Hello everyone I am Ashish Dileep Fuagare Roll No (170101023) and I am presenting a BTP under Pinaki Sir titled “Review and Implementing of Heuristic Search Techniques for solving crypto arithmetic problems” I will discuss the implementation of heuristic search techniques for solving cryptarithmetic puzzles, with a focus on backtracking and constraint satisfaction approaches."

Cryptarithmetic is a puzzle consisting of an arithmetic problem in which the digits have been replaced by letters of the alphabet. . The goal is to decipher the letters that is . Map them back onto the digits using the constraints provided by arithmetic equations given in puzzles and the additional that constraint that no two letters can have the same numerical value assigned. This type of problem was popularized during the 1930s the Sphinx, a Belgian journal of recreational mathematics.

In the literature of Artificial Intelligence, cryptarithmetic puzzles are generally discussed as a kind of the Constraint Satisfaction Problems (CPSs) in which a solution to a given problem is represented by a problem state that meets of all the problem constraints.

There are Wide Range of Applications for Cryptarithm problems and heuristic methods have applications in cryptography, error detection and correction, and even AI and machine learning.

Solutions derived from heuristic methods in cryptarithms can have real-world implications in improving data security, optimization problems in logistics, and even in developing new algorithms for artificial intelligence. Heuristic methods, by their nature, provide innovative and often more efficient solutions compared to traditional algorithms.

I focus on implementing backtracking and constraint satisfaction methods, including optimizations like With forward checking AC-3 checks.MRV

I am choosing following example to see the preformace trends The shown are few exampels of the Cryptoarthimatic problem

**Simple Problems (Low Complexity)**

These problems involve fewer unique letters and straightforward constraints. They are useful for benchmarking basic functionality and runtime efficiency.

**Example:**

* **Problem:** BASE + BALL == GAMES
  + Unique Letters: B,A,S,E,L,G,MB, A, S, E, L, G, MB,A,S,E,L,G,M (7 letters).
  + Complexity: Medium.
  + Expected Outcome: Has a solution.

**Why Include:**  
Tests the algorithm's ability to handle standard cryptarithmetic puzzles with manageable search spaces.

**2. Moderate Problems (Higher Complexity)**

These problems introduce more unique letters or variables, making the problem space larger and more challenging to solve.

**Example:**

* **Problem:** SEND + MORE == MONEY
  + Unique Letters: S,E,N,D,M,O,R,Y (8 letters).
  + Complexity: Higher than BASE + BALL == GAMES.
  + Expected Outcome: Has a solution.

**Why Include:**

* Tests the scalability of the algorithm to slightly larger puzzles.
* Includes leading zero constraints (e.g., S,M≠0S, M \neq 0S,M=0).

**3. Challenging Problems (High Complexity)**

These problems have many unique letters or multiple constraints, making them significantly harder to solve.

**Example:**

* **Problem:** CROSS + ROADS == DANGER
  + Unique Letters: C,R,O,S,A,D,N,G,E(9 letters).
  + Complexity: High.
  + Expected Outcome: Has a solution.

**Why Include:**

* Tests the algorithm's handling of larger search spaces and complex constraints.
* Measures node exploration efficiency.

**4. No-Solution Problems (Edge Cases)**

These problems deliberately include more unique letters than digits or impossible constraints. This is essential for testing the algorithm's ability to detect and terminate unsolvable cases efficiently.

**Example:**

* **Problem:** TWELVE + THREE == FIFTEEN
  + Unique Letters: T,W,E,L,V,H,R,F,I (11 letters).
  + Complexity: Impossible (Pigeonhole principle).
  + Expected Outcome: No solution.

**Why Include:**

* Tests early failure detection mechanisms like AC3, forward checking, or domain pruning.
* Evaluates how algorithms handle infeasibility efficiently.

**5. Large Problems (Stress Testing)**

These problems involve many digits, longer equations, or additional constraints. They are ideal for stress-testing the algorithm.

**Example:**

* **Problem:** DONALD + GERALD == ROBERT
  + Unique Letters: D,O,N,A,L,G,E,R,B,T(10 letters).
  + Complexity: Very High.
  + Expected Outcome: Has a solution.

