

Total No. of Pages: 02
 THIRD SEMESTER
 END SEMESTER EXAMINATION

MC/009
 Roll No.
 B. Tech.[MC]

November, 2013

MC-201, Mathematics-III

Time: 3 Hours

Max. Marks: 70

Note: Attempt all questions selecting any two parts from each question. Assume suitable missing data, if any.

- (1) (a) Drive the polar form of the Cauchy Riemann (CR) equations. [7]
 (b) Show that the function

$$f(z) = \begin{cases} \frac{x^3(1+i)-y^3(1-i)}{x^2+y^2} & \text{when } z \neq 0, \\ 0 & \text{when } z = 0. \end{cases}$$

satisfies the Cauchy Riemann equations at $z = 0$ but $f(z)$ is not differentiable at $z = 0$. [7]

- (c) Discuss the convergence/divergence of the following integrals [3+4]
 (i) $\int_0^\infty x^{p-1} e^{-x} dx, p > 0$
 (ii) $\int_1^\infty \frac{x \tan^{-1}(x)}{\sqrt{4+x^2}} dx$
- (2) (a) Find all Taylor and Laurent series of $f(z) = \frac{-2z+3}{z^2-3z+2}$ with center 0. [7]
 (b) Evaluate the integral $\int_0^\infty \frac{\sin(x)}{x} dx$ using contour integration. [7]
 (c) Solve the followings: [7]
 (i) Find all solutions of $\sin(z) = 100$.
 (ii) Verify that $f(z) = \cos(z)$ is an analytic function and hence find the derivative.

- (3) Evaluate the integral $I = \oint_C \frac{dz}{z(z+2)}$ where C is any rectangle containing the points $z = 0$ and $z = -2$ inside it. [7]
 (b) Show that the function $u(x, y) = 2x + y^3 - 3x^2y$ is harmonic. Find the conjugate harmonic function $v(x, y)$ and the corresponding analytic function $f(z)$. [7]
 (c) Evaluate the integral $I = \oint_C \frac{e^z}{z^2(z+1)^3} dz$ where $C : |z| = 2$. [7]

- (4) (a) Evaluate the integral $I = \int_0^\pi \sin^4(\theta) d\theta$ using residues. [7]
 (b) Solve [7]
 (i) Find the Z-transform $Z[e^{\alpha n} \cos(\beta n)]$.
 (ii) Find the inverse Z-transform $Z^{-1}\left[\frac{z^2}{z^2+1}\right]$.
 (c) Find the image of the region bounded by the lines $x - y < 2$ and $x + y > 2$ under the mapping $W = \frac{1}{z}$ and draw the region. [7]

2

- (5) (a) Solve the difference equation $y_{n+2} - 3y_{n+1} + 2y_n = 0, y_0 = -1, y_1 = 2$ using
Z transformation. [7]
- (b) Evaluate the integral $I = \int_{-\infty}^{\infty} \frac{\sin^2(2x)}{1+x^2} dx.$ [7]
- (c) State and prove Cauchy integral theorem. [7]
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Total No. of Pages: 02

Roll No. MC/009

THIRD SEMESTER

B.Tech.

END SEMESTER EXAMINATION

(November - 2013)

MC- 202 Differential Equations

Time: 3 Hours

Max. Marks: 70

Note: Answer any ten questions.

All questions carry equal marks.

Assume suitable missing data, if any.

1. Derive Jacobi's series from the generating function of Bessel's function. Hence show

that

$$\cos(x\cos \theta) = J_0 - 2J_2 \cos 2\theta + 2J_4 \cos 4\theta - \dots \dots \dots$$

$$\sin(x\cos \theta) = 2J_1 \cos \theta - 2J_3 \cos 3\theta + 2J_5 \cos 5\theta - \dots \dots \dots$$

2. Find the Green's function for the differential equation

$$u'' + k^2 u = f(x), u(0) = 0, u\left(\frac{\pi}{2k}\right) = 0.$$

✓ 3. Find all eigen values and eigen functions of the following differential equation

$$y'' + \lambda y = 0, \text{ given that } y(0) + y'(0) = 0, y(1) + y'(1) = 0$$

✓ 4. Solve $(x^2 - y^2 - z^2)p + 2xyq = 2xz$.

5. Solve $(y - x)(qy - px) = (p - q)^2$.

6. Solve $z^2(p^2 + q^2) = x^2 + y^2$.

7. Solve by Charpit's method $q + xp = p^2$.

8. Solve $r - 4s + 4t = e^{2x+y}$.

9. Reduce $z_{xx} = x^2 z_{yy}$ to canonical form and hence find the solution.

10. A bar 10 cm long, with insulated sides, has its ends A and B maintained at temperature 50°C and 100°C respectively, until steady state condition prevail. The temperature at A is suddenly raised to 90°C and at the same time that at B lowered to 60°C. Find the temperature distribution in the bar at time t.

