

Total No. of Pages: 08

Roll No.: MC/059

THIRD SEMESTER

B.TECH (MC)

END SEMESTER EXAMINATION

(Nov., 2014)

MC-201

MATHEMATICS - III

Time: 3 Hours

Max. Marks: 70

Note: Attempt all the questions by selecting any TWO parts from each question.

- (1) (a) Discuss the convergence or divergence of the following improper integrals. Find its value if it exists.

(i) $\int_0^{\pi/2} \frac{\sin^n x}{x^m} dx$ (ii) $\int_{-\infty}^{\infty} \frac{dx}{x^2+2x+2}$

- (b) Discuss the convergence or divergence of the following improper integrals. Find its value if it exists.

(i) $\int_0^{\infty} \frac{dx}{(1+x)^{2/3}}$ (ii) $\int_0^{\infty} e^{-ax} \sin bx dx, a > 0$

- (c) If $f(z)$ is analytic at every point of a ring shaped region bounded by two closed curves C and C_1 , prove that $\oint_C f(z) dz = \oint_{C_1} f(z) dz$. (7, 7)

- (2) (a) If $f(z) = u + iv$ is an analytic function of $z = x + iy$ and $u - v = e^{-x}[(x - y) \sin y - (x + y) \cos y]$, find $f(z)$ in terms of z .

- (b) Evaluate $\int_0^{2+i} (\bar{z})^2 dz$ along (i) the line $y = x/2$, and (ii) the real axis to 2 and then vertically to $2 + i$.

- (c) If $f(\xi) = \oint_C \frac{3z^2+7z+1}{z-\xi} dz$, where C is the circle $x^2+y^2=4$, find $f(3)$, $f'(1-i)$ and $f''(1-i)$. (7, 7)

- (3) (a) Find the residues at all its poles of (i) $f(z) = \frac{1}{z^3+z^5}$ and (ii) $f(z) = \frac{z^2}{(z^2+1)^2}$.
 $-1, \frac{1}{2}, \frac{1}{2}$ $44, -44$

- (b) State Cauchy's residue theorem and hence evaluate $\oint_C \frac{dz}{z^2 \sinh z}$, where $C: |z-1|=2$.

- (c) Evaluate the integral $I = \int_0^{\infty} \frac{\sin ax}{x(x^2+b^2)} dx, a > 0, b > 0$ by using contour integration. (7, 7)

- (4) (a) Discuss the transformation $w = 1/z$ and hence show that under this transformation, the image of the hyperbola $x^2 - y^2 = 1$ is the lemniscate $R^2 = \cos 2\phi$.

- (b) Define bilinear transformation and hence find the transformation which maps the points $1, -i, -1$ of z -plane into $i, 0, -i$ of the w -plane respectively.

2

(c) Solve the difference equation $y_{n+2} - 5y_{n+1} + 6y_n = 4^n(n^2 - n + 5)$. (7, 7)

(5) (a) Find the Z- transforms of the following sequences:

(i) $a^n \sinh n\theta$, (ii) $e^t \sin 2t$.

(b) Find Inverse Z-transform of the following sequences:

(i) $\frac{9z^3}{(3z-1)^2(z-2)}$, (ii) $\frac{1}{(z-\frac{1}{2})(z-\frac{1}{3})}$, when $\frac{1}{2} < |z|$.

(c) Solve the following difference equation using Z-transform.

$y_{n+2} + 5y_{n+1} + 4y_n = 2^n$ with $y_0 = 1$, $y_1 = -4$. (7, 7)

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IIIrd SEMESTER

END SEMESTER EXAMINATION

MC - 202 Differential Equations

Roll No.

MC/059

B. Tech.(MC)

November 2014

Time: 3 Hours

Max. Marks: 70

Note: Attempt all the questions by selecting any two parts from each question.

(1) (a) Prove that

$$\frac{d}{dx}(x^{-n}J_n) = -x^{-n}J_{n+1}.$$

(b) Determine the Jacobi series and show that

$$J_n(x) = \frac{1}{\pi} \int_0^\pi \cos(n\vartheta - x \sin \vartheta) d\vartheta.$$

(c) Find the Generating function for $J_n(x)$. (14)

(2) (a) Solve $(D^2 - DD' - 2D'^2)z = (y-1)e^x$.

