistical test

$$q(\mu, \alpha) = -2 \ln \lambda \quad (4.4)$$

High values of  $q$  represent greater incompatibility between the data and the fit model.  $q$  is a random variable with a  $\chi^2$  distribution [18]. In order to find the upper limit value of  $\mu$ , we search for the largest value of  $\mu$  such that  $q$  remains below the shaded region shown in Fig. 4.2, where  $\alpha=0.05$  in this case. It is useful to scan  $q$  as function of  $\mu$ , which is normally a parabolic function as shown in 4.3. In the SM case, the scan has a wider shape in comparison to the  $k_t=-1$ . This is due to larger statistical uncertainty on the  $\mu$  parameter. Using the above technique, we find an expected upper limit  $\mu < 17.0$  for SM model and  $\mu < 2.3$  for  $k_t=-1$  model.

## 4.3 Extrapolation for higher luminosities

In the past section, it was shown that the signal strength with an integrated luminosity of  $35.9 \text{ fb}^{-1}$  is not significant. The LHC has already collected data for integrated luminosity of  $150 \text{ fb}^{-1}$  for run 2 finished in 2018. Run 3 that starts in 2021 and is expected an integrated luminosity of  $300 \text{ fb}^{-1}$  and for finally the high luminosity phase of LHC, is expected to collect  $3000 \text{ fb}$ . Therefore, is interesting to study the sensibility of the signal strength to higher luminosities. An extrapolation (scale) is applied to the Asimov data and the statistical uncertainty is adjusted accordingly. For this analysis we assume the systematic uncertainties assigned to the background are the same.

In the figures 4.4, it is shown an estimated prediction of the signal and background events for higher luminosities. As the luminosity increases, the number of events for both

signal and backgrounds also increases, as previously explained in the chapter 1. But even with the increase of number of events, the statistical uncertainty of the data is too large with respect to the expected signal. In the likelihood scan for higher luminosities, as the luminosity increases, the relative uncertainty on the signal strength  $\mu$  decreases. This is caused by the increased events available for the fit.

The same method is used for the model  $k_t=-1$  in the figure 4.5. In this case, the number of events for the signal is more appreciable in comparison with the SM model. However, the likelihood scan parabolas are very closed and that indicates a decrease in the uncertainty of  $\mu$  more notable than SM case. Of course, as the previous section, we estimate the upper limit for these cases and they are shown in the Table 4.3

However, the uncertainty of  $\mu$  for the SM case is still very big, with a error of 170% in the  $3000 \text{ fb}^{-1}$  case, indicating that it is necessary to increase the luminosity even higher to get better results to reduce the uncertainty on  $\mu$  for a possible observation of the signal. In the  $k_t=-1$  model the uncertainty of  $\mu$  decreases with higher luminosities giving better results; upper limits in this models are small. This is caused by the fact that the number of events for the  $tH$  signal is greater. For  $3000 \text{ fb}^{-1}$  the uncertainty on  $\mu$  is 10%, this indicates that for large integrated luminosities this model can be fully ruled out.

Table 4.3 Results of  $\mu$  and upper limits for Asimov extrapolations for SM and  $k_t=-1$  models.

Luminosity ( $\text{fb}^{-1}$ )	$\mu$ (SM)	$\mu$ (SM) upper limit	$\mu$ ( $k_t=-1$ )	$\mu$ ( $k_t=-1$ ) upper limit
35.9	$1.0 \pm 7.7$	17	$1.0 \pm 0.5$	2.3
150	$1.0 \pm 6.7$	11	$1.0 \pm 0.4$	1.8
300	$1.0 \pm 4.3$	8.7	$1.0 \pm 0.3$	1.5
3000	$1.0 \pm 1.7$	4.3	$1.0 \pm 0.1$	1.1

