

PROBABILITY

Probability is a branch of mathematics concerned with the analysis of random phenomena. The outcome of a random event cannot be determined before it occurs, but it may be any one of several possible outcomes. The actual outcome is considered to be determined by chance.

$$\text{Probability of an event} = \frac{\text{Number of favourable events}}{\text{Number of all possible outcomes}}$$

Example-1:

Find the probability of getting 5 when a die is thrown .

Solution:

Number of favourable events = 1

Total number of outcomes = 6

Therefore, probability = 1/6

Example-2:

There are 10 marbles in a bag: 6 are blue, and 4 are red. What is the probability that a red marble gets picked?

Solution:

Number of favourable event = : 4

Total number of outcomes: 10

Therefore, probability = 4/10 = 2/5

Sample Space:

All the possible outcomes of an experiment is known as sample space.

Example-3:

Choosing a card from a pack of cards. There are 52 cards in a deck .

So the Sample Space is all 52 possible cards:

Event: A single result of an experiment is known as event.

Example of Events:

- i) Getting a Tail when tossing a coin is an event
- ii) Choosing a "Jack" from a pack of cards (any of the 4 Jacks) is an event.
- iii) Rolling an "odd number" (1, 3 or 5) is also an event.

Types of events :

- **Sure event:** The sure event, **S**, is formed by all possible results of the sample space. The event which has the probability of 1 i.e. 100 %, then it is a Sure Event.
- **Impossible Event:** An event denoted as ‘ \varnothing ’ is called Impossible Event when we find that the experiment checked never occurs. It means the probability of such event is 0%.
- **Disjoint or Mutually Exclusive Events:** Two events, A and B, are disjointed or mutually exclusive when they don't have an element in common. If outcome A is to obtain an even number from a die and B is to obtain a multiple of 5, A and B are mutually exclusive events.
- **Exhaustive events:** A set of events, where the union of all the experiments in the space is the complete space itself, is called Exhaustive Events.
- **Independent Events:** Two events, A and B are independent if the probability of the succeeding event is not affected by the outcome of the preceding event e.g. by rolling a die twice, the results are independent.
- **Dependent Events:** Two events, A and B are dependent if the probability of the succeeding event is affected by the outcome of the preceding event. For example, two dependent events would be drawing two cards (one at a time) without returning them to the deck.
- **Complementary Event:** The complementary event of **A** is another event that is realized when **A** is not realized. It is denoted by \bar{A} or A' . For example, **the complementary event of** obtaining an even number when rolling a die is obtaining an odd number.

Example-4: A die is rolled. Find the probability of obtaining:

- i) 7
- ii) a number less than or equal to 6

Solution:

$$\text{Sample Space} = S = \{1, 2, 3, 4, 5, 6\}$$

- i) Getting 7 impossible, when a die is thrown. Therefore probability = 0

ii)

Let A be the event that a number less than or equal to 6 is obtained. Then:

$$A = \{1, 2, 3, 4, 5, 6\}$$

$$\begin{aligned}\text{Now, } \Pr(A) &= \frac{\text{Number of outcomes in } A}{\text{Number of outcomes in } S} \\ &= \frac{6}{6} \\ &= 1\end{aligned}$$

CARDS: A pack of 52 playing cards consists of:

Four suits, i.e. clubs, spades, diamonds and hearts.

Each suit has 13 cards which are the 2, 3, 4, 5, 6, 7, 8, 9, 10, jack, queen, king and the ace card.

Clubs and spades are of black colour whereas diamonds and hearts are of red colour.

So, there are 26 red cards and 26 black cards.

Example-5:

Find the probability of drawing from a well-shuffled pack of cards:

a black card

b. King of diamonds

c. a jack

Solution:

a. A pack of 52 cards has 26 black cards.

$$\begin{aligned}\therefore \Pr(\text{a black card}) &= \frac{26}{52} \\ &= \frac{1}{2}\end{aligned}$$

b. A pack of 52 cards has 1 king of diamonds.

$$\therefore \Pr(\text{the king of diamonds}) = \frac{1}{52}$$

c. A pack of 52 cards has 4 jacks.

$$\begin{aligned}\therefore \Pr(\text{a jack}) &= \frac{4}{52} \\ &= \frac{1}{13}\end{aligned}$$

Example-6:

The probability that a train will be late is $1/100$. Find the probability that it will be on time.