(b) Find the complete integral of $(x+y)(p+q)^2 + (x-y)(p-q)^2 = 1$.

(c) Find the complete integral of the partial differential equation

$$(p^2 + q^2)x = pz,$$

by using Charpit's equation. (14)

(3) (a) Define the classification of second-order partial differential equation, Dirichlet and Neumann conditions with examples.

(b) Find all the eigen values and eigen functions of the Sturm-Liouville problem

$$\frac{d^2y}{dx^2} + \lambda y = 0, \quad y(0) = 0, \quad y'\left(\frac{\pi}{2}\right) = 0.$$

(c) Find the Green's function for the BVP

$$y'' + 4y = 3, \quad y'(0) = 0, \quad y(\pi/2) = 0$$

(14).

and solve it.

(4) (a) Solve $(D^2 + DD' + D' - 1)z = \sin(x+2y)$

(b) Solve $(D^2 - DD' + D' - 1)z = x^2y$

2

~~(c)~~ Solve the Lagrange's partial differential equation

$$(z^2 - 2yz - y^2)p + (xy + xz)q = xy - xz.$$

If the solution of the above equation represents a sphere, what will be the co-ordinates of its centre. (14)

(5) (a) Solve the Laplace's equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \quad 0 < x < a, \quad 0 < y < b$$

$$u(0, y) = 0, \quad u(a, y) = 0,$$

$$u(x, 0) = f(x), \quad u(x, b) = 0.$$

(b) Solve the BVP

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}, \quad 0 < x < L, \quad t > 0$$

$$u(0, t) = 0, \quad u(L, t) = 0, \quad \text{for } t \geq 0,$$

$$u(x, 0) = \frac{1}{4}x(L - x), \quad \frac{\partial u}{\partial t}(x, 0) = 0.$$

(c) Find a solution of BVP

$$\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2}, \quad u(x, 0) = 20, \quad u(0, t) = 0, \quad u(L, t) = 0. \quad (14)$$

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**THIRD SEMESTER
END SEMESTER EXAMINATION**

**B.TECH (MC)
NOVEMBER 2014**

MC-203 DISCRETE MATHEMATICS

Time: 3 Hours

Maximum Marks : 70

Note: Answer **ALL** by selecting any **TWO** parts from each question. All questions carry equal marks.

Q1. (a) A relation R is called circular if $(a, b) \in R, (b, c) \in R \Rightarrow (c, a) \in R$. Show that a relation is reflexive and circular iff it is an equivalence relation.

(b) If $f: A \rightarrow B$ and $g: B \rightarrow C$ are bijective mappings then prove that

$$(g \circ f)^{-1} = f^{-1} \circ g^{-1}.$$

(c) Let (A, R) and (B, R') be two posets. Then show that $((A \times B), R'')$ is a poset with partial-order R'' defined by $(a, b) R'' (a', b')$ if $(a, a') \in R$ and $(b, b') \in R'$.

Q2. (a) (I) Give the converse and contrapositive of the following:

- (i) If it is hot, then I take cold drinks.
- (ii) If today is Monday, then tomorrow is Tuesday.
- (iii) If it rains, then they do not go for a walk.

(II) Show that the logical expression

$$\{ [p \rightarrow (q \vee r)] \wedge (\sim q) \} \rightarrow (p \rightarrow r) \text{ is a tautology.}$$

(b) (I) Using the following true statements, locate the treasure hidden in the estate:

- (i) If the house is next to a lake, then the treasure is not in the kitchen.
- (ii) If the tree in front of yard is a mango tree, then the treasure is in the kitchen.
- (iii) The house is next to a lake.
- (iv) The tree in the front yard is a mango tree or the treasure is placed behind the mirror in the wall of the bed room.
- (v) If the tree in the back yard is an oak tree, then the treasure is in the drawing room.

(II) If there is a gas in the car, then John will go to a store. If John goes to the store, then he will get a soda. There is gas in the car. Will John get a soda?

(c) Let a & b be positive numbers with $a > b$. Let $a_1 = \frac{a+b}{2}$ and $b_1 = \sqrt{ab}$. Repeat this process so that, in general $a_{n+1} = \frac{1}{2} [a_n + b_n]$ and $b_{n+1} = \sqrt{a_n b_n}$. Prove by Mathematical Induction that $a_n > a_{n+1} > b_{n+1} > b_n$.

