Review and Implementation Heuristic Search Methods for Solving Cryptoarithmatic Problems

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Certificate

This is certificate for the following, "**Review and Implementation Heuristic Search Methods for Solving Cryptoarithmatic Problems,**",completed by Ashish D. Fugare(Roll No:170101023) completed under my supervision in the Department of Computer Science and Engineering at the Indian Institute of Technology Guwahati is authentic and hasn't be submitted elsewhere for thesis.

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Abstract

This thesis explores heuristic search methods for solving Cryptarithmetic problems, focusing on the implementation and evaluation of Constraint Satisfaction Problem (CSP) techniques. Cryptarithmetic problems are mathematical puzzles where digits are represented by letters of the alphabet, posing a significant challenge for computational algorithms. Traditional brute-force methods quickly become unmanageable as the problem size increases, leading to inefficiencies in time and space complexity. Our study leverages the inherent constraints within these puzzles to reduce the search space and enhance computational efficiency. We review existing literature on heuristic search methods and implement various algorithms to analyze their performance. Our experiments demonstrate that the heuristic CSP method significantly outperforms the brute-force approach, yielding faster and more effective solutions. This research aims to contribute to the field by providing a more efficient approach to solving Cryptarithmetic problems, especially as the size and complexity of the problems increase.

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1.Introduction

Cryptoarithmetic problems are a type of mathematical puzzle where digits are represented by letters of the alphabets.These problems have intrigued both amatures of all ages and professionals in the field such as mathematicians. These puzzles are an engaging mental exercise but also represent a significant challenge for solving these by computational algorithms,. The aim of this thesis is to explore heuristic search methods for solving cryptoarithmetic problems, with a focus on constraint propagation techniques. By leveraging the inherent constraints within these puzzles, we can significantly reduce the search space, leading to a more efficient solution with better time and space .We will also implennt this technqiues. This study will review existing literature on heuristic search methods and implement various algorithms and analyze their performance from a baseline. This research aims to contribute to the field by providing in solving these intricate puzzles through heuristic approaches.

1.1.Cryptarithmetic Puzzle

Cryptoarithmatics is a genre of mathematical puzzle in which the digits are replaced by symbols or by letters of the alphabet or etc.The aim of solving these puzzels is to guess the unique numbers representing aplahabets in the puzzle conting simple arithmetic operators such as +,*,-,/. The puzzle shown in figure 1.1 is a Classic example in this field. Cryptarithmetic has already become a standard AI problem because it characterizes numbers of important problems in the computer science field.

```
  S E N D
+ M O R E
-----------------
M O N E Y
```

Figure 1.1

1.2.Constraint Satisfaction

Constraint Satisfaction propagation is a very powerful technique that is often used in solving constraint satisfaction problems. One example of which is cryptoarithmetic puzzlesThe search process known as CSP works in a space of constraint sets.The constraints that were first stated in the problem description are present in the initial state..A goal state refers to a state that has undergone sufficient constraint, where enough conditions have been established to define the solution for each specific problem.Like in example of Cryptoarthmatic puzzle each leatter must be unquiely assigneed. The procedure of satisfying constraints involves two steps.The system's initial restrictions are identified and spread as widely as feasible. If there is still no answer, the search starts. The next step is to make a hypothesis regarding alphabet and add it as a new restriction.. By systematically applying constraints across the search space, the method can eliminate large sections of the search tree, reducing the overall search effort and leading to faster and more effective solutions.

This process can significantly reduce the complexity of the problem by eliminating inconsistent values early on.

3.Review of Prior Works

Researchers have increasingly used Constraint Satisfaction Problem (CSP) techniques to solve cryptarithmetic puzzles, which are mathematical problems where each letter in an equation represents a unique numerical digit. Earlier approaches often relied on brute-force methods, where every possible combination of digit was tested, but this approach quickly becames not manageable as the number of letters grew leading to increases in time and space complexity. Unlike brute-force approaches that try every possible digit assignment which are computationally intensive and costly, heuristic search methods incorporate strategies to prioritize certain paths over others, aiming to reduce the number of guesses needed. For cryptarithmetic puzzles, in paers researchers have also experimented for cryptoarthimatic problems to optimize with heuristics like **Minimum Remaining Values (MRV**), which first selects variables with the fewest possible digit , and **Least Constraining Value (LCV**), which chooses values that impose the fewest restrictions on other variables in puzzle . Some papers such as [SolFeng] which, have instead used a Logical Programming approach to solve these types of problems. Others are using Heuristic algorithms like Genetic Algorithm [SolvPara] to solve this problem, which also gives very efficient results

4.Proposed Solution

Constraint Satisfaction Propagation

In this part we are looking at the Psudocode of CSP that we will be implementing we are also going to its step using th example of Puzzle from Figure 1.1