Solution:

Let A be the event that a train will be late.

Then A' is the event that it will be on time.

$$\begin{aligned}\Pr(A) + \Pr(A') &= 1 \\ \therefore \frac{1}{100} + \Pr(A') &= 1 \\ \therefore \Pr(A') &= 1 - \frac{1}{100} \\ &= \frac{100 - 1}{100} \\ &= \frac{99}{100}\end{aligned}$$

NOTE:

Odds in favour = Favourable events/ Unfavourable events

Odds against = Unfavourable events/ Favourable events

Example-7:

If odds in favor of X solving a problem are 4 to 3 and odds against Y solving the same problem are 2 to 6.

Find probability for:

(i) X solving the problem

(ii) Y solving the problem

Solution:

Probability of the event = (Number of favorable outcomes) / (Number of favorable outcomes + Number of unfavorable outcomes)

Given odds in favour of X solving a problem are 4 to 3.

Number of favorable outcomes = 4

Number of unfavorable outcomes = 3

(i) X solving the problem

$$P(X) = P(\text{solving the problem}) = 4/(4 + 3) = 4/7$$

Given odds against Y solving the problem are 2 to 6

Number of favorable outcomes = 6

Number of unfavorable outcomes = 2

(ii) Y solving the problem

$$P(Y) = P(\text{solving the problem}) = 6/(2 + 6) = 6/8 = 3/4$$

Example-8: You toss a fair coin three times:

- i) What is the probability of getting three heads?
- ii) What is the probability that you observe exactly one heads?
- iii) Given that you have observed *at least* one heads, what is the probability that you observe at least two heads?

Solution:

We assume that the coin tosses are independent.

i) $P(HHH)=P(H)\cdot P(H)\cdot P(H)= (\frac{1}{2})^3=1/8$

ii) To find the probability of exactly one heads, we can write

$$\begin{aligned}P(\text{One heads}) &= P(\text{HTT} \cup \text{THT} \cup \text{TTH}) \\&= P(\text{HTT})+P(\text{THT})+P(\text{TTH}) \\&= 1/8+1/8+1/8 \\&= 3/8.\end{aligned}$$

iii) Let A_1 be the event that you observe at least one heads, and A_2 be the event that you observe at least two heads. Then $P(A_1)= 1 - 1/8 = 7/8$; and $A_2=\{\text{HHT}, \text{HTH}, \text{THH}, \text{HHH}\}$, and $P(A_2)=4/8$.

Thus, we can write

$$\begin{aligned}P(A_2|A_1) &= P(A_2 \cap A_1)/P(A_1) \\&= P(A_2)/P(A_1) \\&= 4/8 \times 8/7 = 4/7.\end{aligned}$$

PREVIOUS YEAR QUESTIONS

CSAT-2016

A round archery target of diameter 1 m is marked with four scoring regions from the centre outwards as red, blue, yellow and white. The radius of the red band is 0.20 m. The width of all the remaining bands is equal. If archers throw arrows towards the target, what is the probability, that the arrows fall in the red region of the archery target?

- (a) 0.40 (b) 0.20 (c) 0.16 (d) 0.04

Answer. c

Radius of archery target = 0.5 m

$$\pi r^2 = \pi (0.5)^2 = 0.25 \pi$$

Total area of archery target =

Sol. Ans (c)

$$\text{Total area of archery target} = \pi r^2 = \pi (0.5)^2 = 0.25 \pi$$

$$\text{Area of red band} = \pi (0.2)^2 = 0.04 \pi$$

$$\text{Probability that the arrows fall in red region} = \frac{\text{Area of red band}}{\text{Total area}} = \frac{0.04\pi}{0.25\pi} = \frac{4}{25} = \frac{16}{100} = 0.16$$

PROBABILITY

1Q: Two brothers X and Y appeared for an exam. The probability of selection of X is $1/7$ and that of Y is $2/9$. Find the probability that both of them are selected.

a) $1/63$

b) $2/63$

c) $3/63$

d) $4/63$

Solution: b);

Both the events are mutually independent. Hence, the probability that both of them are selected = $(1/7) \times (2/9) = 2/63$

