

Saturation of Learning Curves in Defence Acquisition Projects

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Abstract—Recent empirical evidence has shown that learning is a central cost risk factor in defence acquisition projects. The military learning-by-doing literature implicitly assumes that the marginal cost (or time) required to produce the n th unit (e.g., aircraft, ship) will asymptotically approach zero as n increases. It's assumed, in this paper, that the unit cost reaches a steady state where it remains constant over time. A new method that combines statistical analysis and stochastic simulation is suggested to estimate the distribution of the steady-state value. A case study on the acquisition of new warships is used to illustrate the approach. This model is general and can be applied to various defence acquisition projects.

Keywords— *learning curves; Plateau model; cost risk profile; simulation.*

I. INTRODUCTION

Defence organizations are extensively involved with the management of large scale projects and their risks (Ghanmi et al., 2014) [1]. Typical risk factors related to defence acquisition programs may include inflation, foreign exchange, technology maturity, and learning effect. Learning risk is defined as a measure of the potential variation in achieving the expected production efficiency (Sokri and Ghanmi, 2017) [2].

A growing body of literature has begun to use learning curves for proactive decision making in military project evaluation. They are mainly used as approved models in advanced quantitative risk analysis (Thompson, 2001 [3]; Arena et al., 2008 [4]; Kaluzny, 2011 [5]; Sokri and Ghanmi, 2015 [6]; Sokri and Ghanmi, 2017 [2]). These models assume that the cost (or related time) required to produce individual units decreases as production volumes increase. This human performance is due to the efficiency gained by learning and accumulated experience in producing the same system. Learning curve models are fundamentally characterized by two elements:

- 1) *the cost (or time) required to complete the first unit, and*
- 2) *the learning rate at which the unit cost (or time) is expected to decrease over time.*

There are, however, two important limitations to these models that should be recognized: The first is related to the forgetting effect. The second is about their asymptotical values.

A. Forgetting effect

If learning is the essence of progress, forgetting or negative learning is the root of regression. In a given production process, forgetting can reduce production rate and deteriorate the quality of products (Badiru, 1995 [7]; Anzanello and Fogliatto, 2011 [8]). This knowledge depreciation can be attributed to (1) a long production break, (2) changes in products, personnel, tooling, or methods, and (3) interference between old and new learning. The existing literature considers frequent interruptions in the production process as the main cause of forgetting (Hewitt et al, 1992 [9]; Jaber, 2006 [10]; Anzanello and Fogliatto, 2011 [8]).

B. Asymptotical values

In any production process, there is a saturation phase where learning is concluded and costs cannot be improved. However, all the known log-linear models that have been used to assess defence acquisition projects assume that the marginal cost (or time) required to complete the n th unit will asymptotically approach zero as n increases. This unrealistic assumption states that the lower bound of unit cost (or time) is zero.

To illustrate this problem, Figure 1 presents two learning scenarios where the unit cost is expressed in terms of percentage of the first unit cost. In the first scenario, the learning curve tends asymptotically to zero. In the second, the steady-state is reached after a certain number of units.

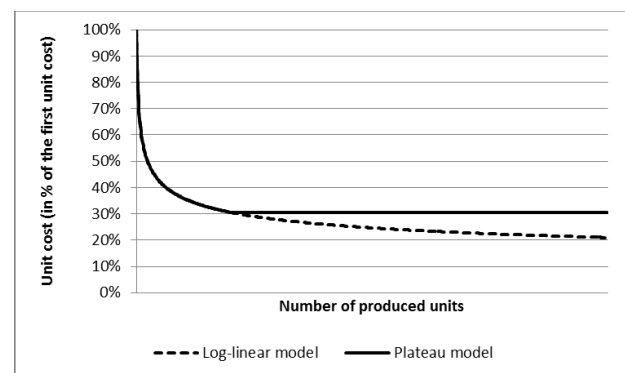


Fig.1. Learning curve with and without saturation phase

To address this limitation, DeJong (1957) [11] was the first to introduce the Plateau model where an additive constant describing the steady-state cost (or time) is added to the log-linear model. However, determining the accurate number of units to reach the standard cost (or time) has always been a perplexing problem. DeJong (1957) [11] and Globerson and Crossman (1976) [12] assume a value of 1000 units or cycles; but there is no basis for their assumption (Dar-El, 2000 [13]; Jaber, 2006 [10]).