Algorithm for CSP

```
Alorithm for CSP

Psudocode for CSP Algo
Algorithm Constraint_Satisfaction

# Initialize  the collection of all items that need values in order for a solution to be complete.
OPEN = {set of all objects}

# Loop until OPEN is empty or an inconsistency is found
while OPEN is not empty:
    # Choose an OB object from OPEN
    OB = select an object from OPEN

    # As much as possible, make the set of restrictions that apply to OB stronger.
    strengthen_constraints(OB)

    # Verify whether the constraints in place now differ from those in place when OB was last
reviewed
    if constraints_set(OB is being examined for the first time or has changed since the last
examination:
        # All objects that share any constraints with OB should be added to OPEN
        for each object X that shares constraints with OB:
            add X to OPEN

    # Remove OB from OPEN
    remove OB from OPEN
```

The algorithm is expressed in the broadest possible terms for general application. Two types of rules are needed to apply it in the problem domain: rules that specify the constraints that can be legitimately propagated and rules that make guesses when needed. Generally speaking, the more effective the rules are at propagating constraints, the less guesswork is required,

We can explain this using the example from figure 1.1Assuming C1 ,C2,C2,C4 be, respectively first,second, third and fourth carry bit. The rules for propagating constraints initially produce the following extra constraints:

- M=1,since 2 single-digit no. plus a carry cannot exceed more than 19
- S=8 or 0 since S+M+C3 >9 and M=1,S+1+C3>9 or S+C>8 and C3 is at most 1
- O=0 since S+M(1) + C3 ≤ 1 must be atleast 10 to generate a carry and it can at most be 11.But M is already 1 that implies O must be 0.
- N=E or E+1 depending on value of C2.But N can't be having same valur as E so N=E+1 and C2 = 1
- In order for C2 be 1 sum of N+R+C1 must be > 9, so N+R must be > 8.
- N+R can't be greater than 18 even with carry so E can't exceed 9

Assume for the moment that no more constraints can be produced. Then we have to guess in order to reach this point. Let's assume that E is gussed as value 2. The following cycle has now started

Now the constraint that are propagated are observed as
- N = E+1, so N =3
- R+N(3) + C1 (1 or 0) = 2 or 12 and R=8 or 9. However, since N is already three from the previous step, the total of these non-negative numbers cannot be less than three. Consequently, R = 8 or 9 and R+3+ (0 or 1) = 12
- This implies that 2+ D= Y or 2+ D = 10+Y

Now we need another guess suppose C1.If we try the value 1then we eventually reach a dead end,,when this happens, we backtrack and try C1=0.