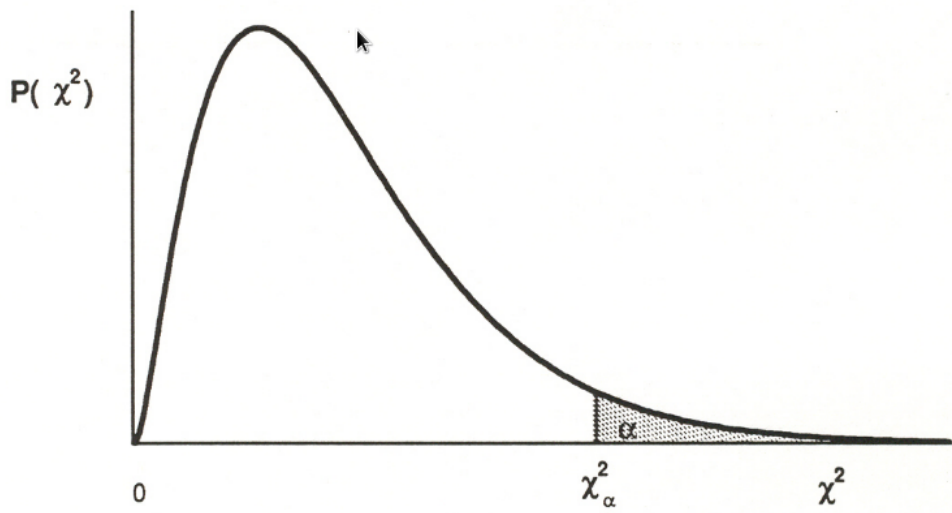


Fig. 4.2 Illustration of the  $\chi^2$  distribution with number of degrees of freedom of 7 used for the limit estimation

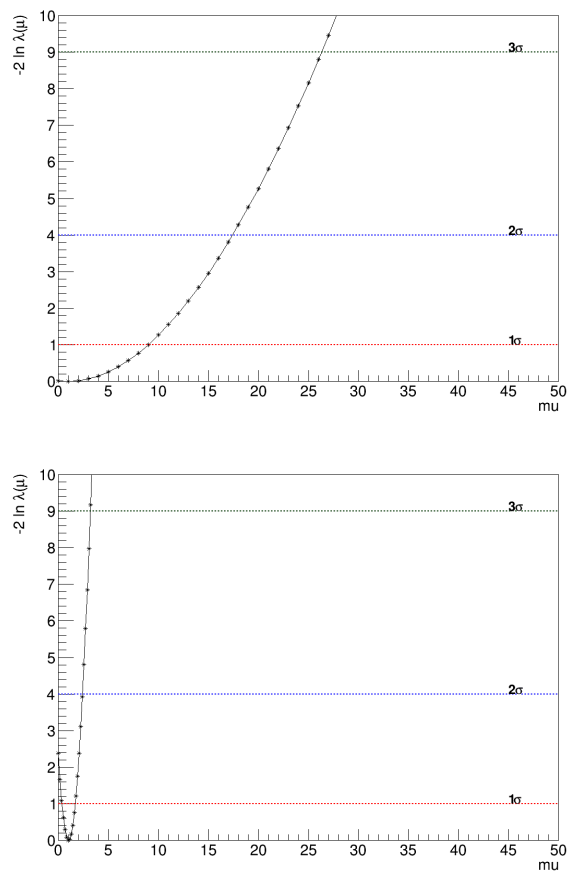


Fig. 4.3 Likelihood scan for SM (up) and  $k_t = -1$  (Down).