2Q: Four dice are thrown simultaneously. Find the probability that all of them show the same face.

a) $1/216$

b) $2/216$

c) $3/216$

d) $4/216$

Solution: a);

$$\text{Total number of events} = 6^4$$

$$\text{Number of favourable events} = 6$$

$$\text{Probability that all of them show the same face} = 6/6^4 = 1/216$$

3Q: A bag contains 21 toys numbered 1 to 21. A toy is drawn and then another toy is drawn without replacement. Find the probability that both toys will show even numbers.

a) $5/21$

b) $9/42$

c) $3/52$

d) $8/9$

Solution: b);

the probability of first toy being even numbered = $10/21$

The probability of second toy being even numbered = $9/20$

The probability that both toys will show even numbers = $(10/21) \times (9/20) = 3/14$

4Q: A speaks truth in 75% of cases and B in 80% of cases. In what percent of cases are they likely to contradict each other in narrating the same event?

a) 45%

b) 35%

c) 65%

d) 85%

Solution: b);

The contradiction will take place only one of them speaks truth and other one speaks a lie. The probability of A speaking truth = 75%

The probability of B speaking lie = $(100 - 80) \% = 20\%$

The probability of B speaking truth = 80%

The probability of A speaking lie = $(100 - 75) \% = 25\%$

Hence the percentage of cases where they are likely to contradict each other in narrating the same event = $75\% \times 20\% + 80\% \times 25\% = 35\%$

5Q: In a charity show tickets numbered consecutively from 101 through 350 are placed in a box. What is the probability that a ticket selected at random (blindly) will have a number with a hundredth digit of 2?

a) 0.30

b) 0.20

c) 0.40

d) 0.50

Solution: c);

The number of ways of selecting a number with a hundredth digit of 2 = ${}^{100}C_1 = 100$

Hence, the probability of selecting such a number = $100/250 = 0.4$

6Q: A special lottery is to be held to select a student who will live in the only deluxe room in a hostel. There are 100 Year-III, 150 Year-II and 200 Year-I students who applied. Each Year-III's name is placed in the lottery 3 times; each Year-II's name, 2 times and Year-I's name, 1 time. What is the probability that a Year-III's name will be chosen?

a) 1/8

b) 2/8

c) 3/8

d) 5/8

Solution: c);

The total number of ways for a student of Year-III being chosen = $100 \times 3 = 300$

The total number of ways for a student of Year-II being chosen = $150 \times 2 = 300$

The total number of ways for a student of Year-I being chosen = $200 \times 1 = 200$

Hence, the probability that a student of Year-III being chosen =

$$= 300 / (300+300+200) = 3/8$$

7Q: There are four hotels in a town. If 3 men check into the hotels in a day then what is the probability that each checks into a different hotel?

a) 1/8

b) 3/8

c) 7/8

d) 1/4

Solution: b);

The number of ways of checking into a different hotel = ${}^4P_3 = 24$

The total number of ways = $4^3 = 64$

Hence, the probability that each checks into a different hotel = $24/64 = 3/8$

8Q: I forgot the last digit of a 8-digit telephone number. If one randomly dials the final 4 digits after correctly dialing the first four, then what is the chance of dialing the correct number?

a) 1/100

b) 1/1000

c) 1/10000

d) 1/5

Solution: c);

It is given that last four digits are randomly dialed after dialing first four digits correctly then each of the last 4 digits can be selected out of 10 digits in 10 ways.

Hence required probability = $(1/10)^4 = 1/10000$

9Q: The letters B, G, I, N and R are rearranged to form the word 'Bring'. Find its probability?

a) 1/220

b) 1/120

c) 1/320

d) 1/420

Solution:b);

The number of ways of arranging 5 letters = $5! = 120$

Probability = $1/120$

10Q: The probability of success of three students X, Y and Z in the one examination are $1/5$, $1/4$ and $1/3$ respectively. Find the probability of success of at least two.

a) $1/7$

b) $1/8$

c) $13/60$

d) $1/4$

Solution: c);