11. Solve $\frac{\partial^2 u}{\partial x^2} = c^2 \frac{\partial^2 u}{\partial t^2}$ given that $u(0,t) = 0$, $u(a,t) = 0$ by method of separation of variable.

12. An infinite long string having one end at $x=0$ is initially at rest on the x-axis. The end $x = 0$ undergoes a periodic transverse displacement given by $A_0 \sin nt$, $t > 0$. Find the displacement of any point on the string $t > 0$.

No. of Pages : 02
MC-009

B.TECH (MC)

NOV. - DEC. 2013

MC-203 DISCRETE MATHEMATICS

Time : 3 Hours

Maximum Marks : 70

Note: Answer ALL by selecting any TWO parts from each question. All questions carry equal marks.

- Q1.** (a) If A, B and C are sets then prove that
 (i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$
 (ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$
 (b) Let $N = \{1, 2, 3, \dots\}$ and a relation R is defined on $N \times N$ as follows:
 (a, b) R (c, d) iff ad = bc then show whether R is an equivalence relation or not.
 (c) Let f and g both are functions from $N \times N$ to N given by $f(x, y) = (x+y)$ and $g(x, y) = xy$. Show that they are onto but not one-one.
- Q2.** (a) Show that the set of cube roots of unity is an abelian group w.r.t. multiplication.
 (b) Prove that an isomorphic image of a group is a group.
 (c) Use generating function to solve the recurrence relation

$$a_{n+2} - 2a_{n+1} + a_n = 2^n, a_0 = 2, a_1 = 1$$

- Q3.** (a) Test the validity of the following arguments:
 (i) If 8 is even then 2 does not divide 9
 Either 7 is not prime or 2 divides 9
 But 7 is prime
 Therefore 8 is odd.
 (ii) I will become famous or I will be writer
 I will not be a writer
 I will become famous.
- (b) (i) Show that any implication is logically equivalent to its contrapositive.
 (ii) Using algebra of proposition show that

$$(p \Rightarrow q) \wedge (r \Rightarrow q) \equiv (p \wedge r) \Rightarrow q$$

(c) What are normal forms? Find PCNF and PDNF of the following:

$$(p \Rightarrow (q \wedge r)) \wedge (\neg p \Rightarrow (\neg q \wedge \neg r))$$

- Q4.** (a) Let X be the set of all 2×2 matrices. Let $A = [a_{ij}]$ and $B = [b_{ij}]$ be any two elements of X. Define a relation ' \leq ' on X s.t.

$$A \leq B \text{ iff } a_{11} + a_{12} + a_{21} + a_{22} \leq b_{11} + b_{12} + b_{21} + b_{22}.$$

 Is this a partial order relation on X?
 (b) Let (L, \leq) be a lattice and $a, b, c \in L$. Then show that
 (i) $a \leq b$ and $a \leq c \rightarrow a \leq b \vee c$
 (ii) $a \leq b$ and $a \leq c \rightarrow a \leq b \wedge c$
 (c) In a Boolean algebra, if $ax = bx$ and $ax' = bx'$, then show that $a=b$

Q5

(a) Prove that

(i) The sum of the degrees of all vertices in a graph G is equal to twice the number of edges in G.

(ii) The vertices of odd degree in a graph is always even.

(b) Define the following with suitable examples:

 Digraph, connected graph, complete graph, bipartite graph, tree, walk, path.

(c) Prove by mathematical induction that a tree with n vertices has $(n-1)$ edges.

Total No. of Pages: 3

IIIRD SEMESTER

B.Tech.(Mathemtics and Computing)

END SEMESTER EXAMINATION

Roll No. MC/009

(Nov. – 2013)

Paper Code: MC-204

Time: 3:00 Hours

Subject: Data Structures

Max. Marks: 70

Note: Answer any five questions.

Assume suitable missing data, if any.

1. (a) What Is a Binary Search Tree (BST)? Make a BST for the following sequence of numbers.