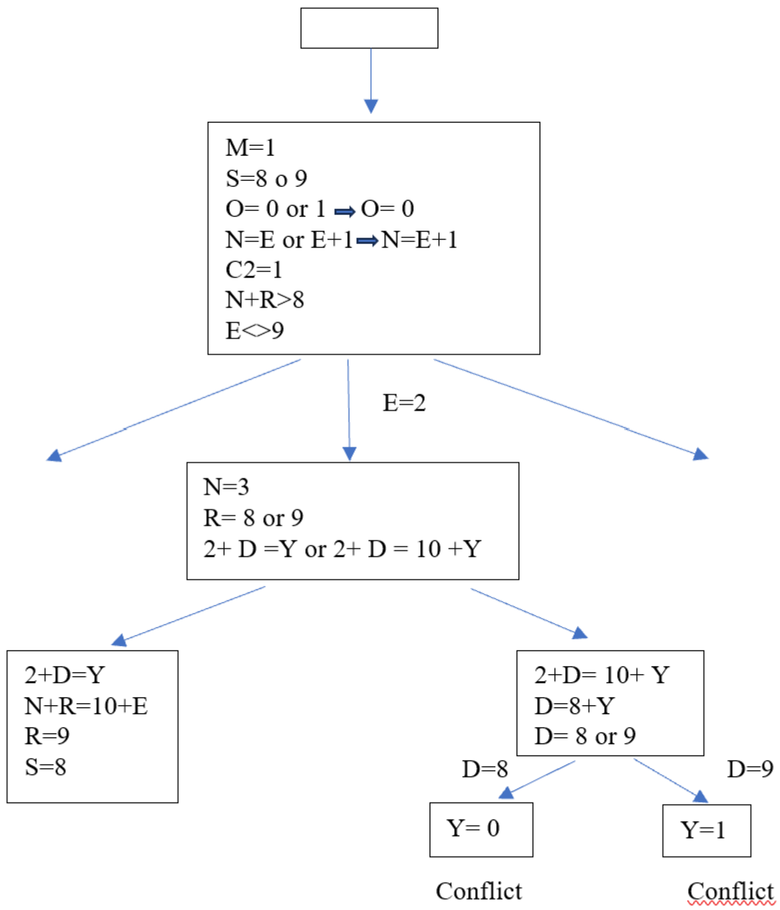


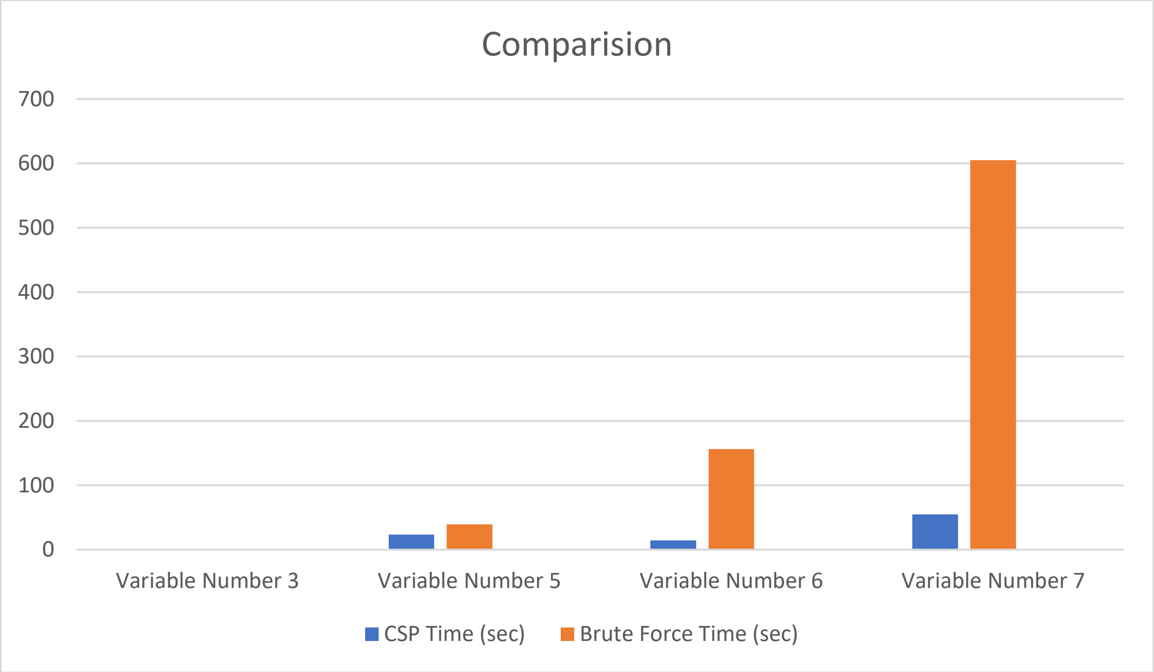
Figure 1.2 Solving a Cryptoarithmatic Problem

5 .Experiments

In this section, we evaluate the performance of our implemented code using different numbers of alphabets, utilizing both the Constraint Satisfaction Problem (CSP) and brute force methods for comparison. The algorithm was implemented in Python 3 and applied to various Cryptarithmetic problems with differing numbers of symbols and numbers. The performance was tested on a machine with a 13th Gen Intel(R) Core(TM) i5-13500H 2.60 GHz processor, 16 GB RAM, and Windows 11 OS. Our results indicate that the heuristic method of CSP yields significantly faster results compared to the brute force method, due to its ability to efficiently prune the search space and reduce the number of potential solutions that need to be evaluated. The comparison of performance metrics shows that the CSP method consistently outperforms the brute force approach, especially as the size of the problem increases. This efficiency makes CSP the preferred method for solving larger and more complex Cryptarithmetic problems.

Variable Number	3	5	6	7
CSP Time	1.26	23.6553	14.4586	54.556
Brute Force Time	1.27	39.1570	156.842	604.8

Table 1.1



Graph 1.1 By the comparison shown above we can see we have a efficient method with CSP rather than Brute force method.

So we see that Constraints Satisfaction Propagation is indeed efficient to the Brute Force method of using permutation for solution

Number of Variables	Time For Execution(sec)
3	1.25844
5	23.6353
6	14.4586
7	54.5567

Table 1.2

The above table also shows the average executing time for solving the problem with different number of variables in the puzzle.

6.Conclusion

My experiments demonstrate that the heuristic method of Constraint Satisfaction Problem (CSP) significantly outperformed the brute force method for solving Cryptarithmetic problems. The efficiency of the CSP approach is shown in its ability to quickly and effectively reduce the search space, thereby reducing the computational time and resources required. This advantage becomes increasingly evident as the problem size grows, making CSP the preferred method for larger and more complex scenarios. Overall, the heuristic method of CSP proves to be a more efficient and scalable solution compared to the traditional brute force approach.

7.Future Work

In future work, the application of Constraint Satisfaction Problems (CSP) for solving cryptarithmetic puzzles can be expanded in several ways to improve performance and broaden the scope of problems that can be addressed.

1. **Using Optimization Techniques**:
   In future we can use advanced heuristic methods such as, such as dynamic variable ordering and adaptive constraint propagation, which can increase performance. Techniques like forward checking or constraint propagation based on consistency can also be further worked for specific characteristics of cryptarithmetic puzzles.
2. **Using Parallel or Distributed Computing Approaches**: Implementing parallel or distributed approach of CSP-based solvers could allow for handling puzzles with a more number of variables or more complex constraints. This would involve distributing constraint propagation or search tasks across multiple processors, potentially speeding up computation significantly.
3. We can also experiment with mix of other Heuristic Techniques such as Greedy Method, Genetic Algorithm ,etc

Refrence

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