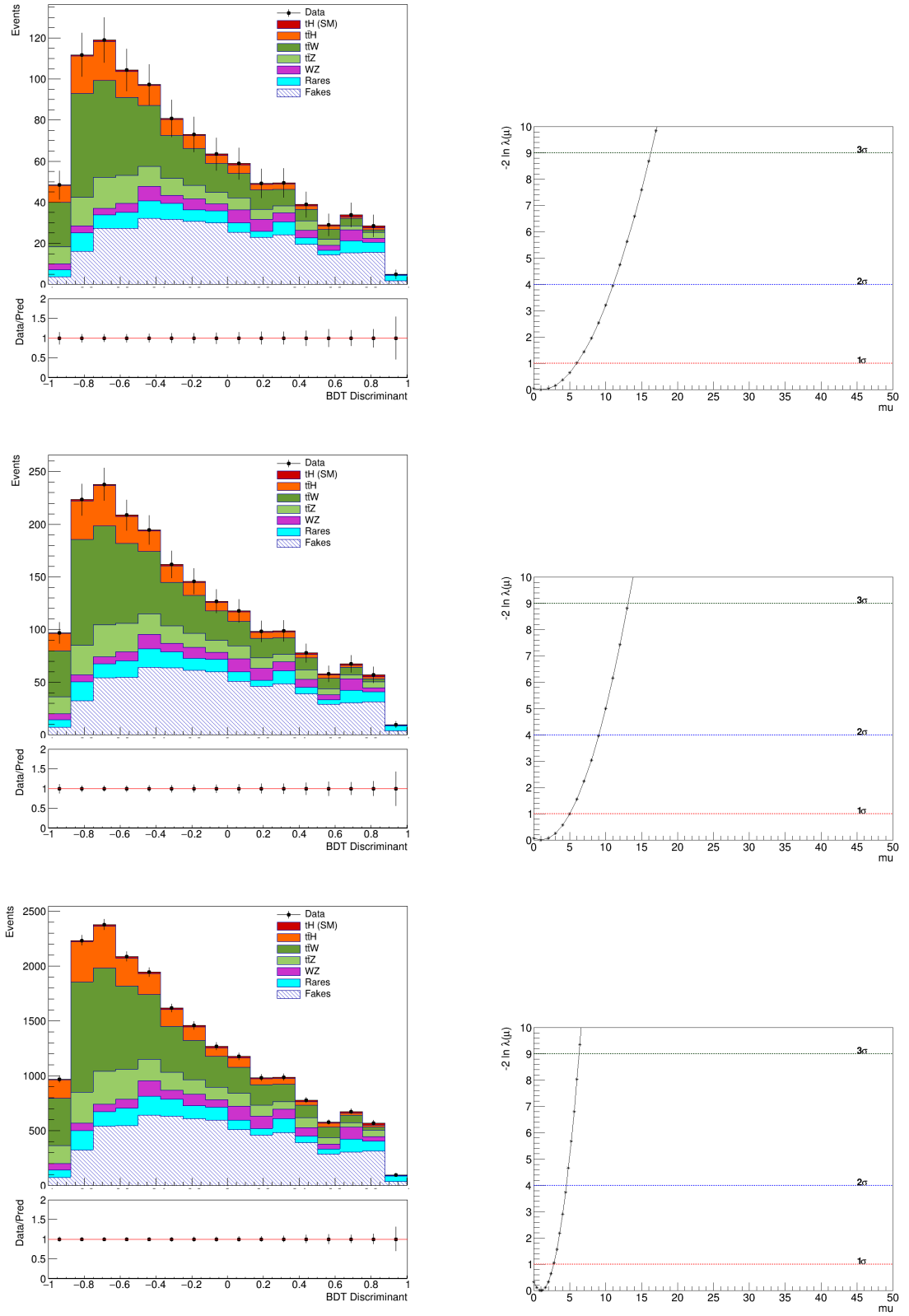


Fig. 4.4 Fit and likelihood scan for 150 fb<sup>-1</sup>, 300 fb<sup>-1</sup> and 3000 fb<sup>-1</sup> in SM model.