45, 36, 76, 23, 89, 115, 98, 39, 41, 56, 69, 48

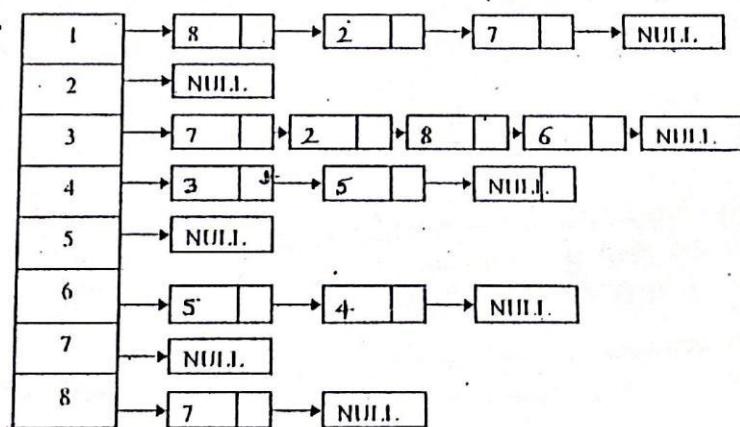
Traverse the tree In Preorder, Inorder and postorder.

- (b) Draw the expression tree of the following Infix expression. Convert It In to Prefix and Postfix expressions.

$((a + b) + c * (d + e) + f)^* (g + h)$

(7+7=14)

2. (a) Consider a graph having following representation. Draw the correponding graph. Run Depth First Search (DFS) on this graph starting from node 3 and show the result.



- (b) Draw a B-tree of order 3 for the following sequence of keys:

2, 4, 9, 8, 7, 6, 3, 1, 5, 10

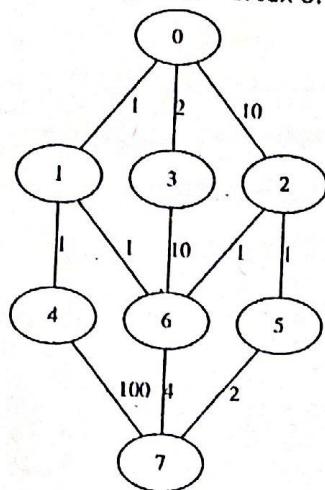
(7+7=14)

✓ 3.

(a) Reverse the order of elements on a stack S

- (i) using two additional stacks.
- (ii) using one additional queue.

✓ (b) Find the minimum spanning tree for the following graph using Prim's algorithm starting from vertex 0. Show every step of algorithm.



(6+8=14)

✓ 4.

(a) Define the term array. How are two-dimensional arrays represented in memory? Explain how address of an element is calculated in a two dimensional array.

✓ (b) Which sorting algorithm is easily adaptable to singly linked lists? Explain your answer.

(7+7=14)

5. (a) Two Binary Trees are similar if they are both empty or if they are both nonempty and left and right sub trees are similar. Write an algorithm to determine if two Binary Trees are similar.

(b) How can stacks be used to check whether an expression is correctly parenthesized or not. For eg (()) is well formed but () or)() is not.

(7+7=14)

6.

(a) Write an algorithm to convert an unsorted array into a min-heap.

- (b) Illustrate your algorithm with the following sequence of keys:
8, 20, 9, 4, 15, 10, 7, 22, 3, 12
- (c) Write down delete-Min() algorithm and
- (d) Show the heap structure after each delete-Min() operation on heap constructed above (perform two such operations).

(3+4+3+4=14)

7. (a) Construct a binary tree whose nodes in inorder and preorder are given as follows:

Inorder : 10, 15, 17, 18, 20, 25, 30, 35, 38, 40, 50
Preorder: 20, 15, 10, 18, 17, 30, 25, 40, 35, 38, 50

- (b) Delete the keys "30" from the constructed tree and give the resultant tree.

(7+7=14)

8. Write short notes on following

- (i) B Tree.
(ii) Time Complexity, Big O notation.
(iii) Merge Sort.
(iv) Breadth First Traversal.

(4x3.5=14)

Total no. of pages: 03

Roll no.....ME009

3rd Semester B. Tech Mathematics & Computing

End Semester Examination November 2013

MC – 205 Probability and Statistics

Time: 3 hrs.

Max. Marks: 70

Note: Answer all questions by selecting any two parts out of the three set in each question. All questions carry equal marks. You can assume for the missing data, if any. You can ask for the statistical tables.

1(a) State and prove multiplication rule of probability. A town has two fire engines operating independently. The probability that a specific engine is available when needed is 0.96. What is the probability that neither engine is available when needed? What is the probability that a fire engine is available when needed?

(b) Prove Baye's formula. A population can be divided into two subgroups that occur with probabilities 60% and 40% respectively. An event A occurs 30% of the time in the first subgroup and 50% of the time in the second subgroup. What is the unconditional probability of the event A regardless of which subgroup it comes from?

