SEVENTH SEMESTER **B.TECH (MC)**

END SEMESTER EXAMINATION **NOVEMBER 2014**

MC-402 APPLIED GRAPH THEORY

Time: 3 Hours **Maximum Marks: 70**

Note: Answer **ALL** by selecting any **TWO** parts from each question. All questions carry equal marks.

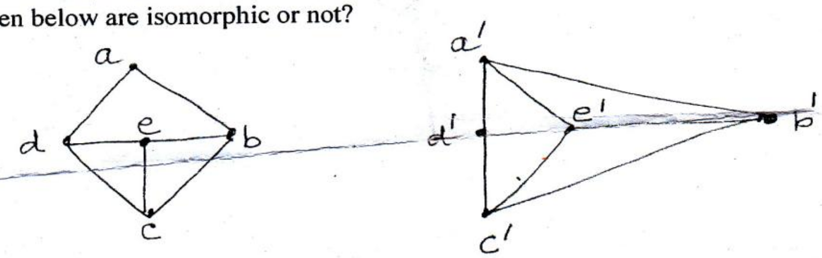
Q1.(a) (i) Prove that if a graph G has more than two vertices of odd degree, then there can be no Euler path in G.

(ii) Prove that if G is a connected graph and has exactly two vertices of odd degree, then there is an Euler path in G.

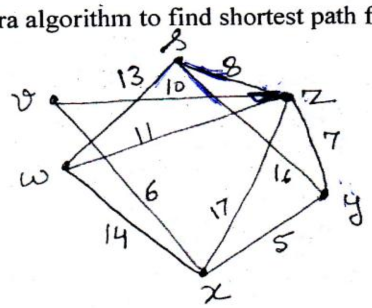
(b) Prove that a connected graph G is an Euler graph iff it can be decomposed into circuits.

(c) Define the Ring sum of two graphs and complement of a graph. Show that a graph is self complementary if it has 4n or 4n+1 vertices.

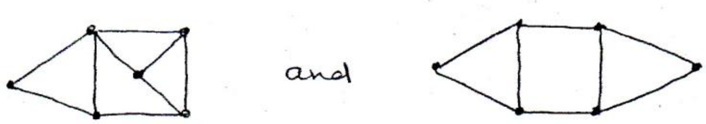
Q2.(a) Define adjacency matrix of a graph. Using adjacency matrix verify whether the two graphs given below are isomorphic or not?



(b) Apply Dijkstra algorithm to find shortest path from s to x in the graph given below:



(c) Define Isomorphism of a graph. Examine the following pairs for isomorphism:



Q3. (a) For an n-vertex graph G, show that the following are equivalent:

(i) G is connected and has no cycles.

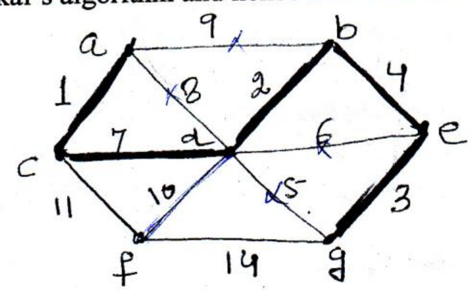
(ii) G is connected and has n-1 edges.

(iii) G has n-1 edges and no cycles.

(iv) For any two vertices u and v of G there is exactly one u-v path.

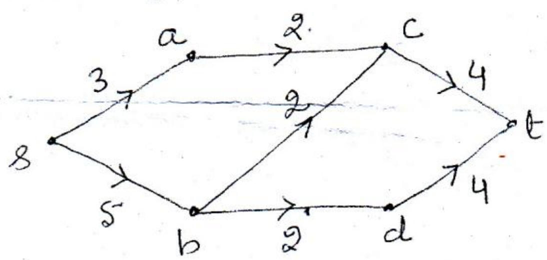
(b) Define binary tree. Prove by mathematical induction that the maximum number of vertices on level n of a binary tree is 2^n.

(c) Explain Kurskal's algorithm and hence find a minimal spanning tree of the graph given below.

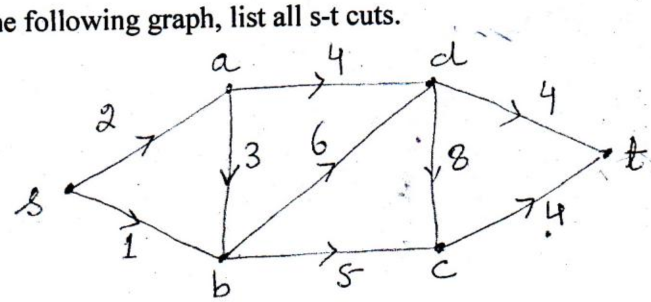


Q4. (a) Define edge connectivity of a graph. Show that the edge connectivity of a graph G cannot exceed the minimum degree of a vertex in G.

(b) Use Ford-Fulkerson algorithm to find the maximum flow for the following network.



(c) For the following graph, list all s-t cuts.



Q5. (a) Find the number of perfect matching in the complete bipartite graph $K_{n,n}$.

(b) Let G be a K-regular bipartite graph with K>0. Then show that G has a perfect matching.

(c) Prove that a planar graph is 5-colorable.