8.2 **Robot in a Grid:** Imagine a robot sitting on the upper left corner of grid with r rows and c columns. The robot can only move in two directions, right and down, but certain cells are "off limits" such that the robot cannot step on them. Design an algorithm to find a path for the robot from the top left to the bottom right.

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SOLUTION

If we picture this grid, the only way to move to spot `(r,c)` is by moving to one of the adjacent spots: `(r-1,c)` or `(r,c-1)`. So, we need to find a path to either `(r-1,c)` or `(r,c-1)`.

How do we find a path to those spots? To find a path to `(r-1,c)` or `(r,c-1)`, we need to move to one of its adjacent cells. So, we need to find a path to a spot adjacent to `(r-1,c)`, which are coordinates `(r-2,c)` and `(r-1,c-1)`, or a spot adjacent to `(r,c-1)`, which are spots `(r-1,c-1)` and `(r,c-2)`. Observe that we list the point `(r-1,c-1)` twice; we'll discuss that issue later.

> Tip: A lot of people use the variable names x and y when dealing with two-dimensional arrays. This can actually cause some bugs. People tend to think about x as the first coordinate in the matrix and y as the second coordinate (e.g., `matrix[x][y]`). But, this isn't really correct. The first coordinate is usually thought of as the row number, which is in fact the y value (it goes vertically!). You should write `matrix[y][x]`. Or, just make your life easier by using r (row) and c (column) instead.

So then, to find a path from the origin, we just work backwards like this. Starting from the last cell, we try to find a path to each of its adjacent cells. The recursive code below implements this algorithm.

```
ArrayList<Point> getPath(boolean[][] maze) {
  if (maze == null || maze.length == 0) return null;
  ArrayList<Point> path = new ArrayList<Point>();
  if (getPath(maze, maze.length - 1, maze[0].length - 1, path)) {
    return path;
  }
  return null;
}

boolean getPath(boolean[][] maze, int row, int col, ArrayList<Point> path) {
  /* If out of bounds or not available, return.*/
  if (col < 0 || row < 0 || !maze[row][col]) {
    return false;
  }

  boolean isAtOrigin = (row == 0) && (col == 0);

  /* If there's a path from the start to here, add my location. */
  if (isAtOrigin || getPath(maze, row, col - 1, path) ||
      getPath(maze, row - 1, col, path)) {
    Point p = new Point(row, col);
    path.add(p);
    return true;
  }

  return false;
}
```

This solution is $O(2^{r+c})$, since each path has r+c steps and there are two choices we can make at each step.

We should look for a faster way.

Often, we can optimize exponential algorithms by finding duplicate work. What work are we repeating?

If we walk through the algorithm, we'll see that we are visiting squares multiple times. In fact, we visit each square many, many times. After all, we have `rc` squares but we're doing $O(2^{r+c})$ work. If we were only visiting each square once, we would probably have an algorithm that was `O(rc)` (unless we were somehow doing a lot of work during each visit).

How does our current algorithm work? To find a path to `(r,c)`, we look for a path to an adjacent coordinate: `(r-1,c)` or `(r,c-1)`. Of course, if one of those squares is off limits, we ignore it. Then, we look at their adjacent coordinates: `(r-2,c)`, `(r-1,c-1)`, `(r-1,c-1)`, and `(r,c-2)`. The spot `(r-1,c-1)` appears twice, which means that we're duplicating effort. Ideally, we should remember that we already visited `(r-1,c-1)` so that we don't waste our time.

This is what the dynamic programming algorithm below does.

```
ArrayList<Point> getPath(boolean[][] maze) {
  if (maze = =null || maze.length == 0) return null;
  ArrayList<Point> path = new ArrayList<Point>();
  HashSet<Point> failedPoints = new HashSet<Point>();
  if (getPath(maze, maze.length - 1, maze[0].length - 1, path, failedPoints)) {
    return path;
  }
  return null;
}

boolean getPath(boolean[][] maze, int row, int col, ArrayList<Point> path,
                HashSet<Point> failedPoints) {
  /* If out of bounds or not available, return.*/
  if (col < 0 || row < 0 || !maze[row][col]) {
    return false;
  }

  Point p = new Point(row, col);

  /* If we've already visited this cell, return. */
  if (failedPoints.contains(p)) {
    return false;
  }

  boolean isAtOrigin = (row == 0) && (col == 0);

  /* If there's a path from start to my current location, add my location.*/
  if (isAtOrigin || getPath(maze, row, col - 1, path, failedPoints) ||
      getPath(maze, row - 1, col, path, failedPoints)) {
    path.add(p);
    return true;
  }

  failedPoints.add(p); // Cache result
  return false;
}
```

This simple change will make our code run substantially faster. The algorithm will now take `O(XY)` time because we hit each cell just once.

8.3 Magic Index: A magic index in an array A[1...n-1] is defined to be an index such that A[i] = i. Given a sorted array of distinct integers, write a method to find a magic index, if one exists, in array A.

FOLLOW UP

What if the values are not distinct?

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SOLUTION

Immediately, the brute force solution should jump to mind—and there's no shame in mentioning it. We simply iterate through the array, looking for an element which matches this condition.

```
int magicSlow(int[] array) {
  for (int i = 0; i < array.length; i++) {
    if (array[i] == i) {
      return i;
    }
  }
  return -1;
}
```

Given that the array is sorted, though, it's very likely that we're supposed to use this condition.

We may recognize that this problem sounds a lot like the classic binary search problem. Leveraging the Pattern Matching approach for generating algorithms, how might we apply binary search here?

In binary search, we find an element k by comparing it to the middle element, x, and determining if k would land on the left or the right side of x.

Building off this approach, is there a way that we can look at the middle element to determine where a magic index might be? Let's look at a sample array:

-40	-20	-1	1	2	3	5	7	9	12	13
0	1	2	3	4	5	6	7	8	9	10

When we look at the middle element A[5] = 3, we know that the magic index must be on the right side, since A[mid] < mid.

Why couldn't the magic index be on the left side? Observe that when we move from i to i-1, the value at this index must decrease by at least 1, if not more (since the array is sorted and all the elements are distinct). So, if the middle element is already too small to be a magic index, then when we move to the left, subtracting k indexes and (at least) k values, all subsequent elements will also be too small.

We continue to apply this recursive algorithm, developing code that looks very much like binary search.

```
int magicFast(int[] array) {
  return magicFast(array, 0, array.length - 1);
}

int magicFast(int[] array, int start, int end) {
  if (end < start) {
    return -1;
  }
  int mid = (start + end) / 2;
  if (array[mid] == mid) {
    return mid;
  } else if (array[mid] > mid){
```

```
      return magicFast(array, start, mid - 1);
    } else {
      return magicFast(array, mid + 1, end);
    }
  }
```

Follow Up: What if the elements are not distinct?

If the elements are not distinct, then this algorithm fails. Consider the following array:

-10	-5	2	2	2	3	4	7	9	12	13
0	1	2	3	4	5	6	7	8	9	10

When we see that `A[mid] < mid`, we cannot conclude which side the magic index is on. It could be on the right side, as before. Or, it could be on the left side (as it, in fact, is).

Could it be *anywhere* on the left side? Not exactly. Since `A[5] = 3`, we know that `A[4]` couldn't be a magic index. `A[4]` would need to be 4 to be the magic index, but `A[4]` must be less than or equal to `A[5]`.

In fact, when we see that `A[5] = 3`, we'll need to recursively search the right side as before. But, to search the left side, we can skip a bunch of elements and only recursively search elements `A[0]` through `A[3]`. `A[3]` is the first element that could be a magic index.

The general pattern is that we compare `midIndex` and `midValue` for equality first. Then, if they are not equal, we recursively search the left and right sides as follows:

- Left side: search indices `start` through `Math.min(midIndex - 1, midValue)`.
- Right side: search indices `Math.max(midIndex + 1, midValue)` through `end`.

The code below implements this algorithm.

```
int magicFast(int[] array) {
  return magicFast(array, 0, array.length - 1);
}

int magicFast(int[] array, int start, int end) {
  if (end < start) return -1;

  int midIndex = (start + end) / 2;
  int midValue = array[midIndex];
  if (midValue == midIndex) {
    return midIndex;
  }

  /* Search left * /
  int leftIndex = Math.min(midIndex - 1, midValue);
  int left = magicFast(array, start, leftIndex);
  if (left >= 0) {
    return left;
  }

  /* Search right * /
  int rightIndex = Math.max(midIndex + 1, midValue);
  int right = magicFast(array, rightIndex, end);

  return right;
}
```

Note that in the above code, if the elements are all distinct, the method operates almost identically to the first solution.

8.4 Power Set: Write a method to return all subsets of a set.

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SOLUTION

We should first have some reasonable expectations of our time and space complexity.

How many subsets of a set are there? When we generate a subset, each element has the "choice" of either being in there or not. That is, for the first element, there are two choices: it is either in the set, or it is not. For the second, there are two, etc. So, doing `{2 * 2 * ... }` n times gives us 2^n subsets.

Assuming that we're going to be returning a list of subsets, then our best case time is actually the total number of elements across all of those subsets. There are 2^n subsets and each of the n elements will be contained in half of the subsets (which 2^{n-1} subsets). Therefore, the total number of elements across all of those subsets is $n * 2^{n-1}$.

We will not be able to beat $O(n2^n)$ in space or time complexity.

The subsets of $\{a_1, a_2, \ldots, a_n\}$ are also called the powerset, $P(\{a_1, a_2, \ldots, a_n\})$, or just $P(n)$.

Solution #1: Recursion

This problem is a good candidate for the Base Case and Build approach. Imagine that we are trying to find all subsets of a set like $S = \{a_1, a_2, \ldots, a_n\}$. We can start with the Base Case.

Base Case: n = 0.

There is just one subset of the empty set: {}.

Case: n = 1.

There are two subsets of the set $\{a_1\}$: $\{\}$, $\{a_1\}$.

Case: n = 2.

There are four subsets of the set $\{a_1, a_2\}$: $\{\}$, $\{a_1\}$, $\{a_2\}$, $\{a_1, a_2\}$.

Case: n = 3.

Now here's where things get interesting. We want to find a way of generating the solution for n = 3 based on the prior solutions.

What is the difference between the solution for n = 3 and the solution for n = 2? Let's look at this more deeply:

$$P(2) = \{\}, \{a_1\}, \{a_2\}, \{a_1, a_2\}$$
$$P(3) = \{\}, \{a_1\}, \{a_2\}, \{a_3\}, \{a_1, a_2\}, \{a_1, a_3\}, \{a_2, a_3\}, \{a_1, a_2, a_3\}$$

The difference between these solutions is that P(2) is missing all the subsets containing a_3.

$$P(3) - P(2) = \{a_3\}, \{a_1, a_3\}, \{a_2, a_3\}, \{a_1, a_2, a_3\}$$

How can we use P(2) to create P(3)? We can simply clone the subsets in P(2) and add a_3 to them:

$$P(2) = \{\}, \{a_1\}, \{a_2\}, \{a_1, a_2\}$$
$$P(2) + a_3 = \{a_3\}, \{a_1, a_3\}, \{a_2, a_3\}, \{a_1, a_2, a_3\}$$

When merged together, the lines above make P(3).

Case: n > 0

Generating P(n) for the general case is just a simple generalization of the above steps. We compute P(n-1), clone the results, and then add a_n to each of these cloned sets.

The following code implements this algorithm:

```
ArrayList<ArrayList<Integer>> getSubsets(ArrayList<Integer> set, int index) {
  ArrayList<ArrayList<Integer>> allsubsets;
  if (set.size() == index) { // Base case - add empty set
    allsubsets = new ArrayList<ArrayList<Integer>>();
    allsubsets.add(new ArrayList<Integer>()); // Empty set
  } else {
    allsubsets = getSubsets(set, index + 1);
    int item = set.get(index);
    ArrayList<ArrayList<Integer>> moresubsets =
      new ArrayList<ArrayList<Integer>>();
    for (ArrayList<Integer> subset : allsubsets) {
      ArrayList<Integer> newsubset = new ArrayList<Integer>();
      newsubset.addAll(subset); //
      newsubset.add(item);
      moresubsets.add(newsubset);
    }
    allsubsets.addAll(moresubsets);
  }
  return allsubsets;
}
```

This solution will be $O(n2^n)$ in time and space, which is the best we can do. For a slight optimization, we could also implement this algorithm iteratively.

Solution #2: Combinatorics

While there's nothing wrong with the above solution, there's another way to approach it.

Recall that when we're generating a set, we have two choices for each element: (1) the element is in the set (the "yes" state) or (2) the element is not in the set (the "no" state). This means that each subset is a sequence of yeses / nos—e.g., "yes, yes, no, no, yes, no"

This gives us 2^n possible subsets. How can we iterate through all possible sequences of "yes" / "no" states for all elements? If each "yes" can be treated as a 1 and each "no" can be treated as a 0, then each subset can be represented as a binary string.

Generating all subsets, then, really just comes down to generating all binary numbers (that is, all integers). We iterate through all numbers from 0 to 2^n (exclusive) and translate the binary representation of the numbers into a set. Easy!

```
ArrayList<ArrayList<Integer>> getSubsets2(ArrayList<Integer> set) {
  ArrayList<ArrayList<Integer>> allsubsets = new ArrayList<ArrayList<Integer>>();
  int max = 1 << set.size(); /* Compute 2^n */
  for (int k = 0; k < max; k++) {
    ArrayList<Integer> subset = convertIntToSet(k, set);
    allsubsets.add(subset);
  }
  return allsubsets;
}
```

```

ArrayList<Integer> convertIntToSet(int x, ArrayList<Integer> set) {
   ArrayList<Integer> subset = new ArrayList<Integer>();
   int index = 0;
   for (int k = x; k > 0; k >>= 1) {
      if ((k & 1) == 1) {
         subset.add(set.get(index));
      }
      index++;
   }
   return subset;
}
```

There's nothing substantially better or worse about this solution compared to the first one.

8.5 Recursive Multiply: Write a recursive function to multiply two positive integers without using the * operator (or / operator). You can use addition, subtraction, and bit shifting, but you should minimize the number of those operations.

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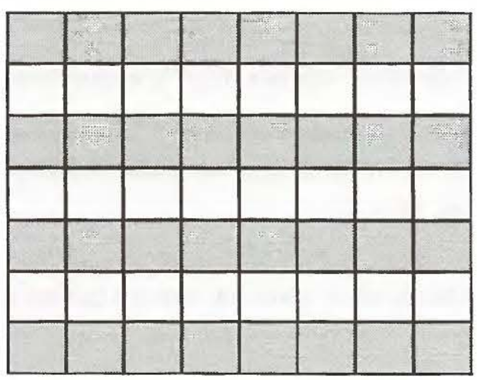
SOLUTION

Let's pause for a moment and think about what it means to do multiplication.

> This is a good approach for a lot of interview questions. It's often useful to think about what it really means to do something, even when it's pretty obvious.

We can think about multiplying 8x7 as doing 8+8+8+8+8+8+8 (or adding 7 eight times). We can also think about it as the number of squares in an 8x7 grid.



Solution #1

How would we count the number of squares in this grid? We could just count each cell. That's pretty slow, though.

Alternatively, we could count half the squares and then double it (by adding this count to itself). To count half the squares, we repeat the same process.

Of course, this "doubling" only works if the number is in fact even. When it's not even, we need to do the counting/summing from scratch.

```
int minProduct(int a, int b) {
   int bigger = a < b ? b : a;
```

```
  int smaller = a < b ? a : b;
  return minProductHelper(smaller, bigger);
}

int minProductHelper(int smaller, int bigger) {
  if (smaller == 0) { // 0 x bigger = 0
    return 0;
  } else if (smaller == 1) { // 1 x bigger = bigger
    return bigger;
  }

  /* Compute half. If uneven, compute other half. If even, double it. */
  int s = smaller >> 1; // Divide by 2
  int side1 = minProduct(s, bigger);
  int side2 = side1;
  if (smaller % 2 == 1) {
    side2 = minProductHelper(smaller - s, bigger);
  }

  return side1 + side2;
}
```

Can we do better? Yes.

Solution #2

If we observe how the recursion operates, we'll notice that we have duplicated work. Consider this example:

```
minProduct(17, 23)
      minProduct(8, 23)
          minProduct(4, 23) * 2
                ...
    + minProduct(9, 23)
          minProduct(4, 23)
                ...
        + minProduct(5, 23)
                ...
```

The second call to `minProduct(4, 23)` is unaware of the prior call, and so it repeats the same work. We should cache these results.

```
int minProduct(int a, int b) {
  int bigger = a < b ? b : a;
  int smaller = a < b ? a : b;

  int memo[] = new int[smaller + 1];
  return minProduct(smaller, bigger, memo);
}

int minProduct(int smaller, int bigger, int[] memo) {
  if (smaller == 0) {
    return 0;
  } else if (smaller == 1) {
    return bigger;
  } else if (memo[smaller] > 0) {
    return memo[smaller];
  }

  /* Compute half. If uneven, compute other half. If even, double it. */
```

```
    int s = smaller >> 1; // Divide by 2
    int side1 = minProduct(s, bigger, memo); // Compute half
    int side2 = side1;
    if (smaller % 2 == 1) {
      side2 = minProduct(smaller - s, bigger, memo);
    }

    /* Sum and cache.*/
    memo[smaller] = side1 + side2;
    return memo[smaller];
}
```

We can still make this a bit faster.

Solution #3

One thing we might notice when we look at this code is that a call to `minProduct` on an even number is much faster than one on an odd number. For example, if we call `minProduct(30, 35)`, then we'll just do `minProduct(15, 35)` and double the result. However, if we do `minProduct(31, 35)`, then we'll need to call `minProduct(15, 35)` and `minProduct(16, 35)`.

This is unnecessary. Instead, we can do:

```
minProduct(31, 35) = 2 * minProduct(15, 35) + 35
```

After all, since `31 = 2*15+1`, then `31x35 = 2*15*35+35`.

The logic in this final solution is that, on even numbers, we just divide `smaller` by 2 and double the result of the recursive call. On odd numbers, we do the same, but then we also add `bigger` to this result.

In doing so, we have an unexpected "win." Our `minProduct` function just recurses straight downwards, with increasingly small numbers each time. It will never repeat the same call, so there's no need to cache any information.

```
int minProduct(int a, int b) {
  int bigger = a < b ? b : a;
  int smaller = a < b ? a : b;
  return minProductHelper(smaller, bigger);
}

int minProductHelper(int smaller, int bigger) {
  if (smaller == 0) return 0;
  else if (smaller == 1) return bigger;

  int s = smaller >> 1; // Divide by 2
  int halfProd = minProductHelper(s, bigger);

  if (smaller % 2 == 0) {
    return halfProd + halfProd;
  } else {
    return halfProd + halfProd + bigger;
  }
}
```

This algorithm will run in `O(log s)` time, where `s` is the smaller of the two numbers.

8.6 **Towers of Hanoi:** In the classic problem of the Towers of Hanoi, you have 3 towers and N disks of different sizes which can slide onto any tower. The puzzle starts with disks sorted in ascending order of size from top to bottom (i.e., each disk sits on top of an even larger one). You have the following constraints:

(1) Only one disk can be moved at a time.

(2) A disk is slid off the top of one tower onto another tower.

(3) A disk cannot be placed on top of a smaller disk.

Write a program to move the disks from the first tower to the last using Stacks.

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SOLUTION

This problem sounds like a good candidate for the Base Case and Build approach.

Let's start with the smallest possible example: `n = 1`.

Case `n = 1`. Can we move Disk 1 from Tower 1 to Tower 3? Yes.

1. We simply move Disk 1 from Tower 1 to Tower 3.

Case `n = 2`. Can we move Disk 1 and Disk 2 from Tower 1 to Tower 3? Yes.

1. Move Disk 1 from Tower 1 to Tower 2
2. Move Disk 2 from Tower 1 to Tower 3
3. Move Disk 1 from Tower 2 to Tower 3

Note how in the above steps, Tower 2 acts as a buffer, holding a disk while we move other disks to Tower 3.

Case `n = 3`. Can we move Disk 1, 2, and 3 from Tower 1 to Tower 3? Yes.

1. We know we can move the top two disks from one tower to another (as shown earlier), so let's assume we've already done that. But instead, let's move them to Tower 2.
2. Move Disk 3 to Tower 3.
3. Move Disk 1 and Disk 2 to Tower 3. We already know how to do this—just repeat what we did in Step 1.

Case `n = 4`. Can we move Disk 1, 2, 3 and 4 from Tower 1 to Tower 3? Yes.

1. Move Disks 1, 2, and 3 to Tower 2. We know how to do that from the earlier examples.
2. Move Disk 4 to Tower 3.
3. Move Disks 1, 2 and 3 back to Tower 3.

Remember that the labels of Tower 2 and Tower 3 aren't important. They're equivalent towers. So, moving disks to Tower 3 with Tower 2 serving as a buffer is equivalent to moving disks to Tower 2 with Tower 3 serving as a buffer.

This approach leads to a natural recursive algorithm. In each part, we are doing the following steps, outlined below with pseudocode:

```
moveDisks(int n, Tower origin, Tower destination, Tower buffer) {
  /* Base case */
  if (n <= 0) return;

  /* move top n - 1 disks from origin to buffer, using destination as a buffer. */
  moveDisks(n - 1, origin, buffer, destination);

  /* move top from origin to destination
  moveTop(origin, destination);

  /* move top n - 1 disks from buffer to destination, using origin as a buffer. */
  moveDisks(n - 1, buffer, destination, origin);
}
```

The following code provides a more detailed implementation of this algorithm, using concepts of object-oriented design.

```
void main(String[] args) {
  int n = 3;
  Tower[] towers = new Tower[n];
  for (int i = 0; i < 3; i++) {
    towers[i] = new Tower(i);
  }

  for (int i = n - 1; i >= 0; i--) {
    towers[0].add(i);
  }
  towers[0].moveDisks(n, towers[2], towers[1]);
}

class Tower {
  private Stack<Integer> disks;
  private int index;
  public Tower(int i) {
    disks = new Stack<Integer>();
    index = i;
  }

  public int index() {
    return index;
  }

  public void add(int d) {
    if (!disks.isEmpty() && disks.peek() <= d) {
      System.out.println("Error placing disk " + d);
    } else {
      disks.push(d);
    }
  }

  public void moveTopTo(Tower t) {
    int top = disks.pop();
    t.add(top);
  }

```

```
  public void moveDisks(int n, Tower destination, Tower buffer) {
    if (n > 0) {
      moveDisks(n - 1, buffer, destination);
      moveTopTo(destination);
      buffer.moveDisks(n - 1, destination, this);
    }
  }
}
```

Implementing the towers as their own objects is not strictly necessary, but it does help to make the code cleaner in some respects.

8.7 Permutations without Dups: Write a method to compute all permutations of a string of unique characters.

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SOLUTION

Like in many recursive problems, the Base Case and Build approach will be useful. Assume we have a string S represented by the characters $a_1a_2 \ldots a_n$.

Approach 1: Building from permutations of first n-1 characters.

Base Case: permutations of first character substring

The only permutation of a_1 is the string a_1. So:

$P(a_1) = a_1$

Case: permutations of a_1a_2

$P(a_1a_2) = a_1a_2$ and a_2a_1

Case: permutations of $a_1a_2a_3$

$P(a_1a_2a_3) = a_1a_2a_3,\ a_1a_3a_2,\ a_2a_1a_3,\ a_2a_3a_1,\ a_3a_1a_2,\ a_3a_2a_1,$

Case: permutations of $a_1a_2a_3a_4$

This is the first interesting case. How can we generate permutations of $a_1a_2a_3a_4$ from $a_1a_2a_3$?

Each permutation of $a_1a_2a_3a_4$ represents an ordering of $a_1a_2a_3$. For example, $a_2a_4a_1a_3$ represents the order $a_2a_1a_3$.

Therefore, if we took all the permutations of $a_1a_2a_3$ and added a_4 into all possible locations, we would get all permutations of $a_1a_2a_3a_4$.

$a_1a_2a_3 \rightarrow a_4a_1a_2a_3,\ a_1a_4a_2a_3,\ a_1a_2a_4a_3,\ a_1a_2a_3a_4$
$a_1a_3a_2 \rightarrow a_4a_1a_3a_2,\ a_1a_4a_3a_2,\ a_1a_3a_4a_2,\ a_1a_3a_2a_4$
$a_3a_1a_2 \rightarrow a_4a_3a_1a_2,\ a_3a_4a_1a_2,\ a_3a_1a_4a_2,\ a_3a_1a_2a_4$
$a_2a_1a_3 \rightarrow a_4a_2a_1a_3,\ a_2a_4a_1a_3,\ a_2a_1a_4a_3,\ a_2a_1a_3a_4$
$a_2a_3a_1 \rightarrow a_4a_2a_3a_1,\ a_2a_4a_3a_1,\ a_2a_3a_4a_1,\ a_2a_3a_1a_4$
$a_3a_2a_1 \rightarrow a_4a_3a_2a_1,\ a_3a_4a_2a_1,\ a_3a_2a_4a_1,\ a_3a_2a_1a_4$

We can now implement this algorithm recursively.

```
ArrayList<String> getPerms(String str) {
  if (str == null) return null;

  ArrayList<String> permutations = new ArrayList<String>();
  if (str.length() == 0) { // base case
    permutations.add("");
```

```
      return permutations;
    }

    char first = str.charAt(0); // get the first char
    String remainder = str.substring(1); // remove the first char
    ArrayList<String> words = getPerms(remainder);
    for (String word : words) {
      for (int j = 0; j <= word.length(); j++) {
        String s = insertCharAt(word, first, j);
        permutations.add(s);
      }
    }
    return permutations;
}

/* Insert char c at index i in word. */
String insertCharAt(String word, char c, int i) {
    String start = word.substring(0, i);
    String end = word.substring(i);
    return start + c + end;
}
```

Approach 2: Building from permutations of all n-1 character substrings.

Base Case: single-character strings

The only permutation of a_1 is the string a_1. So:

$$P(a_1) = a_1$$

Case: two-character strings

$$P(a_1a_2) = a_1a_2 \text{ and } a_2a_1.$$
$$P(a_2a_3) = a_2a_3 \text{ and } a_3a_2.$$
$$P(a_1a_3) = a_1a_3 \text{ and } a_3a_1.$$

Case: three-character strings

Here is where the cases get more interesting. How can we generate all permutations of three-character strings, such as $a_1a_2a_3$, given the permutations of two-character strings?

Well, in essence, we just need to "try" each character as the first character and then append the permutations.

$$P(a_1a_2a_3) = \{a_1 + P(a_2a_3)\} + a_2 + P(a_1a_3)\} + \{a_3 + P(a_1a_2)\}$$
$$\{a_1 + P(a_2a_3)\} \text{ -> } a_1a_2a_3,\ a_1a_3a_2$$
$$\{a_2 + P(a_1a_3)\} \text{ -> } a_2a_1a_3,\ a_2a_3a_1$$
$$\{a_3 + P(a_1a_2)\} \text{ -> } a_3a_1a_2,\ a_3a_2a_1$$

Now that we can generate all permutations of three-character strings, we can use this to generate permutations of four-character strings.

$$P(a_1a_2a_3a_4) = \{a_1 + P(a_2a_3a_4)\} + \{a_2 + P(a_1a_3a_4)\} + \{a_3 + P(a_1a_2a_4)\} + \{a_4 + P(a_1a_2a_3)\}$$

This is now a fairly straightforward algorithm to implement.

```
ArrayList<String> getPerms(String remainder) {
  int len = remainder.length();
  ArrayList<String> result = new ArrayList<String>();

  /* Base case. */
```

```
    if (len == 0) {
      result.add(“”); // Be sure to return empty string!
      return result;
    }


    for (int i = 0; i < len; i++) {
      /* Remove char i and find permutations of remaining chars.*/
      String before = remainder.substring(0, i);
      String after = remainder.substring(i + 1, len);
      ArrayList<String> partials = getPerms(before + after);

      /* Prepend char i to each permutation.*/
      for (String s : partials) {
        result.add(remainder.charAt(i) + s);
      }
    }

    return result;
  }
```

Alternatively, instead of passing the permutations back up the stack, we can push the prefix down the stack. When we get to the bottom (base case), `prefix` holds a full permutation.

```
ArrayList<String> getPerms(String str) {
  ArrayList<String> result = new ArrayList<String>();
  getPerms(“”, str, result);
  return result;
}

void getPerms(String prefix, String remainder, ArrayList<String> result) {
  if (remainder.length() == 0) result.add(prefix);

  int len = remainder.length();
  for (int i = 0; i < len; i++) {
    String before = remainder.substring(0, i);
    String after = remainder.substring(i + 1, len);
    char c = remainder.charAt(i);
    getPerms(prefix + c, before + after, result);
  }
}
```

For a discussion of the runtime of this algorithm, see Example 12 on page 51.

8.8 **Permutations with Duplicates:** Write a method to compute all permutations of a string whose characters are not necessarily unique. The list of permutations should not have duplicates.

pg 135

SOLUTION

This is very similar to the previous problem, except that now we could potentially have duplicate characters in the word.

One simple way of handling this problem is to do the same work to check if a permutation has been created before and then, if not, add it to the list. A simple hash table will do the trick here. This solution will take O(n!) time in the worst case (and, in fact, in all cases).

While it's true that we can't beat this worst case time, we should be able to design an algorithm to beat this in many cases. Consider a string with all duplicate characters, like aaaaaaaaaaaaa. This will take an extremely long time (since there are over 6 billion permutations of a 13-character string), even though there is only one unique permutation.

Ideally, we would like to only create the unique permutations, rather than creating every permutation and then ruling out the duplicates.

We can start with computing the count of each letter (easy enough to get this—just use a hash table). For a string such as aabbbbc, this would be:

```
a->2 | b->4 | c->1
```

Let's imagine generating a permutation of this string (now represented as a hash table). The first choice we make is whether to use an a, b, or c as the first character. After that, we have a subproblem to solve: find all permutations of the remaining characters, and append those to the already picked "prefix."

```
P(a->2 | b->4 | c->1) = {a + P(a->1 | b->4 | c->1)} +
                        {b + P(a->2 | b->3 | c->1)} +
                        {c + P(a->2 | b->4 | c->0)}
    P(a->1 | b->4 | c->1) = {a + P(a->0 | b->4 | c->1)} +
                            {b + P(a->1 | b->3 | c->1)} +
                            {c + P(a->1 | b->4 | c->0)}
    P(a->2 | b->3 | c->1) = {a + P(a->1 | b->3 | c->1)} +
                            {b + P(a->2 | b->2 | c->1)} +
                            {c + P(a->2 | b->3 | c->0)}
    P(a->2 | b->4 | c->0) = {a + P(a->1 | b->4 | c->0)} +
                            {b + P(a->2 | b->3 | c->0)}
```

Eventually, we'll get down to no more characters remaining.

The code below implements this algorithm.

```
ArrayList<String> printPerms(String s) {
  ArrayList<String> result = new ArrayList<String>();
  HashMap<Character, Integer> map = buildFreqTable(s);
  printPerms(map, "", s.length(), result);
  return result;
}

HashMap<Character, Integer> buildFreqTable(String s) {
  HashMap<Character, Integer> map = new HashMap<Character, Integer>();
  for (char c : s.toCharArray()) {
    if (!map.containsKey(c)) {
      map.put(c, 0);
    }
    map.put(c, map.get(c) + 1);
  }
  return map;
}

void printPerms(HashMap<Character, Integer> map, String prefix, int remaining,
                ArrayList<String> result) {
  /* Base case. Permutation has been completed. */
  if (remaining == 0) {
    result.add(prefix);
    return;
  }

  /* Try remaining letters for next char, and generate remaining permutations. */
```

```
    for (Character c : map.keySet()) {
      int count = map.get(c);
      if (count > 0) {
        map.put(c,  count - 1);
        printPerms(map, prefix + c, remaining - 1, result);
        map.put(c,  count);
      }
    }
  }
```

In situations where the string has many duplicates, this algorithm will run a lot faster than the earlier algorithm.

8.9 Parens: Implement an algorithm to print all valid (i.e., properly opened and closed) combinations of n pairs of parentheses.

EXAMPLE

Input: 3

Output: `((())), (()()), (())(), ()(()), ()()()`

pg 136

SOLUTION

Our first thought here might be to apply a recursive approach where we build the solution for `f(n)` by adding pairs of parentheses to `f(n-1)`. That's certainly a good instinct.

Let's consider the solution for `n = 3`:

```
(()())    ((()))    ()(())    (())()    ()()()
```

How might we build this from `n = 2`?

```
(())    ()()
```

We can do this by inserting a pair of parentheses inside every existing pair of parentheses, as well as one at the beginning of the string. Any other places that we could insert parentheses, such as at the end of the string, would reduce to the earlier cases.

So, we have the following:

```
(()) -> (()()) /* inserted pair after 1st left paren */
     -> ((())) / *inserted pair after 2nd left paren */
     -> ()(()) / *inserted pair at beginning of string */
()() -> (())() / *inserted pair after 1st left paren */
     -> ()(()) / *inserted pair after 2nd left paren */
     -> ()()() / *inserted pair at beginning of string */
```

But wait—we have some duplicate pairs listed. The string `()(())` is listed twice.

If we're going to apply this approach, we'll need to check for duplicate values before adding a string to our list.

```
Set<String> generateParens(int remaining) {
  Set<String> set = new HashSet<String>();
  if (remaining == 0) {
    set.add("");
  } else {
    Set<String> prev = generateParens(remaining - 1);
    for (String str : prev) {
      for (int i = 0; i < str.length(); i++) {
```

```
                if (str.charAt(i) == '(') {
                    String s = insertInside(str, i);
                    /* Add s to set if it's not already in there. Note:  HashSet
                     * automatically checks for duplicates before adding, so an explicit
                     * check is not necessary. */
                    set.add(s);
                }
            }
            set.add("()" + str);
        }
    }
    return set;
}

String insertInside(String str, int leftIndex) {
    String left = str.substring(0, leftIndex + 1);
    String right = str.substring(leftIndex + 1, str.length());
    return left + "()" + right;
}
```

This works, but it's not very efficient. We waste a lot of time coming up with the duplicate strings.

We can avoid this duplicate string issue by building the string from scratch. Under this approach, we add left and right parens, as long as our expression stays valid.

On each recursive call, we have the index for a particular character in the string. We need to select either a left or a right paren. When can we use a left paren, and when can we use a right paren?

1. *Left Paren:* As long as we haven't used up all the left parentheses, we can always insert a left paren.
2. *Right Paren:* We can insert a right paren as long as it won't lead to a syntax error. When will we get a syntax error? We will get a syntax error if there are more right parentheses than left.

So, we simply keep track of the number of left and right parentheses allowed. If there are left parens remaining, we'll insert a left paren and recurse. If there are more right parens remaining than left (i.e., if there are more left parens in use than right parens), then we'll insert a right paren and recurse.

```
void addParen(ArrayList<String> list, int leftRem, int rightRem, char[] str,
              int index) {
    if (leftRem < 0 || rightRem < leftRem) return; // invalid state

    if (leftRem == 0 && rightRem == 0) { /* Out of left and right parentheses */
        list.add(String.copyValueOf(str));
    } else {
        str[index] = '('; // Add left and recurse
        addParen(list, leftRem - 1, rightRem, str, index + 1);

        str[index] = ')'; // Add right and recurse
        addParen(list, leftRem, rightRem - 1, str, index + 1);
    }
}

ArrayList<String> generateParens(int count) {
    char[] str = new char[count*2];
    ArrayList<String> list = new ArrayList<String>();
    addParen(list, count, count, str, 0);
    return list;
}
```

Because we insert left and right parentheses at each index in the string, and we never repeat an index, each string is guaranteed to be unique.

8.10 **Paint Fill:** Implement the "paint fill" function that one might see on many image editing programs. That is, given a screen (represented by a two-dimensional array of colors), a point, and a new color, fill in the surrounding area until the color changes from the original color.

pg 136

SOLUTION

First, let's visualize how this method works. When we call `paintFill` (i.e., "click" paint fill in the image editing application) on, say, a green pixel, we want to "bleed" outwards. Pixel by pixel, we expand outwards by calling `paintFill` on the surrounding pixel. When we hit a pixel that is not green, we stop.

We can implement this algorithm recursively:

```
enum Color { Black, White, Red, Yellow, Green }

boolean PaintFill(Color[][] screen, int r, int c, Color ncolor) {
  if (screen[r][c] == ncolor) return false;
  return PaintFill(screen, r, c, screen[r][c], ncolor);
}

boolean PaintFill(Color[][] screen, int r, int c, Color ocolor, Color ncolor) {
  if (r < 0 || r >= screen.length || c < 0 || c >= screen[0].length) {
    return false;
  }

  if (screen[r][c] == ocolor) {
    screen[r][c] = ncolor;
    PaintFill(screen, r - 1, c, ocolor, ncolor); // up
    PaintFill(screen, r + 1, c, ocolor, ncolor); // down
    PaintFill(screen, r, c - 1, ocolor, ncolor); // left
    PaintFill(screen, r, c + 1, ocolor, ncolor); // right
  }
  return true;
}
```

If you used the variable names x and y to implement this, be careful about the ordering of the variables in `screen[y][x]`. Because x represents the *horizontal* axis (that is, it's left to right), it actually corresponds to the column number, not the row number. The value of y equals the number of rows. This is a very easy place to make a mistake in an interview, as well as in your daily coding. It's typically clearer to use `row` and `column` instead, as we've done here.

Does this algorithm seem familiar? It should! This is essentially depth-first search on a graph. At each pixel, we are searching outwards to each surrounding pixel. We stop once we've fully traversed all the surrounding pixels of this color.

We could alternatively implement this using breadth-first search.

8.11 **Coins:** Given an infinite number of quarters (25 cents), dimes (10 cents), nickels (5 cents), and pennies (1 cent), write code to calculate the number of ways of representing n cents.

pg 136

SOLUTION

This is a recursive problem, so let's figure out how to compute `makeChange(n)` using prior solutions (i.e., subproblems).

Let's say `n = 100`. We want to compute the number of ways of making change for 100 cents. What is the relationship between this problem and its subproblems?

We know that making change for 100 cents will involve either 0, 1, 2, 3, or 4 quarters. So:

```
makeChange(100) = makeChange(100 using 0 quarters) +
                  makeChange(100 using 1 quarter)  +
                  makeChange(100 using 2 quarters) +
                  makeChange(100 using 3 quarters) +
                  makeChange(100 using 4 quarters)
```

Inspecting this further, we can see that some of these problems reduce. For example, `makeChange(100 using 1 quarter)` will equal `makeChange(75 using 0 quarters)`. This is because, if we must use exactly one quarter to make change for 100 cents, then our only remaining choices involve making change for the remaining 75 cents.

We can apply the same logic to `makeChange(100 using 2 quarters)`, `makeChange(100 using 3 quarters)` and `makeChange(100 using 4 quarters)`. We have thus reduced the above statement to the following.

```
makeChange(100) = makeChange(100 using 0 quarters) +
                  makeChange(75 using 0 quarters) +
                  makeChange(50 using 0 quarters) +
                  makeChange(25 using 0 quarters) +
                  1
```

Note that the final statement from above, `makeChange(100 using 4 quarters)`, equals 1. We call this "fully reduced."

Now what? We've used up all our quarters, so now we can start applying our next biggest denomination: dimes.

Our approach for quarters applies to dimes as well, but we apply this for *each* of the four of five parts of the above statement. So, for the first part, we get the following statements:

```
makeChange(100 using 0 quarters) = makeChange(100 using 0 quarters, 0 dimes) +
                                   makeChange(100 using 0 quarters, 1 dime)  +
                                   makeChange(100 using 0 quarters, 2 dimes) +
                                   ...
                                   makeChange(100 using 0 quarters, 10 dimes)

makeChange(75 using 0 quarters) = makeChange(75 using 0 quarters, 0 dimes) +
                                  makeChange(75 using 0 quarters, 1 dime)  +
                                  makeChange(75 using 0 quarters, 2 dimes) +
                                  ...
                                  makeChange(75 using 0 quarters, 7 dimes)

makeChange(50 using 0 quarters) = makeChange(50 using 0 quarters, 0 dimes) +
                                  makeChange(50 using 0 quarters, 1 dime)  +
                                  makeChange(50 using 0 quarters, 2 dimes) +
```

```
                                            ...
                                            makeChange(50 using 0 quarters, 5 dimes)

makeChange(25 using 0 quarters) = makeChange(25 using 0 quarters, 0 dimes) +
                                  makeChange(25 using 0 quarters, 1 dime)  +
                                  makeChange(25 using 0 quarters, 2 dimes)
```

Each one of these, in turn, expands out once we start applying nickels. We end up with a tree-like recursive structure where each call expands out to four or more calls.

The base case of our recursion is the fully reduced statement. For example, `makeChange(50 using 0 quarters, 5 dimes)` is fully reduced to 1, since 5 dimes equals 50 cents.

This leads to a recursive algorithm that looks like this:

```
int makeChange(int amount, int[] denoms, int index) {
  if (index >= denoms.length - 1) return 1; // last denom
  int denomAmount = denoms[index];
  int ways = 0;
  for (int i = 0; i * denomAmount <= amount; i++) {
    int amountRemaining = amount - i * denomAmount;
    ways += makeChange(amountRemaining, denoms, index + 1);
  }
  return ways;
}

int makeChange(int n) {
  int[] denoms = {25, 10, 5, 1};
  return makeChange(n, denoms, 0);
}
```

This works, but it's not as optimal as it could be. The issue is that we will be recursively calling `makeChange` several times for the same values of `amount` and `index`.

We can resolve this issue by storing the previously computed values. We'll need to store a mapping from each pair (`amount, index`) to the precomputed result.

```
int makeChange(int n) {
  int[] denoms = {25, 10, 5, 1};
  int[][] map = new int[n + 1][denoms.length]; // precomputed vals
  return makeChange(n, denoms, 0, map);
}

int makeChange(int amount, int[] denoms, int index, int[][] map) {
  if (map[amount][index] > 0) { // retrieve value
    return map[amount][index];
  }
  if (index >= denoms.length - 1) return 1; // one denom remaining
  int denomAmount = denoms[index];
  int ways = 0;
  for (int i = 0; i * denomAmount <= amount; i++) {
    // go to next denom, assuming i coins of denomAmount
    int amountRemaining = amount - i * denomAmount;
    ways += makeChange(amountRemaining, denoms, index + 1, map);
  }
  map[amount][index] = ways;
  return ways;
}
```

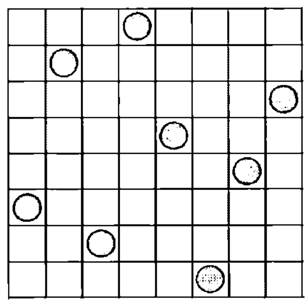
Note that we've used a two-dimensional array of integers to store the previously computed values. This is simpler, but takes up a little extra space. Alternatively, we could use an actual hash table that maps from `amount` to a new hash table, which then maps from `denom` to the precomputed value. There are other alternative data structures as well.

8.12 **Eight Queens:** Write an algorithm to print all ways of arranging eight queens on an 8x8 chess board so that none of them share the same row, column, or diagonal. In this case, "diagonal" means all diagonals, not just the two that bisect the board.

pg 136

SOLUTION

We have eight queens which must be lined up on an 8x8 chess board such that none share the same row, column or diagonal. So, we know that each row and column (and diagonal) must be used exactly once.



A "Solved" Board with 8 Queens

Picture the queen that is placed last, which we'll assume is on row 8. (This is an okay assumption to make since the ordering of placing the queens is irrelevant.) On which cell in row 8 is this queen? There are eight possibilities, one for each column.

So if we want to know all the valid ways of arranging 8 queens on an 8x8 chess board, it would be:

```
ways to arrange 8 queens on an 8x8 board =
   ways to arrange 8 queens on an 8x8 board with queen at (7, 0) +
   ways to arrange 8 queens on an 8x8 board with queen at (7, 1) +
   ways to arrange 8 queens on an 8x8 board with queen at (7, 2) +
   ways to arrange 8 queens on an 8x8 board with queen at (7, 3) +
   ways to arrange 8 queens on an 8x8 board with queen at (7, 4) +
   ways to arrange 8 queens on an 8x8 board with queen at (7, 5) +
   ways to arrange 8 queens on an 8x8 board with queen at (7, 6) +
   ways to arrange 8 queens on an 8x8 board with queen at (7, 7)
```

We can compute each one of these using a very similar approach:

```
ways to arrange 8 queens on an 8x8 board with queen at (7, 3) =
   ways to ... with queens at (7, 3) and (6, 0) +
   ways to ... with queens at (7, 3) and (6, 1) +
   ways to ... with queens at (7, 3) and (6, 2) +
   ways to ... with queens at (7, 3) and (6, 4) +
   ways to ... with queens at (7, 3) and (6, 5) +
   ways to ... with queens at (7, 3) and (6, 6) +
   ways to ... with queens at (7, 3) and (6, 7)
```

Note that we don't need to consider combinations with queens at `(7, 3)` and `(6, 3)`, since this is a violation of the requirement that every queen is in its own row, column and diagonal.

Implementing this is now reasonably straightforward.

```
int GRID_SIZE = 8;

void placeQueens(int row, Integer[] columns, ArrayList<Integer[]> results) {
  if (row == GRID_SIZE) { // Found valid placement
    results.add(columns.clone());
  } else {
    for (int col = 0; col < GRID_SIZE; col++) {
      if (checkValid(columns, row, col)) {
        columns[row] = col;  // Place queen
        placeQueens(row + 1, columns, results);
      }
    }
  }
}

/* Check if (row1, column1) is a valid spot for a queen by checking if there is a
 * queen in the same column or diagonal. We don't need to check it for queens in
 * the same row because the calling placeQueen only attempts to place one queen at
 * a time. We know this row is empty. */
boolean checkValid(Integer[] columns, int row1, int column1) {
  for (int row2 = 0; row2 < row1; row2++) {
    int column2 = columns[row2];
    /* Check if (row2, column2) invalidates (row1, column1) as a
     * queen spot. */

    /* Check if rows have a queen in the same column */
    if (column1 == column2) {
      return false;
    }

    /* Check diagonals: if the distance between the columns equals the distance
     * between the rows, then they're in the same diagonal. */
    int columnDistance = Math.abs(column2 - column1);

    /* row1 > row2, so no need for abs */
    int rowDistance = row1 - row2;
    if (columnDistance == rowDistance) {
      return false;
    }
  }
  return true;
}
```

Observe that since each row can only have one queen, we don't need to store our board as a full 8x8 matrix. We only need a single array where `column[r] = c` indicates that row r has a queen at column c.

8.13 **Stack of Boxes:** You have a stack of n boxes, with widths w_i, heights h_i, and depths d_i. The boxes cannot be rotated and can only be stacked on top of one another if each box in the stack is strictly larger than the box above it in width, height, and depth. Implement a method to compute the height of the tallest possible stack. The height of a stack is the sum of the heights of each box.

pg 136

SOLUTION

To tackle this problem, we need to recognize the relationship between the different subproblems.

Solution #1

Imagine we had the following boxes: b_1, b_2, ..., b_n. The biggest stack that we can build with all the boxes equals the max of (biggest stack with bottom b_1, biggest stack with bottom b_2, ..., biggest stack with bottom b_n). That is, if we experimented with each box as a bottom and built the biggest stack possible with each, we would find the biggest stack possible.

But, how would we find the biggest stack with a particular bottom? Essentially the same way. We experiment with different boxes for the second level, and so on for each level.

Of course, we only experiment with valid boxes. If b_5 is bigger than b_1, then there's no point in trying to build a stack that looks like {b_1, b_5, ...}. We already know b_1 can't be below b_5.

We can perform a small optimization here. The requirements of this problem stipulate that the lower boxes must be strictly greater than the higher boxes in all dimensions. Therefore, if we sort (descending order) the boxes on a dimension—any dimension—then we know we don't have to look backwards in the list. The box b_1 cannot be on top of box b_5, since its height (or whatever dimension we sorted on) is greater than b_5's height.

The code below implements this algorithm recursively.

```
int createStack(ArrayList<Box> boxes) {
  /* Sort in decending order by height. */
  Collections.sort(boxes, new BoxComparator());
  int maxHeight = 0;
  for (int i = 0; i < boxes.size(); i++) {
    int height = createStack(boxes, i);
    maxHeight = Math.max(maxHeight, height);
  }
  return maxHeight;
}

int createStack(ArrayList<Box> boxes, int bottomIndex) {
  Box bottom = boxes.get(bottomIndex);
  int maxHeight = 0;
  for (int i = bottomIndex + 1; i < boxes.size(); i++) {
    if (boxes.get(i).canBeAbove(bottom)) {
      int height = createStack(boxes, i);
      maxHeight = Math.max(height, maxHeight);
    }
  }
  maxHeight += bottom.height;
  return maxHeight;
}

class BoxComparator implements Comparator<Box> {
```

```
    @Override
    public int compare(Box x, Box y){
      return y.height - x.height;
    }
}
```

The problem in this code is that it gets very inefficient. We try to find the best solution that looks like {b_3, b_4, ...} even though we may have already found the best solution with b_4 at the bottom. Instead of generating these solutions from scratch, we can cache these results using memoization.

```
int createStack(ArrayList<Box> boxes) {
  Collections.sort(boxes, new BoxComparator());
  int maxHeight = 0;
  int[] stackMap = new int[boxes.size()];
  for (int i = 0; i < boxes.size(); i++) {
    int height = createStack(boxes, i, stackMap);
    maxHeight = Math.max(maxHeight, height);
  }
  return maxHeight;
}

int createStack(ArrayList<Box> boxes, int bottomIndex, int[] stackMap) {
  if (bottomIndex < boxes.size() && stackMap[bottomIndex] > 0) {
    return stackMap[bottomIndex];
  }

  Box bottom = boxes.get(bottomIndex);
  int maxHeight = 0;
  for (int i = bottomIndex + 1; i < boxes.size(); i++) {
    if (boxes.get(i).canBeAbove(bottom)) {
      int height = createStack(boxes, i, stackMap);
      maxHeight = Math.max(height, maxHeight);
    }
  }
  maxHeight += bottom.height;
  stackMap[bottomIndex] = maxHeight;
  return maxHeight;
}
```

Because we're only mapping from an index to a height, we can just use an integer array for our "hash table."

Be very careful here with what each spot in the hash table represents. In this code, `stackMap[i]` represents the tallest stack with box `i` at the bottom. Before pulling the value from the hash table, you have to ensure that box `i` can be placed on top of the current bottom.

It helps to keep the line that recalls from the hash table symmetric with the one that inserts. For example, in this code, we recall from the hash table with `bottomIndex` at the start of the method. We insert into the hash table with `bottomIndex` at the end.

Solution #2

Alternatively, we can think about the recursive algorithm as making a choice, at each step, whether to put a particular box in the stack. (We will again sort our boxes in descending order by a dimension, such as height.)

First, we choose whether or not to put box 0 in the stack. Take one recursive path with box 0 at the bottom and one recursive path without box 0. Return the better of the two options.

Then, we choose whether or not to put box 1 in the stack. Take one recursive path with box 1 at the bottom and one path without box 1. Return the better of the two options.

We will again use memoization to cache the height of the tallest stack with a particular bottom.

```
1  int createStack(ArrayList<Box> boxes) {
2    Collections.sort(boxes, new BoxComparator());
3    int[] stackMap = new int[boxes.size()];
4    return createStack(boxes, null, 0, stackMap);
5  }
6
7  int createStack(ArrayList<Box> boxes, Box bottom, int offset, int[] stackMap) {
8    if (offset >= boxes.size()) return 0; // Base case
9
10   / *height with this bottom */
11   Box newBottom = boxes.get(offset);
12   int heightWithBottom = 0;
13   if (bottom == null || newBottom.canBeAbove(bottom)) {
14     if (stackMap[offset] == 0) {
15       stackMap[offset] = createStack(boxes, newBottom, offset + 1, stackMap);
16       stackMap[offset] += newBottom.height;
17     }
18     heightWithBottom = stackMap[offset];
19   }
20
21   / *without this bottom */
22   int heightWithoutBottom = createStack(boxes, bottom, offset + 1, stackMap);
23
24   /* Return better of two options. */
25   return Math.max(heightWithBottom, heightWithoutBottom);
26 }
```

Again, pay close attention to when you recall and insert values into the hash table. It's typically best if these are symmetric, as they are in lines 15 and 16-18.

8.14 Boolean Evaluation: Given a boolean expression consisting of the symbols `0` (false), `1` (true), `&` (AND), `|` (OR), and `^` (XOR), and a desired boolean result value `result`, implement a function to count the number of ways of parenthesizing the expression such that it evaluates to `result`. The expression should be fully parenthesized (e.g., `(0)^(1)`) but not extraneously (e.g., `(((0))^(1))`).

EXAMPLE

```
countEval("1^0|0|1", false) -> 2
countEval("0&0&0&1^1|0", true) -> 10
```

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SOLUTION

As in other recursive problems, the key to this problem is to figure out the relationship between a problem and its subproblems.

Brute Force

Consider an expression like `0^0&0^1|1` and the target result `true`. How can we break down `countEval(0^0&0^1|1, true)` into smaller problems?

We could just essentially iterate through each possible place to put a parenthesis.

```
countEval(0^0&0^1|1, true) =
    countEval(0^0&0^1|1 where paren around char 1, true)
  + countEval(0^0&0^1|1 where paren around char 3, true)
  + countEval(0^0&0^1|1 where paren around char 5, true)
  + countEval(0^0&0^1|1 where paren around char 7, true)
```

Now what? Let's look at just one of those expressions—the paren around char 3. This gives us `(0^0)&(0^1)`.

In order to make that expression true, both the left and right sides must be true. So:

```
left = "0^0"
right = "0^1|1"
countEval(left & right, true) = countEval(left, true) * countEval(right, true)
```

The reason we multiply the results of the left and right sides is that each result from the two sides can be paired up with each other to form a unique combination.

Each of those terms can now be decomposed into smaller problems in a similar process.

What happens when we have an "|" (OR)? Or an "^" (XOR)?

If it's an OR, then either the left or the right side must be true—or both.

```
countEval(left | right, true) = countEval(left, true)  * countEval(right, false)
                              + countEval(left, false) * countEval(right, true)
                              + countEval(left, true)  * countEval(right, true)
```

If it's an XOR, then the left or the right side can be true, but not both.

```
countEval(left ^ right, true) = countEval(left, true) * countEval(right, false)
                              + countEval(left, false) * countEval(right, true)
```

What if we were trying to make the result `false` instead? We can switch up the logic from above:

```
countEval(left & right, false) = countEval(left, true)  * countEval(right, false)
                               + countEval(left, false) * countEval(right, true)
                               + countEval(left, false) * countEval(right, false)
countEval(left | right, false) = countEval(left, false) * countEval(right, false)
countEval(left ^ right, false) = countEval(left, false) * countEval(right, false)
                               + countEval(left, true)  * countEval(right, true)
```

Alternatively, we can just use the same logic from above and subtract it out from the total number of ways of evaluating the expression.

```
totalEval(left) = countEval(left, true) + countEval(left, false)
totalEval(right) = countEval(right, true) + countEval(right, false)
totalEval(expression) = totalEval(left) * totalEval(right)
countEval(expression, false) = totalEval(expression) - countEval(expression, true)
```

This makes the code a bit more concise.

```
int countEval(String s, boolean result) {
  if (s.length() == 0) return 0;
  if (s.length() == 1) return stringToBool(s) == result ? 1 : 0;

  int ways = 0;
  for (int i = 1; i < s.length(); i += 2) {
    char c = s.charAt(i);
    String left = s.substring(0, i);
    String right = s.substring(i + 1, s.length());

    /* Evaluate each side for each result. */
    int leftTrue = countEval(left, true);
    int leftFalse = countEval(left, false);
    int rightTrue = countEval(right, true);
```

```
    int rightFalse = countEval(right, false);
    int total = (leftTrue + leftFalse) * (rightTrue + rightFalse);

    int totalTrue = 0;
    if (c == '^') { // required: one true and one false
      totalTrue = leftTrue * rightFalse + leftFalse * rightTrue;
    } else if (c == '&') { // required: both true
      totalTrue = leftTrue * rightTrue;
    } else if (c == '|') { // required: anything but both false
      totalTrue = leftTrue * rightTrue + leftFalse * rightTrue +
                  leftTrue * rightFalse;
    }

    int subWays = result ? totalTrue : total - totalTrue;
    ways += subWays;
  }

  return ways;
}

boolean stringToBool(String c) {
  return c.equals("1") ? true : false;
}
```

Note that the tradeoff of computing the `false` results from the `true` ones, and of computing the `{leftTrue, rightTrue, leftFalse, and rightFalse}` values upfront, is a small amount of extra work in some cases. For example, if we're looking for the ways that an AND (&) can result in `true`, we never would have needed the `leftFalse` and `rightFalse` results. Likewise, if we're looking for the ways that an OR (|) can result in `false`, we never would have needed the `leftTrue` and `rightTrue` results.

Our current code is blind to what we do and don't actually need to do and instead just computes all of the values. This is probably a reasonable tradeoff to make (especially given the constraints of whiteboard coding) as it makes our code substantially shorter and less tedious to write. Whichever approach you make, you should discuss the tradeoffs with your interviewer.

That said, there are more important optimizations we can make.

Optimized Solutions

If we follow the recursive path, we'll note that we end up doing the same computation repeatedly.

Consider the expression `0^0&0^1|1` and these recursion paths:

- Add parens around char 1. `(0)^(0&0^1|1)`
  - » Add parens around char 3. `(0)^((0)&(0^1|1))`
- Add parens around char 3. `(0^0)&(0^1|1)`
  - » Add parens around char 1. `((0)^(0))&(0^1|1)`

Although these two expressions are different, they have a similar component: `(0^1|1)`. We should reuse our effort on this.

We can do this by using memoization, or a hash table. We just need to store the result of `countEval(expression, result)` for each expression and result. If we see an expression that we've calculated before, we just return it from the cache.

```
int countEval(String s, boolean result, HashMap<String, Integer> memo) {
  if (s.length() == 0) return 0;
  if (s.length() == 1) return stringToBool(s) == result ? 1 : 0;
```

```
    if (memo.containsKey(result + s)) return memo.get(result + s);

    int ways = 0;

    for (int i = 1; i < s.length(); i += 2) {
      char c = s.charAt(i);
      String left = s.substring(0, i);
      String right = s.substring(i + 1, s.length());
      int leftTrue = countEval(left, true, memo);
      int leftFalse = countEval(left, false, memo);
      int rightTrue = countEval(right, true, memo);
      int rightFalse = countEval(right, false, memo);
      int total = (leftTrue + leftFalse) * (rightTrue + rightFalse);

      int totalTrue = 0;
      if (c == '^') {
        totalTrue = leftTrue * rightFalse + leftFalse * rightTrue;
      } else if (c == '&') {
        totalTrue = leftTrue * rightTrue;
      } else if (c == '|') {
        totalTrue = leftTrue * rightTrue + leftFalse * rightTrue +
                    leftTrue * rightFalse;
      }

      int subWays = result ? totalTrue : total - totalTrue;
      ways += subWays;
    }

    memo.put(result + s, ways);
    return ways;
  }
```

The added benefit of this is that we could actually end up with the same substring in multiple parts of the expression. For example, an expression like `0^1^0&0^1^0` has two instances of `0^1^0`. By caching the result of the substring value in a memoization table, we'll get to reuse the result for the right part of the expression after computing it for the left.

There is one further optimization we can make, but it's far beyond the scope of the interview. There *is* a closed form expression for the number of ways of parenthesizing an expression, but you wouldn't be expected to know it. It is given by the Catalan numbers, where n is the number of operators:

$$C_n = \frac{(2n)!}{(n+1)!n!}$$

We could use this to compute the total ways of evaluating the expression. Then, rather than computing `leftTrue` and `leftFalse`, we just compute one of those and calculate the other using the Catalan numbers. We would do the same thing for the right side.

9

Solutions to System Design and Scalability

9.1 Stock Data: Imagine you are building some sort of service that will be called by up to 1,000 client applications to get simple end-of-day stock price information (open, close, high, low). You may assume that you already have the data, and you can store it in any format you wish. How would you design the client-facing service that provides the information to client applications? You are responsible for the development, rollout, and ongoing monitoring and maintenance of the feed. Describe the different methods you considered and why you would recommend your approach. Your service can use any technologies you wish, and can distribute the information to the client applications in any mechanism you choose.

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SOLUTION

From the statement of the problem, we want to focus on how we actually distribute the information to clients. We can assume that we have some scripts that magically collect the information.

We want to start off by thinking about what the different aspects we should consider in a given proposal are:

- *Client Ease of Use:* We want the service to be easy for the clients to implement and useful for them.
- *Ease for Ourselves:* This service should be as easy as possible for us to implement, as we shouldn't impose unnecessary work on ourselves. We need to consider in this not only the cost of implementing, but also the cost of maintenance.
- *Flexibility for Future Demands:* This problem is stated in a "what would you do in the real world" way, so we should think like we would in a real-world problem. Ideally, we do not want to overly constrain ourselves in the implementation, such that we can't be flexible if the requirements or demands change.
- *Scalability and Efficiency:* We should be mindful of the efficiency of our solution, so as not to overly burden our service.

With this framework in mind, we can consider various proposals.

Proposal #1

One option is that we could keep the data in simple text files and let clients download the data through some sort of FTP server. This would be easy to maintain in some sense, since files can be easily viewed and backed up, but it would require more complex parsing to do any sort of query. And, if additional data were added to our text file, it might break the clients' parsing mechanism.

Proposal #2

We could use a standard SQL database, and let the clients plug directly into that. This would provide the following benefits:

- Facilitates an easy way for the clients to do query processing over the data, in case there are additional features we need to support. For example, we could easily and efficiently perform a query such as "return all stocks having an open price greater than N and a closing price less than M."
- Rolling back, backing up data, and security could be provided using standard database features. We don't have to "reinvent the wheel," so it's easy for us to implement.
- Reasonably easy for the clients to integrate into existing applications. SQL integration is a standard feature in software development environments.

What are the disadvantages of using a SQL database?

- It's much heavier weight than we really need. We don't necessarily need all the complexity of a SQL backend to support a feed of a few bits of information.
- It's difficult for humans to be able to read it, so we'll likely need to implement an additional layer to view and maintain the data. This increases our implementation costs.
- Security: While a SQL database offers pretty well defined security levels, we would still need to be very careful to not give clients access that they shouldn't have. Additionally, even if clients aren't doing anything "malicious," they might perform expensive and inefficient queries, and our servers would bear the costs of that.

These disadvantages don't mean that we shouldn't provide SQL access. Rather, they mean that we should be aware of the disadvantages.

Proposal #3

XML is another great option for distributing the information. Our data has fixed format and fixed size: `company_name`, `open`, `high`, `low`, `closing price`. The XML could look like this:

```
<root>
  <date value="2008-10-12">
    <company name="foo">
      <open>126.23</open>
      <high>130.27</high>
      <low>122.83</low>
      <closingPrice>127.30</closingPrice>
    </company>
    <company name="bar">
      <open>52.73</open>
      <high>60.27</high>
      <low>50.29</low>
      <closingPrice>54.91</closingPrice>
    </company>
  </date>
  <date value="2008-10-11"> . . . </date>
</root>
```

The advantages of this approach include the following:

- It's very easy to distribute, and it can also be easily read by both machines and humans. This is one reason that XML is a standard data model to share and distribute data.
- Most languages have a library to perform XML parsing, so it's reasonably easy for clients to implement.

- We can add new data to the XML file by adding additional nodes. This would not break the client's parser (provided they have implemented their parser in a reasonable way).
- Since the data is being stored as XML files, we can use existing tools for backing up the data. We don't need to implement our own backup tool.

The disadvantages may include:

- This solution sends the clients all the information, even if they only want part of it. It is inefficient in that way.
- Performing any queries on the data requires parsing the entire file.

Regardless of which solution we use for data storage, we could provide a web service (e.g., SOAP) for client data access. This adds a layer to our work, but it can provide additional security, and it may even make it easier for clients to integrate the system.

However—and this is a pro and a con—clients will be limited to grabbing the data only how we expect or want them to. By contrast, in a pure SQL implementation, clients could query for the highest stock price, even if this wasn't a procedure we "expected" them to need.

So which one of these would we use? There's no clear answer. The pure text file solution is probably a bad choice, but you can make a compelling argument for the SQL or XML solution, with or without a web service.

The goal of a question like this is not to see if you get the "correct" answer (there is no single correct answer). Rather, it's to see how you design a system, and how you evaluate trade-offs.

9.2 **Social Network:** How would you design the data structures for a very large social network like Facebook or LinkedIn? Describe how you would design an algorithm to show the shortest path between two people (e.g., Me -> Bob -> Susan -> Jason -> You).

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SOLUTION

A good way to approach this problem is to remove some of the constraints and solve it for that situation first.

Step 1: Simplify the Problem—Forget About the Millions of Users

First, let's forget that we're dealing with millions of users. Design this for the simple case.

We can construct a graph by treating each person as a node and letting an edge between two nodes indicate that the two users are friends.

If I wanted to find the path between two people, I could start with one person and do a simple breadth-first search.

Why wouldn't a depth-first search work well? First, depth-first search would just find a path. It wouldn't necessarily find the shortest path. Second, even if we just needed any path, it would be very inefficient. Two users might be only one degree of separation apart, but I could search millions of nodes in their "subtrees" before finding this relatively immediate connection.

Alternatively, I could do what's called a bidirectional breadth-first search. This means doing two breadth-first searches, one from the source and one from the destination. When the searches collide, we know we've found a path.

In the implementation, we'll use two classes to help us. BFSData holds the data we need for a breadth-first search, such as the isVisited hash table and the toVisit queue. PathNode will represent the path as we're searching it, storing each Person and the previousNode we visited in this path.

```
LinkedList<Person> findPathBiBFS(HashMap<Integer, Person> people, int source,
                                 int destination) {
  BFSData sourceData = new BFSData(people.get(source));
  BFSData destData = new BFSData(people.get(destination));

  while (!sourceData.isFinished() && !destData.isFinished()) {
    /* Search out from source. */
    Person collision = searchLevel(people, sourceData, destData);
    if (collision != null) {
      return mergePaths(sourceData, destData, collision.getID());
    }

    /* Search out from destination. */
    collision = searchLevel(people, destData, sourceData);
    if (collision != null) {
      return mergePaths(sourceData, destData, collision.getID());
    }
  }
  return null;
}

/* Search one level and return collision, if any. */
Person searchLevel(HashMap<Integer, Person> people, BFSData primary,
                   BFSData secondary) {
  /* We only want to search one level at a time. Count how many nodes are
   * currently in the primary's level and only do that many nodes. We'll continue
   * to add nodes to the end. */
  int count = primary.toVisit.size();
  for (int i = 0; i < count; i++) {
    /* Pull out first node. */
    PathNode pathNode = primary.toVisit.poll();
    int personId = pathNode.getPerson().getID();

    /* Check if it's already been visited. */
    if (secondary.visited.containsKey(personId)) {
      return pathNode.getPerson();
    }

    /* Add friends to queue. */
    Person person = pathNode.getPerson();
    ArrayList<Integer> friends = person.getFriends();
    for (int friendId : friends) {
      if (!primary.visited.containsKey(friendId)) {
        Person friend = people.get(friendId);
        PathNode next = new PathNode(friend, pathNode);
        primary.visited.put(friendId, next);
        primary.toVisit.add(next);
      }
    }
  }
  return null;
}

```

```
/* Merge paths where searches met at connection. */
LinkedList<Person> mergePaths(BFSData bfs1, BFSData bfs2, int connection) {
  PathNode end1 = bfs1.visited.get(connection); // end1 -> source
  PathNode end2 = bfs2.visited.get(connection); // end2 -> dest
  LinkedList<Person> pathOne = end1.collapse(false);
  LinkedList<Person> pathTwo = end2.collapse(true); // reverse
  pathTwo.removeFirst(); // remove connection
  pathOne.addAll(pathTwo); // add second path
  return pathOne;
}

class PathNode {
  private Person person = null;
  private PathNode previousNode = null;
  public PathNode(Person p, PathNode previous) {
    person = p;
    previousNode = previous;
  }

  public Person getPerson() { return person; }

  public LinkedList<Person> collapse(boolean startsWithRoot) {
    LinkedList<Person> path = new LinkedList<Person>();
    PathNode node = this;
    while (node != null) {
      if (startsWithRoot) {
        path.addLast(node.person);
      } else {
        path.addFirst(node.person);
      }
      node = node.previousNode;
    }
    return path;
  }
}

class BFSData {
  public Queue<PathNode> toVisit = new LinkedList<PathNode>();
  public HashMap<Integer, PathNode> visited =
    new HashMap<Integer, PathNode>();

  public BFSData(Person root) {
    PathNode sourcePath = new PathNode(root, null);
    toVisit.add(sourcePath);
    visited.put(root.getID(), sourcePath);
  }

  public boolean isFinished() {
    return toVisit.isEmpty();
  }
}
```

Many people are surprised that this is faster. Some quick math can explain why.