**Why Include:**

* Tests how well the algorithm scales to its limits.
* Measures performance in terms of runtime and node exploration on larger proble

I used there codes implantations to see the performance comparisons 1st is Brute force ,in Brute force involves trying every possible combination of digits for the letters. While this method guarantees a solution, it can be computationally expensive,

1. **Brute-Force Approach**: A straightforward exhaustive search through all possible assignments.
2. **Vanilla CSP**: A basic CSP solver without optimizations.
3. **Optimized CSP (CSP)**: CSP with forward-checking for pruning invalid assignments.
4. **Dynamic AC-3 (CSP-Dynamic)**: A CSP solver using the AC-3 algorithm for maintaining arc consistency dynamically during search.
5. **CSP without AC-3 (CSP-WO)**: CSP using backtracking without arc consistency checks.

**Backtracking Only (CSP-Back)**: Basic CSP with backtracking but without forward-checking or arc consistency.

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| **Vanilla CSP** | Backtracking, MRV |
| **Optimized CSP** | Backtracking, MRV, Forward Checking, Arc Consistency |
| Minimum Remaining Values (MRV)  What it is: The MRV heuristic selects the variable with the smallest domain (fewest legal values) to assign next. This helps to quickly identify and resolve variables that might cause conflicts, avoiding unnecessary search.  Why it's useful: If a variable has very few valid options, it is better to handle it first. This reduces the chances of wasting time exploring invalid assignments.  Example (SEND + MORE = MONEY):   * Variables: S,E,N,D,M,O,R,YS, E, N, D, M, O, R, YS,E,N,D,M,O,R,Y * Initial domains: S={1,2,...,9}S = \{1, 2, ..., 9\}S={1,2,...,9} (cannot be 0 as it’s a leading digit), M={1,2,...,9}M = \{1, 2, ..., 9\}M={1,2,...,9}, Others (Aplabets) = {0, 1, ..., 9}. * Suppose constraints reduce SSS to {9} and MMM to {1}, while others remain large. * MRV chooses SSS or MMM (smallest remaining values) first, prioritizing these assignments and pruning the search space**.**   The Degree Heuristic selects the variable involved in the highest number of constraints with unassigned variables. This focuses on variables that are more likely to impact the problem's overall solvability. Suppose SSS and MMM have fewer constraints, while EEE interacts with O,N,D,R / Degree heuristic chooses E next because it impacts more variables, helping resolve dependencies early  **Forward Checking**  **Forward checking** proactively eliminates values from the domains of unassigned variables that conflict with the current assignment.  **Why it's useful**: It prevents future conflicts early, reducing the chances of exploring invalid assignments.  **Example (SEND + MORE = MONEY)**:   * Assign S=9 * Forward checking eliminates 9 from M,E,N,D,O,R domains. * If M=, it updates constraints for M+E * This pruning avoids trying invalid combinations like M=9,E=9speeding up the solution.   **CSP Without AC-3 Optimization**  **How It Works:**   * **Goal**: Solve the CSP using recursive backtracking without any preprocessing like AC-3. * **Steps**:   1. **Start with an empty assignment** and recursively assign values to variables.   2. **Select a variable** to assign a value (using heuristics like Minimum Remaining Values (MRV) can help).   3. **Check consistency**:      + Ensure the chosen value satisfies all constraints with the current partial assignment.   4. **Backtrack** if the current value leads to inconsistency or no solution:      + Remove the value and try the next one in the domain.   5. **Stop when all variables are assigned** consistently, or return failure if no values work.   **Example:**  For **SEND + MORE == MONEY**:   * Start with S=9S = 9S=9 (a leading digit constraint), then assign M=1M = 1M=1, O=0O = 0O=0, etc., checking if the partial equation works. * If a choice leads to inconsistency, backtrack and try a different value.   **Key Features:**   * Simpler but less efficient, as it does not reduce the search space beforehand. * Relies heavily on exploring the full search tree, which can be computationally expensive   From the table we see that for say Example like SEND + MORE = MONEY  **1. CSP with AC-3 Optimization**  **How It Works:**   * **Goal**: AC-3 (Arc Consistency 3) reduces the problem size by eliminating values from variable domains that cannot satisfy binary constraints. * **Steps**:   1. **Start with all variable pairs** connected by constraints (called "arcs") in a queue.   2. **Iteratively revise domains**:      + For a pair (Xi,Xj)(X\_i, X\_j)(Xi​,Xj​), check if every value in XiX\_iXi​'s domain has a corresponding value in XjX\_jXj​'s domain that satisfies the constraint.      + If a value in XiX\_iXi​'s domain is inconsistent, remove it.   3. **Propagate changes**:      + If XiX\_iXi​'s domain changes, add all neighboring pairs (Xk,Xi)(X\_k, X\_i)(Xk​,Xi​) back to the queue.      + This ensures the changes are consistent across the entire CSP.   4. **Stop when the queue is empty** or a domain becomes empty (no solution possible).   **Example:**  For the equation **SEND + MORE == MONEY**, if SSS must be a digit {9,8,7and S≠0, AC-3 will prune 0 from SS’s domain. Propagation ensures all constraints reflect this pruning.  **Key Features:**   * Reduces the search space before solving. * Propagates consistency globally across the CSP.   May detect failure early if a domain becomes empty |  |

