In[1]:= A = - {{0, 0, 0, λ + 2 μ, 0}, {0, 0, 0, λ, 0},
    {0, 0, 0, 0, μ}, {1 / ρ, 0, 0, 0, 0}, {0, 0, 1 / ρ, 0, 0}}
B = -{{0, 0, 0, 0, λ }, {0, 0, 0, 0, λ + 2 μ}, {0, 0, 0, μ, 0},
    {0, 0, 1 / ρ, 0, 0}, {0, 1 / ρ, 0, 0, 0}}

Out[1]= $\left\{\{0, 0, 0, -\lambda - 2\mu, 0\}, \{0, 0, 0, -\lambda, 0\}, \{0, 0, 0, 0, -\mu\}, \left\{-\frac{1}{\rho}, 0, 0, 0, 0\right\}, \left\{0, 0, -\frac{1}{\rho}, 0, 0\right\}\right\}$

Out[2]= $\left\{\{0, 0, 0, 0, -\lambda\}, \{0, 0, 0, 0, -\lambda - 2\mu\}, \{0, 0, 0, -\mu, 0\}, \left\{0, 0, -\frac{1}{\rho}, 0, 0\right\}, \left\{0, -\frac{1}{\rho}, 0, 0, 0\right\}\right\}$

In[3]:= constants = {Sqrt [μ] / Sqrt [ρ] → cs, Sqrt [(λ + 2 μ)] / Sqrt [ρ] → cp,
$\frac{\lambda \sqrt{\rho}}{\sqrt{\lambda + 2\mu}} \to \lambda / \mathrm{cp}, \sqrt{\mu}\ \sqrt{\rho} \to \mathrm{cs}\ \rho, \sqrt{\lambda + 2\mu}\ \sqrt{\rho} \to \mathrm{cp}\ \rho\}$

Out[3]= $\left\{\frac{\sqrt{\mu}}{\sqrt{\rho}} \to \mathrm{cs}, \frac{\sqrt{\lambda + 2\mu}}{\sqrt{\rho}} \to \mathrm{cp}, \frac{\lambda \sqrt{\rho}}{\sqrt{\lambda + 2\mu}} \to \frac{\lambda}{\mathrm{cp}}, \sqrt{\mu}\ \sqrt{\rho} \to \mathrm{cs}\ \rho, \sqrt{\lambda + 2\mu}\ \sqrt{\rho} \to \mathrm{cp}\ \rho\right\}$

In[5]:= α = Eigenvalues [A] //. constants
Eigenvalues [B] //. constants

Out[5]= {0, - cs, cs, - cp, cp}

Out[6]= {0, - cs, cs, - cp, cp}

In[7]:= S1 = Eigenvectors [A] //. constants
S2 = Eigenvectors [B] //. constants

Out[7]= $\left\{\{0, 1, 0, 0, 0\}, \{0, 0, \mathrm{cs}\ \rho, 0, 1\}, \{0, 0, -\mathrm{cs}\ \rho, 0, 1\}, \left\{\mathrm{cp}\ \rho, \frac{\lambda}{\mathrm{cp}}, 0, 1, 0\right\}, \left\{-\mathrm{cp}\ \rho, -\frac{\lambda}{\mathrm{cp}}, 0, 1, 0\right\}\right\}$

Out[8]= $\left\{\{1, 0, 0, 0, 0\}, \{0, 0, \mathrm{cs}\ \rho, 1, 0\}, \{0, 0, -\mathrm{cs}\ \rho, 1, 0\}, \left\{\frac{\lambda}{\mathrm{cp}}, \mathrm{cp}\ \rho, 0, 0, 1\right\}, \left\{-\frac{\lambda}{\mathrm{cp}}, -\mathrm{cp}\ \rho, 0, 0, 1\right\}\right\}$

Reorder eigenvectors according to $\alpha 1 > \alpha 2 > \alpha 3$ ...