Suppose every person has k friends, and node S and node D have a friend C in common.

- Traditional breadth-first search from S to D: We go through roughly k+k*k nodes: each of S's k friends, and then each of their k friends.

- Bidirectional breadth-first search: We go through 2k nodes: each of S's k friends and each of D's k friends.

Of course, 2k is much less than k+k*k.

Generalizing this to a path of length q, we have this:

- BFS: $O(k^q)$
- Bidirectional BFS: $O(k^{q/2} + k^{q/2})$, which is just $O(k^{q/2})$

If you imagine a path like A->B->C->D->E where each person has 100 friends, this is a big difference. BFS will require looking at 100 million (100^4) nodes. A bidirectional BFS will require looking at only 20,000 nodes (2×100^2).

A bidirectional BFS will generally be faster than the traditional BFS. However, it requires actually having access to both the source node and the destination nodes, which is not always the case.

Step 2: Handle the Millions of Users

When we deal with a service the size of LinkedIn or Facebook, we cannot possibly keep all of our data on one machine. That means that our simple Person data structure from above doesn't quite work—our friends may not live on the same machine as we do. Instead, we can replace our list of friends with a list of their IDs, and traverse as follows:

1. For each friend ID: `int machine_index = getMachineIDForUser(personID);`
2. Go to machine `#machine_index`
3. On that machine, do: `Person friend = getPersonWithID(person_id);`

The code below outlines this process. We've defined a class Server, which holds a list of all the machines, and a class Machine, which represents a single machine. Both classes have hash tables to efficiently lookup data.

```
class Server {
  HashMap<Integer, Machine> machines = new HashMap<Integer, Machine>();
  HashMap<Integer, Integer> personToMachineMap = new HashMap<Integer, Integer>();

  public Machine getMachineWithId(int machineID) {
    return machines.get(machineID);
  }

  public int getMachineIDForUser(int personID) {
    Integer machineID = personToMachineMap.get(personID);
    return machineID == null ? -1 : machineID;
  }

  public Person getPersonWithID(int personID) {
    Integer machineID = personToMachineMap.get(personID);
    if (machineID == null) return null;

    Machine machine = getMachineWithId(machineID);
    if (machine == null) return null;

    return machine.getPersonWithID(personID);
  }
}

class Person {
```

```
    private ArrayList<Integer> friends = new ArrayList<Integer>();
    private int personID;
    private String info;

    public Person(int id) { this.personID = id; }
    public String getInfo() { return info; }
    public void setInfo(String info) { this.info = info; }
    public ArrayList<Integer> getFriends() { return friends; }
    public int getID() { return personID; }
    public void addFriend(int id) { friends.add(id); }
}
```

There are more optimizations and follow-up questions here than we could possibly discuss, but here are just a few possibilities.

Optimization: Reduce machine jumps

Jumping from one machine to another is expensive. Instead of randomly jumping from machine to machine with each friend, try to batch these jumps—e.g., if five of my friends live on one machine, I should look them up all at once.

Optimization: Smart division of people and machines

People are much more likely to be friends with people who live in the same country as they do. Rather than randomly dividing people across machines, try to divide them by country, city, state, and so on. This will reduce the number of jumps.

Question: Breadth-first search usually requires "marking" a node as visited. How do you do that in this case?

Usually, in BFS, we mark a node as visited by setting a `visited` flag in its node class. Here, we don't want to do that. There could be multiple searches going on at the same time, so it's a bad idea to just edit our data.

Instead, we could mimic the marking of nodes with a hash table to look up a node id and determine whether it's been visited.

Other Follow-Up Questions:

- In the real world, servers fail. How does this affect you?
- How could you take advantage of caching?
- Do you search until the end of the graph (infinite)? How do you decide when to give up?
- In real life, some people have more friends of friends than others, and are therefore more likely to make a path between you and someone else. How could you use this data to pick where to start traversing?

These are just a few of the follow-up questions you or the interviewer could raise. There are many others.

9.3 Web Crawler: If you were designing a web crawler, how would you avoid getting into infinite loops?

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SOLUTION

The first thing to ask ourselves in this problem is how an infinite loop might occur. The simplest answer is that, if we picture the web as a graph of links, an infinite loop will occur when a cycle occurs.

To prevent infinite loops, we just need to detect cycles. One way to do this is to create a hash table where we set `hash[v]` to `true` after we visit page v.

We can crawl the web using breadth-first search. Each time we visit a page, we gather all its links and insert them at the end of a queue. If we've already visited a page, we ignore it.

This is great—but what does it mean to visit page v? Is page v defined based on its content or its URL?

If it's defined based on its URL, we must recognize that URL parameters might indicate a completely different page. For example, the page `www.careercup.com/page?pid=microsoft-interview-questions` is totally different from the page `www.careercup.com/page?pid=google-interview-questions`. But, we can also append URL parameters arbitrarily to any URL without truly changing the page, provided it's not a parameter that the web application recognizes and handles. The page `www.careercup.com?foobar=hello` is the same as `www.careercup.com`.

"Okay, then," you might say, "let's define it based on its content." That sounds good too, at first, but it also doesn't quite work. Suppose I have some randomly generated content on the careercup.com home page. Is it a different page each time you visit it? Not really.

The reality is that there is probably no perfect way to define a "different" page, and this is where this problem gets tricky.

One way to tackle this is to have some sort of estimation for degree of similarity. If, based on the content and the URL, a page is deemed to be sufficiently similar to other pages, we *deprioritize* crawling its children. For each page, we would come up with some sort of signature based on snippets of the content and the page's URL.

Let's see how this would work.

We have a database which stores a list of items we need to crawl. On each iteration, we select the highest priority page to crawl. We then do the following:

1. Open up the page and create a signature of the page based on specific subsections of the page and its URL.
2. Query the database to see whether anything with this signature has been crawled recently.
3. If something with this signature has been recently crawled, insert this page back into the database at a low priority.
4. If not, crawl the page and insert its links into the database.

Under the above implementation, we never "complete" crawling the web, but we will avoid getting stuck in a loop of pages. If we want to allow for the possibility of "finishing" crawling the web (which would clearly happen only if the "web" were actually a smaller system, like an intranet), then we can set a minimum priority that a page must have to be crawled.

This is just one, simplistic solution, and there are many others that are equally valid. A problem like this will more likely resemble a conversation with your interviewer which could take any number of paths. In fact, the discussion of this problem could have taken the path of the very next problem.

9.4 **Duplicate URLs:** You have 10 billion URLs. How do you detect the duplicate documents? In this case, assume "duplicate" means that the URLs are identical.

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SOLUTION

Just how much space do 10 billion URLs take up? If each URL is an average of 100 characters, and each character is 4 bytes, then this list of 10 billion URLs will take up about 4 terabytes. We are probably not going to hold that much data in memory.

But, let's just pretend for a moment that we were miraculously holding this data in memory, since it's useful to first construct a solution for the simple version. Under this version of the problem, we would just create a hash table where each URL maps to `true` if it's already been found elsewhere in the list. (As an alternative solution, we could sort the list and look for the duplicate values that way. That will take a bunch of extra time and offers few advantages.)

Now that we have a solution for the simple version, what happens when we have all 4000 gigabytes of data and we can't store it all in memory? We could solve this either by storing some of the data on disk or by splitting up the data across machines.

Solution #1: Disk Storage

If we stored all the data on one machine, we would do two passes of the document. The first pass would split the list of URLs into 4000 chunks of 1 GB each. An easy way to do that might be to store each URL u in a file named `<x>.txt` where `x = hash(u) % 4000`. That is, we divide up the URLs based on their hash value (modulo the number of chunks). This way, all URLs with the same hash value would be in the same file.

In the second pass, we would essentially implement the simple solution we came up with earlier: load each file into memory, create a hash table of the URLs, and look for duplicates.

Solution #2: Multiple Machines

The other solution is to perform essentially the same procedure, but to use multiple machines. In this solution, rather than storing the data in file `<x>.txt`, we would send the URL to machine `x`.

Using multiple machines has pros and cons.

The main pro is that we can parallelize the operation, such that all 4000 chunks are processed simultaneously. For large amounts of data, this might result in a faster solution.

The disadvantage though is that we are now relying on 4000 different machines to operate perfectly. That may not be realistic (particularly with more data and more machines), and we'll need to start considering how to handle failure. Additionally, we have increased the complexity of the system simply by involving so many machines.

Both are good solutions, though, and both should be discussed with your interviewer.

9.5 **Cache:** Imagine a web server for a simplified search engine. This system has 100 machines to respond to search queries, which may then call out using `processSearch(string query)` to another cluster of machines to actually get the result. The machine which responds to a given query is chosen at random, so you cannot guarantee that the same machine will always respond to the same request. The method `processSearch` is very expensive. Design a caching mechanism to cache the results of the most recent queries. Be sure to explain how you would update the cache when data changes.

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SOLUTION

Before getting into the design of this system, we first have to understand what the question means. Many of the details are somewhat ambiguous, as is expected in questions like this. We will make reasonable assumptions for the purposes of this solution, but you should discuss these details—in depth—with your interviewer.

Assumptions

Here are a few of the assumptions we make for this solution. Depending on the design of your system and how you approach the problem, you may make other assumptions. Remember that while some approaches are better than others, there is no one "correct" approach.

- Other than calling out to `processSearch` as necessary, all query processing happens on the initial machine that was called.
- The number of queries we wish to cache is large (millions).
- Calling between machines is relatively quick.
- The result for a given query is an ordered list of URLs, each of which has an associated 50 character title and 200 character summary.
- The most popular queries are extremely popular, such that they would always appear in the cache.

Again, these aren't the *only* valid assumptions. This is just one reasonable set of assumptions.

System Requirements

When designing the cache, we know we'll need to support two primary functions:

- Efficient lookups given a key.
- Expiration of old data so that it can be replaced with new data.

In addition, we must also handle updating or clearing the cache when the results for a query change. Because some queries are very common and may permanently reside in the cache, we cannot just wait for the cache to naturally expire.

Step 1: Design a Cache for a Single System

A good way to approach this problem is to start by designing it for a single machine. So, how would you create a data structure that enables you to easily purge old data and also efficiently look up a value based on a key?

- A linked list would allow easy purging of old data, by moving "fresh" items to the front. We could implement it to remove the last element of the linked list when the list exceeds a certain size.

- A hash table allows efficient lookups of data, but it wouldn't ordinarily allow easy data purging.

How can we get the best of both worlds? By merging the two data structures. Here's how this works:

Just as before, we create a linked list where a node is moved to the front every time it's accessed. This way, the end of the linked list will always contain the stalest information.

In addition, we have a hash table that maps from a query to the corresponding node in the linked list. This allows us to not only efficiently return the cached results, but also to move the appropriate node to the front of the list, thereby updating its "freshness."

For illustrative purposes, abbreviated code for the cache is below. The code attachment provides the full code for this part. Note that in your interview, it is unlikely that you would be asked to write the full code for this as well as perform the design for the larger system.

```
public class Cache {
  public static int MAX_SIZE = 10;
  public Node head, tail;
  public HashMap<String, Node> map;
  public int size = 0;

  public Cache() {
    map = new HashMap<String, Node>();
  }

  /* Moves node to front of linked list */
  public void moveToFront(Node node) { ... }
  public void moveToFront(String query) { ... }

  /* Removes node from linked list */
  public void removeFromLinkedList(Node node) { ... }

  /* Gets results from cache, and updates linked list */
  public String[] getResults(String query) {
    if (!map.containsKey(query)) return null;

    Node node = map.get(query);
    moveToFront(node); // update freshness
    return node.results;
  }

  /* Inserts results into linked list and hash */
  public void insertResults(String query, String[] results) {
    if (map.containsKey(query)) { // update values
      Node node = map.get(query);
      node.results = results;
      moveToFront(node); // update freshness
      return;
    }

    Node node = new Node(query, results);
    moveToFront(node);
    map.put(query, node);

    if (size > MAX_SIZE) {
      map.remove(tail.query);
      removeFromLinkedList(tail);
    }
```

```
    }
}
```

Step 2: Expand to Many Machines

Now that we understand how to design this for a single machine, we need to understand how we would design this when queries could be sent to many different machines. Recall from the problem statement that there's no guarantee that a particular query will be consistently sent to the same machine.

The first thing we need to decide is to what extent the cache is shared across machines. We have several options to consider.

Option 1: Each machine has its own cache.

A simple option is to give each machine its own cache. This means that if "foo" is sent to machine 1 twice in a short amount of time, the result would be recalled from the cache on the second time. But, if "foo" is sent first to machine 1 and then to machine 2, it would be treated as a totally fresh query both times.

This has the advantage of being relatively quick, since no machine-to-machine calls are used. The cache, unfortunately, is somewhat less effective as an optimization tool as many repeat queries would be treated as fresh queries.

Option 2: Each machine has a copy of the cache.

On the other extreme, we could give each machine a complete copy of the cache. When new items are added to the cache, they are sent to all machines. The entire data structure—linked list and hash table—would be duplicated.

This design means that common queries would nearly always be in the cache, as the cache is the same everywhere. The major drawback however is that updating the cache means firing off data to N different machines, where N is the size of the response cluster. Additionally, because each item effectively takes up N times as much space, our cache would hold much less data.

Option 3: Each machine stores a segment of the cache.

A third option is to divide up the cache, such that each machine holds a different part of it. Then, when machine `i` needs to look up the results for a query, machine `i` would figure out which machine holds this value, and then ask this other machine (machine `j`) to look up the query in `j`'s cache.

But how would machine `i` know which machine holds this part of the hash table?

One option is to assign queries based on the formula `hash(query) % N`. Then, machine `i` only needs to apply this formula to know that machine `j` should store the results for this query.

So, when a new query comes in to machine `i`, this machine would apply the formula and call out to machine `j`. Machine `j` would then return the value from its cache or call `processSearch(query)` to get the results. Machine `j` would update its cache and return the results back to `i`.

Alternatively, you could design the system such that machine `j` just returns `null` if it doesn't have the `query` in its current cache. This would require machine `i` to call `processSearch` and then forward the results to machine `j` for storage. This implementation actually increases the number of machine-to-machine calls, with few advantages.

Step 3: Updating results when contents change

Recall that some queries may be so popular that, with a sufficiently large cache, they would permanently be cached. We need some sort of mechanism to allow cached results to be refreshed, either periodically or "on-demand" when certain content changes.

To answer this question, we need to consider when results would change (and you need to discuss this with your interviewer). The primary times would be when:

1. The content at a URL changes (or the page at that URL is removed).
2. The ordering of results change in response to the rank of a page changing.
3. New pages appear related to a particular query.

To handle situations #1 and #2, we could create a separate hash table that would tell us which cached queries are tied to a specific URL. This could be handled completely separately from the other caches, and reside on different machines. However, this solution may require a lot of data.

Alternatively, if the data doesn't require instant refreshing (which it probably doesn't), we could periodically crawl through the cache stored on each machine to purge queries tied to the updated URLs.

Situation #3 is substantially more difficult to handle. We could update single word queries by parsing the content at the new URL and purging these one-word queries from the caches. But, this will only handle the one-word queries.

A good way to handle Situation #3 (and likely something we'd want to do anyway) is to implement an "automatic time-out" on the cache. That is, we'd impose a time out where *no* query, regardless of how popular it is, can sit in the cache for more than x minutes. This will ensure that all data is periodically refreshed.

Step 4: Further Enhancements

There are a number of improvements and tweaks you could make to this design depending on the assumptions you make and the situations you optimize for.

One such optimization is to better support the situation where some queries are very popular. For example, suppose (as an extreme example) a particular string constitutes 1% of all queries. Rather than machine `i` forwarding the request to machine `j` every time, machine `i` could forward the request just once to `j`, and then `i` could store the results in its own cache as well.

Alternatively, there may also be some possibility of doing some sort of re-architecture of the system to assign queries to machines based on their hash value (and therefore the location of the cache), rather than randomly. However, this decision may come with its own set of trade-offs.

Another optimization we could make is to the "automatic time out" mechanism. As initially described, this mechanism purges any data after X minutes. However, we may want to update some data (like current news) much more frequently than other data (like historical stock prices). We could implement timeouts based on topic or based on URLs. In the latter situation, each URL would have a time out value based on how frequently the page has been updated in the past. The time out for the query would be the minimum of the time outs for each URL.

These are just a few of the enhancements we can make. Remember that in questions like this, there is no single correct way to solve the problem. These questions are about having a discussion with your interviewer about design criteria and demonstrating your general approach and methodology.

9.6 **Sales Rank:** A large eCommerce company wishes to list the best-selling products, overall and by category. For example, one product might be the #1056th best-selling product overall but the #13th best-selling product under "Sports Equipment" and the #24th best-selling product under "Safety." Describe how you would design this system.

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SOLUTION

Let's first start off by making some assumptions to define the problem.

Step 1: Scope the Problem

First, we need to define what exactly we're building.

- We'll assume that we're only being asked to design the components relevant to this question, and not the entire eCommerce system. In this case, we might touch the design of the frontend and purchase components, but only as it impacts the sales rank.
- We should also define what the sales rank means. Is it total sales over all time? Sales in the last month? Last week? Or some more complicated function (such as one involving some sort of exponential decay of sales data)? This would be something to discuss with your interviewer. We will assume that it is simply the total sales over the past week.
- We will assume that each product can be in multiple categories, and that there is no concept of "subcategories."

This part just gives us a good idea of what the problem, or scope of features, is.

Step 2: Make Reasonable Assumptions

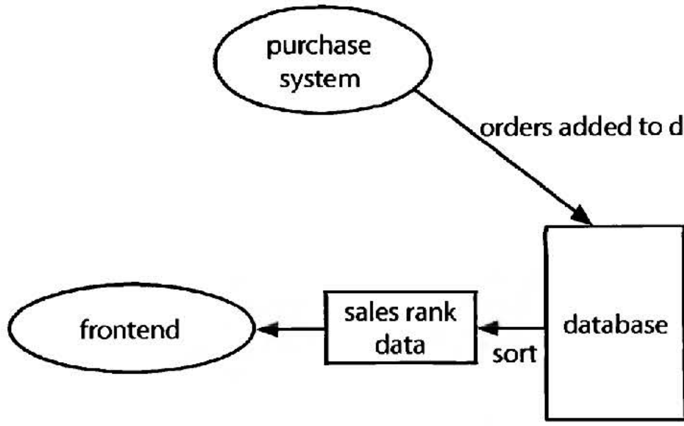
These are the sorts of things you'd want to discuss with your interviewer. Because we don't have an interviewer in front of us, we'll have to make some assumptions.

- We will assume that the stats do not need to be 100% up-to-date. Data can be up to an hour old for the most popular items (for example, top 100 in each category), and up to one day old for the less popular items. That is, few people would care if the #2,809,132th best-selling item should have actually been listed as #2,789,158th instead.
- Precision is important for the most popular items, but a small degree of error is okay for the less popular items.
- We will assume that the data should be updated every hour (for the most popular items), but the time range for this data does not need to be precisely the last seven days (168 hours). If it's sometimes more like 150 hours, that's okay.
- We will assume that the categorizations are based strictly on the origin of the transaction (i.e., the seller's name), not the price or date.

The important thing is not so much which decision you made at each possible issue, but whether it occurred to you that these are assumptions. We should get out as many of these assumptions as possible in the beginning. It's possible you will need to make other assumptions along the way.

Step 3: Draw the Major Components

We should now design just a basic, naive system that describes the major components. This is where you would go up to a whiteboard.



In this simple design, we store every order as soon as it comes into the database. Every hour or so, we pull sales data from the database by category, compute the total sales, sort it, and store it in some sort of sales rank data cache (which is probably held in memory). The frontend just pulls the sales rank from this table, rather than hitting the standard database and doing its own analytics.

Step 4: Identify the Key Issues

Analytics are Expensive

In the naive system, we periodically query the database for the number of sales in the past week for each product. This will be fairly expensive. That's running a query over all sales for all time.

Our database just needs to track the total sales. We'll assume (as noted in the beginning of the solution) that the general storage for purchase history is taken care of in other parts of the system, and we just need to focus on the sales data analytics.

Instead of listing every purchase in our database, we'll store just the total sales from the last week. Each purchase will just update the total weekly sales.

Tracking the total sales takes a bit of thought. If we just use a single column to track the total sales over the past week, then we'll need to re-compute the total sales every day (since the specific days covered in the last seven days change with each day). That is unnecessarily expensive.

Instead, we'll just use a table like this.

Prod ID	Total	Sun	Mon	Tues	Wed	Thurs	Fri	Sat

This is essentially like a circular array. Each day, we clear out the corresponding day of the week. On each purchase, we update the total sales count for that product on that day of the week, as well as the total count.

We will also need a separate table to store the associations of product IDs and categories.

Prod ID	Category ID

To get the sales rank per category, we'll need to join these tables.

Database Writes are Very Frequent

Even with this change, we'll still be hitting the database very frequently. With the amount of purchases that could come in every second, we'll probably want to batch up the database writes.

Instead of immediately committing each purchase to the database, we could store purchases in some sort of in-memory cache (as well as to a log file as a backup). Periodically, we'll process the log / cache data, gather the totals, and update the database.

> We should quickly think about whether or not it's feasible to hold this in memory. If there are 10 million products in the system, can we store each (along with a count) in a hash table? Yes. If each product ID is four bytes (which is big enough to hold up to 4 billion unique IDs) and each count is four bytes (more than enough), then such a hash table would only take about 40 megabytes. Even with some additional overhead and substantial system growth, we would still be able to fit this all in memory.

After updating the database, we can re-run the sales rank data.

We need to be a bit careful here, though. If we process one product's logs before another's, and re-run the stats in between, we could create a bias in the data (since we're including a larger timespan for one product than its "competing" product).

We can resolve this by either ensuring that the sales rank doesn't run until all the stored data is processed (difficult to do when more and more purchases are coming in), or by dividing up the in-memory cache by some time period. If we update the database for all the stored data up to a particular moment in time, this ensures that the database will not have biases.

Joins are Expensive

We have potentially tens of thousands of product categories. For each category, we'll need to first pull the data for its items (possibly through an expensive join) and then sort those.

Alternatively, we could just do one join of products and categories, such that each product will be listed once per category. Then, if we sorted that on category and then product ID, we could just walk the results to get the sales rank for each category.

Prod ID	Category	Total	Sun	Mon	Tues	Wed	Thurs	Fri	Sat
1423	sportseq	13	4	1	4	19	322	32	232
1423	safety	13	4	1	4	19	322	32	232

Rather than running thousands of queries (one for each category), we could sort the data on the category first and then the sales volume. Then, if we walked those results, we would get the sales rank for each category. We would also need to do one sort of the entire table on just sales number, to get the overall rank.

We could also just keep the data in a table like this from the beginning, rather than doing joins. This would require us to update multiple rows for each product.

Database Queries Might Still Be Expensive

Alternatively, if the queries and writes get very expensive, we could consider forgoing a database entirely and just using log files. This would allow us to take advantage of something like MapReduce.

Under this system, we would write a purchase to a simple text file with the product ID and time stamp. Each category has its own directory, and each purchase gets written to all the categories associated with that product.

We would run frequent jobs to merge files together by product ID and time ranges, so that eventually all purchases in a given day (or possibly hour) were grouped together.

```
/sportsequipment
    1423,Dec 13 08:23-Dec 13 08:23,1
    4221,Dec 13 15:22-Dec 15 15:45,5
    ...
/safety
    1423,Dec 13 08:23-Dec 13 08:23,1
    5221,Dec 12 03:19-Dec 12 03:28,19
...
```

To get the best-selling products within each category, we just need to sort each directory.

How do we get the overall ranking? There are two good approaches:

- We could treat the general category as just another directory, and write every purchase to that directory. That would mean a lot of files in this directory.
- Or, since we'll already have the products sorted by sales volume order for each category, we can also do an N-way merge to get the overall rank.

Alternatively, we can take advantage of the fact that the data doesn't need (as we assumed earlier) to be 100% up-to-date. We just need the most popular items to be up-to-date.

We can merge the most popular items from each category in a pairwise fashion. So, two categories get paired together and we merge the most popular items (the first 100 or so). After we have 100 items in this sorted order, we stop merging this pair and move onto the next pair.

To get the ranking for all products, we can be much lazier and only run this work once a day.

One of the advantages of this is that it scales nicely. We can easily divide up the files across multiple servers, as they aren't dependent on each other.

Follow Up Questions

The interviewer could push this design in any number of directions.

- Where do you think you'd hit the next bottlenecks? What would you do about that?
- What if there were subcategories as well? So items could be listed under "Sports" and "Sports Equipment" (or even "Sports" > "Sports Equipment" > "Tennis" > "Rackets")?
- What if data needed to be more accurate? What if it needed to be accurate within 30 minutes for all products?

Think through your design carefully and analyze it for the tradeoffs. You might also be asked to go into more detail on any specific aspect of the product.

9.7 **Personal Financial Manager**: Explain how you would design a personal financial manager (like Mint.com). This system would connect to your bank accounts, analyze your spending habits, and make recommendations.

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SOLUTION

The first thing we need to do is define what it is exactly that we are building.

Step 1: Scope the Problem

Ordinarily, you would clarify this system with your interviewer. We'll scope the problem as follows:

- You create an account and add your bank accounts. You can add multiple bank accounts. You can also add them at a later point in time.
- It pulls in all your financial history, or as much of it as your bank will allow.
- This financial history includes outgoing money (things you bought or paid for), incoming money (salary and other payments), and your current money (what's in your bank account and investments).
- Each payment transaction has a "category" associated with it (food, travel, clothing, etc.).
- There is some sort of data source provided that tells the system, with some reliability, which category a transaction is associated with. The user might, in some cases, override the category when it's improperly assigned (e.g., eating at the cafe of a department store getting assigned to "clothing" rather than "food").
- Users will use the system to get recommendations on their spending. These recommendations will come from a mix of "typical" users ("people generally shouldn't spend more than X% of their income on clothing"), but can be overridden with custom budgets. This will not be a primary focus right now.
- We assume this is just a website for now, although we could potentially talk about a mobile app as well.
- We probably want email notifications either on a regular basis, or on certain conditions (spending over a certain threshold, hitting a budget max, etc.).
- We'll assume that there's no concept of user-specified rules for assigning categories to transactions.

This gives us a basic goal for what we want to build.

Step 2: Make Reasonable Assumptions

Now that we have the basic goal for the system, we should define some further assumptions about the characteristics of the system.

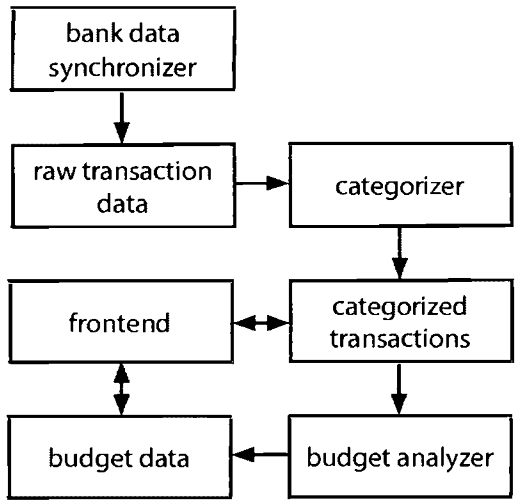
- Adding or removing bank accounts is relatively unusual.
- The system is write-heavy. A typical user may make several new transactions daily, although few users would access the website more than once a week. In fact, for many users, their primary interaction might be through email alerts.
- Once a transaction is assigned to a category, it will only be changed if the user asks to change it. The system will never reassign a transaction to a different category "behind the scenes", even if the rules change. This means that two otherwise identical transactions could be assigned to different categories if the rules changed in between each transaction's date. We do this because it may confuse users if their spending per category changes with no action on their part.
- The banks probably won't push data to our system. Instead, we will need to pull data from the banks.
- Alerts on users exceeding budgets probably do not need to be sent instantaneously. (That wouldn't be realistic anyway, since we won't get the transaction data instantaneously.) It's probably pretty safe for them to be up to 24 hours delayed.

It's okay to make different assumptions here, but you should explicitly state them to your interviewer.

Step 3: Draw the Major Components

The most naive system would be one that pulls bank data on each login, categorizes all the data, and then analyzes the user's budget. This wouldn't quite fit the requirements, though, as we want email notifications on particular events.

We can do a bit better.



With this basic architecture, the bank data is pulled at periodic times (hourly or daily). The frequency may depend on the behavior of the users. Less active users may have their accounts checked less frequently.

Once new data arrives, it is stored in some list of raw, unprocessed transactions. This data is then pushed to the categorizer, which assigns each transaction to a category and stores these categorized transactions in another datastore.

The budget analyzer pulls in the categorized transactions, updates each user's budget per category, and stores the user's budget.

The frontend pulls data from both the categorized transactions datastore as well as from the budget datastore. Additionally, a user could also interact with the frontend by changing the budget or the categorization of their transactions.

Step 4: Identify the Key Issues

We should now reflect on what the major issues here might be.

This will be a very data-heavy system. We want it to feel snappy and responsive, though, so we'll want as much processing as possible to be asynchronous.

We will almost certainly want at least one task queue, where we can queue up work that needs to be done. This work will include tasks such as pulling in new bank data, re-analyzing budgets, and categorizing new bank data. It would also include re-trying tasks that failed.

These tasks will likely have some sort of priority associated with them, as some need to be performed more often than others. We want to build a task queue system that can prioritize some task types over others, while still ensuring that all tasks will be performed eventually. That is, we wouldn't want a low priority task to essentially "starve" because there are always higher priority tasks.

One important part of the system that we haven't yet addressed will be the email system. We could use a task to regularly crawl user's data to check if they're exceeding their budget, but that means checking every single user daily. Instead, we'll want to queue a task whenever a transaction occurs that potentially exceeds a budget. We can store the current budget totals by category to make it easy to understand if a new transaction exceeds the budget.

We should also consider incorporating the knowledge (or assumption) that a system like this will probably have a large number of inactive users—users who signed up once and then haven't touched the system since. We may want to either remove them from the system entirely or deprioritize their accounts. We'll want some system to track their account activity and associate priority with their accounts.

The biggest bottleneck in our system will likely be the massive amount of data that needs to be pulled and analyzed. We should be able to fetch the bank data asynchronously and run these tasks across many servers. We should drill a bit deeper into how the categorizer and budget analyzer work.

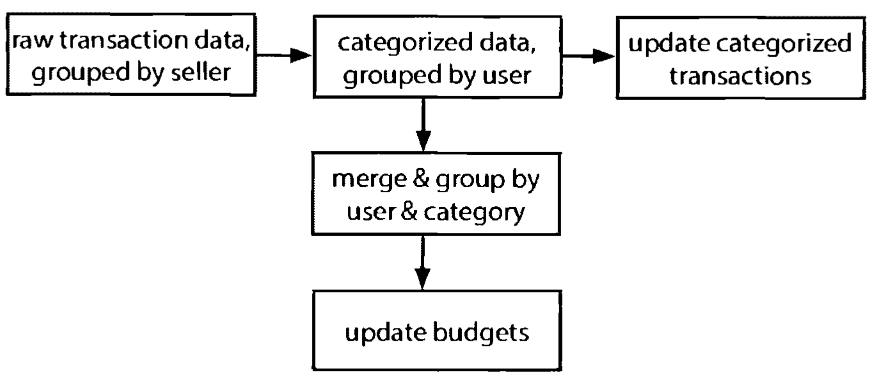
Categorizer and Budget Analyzer

One thing to note is that transactions are not dependent on each other. As soon as we get a transaction for a user, we can categorize it and integrate this data. It might be inefficient to do so, but it won't cause any inaccuracies.

Should we use a standard database for this? With lots of transactions coming in at once, that might not be very efficient. We certainly don't want to do a bunch of joins.

It may be better instead to just store the transactions to a set of flat text files. We assumed earlier that the categorizations are based on the seller's name alone. If we're assuming a lot of users, then there will be a lot of duplicates across the sellers. If we group the transaction files by seller's name, we can take advantage of these duplicates.

The categorizer can do something like this:



It first gets the raw transaction data, grouped by seller. It picks the appropriate category for the seller (which might be stored in a cache for the most common sellers), and then applies that category to all those transactions.

After applying the category, it re-groups all the transactions by user. Then, those transactions are inserted into the datastore for this user.

<table>
<tr><th>before categorizer</th><th>after categorizer</th></tr>
<tr><td><pre>amazon/
   user121,$5.43,Aug 13
   user922,$15.39,Aug 27
   ...
comcast/
   user922,$9.29,Aug 24
   user248,$40.13,Aug 18
   ...</pre></td><td><pre>user121/
   amazon,shopping,$5.43,Aug 13
   ...
user922/
   amazon,shopping,$15.39,Aug 27
   comcast,utilities,$9.29,Aug 24
   ...
user248/
   comcast,utilities,$40.13,Aug 18
   ...</pre></td></tr>
</table>

Then, the budget analyzer comes in. It takes the data grouped by user, merges it across categories (so all Shopping tasks for this user in this timespan are merged), and then updates the budget.

Most of these tasks will be handled in simple log files. Only the final data (the categorized transactions and the budget analysis) will be stored in a database. This minimizes writing and reading from the database.

User Changing Categories

The user might selectively override particular transactions to assign them to a different category. In this case, we would update the datastore for the categorized transactions. It would also signal a quick recomputation of the budget to decrement the item from the old category and increment the item in the other category.

We could also just recompute the budget from scratch. The budget analyzer is fairly quick as it just needs to look over the past few weeks of transactions for a single user.

Follow Up Questions

- How would this change if you also needed to support a mobile app?
- How would you design the component which assigns items to each category?
- How would you design the recommended budgets feature?
- How would you change this if the user could develop rules to categorize all transactions from a particular seller differently than the default?

9.8 **Pastebin:** Design a system like Pastebin, where a user can enter a piece of text and get a randomly generated URL for public access.

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SOLUTION

We can start with clarifying the specifics of this system.

Step 1: Scope the Problem

- The system does not support user accounts or editing documents.
- The system tracks analytics of how many times each page is accessed.
- Old documents get deleted after not being accessed for a sufficiently long period of time.
- While there isn't true authentication on accessing documents, users should not be able to "guess" docu-

ment URLs easily.

- The system has a frontend as well as an API.
- The analytics for each URL can be accessed through a "stats" link on each page. It is not shown by default, though.

Step 2: Make Reasonable Assumptions

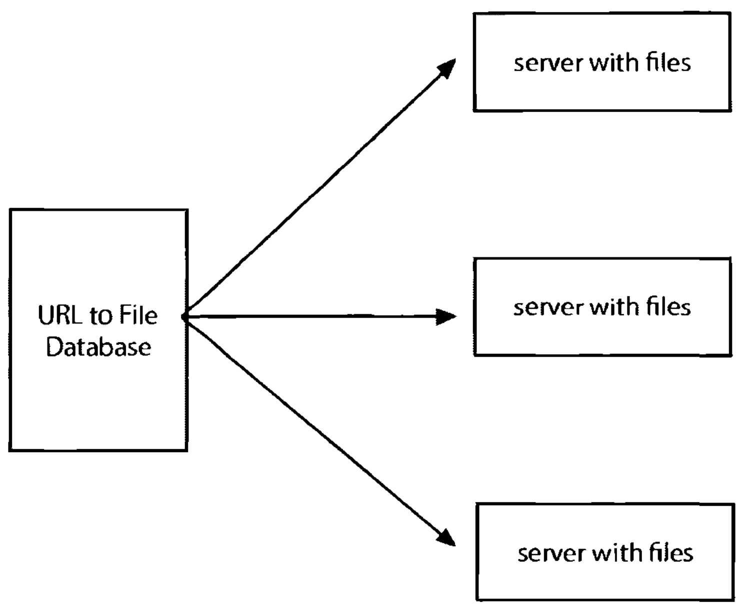
- The system gets heavy traffic and contains many millions of documents.
- Traffic is not equally distributed across documents. Some documents get much more access than others.

Step 3: Draw the Major Components

We can sketch out a simple design. We'll need to keep track of URLs and the files associated with them, as well as analytics for how often the files have been accessed.

How should we store the documents? We have two options: we can store them in a database or we can store them on a file. Since the documents can be large and it's unlikely we need searching capabilities, storing them on a file is probably the better choice.

A simple design like this might work well:



Here, we have a simple database that looks up the location (server and path) of each file. When we have a request for a URL, we look up the location of the URL within the datastore and then access the file.

Additionally, we will need a database that tracks analytics. We can do this with a simple datastore that adds each visit (including timestamp, IP address, and location) as a row in a database. When we need to access the stats of each visit, we pull the relevant data in from this database.

Step 4: Identify the Key Issues

The first issue that comes to mind is that some documents will be accessed much more frequently than others. Reading data from the filesystem is relatively slow compared with reading from data in memory. Therefore, we probably want to use a cache to store the most recently accessed documents. This will ensure that items accessed very frequently (or very recently) will be quickly accessible. Since documents cannot be edited, we will not need to worry about invalidating this cache.

We should also potentially consider sharding the database. We can shard it using some mapping from the URL (for example, the URL's hash code modulo some integer), which will allow us to quickly locate the database which contains this file.

In fact, we could even take this a step further. We could skip the database entirely and just let a hash of the URL indicate which server contains the document. The URL itself could reflect the location of the document. One potential issue from this is that if we need to add servers, it could be difficult to redistribute the documents.

Generating URLs

We have not yet discussed how to actually generate the URLs. We probably do not want a monotonically increasing integer value, as this would be easy for a user to "guess." We want URLs to be difficult to access without being provided the link.

One simple path is to generate a random GUID (e.g., 5d50e8ac-57cb-4a0d-8661-bcdee2548979). This is a 128-bit value that, while not strictly guaranteed to be unique, has low enough odds of a collision that we can treat it as unique. The drawback of this plan is that such a URL is not very "pretty" to the user. We could hash it to a smaller value, but then that increases the odds of collision.

We could do something very similar, though. We could just generate a 10-character sequence of letters and numbers, which gives us 36^{10} possible strings. Even with a billion URLs, the odds of a collision on any specific URL are very low.

> This is not to say that the odds of a collision over the whole system are low. They are not. Any one specific URL is unlikely to collide. However, after storing a billion URLs, we are very likely to have a collision at some point.

Assuming that we aren't okay with periodic (even if unusual) data loss, we'll need to handle these collisions. We can either check the datastore to see if the URL exists yet or, if the URL maps to a specific server, just detect whether a file already exists at the destination.

When a collision occurs, we can just generate a new URL. With 36^{10} possible URLs, collisions would be rare enough that the lazy approach here (detect collisions and retry) is sufficient.

Analytics

The final component to discuss is the analytics piece. We probably want to display the number of visits, and possibly break this down by location or time.

We have two options here:

- Store the raw data from each visit.
- Store just the data we know we'll use (number of visits, etc.).

You can discuss this with your interviewer, but it probably makes sense to store the raw data. We never know what features we'll add to the analytics down the road. The raw data allows us flexibility.

This does not mean that the raw data needs to be easily searchable or even accessible. We can just store a log of each visit in a file, and back this up to other servers.

One issue here is that this amount of data could be substantial. We could potentially reduce the space usage considerably by storing data only probabilistically. Each URL would have a `storage_probability` associated with it. As the popularity of a site goes up, the `storage_probability` goes down. For example, a popular document might have data logged only one out of every ten times, at random. When we look up the number of visits for the site, we'll need to adjust the value based on the probability (for example, by multiplying it by 10). This will of course lead to a small inaccuracy, but that may be acceptable.

The log files are not designed to be used frequently. We will want to also store this precomputed data in a datastore. If the analytics just displays the number of visits plus a graph over time, this could be kept in a separate database.

URL	Month and Year	Visits
12ab31b92p	December 2013	242119
12ab31b92p	January 2014	429918
...	...	...

Every time a URL is visited, we can increment the appropriate row and column. This datastore can also be sharded by the URL.

As the stats are not listed on the regular pages and would generally be of less interest, it should not face as heavy of a load. We could still cache the generated HTML on the frontend servers, so that we don't continuously reaccess the data for the most popular URLs.

Follow-Up Questions

- How would you support user accounts?
- How would you add a new piece of analytics (e.g., referral source) to the stats page?
- How would your design change if the stats were shown with each document?

10

Solutions to Sorting and Searching

10.1 Sorted Merge: You are given two sorted arrays, A and B, where A has a large enough buffer at the end to hold B. Write a method to merge B into A in sorted order.

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SOLUTION

Since we know that A has enough buffer at the end, we won't need to allocate additional space. Our logic should involve simply comparing elements of A and B and inserting them in order, until we've exhausted all elements in A and in B.

The only issue with this is that if we insert an element into the front of A, then we'll have to shift the existing elements backwards to make room for it. It's better to insert elements into the back of the array, where there's empty space.

The code below does just that. It works from the back of A and B, moving the largest elements to the back of A.

```
void merge(int[] a, int[] b, int lastA, int lastB) {
   int indexA = lastA - 1; /* Index of last element in array a */
   int indexB = lastB - 1; /* Index of last element in array b */
   int indexMerged = lastB + lastA - 1; /* end of merged array */

   /* Merge a and b, starting from the last element in each */
   while (indexB >= 0) {
      /* end of a is > than end of b */
      if (indexA >= 0 && a[indexA] > b[indexB]) {
         a[indexMerged] = a[indexA]; // copy element
         indexA--;
      } else {
         a[indexMerged] = b[indexB]; // copy element
         indexB--;
      }
      indexMerged--; // move indices
   }
}
```

Note that you don't need to copy the contents of A after running out of elements in B. They are already in place.

10.2 Group Anagrams: Write a method to sort an array of strings so that all the anagrams are next to each other.

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SOLUTION

This problem asks us to group the strings in an array such that the anagrams appear next to each other. Note that no specific ordering of the words is required, other than this.

We need a quick and easy way of determining if two strings are anagrams of each other. What defines if two words are anagrams of each other? Well, anagrams are words that have the same characters but in different orders. It follows then that if we can put the characters in the same order, we can easily check if the new words are identical.

One way to do this is to just apply any standard sorting algorithm, like merge sort or quick sort, and modify the comparator. This comparator will be used to indicate that two strings which are anagrams of each other are equivalent.

What's the easiest way of checking if two words are anagrams? We could count the occurrences of the distinct characters in each string and return `true` if they match. Or, we could just sort the string. After all, two words which are anagrams will look the same once they're sorted.

The code below implements the comparator.

```
class AnagramComparator implements Comparator<String> {
  public String sortChars(String s) {
    char[] content = s.toCharArray();
    Arrays.sort(content);
    return new String(content);
  }

  public int compare(String s1, String s2) {
    return sortChars(s1).compareTo(sortChars(s2));
  }
}
```

Now, just sort the arrays using this `compareTo` method instead of the usual one.

```
Arrays.sort(array, new AnagramComparator());
```

This algorithm will take `O(n log(n))` time.

This may be the best we can do for a general sorting algorithm, but we don't actually need to fully sort the array. We only need to *group* the strings in the array by anagram.

We can do this by using a hash table which maps from the sorted version of a word to a list of its anagrams. So, for example, `acre` will map to the list `{acre, race, care}`. Once we've grouped all the words into these lists by anagram, we can then put them back into the array.

The code below implements this algorithm.

```
void sort(String[] array) {
  HashMapList<String, String> mapList = new HashMapList<String, String>();

  /* Group words by anagram */
  for (String s : array) {
    String key = sortChars(s);
    mapList.put(key, s);
  }
```

```

    / *Convert hash table to array */
    int index = 0;
    for (String key : mapList.keySet()) {
      ArrayList<String> list = mapList.get(key);
      for (String t : list) {
        array[index] = t;
        index++;
      }
    }
}

String sortChars(String s) {
  char[] content = s.toCharArray();
  Arrays.sort(content);
  return new String(content);
}

/ *HashMapList<String, Integer> is a HashMap that maps from Strings to
 * ArrayList<Integer>. See appendix for implementation. */
```

You may notice that the algorithm above is a modification of bucket sort.

10.3 Search in Rotated Array: Given a sorted array of n integers that has been rotated an unknown number of times, write code to find an element in the array. You may assume that the array was originally sorted in increasing order.

EXAMPLE

Input: find 5 in `{15, 16, 19, 20, 25, 1, 3, 4, 5, 7, 10, 14}`

Output: 8 (the index of 5 in the array)

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SOLUTION

If this problem smells like binary search to you, you're right!

In classic binary search, we compare x with the midpoint to figure out if x belongs on the left or the right side. The complication here is that the array is rotated and may have an inflection point. Consider, for example, the following two arrays:

```
Array1: {10, 15, 20,  0,  5}
Array2: {50,  5, 20, 30, 40}
```

Note that both arrays have a midpoint of 20, but 5 appears on the left side of one and on the right side of the other. Therefore, comparing x with the midpoint is insufficient.

However, if we look a bit deeper, we can see that one half of the array must be ordered normally (in increasing order). We can therefore look at the normally ordered half to determine whether we should search the left or right half.

For example, if we are searching for 5 in `Array1`, we can look at the left element (10) and middle element (20). Since 10 < 20, the left half must be ordered normally. And, since 5 is not between those, we know that we must search the right half.

In `Array2`, we can see that since 50 > 20, the right half must be ordered normally. We turn to the middle (20) and right (40) element to check if 5 would fall between them. The value 5 would not; therefore, we search the left half.

The tricky condition is if the left and the middle are identical, as in the example array `{2, 2, 2, 3, 4, 2}`. In this case, we can check if the rightmost element is different. If it is, we can search just the right side. Otherwise, we have no choice but to search both halves.

```
int search(int a[], int left, int right, int x) {
  int mid = (left + right) / 2;
  if (x == a[mid]) { // Found element
    return mid;
  }
  if (right < left) {
    return -1;
  }

  /* Either the left or right half must be normally ordered. Find out which side
   * is normally ordered, and then use the normally ordered half to figure out
   * which side to search to find x. */
  if (a[left] < a[mid]) { // Left is normally ordered.
    if (x >= a[left] && x < a[mid]) {
      return search(a, left, mid - 1, x); // Search left
    } else {
      return search(a, mid + 1, right, x); // Search right
    }
  } else if (a[mid] < a[left]) { // Right is normally ordered.
    if (x > a[mid] && x <= a[right]) {
      return search(a, mid + 1, right, x); // Search right
    } else {
      return search(a, left, mid - 1, x); // Search left
    }
  } else if (a[left] == a[mid]) { // Left or right half is all repeats
    if (a[mid] != a[right]) { // If right is different, search it
      return search(a, mid + 1, right, x); // search right
    } else { // Else, we have to search both halves
      int result = search(a, left, mid - 1, x); // Search left
      if (result == -1) {
        return search(a, mid + 1, right, x); // Search right
      } else {
        return result;
      }
    }
  }
  return -1;
}
```

This code will run in `O(log n)` if all the elements are unique. However, with many duplicates, the algorithm is actually `O(n)`. This is because with many duplicates, we will often have to search both the left and right sides of the array (or subarrays).

Note that while this problem is not conceptually very complex, it is actually very difficult to implement flawlessly. Don't feel bad if you had trouble implementing it without a few bugs. Because of the ease of making off-by-one and other minor errors, you should make sure to test your code very thoroughly.

10.4 **Sorted Search, No Size:** You are given an array-like data structure `Listy` which lacks a size method. It does, however, have an `elementAt(i)` method that returns the element at index `i` in `O(1)` time. If `i` is beyond the bounds of the data structure, it returns `-1`. (For this reason, the data structure only supports positive integers.) Given a `Listy` which contains sorted, positive integers, find the index at which an element x occurs. If x occurs multiple times, you may return any index.

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SOLUTION

Our first thought here should be binary search. The problem is that binary search requires us knowing the length of the list, so that we can compare it to the midpoint. We don't have that here.

Could we compute the length? Yes!

We know that `elementAt` will return `-1` when `i` is too large. We can therefore just try bigger and bigger values until we exceed the size of the list.

But how much bigger? If we just went through the list linearly—1, then 2, then 3, then 4, and so on—we'd wind up with a linear time algorithm. We probably want something faster than this. Otherwise, why would the interviewer have specified the list is sorted?

It's better to back off exponentially. Try 1, then 2, then 4, then 8, then 16, and so on. This ensures that, if the list has length n, we'll find the length in at most `O(log n)` time.

> Why `O(log n)`? Imagine we start with pointer q at q = `1`. At each iteration, this pointer q doubles, until q is bigger than the length n. How many times can q double in size before it's bigger than n? Or, in other words, for what value of k does $2^k = n$? This expression is equal when k = `log n`, as this is precisely what `log` means. Therefore, it will take `O(log n)` steps to find the length.

Once we find the length, we just perform a (mostly) normal binary search. I say "mostly" because we need to make one small tweak. If the mid point is -1, we need to treat this as a "too big" value and search left. This is on line 16 below.

There's one more little tweak. Recall that the way we figure out the length is by calling `elementAt` and comparing it to -1. If, in the process, the element is bigger than the value x (the one we're searching for), we'll jump over to the binary search part early.

```
1  int search(Listy list, int value) {
2    int index = 1;
3    while (list.elementAt(index) != -1 && list.elementAt(index) < value) {
4      index *= 2;
5    }
6    return binarySearch(list, value, index / 2, index);
7  }
8
9  int binarySearch(Listy list, int value, int low, int high) {
10   int mid;
11
12   while (low <= high) {
13     mid = (low + high) / 2;
14     int middle = list.elementAt(mid);
15     if (middle > value || middle == -1) {
16       high = mid - 1;
17     } else if (middle < value) {
```

```
            low = mid + 1;
        } else {
            return mid;
        }
    }
    return -1;
}
```

It turns out that not knowing the length didn't impact the runtime of the search algorithm. We find the length in `O(log n)` time and then do the search in `O(log n)` time. Our overall runtime is `O(log n)`, just as it would be in a normal array.

10.5 **Sparse Search:** Given a sorted array of strings that is interspersed with empty strings, write a method to find the location of a given string.

EXAMPLE

Input: ball, {“at”, “”, “”, “”, “ball”, “”, “”, “car”, “”, “”, “dad”, “”, “”}

Output: 4

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SOLUTION

If it weren't for the empty strings, we could simply use binary search. We would compare the string to be found, `str`, with the midpoint of the array, and go from there.

With empty strings interspersed, we can implement a simple modification of binary search. All we need to do is fix the comparison against `mid`, in case `mid` is an empty string. We simply move `mid` to the closest non-empty string.

The recursive code below to solve this problem can easily be modified to be iterative. We provide such an implementation in the code attachment.

```
int search(String[] strings, String str, int first, int last) {
    if (first > last) return -1;
    /* Move mid to the middle * /
    int mid = (last + first) / 2;

    /* If mid is empty, find closest non-empty string. * /
    if (strings[mid].isEmpty()) {
        int left = mid - 1;
        int right = mid + 1;
        while (true) {
            if (left < first && right > last) {
                return -1;
            } else if (right <= last && !strings[right].isEmpty()) {
                mid = right;
                break;
            } else if (left >= first && !strings[left].isEmpty()) {
                mid = left;
                break;
            }
            right++;
            left--;
        }
    }
```

```

  /* Check for string, and recurse if necessary */
  if (str.equals(strings[mid])) { // Found it!
    return mid;
  } else if (strings[mid].compareTo(str) < 0) { // Search right
    return search(strings, str, mid + 1, last);
  } else { // Search left
    return search(strings, str, first, mid - 1);
  }
}

int search(String[] strings, String str) {
  if (strings == null || str == null || str == “”) {
    return -1;
  }
  return search(strings, str, 0, strings.length - 1);
}
```

The worst-case runtime for this algorithm is O(n). In fact, it's impossible to have an algorithm for this problem that is better than O(n) in the worst case. After all, you could have an array of all empty strings except for one non-empty string. There is no "smart" way to find this non-empty string. In the worst case, you will need to look at every element in the array.

Careful consideration should be given to the situation when someone searches for the empty string. Should we find the location (which is an O(n) operation)? Or should we handle this as an error?

There's no correct answer here. This is an issue you should raise with your interviewer. Simply asking this question will demonstrate that you are a careful coder.

10.6 Sort Big File: Imagine you have a 20 GB file with one string per line. Explain how you would sort the file.

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SOLUTION

When an interviewer gives a size limit of 20 gigabytes, it should tell you something. In this case, it suggests that they don't want you to bring all the data into memory.

So what do we do? We only bring part of the data into memory.

We'll divide the file into chunks, which are x megabytes each, where x is the amount of memory we have available. Each chunk is sorted separately and then saved back to the file system.

Once all the chunks are sorted, we merge the chunks, one by one. At the end, we have a fully sorted file.

This algorithm is known as external sort.

10.7 **Missing Int:** Given an input file with four billion non-negative integers, provide an algorithm to generate an integer that is not contained in the file. Assume you have 1 GB of memory available for this task.

FOLLOW UP

What if you have only 10 MB of memory? Assume that all the values are distinct and we now have no more than one billion non-negative integers.

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SOLUTION

There are a total of 2^{32}, or 4 billion, distinct integers possible and 2^{31} non-negative integers. Therefore, we know the input file (assuming it is `int`s rather than `long`s) contains some duplicates.

We have 1 GB of memory, or 8 billion bits. Thus, with 8 billion bits, we can map all possible integers to a distinct bit with the available memory. The logic is as follows:

1. Create a bit vector (BV) with 4 billion bits. Recall that a bit vector is an array that compactly stores boolean values by using an array of ints (or another data type). Each int represents 32 boolean values.
2. Initialize BV with all 0s.
3. Scan all numbers (`num`) from the file and call `BV.set(num, 1)`.
4. Now scan again BV from the 0th index.
5. Return the first index which has a value of 0.

The following code demonstrates our algorithm.

```
long numberOfInts = ((long) Integer.MAX_VALUE) + 1;
byte[] bitfield = new byte [(int) (numberOfInts / 8)];
String filename = ...

void findOpenNumber() throws FileNotFoundException {
  Scanner in = new Scanner(new FileReader(filename));
  while (in.hasNextInt()) {
    int n = in.nextInt ();
    /* Finds the corresponding number in the bitfield by using the OR operator to
     * set the nth bit of a byte (e.g., 10 would correspond to the 2nd bit of
     * index 2 in the byte array). */
    bitfield [n / 8] |= 1 << (n % 8);
  }

  for (int i = 0; i < bitfield.length; i++) {
    for (int j = 0; j < 8; j++) {
      /* Retrieves the individual bits of each byte. When 0 bit is found, print
       * the corresponding value. */
      if ((bitfield[i] & (1 << j)) == 0) {
        System.out.println (i * 8 + j);
        return;
      }
    }
  }
}
```

Follow Up: What if we have only 10 MB memory?

It's possible to find a missing integer with two passes of the data set. We can divide up the integers into blocks of some size (we'll discuss how to decide on a size later). Let's just assume that we divide up the integers into blocks of 1000. So, block 0 represents the numbers 0 through 999, block 1 represents numbers 1000 - 1999, and so on.

Since all the values are distinct, we know how many values we *should* find in each block. So, we search through the file and count how many values are between 0 and 999, how many are between 1000 and 1999, and so on. If we count only 999 values in a particular range, then we know that a missing int must be in that range.

In the second pass, we'll actually look for which number in that range is missing. We use the bit vector approach from the first part of this problem. We can ignore any number outside of this specific range.

The question, now, is what is the appropriate block size? Let's define some variables as follows:

- Let `rangeSize` be the size of the ranges that each block in the first pass represents.
- Let `arraySize` represent the number of blocks in the first pass. Note that `arraySize` = $2^{31}/_{\texttt{rangeSize}}$ since there are 2^{31} non-negative integers.

We need to select a value for `rangeSize` such that the memory from the first pass (the array) and the second pass (the bit vector) fit.

First Pass: The Array

The array in the first pass can fit in 10 megabytes, or roughly 2^{23} bytes, of memory. Since each element in the array is an int, and an int is 4 bytes, we can hold an array of at most about 2^{21} elements. So, we can deduce the following:

$$\texttt{arraySize} = \frac{2^{31}}{\texttt{rangeSize}} \leq 2^{21}$$

$$\texttt{rangeSize} \geq \frac{2^{31}}{2^{21}}$$

$$\texttt{rangeSize} \geq 2^{10}$$

Second Pass: The Bit Vector

We need to have enough space to store `rangeSize` bits. Since we can fit 2^{23} bytes in memory, we can fit 2^{26} bits in memory. Therefore, we can conclude the following:

$$2^{11} \texttt{ <= rangeSize <= } 2^{26}$$

These conditions give us a good amount of "wiggle room," but the nearer to the middle that we pick, the less memory will be used at any given time.

The below code provides one implementation for this algorithm.

```
int findOpenNumber(String filename) throws FileNotFoundException {
  int rangeSize = (1 << 20); // 2^20 bits (2^17 bytes)

  /* Get count of number of values within each block. */
  int[] blocks = getCountPerBlock(filename, rangeSize);

  /* Find a block with a missing value. */
  int blockIndex = findBlockWithMissing(blocks, rangeSize);
  if (blockIndex < 0) return -1;
```

```

  /* Create bit vector for items within this range. */
  byte[] bitVector = getBitVectorForRange(filename, blockIndex, rangeSize);

  /* Find a zero in the bit vector */
  int offset = findZero(bitVector);
  if (offset < 0) return -1;

  /* Compute missing value. */
  return blockIndex * rangeSize + offset;
}

/* Get count of items within each range. */
int[] getCountPerBlock(String filename, int rangeSize)
    throws FileNotFoundException {
  int arraySize = Integer.MAX_VALUE / rangeSize + 1;
  int[] blocks = new int[arraySize];

  Scanner in = new Scanner (new FileReader(filename));
  while (in.hasNextInt()) {
      int value = in.nextInt();
      blocks[value / rangeSize]++;
  }
  in.close();
  return blocks;
}

/* Find a block whose count is low. */
int findBlockWithMissing(int[] blocks, int rangeSize) {
  for (int i = 0; i < blocks.length; i++) {
    if (blocks[i] < rangeSize){
      return i;
    }
  }
  return -1;
}

/* Create a bit vector for the values within a specific range. */
byte[] getBitVectorForRange(String filename, int blockIndex, int rangeSize)
    throws FileNotFoundException {
  int startRange = blockIndex * rangeSize;
  int endRange = startRange + rangeSize;
  byte[] bitVector = new byte[rangeSize/Byte.SIZE];

  Scanner in = new Scanner(new FileReader(filename));
  while (in.hasNextInt()) {
    int value = in.nextInt();
    /* If the number is inside the block that's missing numbers, we record it */
    if (startRange <= value && value < endRange) {
      int offset = value - startRange;
      int mask = (1 << (offset % Byte.SIZE));
      bitVector[offset / Byte.SIZE] |= mask;
    }
  }
  in.close();
  return bitVector;
```

```
}

/* Find bit index that is 0 within byte. */
int findZero(byte b) {
   for (int i = 0; i < Byte.SIZE; i++) {
      int mask = 1 << i;
      if ((b & mask) == 0) {
         return i;
      }
   }
   return -1;
}

/* Find a zero within the bit vector and return the index. */
int findZero(byte[] bitVector) {
   for (int i = 0; i < bitVector.length; i++) {
      if (bitVector[i] != ~0) { // If not all 1s
         int bitIndex = findZero(bitVector[i]);
         return i * Byte.SIZE + bitIndex;
      }
   }
   return -1;
}
```

What if, as a follow up question, you are asked to solve the problem with even less memory? In this case, we can do repeated passes using the approach from the first step. We'd first check to see how many integers are found within each sequence of a million elements. Then, in the second pass, we'd check how many integers are found in each sequence of a thousand elements. Finally, in the third pass, we'd apply the bit vector.

10.8 Find Duplicates: You have an array with all the numbers from 1 to N, where N is at most 32,000. The array may have duplicate entries and you do not know what N is. With only 4 kilobytes of memory available, how would you print all duplicate elements in the array?

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SOLUTION

We have 4 kilobytes of memory which means we can address up to $8 * 4 * 2^{10}$ bits. Note that $32 * 2^{10}$ bits is greater than 32000. We can create a bit vector with 32000 bits, where each bit represents one integer.

Using this bit vector, we can then iterate through the array, flagging each element v by setting bit v to 1. When we come across a duplicate element, we print it.

```
void checkDuplicates(int[] array) {
   BitSet bs = new BitSet(32000);
   for (int i = 0; i < array.length; i++) {
      int num = array[i];
      int num0 = num - 1; // bitset starts at 0, numbers start at 1
      if (bs.get(num0)) {
         System.out.println(num);
      } else {
         bs.set(num0);
      }
   }
}

class BitSet {
```

```
    int[] bitset;

    public BitSet(int size) {
      bitset = new int[(size >> 5) + 1]; // divide by 32
    }

    boolean get(int pos) {
      int wordNumber = (pos >> 5); // divide by 32
      int bitNumber = (pos & 0x1F); // mod 32
      return (bitset[wordNumber] & (1 << bitNumber)) != 0;
    }

    void set(int pos) {
      int wordNumber = (pos >> 5); // divide by 32
      int bitNumber = (pos & 0x1F); // mod 32
      bitset[wordNumber] |= 1 << bitNumber;
    }
  }
```

Note that while this isn't an especially difficult problem, it's important to implement this cleanly. This is why we defined our own bit vector class to hold a large bit vector. If our interviewer lets us (she may or may not), we could have of course used Java's built in `BitSet` class.

10.9 Sorted Matrix Search: Given an M x N matrix in which each row and each column is sorted in ascending order, write a method to find an element.

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SOLUTION

We can approach this in two ways: a more naive solution that only takes advantage of part of the sorting, and a more optimal way that takes advantage of both parts of the sorting.

Solution #1: Naive Solution

As a first approach, we can do binary search on every row to find the element. This algorithm will be `O(M log(N))`, since there are M rows and it takes `O(log(N))` time to search each one. This is a good approach to mention to your interviewer before you proceed with generating a better algorithm.

To develop an algorithm, let's start with a simple example.

15	20	40	85
20	35	80	95
30	55	95	105
40	80	100	120

Suppose we are searching for the element 55. How can we identify where it is?

If we look at the start of a row or the start of a column, we can start to deduce the location. If the start of a column is greater than 55, we know that 55 can't be in that column, since the start of the column is always the minimum element. Additionally, we know that 55 can't be in any columns on the right, since the first element of each column must increase in size from left to right. Therefore, if the start of the column is greater than the element x that we are searching for, we know that we need to move further to the left.

For rows, we use identical logic. If the start of a row is bigger than x, we know we need to move upwards.

Observe that we can also make a similar conclusion by looking at the ends of columns or rows. If the end of a column or row is less than x, then we know that we must move down (for rows) or to the right (for columns) to find x. This is because the end is always the maximum element.

We can bring these observations together into a solution. The observations are the following:

- If the start of a column is greater than x, then x is to the left of the column.
- If the end of a column is less than x, then x is to the right of the column.
- If the start of a row is greater than x, then x is above that row.
- If the end of a row is less than x, then x is below that row.

We can begin in any number of places, but let's begin with looking at the starts of columns.

We need to start with the greatest column and work our way to the left. This means that our first element for comparison is `array[0][c-1]`, where c is the number of columns. By comparing the start of columns to x (which is 55), we'll find that x must be in columns 0, 1, or 2. We will have stopped at `array[0][2]`.

This element may not be the end of a row in the full matrix, but it is an end of a row of a submatrix. The same conditions apply. The value at `array[0][2]`, which is 40, is less than 55, so we know we can move downwards.

We now have a submatrix to consider that looks like the following (the gray squares have been eliminated).

15	20	40	85
20	35	80	95
30	55	95	105
40	80	100	120

We can repeatedly apply these conditions to search for 55. Note that the only conditions we actually use are conditions 1 and 4.

The code below implements this elimination algorithm.

```
boolean findElement(int[][] matrix, int elem) {
   int row = 0;
   int col = matrix[0].length - 1;
   while (row < matrix.length && col >= 0) {
      if (matrix[row][col] == elem) {
         return true;
      } else if (matrix[row][col] > elem) {
         col--;
      } else {
         row++;
      }
   }
   return false;
}
```

Alternatively, we can apply a solution that more directly looks like binary search. The code is considerably more complicated, but it applies many of the same learnings.

Solution #2: Binary Search

Let's again look at a simple example.

15	20	70	85
20	35	80	95
30	55	95	105
40	80	100	120

We want to be able to leverage the sorting property to more efficiently find an element. So, we might ask ourselves, what does the unique ordering property of this matrix imply about where an element might be located?

We are told that every row and column is sorted. This means that element `a[i][j]` will be greater than the elements in row `i` between columns 0 and `j - 1` and the elements in column `j` between rows 0 and `i - 1`.

Or, in other words:

```
a[i][0] <= a[i][1] <= ... <= a[i][j-1] <= a[i][j]
a[0][j] <= a[1][j] <= ... <= a[i-1][j] <= a[i][j]
```

Looking at this visually, the dark gray element below is bigger than all the light gray elements.

15	20	70	85
20	35	80	95
30	55	95	105
40	80	100	120

The light gray elements also have an ordering to them: each is bigger than the elements to the left of it, as well as the elements above it. So, by transitivity, the dark gray element is bigger than the entire square.

15	20	70	85
20	35	80	95
30	55	95	105
40	80	100	120

This means that for any rectangle we draw in the matrix, the bottom right hand corner will always be the biggest.

Likewise, the top left hand corner will always be the smallest. The colors below indicate what we know about the ordering of elements (light gray < dark gray < black):

15	20	70	85
20	35	80	95
30	55	95	105
40	80	120	120

Let's return to the original problem: suppose we were searching for the value 85. If we look along the diagonal, we'll find the elements 35 and 95. What does this tell us about the location of 85?