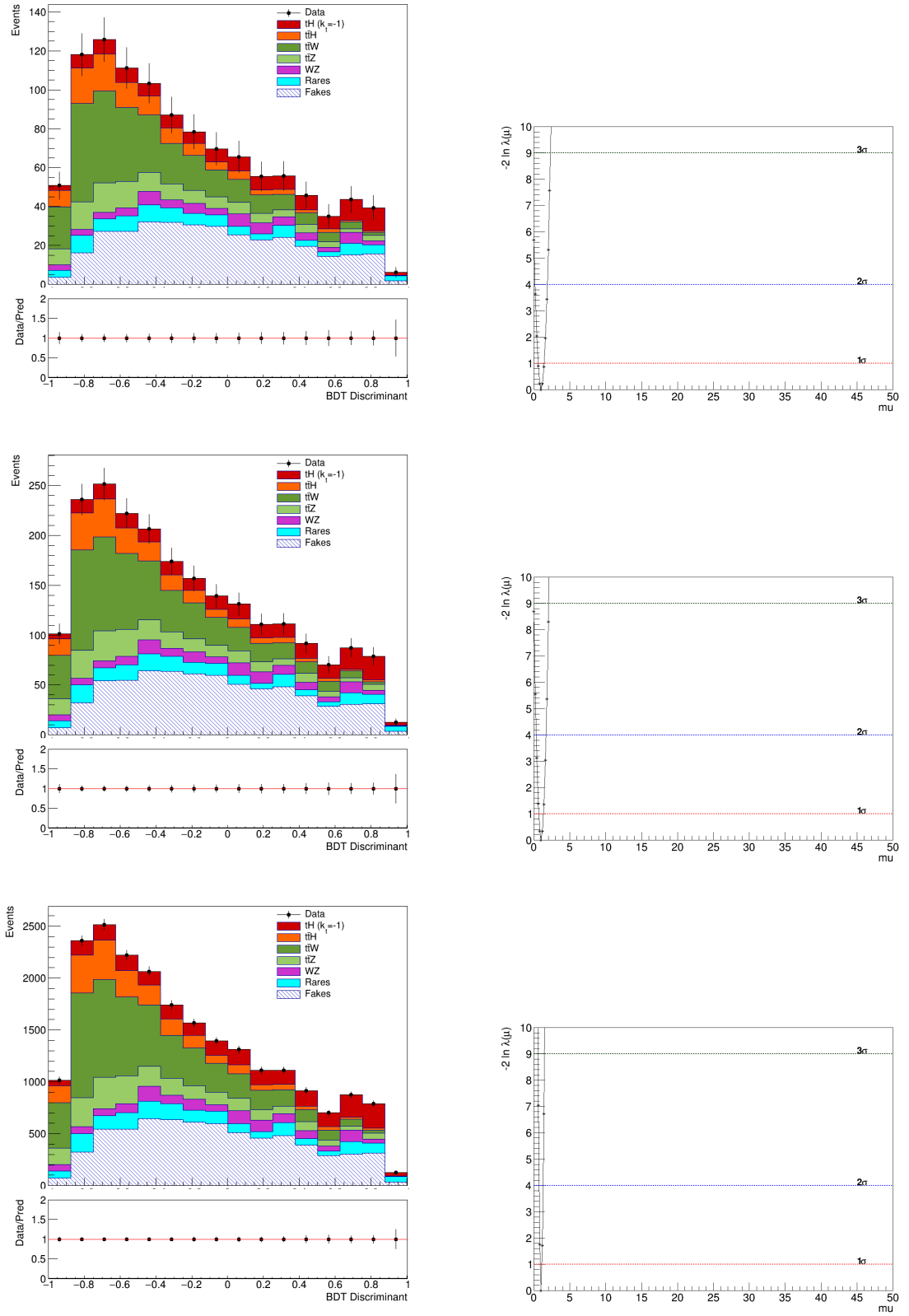


Fig. 4.5 Fit and likelihood scan for  $150 \text{ fb}^{-1}$ ,  $300 \text{ fb}^{-1}$  and  $3000 \text{ fb}^{-1}$  in  $k_t = -1$  model.

# Chapter 5

## Conclusion and outlook

In this work, we explored the  $tH$  processes and its different decay rates. The  $tH$  process is one of the processes that comes from the proton-proton collisions in the LHC. This process has not been observed experimentally. Recently CMS published a search for  $tH$  process using  $35.9 \text{ fb}^{-1}$  from Run 2 in 2018.

MC simulation and real data or even only MC can be used to study sensitivity different signal processes. In this work we used the information from CMS to study the expected sensitivity using an Asimov data and systematic uncertainties in the final state of a pair of same sign muons.

In the SM case, the signal strength for the signal  $tH$  is very low in comparison to the backgrounds, resulting in large uncertainties. However, for the inverted coupling scenario ( $k_t=-1$ ), the uncertainty for the model after the fit was better than the SM case, this is due to the larger expected number of signal events.

We also studied the sensitivity for larger integrated luminosities expected in future runs of the LHC. We find that for the SM scenario the expected uncertainty even at the largest luminosities is not enough to observe the signal. In the case of the inverted coupling scenario, a possible signal can be observed with 10% uncertainty at  $3000 \text{ fb}^{-1}$ .

All the research made in the CMS at CERN has let us discover the nature of the matter and try to answer questions about the origin of the universe, including the origin of the mass of the things.