Q3. (a) Solve the recurrence relation $a_n - 6a_{n-1} + 9a_{n-2} = n \cdot 3^n$.

(b) Using generating function, solve the recurrence relation $a_n - 4a_{n-1} = 6 \cdot 4^n$,
 $a_0 = 1$.

(c) Show that a non-empty set H of a group G is a subgroup of G iff
$$a, b \in H \Rightarrow ab^{-1} \in H.$$

Q4. (a) Define Lattice as a poset. Let D_{30} be the set of all positive divisor of 30. Show that it is a lattice under the relation of divisibility. Draw its Hasse diagram.

(b) Let $(L, \cdot, +)$ be a lattice and $a, b \in L$. Then show that $a \cdot b$ is the glb of a and b and $a + b$ is the lub of a and b .

(c) Define Boolean Algebra. Write the following Boolean expression in its Sum of Product form

- (i) $A+B$
- (ii) $AB+C$

Q5 (a) Prove that

(i) A simple graph with at least two vertices has at least two vertices of same degree.

(ii) The vertices of odd degree in a graph is always even.

(b) Define the following with suitable examples:

Connected graph, Complete graph, Tree, Spanning tree, Regular graph, Walk, Path.

(c) Prove that any connected graph with n vertices and $(n-1)$ edges is a tree.

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Roll No. MC-059.....

III RD SEMESTER

B.Tech.[MC]

END SEMESTER EXAMINATION

(Nov-2014)

MC-204

DATA STRUCTURES

Time: 3:00 Hours

Max. Marks : 70

Note: Answer any FIVE questions. All Questions carry Equal Marks.

Assume suitable missing data, if any.

Q.1(a) Explain Asymptotic Notations for Complexity of Algorithms:

Omega(Ω), Theta(Θ) and Big O.

(7)

(b) What is Reverse Polish Notation? Evaluate the following postfix expression using STACK:

12, 2, 5, \uparrow , +, 13, 2, -, /, 1, +

(7)

Q.2(a) Explain Heap-sort Algorithm with the help of suitable example.

(7)

(b) Sort the following array using Merge Sort:

66, 33, 40, 22, 55, 88, 60, 11, 80, 20, 50, 44, 77, 3

Mentions the required steps to solve the question.

(7)

Q.3(a) Compare Binary Tree, Complete Binary Tree, Extended Binary Tree, AVL trees and B-trees with the help of an example.

(7)

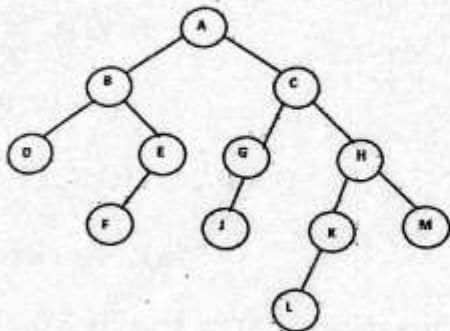
(b) Write an Algorithm/Pseudocode to insert a node at Binary Search Tree with the help of suitable example.

(7)

Q.4(a) Briefly explain the Linked and Sequential Representation of trees in detail followed by suitable examples and figures if needed.

(7)

(b) Write a stepwise procedure to find Postorder traversals of the following tree using STACK:



(7)

Q.5(a) Write a difference between Multi-graph and Directed Acyclic Graph with the help of suitable example.

(7)

(b) Explain Prim's Algorithm with the help of suitable example.

(7)

Q.6 Explain Warshall Algorithm to find a shortest path in detail with the help of a suitable example.

(14)

Q.7 Write a short note on any TWO:

(14)

- (i) Memory Allocation and Garbage Collection
- (ii) File Organization and Indexing
- (iii) Directed Acyclic Graphs(DAG) and Topological Sort
- (iv) DFS and BFS traversals of Graph

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THIRD SEMESTER
END SEMESTER EXAMINATION

Roll No: MC 059
B.Tech. [MC]
NOV. 2014

MC-205, Probability and Statistics

Time: 3.0 Hours

Max. Marks: 70

Note: Attempt **ANY** two parts from each questions. All questions carry equal marks.
 Assume suitable missing data, if any. Distribution table will be provided.