(c) Suppose that 3 batteries are randomly chosen from a group of 3 new and 4 used but still working, and 5 defective batteries. If X and Y denote respectively the number of new and used but still working batteries chosen, then find the joint probability mass function of X and Y. Also find the marginal distributions for X and Y. Are the two variables X and Y independent?

2(a) What is the difference between the problem of 'correlation' and 'regression'? Why are there in general two regression lines? Derive the equation of the line of regression of y on x. Where do they intersect?

(b) For the following data giving the effect of extraction on the efficiency of an extraction process obtained from a chemical industry, find the extraction efficiency (y in %) expected when the extraction time (x in minutes) is 40:

x: 27	45	41	19	35	39	19	49	15	31
y: 57	64	80	46	62	72	52	77	57	68

(c) If the joint distribution of two random variables X and Y is given by

$$f(x, y) = Kxye^{-(x^2+y^2)} ; x \geq 0, y \geq 0,$$

then find K. Test whether X and Y are independent. Also find the conditional density of X given $Y=y$.

3(a) What are specific applications of a Poisson variate? Show that Poisson distribution is a limiting case of binomial distribution. What are mean, variance and mode of a Poisson variate with parameter λ ? What is a standard Poisson variate?

(b)(i) A multiple-choice quiz has 200 questions each with 4 possible answers of which only 1 is the correct answer. What is the probability that sheer guess-work yields from 25 to 30 correct answer from 80 of the 200 problems about which the student has no knowledge ?

(ii) Show that the failure rate in case of exponential distribution is constant.

(c) Define Weibull distribution. Find its mean and variance. Give a few of its specific applications. How is it related to exponential distribution?

4(a) What are the chief characteristics of a normal probability curve? Discuss specifically its area property and its applications in large sampling testing.

(b) State and prove weak law of large numbers. Mention two situations when WLLN is (i) applicable, (ii) not applicable.

(c) What is hypothesis testing? What are the two types of errors that arise in testing? Show that sample mean is an unbiased estimate of the population mean. What about sample variance?

5(a)(i) The mean heights in two large samples of 1000 and 2000 men are 67.5 inches and 68.0 inches respectively. Can the two samples be regarded as drawn from the same population of S.D. 2.5 inches?

(ii) A sleep inducing tablet when administered to 50 insomniacs was found to be effective on 37 patients. Test the hypothesis at $\alpha = 0.05$ that the tablet was effective in at least 80 % cases.

(b)(i) A survey of 800 families with four children each recorded the following distribution:

No. of boys:	0	1	2	3	4
No. of girls:	4	3	2	1	0
No. of families:	32	178	290	236	64

Test the hypothesis that male and female births are equally likely.

(ii) To test the claim that the resistance of electric wire can be reduced by at least 0.05 ohm by alloying, 25 measurements obtained for each alloyed wire and standard wire produced the following results:

	Mean	S.D.
Alloyed wire (x)	0.083 ohm	0.003 ohm
Standard wire (y)	0.136 ohm	0.002 ohm

Test the claim at 5 % level of significance.

(c) What is ANOVA and when is it specifically used? Following data represents the total mileages obtained by the vehicle on the same type of the run on three different gas fuels

Gas 1:	220	251	226	246	260
Gas 2:	244	235	232	242	225
Gas 3:	252	272	250	238	256

At $\alpha = 0.05$, test the hypothesis that the average mileage obtained is not affected by the type of the gas used.