# References

- [1] Search for the  $t\bar{t}(H \rightarrow b\bar{b})$  process in pp collisions at  $\sqrt{s} = 13$  TeV and study of Higgs boson couplings. Technical Report CMS-PAS-HIG-17-016, CERN, Geneva, 2018.
- [2] O. Brüning P. Fessia M. Lamont L. Rossi L. Taviani G. Apollinari, I. Béjar Alonso. High-luminosity large hadron collider (hl-lhc). technical design report v. 0.1,. *CERN Yellow*, (CERN-2017-007-M), 2017.
- [3] G L Bayatian, S Chatrchyan, and Et Al. Hmayakyan. *CMS Physics: Technical Design Report Volume 1: Detector Performance and Software*. Technical Design Report CMS. CERN, Geneva, 2006.
- [4] L.D. Landau y E.M. Lifschitz. *Mechanics*, volume 3 of *Course of Theoretical Physics*. Butterworth Heinemann, third edition, 1966.
- [5] Frank Gross. *Relativistic quantum mechanics and field theory*. WILEY-VCH Verlag GmbH Co. KGaA, first edition, 1998.
- [6] Tanabashi M. Et Al. *Review of Particle Physics*, volume 98. American Physical Society, Aug 2018.
- [7] D.J. Griffiths. *Introduction to elementary particles*. WILEY-VCH Verlag GmbH Co. KGaA, second edition, 2008.
- [8] Mark Thompson. *Modern particle physics*. Cambridge University press, first edition, 2013.
- [9] Chatrchyan et al. Observation of a new boson at a mass of 125 gev with the cms experiment at the lhc. *Physics Letters B*, 716(1):30 – 61, 2012.
- [10] Stephen Parke Tom Schwarz Frédéric Déliot, Nicholas Hadley. Properties of top quark. *arXiv.org*, (arXiv:1803.00656):45, 2018.
- [11] The CMS Collaboration. Search for production of a higgs boson and a single top quark in multilepton final states in proton collisions at  $\sqrt{s} = 13$  tev. *The CMS Collaboration*, (CMS PAS HIG-17-005), May 2017.
- [12] S. Dawson. Introduction to electroweak symmetry breaking. *arXiv.org*, (arXiv:hep-ph/9901280):30, 1999.

- 
- [13] PAUL LANGACKER. Introduction to the standard model and electroweak physics. *arXiv.org*, (arXiv:0901.0241):45, 2009.
- [14] LHC experiment. The large hadron collider. <https://home.cern/science/accelerators/large-hadron-collider>, 2019.
- [15] CMS experiment. The cms detector. <https://cms.cern/detector>, 2019.
- [16] The CMS collaboration. The performance of the cms muon detector in proton-proton collisions at  $\sqrt{s}=7$  tev at the lhc. *Journal of Instrumentation*, 8(11):P11002–P11002, nov 2013.
- [17]
- [18] Eilam Gross Ofer Vitells Glen Cowan, Kyle Cranmer. Asymptotic formulae for likelihood-based tests of new physics. *arXiv.org*, (arXiv:1007.1727v3):32, 2013.
- [19] Charles Poole Herbert Goldstein and John Safko. *Classical Mechanics*. Addison Wesley Longman Inc., 3 edition, 2002.

# Appendix A

## Scattering and cross section

In classical mechanics, scattering is a set of particles which deviate from their original trajectory to one or many ways. Examples of scattering are particle collisions and particles that deviates in a potential field.

The cross section in a scattering is the area transverse to the relative motion between two object to scatter from each other.[19] For a particle which deviates from its original way due to a potential whose center is at rest, the scattering angle is given by

$$\theta = \pi - 2\phi_0 \quad (\text{A.1})$$

with

$$\phi_0 = \int_{r_0}^{\infty} \frac{M/r^2 dr}{\sqrt{2m(E - U(r) - M^2/r^2)}} \quad (\text{A.2})$$

with  $r_0$  the nearest position to the center of potential and  $U(r)$  is the potential. Now for great velocity , if we change  $E = \frac{1}{2}mv^2$  and  $M = mpv$  and substitute in A.2, we have

$$\phi_0 = \int_{r_0}^{\infty} \frac{\rho/r^2 dr}{\sqrt{1 - \frac{\rho^2}{r^2} - 2U(r)/mv^2}} \quad (\text{A.3})$$

in terms of parameter of impact  $\rho$  and the position[4]. In a scattering of a beam of identical particles with velocity of  $v$  , let  $dN$  the number of particles scattered per unit of time through the angles  $\theta$  and  $\theta + d\theta$  The effective cross section is