The probability of X and Y succeeding together = $(1/5) \times (1/4) = 1/20$

The probability of Y and Z succeeding together = $(1/4) \times (1/3) = 1/12$

The probability of X and Z succeeding together = $(1/5) \times (1/3) = 1/15$

The probability of X, Y and Z succeeding together = $(1/5) \times (1/4) \times (1/3) = 1/60$

Hence, the probability of success of at least two = $1/20 + 1/12 + 1/15 + 1/60$

$$= 13/60$$

11Q: Four teams Australia Vs Pakistan and England Vs India are participating in the semi-final of a cricket tournament. The odds in favour of team Australia being finalist is 5 to 3, and the odds against of team India being finalist is 1 to 4. What are the odds in favour of Australia and India play final?

a) $11/40$

b) $22/40$

c) $33/40$

d) $1/8$

Solution: d);

Odds in favour = favourable events/ Unfavourable events

Odds against = Unfavourable events/ Favourable events

Probability of Australia winning semi final = $5/(5+3) = 5/8$

Probability of India winning semi final = $1/(1+4) = 1/5$

Probability of India and Australia play final = $5/8 \times 1/5 = 1/8$

12Q: A box contains 20 balls, numbered from 1 to 20. If three balls are selected at random and with replacement from the box, what is the probability that the sum of the three numbers on the balls selected from the box will be odd?

- a) 1/2 b) 3/5 c) 6/7 d) 9/11

Solution: a);

The sum of the three numbers on the balls selected from the box can be odd only when (i) all three balls selected are odd numbered or (ii) only one of them is odd numbered i.e. ODD EVEN, EVEN; EVEN, ODD,EVEN; EVEN, EVEN, EVEN, ODD. Therefore the probability = $1/2 \times 1/2 \times 1/2 + 1/2 \times 1/2 \times 1/2 + 1/2 \times 1/2 \times 1/2 + 1/2 \times 1/2 \times 1/2 = 4/8 = 1/2$

13Q: Set A = {2,3,4,5} Set B = {4,5,6,7,8} Two integers will be randomly selected from the sets above, one integer from Set A and one integer from Set B. What is the probability that the sum of the two integers will equal 9?

- a) 0.10 b) 0.30 c) 0.20 d) 0.40

Solution:c);

If we take any element from Set A, We have exactly one element in Set B to make sum 9. Hence, favourable events = ${}^4C_1 \times {}^1C_1 = 4$

Total events = ${}^4C_1 \times {}^5C_1 = 20$

Probability = $4/20 = 1/5$

14Q: If 6 boys and 6 girls (including Sunita and Geeta) are to stand in row, then what is the probability that the two particular girls Sunita and Geeta are not standing together?

- a) $\frac{5}{12}$ b) $\frac{5}{36}$ c) $\frac{1}{24}$ d) None of these

Solution: d);

Number of ways in which 6 boys and 6 girls can stand in a row

(without any constraints) = $12!$

Number of ways in which 6 boys and 6 girls can stand in a row such that Sunita and Geeta are always together = $2! \times 11!$

$$\therefore \text{Required probability} = 1 - \frac{2! \times 11!}{12!} = \frac{5}{6}$$

15Q: What is the probability that a two digit number selected at random will be a multiple of '3' and not a multiple of '5'?

- a) 2/15 b) 4/15 c) 1/15 d) 6/15

Solution: b);

Number of two digit numbers which are multiples of 3 = 30

Number of two digit numbers which are multiples of 3 and 5 = 6

Number of two digit numbers which are multiples of 3 but not of 5 = $30 - 6 = 24$

Total number of two digit numbers = 90

Probability = $24/90 = 4/15$

16Q: An anti-aircraft gun can take a maximum of four shots at an enemy plane moving away from it. The probability of hitting the plane at the first, second, third and fourth shots are 0.4, 0.3, 0.2 and 0.1 respectively. The plane gets crashed as it is hit by any shot. What is the probability that the plane is hit when all the four shots are fired?