15	20	70	85
25	35	80	95
30	55	95	105
40	80	120	120

85 can't be in the black area, since 95 is in the upper left hand corner and is therefore the smallest element in that square.

85 can't be in the light gray area either, since 35 is in the lower right hand corner of that square.

85 must be in one of the two white areas.

So, we partition our grid into four quadrants and recursively search the lower left quadrant and the upper right quadrant. These, too, will get divided into quadrants and searched.

Observe that since the diagonal is sorted, we can efficiently search it using binary search.

The code below implements this algorithm.

```
Coordinate findElement(int[][] matrix, Coordinate origin, Coordinate dest, int x){
  if (!origin.inbounds(matrix) || !dest.inbounds(matrix)) {
    return null;
  }
  if (matrix[origin.row][origin.column] == x) {
    return origin;
  } else if (!origin.isBefore(dest)) {
    return null;
  }

  /* Set start to start of diagonal and end to the end of the diagonal. Since the
   * grid may not be square, the end of the diagonal may not equal dest. */
  Coordinate start = (Coordinate) origin.clone();
  int diagDist = Math.min(dest.row - origin.row, dest.column - origin.column);
  Coordinate end = new Coordinate(start.row + diagDist, start.column + diagDist);
  Coordinate p = new Coordinate(0, 0);

  /* Do binary search on the diagonal, looking for the first element > x */
  while (start.isBefore(end)) {
    p.setToAverage(start, end);
    if (x > matrix[p.row][p.column]) {
      start.row = p.row + 1;
      start.column = p.column + 1;
    } else {
      end.row = p.row - 1;
      end.column = p.column - 1;
    }
  }

  /* Split the grid into quadrants. Search the bottom left and the top right. */
  return partitionAndSearch(matrix, origin, dest, start, x);
}

Coordinate partitionAndSearch(int[][] matrix, Coordinate origin, Coordinate dest,
                              Coordinate pivot, int x) {
  Coordinate lowerLeftOrigin = new Coordinate(pivot.row, origin.column);
  Coordinate lowerLeftDest = new Coordinate(dest.row, pivot.column - 1);
  Coordinate upperRightOrigin = new Coordinate(origin.row, pivot.column);
  Coordinate upperRightDest = new Coordinate(pivot.row - 1, dest.column);

  Coordinate lowerLeft = findElement(matrix, lowerLeftOrigin, lowerLeftDest, x);
  if (lowerLeft == null) {
    return findElement(matrix, upperRightOrigin, upperRightDest, x);
  }
```

```
    return lowerLeft;
}

Coordinate findElement(int[][] matrix, int x) {
  Coordinate origin = new Coordinate(0, 0);
  Coordinate dest = new Coordinate(matrix.length - 1, matrix[0].length - 1);
  return findElement(matrix, origin, dest, x);
}

public class Coordinate implements Cloneable {
  public int row, column;
  public Coordinate(int r, int c) {
    row = r;
    column = c;
  }

  public boolean inbounds(int[][] matrix) {
    return  row >= 0 && column >= 0 &&
            row < matrix.length && column < matrix[0].length;
  }

  public boolean isBefore(Coordinate p) {
    return row <= p.row && column <= p.column;
  }

  public Object clone() {
    return new Coordinate(row, column);
  }

  public void setToAverage(Coordinate min, Coordinate max) {
    row = (min.row + max.row) / 2;
    column = (min.column + max.column) / 2;
  }
}
```

If you read all this code and thought, "there's no way I could do all this in an interview!" you're probably right. You couldn't. But, your performance on any problem is evaluated compared to other candidates on the same problem. So while you couldn't implement all this, neither could they. You are at no disadvantage when you get a tricky problem like this.

You help yourself out a bit by separating code out into other methods. For example, by pulling `partitionAndSearch` out into its own method, you will have an easier time outlining key aspects of the code. You can then come back to fill in the body for `partitionAndSearch` if you have time.

10.10 Rank from Stream: Imagine you are reading in a stream of integers. Periodically, you wish to be able to look up the rank of a number x (the number of values less than or equal to x). Implement the data structures and algorithms to support these operations. That is, implement the method `track(int x)`, which is called when each number is generated, and the method `getRankOfNumber(int x)`, which returns the number of values less than or equal to x (not including x itself).

EXAMPLE

Stream (in order of appearance): `5, 1, 4, 4, 5, 9, 7, 13, 3`

```
getRankOfNumber(1) = 0
getRankOfNumber(3) = 1
getRankOfNumber(4) = 3
```

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SOLUTION

A relatively easy way to implement this would be to have an array that holds all the elements in sorted order. When a new element comes in, we would need to shift the other elements to make room. Implementing `getRankOfNumber` would be quite efficient, though. We would simply perform a binary search for n, and return the index.

However, this is very inefficient for inserting elements (that is, the `track(int x)` function). We need a data structure which is good at keeping relative ordering, as well as updating when we insert new elements. A binary search tree can do just that.

Instead of inserting elements into an array, we insert elements into a binary search tree. The method `track(int x)` will run in `O(log n)` time, where n is the size of the tree (provided, of course, that the tree is balanced).

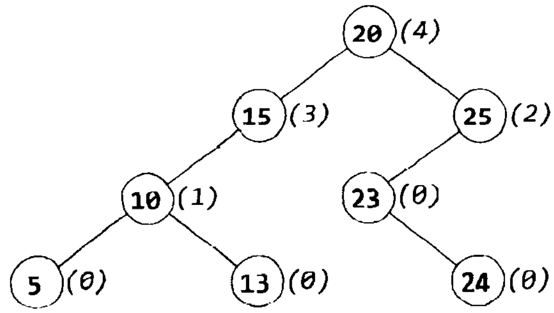
To find the rank of a number, we could do an in-order traversal, keeping a counter as we traverse. The goal is that, by the time we find x, `counter` will equal the number of elements less than x.

As long as we're moving left during searching for x, the counter won't change. Why? Because all the values we're skipping on the right side are greater than x. After all, the very smallest element (with rank of 1) is the leftmost node.

When we move to the right though, we skip over a bunch of elements on the left. All of these elements are less than x, so we'll need to increment `counter` by the number of elements in the left subtree.

Rather than counting the size of the left subtree (which would be inefficient), we can track this information as we add new elements to the tree.

Let's walk through an example on the following tree. In the below example, the value in parentheses indicates the number of nodes in the left subtree (or, in other words, the rank of the node *relative* to its subtree).



Suppose we want to find the rank of 24 in the tree above. We would compare 24 with the root, 20, and find that 24 must reside on the right. The root has 4 nodes in its left subtree, and when we include the root itself, this gives us five total nodes smaller than 24. We set `counter` to 5.

Then, we compare 24 with node 25 and find that 24 must be on the left. The value of `counter` does not update, since we're not "passing over" any smaller nodes. The value of `counter` is still 5.

Next, we compare 24 with node 23, and find that 24 must be on the right. `Counter` gets incremented by just 1 (to 6), since 23 has no left nodes.

Finally, we find 24 and we return `counter`: 6.

Recursively, the algorithm is the following:

```
int getRank(Node node, int x) {
  if x is node.data, return node.leftSize()
  if x is on left of node, return getRank(node.left, x)
  if x is on right of node, return node.leftSize() + 1 + getRank(node.right, x)
}
```

The full code for this is below.

```
RankNode root = null;

void track(int number) {
  if (root == null) {
    root = new RankNode(number);
  } else {
    root.insert(number);
  }
}

int getRankOfNumber(int number) {
  return root.getRank(number);
}


public class RankNode {
  public int left_size = 0;
  public RankNode left, right;
  public int data = 0;
  public RankNode(int d) {
    data = d;
  }

  public void insert(int d) {
    if (d <= data) {
```

```
            if (left != null) left.insert(d);
            else left = new RankNode(d);
            left_size++;
        } else {
            if (right != null) right.insert(d);
            else right = new RankNode(d);
        }
    }

    public int getRank(int d) {
        if (d == data) {
            return left_size;
        } else if (d < data) {
            if (left == null) return -1;
            else return left.getRank(d);
        } else {
            int right_rank = right == null ? -1 : right.getRank(d);
            if (right_rank == -1) return -1;
            else return left_size + 1 + right_rank;
        }
    }
}
```

The `track` method and the `getRankOfNumber` method will both operate in `O(log N)` on a balanced tree and `O(N)` on an unbalanced tree.

Note how we've handled the case in which d is not found in the tree. We check for the -1 return value, and, when we find it, return -1 up the tree. It is important that you handle cases like this.

10.11 Peaks and Valleys: In an array of integers, a "peak" is an element which is greater than or equal to the adjacent integers and a "valley" is an element which is less than or equal to the adjacent integers. For example, in the array {5, 8, 6, 2, 3, 4, 6}, {8, 6} are peaks and {5, 2} are valleys. Given an array of integers, sort the array into an alternating sequence of peaks and valleys.

EXAMPLE

Input: {5, 3, 1, 2, 3}

Output: {5, 1, 3, 2, 3}

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SOLUTION

Since this problem asks us to sort the array in a particular way, one thing we can try is doing a normal sort and then "fixing" the array into an alternating sequence of peaks and valleys.

Suboptimal Solution

Imagine we were given an unsorted array and then sort it to become the following:

```
0   1   4   7   8   9
```

We now have an ascending list of integers.

How can we rearrange this into a proper alternating sequence of peaks and valleys? Let's walk through it and try to do that.

- The `0` is okay.

- The 1 is in the wrong place. We can swap it with either the 0 or 4. Let's swap it with the 0.

  `1 0 4 7 8 9`

- The 4 is okay.
- The 7 is in the wrong place. We can swap it with either the 4 or the 8. Let's swap it with the 4.

  `1 0 7 4 8 9`

- The 9 is in the wrong place. Let's swap it with the 8.

  `1 0 7 4 9 8`

Observe that there's nothing special about the array having these values. The relative order of the elements matters, but all sorted arrays will have the same relative order. Therefore, we can take this same approach on any sorted array.

Before coding, we should clarify the exact algorithm, though.

1. Sort the array in ascending order.
2. Iterate through the elements, starting from index 1 (not 0) and jumping two elements at a time.
3. At each element, swap it with the previous element. Since every three elements appear in the order `small <= medium <= large`, swapping these elements will always put medium as a peak: `medium <= small <= large`.

This approach will ensure that the peaks are in the right place: indexes 1, 3, 5, and so on. As long as the odd-numbered elements (the peaks) are bigger than the adjacent elements, then the even-numbered elements (the valleys) must be smaller than the adjacent elements.

The code to implement this is below.

```
void sortValleyPeak(int[] array) {
  Arrays.sort(array);
  for (int i = 1; i < array.length; i += 2) {
    swap(array, i - 1, i);
  }
}

void swap(int[] array, int left, int right) {
  int temp = array[left];
  array[left] = array[right];
  array[right] = temp;
}
```

This algorithm runs in `O(n log n)` time.

Optimal Solution

To optimize past the prior solution, we need to cut out the sorting step. The algorithm must operate on an unsorted array.

Let's revisit an example.

```
9 1 0 4 8 7
```

For each element, we'll look at the adjacent elements. Let's imagine some sequences. We'll just use the numbers 0, 1 and 2. The specific values don't matter.

```
0 1 2
0 2 1     // peak
1 0 2
1 2 0     // peak
2 1 0
```

```
2   0   1
```

If the center element needs to be a peak, then two of those sequences work. Can we fix the other ones to make the center element a peak?

Yes. We can fix this sequence by swapping the center element with the largest adjacent element.

```
0   1   2   ->  0   2   1
0   2   1       // peak
1   0   2   ->  1   2   0
1   2   0   // peak
2   1   0   ->  1   2   0
2   0   1   ->  0   2   1
```

As we noted before, if we make sure the peaks are in the right place then we know the valleys are in the right place.

> We should be a little cautious here. Is it possible that one of these swaps could "break" an earlier part of the sequence that we'd already processed? This is a good thing to worry about, but it's not an issue here. If we're swapping `middle` with `left`, then `left` is currently a valley. `Middle` is smaller than `left`, so we're putting an even smaller element as a valley. Nothing will break. All is good!

The code to implement this is below.

```
void sortValleyPeak(int[] array) {
   for (int i = 1; i < array.length; i += 2) {
      int biggestIndex = maxIndex(array, i - 1, i, i + 1);
      if (i != biggestIndex) {
         swap(array, i, biggestIndex);
      }
   }
}

int maxIndex(int[] array, int a, int b, int c) {
   int len = array.length;
   int aValue = a >= 0 && a < len ? array[a] : Integer.MIN_VALUE;
   int bValue = b >= 0 && b < len ? array[b] : Integer.MIN_VALUE;
   int cValue = c >= 0 && c < len ? array[c] : Integer.MIN_VALUE;

   int max = Math.max(aValue, Math.max(bValue, cValue));
   if (aValue == max) return a;
   else if (bValue == max) return b;
   else return c;
}
```

This algorithm takes `O(n)` time.

11

Solutions to Testing

11.1 **Mistake:** Find the mistake(s) in the following code:

```
unsigned int i;
for (i = 100; i >= 0; --i)
  printf("%d\n", i);
```

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SOLUTION

There are two mistakes in this code.

First, note that an `unsigned int` is, by definition, always greater than or equal to zero. The for loop condition will therefore always be true, and it will loop infinitely.

The correct code to print all numbers from 100 to 1, is `i > 0`. If we truly wanted to print zero, we could add an additional `printf` statement after the for loop.

```
unsigned int i;
for (i = 100; i > 0; --i)
  printf("%d\n", i);
```

One additional correction is to use %u in place of %d, as we are printing `unsigned int`.

```
unsigned int i;
for (i = 100; i > 0; --i)
  printf("%u\n", i);
```

This code will now correctly print the list of all numbers from 100 to 1, in descending order.

11.2 **Random Crashes:** You are given the source to an application which crashes when it is run. After running it ten times in a debugger, you find it never crashes in the same place. The application is single threaded, and uses only the C standard library. What programming errors could be causing this crash? How would you test each one?

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SOLUTION

The question largely depends on the type of application being diagnosed. However, we can give some general causes of random crashes.

1. *"Random Variable:"* The application may use some random number or variable component that may not be fixed for every execution of the program. Examples include user input, a random number generated by the program, or the time of day.

2. *Uninitialized Variable:* The application could have an uninitialized variable which, in some languages, may cause it to take on an arbitrary value. The values of this variable could result in the code taking a slightly different path each time.
3. *Memory Leak:* The program may have run out of memory. Other culprits are totally random for each run since it depends on the number of processes running at that particular time. This also includes heap overflow or corruption of data on the stack.
4. *External Dependencies:* The program may depend on another application, machine, or resource. If there are multiple dependencies, the program could crash at any point.

To track down the issue, we should start with learning as much as possible about the application. Who is running it? What are they doing with it? What kind of application is it?

Additionally, although the application doesn't crash in exactly the same place, it's possible that it is linked to specific components or scenarios. For example, it could be that the application never crashes if it's simply launched and left untouched, and that crashes only appear at some point after loading a file. Or, it may be that all the crashes take place within the lower level components, such as file I/O.

It may be useful to approach this by elimination. Close down all other applications on the system. Track resource use very carefully. If there are parts of the program we can disable, do so. Run it on a different machine and see if we experience the same issue. The more we can eliminate (or change), the easier we can track down the issue.

Additionally, we may be able to use tools to check for specific situations. For example, to investigate issue #2, we can utilize runtime tools which check for uninitialized variables.

These problems are as much about your brainstorming ability as they are about your approach. Do you jump all over the place, shouting out random suggestions? Or do you approach it in a logical, structured manner? Hopefully, it's the latter.

11.3 **Chess Test:** We have the following method used in a chess game: boolean `canMoveTo(int x, int y)`. This method is part of the `Piece` class and returns whether or not the piece can move to position `(x, y)`. Explain how you would test this method.

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SOLUTION

In this problem, there are two primary types of testing: extreme case validation (ensuring that the program doesn't crash on bad input), and general case testing. We'll start with the first type.

Testing Type #1: Extreme Case Validation

We need to ensure that the program handles bad or unusual input gracefully. This means checking the following conditions:

- Test with negative numbers for x and y
- Test with x larger than the width
- Test with y larger than the height
- Test with a completely full board
- Test with an empty or nearly empty board
- Test with far more white pieces than black

- Test with far more black pieces than white

For the error cases above, we should ask our interviewer whether we want to return false or throw an exception, and we should test accordingly.

Testing Type #2: General Testing:

General testing is much more expansive. Ideally, we would test every possible board, but there are far too many boards. We can, however, perform a reasonable coverage of different boards.

There are 6 pieces in chess, so we can test each piece against every other piece, in every possible direction. This would look something like the below code:

```
foreach piece a:
   for each other type of piece b (6 types + empty space)
      foreach direction d
         Create a board with piece a.
         Place piece b in direction d.
         Try to move - check return value.
```

The key to this problem is recognizing that we can't test every possible scenario, even if we would like to. So, instead, we must focus on the essential areas.

11.4 No Test Tools: How would you load test a webpage without using any test tools?

pg 157

SOLUTION

Load testing helps to identify a web application's maximum operating capacity, as well as any bottlenecks that may interfere with its performance. Similarly, it can check how an application responds to variations in load.

To perform load testing, we must first identify the performance critical scenarios and the metrics which fulfill our performance objectives. Typical criteria include:

- Response time
- Throughput
- Resource utilization
- Maximum load that the system can bear.

Then, we design tests to simulate the load, taking care to measure each of these criteria.

In the absence of formal testing tools, we can basically create our own. For example, we could simulate concurrent users by creating thousands of virtual users. We would write a multi-threaded program with thousands of threads, where each thread acts as a real-world user loading the page. For each user, we would programmatically measure response time, data I/O, etc.

We would then analyze the results based on the data gathered during the tests and compare it with the accepted values.

11.5 Test a Pen: How would you test a pen?

pg 157

SOLUTION

This problem is largely about understanding the constraints and approaching the problem in a structured manner.

To understand the constraints, you should ask a lot of questions to understand the "who, what, where, when, how and why" of a problem (or as many of those as apply to the problem). Remember that a good tester understands exactly what he is testing before starting the work.

To illustrate the technique in this problem, let us guide you through a mock conversation.

- **Interviewer:** How would you test a pen?
- **Candidate:** Let me find out a bit about the pen. Who is going to use the pen?
- **Interviewer:** Probably children.
- **Candidate:** Okay, that's interesting. What will they be doing with it? Will they be writing, drawing, or doing something else with it?
- **Interviewer:** Drawing.
- **Candidate:** Okay, great. On what? Paper? Clothing? Walls?
- **Interviewer:** On clothing.
- **Candidate:** Great. What kind of tip does the pen have? Felt? Ballpoint? Is it intended to wash off, or is it intended to be permanent?
- **Interviewer:** It's intended to wash off.

Many questions later, you may get to this:

- **Candidate:** Okay, so as I understand it, we have a pen that is being targeted at 5 to 10-year-olds. The pen has a felt tip and comes in red, green, blue and black. It's intended to wash off when clothing is washed. Is that correct?

The candidate now has a problem that is significantly different from what it initially seemed to be. This is not uncommon. In fact, many interviewers intentionally give a problem that seems clear (everyone knows what a pen is!), only to let you discover that it's quite a different problem from what it seemed. Their belief is that users do the same thing, though users do so accidentally.

Now that you understand what you're testing, it's time to come up with a plan of attack. The key here is *structure.*

Consider what the different components of the object or problem, and go from there. In this case, the components might be:

- *Fact check:* Verify that the pen is felt tip and that the ink is one of the allowed colors.
- *Intended use:* Drawing. Does the pen write properly on clothing?
- *Intended use:* Washing. Does it wash off of clothing (even if it's been there for an extended period of time)? Does it wash off in hot, warm and cold water?
- *Safety:* Is the pen safe (non-toxic) for children?
- *Unintended uses:* How else might children use the pen? They might write on other surfaces, so you need to check whether the behavior there is correct. They might also stomp on the pen, throw it, and so on.

You'll need to make sure that the pen holds up under these conditions.

Remember that in any testing question, you need to test both the intended and unintended scenarios. People don't always use the product the way you want them to.

11.6 Test an ATM: How would you test an ATM in a distributed banking system?

pg 157

SOLUTION

The first thing to do on this question is to clarify assumptions. Ask the following questions:

- Who is going to use the ATM? Answers might be "anyone," or it might be "blind people," or any number of other answers.
- What are they going to use it for? Answers might be "withdrawing money," "transferring money," "checking their balance," or many other answers.
- What tools do we have to test? Do we have access to the code, or just to the ATM?

Remember: a good tester makes sure she knows what she's testing!

Once we understand what the system looks like, we'll want to break down the problem into different testable components. These components include:

- Logging in
- Withdrawing money
- Depositing money
- Checking balance
- Transferring money

We would probably want to use a mix of manual and automated testing.

Manual testing would involve going through the steps above, making sure to check for all the error cases (low balance, new account, nonexistent account, and so on).

Automated testing is a bit more complex. We'll want to automate all the standard scenarios, as shown above, and we also want to look for some very specific issues, such as race conditions. Ideally, we would be able to set up a closed system with fake accounts and ensure that, even if someone withdraws and deposits money rapidly from different locations, he never gets money or loses money that he shouldn't.

Above all, we need to prioritize security and reliability. People's accounts must always be protected, and we must make sure that money is always properly accounted for. No one wants to unexpectedly lose money! A good tester understands the system priorities.

12

Solutions to C and C++

12.1 **Last K Lines:** Write a method to print the last K lines of an input file using C++.

pg 163

SOLUTION

One brute force way could be to count the number of lines (N) and then print from N-K to Nth line. But this requires two reads of the file, which is unnecessarily costly. We need a solution which allows us to read just once and be able to print the last K lines.

We can allocate an array for all K lines and the last K lines we've read in the array. , and so on. Each time that we read a new line, we purge the oldest line from the array.

But—you might ask—wouldn't this require shifting elements in the array, which is also very expensive? No, not if we do it correctly. Instead of shifting the array each time, we will use a circular array.

With a circular array, we always replace the oldest item when we read a new line. The oldest item is tracked in a separate variable, which adjusts as we add new items.

The following is an example of a circular array:

```
step 1 (initially): array = {a, b, c, d, e, f}. p = 0
step 2 (insert g):  array = {g, b, c, d, e, f}. p = 1
step 3 (insert h):  array = {g, h, c, d, e, f}. p = 2
step 4 (insert i):  array = {g, h, i, d, e, f}. p = 3
```

The code below implements this algorithm.

```
void printLast10Lines(char* fileName) {
  const int K = 10;
  ifstream file (fileName);
  string L[K];
  int size = 0;

  /* read file line by line into circular array */
  /* peek() so an EOF following a line ending is not considered a separate line */
  while (file.peek() != EOF) {
    getline(file, L[size % K]);
    size++;
  }

  /* compute start of circular array, and the size of it */
  int start = size > K ? (size % K) : 0;
  int count = min(K, size);

```

```
    /* print elements in the order they were read */
    for (int i = 0; i < count; i++) {
      cout << L[(start + i) % K] << endl;
    }
}
```

This solution will require reading in the whole file, but only ten lines will be in memory at any given point.

12.2 Reverse String: Implement a function void reverse(char* str) in C or C++ which reverses a null-terminated string.

pg 163

SOLUTION

This is a classic interview question. The only "gotcha" is to try to do it in place, and to be careful for the null character.

We will implement this in C.

```
void reverse(char *str) {
  char* end = str;
  char tmp;
  if (str) {
    while (*end) { /* find end of the string */
      ++end;
    }
    --end; /* set one char back, since last char is null */

    /* swap characters from start of string with the end of the string, until the
     * pointers meet in middle. */
    while (str < end) {
      tmp = *str;
      *str++ = *end;
      *end-- = tmp;
    }
  }
}
```

This is just one of many ways to implement this solution. We could even implement this code recursively (but we wouldn't recommend it).

12.3 Hash Table vs STL Map: Compare and contrast a hash table and an STL map. How is a hash table implemented? If the number of inputs is small, which data structure options can be used instead of a hash table?

pg 163

SOLUTION

In a hash table, a value is stored by calling a hash function on a key. Values are not stored in sorted order. Additionally, since hash tables use the key to find the index that will store the value, an insert or lookup can be done in amortized O(1) time (assuming few collisions in the hash table). In a hash table, one must also handle potential collisions. This is often done by chaining, which means to create a linked list of all the values whose keys map to a particular index.

An STL map inserts the key/value pairs into a binary search tree based on the keys. There is no need to handle collisions, and, since the tree is balanced, the insert and lookup time is guaranteed to be `O(log N)`.

How is a hash table implemented?

A hash table is traditionally implemented with an array of linked lists. When we want to insert a key/value pair, we map the key to an index in the array using a hash function. The value is then inserted into the linked list at that position.

Note that the elements in a linked list at a particular index of the array do not have the same key. Rather, `hashFunction(key)` is the same for these values. Therefore, in order to retrieve the value for a specific key, we need to store in each node both the exact key and the value.

To summarize, the hash table will be implemented with an array of linked lists, where each node in the linked list holds two pieces of data: the value and the original key. In addition, we will want to note the following design criteria:

1. We want to use a good hash function to ensure that the keys are well distributed. If they are not well distributed, then we would get a lot of collisions and the speed to find an element would decline.
2. No matter how good our hash function is, we will still have collisions, so we need a method for handling them. This often means chaining via a linked list, but it's not the only way.
3. We may also wish to implement methods to dynamically increase or decrease the hash table size depending on capacity. For example, when the ratio of the number of elements to the table size exceeds a certain threshold, we may wish to increase the hash table size. This would mean creating a new hash table and transferring the entries from the old table to the new table. Because this is an expensive operation, we want to be careful to not do it too often.

What can be used instead of a hash table, if the number of inputs is small?

You can use an STL map or a binary tree. Although this takes `O(log(n))` time, the number of inputs may be small enough to make this time negligible.

12.4 Virtual Functions: How do virtual functions work in C++?

pg 164

SOLUTION

A virtual function depends on a "vtable" or "Virtual Table." If any function of a class is declared to be virtual, a vtable is constructed which stores addresses of the virtual functions of this class. The compiler also adds a hidden `vptr` variable in all such classes which points to the vtable of that class. If a virtual function is not overridden in the derived class, the vtable of the derived class stores the address of the function in its parent class. The vtable is used to resolve the address of the function when the virtual function is called. Dynamic binding in C++ is performed through the vtable mechanism.

Thus, when we assign the derived class object to the base class pointer, the `vptr` variable points to the vtable of the derived class. This assignment ensures that the most derived virtual function gets called.

Consider the following code.

```
class Shape {
  public:
   int edge_length;
   virtual int circumference () {
```

```
        cout << "Circumference of Base Class\n";
        return 0;
    }
};

class Triangle: public Shape {
  public:
    int circumference () {
        cout<< "Circumference of Triangle Class\n";
        return 3 * edge_length;
    }
};

void main() {
    Shape * x = new Shape();
    x->circumference(); // "Circumference of Base Class"
    Shape *y = new Triangle();
    y->circumference(); // "Circumference of Triangle Class"
}
```

In the previous example, `circumference` is a virtual function in the Shape class, so it becomes virtual in each of the derived classes (`Triangle`, etc). C++ non-virtual function calls are resolved at compile time with static binding, while virtual function calls are resolved at runtime with dynamic binding.

12.5 Shallow vs Deep Copy: What is the difference between deep copy and shallow copy? Explain how you would use each.

pg 164

SOLUTION

A shallow copy copies all the member values from one object to another. A deep copy does all this and also deep copies any pointer objects.

An example of shallow and deep copy is below.

```
struct Test {
    char * ptr;
};

void shallow_copy(Test & src, Test & dest) {
    dest.ptr = src.ptr;
}

void deep_copy(Test & src, Test & dest) {
    dest.ptr = (char*)malloc(strlen(src.ptr) + 1);
    strcpy(dest.ptr, src.ptr);
}
```

Note that `shallow_copy` may cause a lot of programming runtime errors, especially with the creation and deletion of objects. Shallow copy should be used very carefully and only when a programmer really understands what he wants to do. In most cases, shallow copy is used when there is a need to pass information about a complex structure without actual duplication of data. One must also be careful with destruction of objects in a shallow copy.

In real life, shallow copy is rarely used. Deep copy should be used in most cases, especially when the size of the copied structure is small.

12.6 Volatile: What is the significance of the keyword "volatile" in C?

pg 164

SOLUTION

The keyword `volatile` informs the compiler that the value of variable it is applied to can change from the outside, without any update done by the code. This may be done by the operating system, the hardware, or another thread. Because the value can change unexpectedly, the compiler will therefore reload the value each time from memory.

A volatile integer can be declared by either of the following statements:

```
int volatile x;
volatile int x;
```

To declare a pointer to a volatile integer, we do the following:

```
volatile int * x;
int volatile * x;
```

A volatile pointer to non-volatile data is rare, but can be done.

```
int * volatile x;
```

If you wanted to declare a volatile variable pointer for volatile memory (both pointer address and memory contained are volatile), you would do the following:

```
int volatile * volatile x;
```

Volatile variables are not optimized, which can be very useful. Imagine this function:

```
int opt = 1;
void Fn(void) {
  start:
    if (opt == 1) goto start;
    else break;
}
```

At first glance, our code appears to loop infinitely. The compiler may try to optimize it to:

```
void Fn(void) {
  start:
    int opt = 1;
    if (true)
    goto start;
}
```

This becomes an infinite loop. However, an external operation might write '0' to the location of variable `opt`, thus breaking the loop.

To prevent the compiler from performing such optimization, we want to signal that another element of the system could change the variable. We do this using the `volatile` keyword, as shown below.

```
volatile int opt = 1;
void Fn(void) {
  start:
    if (opt == 1) goto start;
    else break;
}
```

Volatile variables are also useful when multi-threaded programs have global variables and any thread can modify these shared variables. We may not want optimization on these variables.

12.7 Virtual Base Class: Why does a destructor in base class need to be declared virtual?

pg 164

SOLUTION

Let's think about why we have virtual methods to start with. Suppose we have the following code:

```
class Foo {
 public:
   void f();
};

class Bar : public Foo {
 public:
   void f();
}

Foo * p = new Bar();
p->f();
```

Calling `p->f()` will result in a call to `Foo::f()`. This is because p is a pointer to Foo, and `f()` is not virtual.

To ensure that `p->f()` will invoke the most derived implementation of `f()`, we need to declare `f()` to be a virtual function.

Now, let's go back to our destructor. Destructors are used to clean up memory and resources. If Foo's destructor were not virtual, then Foo's destructor would be called, even when p is *really* of type Bar.

This is why we declare destructors to be virtual; we want to ensure that the destructor for the most derived class is called.

12.8 Copy Node: Write a method that takes a pointer to a Node structure as a parameter and returns a complete copy of the passed in data structure. The Node data structure contains two pointers to other Nodes.

pg 164

SOLUTION

The algorithm will maintain a mapping from a node address in the original structure to the corresponding node in the new structure. This mapping will allow us to discover previously copied nodes during a traditional depth-first traversal of the structure. Traversals often mark visited nodes—the mark can take many forms and does not necessarily need to be stored in the node.

Thus, we have a simple recursive algorithm:

```
typedef map<Node*, Node*> NodeMap;

Node * copy_recursive(Node * cur, NodeMap & nodeMap) {
   if (cur == NULL) {
      return NULL;
   }

   NodeMap::iterator i = nodeMap.find(cur);
   if (i != nodeMap.end()) {
      // we've been here before, return the copy
      return i->second;
   }
```

```

  Node * node = new Node;
  nodeMap[cur] = node; // map current before traversing links
  node->ptr1 = copy_recursive(cur->ptr1, nodeMap);
  node->ptr2 = copy_recursive(cur->ptr2, nodeMap);
  return node;
}

Node * copy_structure(Node * root) {
  NodeMap nodeMap; // we will need an empty map
  return copy_recursive(root, nodeMap);
}
```

12.9 Smart Pointer: Write a smart pointer class. A smart pointer is a data type, usually implemented with templates, that simulates a pointer while also providing automatic garbage collection. It automatically counts the number of references to a *SmartPointer<T*>* object and frees the object of type T when the reference count hits zero.

pg 164

SOLUTION

A smart pointer is the same as a normal pointer, but it provides safety via automatic memory management. It avoids issues like dangling pointers, memory leaks and allocation failures. The smart pointer must maintain a single reference count for all references to a given object.

This is one of those problems that seems at first glance pretty overwhelming, especially if you're not a C++ expert. One useful way to approach the problem is to divide the problem into two parts: (1) outline the pseudocode and approach and then (2) implement the detailed code.

In terms of the approach, we need a reference count variable that is incremented when we add a new reference to the object and decremented when we remove a reference. The code should look something like the below pseudocode:

```
template <class T> class SmartPointer {
  /* The smart pointer class needs pointers to both the object itself and to the
   * ref count. These must be pointers, rather than the actual object or ref count
   * value, since the goal of a smart pointer is that the reference count is
   * tracked across multiple smart pointers to one object. */
  T * obj;
  unsigned * ref_count;
}
```

We know we need constructors and a single destructor for this class, so let's add those first.

```
SmartPointer(T * object) {
  /* We want to set the value of T * obj, and set the reference counter to 1. */
}

SmartPointer(SmartPointer<T>& sptr) {
  /* This constructor creates a new smart pointer that points to an existing
   * object. We will need to first set obj and ref_count to pointer to sptr's obj
   * and ref_count. Then, because we created a new reference to obj, we need to
   * increment ref_count. */
}

~SmartPointer(SmartPointer<T> sptr) {
  /* We are destroying a reference to the object. Decrement ref_count. If
```

```
    * ref_count is 0, then free the memory created by the integer and destroy the
    * object. */
}
```

There's one additional way that references can be created: by setting one `SmartPointer` equal to another. We'll want to override the `equal` operator to handle this, but for now, let's sketch the code like this.

```
onSetEquals(SmartPoint<T> ptr1, SmartPoint<T> ptr2) {
  /* If ptr1 has an existing value, decrement its reference count. Then, copy the
   * pointers to obj and ref_count over. Finally, since we created a new
   * reference, we need to increment ref_count. */
}
```

Getting just the approach, even without filling in the complicated C++ syntax, would count for a lot. Finishing out the code is now just a matter of filling the details.

```
template <class T> class SmartPointer {
 public:
  SmartPointer(T * ptr) {
    ref = ptr;
    ref_count = (unsigned*)malloc(sizeof(unsigned));
    *ref_count = 1;
  }

  SmartPointer(SmartPointer<T> & sptr) {
    ref = sptr.ref;
    ref_count = sptr.ref_count;
    ++(*ref_count);
  }

  /* Override the equal operator, so that when you set one smart pointer equal to
   * another the old smart pointer has its reference count decremented and the new
   * smart pointer has its reference count incremememted. */
  SmartPointer<T> & operator=(SmartPointer<T> & sptr) {
    if (this == &sptr) return *this;

    /* If already assigned to an object, remove one reference. */
    if (*ref_count > 0) {
      remove();
    }

    ref = sptr.ref;
    ref_count = sptr.ref_count;
    ++(*ref_count);
    return *this;
  }

  ~SmartPointer() {
    remove(); // Remove one reference to object.
  }

  T getValue() {
    return *ref;
  }

 protected:
  void remove() {
    --(*ref_count);
    if (*ref_count == 0) {
```

```
            delete ref;
            free(ref_count);
            ref = NULL;
            ref_count = NULL;
        }
    }

    T * ref;
    unsigned * ref_count;
};
```

The code for this problem is complicated, and you probably wouldn't be expected to complete it flawlessly.

12.10 Malloc: Write an aligned malloc and free function that supports allocating memory such that the memory address returned is divisible by a specific power of two.

EXAMPLE

`align_malloc(1000,128)` will return a memory address that is a multiple of 128 and that points to memory of size 1000 bytes.

`aligned_free()` will free memory allocated by `align_malloc`.

pg 164

SOLUTION

Typically, with `malloc`, we do not have control over where the memory is allocated within the heap. We just get a pointer to a block of memory which could start at any memory address within the heap.

We need to work with these constraints by requesting enough memory that we can return a memory address which is divisible by the desired value.

Suppose we are requesting a 100-byte chunk of memory, and we want it to start at a memory address that is a multiple of 16. How much extra memory would we need to allocate to ensure that we can do so? We would need to allocate an extra 15 bytes. With these 15 bytes, plus another 100 bytes right after that sequence, we know that we would have a memory address divisible by 16 with space for 100 bytes.

We could then do something like:

```
1   void* aligned_malloc(size_t required_bytes, size_t alignment) {
2      int offset = alignment - 1;
3      void* p = (void*) malloc(required_bytes + offset);
4      void* q = (void*) (((size_t)(p) + offset) & ~(alignment - 1));
5      return q;
6   }
```

Line 4 is a bit tricky, so let's discuss it. Suppose `alignment` is 16. We know that one of the first 16 memory address in the block at p must be divisible by 16. With `(p + 15) & 11...10000` we advance as need to this address. ANDing the last four bits of `p + 15` with `0000` guarantees that this new value will be divisible by 16 (either at the original p or in one of the following 15 addresses).

This solution is *almost* perfect, except for one big issue: how do we free the memory?

We've allocated an extra 15 bytes, in the above example, and we need to free them when we free the "real" memory.

We can do this by storing, in this "extra" memory, the address of where the full memory block begins. We will store this immediately before the aligned memory block. Of course, this means that we now need to allocate even *more* extra memory to ensure that we have enough space to store this pointer.

Therefore, to guarantee both an aligned address and space for this pointer, we will need to allocate an additional `alignment - 1 + sizeof(void*)` bytes.

The code below implements this approach.

```
1   void* aligned_malloc(size_t required_bytes, size_t alignment) {
2     void* p1; // initial block
3     void* p2; // aligned block inside initial block
4     int offset = alignment - 1 + sizeof(void*);
5     if ((p1 = (void*)malloc(required_bytes + offset)) == NULL) {
6       return NULL;
7     }
8     p2 = (void*)(((size_t)(p1) + offset) & ~(alignment - 1));
9     ((void **)p2)[-1] = p1;
10    return p2;
11  }
12
13  void aligned_free(void *p2) {
14    /* for consistency, we use the same names as aligned_malloc*/
15    void* p1 = ((void**)p2)[-1];
16    free(p1);
17  }
```

Let's look at the pointer arithmetic in lines 9 and 15. If we treat p2 as a `void**` (or an array of `void*`'s), we can just look at the index - 1 to retrieve p1.

In `aligned_free`, we take p2 as the same p2 returned from `aligned_malloc`. As before, we know that the value of p1 (which points to the beginning of the full memory block) was stored just before p2. By freeing p1, we deallocate the whole memory block.

12.11 2D Alloc: Write a function in C called `my2DAlloc` which allocates a two-dimensional array. Minimize the number of calls to `malloc` and make sure that the memory is accessible by the notation `arr[i][j]`.

pg 164

SOLUTION

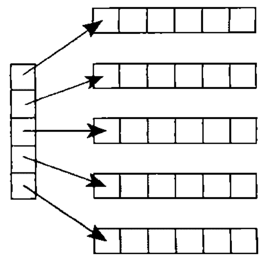
As you may know, a two-dimensional array is essentially an array of arrays. Since we use pointers with arrays, we can use double pointers to create a double array.

The basic idea is to create a one-dimensional array of pointers. Then, for each array index, we create a new one-dimensional array. This gives us a two-dimensional array that can be accessed via array indices.

The code below implements this.

```
int** my2DAlloc(int rows, int cols) {
  int** rowptr;
  int i;
  rowptr = (int**) malloc(rows * sizeof(int*));
  for (i = 0; i < rows; i++)  {
    rowptr[i] = (int*) malloc(cols * sizeof(int));
  }
   return rowptr;
}
```

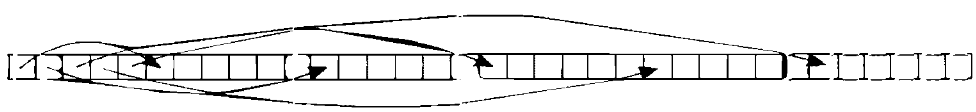
Observe how, in the above code, we've told `rowptr` where exactly each index should point. The following diagram represents how this memory is allocated.



To free this memory, we cannot simply call `free` on `rowptr`. We need to make sure to free not only the memory from the first `malloc` call, but also each subsequent call.

```
void my2DDealloc(int** rowptr, int rows) {
  for (i = 0; i < rows; i++) {
    free(rowptr[i]);
  }
  free(rowptr);
}
```

Rather than allocating the memory in many different blocks (one block for each row, plus one block to specify *where* each row is located), we can allocate this in a consecutive block of memory. Conceptually, for a two-dimensional array with five rows and six columns, this would look like the following.



If it seems strange to view the 2D array like this (and it probably does), remember that this is fundamentally no different than the first diagram. The only difference is that the memory is in a contiguous block, so our first five (in this example) elements point elsewhere in the same block of memory.

To implement this solution, we do the following.

```
1  int** my2DAlloc(int rows, int cols) {
2    int i;
3    int header = rows * sizeof(int*);
4    int data = rows * cols * sizeof(int);
5    int** rowptr = (int**)malloc(header + data);
6    if (rowptr == NULL) return NULL;
7
8    int* buf = (int*) (rowptr + rows);
9    for (i = 0; i < rows; i++) {
10     rowptr[i] = buf + i * cols;
11   }
12   return rowptr;
13 }
```

You should carefully observe what is happening on lines 11 through 13. If there are five rows of six columns each, `array[0]` will point to `array[5]`, `array[1]` will point to `array[11]`, and so on.

Then, when we actually call `array[1][3]`, the computer looks up `array[1]`, which is a pointer to another spot in memory—specifically, a pointer to `array[5]`. This element is treated as its own array, and we then get the third (zero-indexed) element from it.

Constructing the array in a single call to `malloc` has the added benefit of allowing disposal of the array with a single `free` call rather than using a special function to free the remaining data blocks.

13

Solutions to Java

13.1 Private Constructor: In terms of inheritance, what is the effect of keeping a constructor private?

pg 167

SOLUTION

Declaring a constructor `private` on class A means that you can only access the (private) constructor if you could also access A's private methods. Who, other than A, can access A's private methods and constructor? A's inner classes can. Additionally, if A is an inner class of Q, then Q's other inner classes can.

This has direct implications for inheritance, since a subclass calls its parent's constructor. The class A can be inherited, but only by its own or its parent's inner classes.

13.2 Return from Finally: In Java, does the finally block get executed if we insert a return statement inside the try block of a try-catch-finally?

pg 167

SOLUTION

Yes, it will get executed. The `finally` block gets executed when the `try` block exits. Even when we attempt to exit within the `try` block (via a `return` statement, a `continue` statement, a `break` statement or any exception), the `finally` block will still be executed.

Note that there are some cases in which the `finally` block will not get executed, such as the following:

- If the virtual machine exits during `try/catch` block execution.
- If the thread which is executing during the `try/catch` block gets killed.

13.3 Final, etc.: What is the difference between `final`, `finally`, and `finalize`?

pg 167

SOLUTIONS

Despite their similar sounding names, `final`, `finally` and `finalize` have very different purposes. To speak in very general terms, `final` is used to control whether a variable, method, or class is "changeable." The `finally` keyword is used in a `try/ catch` block to ensure that a segment of code is always executed. The `finalize()` method is called by the garbage collector once it determines that no more references exist.

Further detail on these keywords and methods is provided below.

final

The final statement has a different meaning depending on its context.

- When applied to a variable (primitive): The value of the variable cannot change.
- When applied to a variable (reference): The reference variable cannot point to any other object on the heap.
- When applied to a method: The method cannot be overridden.
- When applied to a class: The class cannot be subclassed.

finally keyword

There is an optional `finally` block after the `try` block or after the `catch` block. Statements in the `finally` block will always be executed, even if an exception is thrown (except if Java Virtual Machine exits from the `try` block). The `finally` block is often used to write the clean-up code. It will be executed after the `try` and `catch` blocks, but before control transfers back to its origin.

Watch how this plays out in the example below.

```
public static String lem() {
  System.out.println("lem");
  return "return from lem";
}

public static String foo() {
  int x = 0;
  int y = 5;
  try {
    System.out.println("start try");
    int b = y / x;
    System.out.println("end try");
    return "returned from try";
  } catch (Exception ex) {
    System.out.println("catch");
    return lem() + " | returned from catch";
  } finally {
    System.out.println("finally");
  }
}

public static void bar() {
  System.out.println("start bar");
  String v = foo();
  System.out.println(v);
  System.out.println("end bar");
}

public static void main(String[] args) {
  bar();
}
```

The output for this code is the following:

```
start bar
```

```
2 start try
3 catch
4 lem
5 finally
6 return from lem | returned from catch
7 end bar
```

Look carefully at lines 3 to 5 in the output. The `catch` block is fully executed (including the function call in the `return` statement), then the `finally` block, and then the function actually returns.

finalize()

The automatic garbage collector calls the `finalize()` method just before actually destroying the object. A class can therefore override the `finalize()` method from the `Object` class in order to define custom behavior during garbage collection.

```
protected void finalize() throws Throwable {
  /* Close open files, release resources, etc */
}
```

13.4 Generics vs. Templates: Explain the difference between templates in C++ and generics in Java.

pg 167

SOLUTION

Many programmers consider templates and generics to be essentially equivalent because both allow you to do something like `List<String>`. But, *how* each language does this, and *why*, varies significantly.

The implementation of Java generics is rooted in an idea of "type erasure." This technique eliminates the parameterized types when source code is translated to the Java Virtual Machine (JVM) byte code.

For example, suppose you have the Java code below:

```
Vector<String>  vector = new Vector<String>();
vector.add(new String("hello"));
String  str = vector.get(0);
```

During compilation, this code is re-written into:

```
Vector vector = new Vector();
vector.add(new String("hello"));
String  str = (String) vector.get(0);
```

The use of Java generics didn't really change much about our capabilities; it just made things a bit prettier. For this reason, Java generics are sometimes called "syntactic sugar."

This is quite different from C++. In C++, templates are essentially a glorified macro set, with the compiler creating a new copy of the template code for each type. Proof of this is in the fact that an instance of `MyClass<Foo>` will not share a static variable with `MyClass<Bar>`. Two instances of `MyClass<Foo>`, however, will share a static variable.

To illustrate this, consider the code below:

```
/*** MyClass.h ***/
template<class T> class MyClass {
 public:
  static int val;
  MyClass(int v) { val = v; }
};

```

```
/*** MyClass.cpp ***/
template<typename T>
int MyClass<T>::bar;

template class MyClass<Foo>;
template class MyClass<Bar>;

/*** main.cpp ***/
MyClass<Foo> * foo1 = new MyClass<Foo>(10);
MyClass<Foo> * foo2 = new MyClass<Foo>(15);
MyClass<Bar> * bar1 = new MyClass<Bar>(20);
MyClass<Bar> * bar2 = new MyClass<Bar>(35);

int f1 = foo1->val; // will equal 15
int f2 = foo2->val; // will equal 15
int b1 = bar1->val; // will equal 35
int b2 = bar2->val; // will equal 35
```

In Java, static variables are shared across instances of `MyClass`, regardless of the different type parameters.

Java generics and C++ templates have a number of other differences. These include:

- C++ templates can use primitive types, like `int`. Java cannot and must instead use `Integer`.
- In Java, you can restrict the template's type parameters to be of a certain type. For instance, you might use generics to implement a `CardDeck` and specify that the type parameter must extend from `CardGame`.
- In C++, the type parameter can be instantiated, whereas Java does not support this.
- In Java, the type parameter (i.e., the `Foo` in `MyClass<Foo>`) cannot be used for static methods and variables, since these would be shared between `MyClass<Foo>` and `MyClass<Bar>`. In C++, these classes are different, so the type parameter can be used for static methods and variables.
- In Java, all instances of `MyClass`, regardless of their type parameters, are the same type. The type parameters are erased at runtime. In C++, instances with different type parameters are different types.

Remember: Although Java generics and C++ templates look the same in many ways, they are very different.

13.5 TreeMap, HashMap, LinkedHashMap: Explain the differences between `TreeMap`, `HashMap`, and `LinkedHashMap`. Provide an example of when each one would be best.

pg 167

SOLUTION

All offer a key->value map and a way to iterate through the keys. The most important distinction between these classes is the time guarantees and the ordering of the keys.

- `HashMap` offers `O(1)` lookup and insertion. If you iterate through the keys, though, the ordering of the keys is essentially arbitrary. It is implemented by an array of linked lists.
- `TreeMap` offers `O(log N)` lookup and insertion. Keys are ordered, so if you need to iterate through the keys in sorted order, you can. This means that keys must implement the `Comparable` interface. `TreeMap` is implemented by a Red-Black Tree.
- `LinkedHashMap` offers `O(1)` lookup and insertion. Keys are ordered by their insertion order. It is implemented by doubly-linked buckets.

Imagine you passed an empty `TreeMap`, `HashMap`, and `LinkedHashMap` into the following function:

```
void insertAndPrint(AbstractMap<Integer, String> map) {
  int[] array = {1, -1, 0};
  for (int x : array) {
    map.put(x, Integer.toString(x));
  }

  for (int k : map.keySet()) {
    System.out.print(k + ", ");
  }
}
```

The output for each will look like the results below.

HashMap	LinkedHashMap	TreeMap
(any ordering)	{1, -1, 0}	{-1, 0, 1}

Very important: The output of LinkedHashMap and TreeMap must look like the above. For HashMap, the output was, in my own tests, {0, 1, -1}, but it could be any ordering. There is no *guarantee* on the ordering.

When might you need ordering in real life?

- Suppose you were creating a mapping of names to Person objects. You might want to periodically output the people in alphabetical order by name. A TreeMap lets you do this.
- A TreeMap also offers a way to, given a name, output the next 10 people. This could be useful for a "More" function in many applications.
- A LinkedHashMap is useful whenever you need the ordering of keys to match the ordering of insertion. This might be useful in a caching situation, when you want to delete the oldest item.

Generally, unless there is a reason not to, you would use HashMap. That is, if you need to get the keys back in insertion order, then use LinkedHashMap. If you need to get the keys back in their true/natural order, then use TreeMap. Otherwise, HashMap is probably best. It is typically faster and requires less overhead.

13.6 Object Reflection: Explain what object reflection is in Java and why it is useful.

pg 168

SOLUTION

Object Reflection is a feature in Java that provides a way to get reflective information about Java classes and objects, and perform operations such as:

1. Getting information about the methods and fields present inside the class at runtime.
2. Creating a new instance of a class.
3. Getting and setting the object fields directly by getting field reference, regardless of what the access modifier is.

The code below offers an example of object reflection.

```
/* Parameters */
Object[] doubleArgs = new Object[] { 4.2, 3.9 };

/* Get class */
Class rectangleDefinition = Class.forName("MyProj.Rectangle");

```

```
/* Equivalent: Rectangle rectangle = new Rectangle(4.2, 3.9); */
Class[] doubleArgsClass = new Class[] {double.class, double.class};
Constructor doubleArgsConstructor =
  rectangleDefinition.getConstructor(doubleArgsClass);
Rectangle rectangle = (Rectangle) doubleArgsConstructor.newInstance(doubleArgs);

/* Equivalent: Double area = rectangle.area(); */
Method m = rectangleDefinition.getDeclaredMethod("area");
Double area = (Double) m.invoke(rectangle);
```

This code does the equivalent of:

```
Rectangle rectangle = new Rectangle(4.2, 3.9);
Double area = rectangle.area();
```

Why Is Object Reflection Useful?

Of course, it doesn't seem very useful in the above example, but reflection can be very useful in some cases. Three main reasons are:

1. It can help you observe or manipulate the runtime behavior of applications.
2. It can help you debug or test programs, as you have direct access to methods, constructors, and fields.
3. You can call methods by name when you don't know the method in advance. For example, we may let the user pass in a class name, parameters for the constructor, and a method name. We can then use this information to create an object and call a method. Doing these operations without reflection would require a complex series of if-statements, if it's possible at all.

13.7 Lambda Expressions: There is a class `Country` that has methods `getContinent()` and `getPopulation()`. Write a function `int getPopulation(List<Country> countries, String continent)` that computes the total population of a given continent, given a list of all countries and the name of a continent.

pg 168

SOLUTION

This question really comes in two parts. First, we need to generate a list of the countries in North America. Then, we need to compute their total population.

Without lambda expressions, this is fairly straightforward to do.

```
int getPopulation(List<Country> countries, String continent) {
  int sum = 0;
  for (Country c : countries) {
    if (c.getContinent().equals(continent)) {
      sum += c.getPopulation();
    }
  }
  return sum;
}
```

To implement this with lambda expressions, let's break this up into multiple parts.

First, we use `filter` to get a list of the countries in the specified continent.

```
Stream<Country> northAmerica = countries.stream().filter(
  country -> { return country.getContinent().equals(continent);}
```

```
);
```

Second, we convert this into a list of populations using map.

```
Stream<Integer> populations = northAmerica.map(
  c -> c.getPopulation()
);
```

Third and finally, we compute the sum using reduce.

```
int population = populations.reduce(0, (a, b) -> a + b);
```

This function puts it all together.

```
int getPopulation(List<Country> countries, String continent) {
  /* Filter countries. */
  Stream<Country> sublist = countries.stream().filter(
    country -> { return country.getContinent().equals(continent);}
  );

  /* Convert to list of populations. */
  Stream<Integer> populations = sublist.map(
    c -> c.getPopulation()
  );

  /* Sum list. */
  int population = populations.reduce(0, (a, b) -> a + b);
  return population;
}
```

Alternatively, because of the nature of this specific problem, we can actually remove the `filter` entirely. The `reduce` operation can have logic that maps the population of countries not in the right continent to zero. The sum will effectively disregard countries not within `continent`.

```
int getPopulation(List<Country> countries, String continent) {
  Stream<Integer> populations = countries.stream().map(
    c -> c.getContinent().equals(continent) ? c.getPopulation() : 0);
  return populations.reduce(0, (a, b) -> a + b);
}
```

Lambda functions were new to Java 8, so if you don't recognize them, that's probably why. Now is a great time to learn about them, though!

13.8 Lambda Random: Using Lambda expressions, write a function `List<Integer> getRandomSubset(List<Integer> list)` that returns a random subset of arbitrary size. All subsets (including the empty set) should be equally likely to be chosen.

pg 439

SOLUTION

It's tempting to approach this problem by picking a subset size from 0 to N and then generating a random subset of that size.

That creates two issues:

1. We'd have to weight those probabilities. If N > 1, there are more subsets of size N/2 than there are of subsets of size N (of which there is always only one).
2. It's actually more difficult to generate a subset of a restricted size (e.g., specifically 10) than it is to generate a subset of any size.

Instead, rather than generating a subset based on sizes, let's think about it based on elements. (The fact that we're told to use lambda expressions is also a hint that we should think about some sort of iteration or processing through the elements.)

Imagine we were iterating through `{1, 2, 3}` to generate a subset. Should 1 be in this subset?

We've got two choices: yes or no. We need to weight the probability of "yes" vs. "no" based on the percent of subsets that contain 1. So, what percent of elements contain 1?

For any specific element, there are as many subsets that contain the element as do not contain it. Consider the following:

```
{}          {1}
{2}         {1, 2}
{3}         {1, 3}
{2, 3}      {1, 2, 3}
```

Note how the difference between the subsets on the left and the subsets on the right is the existence of 1. The left and right sides must have the same number of subsets because we can convert from one to the other by just adding an element.

This means that we can generate a random subset by iterating through the list and flipping a coin (i.e., deciding on a 50/50 chance) to pick whether or not each element will be in it.

Without lambda expressions, we can write something like this:

```
List<Integer> getRandomSubset(List<Integer> list) {
  List<Integer> subset = new ArrayList<Integer>();
  Random random = new Random();
  for (int item : list) {
    /* Flip coin. */
    if (random.nextBoolean()) {
      subset.add(item);
    }
  }
  return subset;
}
```

To implement this approach using lambda expressions, we can do the following:

```
List<Integer> getRandomSubset(List<Integer> list) {
  Random random = new Random();
  List<Integer> subset = list.stream().filter(
    k -> { return random.nextBoolean(); /* Flip coin. */
  }).collect(Collectors.toList());
  return subset;
}
```

Or, we can use a predicate (defined within the class or within the function):

```
Random random = new Random();
Predicate<Object> flipCoin = o -> {
  return random.nextBoolean();
};

List<Integer> getRandomSubset(List<Integer> list) {
  List<Integer> subset = list.stream().filter(flipCoin).
    collect(Collectors.toList());
  return subset;
}
```

The nice thing about this implementation is that now we can apply the `flipCoin` predicate in other places.

14

Solutions to Databases

Questions 1 through 3 refer to the following database schema:

Apartments	
AptID	int
UnitNumber	varchar(10)
BuildingID	int

Buildings	
BuildingID	int
ComplexID	int
BuildingName	varchar(100)
Address	varchar(500)

Requests	
RequestID	int
Status	varchar(100)
AptID	int
Description	varchar(500)

Complexes	
ComplexID	int
ComplexName	varchar(100)

AptTenants	
TenantID	int
AptID	int

Tenants	
TenantID	int
TenantName	varchar(100)

Note that each apartment can have multiple tenants, and each tenant can have multiple apartments. Each apartment belongs to one building, and each building belongs to one complex.

14.1 Multiple Apartments: Write a SQL query to get a list of tenants who are renting more than one apartment.

pg 172

SOLUTION

To implement this, we can use the HAVING and GROUP BY clauses and then perform an INNER JOIN with Tenants.

```
SELECT TenantName
FROM Tenants
INNER JOIN
   (SELECT TenantID FROM AptTenants GROUP BY TenantID HAVING count(*) > 1) C
ON Tenants.TenantID = C.TenantID
```

Whenever you write a GROUP BY clause in an interview (or in real life), make sure that anything in the SELECT clause is either an aggregate function or contained within the GROUP BY clause.

14.2 Open Requests: Write a SQL query to get a list of all buildings and the number of open requests (`Requests` in which `status` equals 'Open').

pg 173

SOLUTION

This problem uses a straightforward join of `Requests` and `Apartments` to get a list of building IDs and the number of open requests. Once we have this list, we join it again with the `Buildings` table.

```
SELECT BuildingName, ISNULL(Count, 0) as 'Count'
FROM Buildings
LEFT JOIN
   (SELECT Apartments.BuildingID, count(*) as 'Count'
    FROM Requests INNER JOIN Apartments
    ON Requests.AptID = Apartments.AptID
    WHERE Requests.Status = 'Open'
    GROUP BY Apartments.BuildingID) ReqCounts
ON ReqCounts.BuildingID = Buildings.BuildingID
```

Queries like this that utilize sub-queries should be thoroughly tested, even when coding by hand. It may be useful to test the inner part of the query first, and then test the outer part.

14.3 Close All Requests: Building #11 is undergoing a major renovation. Implement a query to close all requests from apartments in this building.

pg 173

SOLUTION

`UPDATE` queries, like `SELECT` queries, can have `WHERE` clauses. To implement this query, we get a list of all apartment IDs within building #11 and the list of update requests from those apartments.

```
UPDATE Requests
SET Status = 'Closed'
WHERE AptID IN (SELECT AptID FROM Apartments WHERE BuildingID = 11)
```

14.4 Joins: What are the different types of joins? Please explain how they differ and why certain types are better in certain situations.

pg 173

SOLUTION

`JOIN` is used to combine the results of two tables. To perform a `JOIN`, each of the tables must have at least one field that will be used to find matching records from the other table. The join type defines which records will go into the result set.

Let's take for example two tables: one table lists the "regular" beverages, and another lists the calorie-free beverages. Each table has two fields: the beverage name and its product code. The "code" field will be used to perform the record matching.

Regular Beverages:

Name	Code
Budweiser	BUDWEISER
Coca-Cola	COCACOLA

Name	Code
Pepsi	PEPSI

Calorie-Free Beverages:

Name	Code
Diet Coca-Cola	COCACOLA
Fresca	FRESCA
Diet Pepsi	PEPSI
Pepsi Light	PEPSI
Purified Water	Water

If we wanted to join `Beverage` with `Calorie-Free Beverages`, we would have many options. These are discussed below.

- `INNER JOIN`: The result set would contain only the data where the criteria match. In our example, we would get three records: one with a `COCACOLA` code and two with `PEPSI` codes.
- `OUTER JOIN`: An `OUTER JOIN` will always contain the results of `INNER JOIN`, but it may also contain some records that have no matching record in the other table. `OUTER JOIN`s are divided into the following subtypes:
  - `LEFT OUTER JOIN`, or simply `LEFT JOIN`: The result will contain all records from the left table. If no matching records were found in the right table, then its fields will contain the `NULL` values. In our example, we would get four records. In addition to `INNER JOIN` results, `BUDWEISER` would be listed, because it was in the left table.
  - `RIGHT OUTER JOIN`, or simply `RIGHT JOIN`: This type of join is the opposite of `LEFT JOIN`. It will contain every record from the right table; the missing fields from the left table will be `NULL`. Note that if we have two tables, `A` and `B`, then we can say that the statement `A LEFT JOIN B` is equivalent to the statement `B RIGHT JOIN A`. In our example above, we will get five records. In addition to `INNER JOIN` results, `FRESCA` and `WATER` records will be listed.
  - `FULL OUTER JOIN`: This type of join combines the results of the `LEFT` and `RIGHT JOINS`. All records from both tables will be included in the result set, regardless of whether or not a matching record exists in the other table. If no matching record was found, then the corresponding result fields will have a `NULL` value. In our example, we will get six records.

14.5 Denormalization: What is denormalization? Explain the pros and cons.

pg 173

SOLUTION

Denormalization is a database optimization technique in which we add redundant data to one or more tables. This can help us avoid costly joins in a relational database.

By contrast, in a traditional normalized database, we store data in separate logical tables and attempt to minimize redundant data. We may strive to have only one copy of each piece of data in the database.

For example, in a normalized database, we might have a `Courses` table and a `Teachers` table. Each entry in `Courses` would store the `teacherID` for a `Course` but not the `teacherName`. When we need to retrieve a list of all `Courses` with the `Teacher` name, we would do a join between these two tables.

In some ways, this is great; if a teacher changes his or her name, we only have to update the name in one place.

The drawback, however, is that if the tables are large, we may spend an unnecessarily long time doing joins on tables.

Denormalization, then, strikes a different compromise. Under denormalization, we decide that we're okay with some redundancy and some extra effort to update the database in order to get the efficiency advantages of fewer joins.

Cons of Denormalization	Pros of Denormalization
Updates and inserts are more expensive.	Retrieving data is faster since we do fewer joins.
Denormalization can make update and insert code harder to write.	Queries to retrieve can be simpler (and therefore less likely to have bugs), since we need to look at fewer tables.
Data may be inconsistent. Which is the "correct" value for a piece of data?	
Data redundancy necessitates more storage.	

In a system that demands scalability, like that of any major tech companies, we almost always use elements of both normalized and denormalized databases.

14.6 Entity-Relationship Diagram: Draw an entity-relationship diagram for a database with companies, people, and professionals (people who work for companies).

pg 173

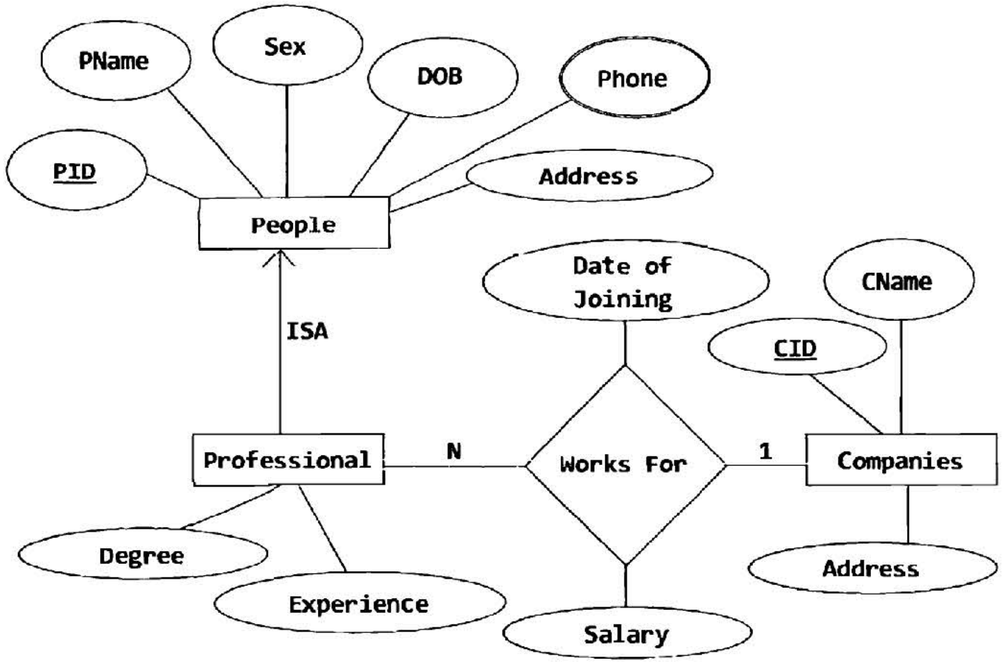
SOLUTION

People who work for `Companies` are `Professionals`. So, there is an `ISA` ("is a") relationship between `People` and `Professionals` (or we could say that a `Professional` is derived from `People`).

Each `Professional` has additional information such as degree and work experiences in addition to the properties derived from `People`.

A `Professional` works for one company at a time (probably—you might want to validate this assumption), but `Companies` can hire many `Professionals`. So, there is a many-to-one relationship between `Professionals` and `Companies`. This "Works For" relationship can store attributes such as an employee's start date and salary. These attributes are defined only when we relate a `Professional` with a `Company`.

A `Person` can have multiple phone numbers, which is why `Phone` is a multi-valued attribute.



14.7 Design Grade Database: Imagine a simple database storing information for students' grades. Design what this database might look like and provide a SQL query to return a list of the honor roll students (top 10%), sorted by their grade point average.

pg 173

SOLUTION

In a simplistic database, we'll have at least three objects: `Students`, `Courses`, and `CourseEnrollment`. `Students` will have at least a student name and ID and will likely have other personal information. `Courses` will contain the course name and ID and will likely contain the course description, professor, and other information. `CourseEnrollment` will pair `Students` and `Courses` and will also contain a field for `CourseGrade`.

Students	
StudentID	int
StudentName	varchar(100)
Address	varchar(500)

Courses	
CourseID	int
CourseName	varchar(100)
ProfessorID	int

CourseEnrollment	
CourseID	int
StudentID	int
Grade	float
Term	int

This database could get arbitrarily more complicated if we wanted to add in professor information, billing information, and other data.

Using the Microsoft SQL Server TOP ... PERCENT function, we might (incorrectly) first try a query like this:

```
SELECT TOP 10 PERCENT AVG(CourseEnrollment.Grade) AS GPA,
                                                    CourseEnrollment.StudentID
FROM CourseEnrollment
GROUP BY CourseEnrollment.StudentID
ORDER BY AVG(CourseEnrollment.Grade)
```

The problem with the above code is that it will return literally the top 10% of rows, when sorted by GPA. Imagine a scenario in which there are 100 students, and the top 15 students all have 4.0 GPAs. The above function will only return 10 of those students, which is not really what we want. In case of a tie, we want to include the students who tied for the top 10% -- even if this means that our honor roll includes more than 10% of the class.