In[9]:=
```
S1 = {S1[[5]], S1[[3]], S1[[1]], S1[[2]], S1[[4]]}
S2 = {S2[[5]], S2[[3]], S2[[1]], S2[[2]], S2[[4]]}
```

Out[9]=
$$\left\{\left\{-\text{cp}\,\rho,\,-\frac{\lambda}{\text{cp}},\,0,\,1,\,0\right\},\,\{0,\,0,\,-\text{cs}\,\rho,\,0,\,1\},\,\{0,\,1,\,0,\,0,\,0\},\,\{0,\,0,\,\text{cs}\,\rho,\,0,\,1\},\,\left\{\text{cp}\,\rho,\,\frac{\lambda}{\text{cp}},\,0,\,1,\,0\right\}\right\}$$

Out[10]=
$$\left\{\left\{-\frac{\lambda}{\text{cp}},\,-\text{cp}\,\rho,\,0,\,0,\,1\right\},\,\{0,\,0,\,-\text{cs}\,\rho,\,1,\,0\},\,\{1,\,0,\,0,\,0,\,0\},\,\{0,\,0,\,\text{cs}\,\rho,\,1,\,0\},\,\left\{\frac{\lambda}{\text{cp}},\,\text{cp}\,\rho,\,0,\,0,\,1\right\}\right\}$$

Solve for x-direction:

In[11]:=
```
leftflux = {σ11l, σ22l, σ12l, ul, vl};
rightflux = {σ11r, σ22r, σ12r, ur, vr};
eq1 = leftflux == γ1 * S1[[1]] + γ2 * S1[[2]] + γ3* S1[[3]] + γ4 * S1[[4]] + γ5 * S1[[5]] ;
eq2 = rightflux == β1 * S1[[1]] + β2 * S1[[2]] + β3* S1[[3]] + β4 * S1[[4]] + β5 * S1[[5]] ;
```

In[15]:=
```
gammas = Solve[eq1 , {γ1, γ2, γ3, γ4, γ5}]
betas = Solve[eq2 , {β1, β2, β3, β4, β5}]
```

Out[15]=
$$\left\{\left\{\gamma1\to-\frac{-\text{cp ul}\,\rho+\sigma11\text{l}}{2\,\text{cp}\,\rho},\,\gamma2\to-\frac{-\text{cs vl}\,\rho+\sigma12\text{l}}{2\,\text{cs}\,\rho},\,\gamma3\to-\frac{\lambda\,\sigma11\text{l}-\text{cp}^2\,\rho\,\sigma22\text{l}}{\text{cp}^2\,\rho},\,\gamma4\to-\frac{-\text{cs vl}\,\rho-\sigma12\text{l}}{2\,\text{cs}\,\rho},\,\gamma5\to-\frac{-\text{cp ul}\,\rho-\sigma11\text{l}}{2\,\text{cp}\,\rho}\right\}\right\}$$

Out[16]=
$$\left\{\left\{\beta1\to-\frac{-\text{cp ur}\,\rho+\sigma11\text{r}}{2\,\text{cp}\,\rho},\,\beta2\to-\frac{-\text{cs vr}\,\rho+\sigma12\text{r}}{2\,\text{cs}\,\rho},\,\beta3\to-\frac{\lambda\,\sigma11\text{r}-\text{cp}^2\,\rho\,\sigma22\text{r}}{\text{cp}^2\,\rho},\,\beta4\to-\frac{-\text{cs vr}\,\rho-\sigma12\text{r}}{2\,\text{cs}\,\rho},\,\beta5\to-\frac{-\text{cp ur}\,\rho-\sigma11\text{r}}{2\,\text{cp}\,\rho}\right\}\right\}$$

In[17]:=
```
γ1 * S1[[1]] + γ2 * S1[[2]] + β3 * S1[[3]] + β4 * S1[[4]] + β5 * S1[[5]] //. gammas [[1]] //. betas [[1]] //
  FullSimplify // MatrixForm
```

Out[17]//MatrixForm=
$$\begin{pmatrix} \frac{1}{2}\,(\text{cp}\,(-\text{ul}+\text{ur})\,\rho+\sigma11\text{l}+\sigma11\text{r}) \\ \frac{\lambda\,(\text{cp}\,(-\text{ul}+\text{ur})\,\rho+\sigma11\text{l}-\sigma11\text{r})}{2\,\text{cp}^2\,\rho}+\sigma22\text{r} \\ \frac{1}{2}\,(\text{cs}\,(-\text{vl}+\text{vr})\,\rho+\sigma12\text{l}+\sigma12\text{r}) \\ \frac{\text{cp}\,(\text{ul}+\text{ur})\,\rho-\sigma11\text{l}+\sigma11\text{r}}{2\,\text{cp}\,\rho} \\ \frac{\text{cs}\,(\text{vl}+\text{vr})\,\rho-\sigma12\text{l}+\sigma12\text{r}}{2\,\text{cs}\,\rho} \end{pmatrix}$$

