

Assignment - 2

Section I

Attempt at least two problems from this section.

Problem 1. Find all non-negative integer tuples (n, a, b, c) which satisfy,

$$2^n = a! + b! + c!$$

Problem 2. If p is a prime, then prove that $(p-1)! \equiv -1 \pmod{p}$.

Problem 3. For all primes p , evaluate $\left(\frac{3}{p}\right)$.

Section II

Attempt at least one problem from this section.

Problem 1. Prove that for all natural numbers n ,

$$n! \mid (2^n - 1)(2^n - 2)(2^n - 4) \cdots (2^n - 2^{n-1})$$

Problem 2. Let m, n be odd primes such that $n = 2m + 1 > 2021$. Evaluate,

$$(a) \left| \prod_{k=1}^{mn} \cos \left(\frac{2021^k \pi}{n} \right) \right| \qquad (b) \prod_{k=1}^{mn} \cos \left(\frac{2021^k \pi}{n} \right)$$

Problem 3. Let n be a natural number. Prove that there exists no prime p such that,

$$6^n \equiv -3 \pmod{p} \quad \text{and} \quad p \equiv -1 \pmod{24}$$

Section III

This section contains a list of CP problems and is completely optional.

- (800) GCD Arrays
- (900) GCD Problem
- (1200) LCM
- (1300) T - Primes
- (1600) X - Magic Pair
- (1800) Remainders Game
- (1800) Same GCDs
- (1900) Lucky Days
- (1900) Lost Arithmetic Progression
- (1900) Madoka and the Best School in Russia
- Sum of GCD of Tuples (Hard)
- (2000) Modular Stability
- (2200) GCD Festival