Total No. of pages. 03
SEVENTH SEMESTER
END SEMESTER EXAMINATION

Roll No.....59...........
B.TECH (MC)
NOVEMBER 2016

MC-402 APPLIED GRAPH THEORY

Time: 3 Hours **Maximum Marks: 70**

Note: Answer **ALL** by selecting any **TWO** parts from each question.
All questions carry equal marks.

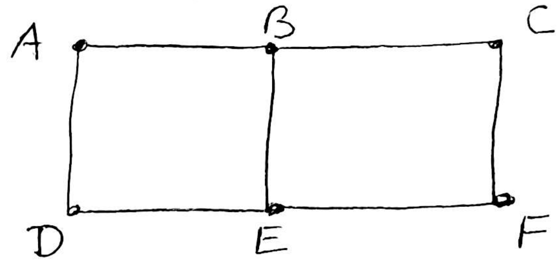
Q1(a) Prove that the maximum number of edges among all p vertices graph with no triangle is $[\frac{p^2}{4}]$, where [r] denotes the greatest integer not exceeding the real number r.

(b) Prove that a graph is bipartite iff all its cycles are even.

(c) Let $G = (p, q)$ graph having p vertices and q edges all of whose vertices have degree k or $k+1$. If G has $p_k > 0$ vertices of degree k and p_{k+1} vertices of degree $k+1$ then show that

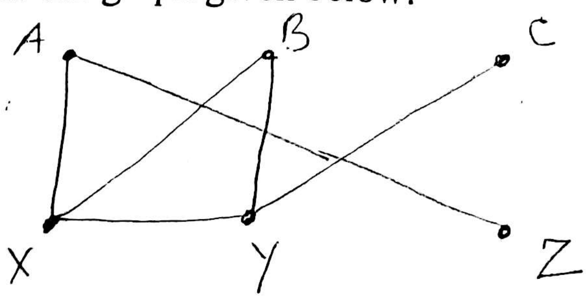
$$p_k = (k+1)p - 2q.$$

Q2(a) (i)Consider the graph given below:



Find the subgraphs obtained when each vertex is deleted. Does these subgraphs have any cut-vertices ? Which of them are isomorphic?

(ii) Consider the graph given below:



Find (a) all paths from A to C, (b) all cycles, (c) subgraph of G generated by { B,C,X,Y }, (d) G-Y, (e) all cut vertices, (f) all bridges.

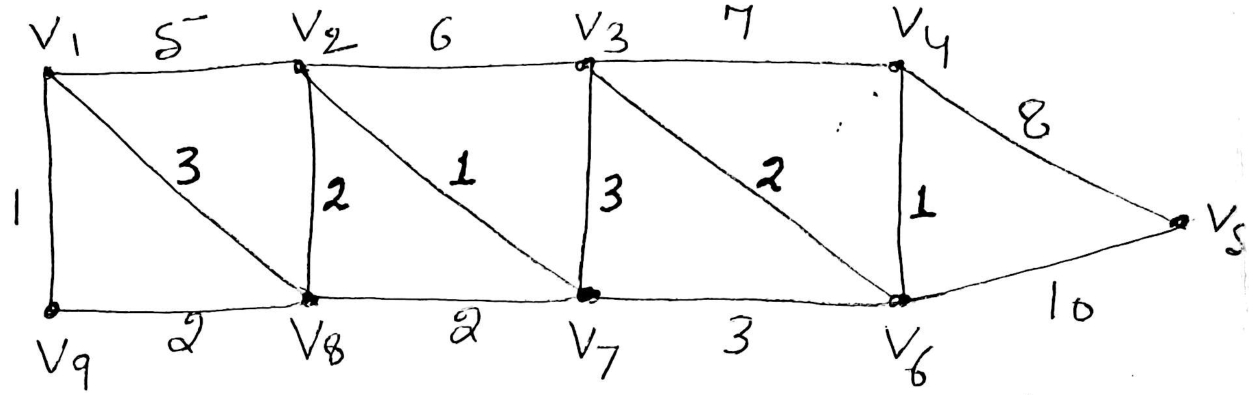
(b) Prove by induction on number of edges that every graph $G = (V, E)$ has at least $|V| - |E|$ connected components.

(c) Prove that in a complete graph with n vertices, there are (n-1)/2 edge disjoint Hamiltonion circuits, if n is an odd number greater than or equal to 3.

Q3. (a) Prove that a graph is a tree iff it is minimally connected.

(b) Define spanning tree. Prove that every connected graph has at least one spanning tree.

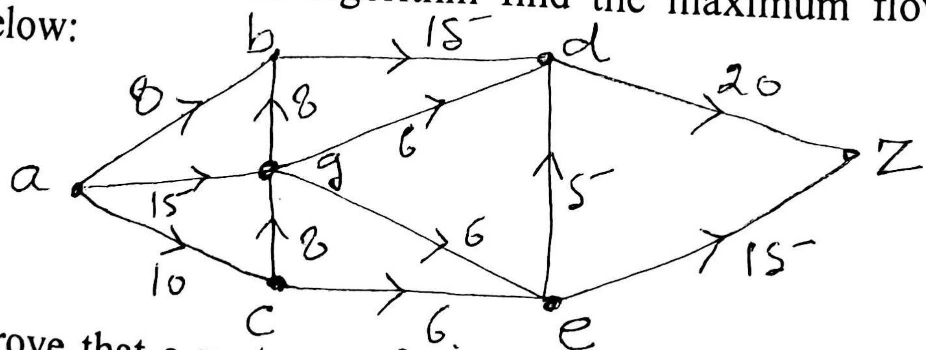
(c) Explain Kurskal's algorithm and hence find a minimal spanning tree of the graph given below.



Q4(a)Let T be a spanning tree of a connected graph G.Then a cut set which contains exactly one branch of T and all chords of T is called a fundamental cut set of G.

For a given spanning tree, let D={ $e_1, e_2, \ldots\ldots\ldots, e_k$ } be a fundamental cut set in which e_1 is a branch and $e_2, \ldots\ldots, e_k$ are chords of the spanning tree. Then show that e_1 is contained in the fundamental circuit corresponding to e_i for $i = 2,3,\ldots\ldots, k$. Moreover e_1 is not contained in any other fundamental circuits.

(b) Explain Ford-Fulkerson algorithm to find the maximum flow for a network. Use this algorithm find the maximum flow for the network below:



(c) Prove that a vertex v of a connected graph G is a cut vertex iff there exist two vertices x and y in G such that every path between them passes through v.

Q5. (a) Define Matching in a graph. Prove that every 3-regular graph without cut edges has a perfect matching.

(b) Let G be a K-regular bipartite graph with $K>0$. Then show that G has a perfect matching.

(c) Show that the following statements are equivalent:

(i) A graph G is 2-colorable

(ii) G is bipartite

(iii) Every cycle of G has even length