1. (a) A and B are two weak students of statistics and their chances of solving a problem in statistics correctly are $\frac{1}{6}$ and $\frac{1}{8}$ respectively. If the probability of their making a common error is $\frac{1}{525}$ and they obtain the same answer, find the probability that their answer is correct.

- (b) The joint density of X and Y is given by

$$f(x, y) = \begin{cases} e^{-(x+y)} & \text{when } x > 0, y > 0, \\ 0 & \text{otherwise.} \end{cases}$$

- (a) Are X and Y independent ?

- (b) Calculate $P(X < Y/X < 2Y)$.

- (c) Calculate $P(1 < X + Y < 2)$. [1+2+4]

- (c) A two dimensional random variable (X,Y) have a joint distribution given as

$$P(x, y) = \frac{x^2 + y}{32}, \text{ for } x = 0, 1, 2, 3 \text{ and } y = 0, 1.$$

- (a) Find the marginal distributions of X and Y.

- (b) Find the conditional distribution of Y for $X = x$. [3+4]

2. (a) (I) A coin is tossed until a head appears. What is the expectation of the number of tosses required?

- (II) A box contains 'a' number of white balls and 'b' number of black balls. Let 'c' balls are drawn at random. Find the expected value of the number of white balls drawn. [3+4]

(b) Discuss Poisson distribution. Find its first three central moments.

(c) In an examination it is laid down that a student passes if he/she secures 30% or more marks. He/She is placed in the first second or third division according as he /she secures 60% or more marks, between 45% and 60% marks and marks between 30% and 45% respectively. He/She gets distinction in case he/she secures 80% or marks. It is noticed from the result that 10% of the students failed in the examination, whereas 5% of them obtained distinction. Calculate the percentage of students placed in the second division.

3. (a) Calculate the coefficient of correlation for the following heights of fathers (X) and their sons (Y)

X(inches)	65	66	67	67	68	69	70	72
Y(inches)	67	68	65	68	72	72	69	71

(b) In a sample of 1000 people in Delhi, 540 are rice eaters and the rest are wheat eaters. Can we assume that both rice and wheat are equally popular in this state at 1% level of significance.

(c) Define an unbiased estimator of a population parameter β . Show that sample mean \bar{x} is an unbiased estimator of the population mean μ but sample variance is not an unbiased estimator of the population variance. [1+3+3]

4. (a) A sample of 100 students of DTU has a mean of their heights 166 cms and standard deviation (s.d.) 25 cms. Is the sample from a large population of mean 162 and s.d. 25 cms? If the population is normal and its mean is unknown, find the 95% fiducial limit of true mean.

(b) The height of 10 males of a given locality are found to be 70, 67, 62, 68, 61, 68, 70, 64, 64, 66 inches. Is it reasonable to believe that the average height is greater than 64 inches? Test at 5% significance level assuming that for 9 degrees of freedom $P(t > 1.83) = 0.05$.

- (c) Calculate the first four moments of the following distribution about the mean and hence find β_1 and β_2

x	0	1	2	3	4	5	6	7	8
y	1	8	28	56	70	56	28	8	1

5. (a) Find the line of best fit for the following data, plot the line and the data points.

x:-	2	3	4	5	6	7
y:-	3.0	5.0	5.5	6.0	8.0	9.5

- (b) Suppose that the lifetime (in hours) of an electronics tube is a random variable X having Weibull distribution with $\alpha = 0.05$ and $\beta = 0.5$. Find

(a) mean lifetime of these tubes

(b) standard deviation of the lifetime

(c) the probability that such a tube will last more than 1000 hrs.

- (c) Discuss negative Binomial distribution. Find its moments generating function, mean and the variance.

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Roll No. MC059

THIRD SEMESTER

B.Tech. (CE/EL/EE/EP/ME/MC/SE)**END SEMESTER EXAMINATION****November-2014****CE/EL/EE/EP/ME/MC/SE -206 ENGINEERING ECONOMICS****Time: 3:00 Hours****Max. Marks : 70**

Note : Answer any **FIVE** questions.
Assume suitable missing data, if any.