• Total Number of Page- 3

Roll No. MC/009

Delhi Technological University
B. Tech, Third Semester

END SEMESTER EXAMINATION

November 2013

CE/EE/EP/EL/MC/SE-206-Engineering Economics

Time: 3 Hour

Max. Marks: 70

Note: Answer any five questions,

Assume suitable missing data, if any

Use of interest rate table is allowed

1.	a	What do you mean by Business Cycle? Discuss the phase of the Business cycle through which the Indian Economy is undergoing at present.	7
	b	Find the price elasticity of demand for the demand function $P = e^k$ and $P = e^k/x$	4
	c	If $TC = ax^2 + bx + c$ is the cost function of a monopolist and $P = \beta + ax$ is the demand law, find the monopoly price and output.	3
2.	a	Discuss role of appropriate technology in the development of a developing economy.	7
	b	A person who is now 35 years old is planning for his retired life. He plans to invest an equal sum of Rs. 10,000 at the end of every year for the next 25 years starting from the end of the next year. The bank gives 20% interest rate, compounded annually. Find the maturity value of his account when he will be 60 years old.	4
	→ c	A company is producing tire. Cost of machine is Rs. 261,000. Cost of labour and raw material is @Rs. 160/tire. Manufacturer sells tire @ Rs.450/tire. What will be the break even point for the company?	3
3.	b	What is the significance of studying Engineering Economics for engineering students? How it can be made more relevant.	7
	→ b	Two years ago, a machine was purchased at a cost of Rs. 2,00,000 to be useful for eight years. Its salvage value at the end of its life is Rs. 25,000. The annual maintenance cost is Rs. 25,000. The market value of the present machine is Rs. 1,20,000. Now, a new machine to cater to the need of the present machine is available at Rs. 1,50,000 to be useful for six years. Its annual maintenance cost is Rs. 14,000. The salvage value of the new machine is Rs. 20,000. Using an interest rate of 12%, find whether it is worth replacing the present machine with the new machine.	7

✓ ✓	Discuss factors which should be considered while deciding price of the product in the market.	7															
✓	Rs. 40,000 were taken as loan for replacement of a machine and it compounds to Rs. 60,000 in 5 years, what will be the rate of interest which is compounded quarterly.	4															
→ c	Following are the demand functions for two commodities x_1 and x_2 : $x_1 = p_1^{-1.7} p_2^{0.8}$ $x_2 = p_1^{0.5} p_2^{-0.2}$ Determine whether the commodities are complementary or competitive.	3															
✓ ✓	Why do businesses such as automobile dealers tend to locate together? Three engineering graduate decides to become automobile dealer, and all three devote their full time to its management. What opportunity cost would you assign to their time?	7															
✓	A company must decide whether to buy Machine A or Machine B by Future Worth method. Rate of interest 12% which is compounded annually.	7															
	<table border="1"> <thead> <tr> <th></th> <th>Machine A</th> <th>Machine B</th> </tr> </thead> <tbody> <tr> <td>Initial Cost in Rs.</td> <td>4,00,000</td> <td>8,00,000</td> </tr> <tr> <td>Useful life in years</td> <td>4</td> <td>4</td> </tr> <tr> <td>Salvage value at the end of machine life in Rs.</td> <td>2,00,000</td> <td>5,00,000</td> </tr> <tr> <td>Annual maintenance Cost in Rs.</td> <td>40,000</td> <td>NIL</td> </tr> </tbody> </table>		Machine A	Machine B	Initial Cost in Rs.	4,00,000	8,00,000	Useful life in years	4	4	Salvage value at the end of machine life in Rs.	2,00,000	5,00,000	Annual maintenance Cost in Rs.	40,000	NIL	
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6. a.	Discuss the nature of Computer Software market in India. What incentive may be given by the Government to encourage Indian producer?	7															
b.	For construction purpose, a concrete aggregate mix is required which should contain at least 34% sand by volume. We have following option available:	4															
	<table border="1"> <thead> <tr> <th></th> <th>Sand</th> <th>Coarse</th> <th>Rate</th> </tr> </thead> <tbody> <tr> <td>One source material</td> <td>30%</td> <td>70%</td> <td>Rs. 3.50 per cubic meter</td> </tr> <tr> <td>Second source material</td> <td>42%</td> <td>58%</td> <td>Rs. 4.25 per cubic meter</td> </tr> </tbody> </table>		Sand	Coarse	Rate	One source material	30%	70%	Rs. 3.50 per cubic meter	Second source material	42%	58%	Rs. 4.25 per cubic meter				
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One source material	30%	70%	Rs. 3.50 per cubic meter														
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	Determine the least cost per cubic meter of blended aggregates.																
c.	A special type of screw is manufactured at a unit cost of Rs. 4 for material and Rs. 5.5 for direct labour. An investment of Rs. 6,00,000 is required to purchase the machine. Other cost is 1.5 times Direct labour cost. The order calls for three million pieces. Half-way through the order, a new method of manufacture can be put into effect which will reduce the unit costs of Rs.3	3															

	for material and Rs. 4.5 for direct labour -but it will require Rs1,00,000.00 for additional machine. Which process should be taken up?	
✓ ✓	Discuss seven salient feature of the Indian Economy.	7
✓ ✓	The green houses have the following cost comparison during their life time production system. Find out which system is more economical if the rate of interest is 15% compounded annually.	7

	System A	System B
Initial Cost in Rs.	30,000	20,000
Maintenance Cost/year in Rs.	1000	2500
Replacement of a component at the end of every third year in Rs.	4000	4200
Salvage value in Rs.	5,000	-
Life in Years	20	20
Benefit from quality control as uniform end of year amount/year	1000	-