To correct this issue, we can build something similar to this query, but instead first get the GPA cut off.

```
DECLARE @GPACutOff float;
SET @GPACutOff = (SELECT min(GPA) as 'GPAMin' FROM (
     SELECT TOP 10 PERCENT AVG(CourseEnrollment.Grade) AS GPA
     FROM CourseEnrollment
     GROUP BY CourseEnrollment.StudentID
     ORDER BY GPA desc) Grades);
```

Then, once we have @GPACutOff defined, selecting the students with at least this GPA is reasonably straightforward.

```
SELECT StudentName, GPA
FROM (SELECT AVG(CourseEnrollment.Grade) AS GPA, CourseEnrollment.StudentID
      FROM CourseEnrollment
      GROUP BY CourseEnrollment.StudentID
      HAVING AVG(CourseEnrollment.Grade) >= @GPACutOff) Honors
INNER JOIN Students ON Honors.StudentID = Student.StudentID
```

Be very careful about what implicit assumptions you make. If you look at the above database description, what potentially incorrect assumption do you see? One is that each course can only be taught by one professor. At some schools, courses may be taught by multiple professors.

However, you *will* need to make some assumptions, or you'd drive yourself crazy. Which assumptions you make is less important than just recognizing *that* you made assumptions. Incorrect assumptions, both in the real world and in an interview, can be dealt with *as long as they are acknowledged.*

Remember, additionally, that there's a trade-off between flexibility and complexity. Creating a system in which a course can have multiple professors does increase the database's flexibility, but it also increases its complexity. If we tried to make our database flexible to every possible situation, we'd wind up with something hopelessly complex.

Make your design reasonably flexible, and state any other assumptions or constraints. This goes for not just database design, but object-oriented design and programming in general.

15

Solutions to Threads and Locks

15.1 Thread vs. Process: What's the difference between a thread and a process?

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SOLUTION

Processes and threads are related to each other but are fundamentally different.

A process can be thought of as an instance of a program in execution. A process is an independent entity to which system resources (e.g., CPU time and memory) are allocated. Each process is executed in a separate address space, and one process cannot access the variables and data structures of another process. If a process wishes to access another process' resources, inter-process communications have to be used. These include pipes, files, sockets, and other forms.

A thread exists within a process and shares the process' resources (including its heap space). Multiple threads within the same process will share the same heap space. This is very different from processes, which cannot directly access the memory of another process. Each thread still has its own registers and its own stack, but other threads can read and write the heap memory.

A thread is a particular execution path of a process. When one thread modifies a process resource, the change is immediately visible to sibling threads.

15.2 Context Switch: How would you measure the time spent in a context switch?

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SOLUTION

This is a tricky question, but let's start with a possible solution.

A context switch is the time spent switching between two processes (i.e., bringing a waiting process into execution and sending an executing process into waiting/terminated state). This happens in multitasking. The operating system must bring the state information of waiting processes into memory and save the state information of the currently running process.

In order to solve this problem, we would like to record the timestamps of the last and first instruction of the swapping processes. The context switch time is the difference in the timestamps between the two processes.

Let's take an easy example: Assume there are only two processes, P_1 and P_2.

P_1 is executing and P_2 is waiting for execution. At some point, the operating system must swap P_1 and P_2—let's assume it happens at the Nth instruction of P_1. If $t_{x,k}$ indicates the timestamp in microseconds of the kth instruction of process x, then the context switch would take $t_{2,1} - t_{1,n}$ microseconds.

The tricky part is this: how do we know when this swapping occurs? We cannot, of course, record the timestamp of every instruction in the process.

Another issue is that swapping is governed by the scheduling algorithm of the operating system and there may be many kernel level threads which are also doing context switches. Other processes could be contending for the CPU or the kernel handling interrupts. The user does not have any control over these extraneous context switches. For instance, if at time $t_{1,n}$ the kernel decides to handle an interrupt, then the context switch time would be overstated.

In order to overcome these obstacles, we must first construct an environment such that after P_1 executes, the task scheduler immediately selects P_2 to run. This may be accomplished by constructing a data channel, such as a pipe, between P_1 and P_2 and having the two processes play a game of ping-pong with a data token.

That is, let's allow P_1 to be the initial sender and P_2 to be the receiver. Initially, P_2 is blocked (sleeping) as it awaits the data token. When P_1 executes, it delivers the token over the data channel to P_2 and immediately attempts to read a response token. However, since P_2 has not yet had a chance to run, no such token is available for P_1 and the process is blocked. This relinquishes the CPU.

A context switch results and the task scheduler must select another process to run. Since P_2 is now in a ready-to-run state, it is a desirable candidate to be selected by the task scheduler for execution. When P_2 runs, the roles of P_1 and P_2 are swapped. P_2 is now acting as the sender and P_1 as the blocked receiver. The game ends when P_2 returns the token to P_1.

To summarize, an iteration of the game is played with the following steps:

1. P_2 blocks awaiting data from P_1.
2. P_1 marks the start time.
3. P_1 sends token to P_2.
4. P_1 attempts to read a response token from P_2. This induces a context switch.
5. P_2 is scheduled and receives the token.
6. P_2 sends a response token to P_1.
7. P_2 attempts read a response token from P_1. This induces a context switch.
8. P_1 is scheduled and receives the token.
9. P_1 marks the end time.

The key is that the delivery of a data token induces a context switch. Let T_d and T_r be the time it takes to deliver and receive a data token, respectively, and let T_c be the amount of time spent in a context switch. At step 2, P_1 records the timestamp of the delivery of the token, and at step 9, it records the timestamp of the response. The amount of time elapsed, T, between these events may be expressed by:

$$T = 2 * (T_d + T_c + T_r)$$

This formula arises because of the following events: P_1 sends a token (3), the CPU context switches (4), P_2 receives it (5). P_2 then sends the response token (6), the CPU context switches (7), and finally P_1 receives it (8).

P_1 will be able to easily compute T, since this is just the time between events 3 and 8. So, to solve for T_c, we must first determine the value of $T_d + T_r$.

How can we do this? We can do this by measuring the length of time it takes P_1 to send and receive a token to itself. This will not induce a context switch since P_1 is running on the CPU at the time it sent the token and will not block to receive it.

The game is played a number of iterations to average out any variability in the elapsed time between steps 2 and 9 that may result from unexpected kernel interrupts and additional kernel threads contending for the CPU. We select the smallest observed context switch time as our final answer.

However, all we can ultimately say that this is an approximation which depends on the underlying system. For example, we make the assumption that P_2 is selected to run once a data token becomes available. However, this is dependent on the implementation of the task scheduler and we cannot make any guarantees.

That's okay; it's important in an interview to recognize when your solution might not be perfect.

15.3 Dining Philosophers: In the famous dining philosophers problem, a bunch of philosophers are sitting around a circular table with one chopstick between each of them. A philosopher needs both chopsticks to eat, and always picks up the left chopstick before the right one. A deadlock could potentially occur if all the philosophers reached for the left chopstick at the same time. Using threads and locks, implement a simulation of the dining philosophers problem that prevents deadlocks.

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SOLUTION

First, let's implement a simple simulation of the dining philosophers problem in which we don't concern ourselves with deadlocks. We can implement this solution by having `Philosopher` extend `Thread`, and `Chopstick` call `lock.lock()` when it is picked up and `lock.unlock()` when it is put down.

```
class Chopstick {
  private Lock lock;

  public Chopstick() {
     lock = new ReentrantLock();
  }

  public void pickUp() {
     void lock.lock();
  }

  public void putDown() {
     lock.unlock();
  }
}

class Philosopher extends Thread {
  private int bites = 10;
  private Chopstick left, right;

  public Philosopher(Chopstick left, Chopstick right) {
     this.left = left;
     this.right = right;
  }
```

```

    public void eat() {
       pickUp();
       chew();
       putDown();
    }

    public void pickUp() {
       left.pickUp();
       right.pickUp();
    }

    public void chew() { }

    public void putDown() {
       right.putDown();
       left.putDown();
    }

    public void run() {
       for (int i = 0; i < bites; i++) {
          eat();
       }
    }
 }
```

Running the above code may lead to a deadlock if all the philosophers have a left chopstick and are waiting for the right one.

Solution #1: All or Nothing

To prevent deadlocks, we can implement a strategy where a philosopher will put down his left chopstick if he is unable to obtain the right one.

```
public class Chopstick {
   /* same as before */

   public boolean pickUp() {
      return lock.tryLock();
   }
}

public class Philosopher extends Thread {
   /* same as before */

   public void eat() {
      if (pickUp()) {
         chew();
         putDown();
      }
   }

   public boolean pickUp() {
      /* attempt to pick up */
      if (!left.pickUp()) {
         return false;
      }
      if (!right.pickUp()) {
```

```
            left.putDown();
            return false;
         }
         return true;
      }
   }
```

In the above code, we need to be sure to release the left chopstick if we can't pick up the right one—and to not call `putDown()` on the chopsticks if we never had them in the first place.

One issue with this is that if all the philosophers were perfectly synchronized, they could simultaneously pick up their left chopstick, be unable to pick up the right one, and then put back down the left one—only to have the process repeated again.

Solution #2: Prioritized Chopsticks

Alternatively, we can label the chopsticks with a number from `0` to `N - 1`. Each philosopher attempts to pick up the lower numbered chopstick first. This essentially means that each philosopher goes for the left chopstick before right one (assuming that's the way you labeled it), except for the last philosopher who does this in reverse. This will break the cycle.

```
public class Philosopher extends Thread {
   private int bites = 10;
   private Chopstick lower, higher;
   private int index;
   public Philosopher(int i, Chopstick left, Chopstick right) {
      index = i;
      if (left.getNumber() < right.getNumber()) {
         this.lower = left;
         this.higher = right;
      } else {
         this.lower = right;
         this.higher = left;
      }
   }

   public void eat() {
      pickUp();
      chew();
      putDown();
   }

   public void pickUp() {
      lower.pickUp();
      higher.pickUp();
   }

   public void chew() { ... }

   public void putDown() {
      higher.putDown();
      lower.putDown();
   }

   public void run() {
      for (int i = 0; i < bites; i++) {
         eat();
```

```
      }
   }
}

public class Chopstick {
   private Lock lock;
   private int number;

   public Chopstick(int n) {
      lock = new ReentrantLock();
      this.number = n;
   }

   public void pickUp() {
      lock.lock();
   }

   public void putDown() {
      lock.unlock();
   }

   public int getNumber() {
      return number;
   }
}
```

With this solution, a philosopher can never hold the larger chopstick without holding the smaller one. This prevents the ability to have a cycle, since a cycle means that a higher chopstick would "point" to a lower one.

15.4 Deadlock-Free Class: Design a class which provides a lock only if there are no possible deadlocks.

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SOLUTION

There are several common ways to prevent deadlocks. One of the popular ways is to require a process to declare upfront what locks it will need. We can then verify if a deadlock would be created by issuing these locks, and we can fail if so.

With these constraints in mind, let's investigate how we can detect deadlocks. Suppose this was the order of locks requested:

```
A = {1, 2, 3, 4}
B = {1, 3, 5}
C = {7, 5, 9, 2}
```

This may create a deadlock because we could have the following scenario:

```
A locks 2, waits on 3
B locks 3, waits on 5
C locks 5, waits on 2
```

We can think about this as a graph, where 2 is connected to 3, 3 is connected to 5, and 5 is connected to 2. A deadlock is represented by a cycle. An edge `(w, v)` exists in the graph if a process declares that it will request lock `v` immediately after lock `w`. For the earlier example, the following edges would exist in the graph: `(1, 2)`, `(2, 3)`, `(3, 4)`, `(1, 3)`, `(3, 5)`, `(7, 5)`, `(5, 9)`, `(9, 2)`. The "owner" of the edge does not matter.

This class will need a `declare` method, which threads and processes will use to declare what order they will request resources in. This declare method will iterate through the declare order, adding each contiguous pair of elements (v, w) to the graph. Afterwards, it will check to see if any cycles have been created. If any cycles have been created, it will backtrack, removing these edges from the graph, and then exit.

We have one final component to discuss: how do we detect a cycle? We can detect a cycle by doing a depth-first search through each connected component (i.e., each connected part of the graph). Complex algorithms exist to find all the connected components of a graph, but our work in this problem does not require this degree of complexity.

We know that if a cycle was created, one of our new edges must be to blame. Thus, as long as our depth-first search touches all of these edges at some point, then we know that we have fully searched for a cycle.

The pseudocode for this special case cycle detection looks like this:

```
boolean checkForCycle(locks[] locks) {
  touchedNodes = hash table(lock -> boolean)
  initialize touchedNodes to false for each lock in locks
  for each (lock x in process.locks) {
    if (touchedNodes[x] == false) {
      if (hasCycle(x, touchedNodes)) {
        return true;
      }
    }
  }
  return false;
}

boolean hasCycle(node x, touchedNodes) {
  touchedNodes[r] = true;
  if (x.state == VISITING) {
    return true;
  } else if (x.state == FRESH) {
    ... (see full code below)
  }
}
```

In the above code, note that we may do several depth-first searches, but `touchedNodes` is only initialized once. We iterate until all the values in `touchedNodes` are false.

The code below provides further details. For simplicity, we assume that all locks and processes (owners) are ordered sequentially.

```
class LockFactory {
  private static LockFactory instance;

  private int numberOfLocks = 5; /* default */
  private LockNode[] locks;

  /* Maps from a process or owner to the order that the owner claimed it would
   * call the locks in */
  private HashMap<Integer, LinkedList<LockNode>> lockOrder;

  private LockFactory(int count) { ... }
  public static LockFactory getInstance() { return instance; }

  public static synchronized LockFactory initialize(int count) {
    if (instance == null) instance = new LockFactory(count);
```

```
        return instance;
    }

    public boolean hasCycle(HashMap<Integer, Boolean> touchedNodes,
                            int[] resourcesInOrder) {
        / *check for a cycle */
        for (int resource : resourcesInOrder) {
            if (touchedNodes.get(resource) == false) {
                LockNode n = locks[resource];
                if (n.hasCycle(touchedNodes)) {
                    return true;
                }
            }
        }
        return false;
    }

    / *To prevent deadlocks, force the processes to declare upfront what order they
     * will need the locks in. Verify that this order does not create a deadlock (a
     * cycle in a directed graph) */
    public boolean declare(int ownerId, int[] resourcesInOrder) {
        HashMap<Integer, Boolean> touchedNodes = new HashMap<Integer, Boolean>();

        / *add nodes to graph */
        int index = 1;
        touchedNodes.put(resourcesInOrder[0], false);
        for (index = 1; index < resourcesInOrder.length; index++) {
            LockNode prev = locks[resourcesInOrder[index - 1]];
            LockNode curr = locks[resourcesInOrder[index]];
            prev.joinTo(curr);
            touchedNodes.put(resourcesInOrder[index], false);
        }

        / *if we created a cycle, destroy this resource list and return false */
        if (hasCycle(touchedNodes, resourcesInOrder)) {
            for (int j = 1; j < resourcesInOrder.length; j++) {
                LockNode p = locks[resourcesInOrder[j - 1]];
                LockNode c = locks[resourcesInOrder[j]];
                p.remove(c);
            }
            return false;
        }

        / *No cycles detected. Save the order that was declared, so that we can
         * verify that the process is really calling the locks in the order it said
         * it would. */
        LinkedList<LockNode> list = new LinkedList<LockNode>();
        for (int i = 0; i < resourcesInOrder.length; i++) {
            LockNode resource = locks[resourcesInOrder[i]];
            list.add(resource);
        }
        lockOrder.put(ownerId, list);

        return true;
    }

```

```
  /* Get the lock, verifying first that the process is really calling the locks in
   * the order it said it would. */
  public Lock getLock(int ownerId, int resourceID) {
    LinkedList<LockNode> list = lockOrder.get(ownerId);
    if (list == null) return null;

    LockNode head = list.getFirst();
    if (head.getId() == resourceID) {
      list.removeFirst();
      return head.getLock();
    }
    return null;
  }
}

public class LockNode {
  public enum VisitState { FRESH, VISITING, VISITED };

  private ArrayList<LockNode> children;
  private int lockId;
  private Lock lock;
  private int maxLocks;

  public LockNode(int id, int max) { ... }

  /* Join "this" to "node", checking that it doesn't create a cycle */
  public void joinTo(LockNode node) { children.add(node); }
  public void remove(LockNode node) { children.remove(node); }

  /* Check for a cycle by doing a depth-first-search. */
  public boolean hasCycle(HashMap<Integer, Boolean> touchedNodes) {
    VisitState[] visited = new VisitState[maxLocks];
    for (int i = 0; i < maxLocks; i++) {
      visited[i] = VisitState.FRESH;
    }
    return hasCycle(visited, touchedNodes);
  }

  private boolean hasCycle(VisitState[] visited,
                           HashMap<Integer, Boolean> touchedNodes) {
    if (touchedNodes.containsKey(lockId)) {
      touchedNodes.put(lockId, true);
    }

    if (visited[lockId] == VisitState.VISITING) {
      /* We looped back to this node while still visiting it, so we know there's
       * a cycle. */
      return true;
    } else if (visited[lockId] == VisitState.FRESH) {
      visited[lockId] = VisitState.VISITING;
      for (LockNode n : children) {
        if (n.hasCycle(visited, touchedNodes)) {
          return true;
        }
      }
      visited[lockId] = VisitState.VISITED;
```

```
    }
    return false;
  }

  public Lock getLock() {
    if (lock == null) lock = new ReentrantLock();
    return lock;
  }

  public int getId() { return lockId; }
}
```

As always, when you see code this complicated and lengthy, you wouldn't be expected to write all of it. More likely, you would be asked to sketch out pseudocode and possibly implement one of these methods.

15.5 Call In Order: Suppose we have the following code:

```
public class Foo {
  public Foo() { ... }
  public void first() { ... }
  public void second() { ... }
  public void third() { ... }
}
```

The same instance of Foo will be passed to three different threads. ThreadA will call first, threadB will call second, and threadC will call third. Design a mechanism to ensure that first is called before second and second is called before third.

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SOLUTION

The general logic is to check if first() has completed before executing second(), and if second() has completed before calling third(). Because we need to be very careful about thread safety, simple boolean flags won't do the job.

What about using a lock to do something like the below code?

```
public class FooBad {
  public int pauseTime = 1000;
  public ReentrantLock lock1, lock2;

  public FooBad() {
    try {
      lock1 = new ReentrantLock();
      lock2 = new ReentrantLock();

      lock1.lock();
      lock2.lock();
    } catch (...) { ... }
  }

  public void first() {
    try {
      ...
      lock1.unlock(); // mark finished with first()
    } catch (...) { ... }
  }
```

```
  public void second() {
    try {
      lock1.lock(); // wait until finished with first()
      lock1.unlock();
      ...

      lock2.unlock(); // mark finished with second()
    } catch (...) { ... }
  }

  public void third() {
    try {
      lock2.lock(); // wait until finished with third()
      lock2.unlock();
      ...
    } catch (...) { ... }
  }
}
```

This code won't actually quite work due to the concept of *lock ownership*. One thread is actually performing the lock (in the FooBad constructor), but different threads attempt to unlock the locks. This is not allowed, and your code will raise an exception. A lock in Java is owned by the same thread which locked it.

Instead, we can replicate this behavior with semaphores. The logic is identical.

```
public class Foo {
  public Semaphore sem1, sem2;

  public Foo() {
    try {
      sem1 = new Semaphore(1);
      sem2 = new Semaphore(1);

      sem1.acquire();
      sem2.acquire();
    } catch (...) { ... }
  }

  public void first() {
    try {
      ...
      sem1.release();
    } catch (...) { ... }
  }

  public void second() {
    try {
      sem1.acquire();
      sem1.release();
      ...
      sem2.release();
    } catch (...) { ... }
  }

  public void third() {
    try {
      sem2.acquire();
```

```
            sem2.release();
            ...
        } catch (...) { ... }
    }
}
```

15.6 Synchronized Methods: You are given a class with `synchronized` method A and a normal method B. If you have two threads in one instance of a program, can they both execute A at the same time? Can they execute A and B at the same time?

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SOLUTION

By applying the word `synchronized` to a method, we ensure that two threads cannot execute synchronized methods *on the same object instance* at the same time.

So, the answer to the first part really depends. If the two threads have the same instance of the object, then no, they cannot simultaneously execute method A. However, if they have different instances of the object, then they can.

Conceptually, you can see this by considering locks. A synchronized method applies a "lock" on *all* synchronized methods in that instance of the object. This blocks other threads from executing synchronized methods within that instance.

In the second part, we're asked if `thread1` can execute synchronized method A while `thread2` is executing non-synchronized method B. Since B is not synchronized, there is nothing to block `thread1` from executing A while `thread2` is executing B. This is true regardless of whether `thread1` and `thread2` have the same instance of the object.

Ultimately, the key concept to remember is that only one synchronized method can be in execution per instance of that object. Other threads can execute non-synchronized methods on that instance, or they can execute any method on a different instance of the object.

15.7 FizzBuzz: In the classic problem FizzBuzz, you are told to print the numbers from 1 to n. However, when the number is divisible by 3, print "Fizz". When it is divisible by 5, print "Buzz". When it is divisible by 3 and 5, print "FizzBuzz". In this problem, you are asked to do this in a multithreaded way. Implement a multithreaded version of FizzBuzz with four threads. One thread checks for divisibility of 3 and prints "Fizz". Another thread is responsible for divisibility of 5 and prints "Buzz". A third thread is responsible for divisibility of 3 and 5 and prints "FizzBuzz". A fourth thread does the numbers.

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SOLUTION

Let's start off with implementing a single threaded version of FizzBuzz.

Single Threaded

Although this problem (in the single threaded version) shouldn't be hard, a lot of candidates overcomplicate it. They look for something "beautiful" that reuses the fact that the divisible by 3 and 5 case ("FizzBuzz") seems to resemble the individual cases ("Fizz" and "Buzz").

In actuality, the best way to do it, considering readability and efficiency, is just the straightforward way.

```
void fizzbuzz(int n) {
```

```
    for (int i = 1; i <= n; i++) {
      if (i % 3 == 0 && i % 5 == 0) {
        System.out.println("FizzBuzz");
      } else if (i % 3 == 0) {
        System.out.println("Fizz");
      } else if (i % 5 == 0) {
        System.out.println("Buzz");
      } else {
        System.out.println(i);
      }
    }
  }
```

The primary thing to be careful of here is the order of the statements. If you put the check for divisibility by 3 before the check for divisibility by 3 and 5, it won't print the right thing.

Multithreaded

To do this multithreaded, we want a structure that looks something like this:

FizzBuzz Thread	Fizz Thread

if i div by 3 && 5
   print FizzBuzz
   increment i
repeat until i > n
	if i div by only 3
	   print Fizz
	   increment i
	repeat until i > n

Buzz Thread	Number Thread

if i div by only 5
   print Buzz
   increment i
repeat until i > n
	if i not div by 3 or 5
	   print i
	   increment i
	repeat until i > n

The code for this will look something like:

```
1  while (true) {
2    if (current > max) {
3      return;
4    }
5    if (/* divisibility test */) {
6      System.out.println(/* print something */);
7      current++;
8    }
9  }
```

We'll need to add some synchronization in the loop. Otherwise, the value of `current` could change between lines 2 - 4 and lines 5 - 8, and we can inadvertently exceed the intended bounds of the loop. Additionally, incrementing is not thread-safe.

To actually implement this concept, there are many possibilities. One possibility is to have four entirely separate thread classes that share a reference to the current variable (which can be wrapped in an object).

The loop for each thread is substantially similar. They just have different target values for the divisibility checks, and different print values.

	FizzBuzz	Fizz	Buzz	Number
current % 3 == 0	true	true	false	false
current % 5 == 0	true	false	true	false
to print	FizzBuzz	Fizz	Buzz	current

For the most part, this can be handled by taking in "target" parameters and the value to print. The output for the `Number` thread needs to be overwritten, though, as it's not a simple, fixed string.

We can implement a `FizzBuzzThread` class which handles most of this. A `NumberThread` class can extend `FizzBuzzThread` and override the `print` method.

```
Thread[] threads = {new FizzBuzzThread(true, true, n, "FizzBuzz"),
                    new FizzBuzzThread(true, false, n, "Fizz"),
                    new FizzBuzzThread(false, true, n, "Buzz"),
                    new NumberThread(false, false, n)};
for (Thread thread : threads) {
  thread.start();
}

public class FizzBuzzThread extends Thread {
  private static Object lock = new Object();
  protected static int current = 1;
  private int max;
  private boolean div3, div5;
  private String toPrint;

  public FizzBuzzThread(boolean div3, boolean div5, int max, String toPrint) {
    this.div3 = div3;
    this.div5 = div5;
    this.max = max;
    this.toPrint = toPrint;
  }

  public void print() {
    System.out.println(toPrint);
  }

  public void run() {
    while (true) {
      synchronized (lock) {
        if (current > max) {
          return;
        }

        if ((current % 3 == 0) == div3 &&
            (current % 5 == 0) == div5) {
          print();
          current++;
        }
      }
    }
  }
}

public class NumberThread extends FizzBuzzThread {
```

```
    public NumberThread(boolean div3, boolean div5, int max) {
        super(div3, div5, max, null);
    }

    public void print() {
        System.out.println(current);
    }
}
```

Observe that we need to put the comparison of `current` and `max` before the if statement, to ensure the value will only get printed when `current` is less than or equal to `max`.

Alternatively, if we're working in a language which supports this (Java 8 and many other languages do), we can pass in a `validate` method and a `print` method as parameters.

```
int n = 100;
Thread[] threads = {
  new FBThread(i -> i % 3 == 0 && i % 5 == 0, i -> "FizzBuzz", n),
  new FBThread(i -> i % 3 == 0 && i % 5 != 0, i -> "Fizz", n),
  new FBThread(i -> i % 3 != 0 && i % 5 == 0, i -> "Buzz", n),
  new FBThread(i -> i % 3 != 0 && i % 5 != 0, i -> Integer.toString(i), n)};
for (Thread thread : threads) {
  thread.start();
}

public class FBThread extends Thread {
  private static Object lock = new Object();
  protected static int current = 1;
  private int max;
  private Predicate<Integer> validate;
  private Function<Integer, String> printer;
  int x = 1;

  public FBThread(Predicate<Integer> validate,
                  Function<Integer, String> printer, int max) {
    this.validate = validate;
    this.printer = printer;
    this.max = max;
  }

  public void run() {
    while (true) {
      synchronized (lock) {
        if (current > max) {
          return;
        }
        if (validate.test(current)) {
          System.out.println(printer.apply(current));
          current++;
        }
      }
    }
  }
}
```

There are of course many other ways of implementing this as well.

16

Solutions to Moderate

16.1 Number Swapper: Write a function to swap a number in place (that is, without temporary variables).

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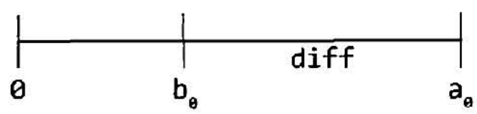
SOLUTION

This is a classic interview problem, and it's a reasonably straightforward one. We'll walk through this using a_0 to indicate the original value of a and b_0 to indicate the original value of b. We'll also use `diff` to indicate the value of a_0 - b_0.

Let's picture these on a number line for the case where a > b.



First, we briefly set a to `diff`, which is the right side of the above number line. Then, when we add b and `diff` (and store that value in b), we get a_0. We now have b = a_0 and a = `diff`. All that's left to do is to set a equal to a_0 - `diff`, which is just b - a.

The code below implements this.

```
// Example for a = 9, b = 4
a = a - b; // a = 9 - 4 = 5
b = a + b; // b = 5 + 4 = 9
a = b - a; // a = 9 - 5
```

We can implement a similar solution with bit manipulation. The benefit of this solution is that it works for more data types than just integers.

```
// Example for a = 101 (in binary) and b = 110
a = a^b; // a = 101^110 = 011
b = a^b; // b = 011^110 = 101
a = a^b; // a = 011^101 = 110
```

This code works by using XORs. The easiest way to see how this works is by focusing on a specific bit. If we can correctly swap two bits, then we know the entire operation works correctly.

Let's take two bits, x and y, and walk through this line by line.

1. `x = x ^ y`

   This line essentially checks if x and y have different values. It will result in 1 if and only if x != y.

2. `y = x ^ y`

Or: `y = {0 if originally same, 1 if different} ^ {original y}`

Observe that XORing a bit with 1 always flips the bit, whereas XORing with 0 will never change it.

Therefore, if we do `y = 1 ^ {original y}` when `x != y`, then y will be flipped and therefore have x's original value.

Otherwise, if `x == y`, then we do `y = 0 ^ {original y}` and the value of y does not change.

Either way, y will be equal to the original value of x.

3. `x = x ^ y`

   Or: `x = {0 if originally same, 1 if different} ^ {original x}`

   At this point, y is equal to the original value of x. This line is essentially equivalent to the line above it, but for different variables.

   If we do `x = 1 ^ {original x}` when the values are different, x will be flipped.

   If we do `x = 0 ^ {original x}` when the values are the same, x will not be changed.

This operation happens for each bit. Since it correctly swaps each bit, it will correctly swap the entire number.

16.2 Word Frequencies: Design a method to find the frequency of occurrences of any given word in a book. What if we were running this algorithm multiple times?

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SOLUTION

Let's start with the simple case.

Solution: Single Query

In this case, we simply go through the book, word by word, and count the number of times that a word appears. This will take O(n) time. We know we can't do better than that since we must look at every word in the book.

```
int getFrequency(String[] book, String word) {
  word = word.trim().toLowerCase();
  int count = 0;
  for (String w : book) {
    if (w.trim().toLowerCase().equals(word)) {
      count++;
    }
  }
  return count;
}
```

We have also converted the string to lowercase and trimmed it. You can discuss with your interviewer if this is necessary (or even desired).

Solution: Repetitive Queries

If we're doing the operation repeatedly, then we can probably afford to take some time and extra memory to do pre-processing on the book. We can create a hash table which maps from a word to its frequency. The frequency of any word can be easily looked up in O(1) time. The code for this is below.

```
HashMap<String, Integer> setupDictionary(String[] book) {
  HashMap<String, Integer> table =
```

```
      new HashMap<String, Integer>();
    for (String word : book) {
      word = word.toLowerCase();
      if (word.trim() != "") {
        if (!table.containsKey(word)) {
          table.put(word, 0);
        }
        table.put(word, table.get(word) + 1);
      }
    }
    return table;
}

int getFrequency(HashMap<String, Integer> table, String word) {
    if (table == null || word == null) return -1;
    word = word.toLowerCase();
    if (table.containsKey(word)) {
      return table.get(word);
    }
    return 0;
}
```

Note that a problem like this is actually relatively easy. Thus, the interviewer is going to be looking heavily at how careful you are. Did you check for error conditions?

16.3 Intersection: Given two straight line segments (represented as a start point and an end point), compute the point of intersection, if any.

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SOLUTION

We first need to think about what it means for two line segments to intersect.

For two infinite lines to intersect, they only have to have different slopes. If they have the same slope, then they must be the exact same line (same y-intercept). That is:

```
slope 1 != slope 2
OR
slope 1 == slope 2 AND intersect 1 == intersect 2
```

For two straight lines to intersect, the condition above must be true, *plus* the point of intersection must be within the ranges of each line segment.

```
extended infinite segments intersect
AND
intersection is within line segment 1 (x and y coordinates)
AND
intersection is within line segment 2 (x and y coordinates)
```

What if the two segments represent the same infinite line? In this case, we have to ensure that some portion of their segments overlap. If we order the line segments by their x locations (start is before end, point 1 is before point 2), then an intersection occurs only if:

```
Assume:
    start1.x < start2.x && start1.x < end1.x && start2.x < end2.x
Then intersection occurs if:
    start2 is between start1 and end1
```

We can now go ahead and implement this algorithm.

```
Point intersection(Point start1, Point end1, Point start2, Point end2) {
  /* Rearranging these so that, in order of x values: start is before end and
   * point 1 is before point 2. This will make some of the later logic simpler. */
  if (start1.x > end1.x) swap(start1, end1);
  if (start2.x > end2.x) swap(start2, end2);
  if (start1.x > start2.x) {
    swap(start1, start2);
    swap(end1, end2);
  }

  /* Compute lines (including slope and y-intercept). */
  Line line1 = new Line(start1, end1);
  Line line2 = new Line(start2, end2);

  /* If the lines are parallel, they intercept only if they have the same y
   * intercept and start 2 is on line 1. */
  if (line1.slope == line2.slope) {
    if (line1.yintercept == line2.yintercept &&
        isBetween(start1, start2, end1)) {
      return start2;
    }
    return null;
  }

  /* Get intersection coordinate. */
  double x = (line2.yintercept - line1.yintercept) / (line1.slope - line2.slope);
  double y = x * line1.slope + line1.yintercept;
  Point intersection = new Point(x, y);

  /* Check if within line segment range. */
  if (isBetween(start1, intersection, end1) &&
      isBetween(start2, intersection, end2)) {
    return intersection;
  }
  return null;
}

/* Checks if middle is between start and end. */
boolean isBetween(double start, double middle, double end) {
  if (start > end) {
    return end <= middle && middle <= start;
  } else {
    return start <= middle && middle <= end;
  }
}

/* Checks if middle is between start and end. */
boolean isBetween(Point start, Point middle, Point end) {
  return isBetween(start.x, middle.x, end.x) &&
         isBetween(start.y, middle.y, end.y);
}

/* Swap coordinates of point one and two. */
void swap(Point one, Point two) {
  double x = one.x;
  double y = one.y;
```

```
    one.setLocation(two.x, two.y);
    two.setLocation(x, y);
}

public class Line {
    public double slope, yintercept;

    public Line(Point start, Point end) {
        double deltaY = end.y - start.y;
        double deltaX = end.x - start.x;
        slope = deltaY / deltaX; // Will be Infinity (not exception) when deltaX = 0
        yintercept = end.y - slope * end.x;
    }

public class Point {
    public double x, y;
    public Point(double x, double y) {
        this.x = x;
        this.y = y;
    }

    public void setLocation(double x, double y) {
        this.x = x;
        this.y = y;
    }
}
```

For simplicity and compactness (it really makes the code easier to read), we've chosen to make the variables within `Point` and `Line` `public`. You can discuss with your interviewer the advantages and disadvantages of this choice.

16.4 Tic Tac Win: Design an algorithm to figure out if someone has won a game of tic-tac-toe.

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SOLUTION

At first glance, this problem seems really straightforward. We're just checking a tic-tac-toe board; how hard could it be? It turns out that the problem is a bit more complex, and there is no single "perfect" answer. The optimal solution depends on your preferences.

There are a few major design decisions to consider:

1. Will `hasWon` be called just once or many times (for instance, as part of a tic-tac-toe website)? If the latter is the case, we may want to add pre-processing time to optimize the runtime of `hasWon`.
2. Do we know the last move that was made?
3. Tic-tac-toe is usually on a 3x3 board. Do we want to design for just that, or do we want to implement it as an NxN solution?
4. In general, how much do we prioritize compactness of code versus speed of execution vs. clarity of code? Remember: The most efficient code may not always be the best. Your ability to understand and maintain the code matters, too.

Solution #1: If `hasWon` is called many times

There are only 3^9, or about 20,000, tic-tac-toe boards (assuming a 3x3 board). Therefore, we can represent our tic-tac-toe board as an `int`, with each digit representing a piece (0 means Empty, 1 means Red, 2 means Blue). We set up a hash table or array in advance with all possible boards as keys and the value indicating who has won. Our function then is simply this:

```
Piece hasWon(int board) {
  return winnerHashtable[board];
}
```

To convert a board (represented by a char array) to an `int`, we can use what is essentially a "base 3" representation. Each board is represented as $3^0v_0 + 3^1v_1 + 3^2v_2 + \ldots + 3^8v_8$, where v_i is a `0` if the space is empty, a 1 if it's a "blue spot" and a 2 if it's a "red spot."

```
enum Piece { Empty, Red, Blue };

int convertBoardToInt(Piece[][] board) {
  int sum = 0;
  for (int i = 0; i < board.length; i++) {
    for (int j = 0; j < board[i].length; j++) {
      /* Each value in enum has an integer associated with it. We
       * can just use that. */
      int value = board[i][j].ordinal();
      sum = sum * 3 + value;
    }
  }
  return sum;
}
```

Now looking up the winner of a board is just a matter of looking it up in a hash table.

Of course, if we need to convert a board into this format every time we want to check for a winner, we haven't saved ourselves any time compared with the other solutions. But, if we can store the board this way from the very beginning, then the lookup process will be very efficient.

Solution #2: If we know the last move

If we know the very last move that was made (and we've been checking for a winner up until now), then we only need to check the row, column, and diagonal that overlaps with this position.

```
Piece hasWon(Piece[][] board, int row, int column) {
  if (board.length != board[0].length) return Piece.Empty;

  Piece piece = board[row][column];

  if (piece == Piece.Empty) return Piece.Empty;

  if (hasWonRow(board, row) || hasWonColumn(board, column)) {
    return piece;
  }

  if (row == column && hasWonDiagonal(board, 1)) {
    return piece;
  }

  if (row == (board.length - column - 1) && hasWonDiagonal(board, -1)) {
    return piece;
  }
```

```java

  return Piece.Empty;
}

boolean hasWonRow(Piece[][] board, int row) {
  for (int c = 1; c < board[row].length; c++) {
    if (board[row][c] != board[row][0]) {
      return false;
    }
  }
  return true;
}

boolean hasWonColumn(Piece[][] board, int column) {
  for (int r = 1; r < board.length; r++) {
    if (board[r][column] != board[0][column]) {
      return false;
    }
  }
  return true;
}

boolean hasWonDiagonal(Piece[][] board, int direction) {
  int row = 0;
  int column = direction == 1 ? 0 : board.length - 1;
  Piece first = board[0][column];
  for (int i = 0; i < board.length; i++) {
    if (board[row][column] != first) {
      return false;
    }
    row += 1;
    column += direction;
  }
  return true;
}
```

There is actually a way to clean up this code to remove some of the duplicated code. We'll see this approach in a later function.

Solution #3: Designing for just a 3x3 board

If we really only want to implement a solution for a 3x3 board, the code is relatively short and simple. The only complex part is trying to be clean and organized, without writing too much duplicated code.

The code below checks each row, column, and diagonal to see if there is a winner.

```java
Piece hasWon(Piece[][] board) {
  for (int i = 0; i < board.length; i++) {
    /* Check Rows */
    if (hasWinner(board[i][0], board[i][1], board[i][2])) {
      return board[i][0];
    }

    /* Check Columns */
    if (hasWinner(board[0][i], board[1][i], board[2][i])) {
      return board[0][i];
    }
```

```
  }

  /* Check Diagonal */
  if (hasWinner(board[0][0], board[1][1], board[2][2])) {
    return board[0][0];
  }

  if (hasWinner(board[0][2], board[1][1], board[2][0])) {
    return board[0][2];
  }

  return Piece.Empty;
}

boolean hasWinner(Piece p1, Piece p2, Piece p3) {
  if (p1 == Piece.Empty) {
    return false;
  }
  return p1 == p2 && p2 == p3;
}
```

This is an okay solution in that it's relatively easy to understand what is going on. The problem is that the values are hard coded. It's easy to accidentally type the wrong indices.

Additionally, it won't be easy to scale this to an NxN board.

Solution #4: Designing for an NxN board

There are a number of ways to implement this on an NxN board.

Nested For-Loops

The most obvious way is through a series of nested for-loops.

```
Piece hasWon(Piece[][] board) {
  int size = board.length;
  if (board[0].length != size) return Piece.Empty;
  Piece first;

  /* Check rows. */
  for (int i = 0; i < size; i++) {
    first = board[i][0];
    if (first == Piece.Empty) continue;
    for (int j = 1; j < size; j++) {
      if (board[i][j] != first) {
        break;
      } else if (j == size - 1) { // Last element
        return first;
      }
    }
  }

  /* Check columns. */
  for (int i = 0; i < size; i++) {
    first = board[0][i];
    if (first == Piece.Empty) continue;
    for (int j = 1; j < size; j++) {
      if (board[j][i] != first) {
```

```
        break;
      } else if (j == size - 1) { // Last element
        return first;
      }
    }
  }

  /* Check diagonals. */
  first = board[0][0];
  if (first != Piece.Empty) {
    for (int i = 1; i < size; i++) {
      if (board[i][i] != first) {
        break;
      } else if (i == size - 1) { // Last element
        return first;
      }
    }
  }

  first = board[0][size - 1];
  if (first != Piece.Empty) {
    for (int i = 1; i < size; i++) {
      if (board[i][size - i - 1] != first) {
        break;
      } else if (i == size - 1) { // Last element
        return first;
      }
    }
  }

  return Piece.Empty;
}
```

This is, to the say the least, pretty ugly. We're doing nearly the same work each time. We should look for a way of reusing the code.

Increment and Decrement Function

One way that we can reuse the code better is to just pass in the values to another function that increments/decrements the rows and columns. The hasWon function now just needs the starting position and the amount to increment the row and column by.

```
class Check {
  public int row, column;
  private int rowIncrement, columnIncrement;
  public Check(int row, int column, int rowI, int colI) {
    this.row = row;
    this.column = column;
    this.rowIncrement = rowI;
    this.columnIncrement = colI;
  }

  public void increment() {
    row += rowIncrement;
    column += columnIncrement;
  }

  public boolean inBounds(int size) {
```

```
        return row >= 0 && column >= 0 && row < size && column < size;
    }
}

Piece hasWon(Piece[][] board) {
    if (board.length != board[0].length) return Piece.Empty;
    int size = board.length;

    /* Create list of things to check. */
    ArrayList<Check> instructions = new ArrayList<Check>();
    for (int i = 0; i < board.length; i++) {
        instructions.add(new Check(0, i, 1, 0));
        instructions.add(new Check(i, 0, 0, 1));
    }
    instructions.add(new Check(0, 0, 1, 1));
    instructions.add(new Check(0, size - 1, 1, -1));

    /* Check them. */
    for (Check instr : instructions) {
        Piece winner = hasWon(board, instr);
        if (winner != Piece.Empty) {
              return winner;
        }
    }
    return Piece.Empty;
}

Piece hasWon(Piece[][] board, Check instr) {
    Piece first = board[instr.row][instr.column];
    while (instr.inBounds(board.length)) {
        if (board[instr.row][instr.column] != first) {
            return Piece.Empty;
        }
        instr.increment();
    }
    return first;
}
```

The Check function is essentially operating as an iterator.

Iterator

Another way of doing it is, of course, to actually build an iterator.

```
Piece hasWon(Piece[][] board) {
    if (board.length != board[0].length) return Piece.Empty;
    int size = board.length;

    ArrayList<PositionIterator> instructions = new ArrayList<PositionIterator>();
    for (int i = 0; i < board.length; i++) {
        instructions.add(new PositionIterator(new Position(0, i), 1, 0, size));
        instructions.add(new PositionIterator(new Position(i, 0), 0, 1, size));
    }
    instructions.add(new PositionIterator(new Position(0, 0), 1, 1, size));
    instructions.add(new PositionIterator(new Position(0, size - 1), 1, -1, size));

    for (PositionIterator iterator : instructions) {
        Piece winner = hasWon(board, iterator);
```

```
      if (winner != Piece.Empty) {
         return winner;
      }
   }
   return Piece.Empty;
}

Piece hasWon(Piece[][] board, PositionIterator iterator) {
   Position firstPosition = iterator.next();
   Piece first = board[firstPosition.row][firstPosition.column];
   while (iterator.hasNext()) {
      Position position = iterator.next();
      if (board[position.row][position.column] != first) {
         return Piece.Empty;
      }
   }
   return first;
}

class PositionIterator implements Iterator<Position> {
   private int rowIncrement, colIncrement, size;
   private Position current;

   public PositionIterator(Position p, int rowIncrement,
                           int colIncrement, int size) {
      this.rowIncrement = rowIncrement;
      this.colIncrement = colIncrement;
      this.size = size;
      current = new Position(p.row - rowIncrement, p.column - colIncrement);
   }

   @Override
   public boolean hasNext() {
      return current.row + rowIncrement < size &&
             current.column + colIncrement < size;
   }

   @Override
   public Position next() {
      current = new Position(current.row + rowIncrement,
                             current.column + colIncrement);
      return current;
   }
}

public class Position {
   public int row, column;
   public Position(int row, int column) {
      this.row = row;
      this.column = column;
   }
}
```

All of this is potentially overkill, but it's worth discussing the options with your interviewer. The point of this problem is to assess your understanding of how to code in a clean and maintainable way.

16.5 Factorial Zeros: Write an algorithm which computes the number of trailing zeros in n factorial.

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SOLUTION

A simple approach is to compute the factorial, and then count the number of trailing zeros by continuously dividing by ten. The problem with this though is that the bounds of an `int` would be exceeded very quickly. To avoid this issue, we can look at this problem mathematically.

Consider a factorial like `19!`:

```
19! = 1*2*3*4*5*6*7*8*9*10*11*12*13*14*15*16*17*18*19
```

A trailing zero is created with multiples of 10, and multiples of 10 are created with pairs of 5-multiples and 2-multiples.

For example, in 19!, the following terms create the trailing zeros:

```
19! = 2 * ... * 5 * ... * 10 * ... * 15 * 16 * ...
```

Therefore, to count the number of zeros, we only need to count the pairs of multiples of 5 and 2. There will always be more multiples of 2 than 5, though, so simply counting the number of multiples of 5 is sufficient.

One "gotcha" here is 15 contributes a multiple of 5 (and therefore one trailing zero), while 25 contributes two (because `25 = 5 * 5`).

There are two different ways to write this code.

The first way is to iterate through all the numbers from 2 through n, counting the number of times that 5 goes into each number.

```
/* If the number is a 5 of five, return which power of 5. For example: 5 -> 1,
 * 25-> 2, etc. */
int factorsOf5(int i) {
  int count = 0;
  while (i % 5 == 0) {
    count++;
    i /= 5;
  }
  return count;
}

int countFactZeros(int num) {
  int count = 0;
  for (int i = 2; i <= num; i++) {
    count += factorsOf5(i);
  }
  return count;
}
```

This isn't bad, but we can make it a little more efficient by directly counting the factors of 5. Using this approach, we would first count the number of multiples of 5 between 1 and n (which is $n/5$), then the number of multiples of 25 ($n/25$), then 125, and so on.

To count how many multiples of m are in n, we can just divide n by m.

```
int countFactZeros(int num) {
  int count = 0;
  if (num < 0) {
    return -1;
  }
```

```
    for (int i = 5; num / i > 0; i *= 5) {
        count += num / i;
    }
    return count;
}
```

This problem is a bit of a brainteaser, but it can be approached logically (as shown above). By thinking through what exactly will contribute a zero, you can come up with a solution. You should be very clear in your rules upfront so that you can implement it correctly.

16.6 Smallest Difference: Given two arrays of integers, compute the pair of values (one value in each array) with the smallest (non-negative) difference. Return the difference.

EXAMPLE

Input: {1, 3, 15, 11, 2}, {23, 127, 235, 19, 8}

Output: 3. That is, the pair (11, 8).

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SOLUTION

Let's start first with a brute force solution.

Brute Force

The simple brute force way is to just iterate through all pairs, compute the difference, and compare it to the current minimum difference.

```
int findSmallestDifference(int[] array1, int[] array2) {
    if (array1.length == 0 || array2.length == 0) return -1;

    int min = Integer.MAX_VALUE;
    for (int i = 0; i < array1.length; i++) {
        for (int j = 0; j < array2.length; j++) {
            if (Math.abs(array1[i] - array2[j]) < min) {
                min = Math.abs(array1[i] - array2[j]);
            }
        }
    }
    return min;
}
```

One minor optimization we could perform from here is to return immediately if we find a difference of zero, since this is the smallest difference possible. However, depending on the input, this might actually be slower.

This will only be faster if there's a pair with difference zero early in the list of pairs. But to add this optimization, we need to execute an additional line of code each time. There's a tradeoff here; it's faster for some inputs and slower for others. Given that it adds complexity in reading the code, it may be best to leave it out.

With or without this "optimization," the algorithm will take O(AB) time.

Optimal

A more optimal approach is to sort the arrays. Once the arrays are sorted, we can find the minimum difference by iterating through the array.

Consider the following two arrays:

```
A: {1, 2, 11, 15}
B: {4, 12, 19, 23, 127, 235}
```

Try the following approach:

1. Suppose a pointer a points to the beginning of A and a pointer b points to the beginning of B. The current difference between a and b is 3. Store this as the `min`.
2. How can we (potentially) make this difference smaller? Well, the value at b is bigger than the value at a, so moving b will only make the difference larger. Therefore, we want to move a.
3. Now a points to 2 and b (still) points to 4. This difference is 2, so we should update `min`. Move a, since it is smaller.
4. Now a points to 11 and b points to 4. Move b.
5. Now a points to 11 and b points to 12. Update `min` to 1. Move b.

And so on.

```
int findSmallestDifference(int[] array1, int[] array2) {
  Arrays.sort(array1);
  Arrays.sort(array2);
  int a = 0;
  int b = 0;
  int difference = Integer.MAX_VALUE;
  while (a < array1.length && b < array2.length) {
    if (Math.abs(array1[a] - array2[b]) < difference) {
      difference = Math.abs(array1[a] - array2[b]);
    }

    /* Move smaller value. */
    if (array1[a] < array2[b]) {
      a++;
    } else {
      b++;
    }
  }
  return difference;
}
```

This algorithm takes `O(A log A + B log B)` time to sort and `O(A + B)` time to find the minimum difference. Therefore, the overall runtime is `O(A log A + B log B)`.

16.7 Number Max: Write a method that finds the maximum of two numbers. You should not use if-else or any other comparison operator.

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SOLUTION

A common way of implementing a `max` function is to look at the sign of `a - b`. In this case, we can't use a comparison operator on this sign, but we *can* use multiplication.

Let k equal the sign of `a - b` such that if `a - b >= 0`, then k is 1. Else, `k = 0`. Let q be the inverse of k.

We can then implement the code as follows:

```
/* Flips a 1 to a 0 and a 0 to a 1 */
int flip(int bit) {
```

```
  return 1^bit;
}

/* Returns 1 if a is positive, and 0 if a is negative */
int sign(int a) {
  return flip((a >> 31) & 0x1);
}

int getMaxNaive(int a, int b) {
  int k = sign(a - b);
  int q = flip(k);
  return a * k + b * q;
}
```

This code almost works. It fails, unfortunately, when a - b overflows. Suppose, for example, that a is INT_MAX - 2 and b is -15. In this case, a - b will be greater than INT_MAX and will overflow, resulting in a negative value.

We can implement a solution to this problem by using the same approach. Our goal is to maintain the condition where k is 1 when a > b. We will need to use more complex logic to accomplish this.

When does a - b overflow? It will overflow only when a is positive and b is negative, or the other way around. It may be difficult to specially detect the overflow condition, but we *can* detect when a and b have different signs. Note that if a and b have different signs, then we want k to equal `sign(a)`.

The logic looks like:

```
if a and b have different signs:
  // if a > 0, then b < 0, and k = 1.
  // if a < 0, then b > 0, and k = 0.
  // so either way, k = sign(a)
  let k = sign(a)
else
  let k = sign(a - b) // overflow is impossible
```

The code below implements this, using multiplication instead of if-statements.

```
int getMax(int a, int b) {
  int c = a - b;

  int sa = sign(a); // if a >= 0, then 1 else 0
  int sb = sign(b); // if b >= 0, then 1 else 0
  int sc = sign(c); // depends on whether or not a - b overflows

  /* Goal: define a value k which is 1 if a > b and 0 if a < b.
   * (if a = b, it doesn't matter what value k is) */

  // If a and b have different signs, then k = sign(a)
  int use_sign_of_a = sa ^ sb;

  // If a and b have the same sign, then k = sign(a - b)
  int use_sign_of_c = flip(sa ^ sb);

  int k = use_sign_of_a * sa + use_sign_of_c * sc;
  int q = flip(k); // opposite of k

  return a * k + b * q;
}
```

Note that for clarity, we split up the code into many different methods and variables. This is certainly not the most compact or efficient way to write it, but it does make what we're doing much cleaner.

16.8 English Int: Given any integer, print an English phrase that describes the integer (e.g., "One Thousand, Two Hundred Thirty Four").

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SOLUTION

This is not an especially challenging problem, but it is a somewhat tedious one. The key is to be organized in how you approach the problem—and to make sure you have good test cases.

We can think about converting a number like 19,323,984 as converting each of three 3-digit segments of the number, and inserting "thousands" and "millions" in between as appropriate. That is,

```
convert(19,323,984) = convert(19) + " million " + convert(323) + " thousand " +
                      convert(984)
```

The code below implements this algorithm.

```
String[] smalls = {"Zero", "One", "Two", "Three", "Four", "Five", "Six", "Seven",
  "Eight", "Nine", "Ten", "Eleven", "Twelve", "Thirteen", "Fourteen", "Fifteen",
  "Sixteen", "Seventeen", "Eighteen", "Nineteen"};
String[] tens = {"", "", "Twenty", "Thirty", "Forty", "Fifty", "Sixty", "Seventy",
  "Eighty", "Ninety"};
String[] bigs = {"", "Thousand", "Million", "Billion"};
String hundred = "Hundred";
String negative = "Negative";

String convert(int num) {
  if (num == 0) {
    return smalls[0];
  } else if (num < 0) {
    return negative + " " + convert(-1 * num);
  }

  LinkedList<String> parts = new LinkedList<String>();
  int chunkCount = 0;

  while (num > 0) {
    if (num % 1000 != 0) {
      String chunk = convertChunk(num % 1000) + " " + bigs[chunkCount];
      parts.addFirst(chunk);
    }
    num /= 1000; // shift chunk
    chunkCount++;
  }

  return listToString(parts);
}

String convertChunk(int number) {
  LinkedList<String> parts = new LinkedList<String>();

  /* Convert hundreds place */
  if (number >= 100) {
    parts.addLast(smalls[number / 100]);
```

```
      parts.addLast(hundred);
      number %= 100;
    }

    /* Convert tens place */
    if (number >= 10 && number <= 19) {
      parts.addLast(smalls[number]);
    } else if (number >= 20) {
      parts.addLast(tens[number / 10]);
      number %= 10;
    }

    /* Convert ones place */
    if (number >= 1 && number <= 9) {
      parts.addLast(smalls[number]);
    }

    return listToString(parts);
  }
  /* Convert a linked list of strings to a string, dividing it up with spaces. */
  String listToString(LinkedList<String> parts) {
    StringBuilder sb = new StringBuilder();
    while (parts.size() > 1) {
      sb.append(parts.pop());
      sb.append(" ");
    }
    sb.append(parts.pop());
    return sb.toString();
  }
```

The key in a problem like this is to make sure you consider all the special cases. There are a lot of them.

16.9 **Operations:** Write methods to implement the multiply, subtract, and divide operations for integers. The results of all of these are integers. Use only the add operator.

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SOLUTION

The only operation we have to work with is the add operator. In each of these problems, it's useful to think in depth about what these operations really do or how to phrase them in terms of other operations (either add or operations we've already completed).

Subtraction

How can we phrase subtraction in terms of addition? This one is pretty straightforward. The operation `a - b` is the same thing as `a + (-1) * b`. However, because we are not allowed to use the `*` (multiply) operator, we must implement a `negate` function.

```
/* Flip a positive sign to negative or negative sign to pos. */
int negate(int a) {
  int neg = 0;
  int newSign = a < 0 ? 1 : -1;
  while (a != 0) {
    neg += newSign;
    a += newSign;
  }
```

```
    return neg;
}

/* Subtract two numbers by negating b and adding them */
int minus(int a, int b) {
    return a + negate(b);
}
```

The negation of the value k is implemented by adding -1 k times. Observe that this will take `O(k)` time.

If optimizing is something we value here, we can try to get a to zero faster. (For this explanation, we'll assume that a is positive.) To do this, we can first reduce a by 1, then 2, then 4, then 8, and so on. We'll call this value `delta`. We want a to reach exactly zero. When reducing a by the next `delta` would change the sign of a, we reset `delta` back to 1 and repeat the process.

For example:

```
a:      29   28   26   22   14   13   11    7    6    4    0
delta:    -1   -2   -4   -8   -1   -2   -4   -1   -2   -4
```

The code below implements this algorithm.

```
int negate(int a) {
    int neg = 0;
    int newSign = a < 0 ? 1 : -1;
    int delta = newSign;
    while (a != 0) {
        boolean differentSigns = (a + delta > 0) != (a > 0);
        if (a + delta != 0 && differentSigns) { // If delta is too big, reset it.
            delta = newSign;
        }
        neg += delta;
        a += delta;
        delta += delta; // Double the delta
    }
    return neg;
}
```

Figuring out the runtime here takes a bit of calculation.

Observe that reducing a by half takes `O(log a)` work. Why? For each round of "reduce a by half", the absolute values of a and `delta` always add up to the same number. The values of `delta` and a will converge at $\frac{a}{2}$. Since `delta` is being doubled each time, it will take `O(log a)` steps to reach half of a.

We do O(log a) rounds.

1. Reducing a to $\frac{a}{2}$ takes `O(log a)` time.
2. Reducing $\frac{a}{2}$ to $\frac{a}{4}$ takes $O(\log \frac{a}{2})$ time.
3. Reducing $\frac{a}{4}$ to $\frac{a}{8}$ takes $O(\log \frac{a}{4})$ time.

... As so on, for `O(log a)` rounds.

The runtime therefore is $O(\log a + \log(\frac{a}{2}) + \log(\frac{a}{4}) + \ldots)$, with `O(log a)` terms in the expression.

Recall two rules of logs:

- $\log(xy) = \log x + \log y$
- $\log(\frac{x}{y}) = \log x - \log y$.

If we apply this to the above expression, we get:

1. `O(log a + log(`$\frac{a}{2}$`) + log(`$\frac{a}{4}$`) + ...)`
2. `O(log a + (log a - log 2) + (log a - log 4) + (log a - log 8) + ...`
3. `O((log a)*(log a) - (log 2 + log 4 + log 8 + ... + log a))` // `O(log a)` terms
4. `O((log a)*(log a) - (1 + 2 + 3 + ... + log a))` // computing the values of logs
5. `O((log a)*(log a) - `$\frac{(\log a)(1+\log a)}{2}$`)` // apply equation for sum of 1 through k
6. `O((log a)`^{2}`)` // drop second term from step 5

Therefore, the runtime is `O((log a)`^{2}`)`.

This math is considerably more complicated than most people would be able to do (or expected to do) in an interview. You could make a simplification: You do `O(log a)` rounds and the longest round takes `O(log a)` work. Therefore, as an upper bound, `negate` takes `O((log a)`^{2}`)` time. In this case, the upper bound happens to be the true time.

There are some faster solutions too. For example, rather than resetting `delta` to 1 at each round, we could change `delta` to its previous value. This would have the effect of `delta` "counting up" by multiples of two, and then "counting down" by multiples of two. The runtime of this approach would be `O(log a)`. However, this implementation would require a stack, division, or bit shifting—any of which might violate the spirit of the problem. You could certainly discuss those implementations with your interviewer though.

Multiplication

The connection between addition and multiplication is equally straightforward. To multiply a by b, we just add a to itself b times.

```
/* Multiply a by b by adding a to itself b times */
int multiply(int a, int b) {
  if (a < b) {
    return multiply(b, a); // algorithm is faster if b < a
  }
  int sum = 0;
  for (int i = abs(b); i > 0; i = minus(i, 1)) {
    sum += a;
  }
  if (b < 0) {
    sum = negate(sum);
  }
  return sum;
}

/* Return absolute value */
int abs(int a) {
  if (a < 0) {
    return negate(a);
  } else {
    return a;
  }
}
```

The one thing we need to be careful of in the above code is to properly handle multiplication of negative numbers. If b is negative, we need to flip the value of `sum`. So, what this code really does is:

```
multiply(a, b) <-- abs(b) * a * (-1 if b < 0).
```

We also implemented a simple `abs` function to help.

Division

Of the three operations, division is certainly the hardest. The good thing is that we can use the `multiply`, `subtract`, and `negate` methods now to implement `divide`.

We are trying to compute x where $x = a/b$. Or, to put this another way, find x where a = bx. We've now changed the problem into one that can be stated with something we know how to do: multiplication.

We could implement this by multiplying b by progressively higher values, until we reach a. That would be fairly inefficient, particularly given that our implementation of `multiply` involves a lot of adding.

Alternatively, we can look at the equation a = xb to see that we can compute x by adding b to itself repeatedly until we reach a. The number of times we need to do that will equal x.

Of course, a might not be evenly divisible by b, and that's okay. Integer division, which is what we've been asked to implement, is supposed to truncate the result.

The code below implements this algorithm.

```
int divide(int a, int b) throws java.lang.ArithmeticException {
  if (b == 0) {
    throw new java.lang.ArithmeticException("ERROR");
  }
  int absa = abs(a);
  int absb = abs(b);

  int product = 0;
  int x = 0;
  while (product + absb <= absa) { /* don't go past a */
    product += absb;
    x++;
  }

  if ((a < 0 && b < 0) || (a > 0 && b > 0)) {
    return x;
  } else {
    return negate(x);
  }
}
```

In tackling this problem, you should be aware of the following:

- A logical approach of going back to what exactly multiplication and division do comes in handy. Remember that. All (good) interview problems can be approached in a logical, methodical way!
- The interviewer is looking for this sort of logical work-your-way-through-it approach.
- This is a great problem to demonstrate your ability to write clean code—specifically, to show your ability to reuse code. For example, if you were writing this solution and didn't put `negate` in its own method, you should move it into its own method once you see that you'll use it multiple times.
- Be careful about making assumptions while coding. Don't assume that the numbers are all positive or that a is bigger than b.

16.10 Living People: Given a list of people with their birth and death years, implement a method to compute the year with the most number of people alive. You may assume that all people were born between 1900 and 2000 (inclusive). If a person was alive during any portion of that year, they should be included in that year's count. For example, Person (birth = 1908, death = 1909) is included in the counts for both 1908 and 1909.

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SOLUTION

The first thing we should do is outline what this solution will look like. The interview question hasn't specified the exact form of input. In a real interview, we could ask the interviewer how the input is structured. Alternatively, you can explicitly state your (reasonable) assumptions.

Here, we'll need to make our own assumptions. We will assume that we have an array of simple `Person` objects:

```
public class Person {
  public int birth;
  public int death;
  public Person(int birthYear, int deathYear) {
    birth = birthYear;
    death = deathYear;
  }
}
```

We could have also given `Person` a `getBirthYear()` and `getDeathYear()` objects. Some would argue that's better style, but for compactness and clarity, we'll just keep the variables public.

The important thing here is to actually use a `Person` object. This shows better style than, say, having an integer array for birth years and an integer array for death years (with an implicit association of `births[i]` and `deaths[i]` being associated with the same person). You don't get a lot of chances to demonstrate great coding style, so it's valuable to take the ones you get.

With that in mind, let's start with a brute force algorithm.

Brute Force

The brute force algorithm falls directly out from the wording of the problem. We need to find the year with the most number of people alive. Therefore, we go through each year and check how many people are alive in that year.

```
int maxAliveYear(Person[] people, int min, int max) {
  int maxAlive = 0;
  int maxAliveYear = min;

  for (int year = min; year <= max; year++) {
    int alive = 0;
    for (Person person : people) {
      if (person.birth <= year && year <= person.death) {
        alive++;
      }
    }
    if (alive > maxAlive) {
      maxAlive = alive;
      maxAliveYear = year;
    }
  }
```

```

  return maxAliveYear;
}
```

Note that we have passed in the values for the min year (1900) and max year (2000). We shouldn't hard code these values.

The runtime of this is O(RP), where R is the range of years (100 in this case) and P is the number of people.

Slightly Better Brute Force

A slightly better way of doing this is to create an array where we track the number of people born in each year. Then, we iterate through the list of people and increment the array for each year they are alive.

```
1  int maxAliveYear(Person[] people, int min, int max) {
2    int[] years = createYearMap(people, min, max);
3    int best = getMaxIndex(years);
4    return best + min;
5  }
6
7  /* Add each person's years to a year map. */
8  int[] createYearMap(Person[] people, int min, int max) {
9    int[] years = new int[max - min + 1];
10   for (Person person : people) {
11     incrementRange(years, person.birth - min, person.death - min);
12   }
13   return years;
14 }
15
16 /* Increment array for each value between left and right. */
17 void incrementRange(int[] values, int left, int right) {
18   for (int i = left; i <= right; i++) {
19     values[i]++;
20   }
21 }
22
23 /* Get index of largest element in array. */
24 int getMaxIndex(int[] values) {
25   int max = 0;
26   for (int i = 1; i < values.length; i++) {
27     if (values[i] > values[max]) {
28       max = i;
29     }
30   }
31   return max;
32 }
```

Be careful on the size of the array in line 9. If the range of years is 1900 to 2000 inclusive, then that's 101 years, not 100. That is why the array has size `max - min + 1`.

Let's think about the runtime by breaking this into parts.

- We create an R-sized array, where R is the min and max years.
- Then, for P people, we iterate through the years (Y) that the person is alive.
- Then, we iterate through the R-sized array again.

The total runtime is O(PY + R). In the worst case, Y is R and we have done no better than we did in the first algorithm.

More Optimal

Let's create an example. (In fact, an example is really helpful in almost all problems. Ideally, you've already done this.) Each column below is matched, so that the items correspond to the same person. For compactness, we'll just write the last two digits of the year.

```
birth: 12  20  10  01  10  23  13  90  83  75
death: 15  90  98  72  98  82  98  98  99  94
```

It's worth noting that it doesn't really matter whether these years are matched up. Every birth adds a person and every death removes a person.

Since we don't actually need to match up the births and deaths, let's sort both. A sorted version of the years might help us solve the problem.

```
birth: 01  10  10  12  13  20  23  75  83  90
death: 15  72  82  90  94  98  98  98  98  99
```

We can try walking through the years.

- At year 0, no one is alive.
- At year 1, we see one birth.
- At years 2 through 9, nothing happens.
- Let's skip ahead until year 10, when we have two births. We now have three people alive.
- At year 15, one person dies. We are now down to two people alive.
- And so on.

If we walk through the two arrays like this, we can track the number of people alive at each point.

```
int maxAliveYear(Person[] people, int min, int max) {
   int[] births = getSortedYears(people, true);
   int[] deaths = getSortedYears(people, false);

   int birthIndex = 0;
   int deathIndex = 0;
   int currentlyAlive = 0;
   int maxAlive = 0;
   int maxAliveYear = min;

   /* Walk through arrays. */
   while (birthIndex < births.length) {
      if (births[birthIndex] <= deaths[deathIndex]) {
         currentlyAlive++; // include birth
         if (currentlyAlive > maxAlive) {
            maxAlive = currentlyAlive;
            maxAliveYear = births[birthIndex];
         }
         birthIndex++; // move birth index
      } else if (births[birthIndex] > deaths[deathIndex]) {
         currentlyAlive--; // include death
         deathIndex++; // move death index
      }
   }

   return maxAliveYear;
}

/* Copy birth years or death years (depending on the value of copyBirthYear into
```

```
30   * integer array, then sort array. */
31  int[] getSortedYears(Person[] people, boolean copyBirthYear) {
32    int[] years = new int[people.length];
33    for (int i = 0; i < people.length; i++) {
34      years[i] = copyBirthYear ? people[i].birth : people[i].death;
35    }
36    Arrays.sort(years);
37    return years;
38  }
```

There are some very easy things to mess up here.

