
Contents

| | |
|---|-----------|
| Preface | xi |
| Authors | xv |
| Acknowledgements | xvii |
| 1 Introduction to mathematical modelling | 1 |
| 1.1 Mathematical models | 1 |
| 1.2 An overview of the book | 2 |
| 1.3 Some modelling approaches | 4 |
| 1.4 Modelling for decision-making | 6 |
| 2 Compartmental models | 9 |
| 2.1 Introduction | 9 |
| 2.2 Exponential decay and radioactivity | 10 |
| 2.3 <i>Case Study: Detecting art forgeries</i> | 17 |
| 2.4 Scenario: Pacific rats colonise New Zealand | 19 |
| 2.5 Lake pollution models | 20 |
| 2.6 <i>Case Study: Lake Burley Griffin</i> | 26 |
| 2.7 Drug assimilation into the blood | 28 |
| 2.8 <i>Case Study: Dull, dizzy or dead?</i> | 33 |
| 2.9 Cascades of compartments | 37 |
| 2.10 First-order linear DEs | 38 |
| 2.11 Equilibrium points and stability | 39 |
| 2.12 <i>Case Study: Money makes the world go around</i> | 41 |
| 2.13 Exercises for Chapter 2 | 44 |
| 3 Models of single populations | 51 |
| 3.1 Exponential growth | 51 |
| 3.2 Density-dependent growth | 56 |
| 3.3 Limited growth with harvesting | 62 |
| 3.4 Scenario: Anchovy wipe-out | 65 |
| 3.5 Scenario: How can 2×10^6 birds mean rare? | 66 |
| 3.6 <i>Case Study: It's a dog's life: The control of stray dogs</i> | 66 |
| 3.7 Discrete population growth and chaos | 68 |
| 3.8 Time-delayed regulation | 75 |
| 3.9 <i>Case Study: Australian blowflies</i> | 77 |
| 3.10 Exercises for Chapter 3 | 78 |
| 4 Numerical solution of differential equations | 85 |
| 4.1 Introduction | 85 |
| 4.2 Basic numerical schemes | 85 |
| 4.3 Computer implementation using Maple TM and MATLAB [®] | 89 |

| | | |
|----------|---|------------|
| 4.4 | Instability | 91 |
| 4.5 | Discussion | 91 |
| 4.6 | Exercises for Chapter 4 | 92 |
| 5 | Interacting population models | 97 |
| 5.1 | Introduction | 97 |
| 5.2 | Model for an influenza outbreak | 99 |
| 5.3 | <i>Case Study: Cholera</i> | 109 |
| 5.4 | Predators and prey | 113 |
| 5.5 | Scenario: Nile Perch catastrophe | 120 |
| 5.6 | <i>Case Study: It's a dog's life: More on the control of stray dogs</i> | 121 |
| 5.7 | Competing species | 124 |
| 5.8 | Scenario: Aggressive protection of lerps and nymphs | 129 |
| 5.9 | Model of a battle | 130 |
| 5.10 | <i>Case Study: Rise and fall of civilisations</i> | 135 |
| 5.11 | Exercises for Chapter 5 | 138 |
| 6 | Phase-plane analysis | 145 |
| 6.1 | Introduction | 145 |
| 6.2 | Phase-plane analysis of epidemic model | 148 |
| 6.3 | Analysis of a battle model | 152 |
| 6.4 | Analysis of a predator-prey model | 157 |
| 6.5 | Analysis of competing species models | 161 |
| 6.6 | Closed trajectories for the predator-prey | 167 |
| 6.7 | <i>Case Study: Bacteria battle in the gut</i> | 169 |
| 6.8 | Exercises for Chapter 6 | 171 |
| 7 | Linearisation analysis | 179 |
| 7.1 | Introduction | 179 |
| 7.2 | Linear theory | 179 |
| 7.3 | Applications of linear theory | 188 |
| 7.4 | Nonlinear theory | 190 |
| 7.5 | Applications of nonlinear theory | 193 |
| 7.6 | Exercises for Chapter 7 | 196 |
| 8 | Some extended population models | 199 |
| 8.1 | Introduction | 199 |
| 8.2 | <i>Case Study: Competition, predation and diversity</i> | 199 |
| 8.3 | Extended predator-prey models | 202 |
| 8.4 | <i>Case Study: Lemming mass suicides?</i> | 205 |
| 8.5 | <i>Case Study: Prickly pear meets its moth</i> | 208 |
| 8.6 | <i>Case Study: Geese defy mathematical convention</i> | 210 |
| 8.7 | <i>Case Study: Possums threaten New Zealand cows</i> | 214 |
| 8.8 | Exercises for Chapter 8 | 220 |
| 9 | Formulating heat and mass transport models | 225 |
| 9.1 | Introduction | 225 |
| 9.2 | Some basic physical laws | 226 |
| 9.3 | Model for a hot water heater | 229 |
| 9.4 | Heat conduction and Fourier's law | 234 |
| 9.5 | Heat conduction through a wall | 236 |

| | | |
|-----------|---|------------|
| 9.6 | Radial heat conduction | 240 |
| 9.7 | Heat fins | 242 |
| 9.8 | Diffusion | 246 |
| 9.9 | Exercises for Chapter 9 | 249 |
| 10 | Solving time-dependent heat problems | 251 |
| 10.1 | The cooling coffee problem revisited | 251 |
| 10.2 | The hot water heater problem revisited | 254 |
| 10.3 | <i>Case Study: It's hot and stuffy in the attic</i> | 258 |
| 10.4 | Spontaneous combustion | 261 |
| 10.5 | Scenario: Fish and chips explode | 267 |
| 10.6 | Exercises for Chapter 10 | 268 |
| 11 | Solving heat conduction and diffusion problems | 271 |
| 11.1 | Boundary value problems | 271 |
| 11.2 | Heat loss through a wall | 273 |
| 11.3 | <i>Case Study: Double glazing: What's it worth?</i> | 278 |
| 11.4 | Insulating a water pipe | 281 |
| 11.5 | Cooling a computer chip | 286 |
| 11.6 | <i>Case Study: Tumour growth</i> | 291 |
| 11.7 | Exercises for Chapter 11 | 294 |
| 12 | Introduction to partial differential equations | 301 |
| 12.1 | The heat conduction equation | 301 |
| 12.2 | Oscillating soil temperatures | 303 |
| 12.3 | <i>Case Study: Detecting land mines</i> | 308 |
| 12.4 | Lake pollution revisited | 311 |
| 12.5 | Exercises for Chapter 12 | 317 |
| A | Differential equations | 319 |
| A.1 | Properties of differential equations | 319 |
| A.2 | Solution by inspection | 320 |
| A.3 | First-order separable equations | 321 |
| A.4 | First-order linear equations and integrating factors | 322 |
| A.5 | Homogeneous equations | 323 |
| A.6 | Inhomogeneous equations | 324 |
| B | Further mathematics | 329 |
| B.1 | Linear algebra | 329 |
| B.2 | Partial derivatives and Taylor expansions | 332 |
| B.3 | Review of complex numbers | 335 |
| B.4 | Hyperbolic functions | 335 |
| B.5 | Integration using partial fractions | 337 |
| C | Notes on Maple and MATLAB | 339 |
| C.1 | Brief introduction to Maple | 339 |
| C.2 | Solving differential equations with Maple | 339 |
| C.3 | Brief introduction to MATLAB | 341 |
| C.4 | Solving differential equations with MATLAB | 342 |
| D | Units and scaling | 347 |
| D.1 | Scaling differential equations | 347 |