This paper suggests a novel method that combines regression analysis and stochastic simulation to estimate the steady-state value of learning curves. The first technique uses different systems to estimate the relationship between the multiplicative inverse or reciprocal for the incompressibility factor and the learning slope. As in DeJong (1957) [11], the incompressibility factor can be defined as the ratio between the standard unit cost (or time) and the first unit cost (or time). This relationship is generally stable and robust (Dar-El, 2000) [13]. The second technique uses simulation to provide a Cumulative Distribution Function (an S-curve) of the steady-state value and insightful sensitivity analysis (Moore et al, 2015) [14].

The paper is organized into four sections. Following the introduction, Section 2 sets up the employed model and indicates its mathematical derivations. Section 3 provides an illustrative example for discussion purposes. Some concluding remarks and directions for future research are indicated in section 4.

II. THE MODEL

This section discusses the learning curve model and presents its mathematical formulation. It also describes how the steady state cost can be derived.

A. The augmented learning curve model

The augmented learning curve model takes into account the effects of learning and production. It includes the rate of production in the current period with the conventional cumulative number of units produced (Sokri and Ghanmi, 2017) [2]. This model can be represented as (Younossi et al., 2007) [15]:

$$LAC_i = C_1 \times (\bar{Q}_i(l))^l \times r_i^p, \quad (1)$$

Where LAC_i denotes the average cost (or related time) of the i^{th} lot and \bar{Q}_i its midpoint. The lot midpoint is the unit whose marginal cost is equal to the lot average cost (Matthew et al., 2003) [16]. r_i is the production rate of lot i . C_1 , l , and p are parameters to be estimated. C_1 represents the cost (or time) of the first unit, l is the learning index and p is the production index. The lot midpoint $\bar{Q}_i(l)$ cannot be calculated without an estimate of the learning index l .

In the shipbuilding sector, ships are generally purchased as individual units and their costs are presented by unit (Sokri,

2015) [17]. In this case, the augmented learning curve model in equation 1 can be modified as follows:

- the lot average cost LAC_i is replaced by the marginal cost required to complete the n^{th} unit (C_n),
- the lot midpoint $\bar{Q}_i(l)$ is replaced by the sequence number of the n^{th} unit in the production run,
- the production rate r_i is equal to one, and
- Equation 1 becomes the most basic log-linear model (Wright, 1936) [18]

$$C_n = C_1 \times n^l. \quad (2)$$

B. The plateau model

As can be seen in equation 3, since $l \leq 0$, the horizontal line at $y = 0$ is a horizontal asymptote of the functions LAC_i and C_n . This states that the average cost of the i^{th} lot and the unit cost of the n^{th} unit asymptotically approach zero as their numbers increase which is unrealistic.

$$\lim_{Q_i \rightarrow \infty} LAC_i = \lim_{n \rightarrow \infty} C_n = 0 \quad (3)$$

The asymptotical value of Wright's model would not be a big issue in the military shipbuilding sector. But it can be a perplexing problem in the aerospace industry, for example. In the military shipbuilding sector, a small number of ships are often purchased as individual units. The sequence number of the n^{th} ship in the production run will never go to "infinity" (Sokri, 2015) [17]. In the aircraft manufacturing industry, hundreds to thousands of aircraft may be produced within the same program (Younossi et al., 2007) [15]. In any production process, there is a saturation phase where learning is concluded and costs cannot be improved.

To extend the log-linear model from a start-up to a plateau model, an additional parameter is used to describe the steady-state. DeJong's model described in equation 4 incorporates the incompressibility factor M ($0 \leq M \leq 1$) that indicates the fraction of the work executed by machines. The other variables are the same as in equation 1.

$$LAC_i = C_1 \times [M + (1 - M) \times (\bar{Q}_i(b))^b] \times r_i^c \quad (4)$$

If $M = 0$, the plateau model reduces to the initial model in equations 1. When $M=1$, there is no learning. In this model, the constant $C = C_1 M$ describes the steady state of workers' performance (Yelle, 1979 [19]; Li and Rajagopalan, 1998 [20]; Anzanello and Fogliatto, 2011 [8]).