On line 13, we need to think carefully about whether this should be a less than (<) or a less than or equals (<=). The scenario we need to worry about is that you see a birth and death in the same year. (It doesn't matter whether the birth and death is from the same person.)

When we see a birth and death from the same year, we want to include the birth *before* we include the death, so that we count this person as alive for that year. That is why we use a <= on line 13.

We also need to be careful about where we put the updating of `maxAlive` and `maxAliveYear`. It needs to be after the `currentAlive++`, so that it takes into account the updated total. But it needs to be before `birthIndex++`, or we won't have the right year.

This algorithm will take `O(P log P)` time, where P is the number of people.

More Optimal (Maybe)

Can we optimize this further? To optimize this, we'd need to get rid of the sorting step. We're back to dealing with unsorted values:

```
birth: 12  20  10  01  10  23  13  90  83  75
death: 15  90  98  72  98  82  98  98  99  94
```

Earlier, we had logic that said that a birth is just adding a person and a death is just subtracting a person. Therefore, let's represent the data using the logic:

```
01: +1     10: +1     10: +1     12: +1     13: +1
15: -1     20: +1     23: +1     72: -1     75: +1
82: -1     83: +1     90: +1     90: -1     94: -1
98: -1     98: -1     98: -1     98: -1     99: -1
```

We can create an array of the years, where the value at `array[year]` indicates how the population changed in that year. To create this array, we walk through the list of people and increment when they're born and decrement when they die.

Once we have this array, we can walk through each of the years, tracking the current population as we go (adding the value at `array[year]` each time).

This logic is reasonably good, but we should think about it more. Does it really work?

One edge case we should consider is when a person dies the same year that they're born. The increment and decrement operations will cancel out to give 0 population change. According to the wording of the problem, this person should be counted as living in that year.

In fact, the "bug" in our algorithm is broader than that. This same issue applies to all people. People who die in 1908 shouldn't be removed from the population count until 1909.

There's a simple fix: instead of decrementing `array[deathYear]`, we should decrement `array[deathYear + 1]`.

```
int maxAliveYear(Person[] people, int min, int max) {
```

```
  /* Build population delta array. */
  int[] populationDeltas = getPopulationDeltas(people, min, max);
  int maxAliveYear = getMaxAliveYear(populationDeltas);
  return maxAliveYear + min;
}

/* Add birth and death years to deltas array. */
int[] getPopulationDeltas(Person[] people, int min, int max) {
  int[] populationDeltas = new int[max - min + 2];
  for (Person person : people) {
    int birth = person.birth - min;
    populationDeltas[birth]++;

    int death = person.death - min;
    populationDeltas[death + 1]--;
  }
  return populationDeltas;
}

/* Compute running sums and return index with max. */
int getMaxAliveYear(int[] deltas) {
  int maxAliveYear = 0;
  int maxAlive = 0;
  int currentlyAlive = 0;
  for (int year = 0; year < deltas.length; year++) {
    currentlyAlive += deltas[year];
    if (currentlyAlive > maxAlive) {
      maxAliveYear = year;
      maxAlive = currentlyAlive;
    }
  }

  return maxAliveYear;
}
```

This algorithm takes `O(R + P)` time, where R is the range of years and P is the number of people. Although `O(R + P)` might be faster than `O(P log P)` for many expected inputs, you cannot directly compare the speeds to say that one is faster than the other.

16.11 Diving Board: You are building a diving board by placing a bunch of planks of wood end-to-end. There are two types of planks, one of length `shorter` and one of length `longer`. You must use exactly K planks of wood. Write a method to generate all possible lengths for the diving board.

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SOLUTION

One way to approach this is to think about the choices we make as we're building a diving board. This leads us to a recursive algorithm.

Recursive Solution

For a recursive solution, we can imagine ourselves building a diving board. We make K decisions, each time choosing which plank we will put on next. Once we've put on K planks, we have a complete diving board and we can add this to the list (assuming we haven't seen this length before).

We can follow this logic to write recursive code. Note that we don't need to track the sequence of planks. All we need to know is the current length and the number of planks remaining.

```
HashSet<Integer> allLengths(int k, int shorter, int longer) {
  HashSet<Integer> lengths = new HashSet<Integer>();
  getAllLengths(k, 0, shorter, longer, lengths);
  return lengths;
}

void getAllLengths(int k, int total, int shorter, int longer,
                   HashSet<Integer> lengths) {
  if (k == 0) {
    lengths.add(total);
    return;
  }
  getAllLengths(k - 1, total + shorter, shorter, longer, lengths);
  getAllLengths(k - 1, total + longer, shorter, longer, lengths);
}
```

We've added each length to a hash set. This will automatically prevent adding duplicates.

This algorithm takes $O(2^K)$ time, since there are two choices at each recursive call and we recurse to a depth of K.

Memoization Solution

As in many recursive algorithms (especially those with exponential runtimes), we can optimize this through memorization (a form of dynamic programming).

Observe that some of the recursive calls will be essentially equivalent. For example, picking plank 1 and then plank 2 is equivalent to picking plank 2 and then plank 1.

Therefore, if we've seen this `(total, plank count)` pair before then we stop this recursive path. We can do this using a `HashSet` with a key of `(total, plank count)`.

> Many candidates will make a mistake here. Rather than stopping only when they've seen `(total, plank count)`, they'll stop whenever they've seen just `total` before. This is incorrect. Seeing two planks of length 1 is not the same thing as one plank of length 2, because there are different numbers of planks remaining. In memoization problems, be very careful about what you choose for your key.

The code for this approach is very similar to the earlier approach.

```
HashSet<Integer> allLengths(int k, int shorter, int longer) {
  HashSet<Integer> lengths = new HashSet<Integer>();
  HashSet<String> visited = new HashSet<String>();
  getAllLengths(k, 0, shorter, longer, lengths, visited);
  return lengths;
}

void getAllLengths(int k, int total, int shorter, int longer,
                   HashSet<Integer> lengths, HashSet<String> visited) {
  if (k == 0) {
    lengths.add(total);
    return;
  }
  String key = k + " " + total;
```

```
    if (visited.contains(key)) {
      return;
    }
    getAllLengths(k - 1, total + shorter, shorter, longer, lengths, visited);
    getAllLengths(k - 1, total + longer, shorter, longer, lengths, visited);
    visited.add(key);
  }
```

For simplicity, we've set the key to be a string representation of `total` and the current plank count. Some people may argue it's better to use a data structure to represent this pair. There are benefits to this, but there are drawbacks as well. It's worth discussing this tradeoff with your interviewer.

The runtime of this algorithm is a bit tricky to figure out.

One way we can think about the runtime is by understanding that we're basically filling in a table of SUMS x PLANK COUNTS. The biggest possible sum is K * LONGER and the biggest possible plank count is K. Therefore, the runtime will be no worse than O(K^2 * LONGER).

Of course, a bunch of those sums will never actually be reached. How many unique sums can we get? Observe that any path with the same number of each type of planks will have the same sum. Since we can have at most K planks of each type, there are only K different sums we can make. Therefore, the table is really KxK, and the runtime is O(K^2).

Optimal Solution

If you re-read the prior paragraph, you might notice something interesting. There are only K distinct sums we can get. Isn't that the whole point of the problem—to find all possible sums?

We don't actually need to go through all arrangements of planks. We just need to go through all unique sets of K planks (sets, not orders!). There are only K ways of picking K planks if we only have two possible types: {0 of type A, K of type B}, {1 of type A, K-1 of type B}, {2 of type A, K-2 of type B}, ...

This can be done in just a simple for loop. At each "sequence", we just compute the sum.

```
HashSet<Integer> allLengths(int k, int shorter, int longer) {
  HashSet<Integer> lengths = new HashSet<Integer>();
  for (int nShorter = 0; nShorter <= k; nShorter++) {
    int nLonger = k - nShorter;
    int length = nShorter * shorter + nLonger * longer;
    lengths.add(length);
  }
  return lengths;
}
```

We've used a `HashSet` here for consistency with the prior solutions. This isn't really necessary though, since we shouldn't get any duplicates. We could instead use an `ArrayList`. If we do this, though, we just need to handle an edge case where the two types of planks are the same length. In this case, we would just return an `ArrayList` of size 1.

16.12 XML Encoding: Since XML is very verbose, you are given a way of encoding it where each tag gets mapped to a pre-defined integer value. The language/grammar is as follows:

```
Element   --> Tag Attributes END Children END
Attribute --> Tag Value
END       --> 0
Tag       --> some predefined mapping to int
Value     --> string value
```

For example, the following XML might be converted into the compressed string below (assuming a mapping of `family -> 1, person ->2, firstName -> 3, lastName -> 4, state -> 5`).

```
<family lastName="McDowell" state="CA">
  <person firstName="Gayle">Some Message</person>
</family>
```

Becomes:

```
1 4 McDowell 5 CA 0 2 3 Gayle 0 Some Message 0 0
```

Write code to print the encoded version of an XML element (passed in `Element` and `Attribute` objects).

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SOLUTION

Since we know the element will be passed in as an `Element` and `Attribute`, our code is reasonably simple. We can implement this by applying a tree-like approach.

We repeatedly call `encode()` on parts of the XML structure, handling the code in slightly different ways depending on the type of the XML element.

```
void encode(Element root, StringBuilder sb) {
  encode(root.getNameCode(), sb);
  for (Attribute a : root.attributes) {
    encode(a, sb);
  }
  encode("0", sb);
  if (root.value != null && root.value != "") {
    encode(root.value, sb);
  } else {
    for (Element e : root.children) {
      encode(e, sb);
    }
  }
  encode("0", sb);
}

void encode(String v, StringBuilder sb) {
  sb.append(v);
  sb.append(" ");
}

void encode(Attribute attr, StringBuilder sb) {
  encode(attr.getTagCode(), sb);
  encode(attr.value, sb);
}

```

```
27 String encodeToString(Element root) {
28    StringBuilder sb = new StringBuilder();
29    encode(root, sb);
30    return sb.toString();
31 }
```

Observe in line 17, the use of the very simple encode method for a string. This is somewhat unnecessary; all it does is insert the string and a space following it. However, using this method is a nice touch as it ensures that every element will be inserted with a space surrounding it. Otherwise, it might be easy to break the encoding by forgetting to append the empty string.

16.13 Bisect Squares: Given two squares on a two-dimensional plane, find a line that would cut these two squares in half. Assume that the top and the bottom sides of the square run parallel to the x-axis.

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SOLUTION

Before we start, we should think about what exactly this problem means by a "line." Is a line defined by a slope and a y-intercept? Or by any two points on the line? Or, should the line be really a line segment, which starts and ends at the edges of the squares?

We will assume, since it makes the problem a bit more interesting, that we mean the third option: that the line should end at the edges of the squares. In an interview situation, you should discuss this with your interviewer.

This line that cuts two squares in half must connect the two middles. We can easily calculate the slope, knowing that slope= $\frac{y1-y2}{x1-x2}$. Once we calculate the slope using the two middles, we can use the same equation to calculate the start and end points of the line segment.

In the below code, we will assume the origin (0, 0) is in the upper left-hand corner.

```
public class Square {
   ...
   public Point middle() {
      return new Point((this.left + this.right) / 2.0,
                       (this.top + this.bottom) / 2.0);
   }

   /* Return the point where the line segment connecting mid1 and mid2 intercepts
    * the edge of square 1. That is, draw a line from mid2 to mid1, and continue it
    * out until the edge of the square. * /
   public Point extend(Point mid1, Point mid2, double size) {
      /* Find what direction the line mid2 -> mid1 goes. * /
      double xdir = mid1.x < mid2.x ? -1 : 1;
      double ydir = mid1.y < mid2.y ? -1 : 1;

      /* If mid1 and mid2 have the same x value, then the slope calculation will
       * throw a divide by 0 exception. So, we compute this specially. * /
      if (mid1.x == mid2.x) {
         return new Point(mid1.x, mid1.y + ydir * size / 2.0);
      }

      double slope = (mid1.y - mid2.y) / (mid1.x - mid2.x);
      double x1 = 0;
      double y1 = 0;

```

```
    /* Calculate slope using the equation (y1 - y2) / (x1 - x2).
     * Note: if the slope is "steep" (>1) then the end of the line segment will
     * hit size / 2 units away from the middle on the y axis. If the slope is
     * "shallow" (<1) the end of the line segment will hit size / 2 units away
     * from the middle on the x axis. * /
    if (Math.abs(slope) == 1) {
      x1 = mid1.x + xdir * size / 2.0;
      y1 = mid1.y + ydir * size / 2.0;
    } else if (Math.abs(slope) < 1) { // shallow slope
      x1 = mid1.x + xdir * size / 2.0;
      y1 = slope * (x1 - mid1.x) + mid1.y;
    } else { // steep slope
      y1 = mid1.y + ydir * size / 2.0;
      x1 = (y1 - mid1.y) / slope + mid1.x;
    }
    return new Point(x1, y1);
  }

  public Line cut(Square other) {
    /* Calculate where a line between each middle would collide with the edges of
     * the squares * /
    Point p1 = extend(this.middle(), other.middle(), this.size);
    Point p2 = extend(this.middle(), other.middle(), -1 * this.size);
    Point p3 = extend(other.middle(), this.middle(), other.size);
    Point p4 = extend(other.middle(), this.middle(), -1 * other.size);

    /* Of above points, find start and end of lines. Start is farthest left (with
     * top most as a tie breaker) and end is farthest right (with bottom most as
     * a tie breaker. * /
    Point start = p1;
    Point end = p1;
    Point[] points = {p2, p3, p4};
    for (int i = 0; i < points.length; i++) {
      if (points[i].x < start.x ||
          (points[i].x == start.x && points[i].y < start.y)) {
        start = points[i];
      } else if (points[i].x > end.x ||
                 (points[i].x == end.x && points[i].y > end.y)) {
        end = points[i];
      }
    }

    return new Line(start, end);
  }
```

The main goal of this problem is to see how careful you are about coding. It's easy to glance over the special cases (e.g., the two squares having the same middle). You should make a list of these special cases before you start the problem and make sure to handle them appropriately. This is a question that requires careful and thorough testing.

16.14 Best Line: Given a two-dimensional graph with points on it, find a line which passes the most number of points.

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SOLUTION

This solution seems quite straightforward at first. And it is—sort of.

We just"draw" an infinite line (that is, not a line segment) between every two points and, using a hash table, track which line is the most common. This will take $O(N^2)$ time, since there are N^2 line segments.

We will represent a line as a slope and y-intercept (as opposed to a pair of points), which allows us to easily check to see if the line from `(x1, y1)` to `(x2, y2)` is equivalent to the line from `(x3, y3)` to `(x4, y4)`.

To find the most common line then, we just iterate through all lines segments, using a hash table to count the number of times we've seen each line. Easy enough!

However, there's one little complication. We're defining two lines to be equal if the lines have the same slope and y-intercept. We are then, furthermore, hashing the lines based on these values (specifically, based on the slope). The problem is that floating point numbers cannot always be represented accurately in binary. We resolve this by checking if two floating point numbers are within an `epsilon` value of each other.

What does this mean for our hash table? It means that two lines with "equal" slopes may not be hashed to the same value. To solve this, we will round the slope down to the next epsilon and use this `flooredSlope` as the hash key. Then, to retrieve all lines that are *potentially* equal, we will search the hash table at three spots: `flooredSlope`, `flooredSlope - epsilon`, and `flooredSlope + epsilon`. This will ensure that we've checked out all lines that might be equal.

```
/* Find line that goes through most number of points. */
Line findBestLine(GraphPoint[] points) {
  HashMapList<Double, Line> linesBySlope = getListOfLines(points);
  return getBestLine(linesBySlope);
}

/* Add each pair of points as a line to the list. */
HashMapList<Double, Line> getListOfLines(GraphPoint[] points) {
  HashMapList<Double, Line> linesBySlope = new HashMapList<Double, Line>();
  for (int i = 0; i < points.length; i++) {
    for (int j = i + 1; j < points.length; j++) {
      Line line = new Line(points[i], points[j]);
      double key = Line.floorToNearestEpsilon(line.slope);
      linesBySlope.put(key, line);
    }
  }
  return linesBySlope;
}

/* Return the line with the most equivalent other lines. */
Line getBestLine(HashMapList<Double, Line> linesBySlope) {
  Line bestLine = null;
  int bestCount = 0;

  Set<Double> slopes = linesBySlope.keySet();

  for (double slope : slopes) {
```

```
    ArrayList<Line> lines = linesBySlope.get(slope);
    for (Line line : lines) {
      /* count lines that are equivalent to current line */
      int count = countEquivalentLines(linesBySlope, line);

      /* if better than current line, replace it */
      if (count > bestCount) {
        bestLine = line;
        bestCount = count;
        bestLine.Print();
        System.out.println(bestCount);
      }
    }
  }
  return bestLine;
}

/* Check hashmap for lines that are equivalent. Note that we need to check one
 * epsilon above and below the actual slope since we're defining two lines as
 * equivalent if they're within an epsilon of each other. */
int countEquivalentLines(HashMapList<Double, Line> linesBySlope, Line line) {
  double key = Line.floorToNearestEpsilon(line.slope);
  int count = countEquivalentLines(linesBySlope.get(key), line);
  count += countEquivalentLines(linesBySlope.get(key - Line.epsilon), line);
  count += countEquivalentLines(linesBySlope.get(key + Line.epsilon), line);
  return count;
}

/* Count lines within an array of lines which are "equivalent" (slope and
 * y-intercept are within an epsilon value) to a given line */
int countEquivalentLines(ArrayList<Line> lines, Line line) {
  if (lines == null) return 0;

  int count = 0;
  for (Line parallelLine : lines) {
    if (parallelLine.isEquivalent(line)) {
      count++;
    }
  }
  return count;
}

public class Line {
  public static double epsilon = .0001;
  public double slope, intercept;
  private boolean infinite_slope = false;

  public Line(GraphPoint p, GraphPoint q) {
    if (Math.abs(p.x - q.x) > epsilon) { // if x's are different
      slope = (p.y - q.y) / (p.x - q.x); // compute slope
      intercept = p.y - slope * p.x; // y intercept from y=mx+b
    } else {
      infinite_slope = true;
      intercept = p.x; // x-intercept, since slope is infinite
    }
  }
```

```

  public static double floorToNearestEpsilon(double d) {
    int r = (int) (d / epsilon);
    return ((double) r) * epsilon;
  }

  public boolean isEquivalent(double a, double b) {
    return (Math.abs(a - b) < epsilon);
  }

  public boolean isEquivalent(Object o) {
    Line l = (Line) o;
    if (isEquivalent(l.slope, slope) && isEquivalent(l.intercept, intercept) &&
        (infinite_slope == l.infinite_slope)) {
      return true;
    }
    return false;
  }
}

/* HashMapList<String, Integer> is a HashMap that maps from Strings to
 * ArrayList<Integer>. See appendix for implementation. */
```

We need to be careful about the calculation of the slope of a line. The line might be completely vertical, which means that it doesn't have a y-intercept and its slope is infinite. We can keep track of this in a separate flag (`infinite_slope`). We need to check this condition in the equals method.

16.15 Master Mind: The Game of Master Mind is played as follows:

The computer has four slots, and each slot will contain a ball that is red (R), yellow (Y), green (G) or blue (B). For example, the computer might have RGGB (Slot #1 is red, Slots #2 and #3 are green, Slot #4 is blue).

You, the user, are trying to guess the solution. You might, for example, guess YRGB.

When you guess the correct color for the correct slot, you get a "hit." If you guess a color that exists but is in the wrong slot, you get a "pseudo-hit." Note that a slot that is a hit can never count as a pseudo-hit.

For example, if the actual solution is RGBY and you guess GGRR, you have one hit and one pseudo-hit.

Write a method that, given a guess and a solution, returns the number of hits and pseudo-hits.

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SOLUTION

This problem is straightforward, but it's surprisingly easy to make little mistakes. You should check your code *extremely* thoroughly, on a variety of test cases.

We'll implement this code by first creating a frequency array which stores how many times each character occurs in `solution`, excluding times when the slot is a "hit." Then, we iterate through `guess` to count the number of pseudo-hits.

The code below implements this algorithm.

```
class Result {
  public int hits = 0;
```

```
  public int pseudoHits = 0;

  public String toString() {
    return "(" + hits + ", " + pseudoHits + ")";
  }
}

int code(char c) {
  switch (c) {
  case 'B':
    return 0;
  case 'G':
    return 1;
  case 'R':
    return 2;
  case 'Y':
    return 3;
  default:
    return -1;
  }
}

int MAX_COLORS = 4;

Result estimate(String guess, String solution) {
  if (guess.length() != solution.length()) return null;

  Result res = new Result();
  int[] frequencies = new int[MAX_COLORS];

  /* Compute hits and build frequency table */
  for (int i = 0; i < guess.length(); i++) {
    if (guess.charAt(i) == solution.charAt(i)) {
      res.hits++;
    } else {
      /* Only increment the frequency table (which will be used for pseudo-hits)
       * if it's not a hit. If it's a hit, the slot has already been "used." */
      int code = code(solution.charAt(i));
      frequencies[code]++;
    }
  }

  /* Compute pseudo-hits */
  for (int i = 0; i < guess.length(); i++) {
    int code = code(guess.charAt(i));
    if (code >= 0 && frequencies[code] > 0 &&
        guess.charAt(i) != solution.charAt(i)) {
      res.pseudoHits++;
      frequencies[code]--;
    }
  }
  return res;
}
```

Note that the easier the algorithm for a problem is, the more important it is to write clean and correct code. In this case, we've pulled `code(char c)` into its own method, and we've created a `Result` class to hold the result, rather than just printing it.

16.16 Sub Sort: Given an array of integers, write a method to find indices m and n such that if you sorted elements m through n, the entire array would be sorted. Minimize n - m (that is, find the smallest such sequence).

EXAMPLE

Input: `1, 2, 4, 7, 10, 11, 7, 12, 6, 7, 16, 18, 19`

Output: `(3, 9)`

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SOLUTION

Before we begin, let's make sure we understand what our answer will look like. If we're looking for just two indices, this indicates that some middle section of the array will be sorted, with the start and end of the array already being in order.

Now, let's approach this problem by looking at an example.

```
1, 2, 4, 7, 10, 11, 8, 12, 5, 6, 16, 18, 19
```

Our first thought might be to just find the longest increasing subsequence at the beginning and the longest increasing subsequence at the end.

```
left:   1, 2, 4, 7, 10, 11
middle: 8, 12
right:  5, 6, 16, 18, 19
```

These subsequences are easy to generate. We just start from the left and the right sides, and work our way inward. When an element is out of order, then we have found the end of our increasing/decreasing subsequence.

In order to solve our problem, though, we would need to be able to sort the middle part of the array and, by doing just that, get all the elements in the array in order. Specifically, the following would have to be true:

```
/* all items on left are smaller than all items in middle */
min(middle) > end(left)

/* all items in middle are smaller than all items in right */
max(middle) < start(right)
```

Or, in other words, for all elements:

```
left < middle < right
```

In fact, this condition will *never* be met. The middle section is, by definition, the elements that were out of order. That is, it is *always* the case that `left.end > middle.start` and `middle.end > right.start`. Thus, you cannot sort the middle to make the entire array sorted.

But, what we can do is *shrink* the left and right subsequences until the earlier conditions are met. We need the left part to be smaller than all the elements in the middle and right side, and the right part to be bigger than all the elements on the left and right side.

Let `min` equal `min(middle and right side)` and `max` equal `max(middle and left side)`. Observe that since the right and left sides are already in sorted order, we only actually need to check their start or end point.

On the left side, we start with the end of the subsequence (value 11, at element 5) and move to the left. The value `min` equals 5. Once we find an element `i` such that `array[i] < min`, we know that we could sort the middle and have that part of the array appear in order.

Then, we do a similar thing on the right side. The value max equals 12. So, we begin with the start of the right subsequence (value 6) and move to the right. We compare the max of 12 to 6, then 7, then 16. When reach 16, we know that no elements smaller than 12 could be after it (since it's an increasing subsequence). Thus, the middle of the array could now be sorted to make the entire array sorted.

The following code implements this algorithm.

```
void findUnsortedSequence(int[] array) {
  // find left subsequence
  int end_left = findEndOfLeftSubsequence(array);
  if (end_left >= array.length - 1) return; // Already sorted

  // find right subsequence
  int start_right = findStartOfRightSubsequence(array);

  // get min and max
  int max_index = end_left; // max of left side
  int min_index = start_right; // min of right side
  for (int i = end_left + 1; i < start_right; i++) {
    if (array[i] < array[min_index]) min_index = i;
    if (array[i] > array[max_index]) max_index = i;
  }

  // slide left until less than array[min_index]
  int left_index = shrinkLeft(array, min_index, end_left);

  // slide right until greater than array[max_index]
  int right_index = shrinkRight(array, max_index, start_right);

  System.out.println(left_index + " " + right_index);
}

int findEndOfLeftSubsequence(int[] array) {
  for (int i = 1; i < array.length; i++) {
    if (array[i] < array[i - 1]) return i - 1;
  }
  return array.length - 1;
}

int findStartOfRightSubsequence(int[] array) {
  for (int i = array.length - 2; i >= 0; i--) {
    if (array[i] > array[i + 1]) return i + 1;
  }
  return 0;
}

int shrinkLeft(int[] array, int min_index, int start) {
  int comp = array[min_index];
  for (int i = start - 1; i >= 0; i--) {
    if (array[i] <= comp) return i + 1;
  }
  return 0;
}

int shrinkRight(int[] array, int max_index, int start) {
  int comp = array[max_index];
  for (int i = start; i < array.length; i++) {
```

```
        if (array[i] >= comp) return i - 1;
    }
    return array.length - 1;
}
```

Note the use of other methods in this solution. Although we could have jammed it all into one method, it would have made the code a lot harder to understand, maintain, and test. In your interview coding, you should prioritize these aspects.

16.17 Contiguous Sequence: You are given an array of integers (both positive and negative). Find the contiguous sequence with the largest sum. Return the sum.

EXAMPLE

Input: `2, -8, 3, -2, 4, -10`

Output: `5 (i.e., {3, -2, 4})`

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SOLUTION

This is a challenging problem, but an extremely common one. Let's approach this by looking at an example:

```
2    3    -8    -1    2    4    -2    3
```

If we think about our array as having alternating sequences of positive and negative numbers, we can observe that we would never include only part of a negative subsequence or part of a positive sequence. Why would we? Including part of a negative subsequence would make things unnecessarily negative, and we should just instead not include that negative sequence at all. Likewise, including only part of a positive subsequence would be strange, since the sum would be even bigger if we included the whole thing.

For the purposes of coming up with our algorithm, we can think about our array as being a sequence of alternating negative and positive numbers. Each number corresponds to the sum of a subsequence of positive numbers of a subsequence of negative numbers. For the array above, our new reduced array would be:

```
5    -9    6    -2    3
```

This doesn't give away a great algorithm immediately, but it does help us to better understand what we're working with.

Consider the array above. Would it ever make sense to have `{5, -9}` in a subsequence? No. These numbers sum to `-4`, so we're better off not including either number, or possibly just having the sequence be just `{5}`).

When would we want negative numbers included in a subsequence? Only if it allows us to join two positive subsequences, each of which have a sum greater than the negative value.

We can approach this in a step-wise manner, starting with the first element in the array.

When we look at 5, this is the biggest sum we've seen so far. We set `maxSum` to 5, and `sum` to 5. Then, we consider -9. If we added it to `sum`, we'd get a negative value. There's no sense in extending the subsequence from 5 to -9 (which "reduces" to a sequence of just -4), so we just reset the value of `sum`.

Now, we consider 6. This subsequence is greater than 5, so we update both `maxSum` and `sum`.

Next, we look at -2. Adding this to 6 will set `sum` to 4. Since this is still a "value add" (when adjoined to another, bigger sequence), we *might* want `{6, -2}` in our max subsequence. We'll update `sum`, but not `maxSum`.

Finally, we look at 3. Adding 3 to sum (4) gives us 7, so we update maxSum. The max subsequence is therefore the sequence {6, -2, 3}.

When we look at this in the fully expanded array, our logic is identical. The code below implements this algorithm.

```
int getMaxSum(int[] a) {
  int maxsum = 0;
  int sum = 0;
  for (int i = 0; i < a.length; i++) {
    sum += a[i];
    if (maxsum < sum) {
      maxsum = sum;
    } else if (sum < 0) {
      sum = 0;
    }
  }
  return maxsum;
}
```

If the array is all negative numbers, what is the correct behavior? Consider this simple array: {-3, -10, -5}. You could make a good argument that the maximum sum is either:

1. -3 (if you assume the subsequence can't be empty)
2. 0 (the subsequence has length 0)
3. MINIMUM_INT (essentially, the error case).

We went with option #2 (maxSum = 0), but there's no "correct" answer. This is a great thing to discuss with your interviewer; it will show how detail-oriented you are.

16.18 Pattern Matching: You are given two strings, pattern and value. The pattern string consists of just the letters a and b, describing a pattern within a string. For example, the string catcatgocatgo matches the pattern aabab (where cat is a and go is b). It also matches patterns like a, ab, and b. Write a method to determine if value matches pattern.

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SOLUTION

As always, we can start with a simple brute force approach.

Brute Force

A brute force algorithm is to just try all possible values for a and b and then check if this works.

We could do this by iterating through all substrings for a and all possible substrings for b. There are $O(n^2)$ substrings in a string of length n, so this will actually take $O(n^4)$ time. But then, for each value of a and b, we need to build the new string of this length and compare it for equality. This building/comparison step takes $O(n)$ time, giving an overall runtime of $O(n^5)$.

```
for each possible substring a
  for each possible substring b
    candidate = buildFromPattern(pattern, a, b)
    if candidate equals value
      return true
```

Ouch.

One easy optimization is to notice that if the pattern starts with 'a', then the a string must start at the beginning of `value`. (Otherwise, the b *string must start at the beginning of* `value`*.)* Therefore, there aren't $O(n^2)$ possible values for a; there are $O(n)$.

The algorithm then is to check if the pattern starts with a or b. If it starts with b, we can "invert" it (flipping each 'a' to a 'b' and each 'b' to an 'a') so that it starts with 'a'. Then, iterate through all possible substrings for a (each of which must begin at index 0) and all possible substrings for b (each of which must begin at some character after the end of a). As before, we then compare the string for this pattern with the original string.

This algorithm now takes $O(n^4)$ time.

There's one more minor (optional) optimization we can make. We don't actually need to do this "inversion" if the string starts with 'b' instead of 'a'. The `buildFromPattern` method can take care of this. We can think about the first character in the pattern as the "main" item and the other character as the alternate character. The `buildFromPattern` method can build the appropriate string based on whether 'a' is the main character or alternate character.

```
boolean doesMatch(String pattern, String value) {
   if (pattern.length() == 0) return value.length() == 0;

   int size = value.length();
   for (int mainSize = 0; mainSize < size; mainSize++) {
      String main = value.substring(0, mainSize);
      for (int altStart = mainSize; altStart <= size; altStart++) {
         for (int altEnd = altStart; altEnd <= size; altEnd++) {
            String alt = value.substring(altStart, altEnd);
            String cand = buildFromPattern(pattern, main, alt);
            if (cand.equals(value)) {
               return true;
            }
         }
      }
   }
   return false;
}

String buildFromPattern(String pattern, String main, String alt) {
   StringBuffer sb = new StringBuffer();
   char first = pattern.charAt(0);
   for (char c : pattern.toCharArray()) {
      if (c == first) {
         sb.append(main);
      } else {
         sb.append(alt);
      }
   }
   return sb.toString();
}
```

We should look for a more optimal algorithm.

Optimized

Let's think through our current algorithm. Searching through all values for the main string is fairly fast (it takes $O(n)$ time). It's the alternate string that is so slow: $O(n^2)$ time. We should study how to optimize that.

Suppose we have a pattern like aabab and we're comparing it to the string catcatgocatgo. Once we've picked "cat" as the value for a to try, then the a strings are going to take up nine characters (three a strings with length three each). Therefore, the b strings must take up the remaining four characters, with each having length two. Moreover, we actually know exactly where they must occur, too. If a is cat, and the pattern is aabab, then b must be go.

In other words, once we've picked a, we've picked b too. There's no need to iterate. Gathering some basic stats on pattern (number of as, number of bs, first occurrence of each) and iterating through values for a (or whichever the main string is) will be sufficient.

```
boolean doesMatch(String pattern, String value) {
  if (pattern.length() == 0) return value.length() == 0;

  char mainChar = pattern.charAt(0);
  char altChar = mainChar == 'a' ? 'b' : 'a';
  int size = value.length();

  int countOfMain = countOf(pattern, mainChar);
  int countOfAlt = pattern.length() - countOfMain;
  int firstAlt = pattern.indexOf(altChar);
  int maxMainSize = size / countOfMain;

  for (int mainSize = 0; mainSize <= maxMainSize; mainSize++) {
    int remainingLength = size - mainSize * countOfMain;
    String first = value.substring(0, mainSize);
    if (countOfAlt == 0 || remainingLength % countOfAlt == 0) {
      int altIndex = firstAlt * mainSize;
      int altSize = countOfAlt == 0 ? 0 : remainingLength / countOfAlt;
      String second = countOfAlt == 0 ? "" :
                      value.substring(altIndex, altSize + altIndex);

      String cand = buildFromPattern(pattern, first, second);
      if (cand.equals(value)) {
        return true;
      }
    }
  }
  return false;
}

int countOf(String pattern, char c) {
  int count = 0;
  for (int i = 0; i < pattern.length(); i++) {
    if (pattern.charAt(i) == c) {
      count++;
    }
  }
  return count;
}

String buildFromPattern(...) {  /* same as before */ }
```

This algorithm takes $O(n^2)$, since we iterate through $O(n)$ possibilities for the main string and do $O(n)$ work to build and compare the strings.

Observe that we've also cut down the possibilities for the main string that we try. If there are three instances of the main string, then its length cannot be any more than one third of value.

Optimized (Alternate)

If you don't like the work of building a string only to compare it (and then destroy it), we can eliminate this.

Instead, we can iterate through the values for a and b as before. But this time, to check if the string matches the pattern (given those values for a and b), we walk through `value`, comparing each substring to the first instance of the a and b strings.

```
boolean doesMatch(String pattern, String value) {
  if (pattern.length() == 0) return value.length() == 0;

  char mainChar = pattern.charAt(0);
  char altChar = mainChar == 'a' ? 'b' : 'a';
  int size = value.length();

  int countOfMain = countOf(pattern, mainChar);
  int countOfAlt = pattern.length() - countOfMain;
  int firstAlt = pattern.indexOf(altChar);
  int maxMainSize = size / countOfMain;

  for (int mainSize = 0; mainSize <= maxMainSize; mainSize++) {
    int remainingLength = size - mainSize * countOfMain;
    if (countOfAlt == 0 || remainingLength % countOfAlt == 0) {
      int altIndex = firstAlt * mainSize;
      int altSize = countOfAlt == 0 ? 0 : remainingLength / countOfAlt;
      if (matches(pattern, value, mainSize, altSize, altIndex)) {
        return true;
      }
    }
  }
  return false;
}

/* Iterates through pattern and value. At each character within pattern, checks if
 * this is the main string or the alternate string. Then checks if the next set of
 * characters in value match the original set of those characters (either the main
 * or the alternate. */
boolean matches(String pattern, String value, int mainSize, int altSize,
                int firstAlt) {
  int stringIndex = mainSize;
  for (int i = 1; i < pattern.length(); i++) {
    int size = pattern.charAt(i) == pattern.charAt(0) ? mainSize : altSize;
    int offset = pattern.charAt(i) == pattern.charAt(0) ? 0 : firstAlt;
    if (!isEqual(value, offset, stringIndex, size)) {
      return false;
    }
    stringIndex += size;
  }
  return true;
}

/* Checks if two substrings are equal, starting at given offsets and continuing to
 * size. */
boolean isEqual(String s1, int offset1, int offset2, int size) {
  for (int i = 0; i < size; i++) {
    if (s1.charAt(offset1 + i) != s1.charAt(offset2 + i)) {
      return false;
```

```
        }
    }
    return true;
}
```

This algorithm will still take $O(n^2)$ time, but the benefit is that it can short circuit when matches fail early (which they usually will). The previous algorithm must go through all the work to build the string before it can learn that it has failed.

16.19 Pond Sizes: You have an integer matrix representing a plot of land, where the value at that location represents the height above sea level. A value of zero indicates water. A pond is a region of water connected vertically, horizontally, or diagonally. The size of the pond is the total number of connected water cells. Write a method to compute the sizes of all ponds in the matrix.

EXAMPLE

Input:

```
0 2 1 0
0 1 0 1
1 1 0 1
0 1 0 1
```

Output: 2, 4, 1 (in any order)

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SOLUTION

The first thing we can try is just walking through the array. It's easy enough to find water: when it's a zero, that's water.

Given a water cell, how can we compute the amount of water nearby? If the cell is not adjacent to any zero cells, then the size of this pond is 1. If it is, then we need to add in the adjacent cells, plus any water cells adjacent to those cells. We need to, of course, be careful to not recount any cells. We can do this with a modified breadth-first or depth-first search. Once we visit a cell, we permanently mark it as visited.

For each cell, we need to check eight adjacent cells. We could do this by writing in lines to check up, down, left, right, and each of the four diagonal cells. It's even easier, though, to do this with a loop.

```
ArrayList<Integer> computePondSizes(int[][] land) {
  ArrayList<Integer> pondSizes = new ArrayList<Integer>();
  for (int r = 0; r < land.length; r++) {
    for (int c = 0; c < land[r].length; c++) {
      if (land[r][c] == 0) { // Optional. Would return anyway.
        int size = computeSize(land, r, c);
        pondSizes.add(size);
      }
    }
  }
  return pondSizes;
}

int computeSize(int[][] land, int row, int col) {
  /* If out of bounds or already visited. */
  if (row < 0 || col < 0 || row >= land.length || col >= land[row].length ||
      land[row][col] != 0) { // visited or not water
    return 0;
  }
```

```
    int size = 1;
    land[row][col] = -1; // Mark visited
    for (int dr = -1; dr <= 1; dr++) {
      for (int dc = -1; dc <= 1; dc++) {
        size += computeSize(land, row + dr, col + dc);
      }
    }
    return size;
  }
```

In this case, we marked a cell as visited by setting its value to -1. This allows us to check, in one line (`land[row][col] != 0`), if the value is valid dry land or visited. In either case, the value will be zero.

You might also notice that the for loop iterates through nine cells, not eight. It includes the current cell. We could add a line in there to not recurse if `dr == 0` and `dc == 0`. This really doesn't save us much. We'll execute this if-statement in eight cells unnecessarily, just to avoid one recursive call. The recursive call returns immediately since the cell is marked as visited.

If you don't like modifying the input matrix, you can create a secondary `visited` matrix.

```
ArrayList<Integer> computePondSizes(int[][] land) {
  boolean[][] visited = new boolean[land.length][land[0].length];
  ArrayList<Integer> pondSizes = new ArrayList<Integer>();
  for (int r = 0; r < land.length; r++) {
    for (int c = 0; c < land[r].length; c++) {
      int size = computeSize(land, visited, r, c);
      if (size > 0) {
        pondSizes.add(size);
      }
    }
  }
  return pondSizes;
}

int computeSize(int[][] land, boolean[][] visited, int row, int col) {
  /* If out of bounds or already visited. */
  if (row < 0 || col < 0 || row >= land.length || col >= land[row].length ||
      visited[row][col] || land[row][col] != 0) {
    return 0;
  }
  int size = 1;
  visited[row][col] = true;
  for (int dr = -1; dr <= 1; dr++) {
    for (int dc = -1; dc <= 1; dc++) {
      size += computeSize(land, visited, row + dr, col + dc);
    }
  }
  return size;
}
```

Both implementations are $O(WH)$, where W is the width of the matrix and H is the height.

> Note: Many people say "$O(N)$" or "$O(N^2)$", as though N has some inherent meaning. It doesn't. Suppose this were a square matrix. You could describe the runtime as $O(N)$ or $O(N^2)$. Both are correct, depending on what you mean by N. The runtime is $O(N^2)$, where N is the length of one side. Or, if N is the number of cells, it is $O(N)$. Be careful by what you mean by N. In fact, it might be safer to just not use N at all when there's any ambiguity as to what it could mean.

Some people will miscompute the runtime to be $O(N^4)$, reasoning that the `computeSize` method could take as long as $O(N^2)$ time and you might call it as much as $O(N^2)$ times (and apparently assuming an NxN matrix, too). While those are both basically correct statements, you can't just multiply them together. That's because as a single call to `computeSize` gets more expensive, the number of times it is called goes down.

For example, suppose the very first call to `computeSize` goes through the entire matrix. That might take $O(N^2)$ time, but then we never call `computeSize` again.

Another way to compute this is to think about how many times each cell is "touched" by either call. Each cell will be touched once by the `computePondSizes` function. Additionally, a cell might be touched once by each of its adjacent cells. This is still a constant number of touches per cell. Therefore, the overall runtime is $O(N^2)$ on an NxN matrix or, more generally, $O(WH)$.

16.20 **T9:** On old cell phones, users typed on a numeric keypad and the phone would provide a list of words that matched these numbers. Each digit mapped to a set of 0 - 4 letters. Implement an algorithm to return a list of matching words, given a sequence of digits. You are provided a list of valid words (provided in whatever data structure you'd like). The mapping is shown in the diagram below:

1	**2** abc	**3** def
4 ghi	**5** jkl	**6** mno
7 pqrs	**8** tuv	**9** wxyz
	0	

EXAMPLE

Input: 8733

Output: `tree, used`

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SOLUTION

We could approach this in a couple of ways. Let's start with a brute force algorithm.

Brute Force

Imagine how you would solve the problem if you had to do it by hand. You'd probably try every possible value for each digit with all other possible values.

This is exactly what we do algorithmically. We take the first digit and run through all the characters that map to that digit. For each character, we add it to a `prefix` variable and recurse, passing the prefix downward. Once we run out of characters, we print `prefix` (which now contains the full word) if the string is a valid word.

We will assume the list of words is passed in as a `HashSet`. A `HashSet` operates similarly to a hash table, but rather than offering key->value lookups, it can tell us if a word is contained in the set in $O(1)$ time.

```
ArrayList<String> getValidT9Words(String number, HashSet<String> wordList) {
  ArrayList<String> results = new ArrayList<String>();
  getValidWords(number, 0, "", wordList, results);
  return results;
}

```

```
void getValidWords(String number, int index, String prefix,
                   HashSet<String> wordSet, ArrayList<String> results) {
  /* If it’s a complete word, print it. */
  if (index == number.length() && wordSet.contains(prefix)) {
    results.add(prefix);
    return;
  }

  /* Get characters that match this digit. */
  char digit = number.charAt(index);
  char[] letters = getT9Chars(digit);

  /* Go through all remaining options. */
  if (letters != null) {
    for (char letter : letters) {
      getValidWords(number, index + 1, prefix + letter, wordSet, results);
    }
  }
}

/* Return array of characters that map to this digit. */
char[] getT9Chars(char digit) {
  if (!Character.isDigit(digit)) {
    return null;
  }
  int dig = Character.getNumericValue(digit) - Character.getNumericValue(‘0’);
  return t9Letters[dig];
}

/* Mapping of digits to letters. */
char[][] t9Letters = {null, null, {‘a’, ‘b’, ‘c’}, {‘d’, ‘e’, ‘f’},
  {‘g’, ‘h’, ‘i’}, {‘j’, ‘k’, ‘l’}, {‘m’, ‘n’, ‘o’}, {‘p’, ‘q’, ‘r’, ‘s’},
  {‘t’, ‘u’, ‘v’},  {‘w’, ‘x’, ‘y’, ‘z’}
};
```

This algorithm runs in $O(4^N)$ time, where N is the length of the string. This is because we recursively branch four times for each call to `getValidWords`, and we recurse until a call stack depth of N.

This is very, very slow on large strings.

Optimized

Let's return to thinking about how you would do this, if you were doing it by hand. Imagine the example of 33835676368 (which corresponds to `development`). If you were doing this by hand, I bet you'd skip over solutions that start with `fftf` [3383], as no valid words start with those characters.

Ideally, we'd like our program to make the same sort of optimization: stop recursing down paths which will obviously fail. Specifically, if there are no words in the dictionary that start with `prefix`, stop recursing.

The `Trie` data structure (see "Tries (Prefix Trees)" on page 105) can do this for us. Whenever we reach a string which is not a valid prefix, we exit.

```
ArrayList<String> getValidT9Words(String number, Trie trie) {
  ArrayList<String> results = new ArrayList<String>();
  getValidWords(number, 0, “”, trie.getRoot(), results);
  return results;
}

```

```
void getValidWords(String number, int index, String prefix, TrieNode trieNode,
                   ArrayList<String> results) {
  /* If it's a complete word, print it. */
  if (index == number.length()) {
    if (trieNode.terminates()) { // Is complete word
      results.add(prefix);
    }
    return;
  }

  /* Get characters that match this digit */
  char digit = number.charAt(index);
  char[] letters = getT9Chars(digit);

  /* Go through all remaining options. */
  if (letters != null) {
    for (char letter : letters) {
      TrieNode child = trieNode.getChild(letter);
      /* If there are words that start with prefix + letter,
       * then continue recursing. */
      if (child != null) {
        getValidWords(number, index + 1, prefix + letter, child, results);
      }
    }
  }
}
```

It's difficult to describe the runtime of this algorithm since it depends on what the language looks like. However, this "short-circuiting" will make it run much, much faster in practice.

Most Optimal

Believe or not, we can actually make it run even faster. We just need to do a little bit of preprocessing. That's not a big deal though. We were doing that to build the trie anyway.

This problem is asking us to list all the words represented by a particular number in T9. Instead of trying to do this "on the fly" (and going through a lot of possibilities, many of which won't actually work), we can just do this in advance.

Our algorithm now has a few steps:

Pre-Computation:

1. Create a hash table that maps from a sequence of digits to a list of strings.
2. Go through each word in the dictionary and convert it to its T9 representation (e.g., `APPLE -> 27753`). Store each of these in the above hash table. For example, 8733 would map to `{used, tree}`.

Word Lookup:

1. Just look up the entry in the hash table and return the list.

That's it!

```
/* WORD LOOKUP */
ArrayList<String> getValidT9Words(String numbers,
                                  HashMapList<String, String> dictionary) {
  return dictionary.get(numbers);
}

```

```
/* PRECOMPUTATION */

/* Create a hash table that maps from a number to all words that have this
 * numerical representation. */
HashMapList<String, String> initializeDictionary(String[] words) {
  /* Create a  hash table that maps from a letter to the digit */
  HashMap<Character, Character> letterToNumberMap = createLetterToNumberMap();

  /* Create word -> number map. */
  HashMapList<String, String> wordsToNumbers = new HashMapList<String, String>();
  for (String word : words) {
    String numbers = convertToT9(word, letterToNumberMap);
    wordsToNumbers.put(numbers, word);
  }
  return wordsToNumbers;
}

/* Convert mapping of number->letters into letter->number. */
HashMap<Character, Character> createLetterToNumberMap() {
  HashMap<Character, Character> letterToNumberMap =
    new HashMap<Character, Character>();
  for (int i = 0; i < t9Letters.length; i++) {
    char[] letters = t9Letters[i];
    if (letters != null) {
      for (char letter : letters) {
        char c = Character.forDigit(i, 10);
        letterToNumberMap.put(letter, c);
      }
    }
  }
  return letterToNumberMap;
}

/* Convert from a string to its T9 representation. */
String convertToT9(String word, HashMap<Character, Character> letterToNumberMap) {
  StringBuilder sb = new StringBuilder();
  for (char c : word.toCharArray()) {
    if (letterToNumberMap.containsKey(c)) {
      char digit = letterToNumberMap.get(c);
      sb.append(digit);
    }
  }
  return sb.toString();
}

char[][] t9Letters = /* Same as before */

/* HashMapList<String, Integer> is a HashMap that maps from Strings to
 * ArrayList<Integer>. See appendix for implementation. */
```

Getting the words that map to this number will run in O(N) time, where N is the number of digits. The O(N) comes in during the hash table look up (we need to convert the number to a hash table). If you know the words are never longer than a certain max size, then you could also describe the runtime as O(1).

Note that it's easy to think, "Oh, linear—that's not that fast." But it depends what it's linear *on*. Linear on the length of the word is extremely fast. Linear on the length of the dictionary is not so fast.

16.21 Sum Swap: Given two arrays of integers, find a pair of values (one value from each array) that you can swap to give the two arrays the same sum.

EXAMPLE

Input: `{4, 1, 2, 1, 1, 2}` and `{3, 6, 3, 3}`

Output: `{1, 3}`

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SOLUTION

We should start by trying to understand what exactly we're looking for.

We have two arrays and their sums. Although we likely aren't given their sums upfront, we can just act like we are for now. After all, computing the sum is an O(N) operation and we know we can't beat O(N) anyway. Computing the sum, therefore, won't impact the runtime.

When we move a (positive) value a from array A to array B, then the sum of A drops by a and the sum of B increases by a.

We are looking for two values, a and b, such that:

```
sumA - a + b = sumB - b + a
```

Doing some quick math:

```
2a - 2b = sumA - sumB
a - b = (sumA - sumB) / 2
```

Therefore, we're looking for two values that have a specific target difference: `(sumA - sumB) / 2`.

Observe that because that the target must be an integer (after all, you can't swap two integers to get a non-integer difference), we can conclude that the difference between the sums must be even to have a valid pair.

Brute Force

A brute force algorithm is simple enough. We just iterate through the arrays and check all pairs of values.

We can either do this the "naive" way (compare the new sums) or by looking for a pair with that difference.

Naive approach:

```
int[] findSwapValues(int[] array1, int[] array2) {
  int sum1 = sum(array1);
  int sum2 = sum(array2);

  for (int one : array1) {
    for (int two : array2) {
      int newSum1 = sum1 - one + two;
      int newSum2 = sum2 - two + one;
      if (newSum1 == newSum2) {
        int[] values = {one, two};
        return values;
      }
    }
  }

  return null;
}
```

Target approach:

```
int[] findSwapValues(int[] array1, int[] array2) {
```

```
  Integer target = getTarget(array1, array2);
  if (target == null) return null;

  for (int one : array1) {
    for (int two : array2) {
      if (one - two == target) {
        int[] values = {one, two};
        return values;
      }
    }
  }

  return null;
}

Integer getTarget(int[] array1, int[] array2) {
  int sum1 = sum(array1);
  int sum2 = sum(array2);

  if ((sum1 - sum2) % 2 != 0) return null;
  return (sum1 - sum2) / 2;
}
```

We've used an `Integer` (a boxed data type) as the return value for `getTarget`. This allows us to distinguish an "error" case.

This algorithm takes O(AB) time.

Optimal Solution

This problem reduces to finding a pair of values that have a particular difference. With that in mind, let's revisit what the brute force does.

In the brute force, we're looping through A and then, for each element, looking for an element in B which gives us the "right" difference. If the value in A is 5 and the target is 3, then we must be looking for the value 2. That's the only value that could fulfill the goal.

That is, rather than writing `one - two == target`, we could have written `two == one - target`. How can we more quickly find an element in B that equals `one - target`?

We can do this very quickly with a hash table. We just throw all the elements in B into a hash table. Then, iterate through A and look for the appropriate element in B.

```
int[] findSwapValues(int[] array1, int[] array2) {
  Integer target = getTarget(array1, array2);
  if (target == null) return null;
  return findDifference(array1, array2, target);
}

/* Find a pair of values with a specific difference. */
int[] findDifference(int[] array1, int[] array2, int target) {
  HashSet<Integer> contents2 = getContents(array2);
  for (int one : array1) {
    int two = one - target;
    if (contents2.contains(two)) {
      int[] values = {one, two};
      return values;
    }
```

```
  }

  return null;
}

/* Put contents of array into hash set. */
HashSet<Integer> getContents(int[] array) {
  HashSet<Integer> set = new HashSet<Integer>();
  for (int a : array) {
    set.add(a);
  }
  return set;
}
```

This solution will take O(A+B) time. This is the Best Conceivable Runtime (BCR), since we have to at least touch every element in the two arrays.

Alternate Solution

If the arrays are sorted, we can iterate through them to find an appropriate pair. This will require less space.

```
int[] findSwapValues(int[] array1, int[] array2) {
  Integer target = getTarget(array1, array2);
  if (target == null) return null;
  return findDifference(array1, array2, target);
}

int[] findDifference(int[] array1, int[] array2, int target) {
  int a = 0;
  int b = 0;

  while (a < array1.length && b < array2.length) {
    int difference = array1[a] - array2[b];
    /* Compare difference to target. If difference is too small, then make it
     * bigger by moving a to a bigger value. If it is too big, then make it
     * smaller by moving b to a bigger value. If it's just right, return this
     * pair. */
    if (difference == target) {
      int[] values = {array1[a], array2[b]};
      return values;
    } else if (difference < target) {
      a++;
    } else {
      b++;
    }
  }

  return null;
}
```

This algorithm takes O(A + B) time but requires the arrays to be sorted. If the arrays aren't sorted, we can still apply this algorithm but we'd have to sort the arrays first. The overall runtime would be O(A log A + B log B).

16.22 Langton's Ant: An ant is sitting on an infinite grid of white and black squares. It initially faces right. At each step, it does the following:

(1) At a white square, flip the color of the square, turn 90 degrees right (clockwise), and move forward one unit.

(2) At a black square, flip the color of the square, turn 90 degrees left (counter-clockwise), and move forward one unit.

Write a program to simulate the first K moves that the ant makes and print the final board as a grid. Note that you are not provided with the data structure to represent the grid. This is something you must design yourself. The only input to your method is K. You should print the final grid and return nothing. The method signature might be something like `void printKMoves(int K)`.

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SOLUTION

At first glance, this problem seems very straightforward: create a grid, remember the ant's position and orientation, flip the cells, turn, and move. The interesting part comes in how to handle an infinite grid.

Solution #1: Fixed Array

Technically, since we're only running the first K moves, we do have a max size for the grid. The ant cannot move more than K moves in either direction. If we create a grid that has width 2K and height 2K (and place the ant at the center), we know it will be big enough.

The problem with this is that it's not very extensible. If you run K moves and then want to run another K moves, you might be out of luck.

Additionally, this solution wastes a good amount of space. The max might be K moves in a particular dimension, but the ant is probably going in circles a bit. You probably won't need all this space.

Solution #2: Resizable Array

One thought is to use a resizable array, such as Java's `ArrayList` class. This allows us to grow an array as necessary, while still offering `O(1)` amortized insertion.

The problem is that our grid needs to grow in two dimensions, but the `ArrayList` is only a single array. Additionally, we need to grow "backward" into negative values. The `ArrayList` class doesn't support this.

However, we take a similar approach by building our own resizable grid. Each time the ant hits an edge, we double the size of the grid in that dimension.

What about the negative expansions? While conceptually we can talk about something being at negative positions, we cannot actually access array indices with negative values.

One way we can handle this is to create "fake indices." Let us treat the ant as being at coordinates `(-3, -10)`, but track some sort of offset or delta to translate these coordinates into array indices.

This is actually unnecessary, though. The ant's location does not need to be publicly exposed or consistent (unless, of course, indicated by the interviewer). When the ant travels into negative coordinates, we can double the size of the array and just move the ant and all cells into the positive coordinates. Essentially, we are relabeling all the indices.

This relabeling will not impact the big O time since we have to create a new matrix anyway.

```
public class Grid {
  private boolean[][] grid;
```

```
    private Ant ant = new Ant();

    public Grid() {
      grid = new boolean[1][1];
    }

    /* Copy old values into new array, with an offset/shift applied to the row and
     * columns. * /
    private void copyWithShift(boolean[][] oldGrid, boolean[][] newGrid,
                               int shiftRow, int shiftColumn) {
      for (int r = 0; r < oldGrid.length; r++) {
        for (int c = 0; c < oldGrid[0].length; c++) {
          newGrid[r + shiftRow][c + shiftColumn] = oldGrid[r][c];
        }
      }
    }

    /* Ensure that the given position will fit on the array. If necessary, double
     * the size of the matrix, copy the old values over, and adjust the ant's
     * position so that it's in a positive range. * /
    private void ensureFit(Position position) {
      int shiftRow = 0;
      int shiftColumn = 0;

      /* Calculate new number of rows. * /
      int numRows = grid.length;
      if (position.row < 0) {
        shiftRow = numRows;
        numRows * =2;
      } else if (position.row >= numRows) {
        numRows * =2;
      }

      /* Calculate new number of columns. * /
      int numColumns = grid[0].length;
      if (position.column < 0) {
        shiftColumn = numColumns;
        numColumns * =2;
      } else if (position.column >= numColumns) {
        numColumns * =2;
      }

      /* Grow array, if necessary. Shift ant's position too. * /
      if (numRows != grid.length || numColumns != grid[0].length) {
        boolean[][] newGrid = new boolean[numRows][numColumns];
        copyWithShift(grid, newGrid, shiftRow, shiftColumn);
        ant.adjustPosition(shiftRow, shiftColumn);
        grid = newGrid;
      }
    }

    /* Flip color of cells. * /
    private void flip(Position position) {
      int row = position.row;
      int column = position.column;
      grid[row][column] = grid[row][column] ? false : true;
```

```
    }

    /* Move ant. */
    public void move() {
       ant.turn(grid[ant.position.row][ant.position.column]);
       flip(ant.position);
       ant.move();
       ensureFit(ant.position); // grow
    }

    /* Print board. */
    public String toString() {
       StringBuilder sb = new StringBuilder();
       for (int r = 0; r < grid.length; r++) {
          for (int c = 0; c < grid[0].length; c++) {
             if (r == ant.position.row && c == ant.position.column) {
                sb.append(ant.orientation);
             } else if (grid[r][c]) {
                sb.append("X");
             } else {
                sb.append("_");
             }
          }
          sb.append("\n");
       }
       sb.append("Ant: " + ant.orientation + ". \n");
       return sb.toString();
    }
 }
```

We pulled the Ant code into a separate class. The nice thing about this is that if we need to have multiple ants for some reason, we can easily extend the code to support this.

```
public class Ant {
   public Position position = new Position(0, 0);
   public Orientation orientation = Orientation.right;

   public void turn(boolean clockwise) {
      orientation = orientation.getTurn(clockwise);
   }

   public void move() {
      if (orientation == Orientation.left) {
         position.column--;
      } else if (orientation == Orientation.right) {
         position.column++;
      } else if (orientation == Orientation.up) {
         position.row--;
      } else if (orientation == Orientation.down) {
         position.row++;
      }
   }

   public void adjustPosition(int shiftRow, int shiftColumn) {
      position.row += shiftRow;
      position.column += shiftColumn;
   }
}
```

`Orientation` is also its own enum, with a few useful functions.

```
public enum Orientation {
  left, up, right, down;

  public Orientation getTurn(boolean clockwise) {
    if (this == left) {
      return clockwise ? up : down;
    } else if (this == up) {
      return clockwise ? right : left;
    } else if (this == right) {
      return clockwise ? down : up;
    } else { // down
      return clockwise ? left : right;
    }
  }

  @Override
  public String toString() {
    if (this == left) {
      return "\u2190";
    } else if (this == up) {
      return "\u2191";
    } else if (this == right) {
      return "\u2192";
    } else { // down
      return "\u2193";
    }
  }
}
```

We've also put `Position` into its own simple class. We could just as easily track the row and column separately.

```
public class Position {
  public int row;
  public int column;

  public Position(int row, int column) {
    this.row = row;
    this.column = column;
  }
}
```

This works, but it's actually more complicated than is necessary.

Solution #3: HashSet

Although it may seem "obvious" that we would use a matrix to represent a grid, it's actually easier not to do that. All we actually need is a list of the white squares (as well as the ant's location and orientation).

We can do this by using a HashSet of the white squares. If a position is in the hash set, then the square is white. Otherwise, it is black.

The one tricky bit is how to print the board. Where do we start printing? Where do we end?

Since we will need to print a grid, we can track what should be top-left and bottom-right corner of the grid. Each time the ant moves, we compare the ant's position to the most top-left position and most bottom-right position, updating them if necessary.

```
public class Board {
  private HashSet<Position> whites = new HashSet<Position>();
  private Ant ant = new Ant();
  private Position topLeftCorner = new Position(0, 0);
  private Position bottomRightCorner = new Position(0, 0);

  public Board() { }

  /* Move ant. */
  public void move() {
    ant.turn(isWhite(ant.position)); // Turn
    flip(ant.position); // flip
    ant.move(); // move
    ensureFit(ant.position);
  }

  /* Flip color of cells. */
  private void flip(Position position) {
    if (whites.contains(position)) {
      whites.remove(position);
    } else {
      whites.add(position.clone());
    }
  }

  /* Grow grid by tracking the most top-left and bottom-right positions.*/
  private void ensureFit(Position position) {
    int row = position.row;
    int column = position.column;

    topLeftCorner.row = Math.min(topLeftCorner.row, row);
    topLeftCorner.column = Math.min(topLeftCorner.column, column);

    bottomRightCorner.row = Math.max(bottomRightCorner.row, row);
    bottomRightCorner.column = Math.max(bottomRightCorner.column, column);
  }

  /* Check if cell is white. */
  public boolean isWhite(Position p) {
    return whites.contains(p);
  }

  /* Check if cell is white. */
  public boolean isWhite(int row, int column) {
    return whites.contains(new Position(row, column));
  }

  /* Print board. */
  public String toString() {
    StringBuilder sb = new StringBuilder();
    int rowMin = topLeftCorner.row;
    int rowMax = bottomRightCorner.row;
    int colMin = topLeftCorner.column;
    int colMax = bottomRightCorner.column;
    for (int r = rowMin; r <= rowMax; r++) {
      for (int c = colMin; c <= colMax; c++) {
```

```
            if (r == ant.position.row && c == ant.position.column) {
               sb.append(ant.orientation);
            } else if (isWhite(r, c)) {
               sb.append("X");
            } else {
               sb.append("_");
            }
         }
         sb.append("\n");
      }
      sb.append("Ant: " + ant.orientation + ". \n");
      return sb.toString();
   }
```

The implementation of Ant and Orientation is the same.

The implementation of Position gets updated slightly, in order to support the HashSet functionality. The position will be the key, so we need to implement a hashCode() function.

```
public class Position {
   public int row;
   public int column;

   public Position(int row, int column) {
      this.row = row;
      this.column = column;
   }

   @Override
   public boolean equals(Object o) {
      if (o instanceof Position) {
         Position p = (Position) o;
         return p.row == row && p.column == column;
      }
      return false;
   }

   @Override
   public int hashCode() {
      /* There are many options for hash functions. This is one. */
      return (row * 31) ^ column;
   }

   public Position clone() {
      return new Position(row, column);
   }
}
```

The nice thing about this implementation is that if we do need to access a particular cell elsewhere, we have consistent row and column labeling.

16.23 Rand7 from Rand5: Implement a method `rand7()` given `rand5()`. That is, given a method that generates a random number between 0 and 4 (inclusive), write a method that generates a random number between 0 and 6 (inclusive).

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SOLUTION

To implement this function correctly, we must have each of the values between 0 and 6 returned with 1/7th probability.

First Attempt (Fixed Number of Calls)

As a first attempt, we might try generating all numbers between 0 and 9, and then mod the resulting value by 7. Our code for it might look something like this:

```
int rand7() {
   int v = rand5() + rand5();
   return v % 7;
}
```

Unfortunately, the above code will not generate the values with equal probability. We can see this by looking at the results of each call to `rand5()` an d the return result of the `rand7()` function.

1st Call	2nd Call	Result
0	0	0
0	1	1
0	2	2
0	3	3
0	4	4
1	0	1
1	1	2
1	2	3
1	3	4
1	4	5
2	0	2
2	1	3
2	2	4
2	3	5
2	4	6
3	0	3
3	1	4
3	2	5
3	3	6
3	4	0
4	0	4
4	1	5
4	2	6
4	3	0
4	4	1

Each individual row has a 1 in 25 chance of occurring, since there are two calls to `rand5()` and each distributes its results with $\frac{1}{5}$th probability. If you count up the number of times each number occurs, you'll note that this `rand7()` function will return 4 with $\frac{5}{25}$th probability but return 0 with just $\frac{3}{25}$th probability. This means that our function has failed; the results do not have probability $\frac{1}{7}$th.

Now, imagine we modify our function to add an if-statement, to change the constant multiplier, or to insert a new call to `rand5()`. We will still wind up with a similar looking table, and the probability of getting any one of those rows will be $\frac{1}{5^k}$, where k is the number of calls to `rand5()` in that row. Different rows may have different number of calls.

The probability of winding up with the result of the `rand7()` function being, say, 6 would be the sum of the probabilities of all rows that result in 6. That is:

$$P(\text{rand7()} = 6) = \frac{1}{5^i} + \frac{1}{5^j} + \ldots + \frac{1}{5^m}$$

We know that, in order for our function to be correct, this probability must equal $\frac{1}{7}$. This is impossible though. Because 5 and 7 are relatively prime, no series of reciprocal powers of 5 will result in $\frac{1}{7}$.

Does this mean the problem is impossible? Not exactly. Strictly speaking, it means that, as long as we can list out the combinations of `rand5()` results that will result in a particular value of `rand7()`, the function will not give well distributed results.

We can still solve this problem. We just have to use a while loop, and realize that there's no telling just how many turns will be required to return a result.

Second Attempt (Nondeterministic Number of Calls)

As soon as we've allowed for a while loop, our work gets much easier. We just need to generate a range of values where each value is equally likely (and where the range has at least seven elements). If we can do this, then we can discard the elements greater than the previous multiple of 7, and mod the rest of them by 7. This will get us a value within the range of 0 to 6, with each value being equally likely.

In the below code, we generate the range 0 through 24 by doing `5 * rand5() + rand5()`. Then, we discard the values between 21 and 24, since they would otherwise make `rand7()` unfairly weighted towards 0 through 3. Finally, we mod by 7 to give us the values in the range 0 to 6 with equal probability.

Note that because we discard values in this approach, we have no guarantee on the number of `rand5()` calls it may take to return a value. This is what is meant by a *nondeterministic* number of calls.

```
int rand7() {
  while (true) {
    int num = 5 * rand5() + rand5();
    if (num < 21) {
      return num % 7;
    }
  }
}
```

Observe that doing `5 * rand5() + rand5()` gives us exactly one way of getting each number in its range (0 to 24). This ensures that each value is equally probable.

Could we instead do `2 * rand5() + rand5()`? No, because the values wouldn't be equally distributed. For example, there would be three ways of getting a 6 (`6 = 2 * 1 + 4`, `6 = 2 * 2 + 2`, and `6 = 2 * 3 + 0`) but only one way of getting a 0 (`0=2*0+0`). The values in the range are not equally probable.

There *is* a way that we can use `2 * rand5()` and still get an identically distributed range, but it's much more complicated. See below.

```
int rand7() {
  while (true) {
    int r1 = 2 * rand5(); / *evens between 0 and 9 */
    int r2 = rand5(); / *used later to generate a 0 or 1 */
    if (r2 != 4) { / *r2 has extra even num-discard the extra */
      int rand1 = r2 % 2; / *Generate 0 or 1 */
      int num = r1 + rand1; / *will be in the range 0 to 9 */
      if (num < 7) {
        return num;
      }
    }
  }
}
```

In fact, there is an infinite number of ranges we can use. The key is to make sure that the range is big enough and that all values are equally likely.