Solve for y-direction:

In[18]:=
```
bottomflux = {σ11b, σ22b, σ12b, ub, vb};
topflux = {σ11t, σ22t, σ12t, ut, vt};
eq3 = bottomflux == μ1 * S2[[1]] + μ2 * S2[[2]] + μ3 * S2[[3]] + μ4 * S2[[4]] + μ5 * S2[[5]] ;
eq4 = topflux == η1 * S2[[1]] + η2 * S2[[2]] + η3 * S2[[3]] + η4 * S2[[4]] + η5 * S2[[5]] ;
```

In[22]:=
```
mus = Solve[eq3 , {μ1, μ2, μ3, μ4, μ5}]
etas = Solve[eq4 , {η1, η2, η3, η4, η5}]
```

Out[22]=

$$\left\{\left\{\mu 1 \to -\frac{-\mathrm{cp}\ \mathrm{vb}\ \rho + \sigma 22\mathrm{b}}{2\ \mathrm{cp}\ \rho},\ \mu 2 \to -\frac{-\mathrm{cs}\ \mathrm{ub}\ \rho + \sigma 12\mathrm{b}}{2\ \mathrm{cs}\ \rho},\ \mu 3 \to -\frac{-\mathrm{cp}^2\ \rho\ \sigma 11\mathrm{b} + \lambda\ \sigma 22\mathrm{b}}{\mathrm{cp}^2\ \rho},\ \mu 4 \to -\frac{-\mathrm{cs}\ \mathrm{ub}\ \rho - \sigma 12\mathrm{b}}{2\ \mathrm{cs}\ \rho},\ \mu 5 \to -\frac{-\mathrm{cp}\ \mathrm{vb}\ \rho - \sigma 22\mathrm{b}}{2\ \mathrm{cp}\ \rho}\right\}\right\}$$

Out[23]=

$$\left\{\left\{\eta 1 \to -\frac{-\mathrm{cp}\ \mathrm{vt}\ \rho + \sigma 22\mathrm{t}}{2\ \mathrm{cp}\ \rho},\ \eta 2 \to -\frac{-\mathrm{cs}\ \mathrm{ut}\ \rho + \sigma 12\mathrm{t}}{2\ \mathrm{cs}\ \rho},\ \eta 3 \to -\frac{-\mathrm{cp}^2\ \rho\ \sigma 11\mathrm{t} + \lambda\ \sigma 22\mathrm{t}}{\mathrm{cp}^2\ \rho},\ \eta 4 \to -\frac{-\mathrm{cs}\ \mathrm{ut}\ \rho - \sigma 12\mathrm{t}}{2\ \mathrm{cs}\ \rho},\ \eta 5 \to -\frac{-\mathrm{cp}\ \mathrm{vt}\ \rho - \sigma 22\mathrm{t}}{2\ \mathrm{cp}\ \rho}\right\}\right\}$$

In[24]:=
```
μ1 * S2[[1]] + μ2 * S2[[2]] + η3 * S2[[3]] + η4 * S2[[4]] + η5 * S2[[5]] //. mus[[1]] //. etas[[1]] //
  FullSimplify // MatrixForm
```

Out[24]//MatrixForm=

$$\begin{pmatrix} \sigma 11\mathrm{t} + \frac{\lambda\ (\mathrm{cp}\ (-\mathrm{vb}+\mathrm{vt})\ \rho + \sigma 22\mathrm{b} - \sigma 22\mathrm{t})}{2\ \mathrm{cp}^2\ \rho} \\ \frac{1}{2}\ (\mathrm{cp}\ (-\mathrm{vb}+\mathrm{vt})\ \rho + \sigma 22\mathrm{b} + \sigma 22\mathrm{t}) \\ \frac{1}{2}\ (\mathrm{cs}\ (-\mathrm{ub}+\mathrm{ut})\ \rho + \sigma 12\mathrm{b} + \sigma 12\mathrm{t}) \\ \frac{\mathrm{cs}\ (\mathrm{ub}+\mathrm{ut})\ \rho - \sigma 12\mathrm{b} + \sigma 12\mathrm{t}}{2\ \mathrm{cs}\ \rho} \\ \frac{\mathrm{cp}\ (\mathrm{vb}+\mathrm{vt})\ \rho - \sigma 22\mathrm{b} + \sigma 22\mathrm{t}}{2\ \mathrm{cp}\ \rho} \end{pmatrix}$$