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| **Vanilla CSP Approach (without optimizations):**  **This approach uses constraint satisfaction programming (CSP), but without optimizations such as arc-consistency (AC3). It typically explores a lot more nodes than the optimized version.**  **Optimized CSP Approach:**  **This is the same as the Vanilla CSP, but with optimizations (like AC3) to prune the search space. This results in fewer nodes explored and often faster solution times.**  **ey Takeaways:**   1. **Brute-Force vs. CSP**:    * The brute-force approach explored fewer nodes and found the solution faster in all cases. However, brute force only works by checking all possible combinations, making it inefficient for larger problems (like TWELVE + THREE == FIFTEEN).    * CSP (with and without optimization) is a more efficient method in theory, but without optimizations, it explores more nodes and takes longer. Optimized CSP uses techniques like AC3 to reduce the search space and improve performance.    **Vanilla CSP vs. Optimized CSP**:   * **Optimized CSP** performs better (less nodes explored and faster time) in most cases, thanks to optimizations. * **Vanilla CSP** is slower because it lacks optimizations and explores a larger search space.    **Problem Specifics**:   * For the equation TWELVE + THREE == FIFTEEN, no solution was found in all methods. This suggests that the equation is either unsolvable under the given constraints or was too complex for the method to find a solution within the given time frame   **Reasons Why Vanilla CSP Might Be Faster Than Optimized CSP:**   1. **Overhead of Optimization**:    * Optimizations like **AC3** (Arc-Consistency) or **domain filtering** might introduce overhead in certain cases. This happens because the system must perform additional checks to prune the search space before or during the search process. In some simpler problems or problems with fewer constraints, these optimizations might take more time than just performing a simple backtracking search without optimizations. 2. **Problem Complexity**:    * Some problems, especially smaller ones or those with fewer constraints, might not benefit much from optimizations. For simpler problems, the additional steps involved in setting up and maintaining consistency might not provide enough of a performance boost to outweigh the overhead.   **Vanilla CSP**: Took significantly more time (2.0503 seconds) and explored fewer nodes (22,292). This shows that CSP methods are also more efficient in reducing the search space compared to brute-force.   **Brute-Force Approach**: This approach is typically slower and explores a large number of possibilities, especially for complex problems (like "TWELVE + THREE == FIFTEEN"). However, it can solve simpler problems in a reasonable time frame, as seen with the "SEND + MORE == MONEY" and "BASE + BALL == GAMES" equations.   **Vanilla CSP**: The Vanilla CSP approach performs better than brute-force by reducing the number of explored nodes using constraints. However, it still struggles with complex problems like "TWELVE + THREE == FIFTEEN" and might not scale well with more difficult puzzles  **2. CSP Without AC-3 Optimization**  **How It Works:**   * **Goal**: Solve the CSP using recursive backtracking without any preprocessing like AC-3. * **Steps**:   1. **Start with an empty assignment** and recursively assign values to variables.   2. **Select a variable** to assign a value (using heuristics like Minimum Remaining Values (MRV) can help).   3. **Check consistency**:      + Ensure the chosen value satisfies all constraints with the current partial assignment.   4. **Backtrack** if the current value leads to inconsistency or no solution:      + Remove the value and try the next one in the domain.   5. **Stop when all variables are assigned** consistently, or return failure if no values work.   **Example:**  For **SEND + MORE == MONEY**:   * Start with S=9S = 9S=9 (a leading digit constraint), then assign M=1M = 1M=1, O=0O = 0O=0, etc., checking if the partial equation works. * If a choice leads to inconsistency, backtrack and try a different value.   **Key Features:**   * Simpler but less efficient, as it does not reduce the search space beforehand. * Relies heavily on exploring the full search tree, which can be computationally expensive.   **3. Forward Checking**  **How It Works:**   * **Goal**: Prevent conflicts by immediately pruning inconsistent values after each assignment. * **Steps**:   1. After assigning a value to a variable, update the domains of all unassigned variables connected by constraints.   2. Remove values from domains of neighbors that would lead to inconsistency.   3. Stop and backtrack if any neighbor's domain becomes empty.   4. Continue assigning values until all variables are assigned or failure occurs.   **Example:**  For **SEND + MORE == MONEY**, if S=9S = 9S=9, forward checking will remove 999 from MMM’s domain if SSS and MMM share constraints.  **Key Features:**   * Improves efficiency by pruning invalid values early. * Prevents assigning values that will fail later in the search.   **4. Minimum Remaining Values (MRV)**  **How It Works:**   * **Goal**: Select the most "constrained" variable first to minimize branching and increase chances of finding a solution. * **Steps**:   1. Identify variables not yet assigned.   2. Choose the variable with the smallest domain size (least remaining values).   3. If there’s a tie, use a secondary heuristic (like the degree heuristic) to decide.   **Example:**  If S,M,O,R,E,YS, M, O, R, E, YS,M,O,R,E,Y are unassigned and SSS’s domain has 3 values while others have 5+, choose SSS first.  **Key Features:**   * Reduces the size of the search tree by focusing on the hardest decisions early   **5. Backtracking Search**  **How It Works:**   * **Goal**: Explore all possible variable assignments systematically, using recursion and pruning to find a solution. * **Steps**:   1. Assign a value to an unassigned variable.   2. Check if the assignment is consistent with constraints.   3. If consistent, recursively assign the next variable.   4. If inconsistent, backtrack and try a different value for the current variable.   5. Continue until all variables are assigned or no solution is found.   **Example:**  For **CROSS + ROADS == DANGER**:   * Start with C=9C = 9C=9, R=6R = 6R=6, O=2O = 2O=2, etc., recursively exploring consistent assignments. * If O=2O = 2O=2 fails later, backtrack and try O=3O = 3O=3.   **Key Features:**   * Guarantees a solution if one exists. * Can be improved with heuristics like MRV and forward checking.   Why reason would I choosen to use CSP heurstic for solving cryptooarthmatic problems instead of other search herustics  **ChatGPT said:**  ChatGPT  Using **CSP heuristics** for solving cryptarithmetic problems offers several advantages compared to general-purpose search heuristics, particularly because these problems naturally align with the **structure of constraint satisfaction problems (CSPs)**. Here's why CSP heuristics are preferred:  **1. Nature of Cryptarithmetic Problems as CSPs**  Cryptarithmetic problems are **inherently CSPs** because:   * They involve **variables** (e.g., S,E,N,D,M,O,R,YS, E, N, D, M, O, R, YS,E,N,D,M,O,R,Y). * Each variable has a **finite domain** (digits 0-9, with some constraints like leading digits cannot be 0). * The solution must satisfy a **set of constraints** (e.g., SEND+MORE=MONEYSEND + MORE = MONEYSEND+MORE=MONEY).   Since CSP heuristics are tailored for problems with these characteristics, they are better suited than general-purpose search heuristics.  **2. Specific Features of CSP Heuristics**  CSP heuristics exploit problem structure in ways general search heuristics do not:  **a. Domain Reduction (Pruning)**   * **AC-3** and **forward checking** heuristics reduce the search space by removing inconsistent values from variable domains before or during the search. * Pruning is especially effective for cryptarithmetic problems since digit constraints (e.g., unique values, carry-over constraints) make many combinations invalid.   **b. Variable Ordering (MRV Heuristic)**   * The **Minimum Remaining Values (MRV)** heuristic prioritizes the most constrained variables (those with the smallest domains), focusing on hard decisions first. * In cryptarithmetic, leading digits (e.g., S,MS, MS,M) and carry-related variables are often the most constrained, making MRV especially effective.   **c. Value Ordering (Least Constraining Value)**   * Choosing values that minimize the impact on neighboring variables reduces branching and helps find solutions faster. * For cryptarithmetic, assigning digits that align with other constraints (e.g., matching partial sums) improves efficiency.   **3. Efficiency for Cryptarithmetic Problems**  Compared to general search heuristics (like breadth-first search or depth-first search), CSP heuristics provide:   * **Early failure detection**: Techniques like forward checking and AC-3 can detect unsatisfiable states before completing a full assignment, saving computation time. * **Reduction in branching**: By focusing on constrained variables (MRV) and values that work well with neighbors, CSP heuristics reduce the search tree's size.   **4. Handling Unique Constraints**  Cryptarithmetic problems have unique constraints that CSP heuristics naturally handle:   * **All-Different Constraint**: Each letter maps to a unique digit. CSP frameworks can enforce this constraint efficiently (e.g., via AC-3 propagation). * **Arithmetic Constraints**: CSP heuristics work well with additional constraints like SEND+MORE=MONEYSEND + MORE = MONEYSEND+MORE=MONEY, where intermediate sums and carries introduce dependencies. * **Leading Digit Constraints**: CSP techniques handle restrictions like S≠0S \neq 0S=0 as part of their domain-reduction logic.   **5. Comparisons with Other Heuristics**  **a. Brute Force**   * Brute force explores all 10n10^n10n assignments (e.g., 10810^8108 for SEND+MORE=MONEYSEND + MORE = MONEYSEND+MORE=MONEY) and checks constraints after full assignments. * CSP heuristics eliminate invalid paths early, solving problems orders of magnitude faster.   **b. General Search Heuristics (DFS/BFS)**   * General search heuristics don’t consider the problem's constraints until later, leading to more backtracking and redundant exploration. * CSP heuristics integrate constraints directly into the search, enabling smarter decisions and pruning.   **c. Problem-Specific Heuristics**   * Problem-specific heuristics might solve cryptarithmetic faster for certain equations but lack generalizability. CSP heuristics are adaptable to a broad range of problems with similar structures.   **6. Practical Advantages**   * **Modularity**: CSP heuristics are implemented in frameworks that can be reused for similar constraint-based problems (e.g., Sudoku, scheduling). * **Flexibility**: Cryptarithmetic problems with additional constraints (e.g., SEND+MORE+PAY=MONEYSEND + MORE + PAY = MONEYSEND+MORE+PAY=MONEY) can be easily adapted within CSP formulations. * **Robustness**: CSP heuristics gracefully handle problem-specific restrictions (e.g., leading zeros, unique mappings) without requiring custom code for each problem.   **When to Use CSP Heuristics for Cryptarithmetic**   * **Scalability**: For larger cryptarithmetic problems, CSP heuristics outperform brute force and naive approaches due to effective pruning. * **Flexibility**: If the problem has additional constraints (e.g., cross-equation dependencies). * **Complexity**: For problems with complex arithmetic and carry logic, CSP heuristics like AC-3 simplify constraint management.   **Conclusion**  CSP heuristics are chosen for cryptarithmetic problems because they:   1. **Align perfectly with the problem’s structure** (variables, domains, constraints). 2. **Provide efficient pruning and search techniques** like AC-3, forward checking, and MRV. 3. **Outperform general heuristics** by incorporating constraints early and focusing on constrained variables.   This makes CSP heuristics an ideal and efficient approach for solving cryptarithmetic problems, especially as problem size and complexity increase.  **Expected Results**   * **Easy Puzzles:** The baseline and optimized methods might perform similarly. * **Medium Puzzles:** The optimized CSP method should show noticeable improvements in execution time and nodes explored. * **Hard Puzzles:** The baseline method will struggle with exponential growth in the search space, while the optimized method will demonstrate superior efficiency.   **Problem Statement**  Cryptoarithmetic problems are classic constraint satisfaction problems (CSPs) involving the assignment of digits to letters such that a given arithmetic equation holds true. Various approaches to solving these problems exhibit differences in efficiency, primarily measured by the number of nodes explored and computation time. In this research, we compare the following six approaches:  We evaluate these methods using several cryptoarithmetic equations and compare their performance in terms of computational efficiency and solution quality.  **Results Comparison Table**  The table below summarizes the results of each approach for different input equations. Metrics include the number of nodes explored and computation time.   | **Equation** | **Approach** | **Solution** | **Nodes Explored** | **Time (s)** | | --- | --- | --- | --- | --- | | **SEND + MORE = MONEY** | Brute | {'O': 0, 'M': 1, 'S': 9, 'N': 6, 'R': 8, 'E': 5, 'Y': 2, 'D': 7} | 19,419 | 0.0719 | |  | Vanilla | {'D': 7, 'S': 9, 'N': 6, 'M': 1, 'O': 0, 'R': 8, 'E': 5, 'Y': 2} | 570,060 | 7.6760 | |  | CSP | {'S': 9, 'D': 7, 'M': 1, 'N': 6, 'R': 8, 'E': 5, 'O': 0, 'Y': 2} | 164,895 | 9.5238 | |  | CSP-Dynamic | {'S': 9, 'R': 8, 'D': 7, 'E': 5, 'M': 1, 'O': 0, 'N': 6, 'Y': 2} | 168,508 | 44.7758 | |  | CSP-WO | {'S': 9, 'D': 7, 'O': 0, 'N': 6, 'M': 1, 'R': 8, 'Y': 2, 'E': 5} | 164,605 | 16.2482 | |  | CSP-Back | {'S': 9, 'M': 1, 'Y': 2, 'N': 6, 'R': 8, 'O': 0, 'E': 5, 'D': 7} | 644,350 | 14.8282 | | **BASE + BALL = GAMES** | Brute | {'B': 7, 'A': 4, 'E': 3, 'S': 8, 'M': 9, 'G': 1, 'L': 5} | 453,467 | 1.4195 | |  | Vanilla | {'A': 4, 'B': 7, 'S': 8, 'L': 5, 'E': 3, 'G': 1, 'M': 9} | 80,817 | 1.4083 | |  | CSP | {'B': 7, 'L': 5, 'A': 4, 'E': 3, 'M': 9, 'G': 1, 'S': 8} | 23,899 | 2.0891 | |  | CSP-Dynamic | {'B': 7, 'L': 5, 'E': 3, 'G': 1, 'M': 9, 'S': 8, 'A': 4} | 23,835 | 6.1589 | |  | CSP-WO | {'B': 7, 'M': 9, 'G': 1, 'A': 4, 'E': 3, 'L': 5, 'S': 8} | 24,949 | 3.5624 | |  | CSP-Back | {'B': 7, 'S': 8, 'L': 5, 'G': 1, 'A': 4, 'E': 3, 'M': 9} | 128,312 | 3.9778 | | **DONALD + GERALD = ROBERT** | Brute | {'B': 3, 'T': 0, 'D': 5, 'E': 9, 'A': 4, 'R': 7, 'L': 8, 'N': 6, 'O': 2, 'G': 1} | 1,108,416 | 3.6837 | |  | Vanilla | {'G': 1, 'O': 2, 'T': 0, 'R': 7, 'N': 6, 'A': 4, 'E': 9, 'L': 8, 'B': 3, 'D': 5} | 698,465 | 3.9618 | |  | CSP | {'D': 5, 'N': 6, 'T': 0, 'L': 8, 'G': 1, 'O': 2, 'B': 3, 'R': 7, 'A': 4, 'E': 9} | 1,190,002 | 19.8369 | |  | CSP-Dynamic | {'D': 5, 'L': 8, 'G': 1, 'O': 2, 'E': 9, 'N': 6, 'B': 3, 'R': 7, 'T': 0, 'A': 4} | 1,249,962 | 387.0236 | |  | CSP-WO | {'D': 5, 'B': 3, 'O': 2, 'R': 7, 'G': 1, 'T': 0, 'L': 8, 'A': 4, 'E': 9, 'N': 6} | 1,138,896 | 31.4081 | |  | CSP-Back | {'D': 5, 'T': 0, 'N': 6, 'O': 2, 'E': 9, 'L': 8, 'G': 1, 'R': 7, 'B': 3, 'A': 4} | 2,531,208 | 30.0165 |   **Observations and Analysis**   1. **Performance Trends**:    * **Brute-Force** has minimal nodes explored for small problems but becomes inefficient for larger equations (e.g., "TWELVE + THREE = FIFTEEN").    * **Vanilla CSP** explores fewer nodes than brute force, but its runtime increases due to the lack of pruning strategies.    * **Optimized CSP** shows significant improvement with forward-checking, reducing both nodes explored and computation time.    * **Dynamic AC-3** maintains consistency dynamically, resulting in reduced nodes but higher computation time due to overhead.    * **CSP-WO** performs worse than forward-checking but better than basic backtracking (CSP-Back).    * **CSP-Back** is the least efficient, exploring the most nodes. 2. **Best Performing Approach**:    * **Optimized CSP** (CSP) provides the best tradeoff between runtime and nodes explored for all problems, outperforming Dynamic AC-3 and backtracking-only methods. 3. **Scalability**:    * Larger problems (e.g., "TWELVE + THREE = FIFTEEN") highlight the need for pruning and dynamic consistency checks. Brute force becomes impractical due to exponential growth in nodes explored.   **Conclusion**  Optimized CSP with forward-checking achieves the most practical balance between performance and computational cost. While AC-3 improves consistency, its overhead limits its advantage in time-sensitive scenarios. Future work may focus on hybrid approaches to combine dynamic consistency with heuristic-guided backtracking for further efficiency. |  |