16.24 Pairs with Sum: Design an algorithm to find *all* pairs of integers within an array which sum to a specified value.

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SOLUTION

Let's start with a definition. If we're trying to find a pair of numbers that sums to z, the *complement* of x will be z - x (that is, the number that can be added to x to make z). For example, if we're trying to find a pair of numbers that sums to 12, the complement of -5 would be 17.

Brute Force

A brute force solution is to just iterate through all pairs and print the pair if its sum matches the target sum.

```
ArrayList<Pair> printPairSums(int[] array, int sum) {
  ArrayList<Pair> result = new ArrayList<Pair>();
  for (int i = 0 ; i < array.length; i++) {
    for (int j = i + 1; j < array.length; j++) {
      if (array[i] + array[j] == sum) {
        result.add(new Pair(array[i], array[j]));
      }
    }
  }
  return result;
}
```

If there are duplicates in the array (e.g., {5, 6, 5}), it might print the same sum twice. You should discuss this with your interviewer.

Optimized Solution

We can optimize this with a hash map, where the value in the hash map reflects the number of "unpaired" instances of a key. We walk through the array. At each element x, check how many unpaired instances of x's complement preceded it in the array. If the count is at least one, then there is an unpaired instance of x's complement. We add this pair and decrement x's complement to signify that this element has been paired. If the count is zero, then increment the value of x in the hash table to signify that x is unpaired.

```
ArrayList<Pair> printPairSums(int[] array, int sum) {
  ArrayList<Pair> result = new ArrayList<Pair>();
  HashMap<Integer, Integer> unpairedCount = new HashMap<Integer, Integer>();
  for (int x : array) {
    int complement = sum - x;
    if (unpairedCount.getOrDefault(complement, 0) > 0) {
      result.add(new Pair(x, complement));
      adjustCounterBy(unpairedCount, complement, -1); // decrement complement
    } else {
      adjustCounterBy(unpairedCount, x, 1); // increment count
    }
  }
  return result;
}

```

```
void adjustCounterBy(HashMap<Integer, Integer> counter, int key, int delta) {
  counter.put(key, counter.getOrDefault(key, 0) + delta);
}
```

This solution will print duplicate pairs, but will not reuse the same instance of an element. It will take O(N) time and O(N) space.

Alternate Solution

Alternatively, we can sort the array and then find the pairs in a single pass. Consider this array:

`{-2, -1, 0, 3, 5, 6, 7, 9, 13, 14}.`

Let `first` point to the head of the array and `last` point to the end of the array. To find the complement of `first`, we just move `last` backwards until we find it. If `first + last < sum`, then there is no complement for `first`. We can therefore move `first` forward. We stop when `first` is greater than `last`.

Why must this find all complements for `first`? Because the array is sorted and we're trying progressively smaller numbers. When the sum of `first` and `last` is less than the sum, we know that trying even smaller numbers (as `last`) won't help us find a complement.

Why must this find all complements for `last`? Because all pairs must be made up of a `first` and a `last`. We've found all complements for `first`, therefore we've found all complements of `last`.

```
void printPairSums(int[] array, int sum) {
  Arrays.sort(array);
  int first = 0;
  int last = array.length - 1;
  while (first < last) {
    int s = array[first] + array[last];
    if (s == sum) {
      System.out.println(array[first] + " " + array[last]);
      first++;
      last--;
    } else {
      if (s < sum) first++;
      else last--;
    }
  }
}
```

This algorithm takes O(N log N) time to sort and O(N) time to find the pairs.

Note that since the array is presumably unsorted, it would be equally fast in terms of big O to just do a binary search at each element for its complement. This would give us a two-step algorithm, where each step is O(N log N).

16.25 LRU Cache: Design and build a "least recently used" cache, which evicts the least recently used item. The cache should map from keys to values (allowing you to insert and retrieve a value associated with a particular key) and be initialized with a max size. When it is full, it should evict the least recently used item. You can assume the keys are integers and the values are strings.

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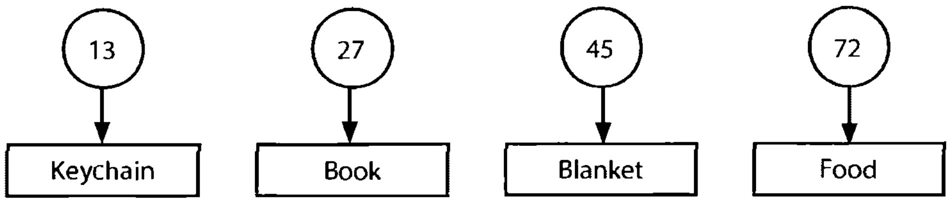
SOLUTION

We should start off by defining the scope of the problem. What exactly do we need to achieve?

- **Inserting Key, Value Pair:** We need to be able to insert a (key, value) pair.

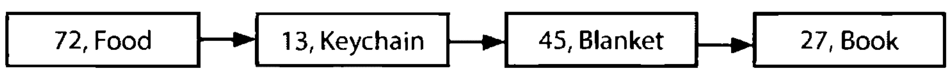
- **Retrieving Value by Key:** We need to be able to retrieve the value using the key.
- **Finding Least Recently Used:** We need to know the least recently used item (and, likely, the usage ordering of all items).
- **Updating Most Recently Used:** When we retrieve a value by key, we need to update the order to be the most recently used item.
- **Eviction:** The cache should have a max capacity and should remove the least recently used item when it hits capacity.

The (key, value) mapping suggests a hash table. This would make it easy to look up the value associated with a particular key.



Unfortunately, a hash table usually would not offer a quick way to remove the most recently used item. We could mark each item with a timestamp and iterate through the hash table to remove the item with the lowest timestamp, but that can get quite slow (O(N) for insertions).

Instead, we could use a linked list, ordered by the most recently used. This would make it easy to mark an item as the most recently used (just put it in the front of the list) or to remove the least recently used item (remove the end).

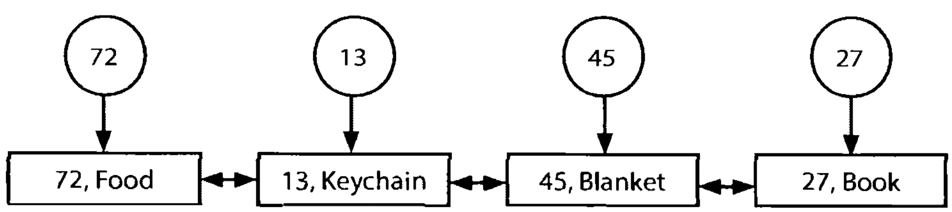


Unfortunately, this does not offer a quick way to look up an item by its key. We could iterate through the linked list and find the item by key. But this could get very slow (O(N) for retrieval).

Each approach does half of the problem (different halves) very well, but neither approach does both parts well.

Can we get the best parts of each? Yes. By using both!

The linked list looks as it did in the earlier example, but now it's a doubly linked list. This allows us to easily remove an element from the middle of the linked list. The hash table now maps to each linked list node rather than the value.



The algorithms now operate as follows:

- **Inserting Key, Value Pair:** Create a linked list node with key, value. Insert into head of linked list. Insert key -> node mapping into hash table.
- **Retrieving Value by Key:** Look up node in hash table and return value. Update most recently used item

(see below).

- **Finding Least Recently Used:** Least recently used item will be found at the end of the linked list.
- **Updating Most Recently Used:** Move node to front of linked list. Hash table does not need to be updated.
- **Eviction:** Remove tail of linked list. Get key from linked list node and remove key from hash table.

The code below implements these classes and algorithms.

```
public class Cache {
  private int maxCacheSize;
  private HashMap<Integer, LinkedListNode> map =
    new HashMap<Integer, LinkedListNode>();
  private LinkedListNode listHead = null;
  public LinkedListNode listTail = null;

  public Cache(int maxSize) {
    maxCacheSize = maxSize;
  }

  /* Get value for key and mark as most recently used. */
  public String getValue(int key) {
    LinkedListNode item = map.get(key);
    if (item == null) return null;

    /* Move to front of list to mark as most recently used. */
    if (item != listHead) {
      removeFromLinkedList(item);
      insertAtFrontOfLinkedList(item);
    }
    return item.value;
  }

  /* Remove node from linked list. */
  private void removeFromLinkedList(LinkedListNode node) {
    if (node == null)  return;

    if (node.prev != null) node.prev.next = node.next;
    if (node.next != null) node.next.prev = node.prev;
    if (node == listTail) listTail = node.prev;
    if (node == listHead) listHead = node.next;
  }

  /* Insert node at front of linked list. */
  private void insertAtFrontOfLinkedList(LinkedListNode node) {
    if (listHead == null) {
      listHead = node;
      listTail = node;
    } else {
      listHead.prev = node;
      node.next = listHead;
      listHead = node;
    }
  }

  /* Remove key/value pair from cache, deleting from hashtable and linked list. */
  public boolean removeKey(int key) {
```

```
        LinkedListNode node = map.get(key);
        removeFromLinkedList(node);
        map.remove(key);
        return true;
    }

    /* Put key, value pair in cache. Removes old value for key if necessary. Inserts
     * pair into linked list and hash table.*/
    public void setKeyValue(int key, String value) {
        /* Remove if already there. */
        removeKey(key);

        /* If full, remove least recently used item from cache. */
        if (map.size() >= maxCacheSize && listTail != null) {
            removeKey(listTail.key);
        }

        /* Insert new node. */
        LinkedListNode node = new LinkedListNode(key, value);
        insertAtFrontOfLinkedList(node);
        map.put(key, node);
    }

    private static class LinkedListNode {
        private LinkedListNode next, prev;
        public int key;
        public String value;
        public LinkedListNode(int k, String v) {
            key = k;
            value = v;
        }
    }
}
```

Note that we've chosen to make LinkedListNode an inner class of Cache, since no other classes should need access to this class and really should only exist within the scope of Cache.

16.26 Calculator: Given an arithmetic equation consisting of positive integers, +, -, * and / (no parentheses), compute the result.

EXAMPLE

Input: 2*3+5/6*3+15

Output: 23.5

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SOLUTION

The first thing we should realize is that the dumb thing—just applying each operator left to right—won't work. Multiplication and division are considered "higher priority" operations, which means that they have to happen before addition.

For example, if you have the simple expression 3+6*2, the multiplication must be performed first, and then the addition. If you just processed the equation left to right, you would end up with the incorrect result, 18, rather than the correct one, 15. You know all of this, of course, but it's worth really spelling out what it means.

Solution #1

We can still process the equation from left to right; we just have to be a little smarter about how we do it. Multiplication and division need to be grouped together such that whenever we see those operations, we perform them immediately on the surrounding terms.

For example, suppose we have this expression:

```
2 - 6 - 7*8/2 + 5
```

It's fine to compute 2 - 6 immediately and store it into a `result` variable. But, when we see `7*(something)`, we know we need to fully process that term before adding it to the result.

We can do this by reading left to right and maintaining two variables.

- The first is `processing`, which maintains the result of the current cluster of terms (both the operator and the value). In the case of addition and subtraction, the cluster will be just the current term. In the case of multiplication and division, it will be the full sequence (until you get to the next addition or subtraction).
- The second is the `result` variable. If the next term is an addition or subtraction (or there is no next term), then `processing` is applied to `result`.

On the above example, we would do the following:

1. Read +2. Apply it to `processing`. Apply processing to `result`. Clear `processing`.
   ```
   processing = {+, 2} --> null
   result = 0          --> 2
   ```
2. Read -6. Apply it to `processing`. Apply processing to `result`. Clear `processing`.
   ```
   processing = {-, 6} --> null
   result = 2          --> -4
   ```
3. Read -7. Apply it to `processing`. Observe next sign is a *. Continue.
   ```
   processing = {-, 7}
   result = -4
   ```
4. Read *8. Apply it to `processing`. Observe next sign is a /. Continue.
   ```
   processing = {-, 56}
   result = -4
   ```
5. Read /2. Apply it to `processing`. Observe next sign is a +, which terminates this multiplication and division cluster. Apply processing to `result`. Clear `processing`.
   ```
   processing = {-, 28}    --> null
   result = -4          --> -32
   ```
6. Read +5. Apply it to `processing`. Apply processing to `result`. Clear `processing`.
   ```
   processing = {+, 5} --> null
   result = -32        --> -27
   ```

The code below implements this algorithm.

```
/* Compute the result of the arithmetic sequence. This works by reading left to
 * right and applying each term to a result. When we see a multiplication or
 * division, we instead apply this sequence to a temporary variable. */
double compute(String sequence) {
  ArrayList<Term> terms = Term.parseTermSequence(sequence);
  if (terms == null) return Integer.MIN_VALUE;

  double result = 0;
  Term processing = null;
  for (int i = 0; i < terms.size(); i++) {
```

```
      Term current = terms.get(i);
      Term next = i + 1 < terms.size() ? terms.get(i + 1) : null;

      /* Apply the current term to "processing". */
      processing = collapseTerm(processing, current);

      /* If next term is + or -, then this cluster is done and we should apply
       * "processing" to "result". */
      if (next == null || next.getOperator() == Operator.ADD
          || next.getOperator() == Operator.SUBTRACT) {
        result = applyOp(result, processing.getOperator(), processing.getNumber());
        processing = null;
      }
    }

    return result;
  }

  /* Collapse two terms together using the operator in secondary and the numbers
   * from each. */
  Term collapseTerm(Term primary, Term secondary) {
    if (primary == null) return secondary;
    if (secondary == null) return primary;

    double value = applyOp(primary.getNumber(), secondary.getOperator(),
                           secondary.getNumber());
    primary.setNumber(value);
    return primary;
  }

  double applyOp(double left, Operator op, double right) {
    if (op == Operator.ADD) return left + right;
    else if (op == Operator.SUBTRACT) return left - right;
    else if (op == Operator.MULTIPLY) return left * right;
    else if (op == Operator.DIVIDE) return left / right;
    else return right;
  }

  public class Term {
    public enum Operator {
      ADD, SUBTRACT, MULTIPLY, DIVIDE, BLANK
    }

    private double value;
    private Operator operator = Operator.BLANK;

    public Term(double v, Operator op) {
      value = v;
      operator = op;
    }

    public double getNumber() { return value; }
    public Operator getOperator() { return operator; }
    public void setNumber(double v) { value = v; }

    /* Parses arithmetic sequence into a list of Terms. For example, 3-5*6 becomes
```

```
     * something like: [{BLANK,3}, {SUBTRACT, 5}, {MULTIPLY, 6}].
     * If improperly formatted, returns null. */
    public static ArrayList<Term> parseTermSequence(String sequence) {
      /* Code can be found in downloadable solutions. */
    }
  }
```

This takes `O(N)` time, where `N` is the length of the initial string.

Solution #2

Alternatively, we can solve this problem using two stacks: one for numbers and one for operators.

```
2 - 6 - 7 * 8 / 2 + 5
```

The processing works as follows:

- Each time we see a number, it gets pushed onto `numberStack`.
- Operators get pushed onto `operatorStack`—as long as the operator has higher priority than the current top of the stack. If `priority(currentOperator) <= priority(operatorStack.top())`, then we "collapse" the top of the stacks:
  - » Collapsing: pop two elements off `numberStack`, pop an operator off `operatorStack`, apply the operator, and push the result onto `numberStack`.
  - » Priority: addition and subtraction have equal priority, which is lower than the priority of multiplication and division (also equal priority).

  This collapsing continues until the above inequality is broken, at which point `currentOperator` is pushed onto `operatorStack`.
- At the very end, we collapse the stack.

Let's see this with an example: `2 - 6 - 7 * 8 / 2 + 5`

	action	numberStack	operatorStack
2	numberStack.push(2)	2	[empty]
-	operatorStack.push(-)	2	-
6	numberStack.push(6)	6, 2	-
-	collapseStacks [2 - 6] operatorStack.push(-)	-4 -4	[empty] -
7	numberStack.push(7)	7, -4	-
*	operatorStack.push(*)	7, -4	*, -
8	numberStack.push(8)	8, 7, -4	*, -
/	collapseStack [7 * 8] numberStack.push(/)	56, -4 56, -4	- /, -
2	numberStack.push(2)	2, 56, -4	/, -
+	collapseStack [56 / 2] collapseStack [-4 - 28] operatorStack.push(+)	28, -4 -32 -32	- [empty] +
5	numberStack.push(5)	5, -32	+
	collapseStack [-32 + 5]	-27	[empty]
	return -27		

The code below implements this algorithm.

```
public enum Operator {
  ADD, SUBTRACT, MULTIPLY, DIVIDE, BLANK
}

double compute(String sequence) {
  Stack<Double> numberStack = new Stack<Double>();
  Stack<Operator> operatorStack = new Stack<Operator>();

  for (int i = 0; i < sequence.length(); i++) {
    try {
      /* Get number and push. */
      int value = parseNextNumber(sequence, i);
      numberStack.push((double) value);

      /* Move to the operator. */
      i += Integer.toString(value).length();
      if (i >= sequence.length()) {
        break;
      }

      /* Get operator, collapse top as needed, push operator. */
      Operator op = parseNextOperator(sequence, i);
      collapseTop(op, numberStack, operatorStack);
      operatorStack.push(op);
    } catch (NumberFormatException ex) {
      return Integer.MIN_VALUE;
    }
  }

  /* Do final collapse. */
  collapseTop(Operator.BLANK, numberStack, operatorStack);
  if (numberStack.size() == 1 && operatorStack.size() == 0) {
    return numberStack.pop();
  }
  return 0;
}

/* Collapse top until priority(futureTop) > priority(top). Collapsing means to pop
 * the top 2 numbers and apply the operator popped from the top of the operator
 * stack, and then push that onto the numbers stack.*/
void collapseTop(Operator futureTop, Stack<Double> numberStack,
                 Stack<Operator> operatorStack) {
  while (operatorStack.size() >= 1 && numberStack.size() >= 2) {
    if (priorityOfOperator(futureTop) <=
        priorityOfOperator(operatorStack.peek())) {
      double second = numberStack.pop();
      double first = numberStack.pop();
      Operator op = operatorStack.pop();
      double collapsed = applyOp(first, op, second);
      numberStack.push(collapsed);
    } else {
      break;
    }
  }
```

```
}

/* Return priority of operator. Mapped so that:
 *     addition == subtraction < multiplication == division. */
int priorityOfOperator(Operator op) {
  switch (op) {
    case ADD: return 1;
    case SUBTRACT: return 1;
    case MULTIPLY: return 2;
    case DIVIDE: return 2;
    case BLANK: return 0;
  }
  return 0;
}

/* Apply operator: left [op] right. * /
double applyOp(double left, Operator op, double right) {
  if (op == Operator.ADD) return left + right;
  else if (op == Operator.SUBTRACT) return left - right;
  else if (op == Operator.MULTIPLY) return left * right;
  else if (op == Operator.DIVIDE) return left / right;
  else return right;
}

/* Return the number that starts at offset. * /
int parseNextNumber(String seq, int offset) {
  StringBuilder sb = new StringBuilder();
  while (offset < seq.length() && Character.isDigit(seq.charAt(offset))) {
    sb.append(seq.charAt(offset));
    offset++;
  }
  return Integer.parseInt(sb.toString());
}

/* Return the operator that occurs as offset. * /
Operator parseNextOperator(String sequence, int offset) {
  if (offset < sequence.length()) {
    char op = sequence.charAt(offset);
    switch(op) {
      case '+': return Operator.ADD;
      case '-': return Operator.SUBTRACT;
      case '*': return Operator.MULTIPLY;
      case '/': return Operator.DIVIDE;
    }
  }
  return Operator.BLANK;
}
```

This code also takes O(N) time, where N is the length of the string.

This solution involves a lot of annoying string parsing code. Remember that getting all these details out is not that important in an interview. In fact, your interviewer might even let you assume the expression is passed in pre-parsed into some sort of data structure.

Focus on modularizing your code from the beginning and "farming out" tedious or less interesting parts of the code to other functions. You want to focus on getting the core `compute` function working. The rest of the details can wait!

17

Solutions to Hard

17.1 Add Without Plus: Write a function that adds two numbers. You should not use + or any arithmetic operators.

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SOLUTION

Our first instinct in problems like these should be that we're going to have to work with bits. Why? Because when you take away the + sign, what other choice do we have? Plus, that's how computers do it!

Our next thought should be to deeply understand how addition works. We can walk through an addition problem to see if we can understand something new—some pattern—and then see if we can replicate that with code.

So let's do just that—let's walk through an addition problem. We'll work in base 10 so that it's easier to see.

To add 759 + 674, I would usually add `digit[0]` from each number, carry the one, add `digit[1]` from each number, carry the one, and so on. You could take the same approach in binary: add each digit, and carry the one as necessary.

Can we make this a little easier? Yes! Imagine I decided to split apart the "addition" and "carry" steps. That is, I do the following:

1. Add 759 + 674, but "forget" to carry. I then get 323.
2. Add 759 + 674 but only do the carrying, rather than the addition of each digit. I then get 1110.
3. Add the result of the first two operations (recursively, using the same process described in step 1 and 2): 1110 + 323 = 1433.

Now, how would we do this in binary?

1. If I add two binary numbers together, but forget to carry, the `i`th bit in the sum will be `0` only if a and b have the same `i`th bit (both `0` or both `1`). This is essentially an XOR.
2. If I add two numbers together but *only* carry, I will have a 1 in the `i`th bit of the sum only if bits `i - 1` of a and b are both 1s. This is an AND, shifted.
3. Now, recurse until there's nothing to carry.

The following code implements this algorithm.

```
int add(int a, int b) {
  if (b == 0) return a;
  int sum = a ^ b; // add without carrying
  int carry = (a & b) << 1; // carry, but don't add
```

```
    return add(sum, carry); // recurse with sum + carry
}
```

Alternatively, you can implement this iteratively.

```
int add(int a, int b) {
  while (b != 0) {
    int sum = a ^ b; // add without carrying
    int carry = (a & b) << 1; // carry, but don't add
    a = sum;
    b = carry;
  }
  return a;
}
```

Problems requiring us to implement core operations like addition and subtraction are relatively common. The key in all of these problems is to dig into how these operations are usually implemented, so that we can re-implement them with the constraints of the given problem.

17.2 Shuffle: Write a method to shuffle a deck of cards. It must be a perfect shuffle—in other words, each of the 52! permutations of the deck has to be equally likely. Assume that you are given a random number generator which is perfect.

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SOLUTION

This is a very well known interview question, and a well known algorithm. If you aren't one of the lucky few to already know this algorithm, read on.

Let's imagine our n-element array. Suppose it looks like this:

```
[1] [2] [3] [4] [5]
```

Using our Base Case and Build approach, we can ask this question: suppose we had a method `shuffle(...)` that worked on n - 1 elements. Could we use this to shuffle n elements?

Sure. In fact, that's quite easy. We would first shuffle the first n - 1 elements. Then, we would take the nth element and randomly swap it with an element in the array. That's it!

Recursively, that algorithm looks like this:

```
/* Random number between lower and higher, inclusive */
int rand(int lower, int higher) {
  return lower + (int)(Math.random() * (higher - lower + 1));
}

int[] shuffleArrayRecursively(int[] cards, int i) {
  if (i == 0) return cards;

  shuffleArrayRecursively(cards, i - 1); // Shuffle earlier part
  int k = rand(0, i); // Pick random index to swap with

  /* Swap element k and i */
  int temp = cards[k];
  cards[k] = cards[i];
  cards[i] = temp;

  /* Return shuffled array */
  return cards;
```

```
}
```

What would this algorithm look like iteratively? Let's think about it. All it does is moving through the array and, for each element i, swapping `array[i]` with a random element between `0` and `i`, inclusive.

This is actually a very clean algorithm to implement iteratively:

```
void shuffleArrayIteratively(int[] cards) {
  for (int i = 0; i < cards.length; i++) {
    int k = rand(0, i);
    int temp = cards[k];
    cards[k] = cards[i];
    cards[i] = temp;
  }
}
```

The iterative approach is usually how we see this algorithm written.

17.3 Random Set: Write a method to randomly generate a set of `m` integers from an array of size n. Each element must have equal probability of being chosen.

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SOLUTION

Like the prior problem which was similar, (problem 17.2 on page 531), we can look at this problem recursively using the Base Case and Build approach.

Suppose we have an algorithm that can pull a random set of `m` elements from an array of size `n - 1`. How can we use this algorithm to pull a random set of `m` elements from an array of size n?

We can first pull a random set of size `m` from the first `n - 1` elements. Then, we just need to decide if `array[n]` should be inserted into our subset (which would require pulling out a random element from it). An easy way to do this is to pick a random number k from `0` through n. If `k < m`, then insert `array[n]` into `subset[k]`. This will both "fairly" (i.e., with proportional probability) insert `array[n]` into the subset and "fairly" remove a random element from the subset.

The pseudocode for this recursive algorithm would look like this:

```
int[] pickMRecursively(int[] original, int m, int i) {
  if (i + 1 == m) { // Base case
    /* return first m elements of original */
  } else if (i + 1 > m) {
    int[] subset = pickMRecursively(original, m, i - 1);
    int k =  random value between 0 and i, inclusive
    if (k < m) {
      subset[k] = original[i];
    }
    return subset;
  }
  return null;
}
```

This is even cleaner to write iteratively. In this approach, we initialize an array `subset` to be the first `m` elements in `original`. Then, we iterate through the array, starting at element `m`, inserting `array[i]` into the subset at (random) position k whenever `k < m`.

```
int[] pickMIteratively(int[] original, int m) {
  int[] subset = new int[m];

```

```
    /* Fill in subset array with first part of original array */
    for (int i = 0; i < m ; i++) {
      subset[i] = original[i];
    }

    /* Go through rest of original array. */
    for (int i = m; i < original.length; i++) {
      int k = rand(0, i); // Random # between 0 and i, inclusive
      if (k < m) {
        subset[k] = original[i];
      }
    }

    return subset;
}
```

Both solutions are, not surprisingly, very similar to the algorithm to shuffle an array.

17.4 Missing Number: An array A contains all the integers from 0 to n, except for one number which is missing. In this problem, we cannot access an entire integer in A with a single operation. The elements of A are represented in binary, and the only operation we can use to access them is "fetch the jth bit of A[i]," which takes constant time. Write code to find the missing integer. Can you do it in O(n) time?

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SOLUTION

You may have seen a very similar sounding problem: Given a list of numbers from 0 to n, with exactly one number removed, find the missing number. This problem can be solved by simply adding the list of numbers and comparing it to the actual sum of 0 through n, which is $^{n(n+1)}/_2$. The difference will be the missing number.

We could solve this by computing the value of each number, based on its binary representation, and calculating the sum.

The runtime of this solution is `n * length(n)`, when `length` is the number of bits in n. Note that `length(n)` = $\log_2(n)$. So, the runtime is actually `O(n log(n))`. Not quite good enough!

So how else can we approach it?

We can actually use a similar approach, but leverage the bit values more directly.

Picture a list of binary numbers (the ----- indicates the value that was removed):

```
00000      00100      01000      01100
00001      00101      01001      01101
00010      00110      01010
-----      00111      01011
```

Removing the number above creates an imbalance of 1s and 0s in the least significant bit, which we'll call LSB_1. In a list of numbers from 0 to n, we would expect there to be the same number of 0s as 1s (if n is odd), or an additional 0 if n is even. That is:

```
if n % 2 == 1 then count(0s) = count(1s)
if n % 2 == 0 then count(0s) = 1 + count(1s)
```

Note that this means that `count(0s)` is always greater than or equal to `count(1s)`.

When we remove a value v from the list, we'll know immediately if v is even or odd just by looking at the least significant bits of all the other values in the list.

	n % 2 == 0 count(0s) = 1 + count(1s)	n % 2 == 1 count(0s) = count(1s)
v % 2 == 0 LSB_1(v) = 0	a 0 is removed. count(0s) = count(1s)	a 0 is removed. count(0s) < count(1s)
v % 2 == 1 LSB_1(v) = 1	a 1 is removed. count(0s) > count(1s)	a 1 is removed. count(0s) > count(1s)

So, if `count(0s) <= count(1s)`, then v is even. If `count(0s) > count(1s)`, then v is odd.

We can now remove all the evens and focus on the odds, or remove all the odds and focus on the evens.

Okay, but how do we figure out what the next bit in v is? If v were contained in our (now smaller) list, then we should expect to find the following (where $count_2$ indicates the number of 0s or 1s in the second least significant bit):

$count_2$(0s) = $count_2$(1s)   OR $count_2$(0s) = 1 + $count_2$(1s)

As in the earlier example, we can deduce the value of the second least significant bit (LSB_2) of v.

	$count_2$(0s) = 1 + $count_2$(1s)	$count_2$(0s) = $count_2$(1s)
LSB_2(v) == 0	a 0 is removed. $count_2$(0s) = $count_2$(1s)	a 0 is removed. $count_2$(0s) < $count_2$(1s)
LSB_2(v) == 1	a 1 is removed. $count_2$(0s) > $count_2$(1s)	a 1 is removed. $count_2$(0s) > $count_2$(1s)

Again, we have the same conclusion:

- If $count_2$(0s) <= $count_2$(1s), then LSB_2(v) = 0.
- If $count_2$(0s) > $count_2$(1s), then LSB_2(v) = 1.

We can repeat this process for each bit. On each iteration, we count the number of 0s and 1s in bit `i` to check if LSB_i(v) is 0 or 1. Then, we discard the numbers where LSB_i(x) != LSB_i(v). That is, if v is even, we discard the odd numbers, and so on.

By the end of this process, we will have computed all bits in v. In each successive iteration, we look at n, then n / 2, then n / 4, and so on, bits. This results in a runtime of O(N).

If it helps, we can also move through this more visually. In the first iteration, we start with all the numbers:

```
00000    00100    01000    01100
00001    00101    01001    01101
00010    00110    01010
-----    00111    01011
```

Since $count_1$(0s) > $count_1$(1s), we know that LSB_1(v) = 1. Now, discard all numbers x where LSB_1(x) != LSB_1(v).

~~00000~~	~~00100~~	~~01000~~	~~01100~~
00001	00101	01001	01101
~~00010~~	~~00110~~	~~01010~~	
-----	00111	01011	

Now, $count_2$(0s) > $count_2$(1s), so we know that LSB_2(v) = 1. Now, discard all numbers x where LSB_2(x) != LSB_2(v).

~~00000~~	~~00100~~	~~01000~~	~~01100~~
~~00001~~	~~00101~~	~~01001~~	~~01101~~
~~00010~~	~~00110~~	~~01010~~	
-----	00111	01011	

This time, count$_3$(0s) <= count$_3$(1s), we know that LSB$_3$(v) = 0. Now, discard all numbers x where LSB$_3$(x) != LSB$_3$(v).

00000	~~00100~~	~~01000~~	~~01100~~
~~00001~~	~~00101~~	~~01001~~	~~01101~~
~~00010~~	~~00110~~	~~01010~~	
-----	~~00111~~	01011	

We're down to just one number. In this case, count$_4$(0s) <= count$_4$(1s), so LSB$_4$(v) = 0.

When we discard all numbers where LSB$_4$(x) != 0, we'll wind up with an empty list. Once the list is empty, then count$_i$(0s) <= count$_i$(1s), so LSB$_i$(v) = 0. In other words, once we have an empty list, we can fill in the rest of the bits of v with 0.

This process will compute that, for the example above, v = 00011.

The code below implements this algorithm. We've implemented the discarding aspect by partitioning the array by bit value as we go.

```
1  int findMissing(ArrayList<BitInteger> array) {
2    /* Start from the least significant bit, and work our way up */
3    return findMissing(array, 0);
4  }
5
6  int findMissing(ArrayList<BitInteger> input, int column) {
7    if (column >= BitInteger.INTEGER_SIZE) { // We're done!
8      return 0;
9    }
10   ArrayList<BitInteger> oneBits = new ArrayList<BitInteger>(input.size()/2);
11   ArrayList<BitInteger> zeroBits = new ArrayList<BitInteger>(input.size()/2);
12
13   for (BitInteger t : input) {
14     if (t.fetch(column) == 0) {
15       zeroBits.add(t);
16     } else {
17       oneBits.add(t);
18     }
19   }
20   if (zeroBits.size() <= oneBits.size()) {
21     int v = findMissing(zeroBits, column + 1);
22     return (v << 1) | 0;
23   } else {
24     int v = findMissing(oneBits, column + 1);
25     return (v << 1) | 1;
26   }
27 }
```

In lines 24 and 27, we recursively calculate the other bits of v. Then, we insert either a 0 or 1, depending on whether or not count$_1$(0s) <= count$_1$(1s).

17.5 Letters and Numbers: Given an array filled with letters and numbers, find the longest subarray with an equal number of letters and numbers.

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SOLUTION

In the introduction, we discussed the importance of creating a really good, general-purpose example. That's absolutely true. It's also important, though, to understand what matters.

In this case, we just want an equal number of letters and numbers. All letters are treated identically and all numbers are treated identically. Therefore, we can use an example with a single letter and a single number—or, for that matter, As and Bs, 0s and 1s, or `Thing1`s and `Thing2`s.

With that said, let's start with an example:

```
[A, B, A, A, A, B, B, B, A, B, A, A, B, B, A, A, A, A, A, A]
```

We're looking for the smallest subarray where `count(A, subarray) = count(B, subarray)`.

Brute Force

Let's start with the obvious solution. Just go through all subarrays, count the number of As and Bs (or letters and numbers), and find the longest one that is equal.

We can make one small optimization to this. We can start with the longest subarray and, as soon as we find one which fits this equality condition, return it.

```
/* Return the largest subarray with equal number of 0s and 1s. Look at each
 * subarray, starting from the longest. As soon as we find one that's equal, we
 * return.
char[] findLongestSubarray(char[] array) {
  for (int len = array.length; len > 1; len--) {
    for (int i = 0; i <= array.length - len; i++) {
      if (hasEqualLettersNumbers(array, i, i + len - 1)) {
        return extractSubarray(array, i, i + len - 1);
      }
    }
  }
  return null;
}

/* Check if subarray has equal number of letters and numbers. */
boolean hasEqualLettersNumbers(char[] array, int start, int end) {
  int counter = 0;
  for (int i = start; i <= end; i++) {
    if (Character.isLetter(array[i])) {
      counter++;
    } else if (Character.isDigit(array[i])) {
      counter--;
    }
  }
  return counter == 0;
}

/* Return subarray of array between start and end (inclusive). */
char[] extractSubarray(char[] array, int start, int end) {
  char[] subarray = new char[end - start + 1];
  for (int i = start; i <= end; i++) {
    subarray[i - start] = array[i];
```

```
    }
    return subarray;
}
```

Despite the one optimization we made, this algorithm is still $O(N^2)$, where N is the length of the array.

Optimal Solution

What we're trying to do is find a subarray where the count of letters equals the count of numbers. What if we just started from the beginning, counting the number of letters and numbers?

	a	a	a	a	1	1	a	1	1	a	a	1	a	a	1	a	a	a	a	a
#a	1	2	3	4	4	4	5	5	5	6	7	7	8	9	9	10	11	12	13	14
#1	0	0	0	0	1	2	2	3	4	4	4	5	5	5	6	6	6	6	6	6

Certainly, whenever the number of letters equals the number of numbers, we can say that from index 0 to that index is an "equal" subarray.

That will only tell us equal subarrays that start at index 0. How can we identify all equal subarrays?

Let's picture this. Suppose we inserted an equal subarray (like `a11a1a`) after an array like `a1aaa1`. How would that impact the counts?

	a	1	a	a	a	1	|	a	1	1	a	1	a
#a	1	1	2	3	4	4	|	5	5	5	6	6	7
#1	0	1	1	1	1	2	|	2	3	4	4	5	5

Study the numbers before the subarray (4, 2) and the end (7, 5). You might notice that, while the values aren't the same, the differences are: `4 - 2 = 7 - 5`. This makes sense. Since they've added the same number of letters and numbers, they should maintain the same difference.

> Observe that when the difference is the same, the subarray starts one after the initial matching index and continues through the final matching index. This explains line 10 in the code below.

Let's update the earlier array with the differences.

	a	a	a	a	1	1	a	1	1	a	a	1	a	a	1	a	a	a	a	a
#a	1	2	3	4	4	4	5	5	5	6	7	7	8	9	9	10	11	12	13	14
#1	0	0	0	0	1	2	2	3	4	4	4	5	5	5	6	6	6	6	6	6
-	1	2	3	4	3	2	3	2	1	2	3	2	3	4	3	4	5	6	7	8

Whenever we return the same difference, then we know we have found an equal subarray. To find the biggest subarray, we just have to find the two indices farthest apart with the same value.

To do so, we use a hash table to store the first time we see a particular difference. Then, each time we see the same difference, we see if this subarray (from first occurrence of this index to current index) is bigger than the current max. If so, we update the max.

```
1  char[] findLongestSubarray(char[] array) {
2    /* Compute deltas between count of numbers and count of letters. */
3    int[] deltas = computeDeltaArray(array);
4
5    /* Find pair in deltas with matching values and largest span. */
6    int[] match = findLongestMatch(deltas);
7
8    /* Return the subarray. Note that it starts one *after* the initial occurence of
9     * this delta. */
10   return extract(array, match[0] + 1, match[1]);
11 }
12
```

```
/* Compute the difference between the number of letters and numbers between the
 * beginning of the array and each index. */
int[] computeDeltaArray(char[] array) {
  int[] deltas = new int[array.length];
  int delta = 0;
  for (int i = 0; i < array.length; i++) {
    if (Character.isLetter(array[i])) {
      delta++;
    } else if (Character.isDigit(array[i])) {
      delta--;
    }
    deltas[i] = delta;
  }
  return deltas;
}

/* Find the matching pair of values in the deltas array with the largest
 * difference in indices. */
int[] findLongestMatch(int[] deltas) {
  HashMap<Integer, Integer> map = new HashMap<Integer, Integer>();
  map.put(0, -1);
  int[] max = new int[2];
  for (int i = 0; i < deltas.length; i++) {
    if (!map.containsKey(deltas[i])) {
      map.put(deltas[i], i);
    } else {
      int match = map.get(deltas[i]);
      int distance = i - match;
      int longest = max[1] - max[0];
      if (distance > longest) {
        max[1] = i;
        max[0] = match;
      }
    }
  }
  return max;
}

char[] extract(char[] array, int start, int end) { /* same */ }
```

This solution takes O(N) time, where N is size of the array.

17.6 Count of 2s: Write a method to count the number of 2s between 0 and n.

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SOLUTION

Our first approach to this problem can be—and probably should be—a brute force solution. Remember that interviewers want to see how you're approaching a problem. Offering a brute force solution is a great way to start.

```
/* Counts the number of '2' digits between 0 and n */
int numberOf2sInRange(int n) {
  int count = 0;
  for (int i = 2; i <= n; i++) { // Might as well start at 2
    count += numberOf2s(i);
```

```
    }
    return count;
}

/* Counts the number of '2' digits in a single number */
int numberOf2s(int n) {
    int count = 0;
    while (n > 0) {
        if (n % 10 == 2) {
            count++;
        }
        n = n / 10;
    }
    return count;
}
```

The only interesting part is that it's probably cleaner to separate out `numberOf2s` into a separate method. This demonstrates an eye for code cleanliness.

Improved Solution

Rather than looking at the problem by ranges of numbers, we can look at the problem digit by digit. Picture a sequence of numbers:

```
  0   1   2   3   4   5   6   7   8   9
 10  11  12  13  14  15  16  17  18  19
 20  21  22  23  24  25  26  27  28  29
...
110 111 112 113 114 115 116 117 118 119
```

We know that roughly one tenth of the time, the last digit will be a 2 since it happens once in any sequence of ten numbers. In fact, any digit is a 2 roughly one tenth of the time.

We say "roughly" because there are (very common) boundary conditions. For example, between 1 and 100, the 10's digit is a 2 exactly $\frac{1}{10}^{th}$ of the time. However, between 1 and 37, the 10's digit is a 2 much more than $1/10^{th}$ of the time.

We can work out what exactly the ratio is by looking at the three cases individually: `digit < 2`, `digit = 2`, and `digit > 2`.

Case digit < 2

Consider the value `x = 61523` and `d = 3`, and observe that `x[d] = 1` (that is, the dth digit of x is 1). There are 2s at the 3rd digit in the ranges `2000 - 2999`, `12000 - 12999`, `22000 - 22999`, `32000 - 32999`, `42000 - 42999`, and `52000 - 52999`. We will not yet have hit the range `62000 - 62999`, so there are 6000 2s total in the 3rd digit. This is the same amount as if we were just counting all the 2s in the 3rd digit between 1 and 60000.

In other words, we can round *down* to the nearest 10^{d+1}, and then divide by 10, to compute the number of 2s in the dth digit.

```
if x[d] < 2: count2sInRangeAtDigit(x, d) =
    let y = round down to nearest 10^(d+1)
    return y / 10
```

Case digit > 2

Now, let's look at the case where dth digit of x is greater than 2 (`x[d] > 2`). We can apply almost the exact same logic to see that there are the same number of 2s in the 3rd digit in the range `0 - 63525` as there as in the range `0 - 70000`. So, rather than rounding down, we round up.

```
if x[d] > 2: count2sInRangeAtDigit(x, d) =
    let y = round up to nearest 10^(d+1)
    return y / 10
```

Case digit = 2

The final case may be the trickiest, but it follows from the earlier logic. Consider `x = 62523` and `d = 3`. We know that there are the same ranges of 2s from before (that is, the ranges `2000 - 2999`, `12000 - 12999`, ..., `52000 - 52999`). How many appear in the 3rd digit in the final, partial range from `62000 - 62523`? Well, that should be pretty easy. It's just 524 (`62000, 62001, ..., 62523`).

```
if x[d] = 2: count2sInRangeAtDigit(x, d) =
    let y = round down to nearest 10^(d+1)
    let z = right side of x (i.e., x % 10^d)
    return y / 10 + z + 1
```

Now, all you need is to iterate through each digit in the number. Implementing this code is reasonably straightforward.

```
int count2sInRangeAtDigit(int number, int d) {
  int powerOf10 = (int) Math.pow(10, d);
  int nextPowerOf10 = powerOf10 * 10;
  int right = number % powerOf10;

  int roundDown = number - number % nextPowerOf10;
  int roundUp = roundDown + nextPowerOf10;

  int digit = (number / powerOf10) % 10;
  if (digit < 2) { // if the digit in spot digit is
    return roundDown / 10;
  } else if (digit == 2) {
    return roundDown / 10 + right + 1;
  } else {
    return roundUp / 10;
  }
}

int count2sInRange(int number) {
  int count = 0;
  int len = String.valueOf(number).length();
  for (int digit = 0; digit < len; digit++) {
    count += count2sInRangeAtDigit(number, digit);
  }
  return count;
}
```

This question requires very careful testing. Make sure to generate a list of test cases, and to work through each of them.

17.7 **Baby Names:** Each year, the government releases a list of the 10,000 most common baby names and their frequencies (the number of babies with that name). The only problem with this is that some names have multiple spellings. For example, "John" and "Jon" are essentially the same name but would be listed separately in the list. Given two lists, one of names/frequencies and the other of pairs of equivalent names, write an algorithm to print a new list of the true frequency of each name. Note that if John and Jon are synonyms, and Jon and Johnny are synonyms, then John and Johnny are synonyms. (It is both transitive and symmetric.) In the final list, any name can be used as the "real" name.

EXAMPLE

Input:

Names: John (15), Jon (12), Chris (13), Kris (4), Christopher (19)

Synonyms: (Jon, John), (John, Johnny), (Chris, Kris), (Chris, Christopher)

Output: John (27), Kris (36)

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SOLUTION

Let's start off with a good example. We want an example with some names with multiple synonyms and some with none. Additionally, we want the synonym list to be diverse in which name is on the left side and which is on the right. For example, we wouldn't want Johnny to always be the name on the left side as we're creating the group of (John, Jonathan, Jon, and Johnny).

This list should work fairly well.

Name	Count
John	10
Jon	3
Davis	2
Kari	3
Johnny	11
Carlton	8
Carleton	2
Jonathan	9
Carrie	5

Name	Alternate
Jonathan	John
Jon	Johnny
Johnny	John
Kari	Carrie
Carleton	Carlton

The final list should be something like: John (33), Kari (8), Davis(2), Carleton (10).

Solution #1

Let's assume our baby names list is given to us as a hash table. (If not, it's easy enough to build one.)

We can start reading pairs in from the synonyms list. As we read the pair (Jonathan, John), we can merge the counts for Jonathan and John together. We'll need to remember, though, that we saw this pair, because, in the future, we could discover that Jonathan is equivalent to something else.

We can use a hash table (`L1`) that maps from a name to its "true" name. We'll also need to know, given a "true" name, all the names equivalent to it. This will be stored in a hash table `L2`. Note that `L2` acts as a reverse lookup of `L1`.

```
READ (Jonathan, John)
```

```
        L1.ADD Jonathan -> John
        L2.ADD John -> Jonathan
    READ (Jon, Johnny)
        L1.ADD Jon -> Johnny
        L2.ADD Johnny -> Jon
    READ (Johnny, John)
        L1.ADD Johnny -> John
        L1.UPDATE Jon -> John
        L2.UPDATE John -> Jonathan, Johnny, Jon
```

If we later find that John is equivalent to, say, Jonny, we'll need to look up the names in L1 and L2 and merge together all the names that are equivalent to them.

This will work, but it's unnecessarily complicated to keep track of these two lists.

Instead, we can think of these names as "equivalence classes." When we find a pair (Jonathan, John), we put these in the same set (or equivalence classes). Each name maps to its equivalence class. All items in the set map to the same instance of the set.

If we need to merge two sets, then we copy one set into the other and update the hash table to point to the new set.

```
    READ (Jonathan, John)
        CREATE Set1 = Jonathan, John
        L1.ADD Jonathan -> Set1
        L1.ADD John -> Set1
    READ (Jon, Johnny)
        CREATE Set2 = Jon, Johnny
        L1.ADD Jon -> Set2
        L1.ADD Johnny -> Set2
    READ (Johnny, John)
        COPY Set2 into Set1.
            Set1 = Jonathan, John, Jon, Johnny
        L1.UPDATE Jon -> Set1
        L1.UPDATE Johnny -> Set1
```

In the last step above, we iterated through all items in Set2 and updated the reference to point to Set1. As we do this, we keep track of the total frequency of names.

```
HashMap<String, Integer> trulyMostPopular(HashMap<String, Integer> names,
                                          String[][] synonyms) {
  /* Parse list and initialize equivalence classes.*/
  HashMap<String, NameSet> groups = constructGroups(names);

  /* Merge equivalence classes together. */
  mergeClasses(groups, synonyms);

  /* Convert back to hash map. */
  return convertToMap(groups);
}

/* This is the core of the algorithm. Read through each pair. Merge their
 * equivalence classes and update the mapping of the secondary class to point to
 * the first set.*/
void mergeClasses(HashMap<String, NameSet> groups, String[][] synonyms) {
  for (String[] entry : synonyms) {
    String name1 = entry[0];
    String name2 = entry[1];
    NameSet set1 = groups.get(name1);
```

```
    NameSet set2 = groups.get(name2);
    if (set1 != set2) {
      /* Always merge the smaller set into the bigger one. */
      NameSet smaller = set2.size() < set1.size() ? set2 : set1;
      NameSet bigger = set2.size() < set1.size() ? set1 : set2;

      /* Merge lists */
      Set<String> otherNames = smaller.getNames();
      int frequency = smaller.getFrequency();
      bigger.copyNamesWithFrequency(otherNames, frequency);

      /* Update mapping */
      for (String name : otherNames) {
        groups.put(name, bigger);
      }
    }
  }
}

/* Read through (name, frequency) pairs and initialize a mapping of names to
 * NameSets (equivalence classes).*/
HashMap<String, NameSet> constructGroups(HashMap<String, Integer> names) {
  HashMap<String, NameSet> groups = new HashMap<String, NameSet>();
  for (Entry<String, Integer> entry : names.entrySet()) {
    String name = entry.getKey();
    int frequency = entry.getValue();
    NameSet group = new NameSet(name, frequency);
    groups.put(name, group);
  }
  return groups;
}

HashMap<String, Integer> convertToMap(HashMap<String, NameSet> groups) {
  HashMap<String, Integer> list = new HashMap<String, Integer>();
  for (NameSet group : groups.values()) {
    list.put(group.getRootName(), group.getFrequency());
  }
  return list;
}

public class NameSet {
  private Set<String> names = new HashSet<String>();
  private int frequency = 0;
  private String rootName;

  public NameSet(String name, int freq) {
    names.add(name);
    frequency = freq;
    rootName = name;
  }

  public void copyNamesWithFrequency(Set<String> more, int freq) {
    names.addAll(more);
    frequency += freq;
  }

```

```
    public Set<String> getNames() { return names; }
    public String getRootName() { return rootName; }
    public int getFrequency() { return frequency; }
    public int size() { return names.size(); }
}
```

The runtime of the algorithm is a bit tricky to figure out. One way to think about it is to think about what the worst case is.

For this algorithm, the worst case is where all names are equivalent—and we have to constantly merge sets together. Also, for the worst case, the merging should come in the worst possible way: repeated pairwise merging of sets. Each merging requires copying the set's elements into an existing set and updating the pointers from those items. It's slowest when the sets are larger.

If you notice the parallel with merge sort (where you have to merge single-element arrays into two-element arrays, and then two-element arrays into four-element arrays, until finally having a full array), you might guess it's `O(N log N)`. That is correct.

If you don't notice that parallel, here's another way to think about it.

Imagine we had the names (a, b, c, d, . . ., z). In our worst case, we'd first pair up the items into equivalence classes: (a, b), (c, d), (e, f), . . ., (y, z). Then, we'd merge pairs of those: (a, b, c, d), (e, f, g, h), . . ., (w, x, y, z). We'd continue doing this until we wind up with just one class.

At each "sweep" through the list where we merge sets together, half of the items get moved into a new set. This takes `O(N)` work per sweep. (There are fewer sets to merge, but each set has grown larger.)

How many sweeps do we do? At each sweep, we have half as many sets as we did before. Therefore, we do `O(log N)` sweeps.

Since we're doing `O(log N)` sweeps and `O(N)` work per sweep, the total runtime is `O(N log N)`.

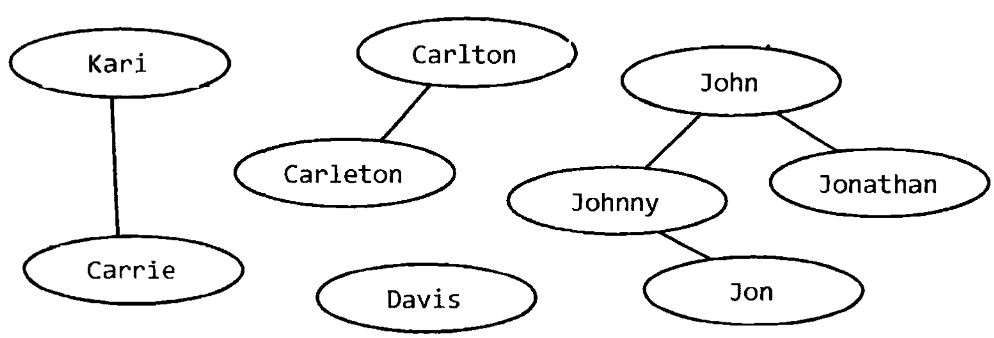
This is pretty good, but let's see if we can make it even faster.

Optimized Solution

To optimize the old solution, we should think about what exactly makes it slow. Essentially, it's the merging and updating of pointers.

So what if we just didn't do that? What if we marked that there was an equivalence relationship between two names, but didn't actually do anything with the information yet?

In this case, we'd be building essentially a graph.



Now what? Visually, it seems easy enough. Each component is an equivalent set of names. We just need to group the names by their component, sum up their frequencies, and return a list with one arbitrarily chosen name from each group.

In practice, how does this work? We could pick a name and do a depth-first (or breadth-first) search to sum the frequencies of all the names in one component. We would have to make sure that we hit each component exactly once. That's easy enough to achieve: mark a node as `visited` after it's discovered in the graph search, and only start the search for nodes where `visited` is false.

```
HashMap<String, Integer> trulyMostPopular(HashMap<String, Integer> names,
                                          String[][] synonyms) {
  /* Create data. */
  Graph graph = constructGraph(names);
  connectEdges(graph, synonyms);

  /* Find components. */
  HashMap<String, Integer> rootNames = getTrueFrequencies(graph);
  return rootNames;
}

/* Add all names to graph as nodes. */
Graph constructGraph(HashMap<String, Integer> names) {
  Graph graph = new Graph();
  for (Entry<String, Integer> entry : names.entrySet()) {
      String name = entry.getKey();
      int frequency = entry.getValue();
      graph.createNode(name, frequency);
  }
  return graph;
}

/* Connect synonymous spellings. */
void connectEdges(Graph graph, String[][] synonyms) {
  for (String[] entry : synonyms) {
      String name1 = entry[0];
      String name2 = entry[1];
      graph.addEdge(name1,  name2);
  }
}

/* Do DFS of each component. If a node has been visited before, then its component
 * has already been computed. */
HashMap<String, Integer> getTrueFrequencies(Graph graph) {
  HashMap<String, Integer> rootNames = new HashMap<String, Integer>();
  for (GraphNode node : graph.getNodes()) {
    if (!node.isVisited()) { // Already visited this component
      int frequency = getComponentFrequency(node);
      String name = node.getName();
      rootNames.put(name, frequency);
    }
  }
  return rootNames;
}

/* Do depth-first search to find the total frequency of this component, and mark
 * each node as visited.*/
int getComponentFrequency(GraphNode node) {
  if (node.isVisited()) return 0; // Already visited

  node.setIsVisited(true);
  int sum = node.getFrequency();
```

```
    for (GraphNode child : node.getNeighbors()) {
      sum += getComponentFrequency(child);
    }
    return sum;
  }

  /* Code for GraphNode and Graph is fairly self-explanatory, but can be found in
   * the downloadable code solutions.*/
```

To analyze the efficiency, we can think about the efficiency of each part of the algorithm.

- Reading in the data is linear with respect to the size of the data, so it takes O(B + P) time, where B is the number of baby names and P is the number of pairs of synonyms. This is because we only do a constant amount of work per piece of input data.
- To compute the frequencies, each edge gets "touched" exactly once across all of the graph searches and each node gets touched exactly once to check if it's been visited. The time of this part is O(B + P).

Therefore, the total time of the algorithm is O(B + P). We know we cannot do better than this since we must at least read in the B + P pieces of data.

17.8 Circus Tower: A circus is designing a tower routine consisting of people standing atop one another's shoulders. For practical and aesthetic reasons, each person must be both shorter and lighter than the person below him or her. Given the heights and weights of each person in the circus, write a method to compute the largest possible number of people in such a tower.

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SOLUTION

When we cut out all the "fluff" to this problem, we can understand that the problem is really the following.

We have a list of pairs of items. Find the longest sequence such that both the first and second items are in non-decreasing order.

One thing we might first try is sorting the items on an attribute. This is useful actually, but it won't get us all the way there.

By sorting the items by height, we have a relative order the items must appear in. We still need to find the longest increasing subsequence of weight though.

Solution 1: Recursive

One approach is to essentially try all possibilities. After sorting by height, we iterate through the array. At each element, we branch into two choices: add this element to the subsequence (if it's valid) or do not.

```
ArrayList<HtWt> longestIncreasingSeq(ArrayList<HtWt> items) {
  Collections.sort(items);
  return bestSeqAtIndex(items, new ArrayList<HtWt>(), 0);
}

ArrayList<HtWt> bestSeqAtIndex(ArrayList<HtWt> array, ArrayList<HtWt> sequence,
                               int index) {
  if (index >= array.size()) return sequence;

  HtWt value = array.get(index);

```

```
  ArrayList<HtWt> bestWith = null;
  if (canAppend(sequence, value)) {
    ArrayList<HtWt> sequenceWith = (ArrayList<HtWt>) sequence.clone();
    sequenceWith.add(value);
    bestWith = bestSeqAtIndex(array, sequenceWith, index + 1);
  }

  ArrayList<HtWt> bestWithout = bestSeqAtIndex(array, sequence, index + 1);

  if (bestWith == null || bestWithout.size() > bestWith.size()) {
    return bestWithout;
  } else {
    return bestWith;
  }
}

boolean canAppend(ArrayList<HtWt> solution, HtWt value) {
  if (solution == null) return false;
  if (solution.size() == 0) return true;

  HtWt last = solution.get(solution.size() - 1);
  return last.isBefore(value);
}

ArrayList<HtWt> max(ArrayList<HtWt> seq1, ArrayList<HtWt> seq2) {
  if (seq1 == null) {
    return seq2;
  } else if (seq2 == null) {
    return seq1;
  }
  return seq1.size() > seq2.size() ? seq1 : seq2;
}

public class HtWt implements Comparable<HtWt> {
  private int height;
  private int weight;
  public HtWt(int h, int w) { height = h; weight = w; }

  public int compareTo(HtWt second) {
    if (this.height != second.height) {
      return ((Integer)this.height).compareTo(second.height);
    } else {
      return ((Integer)this.weight).compareTo(second.weight);
    }
  }

  /* Returns true if "this" should be lined up before "other". Note that it's
   * possible that this.isBefore(other) and other.isBefore(this) are both false.
   * This is different from the compareTo method, where if a < b then b > a. */
  public boolean isBefore(HtWt other) {
    if (height < other.height && weight < other.weight) {
      return true;
    } else {
      return false;
    }
  }
```

```
}
```

This algorithm will take $O(2^n)$ time. We can optimize it using memoization (that is, caching the best sequences).

There's a cleaner way to do this though.

Solution #2: Iterative

Imagine we had the longest subsequence that terminates with each element, A[0] through A[3]. Could we use this to find the longest subsequence that terminates with A[4]?

```
Array: 13, 14, 10, 11, 12
Longest(ending with A[0]): 13
Longest(ending with A[1]): 13, 14
Longest(ending with A[2]): 10
Longest(ending with A[3]): 10, 11
Longest(ending with A[4]): 10, 11, 12
```

Sure. We just append A[4] on to the longest subsequence that it can be appended to.

This is now fairly straightforward to implement.

```
ArrayList<HtWt> longestIncreasingSeq(ArrayList<HtWt> array) {
  Collections.sort(array);

  ArrayList<ArrayList<HtWt>> solutions =  new ArrayList<ArrayList<HtWt>>();
  ArrayList<HtWt> bestSequence = null;

  /* Find the longest subsequence that terminates with each element. Track the
   * longest overall subsequence as we go. */
  for (int i = 0; i < array.size(); i++) {
    ArrayList<HtWt> longestAtIndex = bestSeqAtIndex(array, solutions, i);
    solutions.add(i, longestAtIndex);
    bestSequence = max(bestSequence, longestAtIndex);
  }

  return bestSequence;
}

/* Find the longest subsequence which terminates with this element. */
ArrayList<HtWt> bestSeqAtIndex(ArrayList<HtWt> array,
    ArrayList<ArrayList<HtWt>> solutions, int index) {
  HtWt value = array.get(index);

  ArrayList<HtWt> bestSequence = new ArrayList<HtWt>();

  /* Find the longest subsequence that we can append this element to. */
  for (int i = 0; i < index; i++) {
    ArrayList<HtWt> solution = solutions.get(i);
    if (canAppend(solution, value)) {
      bestSequence = max(solution, bestSequence);
    }
  }

  /* Append element. */
  ArrayList<HtWt> best = (ArrayList<HtWt>) bestSequence.clone();
  best.add(value);

```

```
    return best;
}
```

This algorithm operates in $O(n^2)$ time. An O(n log(n)) algorithm does exist, but it is considerably more complicated and it is highly unlikely that you would derive this in an interview—even with some help. However, if you are interested in exploring this solution, a quick internet search will turn up a number of explanations of this solution.

17.9 **Kth Multiple:** Design an algorithm to find the kth number such that the only prime factors are 3, 5, and 7. Note that 3, 5, and 7 do not have to be factors, but it should not have any other prime factors. For example, the first several multiples would be (in order) 1, 3, 5, 7, 9, 15, 21.

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SOLUTION

Let's first understand what this problem is asking for. It's asking for the kth smallest number that is in the form $3^a * 5^b * 7^c$. Let's start with a brute force way of finding this.

Brute Force

We know that biggest this kth number could be is $3^k * 5^k * 7^k$. So, the "stupid" way of doing this is to compute $3^a * 5^b * 7^c$ for all values of a, b, and c between 0 and k. We can throw them all into a list, sort the list, and then pick the kth smallest value.

```
int getKthMagicNumber(int k) {
  ArrayList<Integer> possibilities = allPossibleKFactors(k);
  Collections.sort(possibilities);
  return possibilities.get(k);
}

ArrayList<Integer> allPossibleKFactors(int k) {
  ArrayList<Integer> values = new ArrayList<Integer>();
  for (int a = 0; a <= k; a++) { // loop 3
    int powA = (int) Math.pow(3, a);
    for (int b = 0; b <= k; b++) { // loop 5
      int powB = (int) Math.pow(5, b);
      for (int c = 0; c <= k; c++) { // loop 7
        int powC = (int) Math.pow(7, c);
        int value = powA * powB * powC;

        /* Check for overflow. */
        if (value < 0 || powA == Integer.MAX_VALUE ||
            powB == Integer.MAX_VALUE ||
            powC == Integer.MAX_VALUE) {
          value = Integer.MAX_VALUE;
        }
        values.add(value);
      }
    }
  }
  return values;
}
```

What is the runtime of this approach? We have nested for loops, each of which runs for k iterations. The runtime of the `allPossibleKFactors` is $O(k^3)$. Then, we sort the k^3 *results in* $O(k^3 \log (k^3))$ time (which is equivalent to $O(k^3 \log k)$. This gives us a runtime of $O(k^3 \log k)$.

There are a number of optimizations you could make to this (and better ways of handling the integer overflow), but honestly this algorithm is fairly slow. We should instead focus on reworking the algorithm.

Improved

Let's picture what our results will look like.

1	-	$3^0 * 5^0 * 7^0$
3	3	$3^1 * 5^0 * 7^0$
5	5	$3^0 * 5^1 * 7^0$
7	7	$3^0 * 5^0 * 7^1$
9	3*3	$3^2 * 5^0 * 7^0$
15	3*5	$3^1 * 5^1 * 7^0$
21	3*7	$3^1 * 5^0 * 7^1$
25	5*5	$3^0 * 5^2 * 7^0$
27	3*9	$3^3 * 5^0 * 7^0$
35	5*7	$3^0 * 5^1 * 7^1$
45	5*9	$3^2 * 5^1 * 7^0$
49	7*7	$3^0 * 5^0 * 7^2$
63	3*21	$3^2 * 5^0 * 7^1$

The question is: what is the next value in the list? The next value will be one of these:

- 3 * (some previous number in list)
- 5 * (some previous number in list)
- 7 * (some previous number in list)

If this doesn't immediately jump out at you, think about it this way: whatever the next value (let's call it nv) is, divide it by 3. Will that number have already appeared? As long as nv has factors of 3 in it, yes. The same can be said for dividing it by 5 and 7.

So, we know A_k can be expressed as (3, 5 or 7) * (some value in $\{A_1, \ldots, A_{k-1}\}$). We also know that A_k is, by definition, the next number in the list. Therefore, A_k will be the smallest "new" number (a number that it's already in $\{A_1, \ldots, A_{k-1}\}$) that can be formed by multiplying each value in the list by 3, 5 or 7.

How would we find A_k? Well, we could actually multiply each number in the list by 3, 5, and 7 and find the smallest element that has not yet been added to our list. This solution is $O(k^2)$. Not bad, but I think we can do better.

Rather than A_k trying to "pull" from a previous element in the list (by multiplying all of them by 3, 5 and 7), we can think about each previous value in the list as "pushing" out three subsequent values in the list. That is, each number A_i will eventually be used later in the list in the following forms:

- $3 * A_i$
- $5 * A_i$
- $7 * A_i$

We can use this thought to plan in advance. Each time we add a number A_i to the list, we hold on to the values $3A_i$, $5A_i$, and $7A_i$ in some sort of temporary list. To generate A_{i+1}, we search through this temporary list to find the smallest value.

Our code looks like this:

```
int removeMin(Queue<Integer> q) {
  int min = q.peek();
  for (Integer v : q) {
    if (min > v) {
      min = v;
    }
  }
  while (q.contains(min)) {
    q.remove(min);
  }
  return min;
}

void addProducts(Queue<Integer> q, int v) {
  q.add(v * 3);
  q.add(v * 5);
  q.add(v * 7);
}

int getKthMagicNumber(int k) {
  if (k < 0) return 0;

  int val = 1;
  Queue<Integer> q = new LinkedList<Integer>();
  addProducts(q, 1);
  for (int i = 0; i < k; i++) {
    val = removeMin(q);
    addProducts(q, val);
  }
  return val;
}
```

This algorithm is certainly much, much better than our first algorithm, but it's still not quite perfect.

Optimal Algorithm

To generate a new element A_i, we are searching through a linked list where each element looks like one of:

- 3 * previous element
- 5 * previous element
- 7 * previous element

Where is there unnecessary work that we might be able to optimize out?

Let's imagine our list looks like:

$$q_6 = \{7A_1, 5A_2, 7A_2, 7A_3, 3A_4, 5A_4, 7A_4, 5A_5, 7A_5\}$$

When we search this list for the min, we check if $7A_1 < \text{min}$, and then later we check if $7A_5 < \text{min}$. That seems sort of silly, doesn't it? Since we know that $A_1 < A_5$, we should only need to check $7A_1$.

If we separated the list from the beginning by the constant factors, then we'd only need to check the first of the multiples of 3, 5 and 7. All subsequent elements would be bigger.

That is, our list above would look like:

$Q36 = \{3A_4\}$
$Q56 = \{5A_2, 5A_4, 5A_5\}$
$Q76 = \{7A_1, 7A_2, 7A_3, 7A_4, 7A_5\}$

To get the min, we only need to look at the fronts of each queue:

```
y = min(Q3.head(), Q5.head(), Q7.head())
```

Once we compute y, we need to insert 3y into Q3, 5y into Q5, and 7y into Q7. But, we only want to insert these elements if they aren't already in another list.

Why might, for example, 3y already be somewhere in the holding queues? Well, if y was pulled from Q7, then that means that y = 7x, for some smaller x. If 7x is the smallest value, we must have already seen 3x. And what did we do when we saw 3x? We inserted 7 * 3x into Q7. Note that 7 * 3x = 3 * 7x = 3y.

To put this another way, if we pull an element from Q7, it will look like 7 * `suffix`, and we know we have already handled 3 * `suffix` and 5 * `suffix`. In handling 3 * `suffix`, we inserted 7 * 3 * `suffix` into a Q7. And in handling 5 * `suffix`, we know we inserted 7 * 5 * `suffix` in Q7. The only value we haven't seen yet is 7 * 7 * suffix, so we just insert 7 * 7 * `suffix` into Q7.