C. The steady-state value

The steady-state value can be determined by estimating the relationship between the standard ratio $\frac{C_1}{C}$ and the learning rate l for other projects, namely

$$C_1/C = \beta_0 + \beta_1 l, \quad (5)$$

where β_0 and β_1 are the regression coefficients. As stated in Dar-El (2000) [13], this relationship is generally strong and linear. Assuming that the value of C_1 is known and having estimated the value of the learning rate, one can obtain the appropriate value of the constant C . Its probability distribution can be generated using stochastic simulation. The incompressibility factor M can also be derived as

$$M = C/C_1. \quad (6)$$

III. ILLUSTRATION

Suppose that the Government is considering the acquisition of new warships. One of the cost risk factors to be analyzed is the impact of learning on production efficiency gains. Assume that the cost of the first unit is estimated to be \$600,030 net of taxes. The learning rate is approximately 85% in the shipbuilding sector. This means that a reduction in cost of 15% can be reached with every doubling of production (Stewart and Wyskida, 1995) [21].

The first step in determining the steady state cost is to apply an Ordinary Least Squares (OLS) regression to equation (5) with an error term where the dependent variable is the standard ratio $\frac{C_1}{C}$ and the independent variable is the learning rate l . Table I lists illustrative values of these variables for different projects.

TABLE I. THE STANDARD RATIO AND THE LEARNING RATE

C_1/C	2.79	2.55	2.29	2.06	...	1.37	1.24	6.24	11.42
l	.92	.93	.94	.9599	1	.85	.80

The steady state cost risk profile was determined using results in table II and a stochastic simulation method. In the simulation, a Program Evaluation and Review Technique (PERT) distribution was used to assess the likely fluctuation of each variable. This three-point estimate approach is used when the minimum, the most likely, and the maximum values are known. It is more adequate than the Triangular distribution

when the distribution is skewed (Sokri and Solomon, 2013) [22].

TABLE II. REGRESSION OUTPUT

	Coef.	Std. Err.	Sig.	Min	Max
Const.	47.88	4.89	.000	15.66	80.09
l	-47.78	5.21	.000	-82.15	-13.41

Dependent Variable: C_1/C

In this approach, the steady state cost is thought to be a continuous random variable. Its cumulative distribution function F (CDF) is given in Figure 2.

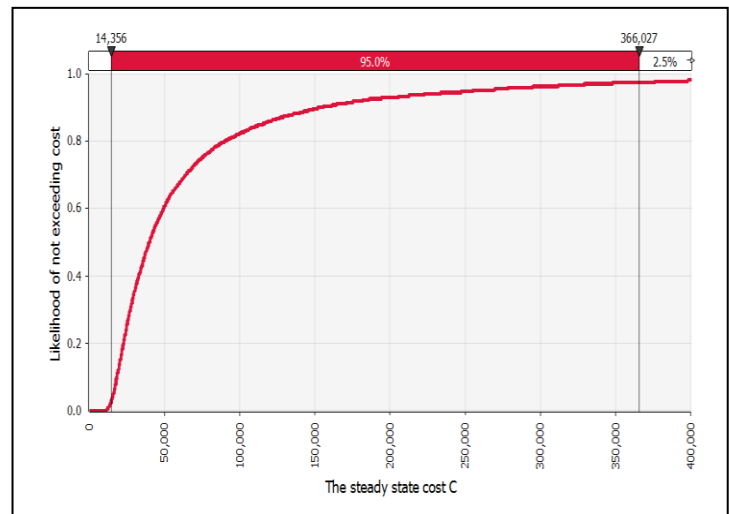


Fig. 2. Distribution of the steady state cost

This graph shows for each cost the likelihood of not exceeding it. It shows, for example, that the probability of achieving a value of \$90,000 is approximately 0.8. In this example, \$90,000 is the 80th percentile. Table III provides the main percentiles and the corresponding cumulative probabilities for the steady state cost variable.

TABLE III. PERCENTILES FOR THE STEADY STATE COST (in \$000)

CDF	5%	25%	50%	60%	80%	99%
Steady state	15.97	24.72	39.82	49.24	90.23	476.57

IV. CONCLUSION

Learning is a fundamental cost risk factor in defence acquisition projects. The existing literature implicitly assumes that unit cost (or time) declines at a uniform rate with a lower bound of zero. The Plateau model has been used to address this limitation. In this model an additive constant describing the standard cost (or time) is added to the log-linear model.

Determining the accurate value of the steady-state cost has, however, been a perplexing problem.

This paper proposes a new method to estimate the steady-state value of learning curves. This approach combines statistical analysis and stochastic simulation within a single mathematical model. The statistical analysis estimates the relationship between the multiplicative inverse for the incompressibility factor and the learning rate. The stochastic simulation provides a Cumulative Distribution Function of the steady-state value. A case study is also used to illustrate the approach.

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