1.a	Discuss production processes in the age of information and Communication Technology (ICT) and enhanced role in such production process for engineers in developing economies like India.	7
1.b	In the design of a jet engine part, the designer has a choice of specifying either an aluminum alloy casting or a steel casting. Either material will provide equal service, but the aluminum casting will weigh 1.2 Kg. as compared with 1.35 Kg. for steel casting. The aluminum can be cost Rs. 80.00 per Kg. and the steel one for Rs. 35.00 per Kg. The cost of machining per unit is Rs. 150.00 for aluminum and Rs. 170.00 for steel. Every Kilogram of excess weight is associated with a penalty of Rs. 1300 due to increased fuel consumption. Which material should be specified and what is the economic advantage of the selection per unit?	3
1.c	<p>The chief engineer of refinery operation is not satisfied with the preliminary design for storage tanks to be used as part of a plant expansion programme. The engineer who submitted the design was called in and asked to reconsider the overall dimensions in the light of an article in the "Chemical Engineer", entitled "How to size future process vessels"?</p> <p>The original design submitted called for 4 tanks 5.2 m in diameter and 7 m in height. From a graph of the article the engineer found that the present ratio of height to diameter of 1.35 is 111% of the minimum cost and that the minimum cost for a tank was when the ratio of height to diameter was 4:1. The cost for the tank design as originally submitted was estimated to be Rs. 9,00,000. What are the optimum tank dimensions if the volume remains the same as the original design? What total savings may be expected through the redesign?</p>	4

2.a	Discuss the nature of software industry in India. What incentive may be given by the Government to encourage the Indian producers?	7															
2.b	Three engineers start their own business after graduation. What is their opportunity cost?	3															
2.c	The market supply and demand functions for videotapes are given by $Q_s = 100 + 20p$ $Q_D = 300 - 5p$ Where Q is the quantity and p is the price of tapes. What are the equilibrium price and the rate of output?	4															
3.a	Explain the difference between labour intensive and capital intensive technology. What kind of technology would be more suitable the Indian economy?	7															
3.b	A firm Excel, has two options for equipment. Cost of both the options is Rs. 23500/- and useful lives for five years and no salvage value. Equipment M will fetch benefit of Rs. 6000 annually. Equipment N will fetch benefit of Rs. 8500 in the first year but will decline Rs. 1000 annually as Rs. 7500 in the second year, Rs. 6500 in the third year, Rs. 5500 in the fourth year and Rs. 4500 in the last year. With interest at 10% which device the firm should purchase.	7															
4.a	Discuss Business Risk in your own words.	7															
4.b	A person plans to save Rs. 15,000 from the next year for next 15 years for his son education. The bank gives 12 % interest rate, compounded annually. What sum of money he will receive after 15 years.	7															
5.a	What do you mean by Inflation? How it can be controlled by monetary and fiscal policy?	7															
5.b	A company invests in one of the two mutually exclusive alternatives. The life of both alternatives is 12 years with following investment annual returns and salvage value. <table border="1"> <thead> <tr> <th></th><th colspan="2">Alternatives</th></tr> <tr> <th></th><th>A</th><th>B</th></tr> </thead> <tbody> <tr> <td>In vestment</td><td>830000</td><td>745000</td></tr> <tr> <td>Annual Equal return</td><td>310000</td><td>270000</td></tr> <tr> <td>Salvage value</td><td>150000</td><td>210000</td></tr> </tbody> </table> Determine the best alternative based on the annual equivalent method by assuming = 10%.		Alternatives			A	B	In vestment	830000	745000	Annual Equal return	310000	270000	Salvage value	150000	210000	7
	Alternatives																
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Salvage value	150000	210000															
6.a	Discuss role of Science and Technology in improving life of poor?	10															
6.b	Following are the demand functions for two commodities x_1 and x_2 : $x_1 = p_1^{-1.7} p_2^{0.8}$ $x_2 = p_1^{0.5} p_2^{-0.2}$ Determine whether the commodities are complementary or competitive.	4															
7.a	Discuss factors which should be considered while launching a new commodity in the market.	7															
7.b	What amount of money saved today will yield Rs. 40,000/- in third year and Rs. 55,000/- after five year at the 12% rate of interest compounded annually?	7															