Let's walk through this with an example to make it really clear.

```
initialize:
        Q3 = 3
        Q5 = 5
        Q7 = 7
remove min = 3. insert 3*3 in Q3, 5*3 into Q5, 7*3 into Q7.
        Q3 = 3*3
        Q5 = 5, 5*3
        Q7 = 7, 7*3
remove min = 5. 3*5 is a dup, since we already did 5*3. insert 5*5 into Q5, 7*5
into Q7.
        Q3 = 3*3
        Q5 = 5*3, 5*5
        Q7 = 7, 7*3, 7*5.
remove min = 7. 3*7 and 5*7 are dups, since we already did 7*3 and 7*5. insert 7*7
into Q7.
        Q3 = 3*3
        Q5 = 5*3, 5*5
        Q7 = 7*3, 7*5, 7*7
remove min = 3*3 = 9. insert 3*3*3 in Q3, 3*3*5 into Q5, 3*3*7 into Q7.
        Q3 = 3*3*3
        Q5 = 5*3, 5*5, 5*3*3
        Q7 = 7*3, 7*5, 7*7, 7*3*3
remove min = 5*3 = 15. 3*(5*3) is a dup, since we already did 5*(3*3). insert
5*5*3 in Q5, 7*5*3 into Q7.
        Q3 = 3*3*3
        Q5 = 5*5, 5*3*3, 5*5*3
        Q7 = 7*3, 7*5, 7*7, 7*3*3, 7*5*3
remove min = 7*3 = 21. 3*(7*3) and 5*(7*3) are dups, since we already did 7*(3*3)
and 7*(5*3). insert 7*7*3 into Q7.
        Q3 = 3*3*3
        Q5 = 5*5, 5*3*3, 5*5*3
        Q7 = 7*5, 7*7, 7*3*3, 7*5*3, 7*7*3
```

Our pseudocode for this problem is as follows:

1. Initialize `array` and queues Q3, Q5, and Q7

2. Insert 1 into `array`.
3. Insert 1*3, 1*5 and 1*7 into Q3, Q5, and Q7 respectively.
4. Let x be the minimum element in Q3, Q5, and Q7. Append x to `magic`.
5. If x was found in:

   Q3 -> append x*3, x*5 and x*7 to Q3, Q5, and Q7. Remove x from Q3.

   Q5 -> append x*5 and x*7 to Q5 and Q7. Remove x from Q5.

   Q7 -> only append x*7 to Q7. Remove x from Q7.
6. Repeat steps 4 - 6 until we've found k elements.

The code below implements this algorithm.

```
int getKthMagicNumber(int k) {
   if (k < 0) {
      return 0;
   }
   int val = 0;
   Queue<Integer> queue3 = new LinkedList<Integer>();
   Queue<Integer> queue5 = new LinkedList<Integer>();
   Queue<Integer> queue7 = new LinkedList<Integer>();
   queue3.add(1);

   /* Include 0th through kth iteration */
   for (int i = 0; i <= k; i++) {
      int v3 = queue3.size() > 0 ? queue3.peek() : Integer.MAX_VALUE;
      int v5 = queue5.size() > 0 ? queue5.peek() : Integer.MAX_VALUE;
      int v7 = queue7.size() > 0 ? queue7.peek() : Integer.MAX_VALUE;
      val = Math.min(v3, Math.min(v5, v7));
      if (val == v3) { // enqueue into queue 3, 5 and 7
         queue3.remove();
         queue3.add(3 * val);
         queue5.add(5 * val);
      } else if (val == v5) { // enqueue into queue 5 and 7
         queue5.remove();
         queue5.add(5 * val);
      } else if (val == v7) { // enqueue into Q7
         queue7.remove();
      }
      queue7.add(7 * val); // Always enqueue into Q7
   }
   return val;
}
```

When you get this question, do your best to solve it—even though it's really difficult. You can start with a brute force approach (challenging, but not quite as tricky), and then you can start trying to optimize it. Or, try to find a pattern in the numbers.

Chances are that your interviewer will help you along when you get stuck. Whatever you do, don't give up! Think out loud, wonder out loud, and explain your thought process. Your interviewer will probably jump in to guide you.

Remember, perfection on this problem is not expected. Your performance is evaluated in comparison to other candidates. Everyone struggles on a tricky problem.

17.10 Majority Element: A majority element is an element that makes up more than half of the items in an array. Given a positive integers array, find the majority element. If there is no majority element, return -1. Do this in O(N) time and O(1) space.

Input: 1 2 5 9 5 9 5 5 5

Output: 5

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SOLUTION

Let's start off with an example:

3 1 7 1 3 7 3 7 1 7 7

One thing we can notice here is that if the majority element (in this case 7) appears less often in the beginning, it must appear much more often toward the end. That's a good observation to make.

This interview question specifically requires us to do this in O(N) time and O(1) space. Nonetheless, sometimes it can be useful to relax one of those requirements and develop an algorithm. Let's try relaxing the time requirement but staying firm on the O(1) space requirement.

Solution #1 (Slow)

One simple way to do this is to just iterate through the array and check each element for whether it's the majority element. This takes $O(N^2)$ time and O(1) space.

```
int findMajorityElement(int[] array) {
   for (int x : array) {
      if (validate(array, x)) {
         return x;
      }
   }
   return -1;
}

boolean validate(int[] array, int majority) {
   int count = 0;
   for (int n : array) {
      if (n == majority) {
         count++;
      }
   }

   return count > array.length / 2;
}
```

This does not fit the time requirements of the problem, but it is potentially a starting point. We can think about optimizing this.

Solution #2 (Optimal)

Let's think about what that algorithm did on a particular example. Is there anything we can get rid of?

3	1	7	1	1	7	7	3	7	7	7
0	1	2	3	4	5	6	7	8	9	10

In the very first validation pass, we select 3 and validate it as the majority element. Several elements later, we've still counted just one 3 and several non-3 elements. Do we need to continue checking for 3?

On one hand, yes. 3 could redeem itself and be the majority element, if there are a bunch of 3s later in the array.

On the other hand, not really. If 3 does redeem itself, then we'll encounter those 3s later on, in a subsequent validation step. We could terminate this `validate(3)` step.

That logic is fine for the first element, but what about the next one? We would immediately terminate `validate(1)`, `validate(7)`, and so on.

Since the logic was okay for the first element, what if we treated all subsequent elements like they're the first element of some new subarray? This would mean that we start `validate(array[1])` at index 1, `validate(array[2])` at index 2, and so on.

What would this look like?

```
validate(3)
    sees 3 -> countYes = 1, countNo = 0
    sees 1 -> countYes = 1, countNo = 1
    TERMINATE. 3 is not majority thus far.
validate(1)
    sees 1 -> countYes = 0, countNo = 0
    sees 7 -> countYes = 1, countNo = 1
    TERMINATE. 1 is not majority thus far.
validate(7)
    sees 7 -> countYes = 1, countNo = 0
    sees 1 -> countYes = 1, countNo = 1
    TERMINATE. 7 is not majority thus far.
validate(1)
    sees 1 -> countYes = 1, countNo = 0
    sees 1 -> countYes = 2, countNo = 0
    sees 7 -> countYes = 2, countNo = 1
    sees 7 -> countYes = 2, countNo = 1
    TERMINATE. 1 is not majority thus far.
validate(1)
    sees 1 -> countYes = 1, countNo = 0
    sees 7 -> countYes = 1, countNo = 1
    TERMINATE. 1 is not majority thus far.
validate(7)
    sees 7 -> countYes = 1, countNo = 0
    sees 7 -> countYes = 2, countNo = 0
    sees 3 -> countYes = 2, countNo = 1
    sees 7 -> countYes = 3, countNo = 1
    sees 7 -> countYes = 4, countNo = 1
    sees 7 -> countYes = 5, countNo = 1
```

Do we know at this point that 7 is the majority element? Not necessarily. We have eliminated everything before that 7, and everything after it. But there could be no majority element. A quick `validate(7)` pass that starts from the beginning can confirm if 7 is actually the majority element. This `validate` step will be `O(N)` time, which is also our Best Conceivable Runtime. Therefore, this final `validate` step won't impact our total runtime.

This is pretty good, but let's see if we can make this a bit faster. We should notice that some elements are being "inspected" repeatedly. Can we get rid of this?

Look at the first `validate(3)`. This fails after the subarray `[3, 1]`, because 3 was not the majority element. But because `validate` fails the instant an element is not the majority element, it also means nothing else in that subarray was the majority element. By our earlier logic, we don't need to call `validate(1)`. We know that 1 did not appear more than half the time. If it is the majority element, it'll pop up later.

Let's try this again and see if it works out.

```
validate(3)
    sees 3 -> countYes = 1, countNo = 0
    sees 1 -> countYes = 1, countNo = 1
    TERMINATE. 3 is not majority thus far.
skip 1
validate(7)
    sees 7 -> countYes = 1, countNo = 0
    sees 1 -> countYes = 1, countNo = 1
    TERMINATE. 7 is not majority thus far.
skip 1
validate(1)
    sees 1 -> countYes = 1, countNo = 0
    sees 7 -> countYes = 1, countNo = 1
    TERMINATE. 1 is not majority thus far.
skip 7
validate(7)
    sees 7 -> countYes = 1, countNo = 0
    sees 3 -> countYes = 1, countNo = 1
    TERMINATE. 7 is not majority thus far.
skip 3
validate(7)
    sees 7 -> countYes = 1, countNo = 0
    sees 7 -> countYes = 2, countNo = 0
    sees 7 -> countYes = 3, countNo = 0
```

Good! We got the right answer. But did we just get lucky?

We should pause for a moment to think what this algorithm is doing.

1. We start off with `[3]` and we expand the subarray until 3 is no longer the majority element. We fail at `[3, 1]`. At the moment we fail, the subarray can have no majority element.
2. Then we go to `[7]` and expand until `[7, 1]`. Again, we terminate and nothing could be the majority element in that subarray.
3. We move to [1] and expand to `[1, 7]`. We terminate. Nothing there could be the majority element.
4. We go to `[7]` and expand to `[7, 3]`. We terminate. Nothing there could be the majority element.
5. We go to `[7]` and expand until the end of the array: `[7, 7, 7]`. We have found the majority element (and now we must validate that).

Each time we terminate the `validate` step, the subarray has no majority element. This means that there are at least as many non-7s as there are 7s. Although we're essentially removing this subarray from the original array, the majority element will still be found in the rest of the array—and will still have majority status. Therefore, at some point, we will discover the majority element.

Our algorithm can now be run in two passes: one to find the possible majority element and another to validate it. Rather than using two variables to count (`countYes` and `countNo`), we'll just use a single `count` variable that increments and decrements.

```
int findMajorityElement(int[] array) {
  int candidate = getCandidate(array);
  return validate(array, candidate) ? candidate : -1;
}

int getCandidate(int[] array) {
  int majority = 0;
```

```
    int count = 0;
    for (int n : array) {
       if (count == 0) { // No majority element in previous set.
          majority = n;
       }
       if (n == majority) {
          count++;
       } else {
          count--;
       }
    }
    return majority;
}

boolean validate(int[] array, int majority) {
    int count = 0;
    for (int n : array) {
       if (n == majority) {
          count++;
       }
    }

    return count > array.length / 2;
}
```

This algorithm runs in O(N) time and O(1) space.

17.11 Word Distance: You have a large text file containing words. Given any two words, find the shortest distance (in terms of number of words) between them in the file. If the operation will be repeated many times for the same file (but different pairs of words), can you optimize your solution?

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SOLUTION

We will assume for this question that it doesn't matter whether word1 or word2 appears first. This is a question you should ask your interviewer.

To solve this problem, we can traverse the file just once. We remember throughout our traversal where we've last seen word1 and word2, storing the locations in location1 and location2. If the current locations are better than our best known location, we update the best locations.

The code below implements this algorithm.

```
LocationPair findClosest(String[] words, String word1, String word2) {
    LocationPair best = new LocationPair(-1, -1);
    LocationPair current = new LocationPair(-1, -1);
    for (int i = 0; i < words.length; i++) {
       String word = words[i];
       if (word.equals(word1)) {
          current.location1 = i;
          best.updateWithMin(current);
       } else if (word.equals(word2)) {
          current.location2 = i;
          best.updateWithMin(current); // If shorter, update values
       }
    }
```

```
  return best;
}

public class LocationPair {
  public int location1, location2;
  public LocationPair(int first, int second) {
    setLocations(first, second);
  }

  public void setLocations(int first, int second) {
    this.location1 = first;
    this.location2 = second;
  }

  public void setLocations(LocationPair loc) {
    setLocations(loc.location1, loc.location2);
  }

  public int distance() {
    return Math.abs(location1 - location2);
  }

  public boolean isValid() {
    return location1 >= 0 && location2 >= 0;
  }

  public void updateWithMin(LocationPair loc) {
    if (!isValid() || loc.distance() < distance()) {
      setLocations(loc);
    }
  }
}
```

If we need to repeat the operation for other pairs of words, we can create a hash table that maps from each word to the locations where it occurs. We'll only need to read through the list of words once. After that point, we can do a very similar algorithm but just iterate through the locations directly.

Consider the following lists of locations.

```
listA: {1, 2, 9, 15, 25}
listB: {4, 10, 19}
```

Picture pointers pA and pB that point to the beginning of each list. Our goal is to make pA and pB point to values as close together as possible.

The first potential pair is `(1, 4)`.

What is the next pair we can find? If we moved pB, then the distance would definitely get larger. If we moved pA, though, we might get a better pair. Let's do that.

The second potential pair is `(2, 4)`. This is better than the previous pair, so let's record this as the best pair.

We move pA again and get `(9, 4)`. This is worse than we had before.

Now, since the value at pA is bigger than the one at pB, we move pB. We get `(9, 10)`.

Next we get `(15, 10)`, then `(15, 19)`, then `(25, 19)`.

We can implement this algorithm as shown below.

```
LocationPair findClosest(String word1, String word2,
```

```
                            HashMapList<String, Integer> locations) {
    ArrayList<Integer> locations1 = locations.get(word1);
    ArrayList<Integer> locations2 = locations.get(word2);
    return findMinDistancePair(locations1, locations2);
}

LocationPair findMinDistancePair(ArrayList<Integer> array1,
                                 ArrayList<Integer> array2) {
    if (array1 == null || array2 == null || array1.size() == 0 ||
        array2.size() == 0) {
        return null;
    }

    int index1 = 0;
    int index2 = 0;
    LocationPair best = new LocationPair(array1.get(0), array2.get(0));
    LocationPair current = new LocationPair(array1.get(0), array2.get(0));

    while (index1 < array1.size() && index2 < array2.size()) {
        current.setLocations(array1.get(index1), array2.get(index2));
        best.updateWithMin(current); // If shorter, update values
        if (current.location1 < current.location2) {
            index1++;
        } else {
            index2++;
        }
    }

    return best;
}

/* Precomputation. */
HashMapList<String, Integer> getWordLocations(String[] words) {
    HashMapList<String, Integer> locations = new HashMapList<String, Integer>();
    for (int i = 0; i < words.length; i++) {
        locations.put(words[i], i);
    }
    return locations;
}

/* HashMapList<String, Integer> is a HashMap that maps from Strings to
 * ArrayList<Integer>. See appendix for implementation. */
```

The precomputation step of this algorithm will take O(N) time, where N is the number of words in the string.

Finding the closest pair of locations will take O(A + B) time, where A is the number of occurrences of the first word and B is the number of occurrences of the second word.

17.12 BiNode: Consider a simple data structure called BiNode, which has pointers to two other nodes. The data structure `BiNode` could be used to represent both a binary tree (where `node1` is the left node and `node2` is the right node) or a doubly linked list (where `node1` is the previous node and `node2` is the next node). Implement a method to convert a binary search tree (implemented with `BiNode`) into a doubly linked list. The values should be kept in order and the operation should be performed in place (that is, on the original data structure).

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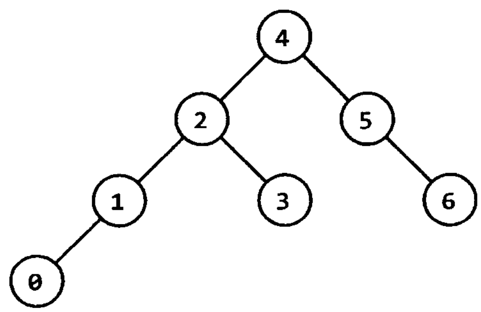
SOLUTION

This seemingly complex problem can be implemented quite elegantly using recursion. You will need to understand recursion very well to solve it.

Picture a simple binary search tree:



The `convert` method should transform it into the below doubly linked list:

`0 <-> 1 <-> 2 <-> 3 <-> 4 <-> 5 <-> 6`

Let's approach this recursively, starting with the root (node 4).

We know that the left and right halves of the tree form their own "sub-parts" of the linked list (that is, they appear consecutively in the linked list). So, if we recursively converted the left and right subtrees to a doubly linked list, could we build the final linked list from those parts?

Yes! We would simply merge the different parts.

The pseudocode looks something like:

```
BiNode convert(BiNode node) {
  BiNode left = convert(node.left);
  BiNode right = convert(node.right);
  mergeLists(left, node, right);
  return left; // front of left
}
```

To actually implement the nitty-gritty details of this, we'll need to get the head and tail of each linked list. We can do this several different ways.

Solution #1: Additional Data Structure

The first, and easier, approach is to create a new data structure called `NodePair` which holds just the head and tail of a linked list. The `convert` method can then return something of type `NodePair`.

The code below implements this approach.

```
private class NodePair {
```

```
  BiNode head, tail;

  public NodePair(BiNode head, BiNode tail) {
    this.head = head;
    this.tail = tail;
  }
}

public NodePair convert(BiNode root) {
  if (root == null) return null;

  NodePair part1 = convert(root.node1);
  NodePair part2 = convert(root.node2);

  if (part1 != null) {
    concat(part1.tail, root);
  }

  if (part2 != null) {
    concat(root, part2.head);
  }

  return new NodePair(part1 == null ? root : part1.head,
                      part2 == null ? root : part2.tail);
}

public static void concat(BiNode x, BiNode y) {
  x.node2 = y;
  y.node1 = x;
}
```

The above code still converts the BiNode data structure in place. We're just using NodePair as a way to return additional data. We could have alternatively used a two-element BiNode array to fulfill the same purposes, but it looks a bit messier (and we like clean code, especially in an interview).

It'd be nice, though, if we could do this without these extra data structures—and we can.

Solution #2: Retrieving the Tail

Instead of returning the head and tail of the linked list with NodePair, we can return just the head, and then we can use the head to find the tail of the linked list.

```
BiNode convert(BiNode root) {
  if (root == null) return null;

  BiNode part1 = convert(root.node1);
  BiNode part2 = convert(root.node2);

  if (part1 != null) {
    concat(getTail(part1), root);
  }

  if (part2 != null) {
    concat(root, part2);
  }

  return part1 == null ? root : part1;
```

```
}

public static BiNode getTail(BiNode node) {
  if (node == null) return null;
  while (node.node2 != null) {
    node = node.node2;
  }
  return node;
}
```

Other than a call to `getTail`, this code is almost identical to the first solution. It is not, however, very efficient. A leaf node at depth d will be "touched" by the `getTail` method d times (one for each node above it), leading to an $O(N^2)$ overall runtime, where N is the number of nodes in the tree.

Solution #3: Building a Circular Linked List

We can build our third and final approach off of the second one.

This approach requires returning the head and tail of the linked list with `BiNode`. We can do this by returning each list as the head of a *circular* linked list. To get the tail, then, we simply call `head.node1`.

```
BiNode convertToCircular(BiNode root) {
  if (root == null) return null;

  BiNode part1 = convertToCircular(root.node1);
  BiNode part3 = convertToCircular(root.node2);

  if (part1 == null && part3 == null) {
    root.node1 = root;
    root.node2 = root;
    return root;
  }
  BiNode tail3 = (part3 == null) ? null : part3.node1;

  /* join left to root */
  if (part1 == null) {
    concat(part3.node1, root);
  } else {
    concat(part1.node1, root);
  }

  / *join right to root */
  if (part3 == null) {
    concat(root, part1);
  } else {
    concat(root, part3);
  }

  / *join right to left */
  if (part1 != null && part3 != null) {
    concat(tail3, part1);
  }

  return part1 == null ? root : part1;
}

/ *Convert list to a circular linked list, then break the circular connection. */
```

```
BiNode convert(BiNode root) {
  BiNode head = convertToCircular(root);
  head.node1.node2 = null;
  head.node1 = null;
  return head;
}
```

Observe that we have moved the main parts of the code into `convertToCircular`. The `convert` method calls this method to get the head of the circular linked list, and then breaks the circular connection.

The approach takes `O(N)` time, since each node is only touched an average of once (or, more accurately, `O(1)` times).

17.13 Re-Space: Oh, no! You have accidentally removed all spaces, punctuation, and capitalization in a lengthy document. A sentence like "`I reset the computer. It still didn't boot!`" became "`iresetthecomputeritstilldidntboot`". You'll deal with the punctuation and capitalization later; right now you need to re-insert the spaces. Most of the words are in a dictionary but a few are not. Given a dictionary (a list of strings) and the document (a string), design an algorithm to unconcatenate the document in a way that minimizes the number of unrecognized characters.

EXAMPLE

Input: `jesslookedjustliketimherbrother`

Output: `jess looked just like tim her brother` (7 unrecognized characters)

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SOLUTION

Some interviewers like to cut to the chase and give you the specific problems. Others, though, like to give you a lot of unnecessary context, like this problem has. It's useful in such cases to boil down the problem to what it's really all about.

In this case, the problem is really about finding a way to break up a string into separate words such that as few characters as possible are "left out" of the parsing.

Note that we do not attempt to "understand" the string. We could just as well parse "`thisisawesome`" to be "`this is a we some`" as we could "`this is awesome`."

Brute Force

The key to this problem is finding a way to define the solution (that is, parsed string) in terms of its subproblems. One way to do this is recursing through the string.

The very first choice we make is where to insert the first space. After the first character? Second character? Third character?

Let's imagine this in terms of a string like `thisismikesfavoritefood`. What is the first space we insert?

- If we insert a space after `t`, this gives us one invalid character.
- After `th` is two invalid characters.
- After `thi` is three invalid characters.
- At `this` we have a complete word. This is zero invalid characters.
- At `thisi` is five invalid characters.
- ... and so on.

After we choose the first space, we can recursively pick the second space, then the third space, and so on, until we are done with the string.

We take the best (fewest invalid characters) out of all these choices and return.

What should the function return? We need both the number of invalid characters in the recursive path as well as the actual parsing. Therefore, we just return both by using a custom-built `ParseResult` class.

```
1  String bestSplit(HashSet<String> dictionary, String sentence) {
2    ParseResult r = split(dictionary, sentence, 0);
3    return r == null ? null : r.parsed;
4  }
5
6  ParseResult split(HashSet<String> dictionary, String sentence, int start) {
7    if (start >= sentence.length()) {
8      return new ParseResult(0, “”);
9    }
10
11   int bestInvalid = Integer.MAX_VALUE;
12   String bestParsing = null;
13   String partial = “”;
14   int index = start;
15   while (index < sentence.length()) {
16     char c = sentence.charAt(index);
17     partial += c;
18     int invalid = dictionary.contains(partial) ? 0 : partial.length();
19     if (invalid < bestInvalid) { // Short circuit
20       /* Recurse, putting a space after this character. If this is better than
21        * the current best option, replace the best option. */
22       ParseResult result = split(dictionary, sentence, index + 1);
23       if (invalid + result.invalid < bestInvalid) {
24         bestInvalid = invalid + result.invalid;
25         bestParsing = partial + “ ” + result.parsed;
26         if (bestInvalid == 0) break; // Short circuit
27       }
28     }
29
30     index++;
31   }
32   return new ParseResult(bestInvalid, bestParsing);
33 }
34
35 public class ParseResult {
36   public int invalid = Integer.MAX_VALUE;
37   public String parsed = “ ”;
38   public ParseResult(int inv, String p) {
39     invalid = inv;
40     parsed = p;
41   }
42 }
```

We've applied two short circuits here.

- Line 22: If the number of current invalid characters exceeds the best known one, then we know this recursive path will not be ideal. There's no point in even taking it.
- Line 30: If we have a path with zero invalid characters, then we know we can't do better than this. We might as well accept this path.

What's the runtime of this? It's difficult to truly describe in practice as it depends on the (English) language.

One way of looking at it is to imagine a bizarre language where essentially all paths in the recursion are taken. In this case, we are making both choices at each character. If there are n characters, this is an $O(2^n)$ runtime.

Optimized

Commonly, when we have exponential runtimes for a recursive algorithm, we optimize them through memoization (that is, caching results). To do so, we need to find the common subproblems.

Where do recursive paths overlap? That is, where are the common subproblems?

Let's again imagine the string `thisismikesfavoritefood`. Again, imagine that everything is a valid word.

In this case, we attempt to insert the first space after `t` as well as after `th` (and many other choices). Think about what the next choice is.

```
split(thisismikesfavoritefood) ->
       t  + split(hisismikesfavoritefood)
    OR th + split(isismikesfavoritefood)
    OR ...

split(hisismikesfavoritefood) ->
       h + split(isismikesfavoritefood)
    OR ...

...
```

Adding a space after `t` and `h` leads to the same recursive path as inserting a space after `th`. There's no sense in computing `split(isismikesfavoritefood)` twice when it will lead to the same result.

We should instead cache the result. We do this using a hash table which maps from the current substring to the `ParseResult` object.

We don't actually need to make the current substring a key. The `start` index in the string sufficiently represents the substring. After all, if we were to use the substring, we'd really be using `sentence.substring(start, sentence.length)`. This hash table will map from a start index to the best parsing from that index to the end of the string.

And, since the start index is the key, we don't need a true hash table at all. We can just use an array of `ParseResult` objects. This will also serve the purpose of mapping from an index to an object.

The code is essentially identical to the earlier function, but now takes in a memo table (a cache). We look up when we first call the function and set it when we return.

```java
String bestSplit(HashSet<String> dictionary, String sentence) {
  ParseResult[] memo = new ParseResult[sentence.length()];
  ParseResult r = split(dictionary, sentence, 0, memo);
  return r == null ? null : r.parsed;
}

ParseResult split(HashSet<String> dictionary, String sentence, int start,
                  ParseResult[] memo) {
  if (start >= sentence.length()) {
    return new ParseResult(0, "");
  } if (memo[start] != null) {
    return memo[start];
  }
```

```
  
  int bestInvalid = Integer.MAX_VALUE;
  String bestParsing = null;
  String partial = "";
  int index = start;
  while (index < sentence.length()) {
    char c = sentence.charAt(index);
    partial += c;
    int invalid = dictionary.contains(partial) ? 0 : partial.length();
    if (invalid < bestInvalid) { // Short circuit
      /* Recurse, putting a space after this character. If this is better than
       * the current best option, replace the best option. */
      ParseResult result = split(dictionary, sentence, index + 1, memo);
      if (invalid + result.invalid < bestInvalid) {
        bestInvalid = invalid + result.invalid;
        bestParsing = partial + " " + result.parsed;
        if (bestInvalid == 0) break; // Short circuit
      }
    }

    index++;
  }
  memo[start] = new ParseResult(bestInvalid, bestParsing);
  return memo[start];
}
```

Understanding the runtime of this is even trickier than in the prior solution. Again, let's imagine the truly bizarre case, where essentially everything looks like a valid word.

One way we can approach it is to realize that `split(i)` will only be computed once for each value of `i`. What happens when we call `split(i)`, assuming we've already called `split(i+1)` through `split(n - 1)`?

```
split(i) -> calls:
    split(i + 1)
    split(i + 2)
    split(i + 3)
    split(i + 4)
    ...
    split(n - 1)
```

Each of the recursive calls has already been computed, so they just return immediately. Doing n - i calls at O(1) time each takes O(n - i) time. This means that `split(i)` takes O(i) time at most.

We can now apply the same logic to `split(i - 1)`, `split(i - 2)`, and so on. If we make 1 call to compute `split(n - 1)`, 2 calls to compute `split(n - 2)`, 3 calls to compute `split(n - 3)`, ..., n calls to compute `split(0)`, how many calls total do we do? This is basically the sum of the numbers from 1 through n, which is $O(n^2)$.

Therefore, the runtime of this function is $O(n^2)$.

17.14 Smallest K: Design an algorithm to find the smallest K numbers in an array.

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SOLUTION

There are a number of ways to approach this problem. We will go through three of them: sorting, max heap, and selection rank.

Some of these algorithms require modifying the array. This is something you should discuss with your interviewer. Note, though, that even if modifying the original array is not acceptable, you can always clone the array and modify the clone instead. This will not impact the overall big O time of any algorithm.

Solution 1: Sorting

We can sort the elements in ascending order and then take the first million numbers from that.

```
int[] smallestK(int[] array, int k) {
  if (k <= 0 || k > array.length) {
    throw new IllegalArgumentException();
  }

  /* Sort array. */
  Arrays.sort(array);

  /* Copy first k elements. */
  int[] smallest = new int[k];
  for (int i = 0; i < k; i++) {
    smallest[i] = array[i];
  }
  return smallest;
}
```

The time complexity is `O(n log(n))`.

Solution 2: Max Heap

We can use a max heap to solve this problem. We first create a max heap (largest element at the top) for the first million numbers.

Then, we traverse through the list. On each element, if it's smaller than the root, we insert it into the heap and delete the largest element (which will be the root).

At the end of the traversal, we will have a heap containing the smallest one million numbers. This algorithm is `O(n log(m))`, where `m` is the number of values we are looking for.

```
int[] smallestK(int[] array, int k) {
  if (k <= 0 || k > array.length) {
    throw new IllegalArgumentException();
  }

  PriorityQueue<Integer> heap = getKMaxHeap(array, k);
  return heapToIntArray(heap);
}

/* Create max heap of smallest k elements. */
PriorityQueue<Integer> getKMaxHeap(int[] array, int k) {
  PriorityQueue<Integer> heap =
```

```
        new PriorityQueue<Integer>(k, new MaxHeapComparator());
    for (int a : array) {
        if (heap.size() < k) { // If space remaining
            heap.add(a);
        } else if (a < heap.peek()) { // If full and top is small
            heap.poll(); // remove highest
            heap.add(a); // insert new element
        }
    }
    return heap;
}

/* Convert heap to int array. */
int[] heapToIntArray(PriorityQueue<Integer> heap) {
    int[] array = new int[heap.size()];
    while (!heap.isEmpty()) {
        array[heap.size() - 1] = heap.poll();
    }
    return array;
}

class MaxHeapComparator implements Comparator<Integer> {
    public int compare(Integer x, Integer y) {
        return y - x;
    }
}
```

Java's uses the `PriorityQueue` class to offer heap-like functionality. By default, it operates as a min heap, with the smallest element on the top. To switch it to the biggest element on the top, we can pass in a different comparator.

Approach 3: Selection Rank Algorithm (if elements are unique)

Selection Rank is a well-known algorithm in computer science to find the `i`th smallest (or largest) element in an array in linear time.

If the elements are unique, you can find the `i`th smallest element in expected `O(n)` time. The basic algorithm operates like this:

1. Pick a random element in the array and use it as a "pivot." Partition elements around the pivot, keeping track of the number of elements on the left side of the partition.
2. If there are exactly `i` elements on the left, then you just return the biggest element on the left.
3. If the left side is bigger than `i`, repeat the algorithm on just the left part of the array.
4. If the left side is smaller than `i`, repeat the algorithm on the right, but look for the element with rank `i - leftSize`.

Once you have found the `i`th smallest element, you know that all elements smaller than this will be to the left of this (since you've partitioned the array accordingly). You can now just return the first `i` elements.

The code below implements this algorithm.

```
int[] smallestK(int[] array, int k) {
    if (k <= 0 || k > array.length) {
        throw new IllegalArgumentException();
    }

```

```
    int threshold = rank(array, k - 1);
    int[] smallest = new int[k];
    int count = 0;
    for (int a : array) {
      if (a <= threshold) {
        smallest[count] = a;
        count++;
      }
    }
    return smallest;
}

/* Get element with rank. */
int rank(int[] array, int rank) {
  return rank(array, 0, array.length - 1, rank);
}

/* Get element with rank between left and right indices. */
int rank(int[] array, int left, int right, int rank) {
  int pivot = array[randomIntInRange(left, right)];
  int leftEnd = partition(array, left, right, pivot);
  int leftSize = leftEnd - left + 1;
  if (rank == leftSize - 1) {
    return max(array, left, leftEnd);
  } else if (rank < leftSize) {
    return rank(array, left, leftEnd, rank);
  } else {
    return rank(array, leftEnd + 1, right, rank - leftSize);
  }
}

/* Partition array around pivot such that all elements <= pivot come before all
 * elements > pivot. */
int partition(int[] array, int left, int right, int pivot) {
  while (left <= right) {
    if (array[left] > pivot) {
      /* Left is bigger than pivot. Swap it to the right side, where we know it
       * should be. */
      swap(array, left, right);
      right--;
    } else if (array[right] <= pivot) {
      /* Right is smaller than the pivot. Swap it to the left side, where we know
       * it should be. */
      swap(array, left, right);
      left++;
    } else {
      /* Left and right are in correct places. Expand both sides. */
      left++;
      right--;
    }
  }
  return left - 1;
}

/* Get random integer within range, inclusive. */
int randomIntInRange(int min, int max) {
```

```
  Random rand = new Random();
  return rand.nextInt(max + 1 - min) + min;
}

/* Swap values at index i and j. */
void swap(int[] array, int i, int j) {
  int t = array[i];
  array[i] = array[j];
  array[j] = t;
}

/* Get largest element in array between left and right indices. */
int max(int[] array, int left, int right) {
  int max = Integer.MIN_VALUE;
  for (int i = left; i <= right; i++) {
    max = Math.max(array[i], max);
  }
  return max;
}
```

If the elements are not unique, we can tweak this algorithm slightly to accommodate this.

Approach 4: Selection Rank Algorithm (if elements are not unique)

The major change that needs to be made is to the `partition` function. When we partition the array around a pivot element, we now partition it into three chunks: less than `pivot`, equal to `pivot`, and greater than `pivot`.

This requires minor tweaks to `rank` as well. We now compare the size of left and middle partitions to `rank`.

```
class PartitionResult {
  int leftSize, middleSize;
  public PartitionResult(int left, int middle) {
    this.leftSize = left;
    this.middleSize = middle;
  }
}

int[] smallestK(int[] array, int k) {
  if (k <= 0 || k > array.length) {
    throw new IllegalArgumentException();
  }

  /* Get item with rank k - 1. */
  int threshold = rank(array, k - 1);

  /* Copy elements smaller than the threshold element. */
  int[] smallest = new int[k];
  int count = 0;
  for (int a : array) {
    if (a < threshold) {
      smallest[count] = a;
      count++;
    }
  }

  /* If there's still room left, this must be for elements equal to the threshold
```

```
   * element. Copy those in. */
  while (count < k) {
    smallest[count] = threshold;
    count++;
  }

  return smallest;
}

/* Find value with rank k in array. */
int rank(int[] array, int k) {
  if (k >= array.length) {
    throw new IllegalArgumentException();
  }
  return rank(array, k, 0, array.length - 1);
}

/* Find value with rank k in sub array between start and end. */
int rank(int[] array, int k, int start, int end) {
  /* Partition array around an arbitrary pivot. */
  int pivot = array[randomIntInRange(start, end)];
  PartitionResult partition = partition(array, start, end, pivot);
  int leftSize = partition.leftSize;
  int middleSize = partition.middleSize;

  /* Search portion of array. */
  if (k < leftSize) { // Rank k is on left half
    return rank(array, k, start, start + leftSize - 1);
  } else if (k < leftSize + middleSize) { // Rank k is in middle
    return pivot; // middle is all pivot values
  } else { // Rank k is on right
    return rank(array, k - leftSize - middleSize, start + leftSize + middleSize,
                end);
  }
}

/* Partition result into < pivot, equal to pivot -> bigger than pivot. */
PartitionResult partition(int[] array, int start, int end, int pivot) {
  int left = start; /* Stays at (right) edge of left side. */
  int right = end;  /* Stays at (left) edge of right side. */
  int middle = start; /* Stays at (right) edge of middle. */
  while (middle <= right) {
    if (array[middle] < pivot) {
      /* Middle is smaller than the pivot. Left is either smaller or equal to
       * the pivot. Either way, swap them. Then middle and left should move by
       * one. */
      swap(array, middle, left);
      middle++;
      left++;
    } else if (array[middle] > pivot) {
      /* Middle is bigger than the pivot. Right could have any value. Swap them,
       * then we know that the new right is bigger than the pivot. Move right by
       * one. */
      swap(array, middle, right);
      right--;
    } else if (array[middle] == pivot) {
```

```
        /* Middle is equal to the pivot. Move by one. */
        middle++;
      }
    }

    /* Return sizes of left and middle. */
    return new PartitionResult(left - start, right - left + 1);
}
```

Notice the change made to smallestK too. We can't simply copy all elements less than or equal to threshold into the array. Since we have duplicates, there could be many more than k elements that are less than or equal to threshold. (We also can't just say "okay, only copy k elements over." We could inadvertently fill up the array early on with "equal" elements, and not leave enough space for the smaller ones.)

The solution for this is fairly simple: only copy over the smaller elements first, then fill up the array with equal elements at the end.

17.15 Longest Word: Given a list of words, write a program to find the longest word made of other words in the list.

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SOLUTION

This problem seems complex, so let's simplify it. What if we just wanted to know the longest word made of *two* other words in the list?

We could solve this by iterating through the list, from the longest word to the shortest word. For each word, we would split it into all possible pairs and check if both the left and right side are contained in the list.

The pseudocode for this would look like the following:

```
String getLongestWord(String[] list) {
  String[] array = list.SortByLength();
  /* Create map for easy lookup */
  HashMap<String, Boolean> map = new HashMap<String, Boolean>;

  for (String str : array) {
    map.put(str, true);
  }

  for (String s : array) {
    // Divide into every possible pair
    for (int i = 1; i < s.length(); i++) {
      String left = s.substring(0, i);
      String right = s.substring(i);
      // Check if both sides are in the array
      if (map[left] == true && map[right] == true) {
        return s;
      }
    }
  }
  return str;
}
```

This works great for when we just want to know composites of two words. But what if a word could be formed by any number of other words?

In this case, we could apply a very similar approach, with one modification: rather than simply looking up if the right side is in the array, we would recursively see if we can build the right side from the other elements in the array.

The code below implements this algorithm:

```
String printLongestWord(String arr[]) {
  HashMap<String, Boolean> map = new HashMap<String, Boolean>();
  for (String str : arr) {
    map.put(str, true);
  }
  Arrays.sort(arr, new LengthComparator()); // Sort by length
  for (String s : arr) {
    if (canBuildWord(s, true, map)) {
      System.out.println(s);
      return s;
    }
  }
  return "";
}

boolean canBuildWord(String str, boolean isOriginalWord,
                     HashMap<String, Boolean> map) {
  if (map.containsKey(str) && !isOriginalWord) {
    return map.get(str);
  }
  for (int i = 1; i < str.length(); i++) {
    String left = str.substring(0, i);
    String right = str.substring(i);
    if (map.containsKey(left) && map.get(left) == true &&
        canBuildWord(right, false, map)) {
      return true;
    }
  }
  map.put(str, false);
  return false;
}
```

Note that in this solution we have performed a small optimization. We use a dynamic programming/memoization approach to cache the results between calls. This way, if we repeatedly need to check if there's any way to build "`testingtester`," we'll only have to compute it once.

A boolean flag `isOriginalWord` is used to complete the above optimization. The method `canBuildWord` is called for the original word and for each substring, and its first step is to check the cache for a previously calculated result. However, for the original words, we have a problem: `map` is initialized to `true` for them, but we don't want to return `true` (since a word cannot be composed solely of itself). Therefore, for the original word, we simply bypass this check using the `isOriginalWord` flag.

17.16 **The Masseuse**: A popular masseuse receives a sequence of back-to-back appointment requests and is debating which ones to accept. She needs a 15-minute break between appointments and therefore she cannot accept any adjacent requests. Given a sequence of back-to-back appointment requests (all multiples of 15 minutes, none overlap, and none can be moved), find the optimal (highest total booked minutes) set the masseuse can honor. Return the number of minutes.

EXAMPLE

Input: `{30, 15, 60, 75, 45, 15, 15, 45}`

Output: `180 minutes ({30, 60, 45, 45})`.

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SOLUTION

Let's start with an example. We'll draw it visually to get a better feel for the problem. Each number indicates the number of minutes in the appointment.

$r_0 = 75$	$r_1 = 105$	$r_2 = 120$	$r_3 = 75$	$r_4 = 90$	$r_5 = 135$

Alternatively, we could have also divided all the values (including the break) by 15 minutes, to give us the array `{5, 7, 8, 5, 6, 9}`. This would be equivalent, but now we would want a 1-minute break.

The best set of appointments for this problem has 330 minutes total, formed with $\{r_0 = 75, r_2 = 120, r_5 = 135\}$. Note that we've intentionally chosen an example in which the best sequence of appointments was not formed through a strictly alternating sequence.

We should also recognize that choosing the longest appointment first (the "greedy" strategy) would not necessarily be optimal. For example, a sequence like `{45, 60, 45, 15}` would not have 60 in the optimal set.

Solution #1: Recursion

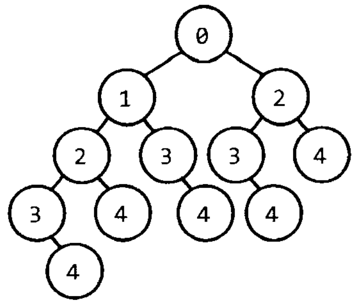
The first thing that may come to mind is a recursive solution. We have essentially a sequence of choices as we walk down the list of appointments: Do we use this appointment or do we not? If we use appointment `i`, we must skip appointment `i + 1` as we can't take back-to-back appointments. Appointment `i + 2` is a possibility (but not necessarily the best choice).

```
int maxMinutes(int[] massages) {
  return maxMinutes(massages, 0);
}

int maxMinutes(int[] massages, int index) {
  if (index >= massages.length) { // Out of bounds
    return 0;
  }

  /* Best with this reservation. */
  int bestWith = massages[index] + maxMinutes(massages, index + 2);

  /* Best without this reservation. */
  int bestWithout = maxMinutes(massages, index + 1);

  /* Return best of this subarray, starting from index. */
  return Math.max(bestWith, bestWithout);
}
```

The runtime of this solution is $O(2^n)$ because at each element we're making two choices and we do this n times (where n is the number of massages).

The space complexity is $O(n)$ due to the recursive call stack.

We can also depict this through a recursive call tree on an array of length 5. The number in each node represents the `index` value in a call to `maxMinutes`. Observe that, for example, `maxMinutes(massages, 0)` calls `maxMinutes(massages, 1)` and `maxMinutes(massages, 2)`.



As with many recursive problems, we should evaluate if there's a possibility to memoize repeated subproblems. Indeed, there is.

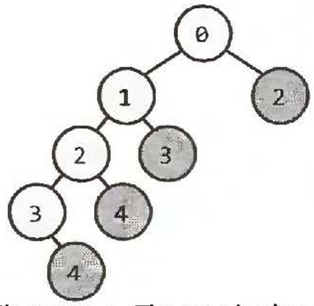
Solution #2: Recursion + Memoization

We will repeatedly call `maxMinutes` on the same inputs. For example, we'll call it on index 2 when we're deciding whether to take appointment 0. We'll also call it on index 2 when we're deciding whether to take appointment 1. We should memoize this.

Our memo table is just a mapping from `index` to the max minutes. Therefore, a simple array will suffice.

```
int maxMinutes(int[] massages) {
  int[] memo = new int[massages.length];
  return maxMinutes(massages, 0, memo);
}

int maxMinutes(int[] massages, int index, int[] memo) {
  if (index >= massages.length) {
    return 0;
  }

  if (memo[index] == 0) {
    int bestWith = massages[index] + maxMinutes(massages, index + 2, memo);
    int bestWithout = maxMinutes(massages, index + 1, memo);
    memo[index] = Math.max(bestWith, bestWithout);
  }

  return memo[index];
}
```

To determine the runtime, we'll draw the same recursive call tree as before but gray-out the calls that will return immediately. The calls that will never happen will be deleted entirely.



If we drew a bigger tree, we'd see a similar pattern. The tree looks very linear, with one branch down to the left. This gives us an O(n) runtime and O(n) space. The space usage comes from the recursive call stack as well as from the memo table.

Solution #3: Iterative

Can we do better? We certainly can't beat the time complexity since we have to look at each appointment. However, we might be able to beat the space complexity. This would mean not solving the problem recursively.

Let's look at our first example again.

r_0 = 30	r_1 = 15	r_2 = 60	r_3 = 75	r_4 = 45	r_5 = 15	r_6 = 15	r_7 = 45

As we noted in the problem statement, we cannot take adjacent appointments.

There's another observation, though, that we can make: We should never *skip* three consecutive appointments. That is, we might skip r_1 and r_2 if we wanted to take r_0 and r_3. But we would never skip r_1, r_2, and r_3. This would be suboptimal since we could always improve our set by grabbing that middle element.

This means that if we take r_0, we know we'll definitely skip r_1 and definitely take either r_2 or r_3. This substantially limits the options we need to evaluate and opens the door to an iterative solution.

Let's think about our recursive + memoization solution and try to reverse the logic; that is, let's try to approach it iteratively.

A useful way to do this is to approach it from the back and move toward the start of the array. At each point, we find the solution for the subarray.

- `best(7)`: What's the best option for {r_7 = 45}? We can get 45 min. if we take r_7, so `best(7) = 45`
- `best(6)`: What's the best option for {r_6 = 15, ...}? Still 45 min., so `best(6) = 45`.
- `best(5)`: What's the best option for {r_5 = 15, ...}? We can either:
  - » take r_5 = 15 and merge it with `best(7) = 45`, or:
  - » take `best(6) = 45`.

  The first gives us 60 minutes, `best(5) = 60`.
- `best(4)`: What's the best option for {r_4 = 45, ...}? We can either:
  - » take r_4 = 45 and merge it with `best(6) = 45`, or:
  - » take `best(5) = 60`.

  The first gives us 90 minutes, `best(4) = 90`.
- `best(3)`: What's the best option for {r_3 = 75, ...}? We can either:
  - » take r_3 = 75 and merge it with `best(5) = 60`, or:

    » take best(4) = 90.

The first gives us 135 minutes, best(3) = 135.

- best(2): What's the best option for {r_2 = 60, ...}? We can either:
    - » take r_2 = 60 and merge it with best(4) = 90, or:
    - » take best(3) = 135.

  The first gives us 150 minutes, best(2) = 150.

- best(1): What's the best option for {r_1 = 15, ...}? We can either:
    - » take r_1 = 15 and merge it with best(3) = 135, or:
    - » take best(2) = 150.

  Either way, best(1) = 150.

- best(0): What's the best option for {r_0 = 30, ...}? We can either:
    - » take r_0 = 30 and merge it with best(2) = 150, or:
    - » take best(1) = 150.

  The first gives us 180 minutes, best(0) = 180.

Therefore, we return 180 minutes.

The code below implements this algorithm.

```
1  int maxMinutes(int[] massages) {
2    /* Allocating two extra slots in the array so we don't have to do bounds
3     * checking on lines 7 and 8. */
4    int[] memo = new int[massages.length + 2];
5    memo[massages.length] = 0;
6    memo[massages.length + 1] = 0;
7    for (int i = massages.length - 1; i >= 0; i--) {
8      int bestWith = massages[i] + memo[i + 2];
9      int bestWithout = memo[i + 1];
10     memo[i] = Math.max(bestWith, bestWithout);
11   }
12   return memo[0];
13 }
```

The runtime of this solution is O(n) and the space complexity is also O(n).

It's nice in some ways that it's iterative, but we haven't actually "won" anything here. The recursive solution had the same time and space complexity.

Solution #4: Iterative with Optimal Time and Space

In reviewing the last solution, we can recognize that we only use the values in the memo table for a short amount of time. Once we are several elements past an index, we never use that element's index again.

In fact, at any given index i, we only need to know the best value from i + 1 and i + 2. Therefore, we can get rid of the memo table and just use two integers.

```
int maxMinutes(int[] massages) {
  int oneAway = 0;
  int twoAway = 0;
  for (int i = massages.length - 1; i >= 0; i--) {
    int bestWith = massages[i] + twoAway;
    int bestWithout = oneAway;
```

```
      int current = Math.max(bestWith, bestWithout);
      twoAway = oneAway;
      oneAway = current;
   }
   return oneAway;
}
```

This gives us the most optimal time and space possible: O(n) time and O(1) space.

Why did we look backward? It's a common technique in many problems to walk backward through an array.

However, we can walk forward if we want. This is easier for some people to think about, and harder for others. In this case, rather than asking "What's the best set that starts with a[i]?", we would ask "What's the best set that ends with a[i]?"

17.17 Multi Search: Given a string b and an array of smaller strings T, design a method to search b for each small string in T.

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SOLUTION

Let's start with an example:

```
T = {"is", "ppi", "hi", "sis", "i", "ssippi"}
b = "mississippi"
```

Note that in our example, we made sure to have some strings (like "is") that appear multiple times in b.

Solution #1

The naive solution is reasonably straightforward. Just search through the bigger string for each instance of the smaller string.

```
HashMapList<String, Integer> searchAll(String big, String[] smalls) {
   HashMapList<String, Integer> lookup =
      new HashMapList<String, Integer>();
   for (String small : smalls) {
      ArrayList<Integer> locations = search(big, small);
      lookup.put(small, locations);
   }
   return lookup;
}

/* Find all locations of the smaller string within the bigger string. */
ArrayList<Integer> search(String big, String small) {
   ArrayList<Integer> locations = new ArrayList<Integer>();
   for (int i = 0; i < big.length() - small.length() + 1; i++) {
      if (isSubstringAtLocation(big, small, i)) {
         locations.add(i);
      }
   }
   return locations;
}

/* Check if small appears at index offset within big. */
boolean isSubstringAtLocation(String big, String small, int offset) {
   for (int i = 0; i < small.length(); i++) {
      if (big.charAt(offset + i) != small.charAt(i)) {
```

```
            return false;
        }
    }
    return true;
}

/* HashMapList<String, Integer> is a HashMap that maps from Strings to
 * ArrayList<Integer>. See appendix for implementation. */
```

We could have also used a `substring` and `equals` function, instead of writing `isAtLocation`. This is slightly faster (though not in terms of big O) because it doesn't require creating a bunch of substrings.

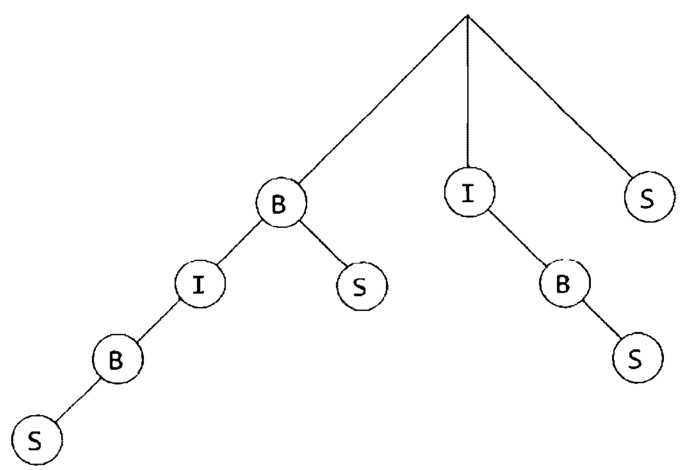
This will take `O(kbt)` time, where k is the length of the longest string in T, b is the length of the bigger string, and t is the number of smaller strings within T.

Solution #2

To optimize this, we should think about how we can tackle all the elements in T at once, or somehow re-use work.

One way is to create a trie-like data structure using each suffix in the bigger string. For the string `bibs`, the suffix list would be: `bibs`, `ibs`, `bs`, `s`.

The tree for this is below.



Then, all you need to do is search in the suffix tree for each string in T. Note that if "B" were a word, you would come up with two locations.

```
HashMapList<String, Integer> searchAll(String big, String[] smalls) {
  HashMapList<String, Integer> lookup = new HashMapList<String, Integer>();
  Trie tree = createTrieFromString(big);
  for (String s : smalls) {
    /* Get terminating location of each occurrence.*/
    ArrayList<Integer> locations = tree.search(s);

    /* Adjust to starting location. */
    subtractValue(locations, s.length());

    /* Insert. */
    lookup.put(s, locations);
  }
  return lookup;
```

```
}

Trie createTrieFromString(String s) {
  Trie trie = new Trie();
  for (int i = 0; i < s.length(); i++) {
    String suffix = s.substring(i);
    trie.insertString(suffix, i);
  }
  return trie;
}

void subtractValue(ArrayList<Integer> locations, int delta) {
  if (locations == null) return;
  for (int i = 0; i < locations.size(); i++) {
    locations.set(i, locations.get(i) - delta);
  }
}

public class Trie {
  private TrieNode root = new TrieNode();

  public Trie(String s) { insertString(s, 0); }
  public Trie() {}

  public ArrayList<Integer> search(String s) {
    return root.search(s);
  }

  public void insertString(String str, int location) {
    root.insertString(str, location);
  }

  public TrieNode getRoot() {
    return root;
  }
}

public class TrieNode {
  private HashMap<Character, TrieNode> children;
  private ArrayList<Integer> indexes;
  private char value;

  public TrieNode() {
    children = new HashMap<Character, TrieNode>();
    indexes = new ArrayList<Integer>();
  }

  public void insertString(String s, int index) {
    indexes.add(index);
    if (s != null && s.length() > 0) {
      value = s.charAt(0);
      TrieNode child = null;
      if (children.containsKey(value)) {
        child = children.get(value);
      } else {
        child = new TrieNode();
```

```
        children.put(value, child);
      }
      String remainder = s.substring(1);
      child.insertString(remainder, index + 1);
    } else {
      children.put('\0', null); // Terminating character
    }
  }

  public ArrayList<Integer> search(String s) {
    if (s == null || s.length() == 0) {
      return indexes;
    } else {
      char first = s.charAt(0);
      if (children.containsKey(first)) {
        String remainder = s.substring(1);
        return children.get(first).search(remainder);
      }
    }
    return null;
  }

  public boolean terminates() {
    return children.containsKey('\0');
  }

  public TrieNode getChild(char c) {
    return children.get(c);
  }
}

/* HashMapList<String, Integer> is a HashMap that maps from Strings to
 * ArrayList<Integer>. See appendix for implementation. */
```

It takes $O(b^2)$ time to create the tree and $O(kt)$ time to search for the locations.

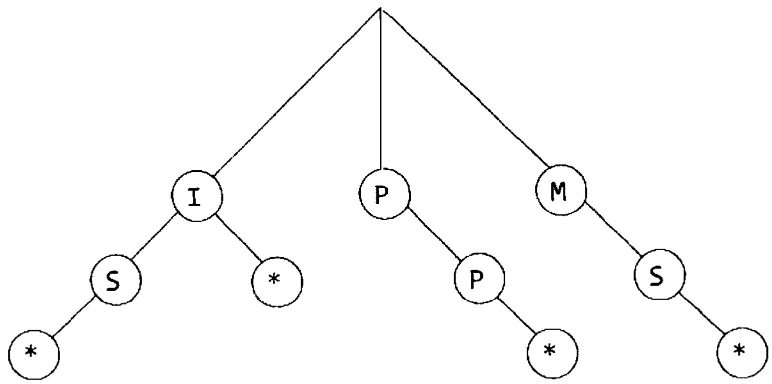
> Reminder: k is the length of the longest string in T, b is the length of the bigger string, and t is the number of smaller strings within T.

The total runtime is $O(b^2 + kt)$.

Without some additional knowledge of the expected input, you cannot directly compare $O(bkt)$, which was the runtime of the prior solution, to $O(b^2 + kt)$. If b is very large, then $O(bkt)$ is preferable. But if you have a lot of smaller strings, then $O(b^2 + kt)$ might be better.

Solution #3

Alternatively, we can add all the smaller strings into a trie. For example, the strings `{i, is, pp, ms}` would look like the trie below. The asterisk (*) hanging from a node indicates that this node completes a word.



Now, when we want to find all words in `mississippi`, we search through this trie starting with each word.

- `m`: We would first look up in the trie starting with `m`, the first letter in `mississippi`. As soon as we go to `mi`, we terminate.
- `i`: Then, we go to `i`, the second character in `mississippi`. We see that `i` is a complete word, so we add it to the list. We also keep going with `i` over to `is`. The string `is` is also a complete word, so we add that to the list. This node has no more children, so we move onto the next character in `mississippi`.
- `s`: We now go to `s`. There is no upper-level node for `s`, so we go onto the next character.
- `s`: Another `s`. Go on to the next character.
- `i`: We see another `i`. We go to the `i` node in the trie. We see that `i` is a complete word, so we add it to the list. We also keep going with `i` over to `is`. The string `is` is also a complete word, so we add that to the list. This node has no more children, so we move onto the next character in `mississippi`.
- `s`: We go to `s`. There is no upper-level node for `s`.
- `s`: Another `s`. Go on to the next character.
- `i`: We go to the `i` node. We see that `i` is a complete word, so we add it to the trie. The next character in `mississippi` is a `p`. There is no node `p`, so we break here.
- `p`: We see a `p`. There is no node `p`.
- `p`: Another `p`.
- `i`: We go to the `i` node. We see that `i` is a complete word, so we add it to the trie. There are no more characters left in `mississippi`, so we are done.

Each time we find a complete "small" word, we add it to a list along with the location in the bigger word (`mississippi`) where we found the small word.

The code below implements this algorithm.

```
HashMapList<String, Integer> searchAll(String big, String[] smalls) {
   HashMapList<String, Integer> lookup = new HashMapList<String, Integer>();
   int maxLen = big.length();
   TrieNode root = createTreeFromStrings(smalls, maxLen).getRoot();

   for (int i = 0; i < big.length(); i++) {
      ArrayList<String> strings = findStringsAtLoc(root, big, i);
      insertIntoHashMap(strings, lookup, i);
   }

   return lookup;
```

```
}

/* Insert each string into trie (provided string is not longer than maxLen). */
Trie createTreeFromStrings(String[] smalls, int maxLen) {
  Trie tree = new Trie("");
  for (String s : smalls) {
    if (s.length() <= maxLen) {
      tree.insertString(s, 0);
    }
  }
  return tree;
}

/* Find strings in trie that start at index "start" within big. */
ArrayList<String> findStringsAtLoc(TrieNode root, String big, int start) {
  ArrayList<String> strings = new ArrayList<String>();
  int index = start;
  while (index < big.length()) {
    root = root.getChild(big.charAt(index));
    if (root == null) break;
    if (root.terminates()) { // Is complete string, add to list
      strings.add(big.substring(start, index + 1));
    }
    index++;
  }
  return strings;
}

/* HashMapList<String, Integer> is a HashMap that maps from Strings to
 * ArrayList<Integer>. See appendix for implementation. */
```

This algorithm takes O(kt) time to create the trie and O(bk) time to search for all the strings.

> Reminder: k is the length of the longest string in T, b is the length of the bigger string, and t is the number of smaller strings within T.

The total time to solve the question is O(kt + bk).

Solution #1 was O(kbt). We know that O(kt + bk) will be faster than O(kbt).

Solution #2 was $O(b^2 + kt)$. Since b will always be bigger than k (or if it's not, then we know this really long string k cannot be found in b), we know Solution #3 is also faster than Solution #2.

17.18 Shortest Supersequence: You are given two arrays, one shorter (with all distinct elements) and one longer. Find the shortest subarray in the longer array that contains all the elements in the shorter array. The items can appear in any order.

EXAMPLE

Input:

`{1, 5, 9}`

`{7, 5, 9, 0, 2, 1, 3, 5, 7, 9, 1, 1, 5, 8, 8, 9, 7}`

Output: `[7, 10] (the underlined portion above)`

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SOLUTIONS

As usual, a brute force approach is a good way to start. Try thinking about it as if you were doing it by hand. How would you do it?

Let's use the example from the problem to walk through this. We'll call the smaller array `smallArray` and the bigger array `bigArray`.

Brute Force

The slow, "easy" way to do this is to iterate through `bigArray` and do repeated small passes through it.

At each index in `bigArray`, scan forward to find the next occurrence of each element in `smallArray`. The largest of these next occurrences will tell us the shortest subarray that starts at that index. (We'll call this concept "closure." That is, the closure is the element that "closes" a complete subarray starting at that index. For example, the closure of index 3—which has value 0—in the example is index 9.)

By finding the closures for each index in the array, we can find the shortest subarray overall.

```
Range shortestSupersequence(int[] bigArray, int[] smallArray) {
   int bestStart = -1;
   int bestEnd = -1;
   for (int i = 0; i < bigArray.length; i++) {
      int end = findClosure(bigArray, smallArray, i);
      if (end == -1) break;
      if (bestStart == -1 || end - i < bestEnd - bestStart) {
         bestStart = i;
         bestEnd = end;
      }
   }
   return new Range(bestStart, bestEnd);
}

/* Given an index, find the closure (i.e., the element which terminates a complete
 * subarray containing all elements in smallArray). This will be the max of the
 * next locations of each element in smallArray. */
int findClosure(int[] bigArray, int[] smallArray, int index) {
   int max = -1;
   for (int i = 0; i < smallArray.length; i++) {
      int next = findNextInstance(bigArray, smallArray[i], index);
      if (next == -1) {
         return -1;
      }
      max = Math.max(next,  max);
   }
```

```
  return max;
}

/* Find next instance of element starting from index. */
int findNextInstance(int[] array, int element, int index) {
  for (int i = index; i < array.length; i++) {
    if (array[i] == element) {
      return i;
    }
  }
  return -1;
}

public class Range {
  private int start;
  private int end;
  public Range(int s, int e) {
    start = s;
    end = e;
  }

  public int length() { return end - start + 1; }
  public int getStart() { return start; }
  public int getEnd() { return end; }

  public boolean shorterThan(Range other) {
    return length() < other.length();
  }
}
```

This algorithm will potentially take $O(SB^2)$ time, where B is the length of `bigString` and S is the length of `smallString`. This is because at each of the B characters, we potentially do $O(SB)$ work: S scans of the rest of the string, which has potentially B characters.

Optimized

Let's think about how we can optimize this. The core reason why it's slow is the repeated searches. Is there a faster way that we can find, given an index, the next occurrence of a particular character?

Let's think about it with an example. Given the array below, is there a way we could quickly find the next 5 from each location?

```
7, 5, 9, 0, 2, 1, 3, 5, 7, 9, 1, 1, 5, 8, 8, 9, 7
```

Yes. Because we're going to have to do this repeatedly, we can precompute this information in just a single (backwards) sweep. Iterate through the array backwards, tracking the last (most recent) occurrence of 5.

value	7	5	9	0	2	1	3	5	7	9	1	1	5	8	8	9	7
index	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
next 5	1	1	7	7	7	7	7	7	12	12	12	12	12	x	x	x	x

Doing this for each of `{1, 5, 9}` takes just 3 backwards sweeps.

Some people want to merge this into one backwards sweep that handles all three values. It feels faster—but it's not really. Doing it in one backwards sweep means doing three comparisons at each iteration. N moves through the list with three comparisons at each move is no better than 3N moves and one comparison at each move. You might as well keep the code clean by doing it in separate sweeps.

value	7	5	9	0	2	1	3	5	7	9	1	1	5	8	8	9	7
index	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
next 1	5	5	5	5	5	5	10	10	10	10	10	11	x	x	x	x	x
next 5	1	1	7	7	7	7	7	7	12	12	12	12	12	x	x	x	x
next 9	2	2	2	9	9	9	9	9	9	9	15	15	15	15	15	15	x

The `findNextInstance` function can now just use this table to find the next occurrence, rather than doing a search.

But, actually, we can make it a bit simpler. Using the table above, we can quickly compute the closure of each index. It's just the max of the column. If a column has an x in it, then there is no closure, at this indicates that there's no next occurrence of that character.

The difference between the index and the closure is the smallest subarray starting at that index.

value	7	5	9	0	2	1	3	5	7	9	1	1	5	8	8	9	7
index	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
next 1	5	5	5	5	5	5	10	10	10	10	10	11	x	x	x	x	x
next 5	1	1	7	7	7	7	7	7	12	12	12	12	12	x	x	x	x
next 9	2	2	2	9	9	9	9	9	9	9	15	15	15	15	15	15	x
closure	5	5	7	9	9	9	10	10	12	12	15	15	x	x	x	x	x
diff.	5	4	5	6	5	4	4	3	4	3	5	4	x	x	x	x	x

Now, all we have to do is to find the minimum distance in this table.

```
Range shortestSupersequence(int[] big, int[] small) {
  int[][] nextElements = getNextElementsMulti(big, small);
  int[] closures = getClosures(nextElements);
  return getShortestClosure(closures);
}

/* Create table of next occurrences. */
int[][] getNextElementsMulti(int[] big, int[] small) {
  int[][] nextElements = new int[small.length][big.length];
  for (int i = 0; i < small.length; i++) {
    nextElements[i] = getNextElement(big, small[i]);
  }
  return nextElements;
}

/* Do backwards sweep to get a list of the next occurrence of value from each
 * index. */
int[] getNextElement(int[] bigArray, int value) {
  int next = -1;
  int[] nexts = new int[bigArray.length];
  for (int i = bigArray.length - 1; i >= 0; i--) {
    if (bigArray[i] == value) {
      next = i;
    }
    nexts[i] = next;
  }
  return nexts;
}

/* Get closure for each index. */
```

```java
int[] getClosures(int[][] nextElements) {
  int[] maxNextElement = new int[nextElements[0].length];
  for (int i = 0; i < nextElements[0].length; i++) {
    maxNextElement[i] = getClosureForIndex(nextElements, i);
  }
  return maxNextElement;
}

/* Given an index and the table of next elements, find the closure for this index
 * (which will be the min of this column). */
int getClosureForIndex(int[][] nextElements, int index) {
  int max = -1;
  for (int i = 0; i < nextElements.length; i++) {
    if (nextElements[i][index] == -1) {
      return -1;
    }
    max = Math.max(max, nextElements[i][index]);
  }
  return max;
}

/* Get shortest closure. */
Range getShortestClosure(int[] closures) {
  int bestStart = -1;
  int bestEnd = -1;
  for (int i = 0; i < closures.length; i++) {
    if (closures[i] == -1) {
      break;
    }
    int current = closures[i] - i;
    if (bestStart == -1 || current < bestEnd - bestStart) {
      bestStart = i;
      bestEnd = closures[i];
    }
  }
  return new Range(bestStart, bestEnd);
}
```

This algorithm will potentially take O(SB) time, where B is the length of `bigString` and S is the length of `smallString`. This is because we do S sweeps through the array to build up the next occurrences table and each sweep takes O(B) time.

It uses O(SB) space.

More Optimized

While our solution is fairly optimal, we can reduce the space usage. Remember the table we created:

value	7	5	9	0	2	1	3	5	7	9	1	1	5	8	8	9	7
index	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
next 1	5	5	5	5	5	5	10	10	10	10	10	11	x	x	x	x	x
next 5	1	1	7	7	7	7	7	7	12	12	12	12	12	x	x	x	x
next 9	2	2	2	9	9	9	9	9	9	9	15	15	15	15	15	15	x
closure	5	5	7	9	9	9	10	10	12	12	15	15	x	x	x	x	x

In actuality, all we need is the closure row, which is the minimum of all the other rows. We don't need to store all the other next occurrence information the entire time.