- a) 0.764 b) 0.864 c) 0.964 d) 0.0336

Solution:d);

Fourth shot will be required only when first three are missed. Hence, the probability that the plane is hit when all the four shots are fired

$$= (1-0.4) \times (1-0.3) \times (1-0.2) \times 0.1$$

$$= 0.6 \times 0.7 \times 0.8 \times 0.1 = 0.0336$$

17Q: Two events A and B have probabilities 0.25 and 0.50 respectively. The probability that both A and B occur simultaneously is 0.14. Then the probability that neither A nor B occurs is

- a) 0.39 b) 0.25 c) 0.11 d) None

Solution: a);

Given $P(A) = 0.25$, $P(B) = 0.5$, $P(A \cap B) = 0.14$

Hence, $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.25 + 0.5 - 0.14 = 0.61$

Then the probability that neither A nor B occurs = $1 - 0.61 = 0.39$

18Q: The probability that an event A happens in one trial of an experiment is 0.4. Three independent trials of the experiment; are performed. The probability that the event A happens at least ones is

- a) 0.936 b) 0.784 c) 0.904 d) None

Solution: b);

Given $P(A) = 0.4$. So $P(\bar{A}) = 1 - 0.4 = 0.6$

$$\begin{aligned}\text{Probability that the event A happens at least ones in three trials} &= 1 - (0.6)^3 \\ &= 1 - 0.216 = 0.784\end{aligned}$$

19Q: Fifteen coupons are numbered 1, 2, 15 respectively. Seven coupons are selected at random one at a time with replacement. The probability that the largest number appearing on a selected coupon is 9 is

- a) $(9/16)^6$ b) $(8/15)^7$ c) $(3/5)^7$ d) none of these

Solution:d);

Since there are 15 possible cases for selecting a coupon and seven coupons are selected, the total number of cases of selecting seven coupons = 15^7

It is given that the number on the selected coupon is 9. Therefore the selection is to be made from the coupons numbered 1 to 9. This can be made in 9^7 ways. Out of these 9^7 cases, 8^7 does not contain the number 9.

Thus, the favourable number of cases = $9^7 - 8^7$

Hence, the required probability = $(9^7 - 8^7) / 15^7$

20Q: You toss a fair coin three times:

- i) What is the probability of getting three heads?
- ii) What is the probability that you observe exactly one heads?
- iii) Given that you have observed *at least* one heads, what is the probability that you

observe at least two heads?

Solution:

We assume that the coin tosses are independent.

$$\text{i) } P(\text{HHH}) = P(H) \cdot P(H) \cdot P(H) = (\frac{1}{2})^3 = 1/8$$

ii) To find the probability of exactly one heads, we can write

$$P(\text{One heads}) = P(\text{HTT} \cup \text{THT} \cup \text{TTH})$$

$$= P(\text{HTT}) + P(\text{THT}) + P(\text{TTH})$$

$$= 1/8 + 1/8 + 1/8 = 3/8$$

iii) Let A_1 be the event that you observe at least one heads, and A_2 be the event that you observe at least two heads. Then $P(A_1) = 1 - 1/8 = 7/8$; and $A_2 = \{\text{HHT}, \text{HTH}, \text{THH}, \text{HHH}\}$, and $P(A_2) = 4/8$.

Thus, we can write

$$P(A_2|A_1) = P(A_2 \cap A_1) / P(A_1)$$

$$= P(A_2) / P(A_1)$$

$$= 4/8 \times 8/7 = 4/7.$$

21Q: If odds in favor of X solving a problem are 4 to 3 and odds against Y solving the same problem are 2 to 6.

Find probability for:

(i) X solving the problem

(ii) Y solving the problem

Solution:

Probability of the event = (Number of favorable outcomes) / (Number of favorable outcomes + Number of unfavorable outcomes)

Given odds in favour of X solving a problem are 4 to 3.

Number of favorable outcomes = 4

Number of unfavorable outcomes = 3

(i) X solving the problem

$$P(X) = P(\text{solving the problem}) = 4/(4 + 3) = 4/7$$

Given odds against Y solving the problem are 2 to 6

Number of favorable outcomes = 6

Number of unfavorable outcomes = 2

(ii) Y solving the problem

$$P(Y) = P(\text{solving the problem}) = 6/(2 + 6) = 6/8 = 3/4$$

UPSC QUESTIONS

CSAT-2016

A round archery target of diameter 1 m is marked with four scoring regions from the centre outwards as red, blue, yellow and white. The radius of the red band is 0.20 m. The width of all the remaining bands is equal. If archers throw arrows towards the target, what is the probability, that the arrows fall in the red region of the archery target?