$$d\sigma = \frac{dN}{n} \quad (\text{A.4})$$

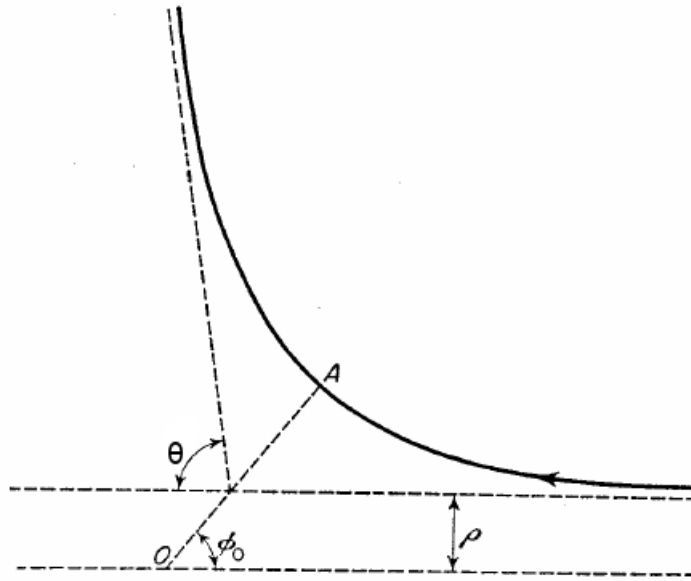


Fig. A.1 Diagram of scattering for a particle in a potential field[4]

$n$  represents the number of particles passing in unit of time in a unit area of beam cross section. If  $\rho$  is  $\theta$  dependent, then the effective cross section is

$$dN = 2\phi\rho n d\rho \quad d\sigma = 2\phi\rho d\rho \quad (\text{A.5})$$

rewriting  $d\sigma$

$$d\sigma = 2\phi\rho \left| \frac{d\rho}{d\theta} \right| d\theta \quad (\text{A.6})$$

the derivative  $\frac{d\rho}{d\theta}$  can be negative ,so it is necessary the modulus [4]. In a three dimension model,  $d\sigma$  is related to the solid angle, instead of  $d\theta$  , so the solid angle between the angles  $\theta$  and  $\theta + d\theta$  is

$$d\Omega = 2\pi \sin\theta d\theta \quad (\text{A.7})$$

thus, the cross section is

$$d\sigma = \frac{\rho(\theta)}{\sin\theta} \left| \frac{d\rho}{d\theta} \right| d\Omega \quad (\text{A.8})$$

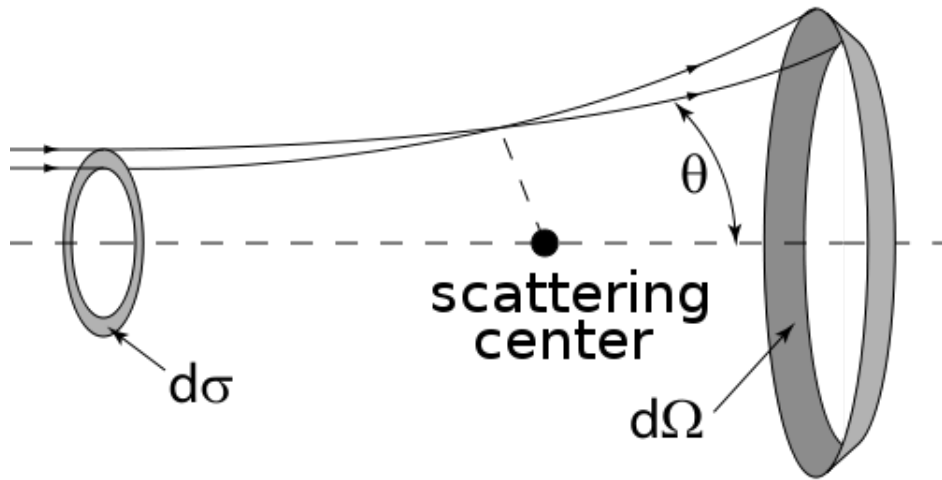


Fig. A.2 Drawing of an idealized scattering process showing the differential solid angle  $d\Omega$  and the scattering angle  $\theta$  [5]

let  $N$  be the number of events. In terms of solid angle

$$dN = L d\sigma = L \frac{d\sigma}{d\Omega} d\Omega \quad \frac{d\sigma}{d\Omega} = \frac{dN}{L d\Omega} \quad (\text{A.9})$$

For a collision of two particles that produce many particles, the cross section is given by

$$\sigma = \frac{S\hbar^2}{4\sqrt{(p_1 \cdot p_2)^2 - (m_1 m_2 c^2)^2}} \int |M|^2 (2\pi)^4 \delta^4(p_1 - p_2 - \dots p_n) \times \prod_{j=2}^n 2\pi \delta(p_j^2 - m_j^2 c^2) \theta(p_j^0) \frac{d^4 p_j}{(2\pi)^4} \quad (\text{A.10})$$

the structure of the equation A.10 is similar to the equation ??[7].

Experimentally

$$d\sigma = \frac{\text{number of particles scattered into solid angle } d\Omega}{(\text{number of particles incident})(\text{scattering centers/area})} \quad (\text{A.11})$$