Instead, as we do each sweep, we just update the closure row with the minimums. The rest of the algorithm works essentially the same way.

```
Range shortestSupersequence(int[] big, int[] small) {
  int[] closures = getClosures(big, small);
  return getShortestClosure(closures);
}

/* Get closure for each index. */
int[] getClosures(int[] big, int[] small) {
  int[] closure = new int[big.length];
  for (int i = 0; i < small.length; i++) {
    sweepForClosure(big, closure, small[i]);
  }
  return closure;
}

/* Do backwards sweep and update the closures list with the next occurrence of
 * value, if it's later than the current closure. */
void sweepForClosure(int[] big, int[] closures, int value) {
  int next = -1;
  for (int i = big.length - 1; i >= 0; i--) {
    if (big[i] == value) {
      next = i;
    }
    if ((next == -1 || closures[i] < next) &&
        (closures[i] != -1)) {
      closures[i] = next;
    }
  }
}

/* Get shortest closure. */
Range getShortestClosure(int[] closures) {
  Range shortest = new Range(0, closures[0]);
  for (int i = 1; i < closures.length; i++) {
    if (closures[i] == -1) {
      break;
    }
    Range range = new Range(i, closures[i]);
    if (!shortest.shorterThan(range)) {
      shortest = range;
    }
  }
  return shortest;
}
```

This still runs in O(SB) time, but it now only takes O(B) additional memory.

Alternative & More Optimal Solution

There's a totally different way to approach it. Let's suppose we had a list of the occurrences of each element in `smallArray`.

value	7	5	9	9	2	1	3	5	7	9	1	1	5	8	8	9	7
index	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

```
1 -> {5, 10, 11}
5 -> {1, 7, 12}
9 -> {2, 3, 9, 15}
```

What is the very first valid subsequence (which contains 1, 5, and 9)? We can just look at the heads of each list to tell us this. The minimum of the heads is the start of the range and the max of the heads is the end of the range. In this case, the first range is [1, 5]. This is currently our "best" subsequence.

How can we find the next one? Well, the next one will not include index 1, so let's remove that from the list.

```
1 -> {5, 10, 11}
5 -> {7, 12}
9 -> {2, 3, 9, 15}
```

The next subsequence is [2, 7]. This is worse than the earlier best, so we can toss it.

Now, what's the next subsequence? We can remove the min from earlier (2) and find out.

```
1 -> {5, 10, 11}
5 -> {7, 12}
9 -> {3, 9, 15}
```

The next subsequence is [3, 7], which is no better or worse than our current best.

We can continue down this path each time, repeating this process. We will end up iterating through all "minimal" subsequences that start from a given point.

1. Current subsequence is [min of heads, max of heads]. Compare to best subsequence and update if necessary.
2. Remove the minimum head.
3. Repeat.

This will give us an `O(SB)` time complexity. This is because for each of B elements, we are doing a comparison to the S other list heads to find the minimum.

This is pretty good, but let's see if we can make that minimum computation faster.

What we're doing in these repeated minimum calls is taking a bunch of elements, finding and removing the minimum, adding in one more element, and then finding the minimum again.

We can make this faster by using a min-heap. First, put each of the heads in a min-heap. Remove the minimum. Look up the list that this minimum came from and add back the new head. Repeat.

To get the list that the minimum element came from, we'll need to use a HeapNode class that stores both the `locationWithinList` (the index) and the `listId`. This way, when we remove the minimum, we can jump back to the correct list and add its new head to the heap.

```java
Range shortestSupersequence(int[] array, int[] elements) {
  ArrayList<Queue<Integer>> locations = getLocationsForElements(array, elements);
  if (locations == null) return null;
  return getShortestClosure(locations);
}

/* Get list of queues (linked lists) storing the indices at which each element in
 * smallArray appears in bigArray. */
ArrayList<Queue<Integer>> getLocationsForElements(int[] big, int[] small) {
  /* Initialize hash map from item value to locations. */
  HashMap<Integer, Queue<Integer>> itemLocations =
```

```
      new HashMap<Integer, Queue<Integer>>();
   for (int s : small) {
      Queue<Integer> queue = new LinkedList<Integer>();
      itemLocations.put(s, queue);
   }

   / *Walk through big array, adding the item locations to hash map */
   for (int i = 0; i < big.length; i++) {
      Queue<Integer> queue = itemLocations.get(big[i]);
      if (queue != null) {
         queue.add(i);
      }
   }

   ArrayList<Queue<Integer>> allLocations = new ArrayList<Queue<Integer>>();
   allLocations.addAll(itemLocations.values());
   return allLocations;
}

Range getShortestClosure(ArrayList<Queue<Integer>> lists) {
   PriorityQueue<HeapNode> minHeap = new PriorityQueue<HeapNode>();
   int max = Integer.MIN_VALUE;

   / *Insert min element from each list. */
   for (int i = 0; i < lists.size(); i++) {
      int head = lists.get(i).remove();
      minHeap.add(new HeapNode(head, i));
      max = Math.max(max, head);
   }

   int min = minHeap.peek().locationWithinList;
   int bestRangeMin = min;
   int bestRangeMax = max;

   while (true) {
      / *Remove min node. */
      HeapNode n = minHeap.poll();
      Queue<Integer> list = lists.get(n.listId);

      / *Compare range to best range. */
      min = n.locationWithinList;
      if (max - min < bestRangeMax - bestRangeMin) {
         bestRangeMax = max;
         bestRangeMin = min;
      }

      / *If there are no more elements, then there's no more subsequences and we
       * can break. */
      if (list.size() == 0) {
         break;
      }

      / *Add new head of list to heap. */
      n.locationWithinList = list.remove();
      minHeap.add(n);
      max = Math.max(max, n.locationWithinList);
```

```
    }

    return new Range(bestRangeMin, bestRangeMax);
}
```

We're going through B elements in `getShortestClosure`, and each time pass in the for loop will take `O(log S)` time (the time to insert/remove from the heap). This algorithm will therefore take `O(B log S)` time in the worst case.

17.19 Missing Two: You are given an array with all the numbers from 1 to N appearing exactly once, except for one number that is missing. How can you find the missing number in `O(N)` time and `O(1)` space? What if there were two numbers missing?

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SOLUTIONS

Let's start with the first part: find a missing number in `O(N)` time and `O(1)` space.

Part 1: Find One Missing Number

We have a very constrained problem here. We can't store all the values (that would take `O(N)` space) and yet, somehow, we need to have a "record" of them such that we can identify the missing number.

This suggests that we need to do some sort of computation with the values. What characteristics does this computation need to have?

- **Unique.** If this computation gives the same result on two arrays (which fit the description in the problem), then those arrays must be equivalent (same missing number). That is, the result of the computation must uniquely correspond to the specific array and missing number.
- **Reversible.** We need some way of getting from the result of the calculation to the missing number.
- **Constant Time:** The calculation can be slow, but it must be constant time per element in the array.
- **Constant Space:** The calculation can require additional memory, but it must be `O(1)` memory.

The "unique" requirement is the most interesting—and the most challenging. What calculations can be performed on a set of numbers such that the missing number will be discoverable?

There are actually a number of possibilities.

We could do something with prime numbers. For example, for each value `x` in the array, we multiply `result` by the xth prime. We would then get some value that is indeed unique (since two different sets of primes can't have the same product).

Is this reversible? Yes. We could take `result` and divide it by each prime number: 2, 3, 5, 7, and so on. When we get a non-integer for the `i`th prime, then we know `i` was missing from our array.

Is it constant time and space, though? Only if we had a way of getting the `i`th prime number in `O(1)` time and `O(1)` space. We don't have that.

What other calculations could we do? We don't even need to do all this prime number stuff. Why not just multiply all the numbers together?

- **Unique?** Yes. Picture `1*2*3*...*n`. Now, imagine crossing off one number. This will give us a different result than if we crossed off any other number.
- **Constant time and space?** Yes.

- **Reversible?** Let's think about this. If we compare what our product is to what it would have been without a number removed, can we find the missing number? Sure. We just divide `full_product` by `actual_product`. This will tell us which number was missing from `actual_product`.

There's just one issue: this product is really, really, really big. If n is 20, the product *will* be somewhere around 2,000,000,000,000,000,000.

We can still approach it this way, but we'll need to use the `BigInteger` class.

```
int missingOne(int[] array) {
  BigInteger fullProduct = productToN(array.length + 1);

  BigInteger actualProduct = new BigInteger("1");
  for (int i = 0; i < array.length; i++) {
    BigInteger value = new BigInteger(array[i] + "");
    actualProduct = actualProduct.multiply(value);
  }

  BigInteger missingNumber = fullProduct.divide(actualProduct);
  return Integer.parseInt(missingNumber.toString());
}

BigInteger productToN(int n) {
  BigInteger fullProduct = new BigInteger("1");
  for (int i = 2; i <= n; i++) {
    fullProduct = fullProduct.multiply(new BigInteger(i + ""));
  }
  return fullProduct;
}
```

There's no need for all of this, though. We can use the sum instead. It too will be unique.

Doing the sum has another benefit: there is already a closed form expression to compute the sum of numbers between 1 and n. This is $\frac{n(n+1)}{2}$.

> Most candidates probably won't remember the expression for the sum of numbers between 1 and n, and that's okay. Your interviewer might, however, ask you to derive it. Here's how to think about that: you can pair up the low and high values in the sequence of `0 + 1 + 2 + 3 + ... + n` to get: `(0, n) + (1, n-1) + (2, n-3)`, and so on. Each of those pairs has a sum of `n` and there are $\frac{n+1}{2}$ pairs. But what if n is even, such that $\frac{n+1}{2}$ is not an integer? In this case, pair up low and high values to get $n/2$ pairs with sum `n+1`. Either way, the math works out to $\frac{n(n+1)}{2}$.

Switching to a sum will delay the overflow issue substantially, but it won't wholly prevent it. You should discuss the issue with your interviewer to see how he/she would like you to handle it. Just mentioning it is plenty sufficient for many interviewers.

Part 2: Find Two Missing Numbers

This is substantially more difficult. Let's start with what our earlier approaches will tell us when we have two missing numbers.

- Sum: Using this approach will give us the sum of the two values that are missing.
- Product: Using this approach will give us the product of the two values that are missing.

Unfortunately, knowing the sum isn't enough. If, for example, the sum is 10, that could correspond to (1, 9), (2, 8), and a handful of other pairs. The same could be said for the product.

We're again at the same point we were in the first part of the problem. We need a calculation that can be applied such that the result is unique across all potential pairs of missing numbers.

Perhaps there is such a calculation (the prime one would work, but it's not constant time), but your interviewer probably doesn't expect you to know such math.

What else can we do? Let's go back to what we can do. We can get x + y and we can also get x * y. Each result leaves us with a number of possibilities. But using both of them narrows it down to the specific numbers.

$$
\begin{aligned}
x + y &= sum && \rightarrow y = sum - x \\
x * y &= product && \rightarrow x(sum - x) = product \\
& && \quad x*sum - x^2 = product \\
& && \quad x*sum - x^2 - product = 0 \\
& && \quad -x^2 + x*sum - product = 0
\end{aligned}
$$

At this point, we can apply the quadratic formula to solve for x. Once we have x, we can then compute y.

There are actually a number of other calculations you can perform. In fact, almost any other calculation (other than "linear" calculations) will give us values for x and y.

For this part, let's use a different calculation. Instead of using the product of 1 * 2 * ... * n, we can use the sum of the squares: $1^2 + 2^2 + \dots + n^2$. This will make the `BigInteger` usage a little less critical, as the code will at least run on small values of n. We can discuss with our interviewer whether or not this is important.

$$
\begin{aligned}
x + y &= s && \rightarrow y = s - x \\
x^2 + y^2 &= t && \rightarrow x^2 + (s-x)^2 = t \\
& && \quad 2x^2 - 2sx + s^2-t = 0
\end{aligned}
$$

Recall the quadratic formula:

$$x = [-b \pm \mathrm{sqrt}(b^2 - 4ac)] / 2a$$

where, in this case:

$$
\begin{aligned}
a &= 2 \\
b &= -2s \\
c &= s^2-t
\end{aligned}
$$

Implementing this is now somewhat straightforward.

```
int[] missingTwo(int[] array) {
  int max_value = array.length + 2;
  int rem_square = squareSumToN(max_value, 2);
  int rem_one = max_value * (max_value + 1) / 2;

  for (int i = 0; i < array.length; i++) {
    rem_square -= array[i] * array[i];
    rem_one -= array[i];
  }

  return solveEquation(rem_one, rem_square);
}

int squareSumToN(int n, int power) {
  int sum = 0;
  for (int i = 1; i <= n; i++) {
    sum += (int) Math.pow(i, power);
  }
  return sum;
}
```

```
int[] solveEquation(int r1, int r2) {
  /* ax^2 + bx + c
   * -->
   * x = [-b +- sqrt(b^2 - 4ac)] / 2a
   * In this case, it has to be a + not a - */
  int a = 2;
  int b = -2 * r1;
  int c = r1 * r1 - r2;

  double part1 = -1 * b;
  double part2 = Math.sqrt(b*b - 4 * a * c);
  double part3 = 2 * a;

  int solutionX = (int) ((part1 + part2) / part3);
  int solutionY = r1 - solutionX;

  int[] solution = {solutionX, solutionY};
  return solution;
}
```

You might notice that the quadratic formula usually gives us two answers (see the + or - part), yet in our code, we only use the `(+)` result. We never checked the `(-)` answer. Why is that?

The existence of the "alternate" solution doesn't mean that one is the correct solution and one is "fake." It means that there are exactly two values for x which will correctly fulfill our equation: $2x^2 - 2sx + (s^2-t) = 0$.

That's true. There are. What's the other one? The other value is y!

If this doesn't immediately make sense to you, remember that x and y are interchangeable. Had we solved for y earlier instead of x, we would have wound up with an identical equation: $2y^2 - 2sy + (s^2-t) = 0$. So of course y could fulfill x's equation and x could fulfill y's equation. They have the exact same equation. Since x and y are both solutions to equations that look like $2[\text{something}]^2 - 2s[\text{something}] + s^2-t = 0$, then the other `something` that fulfills that equation must be y.

Still not convinced? Okay, we can do some math. Let's say we took the alternate value for x: $[-b - \text{sqrt}(b^2 - 4ac)] / 2a$. What's y?

$$
\begin{aligned}
x + y &= r_1 \\
y &= r_1 - x \\
&= r_1 - [-b - \text{sqrt}(b^2 - 4ac)]/2a \\
&= [2a*r_1 + b + \text{sqrt}(b^2 - 4ac)]/2a
\end{aligned}
$$

Partially plug in values for a and b, but keep the rest of the equation as-is:

$$
\begin{aligned}
&= [2(2)*r_1 + (-2r_1) + \text{sqrt}(b^2 - 4ac)]/2a \\
&= [2r_1 + \text{sqrt}(b^2 - 4ac)]/2a
\end{aligned}
$$

Recall that $b = -2r_1$. Now, we wind up with this equation:

$$= [-b + \text{sqrt}(b^2 - 4ac)]/2a$$

Therefore, if we use `x = (part1 + part2) / part3`, then we'll get `(part1 - part2) / part3` for the value for y.

We don't care which one we call x and which one we call y, so we can use either one. It'll work out the same in the end.

17.20 Continuous Median: Numbers are randomly generated and passed to a method. Write a program to find and maintain the median value as new values are generated.

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SOLUTIONS

One solution is to use two priority heaps: a max heap for the values below the median, and a min heap for the values above the median. This will divide the elements roughly in half, with the middle two elements as the top of the two heaps. This makes it trivial to find the median.

What do we mean by "roughly in half," though? "Roughly" means that, if we have an odd number of values, one heap will have an extra value. Observe that the following is true:

- If `maxHeap.size() > minHeap.size()`, `maxHeap.top()` will be the median.
- If `maxHeap.size() == minHeap.size()`, then the average of `maxHeap.top()` and `minHeap.top()` will be the median.

By the way in which we rebalance the heaps, we will ensure that it is always `maxHeap` with extra element.

The algorithm works as follows. When a new value arrives, it is placed in the `maxHeap` if the value is less than or equal to the median, otherwise it is placed into the `minHeap`. The heap sizes can be equal, or the `maxHeap` may have one extra element. This constraint can easily be restored by shifting an element from one heap to the other. The median is available in constant time, by looking at the top element(s). Updates take `O(log(n))` time.

```
Comparator<Integer> maxHeapComparator, minHeapComparator;
PriorityQueue<Integer> maxHeap, minHeap;

void addNewNumber(int randomNumber) {
  /* Note: addNewNumber maintains a condition that
   * maxHeap.size() >= minHeap.size() */
  if (maxHeap.size() == minHeap.size()) {
    if ((minHeap.peek() != null) &&
      randomNumber > minHeap.peek()) {
      maxHeap.offer(minHeap.poll());
      minHeap.offer(randomNumber);
    } else {
      maxHeap.offer(randomNumber);
    }
  } else {
    if(randomNumber < maxHeap.peek()) {
      minHeap.offer(maxHeap.poll());
      maxHeap.offer(randomNumber);
    }
    else {
      minHeap.offer(randomNumber);
    }
  }
}

double getMedian() {
  /* maxHeap is always at least as big as minHeap. So if maxHeap is empty, then
   * minHeap is also. */
  if (maxHeap.isEmpty()) {
    return 0;
  }
```

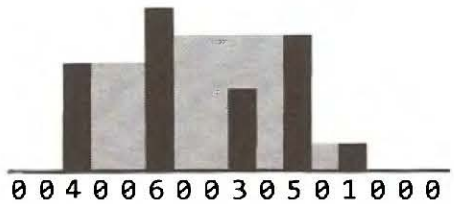
```
    if (maxHeap.size() == minHeap.size()) {
      return ((double)minHeap.peek()+(double)maxHeap.peek()) / 2;
    } else {
      /* If maxHeap and minHeap are of different sizes, then maxHeap must have one
       * extra element. Return maxHeap's top element.*/
      return maxHeap.peek();
    }
  }
```

17.21 Volume of Histogram: Imagine a histogram (bar graph). Design an algorithm to compute the volume of water it could hold if someone poured water across the top. You can assume that each histogram bar has width 1.

EXAMPLE

Input: `{0, 0, 4, 0, 0, 6, 0, 0, 3, 0, 5, 0, 1, 0, 0, 0}`

`(Black bars are the histogram. Gray is water.)`



Output: 26

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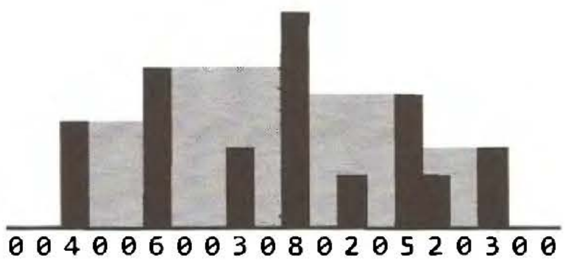
SOLUTION

This is a difficult problem, so let's come up with a good example to help us solve it.



We should study this example to see what we can learn from it. What exactly dictates how big those gray areas are?

Solution #1

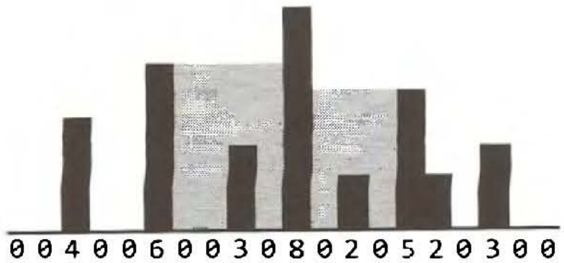
Let's look at the tallest bar, which has size 8. What role does that bar play? It plays an important role for being the highest, but it actually wouldn't matter if that bar instead had height 100. It wouldn't affect the volume.

The tallest bar forms a barrier for water on its left and right. But the volume of water is actually controlled by the next highest bar on the left and right.

- **Water on immediate left of tallest bar:** The next tallest bar on the left has height 6. We can fill up the area in between with water, but we have to deduct the height of each histogram between the tallest and next tallest. This gives a volume on the immediate left of: `(6-0) + (6-0) + (6-3) + (6-0) = 21`.
- **Water on immediate right of tallest bar:** The next tallest bar on the right has height 5. We can now

compute the volume: `(5-0) + (5-2) + (5-0) = 13`.

This just tells us part of the volume.



What about the rest?

We have essentially two subgraphs, one on the left and one on the right. To find the volume there, we repeat a very similar process.

1. Find the max. (Actually, this is given to us. The highest on the left subgraph is the right border (6) and the highest on the right subgraph is the left border (5).)
2. Find the second tallest in each subgraph. In the left subgraph, this is 4. In the right subgraph, this is 3.
3. Compute the volume between the tallest and the second tallest.
4. Recurse on the edge of the graph.

The code below implements this algorithm.

```
int computeHistogramVolume(int[] histogram) {
  int start = 0;
  int end = histogram.length - 1;

  int max = findIndexOfMax(histogram, start, end);
  int leftVolume =  subgraphVolume(histogram, start, max, true);
  int rightVolume = subgraphVolume(histogram, max, end, false);

  return leftVolume + rightVolume;
}

/* Compute the volume of a subgraph of the histogram. One max is at either start
 * or end (depending on isLeft). Find second tallest, then compute volume between
 * tallest and second tallest. Then compute volume of subgraph. */
int subgraphVolume(int[] histogram, int start, int end, boolean isLeft) {
  if (start >= end) return 0;
  int sum = 0;
  if (isLeft) {
    int max = findIndexOfMax(histogram, start, end - 1);
    sum += borderedVolume(histogram, max, end);
    sum += subgraphVolume(histogram, start, max, isLeft);
  } else {
    int max = findIndexOfMax(histogram, start + 1, end);
    sum += borderedVolume(histogram, start, max);
    sum += subgraphVolume(histogram, max, end, isLeft);
  }

  return sum;
}

```

```
/* Find tallest bar in histogram between start and end. */
int findIndexOfMax(int[] histogram, int start, int end) {
  int indexOfMax = start;
  for (int i = start + 1; i <= end; i++) {
    if (histogram[i] > histogram[indexOfMax]) {
      indexOfMax = i;
    }
  }
  return indexOfMax;
}

/* Compute volume between start and end. Assumes that tallest bar is at start and
 * second tallest is at end. */
int borderedVolume(int[] histogram, int start, int end) {
  if (start >= end) return 0;

  int min = Math.min(histogram[start], histogram[end]);
  int sum = 0;
  for (int i = start + 1; i < end; i++) {
    sum += min - histogram[i];
  }
  return sum;
}
```

This algorithm takes $O(N^2)$ time in the worst case, where N is the number of bars in the histogram. This is because we have to repeatedly scan the histogram to find the max height.

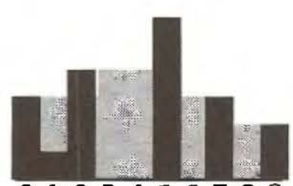
Solution #2 (Optimized)

To optimize the previous algorithm, let's think about the exact cause of the inefficiency of the prior algorithm. The root cause is the perpetual calls to `findIndexOfMax`. This suggests that it should be our focus for optimizing.

One thing we should notice is that we don't pass in arbitrary ranges into the `findIndexOfMax` function. It's actually always finding the max from one point to an edge (either the right edge or the left edge). Is there a quicker way we could know what the max height is from a given point to each edge?

Yes. We could precompute this information in O(N) time.

In two sweeps through the histogram (one moving right to left and the other moving left to right), we can create a table that tells us, from any index `i`, the location of the max index on the right and the max index on the left.



```
         INDEX: 0 1 2 3 4 5 6 7 8 9
        HEIGHT: 3 1 4 0 0 6 0 3 0 2
 INDEX LEFT MAX: 0 0 2 2 2 5 5 5 5 5
INDEX RIGHT MAX: 5 5 5 5 5 5 7 7 9 9
```

The rest of the algorithm precedes essentially the same way.

We've chosen to use a `HistogramData` object to store this extra information, but we could also use a two-dimensional array.

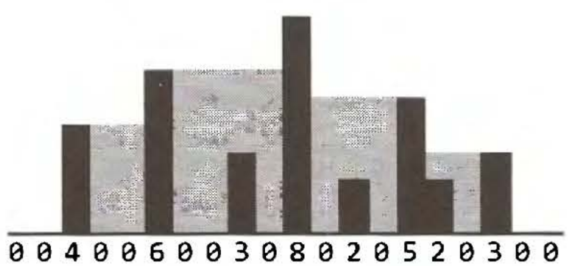
```
int computeHistogramVolume(int[] histogram) {
  int start = 0;
  int end = histogram.length - 1;

  HistogramData[] data = createHistogramData(histogram);

  int max = data[0].getRightMaxIndex(); // Get overall max
  int leftVolume =  subgraphVolume(data, start, max, true);
  int rightVolume = subgraphVolume(data, max, end, false);

  return leftVolume + rightVolume;
}

HistogramData[] createHistogramData(int[] histo) {
  HistogramData[] histogram = new HistogramData[histo.length];
  for (int i = 0; i < histo.length; i++) {
    histogram[i] = new HistogramData(histo[i]);
  }

  /* Set left max index. */
  int maxIndex = 0;
  for (int i = 0; i < histo.length; i++) {
    if (histo[maxIndex] < histo[i]) {
      maxIndex = i;
    }
    histogram[i].setLeftMaxIndex(maxIndex);
  }

  /* Set right max index. */
  maxIndex = histogram.length - 1;
  for (int i = histogram.length - 1; i >= 0; i--) {
    if (histo[maxIndex] < histo[i]) {
      maxIndex = i;
    }
    histogram[i].setRightMaxIndex(maxIndex);
  }

  return histogram;
}

/* Compute the volume of a subgraph of the histogram. One max is at either start
 * or end (depending on isLeft). Find second tallest, then compute volume between
 * tallest and second tallest. Then compute volume of subgraph. */
int subgraphVolume(HistogramData[] histogram, int start, int end,
                   boolean isLeft) {
  if (start >= end) return 0;
  int sum = 0;
  if (isLeft) {
    int max = histogram[end - 1].getLeftMaxIndex();
    sum += borderedVolume(histogram, max, end);
    sum += subgraphVolume(histogram, start, max, isLeft);
  } else {
    int max = histogram[start + 1].getRightMaxIndex();
    sum += borderedVolume(histogram, start, max);
```

```
    sum += subgraphVolume(histogram, max, end, isLeft);
  }

  return sum;
}

/* Compute volume between start and end. Assumes that tallest bar is at start and
 * second tallest is at end. */
int borderedVolume(HistogramData[] data, int start, int end) {
  if (start >= end) return 0;

  int min = Math.min(data[start].getHeight(), data[end].getHeight());
  int sum = 0;
  for (int i = start + 1; i < end; i++) {
    sum += min - data[i].getHeight();
  }
  return sum;
}

public class HistogramData {
  private int height;
  private int leftMaxIndex = -1;
  private int rightMaxIndex = -1;

  public HistogramData(int v) { height = v; }
  public int getHeight() { return height; }
  public int getLeftMaxIndex() { return leftMaxIndex; }
  public void setLeftMaxIndex(int idx) { leftMaxIndex = idx; };
  public int getRightMaxIndex() { return rightMaxIndex; }
  public void setRightMaxIndex(int idx) { rightMaxIndex = idx; };
}
```

This algorithm takes O(N) time. Since we have to look at every bar, we cannot do better than this.

Solution #3 (Optimized & Simplified)

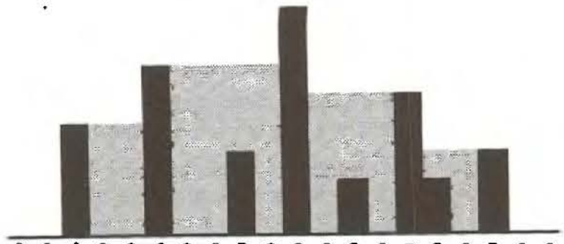
While we can't make the solution faster in terms of big O, we can make it much, much simpler. Let's look at an example again in light of what we've just learned about potential algorithms.



As we've seen, the volume of water in a particular area is determined by the tallest bar to the left and to the right (specifically, by the shorter of the two tallest bars on the left and the tallest bar on the right). For example, water fills in the area between the bar with height 6 and the bar with height 8, up to a height of 6. It's the second tallest, therefore, that determines the height.

The total volume of water is the volume of water above each histogram bar. Can we efficiently compute how much water is above each histogram bar?

Yes. In Solution #2, we were able to precompute the height of the tallest bar on the left and right of each index. The minimums of these will indicate the "water level" at a bar. The difference between the water level and the height of this bar will be the volume of water.



```
   HEIGHT: 0 0 4 0 0 6 0 0 3 0 8 0 2 0 5 2 0 3 0 0
 LEFT MAX: 0 0 4 4 4 6 6 6 6 6 8 8 8 8 8 8 8 8 8 8
RIGHT MAX: 8 8 8 8 8 8 8 8 8 8 8 5 5 5 5 3 3 3 0 0
      MIN: 0 0 4 4 4 6 6 6 6 6 8 5 5 5 5 3 3 3 0 0
    DELTA: 0 0 0 4 4 0 6 6 3 6 0 5 3 5 0 1 3 0 0 0
```

Our algorithm now runs in a few simple steps:

1. Sweep left to right, tracking the max height you've seen and setting left max.
2. Sweep right to left, tracking the max height you've seen and setting right max.
3. Sweep across the histogram, computing the minimum of the left max and right max for each index.
4. Sweep across the histogram, computing the delta between each minimum and the bar. Sum these deltas.

In the actual implementation, we don't need to keep so much data around. Steps 2, 3, and 4 can be merged into the same sweep. First, compute the left maxes in one sweep. Then sweep through in reverse, tracking the right max as you go. At each element, calculate the min of the left and right max and then the delta between that (the "min of maxes") and the bar height. Add this to the sum.

```java
/* Go through each bar and compute the volume of water above it.
 * Volume of water at a bar =
 *   height - min(tallest bar on left, tallest bar on right)
 *   [where above equation is positive]
 * Compute the left max in the first sweep, then sweep again to compute the right
 * max, minimum of the bar heights, and the delta. */
int computeHistogramVolume(int[] histo) {
  /* Get left max */
  int[] leftMaxes = new int[histo.length];
  int leftMax = histo[0];
  for (int i = 0; i < histo.length; i++) {
    leftMax = Math.max(leftMax, histo[i]);
    leftMaxes[i] = leftMax;
  }

  int sum = 0;

  /* Get right max */
  int rightMax = histo[histo.length - 1];
  for (int i = histo.length - 1; i >= 0; i--) {
    rightMax = Math.max(rightMax, histo[i]);
    int secondTallest = Math.min(rightMax, leftMaxes[i]);

    /* If there are taller things on the left and right side, then there is water
     * above this bar. Compute the volume and add to the sum. */
      if (secondTallest > histo[i]) {
```

```
                sum += secondTallest - histo[i];
            }
        }

        return sum;
    }
```

Yes, this really is the entire code! It is still O(N) time, but it's a lot simpler to read and write.

17.22 Word Transformer: Given two words of equal length that are in a dictionary, write a method to transform one word into another word by changing only one letter at a time. The new word you get in each step must be in the dictionary.

EXAMPLE

Input: DAMP, LIKE

Output: DAMP -> LAMP -> LIMP -> LIME -> LIKE

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SOLUTION

Let's start with a naive solution and then work our way to a more optimal solution.

Brute Force

One way of solving this problem is to just transform the words in every possible way (of course checking at each step to ensure each is a valid word), and then see if we can reach the final word.

So, for example, the word bold would be transformed into:

- aold, bold, ..., zold
- bald, bbld, ..., bzld
- boad, bobd, ..., bozd
- bola, bolb, ..., bolz

We will terminate (not pursue this path) if the string is not a valid word or if we've already visited this word.

This is essentially a depth-first search where there is an "edge" between two words if they are only one edit apart. This means that this algorithm will not find the shortest path. It will only find a path.

If we wanted to find the shortest path, we would want to use breadth-first search.

```
LinkedList<String> transform(String start, String stop, String[] words) {
  HashSet<String> dict = setupDictionary(words);
  HashSet<String> visited = new HashSet<String>();
  return transform(visited, start, stop, dict);
}

HashSet<String> setupDictionary(String[] words) {
  HashSet<String> hash = new HashSet<String>();
  for (String word : words) {
    hash.add(word.toLowerCase());
  }
  return hash;
}

LinkedList<String> transform(HashSet<String> visited, String startWord,
```

```
                                String stopWord, Set<String> dictionary) {
  if (startWord.equals(stopWord)) {
    LinkedList<String> path = new LinkedList<String>();
    path.add(startWord);
    return path;
  } else if (visited.contains(startWord) || !dictionary.contains(startWord)) {
    return null;
  }

  visited.add(startWord);
  ArrayList<String> words = wordsOneAway(startWord);

  for (String word : words) {
    LinkedList<String> path = transform(visited, word, stopWord, dictionary);
    if (path != null) {
      path.addFirst(startWord);
      return path;
    }
  }

  return null;
}

ArrayList<String> wordsOneAway(String word) {
  ArrayList<String> words = new ArrayList<String>();
  for (int i = 0; i < word.length(); i++) {
    for (char c = 'a'; c <= 'z'; c++) {
      String w = word.substring(0, i) + c + word.substring(i + 1);
      words.add(w);
    }
  }
  return words;
}
```

One major inefficiency in this algorithm is finding all strings that are one edit away. Right now, we're finding the strings that are one edit away and then eliminating the invalid ones.

Ideally, we want to only go to the ones that are valid.

Optimized Solution

To travel to only valid words, we clearly need a way of going from each word to a list of all the valid related words.

What makes two words "related" (one edit away)? They are one edit away if all but one character is the same. For example, `ball` and `bill` are one edit away, because they are both in the form `b_ll`. Therefore, one approach is to group all words that look like `b_ll` together.

We can do this for the whole dictionary by creating a mapping from a "wildcard word" (like `b_ll`) to a list of all words in this form. For example, for a very small dictionary like `{all, ill, ail, ape, ale}` the mapping might look like this:

```
_il -> ail
_le -> ale
_ll -> all, ill
_pe -> ape
a_e -> ape, ale
a_l -> all, ail
```

```
i_l -> ill
ai_ -> ail
al_ -> all, ale
ap_ -> ape
il_ -> ill
```

Now, when we want to know the words that are one edit away from a word like `ale`, we look up `_le`, `a_e`, and `al_` in the hash table.

The algorithm is otherwise essentially the same.

```
LinkedList<String> transform(String start, String stop, String[] words) {
  HashMapList<String, String> wildcardToWordList = createWildcardToWordMap(words);
  HashSet<String> visited = new HashSet<String>();
  return transform(visited, start, stop, wildcardToWordList);
}

/* Do a depth-first search from startWord to stopWord, traveling through each word
 * that is one edit away. */
LinkedList<String> transform(HashSet<String> visited, String start, String stop,
                             HashMapList<String, String> wildcardToWordList) {
  if (start.equals(stop)) {
    LinkedList<String> path = new LinkedList<String>();
    path.add(start);
    return path;
  } else if (visited.contains(start)) {
    return null;
  }

  visited.add(start);
  ArrayList<String> words = getValidLinkedWords(start, wildcardToWordList);

  for (String word : words) {
    LinkedList<String> path = transform(visited, word, stop, wildcardToWordList);
    if (path != null) {
      path.addFirst(start);
      return path;
    }
  }

  return null;
}

/* Insert words in dictionary into mapping from wildcard form -> word. */
HashMapList<String, String> createWildcardToWordMap(String[] words) {
  HashMapList<String, String> wildcardToWords = new HashMapList<String, String>();
  for (String word : words) {
    ArrayList<String> linked = getWildcardRoots(word);
    for (String linkedWord : linked) {
      wildcardToWords.put(linkedWord, word);
    }
  }
  return wildcardToWords;
}

/* Get list of wildcards associated with word. */
ArrayList<String> getWildcardRoots(String w) {
  ArrayList<String> words = new ArrayList<String>();
```

```
    for (int i = 0; i < w.length(); i++) {
      String word = w.substring(0, i) + "_" + w.substring(i + 1);
      words.add(word);
    }
    return words;
  }

  /* Return words that are one edit away. */
  ArrayList<String> getValidLinkedWords(String word,
      HashMapList<String, String> wildcardToWords) {
    ArrayList<String> wildcards = getWildcardRoots(word);
    ArrayList<String> linkedWords = new ArrayList<String>();
    for (String wildcard : wildcards) {
      ArrayList<String> words = wildcardToWords.get(wildcard);
      for (String linkedWord : words) {
        if (!linkedWord.equals(word)) {
          linkedWords.add(linkedWord);
        }
      }
    }
    return linkedWords;
  }

  /* HashMapList<String, String> is a HashMap that maps from Strings to
   * ArrayList<String>. See appendix for implementation. */
```

This will work, but we can still make it faster.

One optimization is to switch from depth-first search to breadth-first search. If there are zero paths or one path, the algorithms are equivalent speeds. However, if there are multiple paths, breadth-first search may run faster.

Breadth-first search finds the shortest path between two nodes, whereas depth-first search finds any path. This means that depth-first search might take a very long, windy path in order to find a connection when, in fact, the nodes were quite close.

Optimal Solution

As noted earlier, we can optimize this using breadth-first search. Is this as fast as we can make it? Not quite.

Imagine that the path between two nodes has length 4. With breadth-first search, we will visit about 15^4 nodes to find them.

Breadth-first search spans out very quickly.

Instead, what if we searched out from the source and destination nodes simultaneously? In this case, the breadth-first searches would collide after each had done about two levels each.

- Nodes travelled to from source: 15^2
- Nodes travelled to from destination: 15^2
- Total nodes: $15^2 + 15^2$

This is much better than the traditional breadth-first search.

We will need to track the path that we've travelled at each node.

To implement this approach, we've used an additional class BFSData. BFSData helps us keep things a bit clearer, and allows us to keep a similar framework for the two simultaneous breadth-first searches. The alternative is to keep passing around a bunch of separate variables.

```
LinkedList<String> transform(String startWord, String stopWord, String[] words) {
  HashMapList<String, String> wildcardToWordList = getWildcardToWordList(words);

  BFSData sourceData = new BFSData(startWord);
  BFSData destData = new BFSData(stopWord);

  while (!sourceData.isFinished() && !destData.isFinished()) {
    /* Search out from source. */
    String collision = searchLevel(wildcardToWordList, sourceData, destData);
    if (collision != null) {
      return mergePaths(sourceData, destData, collision);
    }

    /* Search out from destination. */
    collision = searchLevel(wildcardToWordList, destData, sourceData);
    if (collision != null) {
      return mergePaths(sourceData, destData, collision);
    }
  }

  return null;
}

/* Search one level and return collision, if any. */
String searchLevel(HashMapList<String, String> wildcardToWordList,
                   BFSData primary, BFSData secondary) {
  /* We only want to search one level at a time. Count how many nodes are
   * currently in the primary's level and only do that many nodes. We'll continue
   * to add nodes to the end. */
  int count = primary.toVisit.size();
  for (int i = 0; i < count; i++) {
    /* Pull out first node. */
    PathNode pathNode = primary.toVisit.poll();
    String word = pathNode.getWord();

    /* Check if it's already been visited. */
    if (secondary.visited.containsKey(word)) {
      return pathNode.getWord();
    }

    /* Add friends to queue. */
    ArrayList<String> words = getValidLinkedWords(word, wildcardToWordList);
    for (String w : words) {
      if (!primary.visited.containsKey(w)) {
        PathNode next = new PathNode(w, pathNode);
        primary.visited.put(w, next);
        primary.toVisit.add(next);
      }
    }
  }
  return null;
}
```

```
LinkedList<String> mergePaths(BFSData bfs1, BFSData bfs2, String connection) {
  PathNode end1 = bfs1.visited.get(connection); // end1 -> source
  PathNode end2 = bfs2.visited.get(connection); // end2 -> dest
  LinkedList<String> pathOne = end1.collapse(false); // forward
  LinkedList<String> pathTwo = end2.collapse(true); // reverse
  pathTwo.removeFirst(); // remove connection
  pathOne.addAll(pathTwo); // add second path
  return pathOne;
}

/* Methods getWildcardRoots, getWildcardToWordList, and getValidLinkedWords are
 * the same as in the earlier solution. */

public class BFSData {
  public Queue<PathNode> toVisit = new LinkedList<PathNode>();
  public HashMap<String, PathNode> visited = new HashMap<String, PathNode>();

  public BFSData(String root) {
    PathNode sourcePath = new PathNode(root, null);
    toVisit.add(sourcePath);
    visited.put(root, sourcePath);
  }

  public boolean isFinished() {
    return toVisit.isEmpty();
  }
}

public class PathNode {
  private String word = null;
  private PathNode previousNode = null;
  public PathNode(String word, PathNode previous) {
    this.word = word;
    previousNode = previous;
  }

  public String getWord() {
    return word;
  }

  /* Traverse path and return linked list of nodes. */
  public LinkedList<String> collapse(boolean startsWithRoot) {
    LinkedList<String> path = new LinkedList<String>();
    PathNode node = this;
    while (node != null) {
      if (startsWithRoot) {
        path.addLast(node.word);
      } else {
        path.addFirst(node.word);
      }
      node = node.previousNode;
    }
    return path;
  }
}
```

```
/* HashMapList<String, Integer> is a HashMap that maps from Strings to
 * ArrayList<Integer>. See appendix for implementation. */
```

This algorithm's runtime is a bit harder to describe since it depends on what the language looks like, as well as the actual source and destination words. One way of expressing it is that if each word has E words that are one edit away and the source and destination are distance D, the runtime is $O(E^{D/2})$. This is how much work each breadth-first search does.

Of course, this is a lot of code to implement in an interview. It just wouldn't be possible. More realistically, you'd leave out a lot of the details. You might write just the skeleton code of `transform` and `searchLevel`, but leave out the rest.

17.23 Max Square Matrix: Imagine you have a square matrix, where each cell (pixel) is either black or white. Design an algorithm to find the maximum subsquare such that all four borders are filled with black pixels.

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SOLUTION

Like many problems, there's an easy way and a hard way to solve this. We'll go through both solutions.

The "Simple" Solution: $O(N^4)$

We know that the biggest possible square has a length of size N, and there is only one possible square of size NxN. We can easily check for that square and return if we find it.

If we do not find a square of size NxN, we can try the next best thing: `(N-1) x (N-1)`. We iterate through all squares of this size and return the first one we find. We then do the same for N-2, N-3, and so on. Since we are searching progressively smaller squares, we know that the first square we find is the biggest.

Our code works as follows:

```
Subsquare findSquare(int[][] matrix) {
  for (int i = matrix.length; i >= 1; i--) {
    Subsquare square = findSquareWithSize(matrix, i);
    if (square != null) return square;
  }
  return null;
}

Subsquare findSquareWithSize(int[][] matrix, int squareSize) {
  /* On an edge of length N, there are (N - sz + 1) squares of length sz. */
  int count = matrix.length - squareSize + 1;

  /* Iterate through all squares with side length squareSize. */
  for (int row = 0; row < count; row++) {
    for (int col = 0; col < count; col++) {
      if (isSquare(matrix, row, col, squareSize)) {
        return new Subsquare(row, col, squareSize);
      }
    }
  }
  return null;
}

boolean isSquare(int[][] matrix, int row, int col, int size) {
```

```
    // Check top and bottom border.
    for (int j = 0; j < size; j++){
      if (matrix[row][col+j] == 1) {
        return false;
      }
      if (matrix[row+size-1][col+j] == 1){
        return false;
      }
    }

    // Check left and right border.
    for (int i = 1; i < size - 1; i++){
      if (matrix[row+i][col] == 1){
        return false;
      }
      if (matrix[row+i][col+size-1] == 1) {
        return false;
      }
    }
    return true;
  }
```

Pre-Processing Solution: $O(N^3)$

A large part of the slowness of the "simple" solution above is due to the fact we have to do $O(N)$ work each time we want to check a potential square. By doing some pre-processing, we can cut down the time of `isSquare` to $O(1)$. The time of the whole algorithm is reduced to $O(N^3)$.

If we analyze what `isSquare` does, we realize that all it ever needs to know is if the next `squareSize` items, on the right of as well as below particular cells, are zeros. We can pre-compute this data in a straightforward, iterative fashion.

We iterate from right to left, bottom to top. At each cell, we do the following computation:

```
if A[r][c] is white, zeros right and zeros below are 0
else A[r][c].zerosRight = A[r][c + 1].zerosRight + 1
     A[r][c].zerosBelow = A[r + 1][c].zerosBelow + 1
```

Below is an example of these values for a potential matrix.

(0s right, 0s below)

0,0	1,3	0,0
2,2	1,2	0,0
2,1	1,1	0,0

Original Matrix

W	B	W
B	B	W
B	B	W

Now, instead of iterating through $O(N)$ elements, the `isSquare` method just needs to check `zerosRight` and `zerosBelow` for the corners.

Our code for this algorithm is below. Note that `findSquare` and `findSquareWithSize` is equivalent, other than a call to `processMatrix` and working with a new data type thereafter.

```
public class SquareCell {
  public int zerosRight = 0;
```

```
  public int zerosBelow = 0;
  /* declaration, getters, setters */
}

Subsquare findSquare(int[][] matrix) {
  SquareCell[][] processed = processSquare(matrix);
  for (int i = matrix.length; i >= 1; i--) {
    Subsquare square = findSquareWithSize(processed, i);
    if (square != null) return square;
  }
  return null;
}

Subsquare findSquareWithSize(SquareCell[][] processed, int size) {
  /* equivalent to first algorithm */
}

boolean isSquare(SquareCell[][] matrix, int row, int col, int sz) {
  SquareCell topLeft = matrix[row][col];
  SquareCell topRight = matrix[row][col + sz - 1];
  SquareCell bottomLeft = matrix[row + sz - 1][col];

  /* Check top, left, right, and bottom edges, respectively. */
  if (topLeft.zerosRight < sz || topLeft.zerosBelow < sz ||
      topRight.zerosBelow < sz || bottomLeft.zerosRight < sz) {
    return false;
  }
  return true;
}

SquareCell[][] processSquare(int[][] matrix) {
  SquareCell[][] processed =
    new SquareCell[matrix.length][matrix.length];

  for (int r = matrix.length - 1; r >= 0; r--) {
    for (int c = matrix.length - 1; c >= 0; c--) {
      int rightZeros = 0;
      int belowZeros = 0;
      // only need to process if it's a black cell
      if (matrix[r][c] == 0) {
        rightZeros++;
        belowZeros++;
        // next column over is on same row
        if (c + 1 < matrix.length) {
          SquareCell previous = processed[r][c + 1];
          rightZeros += previous.zerosRight;
        }
        if (r + 1 < matrix.length) {
          SquareCell previous = processed[r + 1][c];
          belowZeros += previous.zerosBelow;
        }
      }
      processed[r][c] = new SquareCell(rightZeros, belowZeros);
    }
  }
  return processed;
```

```
}
```

17.24 Max Submatrix: Given an NxN matrix of positive and negative integers, write code to find the submatrix with the largest possible sum.

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SOLUTION

This problem can be approached in a variety of ways. We'll start with the brute force solution and then optimize the solution from there.

Brute Force Solution: $O(N^6)$

Like many "maximizing" problems, this problem has a straightforward brute force solution. This solution simply iterates through all possible submatrices, computes the sum, and finds the largest.

To iterate through all possible submatrices (with no duplicates), we simply need to iterate through all ordered pairs of rows, and then all ordered pairs of columns.

This solution is $O(N^6)$, since we iterate through $O(N^4)$ submatrices and it takes $O(N^2)$ time to compute the area of each.

```
SubMatrix getMaxMatrix(int[][] matrix) {
  int rowCount = matrix.length;
  int columnCount = matrix[0].length;
  SubMatrix best = null;
  for (int row1 = 0; row1 < rowCount; row1++) {
    for (int row2 = row1; row2 < rowCount; row2++) {
      for (int col1 = 0; col1 < columnCount; col1++) {
        for (int col2 = col1; col2 < columnCount; col2++) {
          int sum = sum(matrix, row1, col1, row2, col2);
          if (best == null || best.getSum() < sum) {
            best = new SubMatrix(row1, col1, row2, col2, sum);
          }
        }
      }
    }
  }
  return best;
}

int sum(int[][] matrix, int row1, int col1, int row2, int col2) {
  int sum = 0;
  for (int r = row1; r <= row2; r++) {
    for (int c = col1; c <= col2; c++) {
      sum += matrix[r][c];
    }
  }
  return sum;
}

public class SubMatrix {
  private int row1, row2, col1, col2, sum;
  public SubMatrix(int r1, int c1, int r2, int c2, int sm) {
    row1 = r1;
    col1 = c1;
```

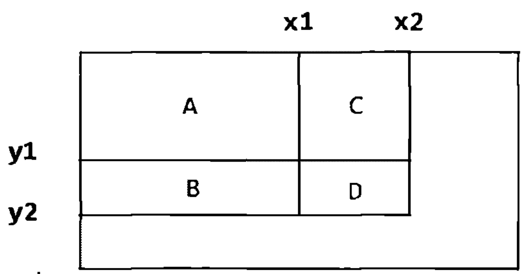
```
        row2 = r2;
        col2 = c2;
        sum = sm;
    }

    public int getSum() {
        return sum;
    }
}
```

It is good practice to pull the sum code into its own function since it's a fairly distinct set of code.

Dynamic Programming Solution: $O(N^4)$

Notice that the earlier solution is made slower by a factor of $O(N^2)$ simply because computing the sum of a matrix is so slow. Can we reduce the time to compute the area? Yes! In fact, we can reduce the time of `computeSum` to `O(1)`.

Consider the following rectangle:



Suppose we knew the following values:

```
ValD = area(point(0, 0) -> point(x2, y2))
ValC = area(point(0, 0) -> point(x2, y1))
ValB = area(point(0, 0) -> point(x1, y2))
ValA = area(point(0, 0) -> point(x1, y1))
```

Each `Val*` starts at the origin and ends at the bottom right corner of a subrectangle.

With these values, we know the following:

```
area(D) = ValD - area(A union C) - area(A union B) + area(A).
```

Or, written another way:

```
area(D) = ValD - ValB - ValC + ValA
```

We can efficiently compute these values for all points in the matrix by using similar logic:

```
Val(x, y) = Val(x-1, y) + Val(y-1, x) - Val(x-1, y-1) + M[x][y]
```

We can precompute all such values and then efficiently find the maximum submatrix.

The following code implements this algorithm.

```
SubMatrix getMaxMatrix(int[][] matrix) {
   SubMatrix best = null;
   int rowCount = matrix.length;
   int columnCount = matrix[0].length;
   int[][] sumThrough = precomputeSums(matrix);

   for (int row1 = 0; row1 < rowCount; row1++) {
      for (int row2 = row1; row2 < rowCount; row2++) {
         for (int col1 = 0; col1 < columnCount; col1++) {
            for (int col2 = col1; col2 < columnCount; col2++) {
```

```
            int sum = sum(sumThrough, row1, col1, row2, col2);
            if (best == null || best.getSum() < sum) {
               best = new SubMatrix(row1, col1, row2, col2, sum);
            }
         }
      }
   }
  }
  return best;
}

int[][] precomputeSums(int[][] matrix) {
  int[][] sumThrough = new int[matrix.length][matrix[0].length];
  for (int r = 0; r < matrix.length; r++) {
     for (int c = 0; c < matrix[0].length; c++) {
        int left = c > 0 ? sumThrough[r][c - 1] : 0;
        int top = r > 0 ? sumThrough[r - 1][c] : 0;
        int overlap = r > 0 && c > 0 ? sumThrough[r-1][c-1] : 0;
        sumThrough[r][c] = left + top - overlap + matrix[r][c];
     }
  }
  return sumThrough;
}

int sum(int[][] sumThrough, int r1, int c1, int r2, int c2) {
  int topAndLeft = r1 > 0 && c1 > 0 ? sumThrough[r1-1][c1-1] : 0;
  int left = c1 > 0 ? sumThrough[r2][c1 - 1] : 0;
  int top = r1 > 0 ? sumThrough[r1 - 1][c2] : 0;
  int full = sumThrough[r2][c2];
  return full - left - top + topAndLeft;
}
```

This algorithm takes $O(N^4)$ time, since it goes through each pair of rows and each pair of columns.

Optimized Solution: $O(N^3)$

Believe it or not, an even more optimal solution exists. If we have R rows and C columns, we can solve it in $O(R^2C)$ time.

Recall the solution to the maximum subarray problem: "Given an array of integers, find the subarray with the largest sum." We can find the maximum subarray in $O(N)$ time. We will leverage this solution for this problem.

Every submatrix can be represented by a contiguous sequence of rows and a contiguous sequence of columns. If we were to iterate through every contiguous sequence of rows, we would then just need to find, for each of those, the set of columns that gives us the highest sum. That is:

```
maxSum = 0
foreach rowStart in rows
  foreach rowEnd in rows
     /* We have many possible submatrices with rowStart and rowEnd as the top and
      * bottom edges of the matrix. Find the colStart and colEnd edges that give
      * the highest sum. */
     maxSum = max(runningMaxSum, maxSum)
return maxSum
```

Now the question is, how do we efficiently find the "best" `colStart` and `colEnd`?

Picture a submatrix:

rowStart

9	-8	1	3	-2
-3	7	6	-2	4
6	-4	-4	8	-7
12	-5	3	9	-5

rowEnd

Given a `rowStart` and `rowEnd`, we want to find the `colStart` and `colEnd` that give us the highest possible sum. To do this, we can sum up each column and then apply the `maximumSubArray` function explained at the beginning of this problem.

For the earlier example, the maximum subarray is the first through fourth columns. This means that the maximum submatrix is (`rowStart, first column`) through (`rowEnd, fourth column`).

We now have pseudocode that looks like the following.

```
1  maxSum = 0
2  foreach rowStart in rows
3    foreach rowEnd in rows
4      foreach col in columns
5        partialSum[col] = sum of matrix[rowStart, col] through matrix[rowEnd, col]
6      runningMaxSum = maxSubArray(partialSum)
7      maxSum = max(runningMaxSum, maxSum)
8  return maxSum
```

The sum in lines 5 and 6 takes R*C time to compute (since it iterates through `rowStart` through `rowEnd`), so this gives us a runtime of $O(R^3C)$. We're not quite done yet.

In lines 5 and 6, we're basically adding up `a[0]...a[i]` from scratch, even though in the previous iteration of the outer for loop, we already added up `a[0]...a[i-1]`. Let's cut out this duplicated effort.

```
maxSum = 0
foreach rowStart in rows
  clear array partialSum
  foreach rowEnd in rows
    foreach col in columns
      partialSum[col] += matrix[rowEnd, col]
    runningMaxSum = maxSubArray(partialSum)
  maxSum = max(runningMaxSum, maxSum)
return maxSum
```

Our full code looks like this:

```
SubMatrix getMaxMatrix(int[][] matrix) {
  int rowCount = matrix.length;
  int colCount = matrix[0].length;
  SubMatrix best = null;

  for (int rowStart = 0; rowStart < rowCount; rowStart++) {
    int[] partialSum = new int[colCount];

    for (int rowEnd = rowStart; rowEnd < rowCount; rowEnd++) {
      /* Add values at row rowEnd. */
      for (int i = 0; i < colCount; i++) {
```

```
                partialSum[i] += matrix[rowEnd][i];
            }

            Range bestRange = maxSubArray(partialSum, colCount);
            if (best == null || best.getSum() < bestRange.sum) {
                best = new SubMatrix(rowStart, bestRange.start, rowEnd,
                                     bestRange.end, bestRange.sum);
            }
        }
    }
    return best;
}

Range maxSubArray(int[] array, int N) {
    Range best = null;
    int start = 0;
    int sum = 0;

    for (int i = 0; i < N; i++) {
        sum += array[i];
        if (best == null || sum > best.sum) {
            best = new Range(start, i, sum);
        }

        /* If running_sum is < 0 no point in trying to continue the series. Reset. */
        if (sum < 0) {
            start = i + 1;
            sum = 0;
        }
    }
    return best;
}

public class Range {
    public int start, end, sum;
    public Range(int start, int end, int sum) {
        this.start = start;
        this.end = end;
        this.sum = sum;
    }
}
```

This was an extremely complex problem. You would not be expected to figure out this entire problem in an interview without a lot of help from your interviewer.

17.25 Word Rectangle: Given a list of millions of words, design an algorithm to create the largest possible rectangle of letters such that every row forms a word (reading left to right) and every column forms a word (reading top to bottom). The words need not be chosen consecutively from the list, but all rows must be the same length and all columns must be the same height.

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SOLUTION

Many problems involving a dictionary can be solved by doing some pre-processing. Where can we do pre-processing?

Well, if we're going to create a rectangle of words, we know that each row must be the same length and each column must be the same length. So let's group the words of the dictionary based on their sizes. Let's call this grouping D, where `D[i]` contains the list of words of length `i`.

Next, observe that we're looking for the largest rectangle. What is the largest rectangle that could be formed? It's `length(largest word)`^{2}.

```
int maxRectangle = longestWord * longestWord;
for z = maxRectangle to 1 {
  for each pair of numbers (i, j) where i*j = z {
    /* attempt to make rectangle. return if successful. */
  }
}
```

By iterating from the biggest possible rectangle to the smallest, we ensure that the first valid rectangle we find will be the largest possible one.

Now, for the hard part: `makeRectangle(int l, int h)`. This method attempts to build a rectangle of words which has length `l` and height h.

One way to do this is to iterate through all (ordered) sets of h words and then check if the columns are also valid words. This will work, but it's rather inefficient.

Imagine that we are trying to build a 6x5 rectangle and the first few rows are:

```
there
queen
pizza
.....
```

At this point, we know that the first column starts with `tqp`. We know—or *should* know—that no dictionary word starts with `tqp`. Why do we bother continuing to build a rectangle when we know we'll fail to create a valid one in the end?

This leads us to a more optimal solution. We can build a trie to easily look up if a substring is a prefix of a word in the dictionary. Then, when we build our rectangle, row by row, we check to see if the columns are all valid prefixes. If not, we fail immediately, rather than continue to try to build this rectangle.

The code below implements this algorithm. It is long and complex, so we will go through it step by step.

First, we do some pre-processing to group words by their lengths. We create an array of tries (one for each word length), but hold off on building the tries until we need them.

```
WordGroup[] groupList = WordGroup.createWordGroups(list);
int maxWordLength = groupList.length;
Trie trieList[] = new Trie[maxWordLength];
```

The `maxRectangle` method is the "main" part of our code. It starts with the biggest possible rectangle area (which is `maxWordLength`^{2}) and tries to build a rectangle of that size. If it fails, it subtracts one from the area and attempts this new, smaller size. The first rectangle that can be successfully built is guaranteed to be the biggest.

```
Rectangle maxRectangle() {
  int maxSize = maxWordLength * maxWordLength;
  for (int z = maxSize; z > 0; z--) { // start from biggest area
    for (int i = 1; i <= maxWordLength; i ++ ) {
      if (z % i == 0) {
        int j = z / i;
        if (j <= maxWordLength) {
          /* Create rectangle of length i and height j. Note that i * j = z. */
          Rectangle rectangle = makeRectangle(i, j);
```

```
                if (rectangle != null) return rectangle;
             }
          }
       }
    }
    return null;
}
```

The `makeRectangle` method is called by `maxRectangle` and tries to build a rectangle of a specific length and height.

```
Rectangle makeRectangle(int length, int height) {
   if (groupList[length-1] == null || groupList[height-1] == null) {
      return null;
   }

   /* Create trie for word length if we haven't yet */
   if (trieList[height - 1] == null) {
      LinkedList<String> words = groupList[height - 1].getWords();
      trieList[height - 1] = new Trie(words);
   }

   return makePartialRectangle(length, height, new Rectangle(length));
}
```

The `makePartialRectangle` method is where the action happens. It is passed in the intended, final length and height, and a partially formed rectangle. If the rectangle is already of the final height, then we just check to see if the columns form valid, complete words, and return.

Otherwise, we check to see if the columns form valid prefixes. If they do not, then we immediately break since there is no way to build a valid rectangle off of this partial one.

But, if everything is okay so far, and all the columns are valid prefixes of words, then we search through all the words of the right length, append each to the current rectangle, and recursively try to build a rectangle off of `{current rectangle with new word appended}`.

```
Rectangle makePartialRectangle(int l, int h, Rectangle rectangle) {
   if (rectangle.height == h) { // Check if complete rectangle
      if (rectangle.isComplete(l, h, groupList[h - 1])) {
         return rectangle;
      }
      return null;
   }

   /* Compare columns to trie to see if potentially valid rect */
   if (!rectangle.isPartialOK(l, trieList[h - 1])) {
      return null;
   }

   /* Go through all words of the right length. Add each one to the current partial
    * rectangle, and attempt to build a rectangle recursively. */
   for (int i = 0; i < groupList[l-1].length(); i++) {
      /* Create a new rectangle which is this rect + new word. */
      Rectangle orgPlus = rectangle.append(groupList[l-1].getWord(i));

      /* Try to build a rectangle with this new, partial rect */
      Rectangle rect = makePartialRectangle(l, h, orgPlus);
      if (rect != null) {
         return rect;
```

```
      }
    }
    return null;
  }
```

The Rectangle class represents a partially or fully formed rectangle of words. The method isPartialOk can be called to check if the rectangle is, thus far, a valid one (that is, all the columns are prefixes of words). The method isComplete serves a similar function, but checks if each of the columns makes a full word.

```
public class Rectangle {
  public int height, length;
  public char[][] matrix;

  / *Construct an "empty" rectangule. Length is fixed, but height varies as we add
   * words. */
  public Rectangle(int l) {
    height = 0;
    length = l;
  }

  / *Construct a rectangular array of letters of the specified length and height,
   * and backed by the specified matrix of letters. (It is assumed that the length
   * and height specified as arguments are consistent with the array argument's
   * dimensions.) */
  public Rectangle(int length, int height, char[][] letters) {
    this.height = letters.length;
    this.length = letters[0].length;
    matrix = letters;
  }

  public char getLetter (int i, int j) { return matrix[i][j]; }
  public String getColumn(int i) { ... }

  / *Check if all columns are valid. All rows are already known to be valid since
   * they were added directly from dictionary. */
  public boolean isComplete(int l, int h, WordGroup groupList) {
    if (height == h) {
      / *Check if each column is a word in the dictionary. */
      for (int i = 0; i < l; i++) {
        String col = getColumn(i);
        if (!groupList.containsWord(col)) {
          return false;
        }
      }
      return true;
    }
    return false;
  }

  public boolean isPartialOK(int l, Trie trie) {
    if (height == 0) return true;
    for (int i = 0; i < l; i++ ) {
      String col = getColumn(i);
      if (!trie.contains(col)) {
        return false;
      }
    }
```

```
      return true;
    }

    / *Create a new Rectangle by taking the rows of the current rectangle and
     * appending s. */
    public Rectangle append(String s) { ... }
}
```

The WordGroup class is a simple container for all words of a specific length. For easy lookup, we store the words in a hash table as well as in an ArrayList.

The lists in WordGroup are created through a static method called createWordGroups.

```
public class WordGroup {
  private HashMap<String, Boolean> lookup = new HashMap<String, Boolean>();
  private ArrayList<String> group = new ArrayList<String>();
  public boolean containsWord(String s) { return lookup.containsKey(s); }
  public int length() { return group.size(); }
  public String getWord(int i) { return group.get(i); }
  public ArrayList<String> getWords() { return group; }

  public void addWord (String s) {
    group.add(s);
    lookup.put(s, true);
  }

  public static WordGroup[] createWordGroups(String[] list) {
    WordGroup[] groupList;
    int maxWordLength = 0;
    / *Find the length of the longest word */
    for (int i = 0; i < list.length; i++) {
      if (list[i].length() > maxWordLength) {
        maxWordLength = list[i].length();
      }
    }

    / *Group the words in the dictionary into lists of words of same length.
     * groupList[i] will contain a list of words, each of length (i+1). */
    groupList = new WordGroup[maxWordLength];
    for (int i = 0; i < list.length; i++) {
      / *We do wordLength - 1 instead of just wordLength since this is used as
       * an index and no words are of length 0 */
      int wordLength = list[i].length() - 1;
      if (groupList[wordLength] == null) {
        groupList[wordLength] = new WordGroup();
      }
      groupList[wordLength].addWord(list[i]);
    }
    return groupList;
  }
}
```

The full code for this problem, including the code for Trie and TrieNode, can be found in the code attachment. Note that in a problem as complex as this, you'd most likely only need to write the pseudocode. Writing the entire code would be nearly impossible in such a short amount of time.

17.26 Sparse Similarity: The similarity of two documents (each with distinct words) is defined to be the size of the intersection divided by the size of the union. For example, if the documents consist of integers, the similarity of `{1, 5, 3}` and `{1, 7, 2, 3}` is `0.4`, because the intersection has size 2 and the union has size 5.

We have a long list of documents (with distinct values and each with an associated ID) where the similarity is believed to be "sparse." That is, any two arbitrarily selected documents are very likely to have similarity 0. Design an algorithm that returns a list of pairs of document IDs and the associated similarity.

Print only the pairs with similarity greater than 0. Empty documents should not be printed at all. For simplicity, you may assume each document is represented as an array of distinct integers.

EXAMPLE

Input:

```
13: {14, 15, 100, 9, 3}
16: {32, 1, 9, 3, 5}
19: {15, 29, 2, 6, 8, 7}
24: {7, 10}
```

Output:

```
ID1, ID2 : SIMILARITY
13, 19   : 0.1
13, 16   : 0.25
19, 24   : 0.14285714285714285
```

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SOLUTION

This sounds like quite a tricky problem, so let's start off with a brute force algorithm. If nothing else, it will help wrap our heads around the problem.

Remember that each document is an array of distinct "words", and each is just an integer.

Brute Force

A brute force algorithm is as simple as just comparing all arrays to all other arrays. At each comparison, we compute the size of the intersection and size of the union of the two arrays.

Note that we only want to print this pair if the similarity is greater than 0. The union of two arrays can never be zero (unless both arrays are empty, in which case we don't want them printed anyway). Therefore, we are really just printing the similarity if the intersection is greater than 0.

How do we compute the size of the intersection and the union?

The intersection means the number of elements in common. Therefore, we can just iterate through the first array (A) and check if each element is in the second array (B). If it is, increment an `intersection` variable.

To compute the union, we need to be sure that we don't double count elements that are in both. One way to do this is to count up all the elements in A that are *not* in B. Then, add in all the elements in B. This will avoid double counting as the duplicate elements are only counted with B.

Alternatively, we can think about it this way. If we *did* double count elements, it would mean that elements in the intersection (in both A and B) were counted twice. Therefore, the easy fix is to just remove these duplicate elements.

```
union(A, B) = A + B - intersection(A, B)
```

This means that all we really need to do is compute the intersection. We can derive the union, and therefore similarity, from that immediately.

This gives us an O(AB) algorithm, just to compare two arrays (or documents).

However, we need to do this for all pairs of D documents. If we assume each document has at most W words then the runtime is $O(D^2 W^2)$.

Slightly Better Brute Force

As a quick win, we can optimize the computation for the similarity of two arrays. Specifically, we need to optimize the intersection computation.

We need to know the number of elements in common between the two arrays. We can throw all of A's elements into a hash table. Then we iterate through B, incrementing `intersection` every time we find an element in A.

This takes O(A + B) time. If each array has size W and we do this for D arrays, then this takes $O(D^2 W)$.

Before implementing this, let's first think about the classes we'll need.

We'll need to return a list of document pairs and their similarities. We'll use a `DocPair` class for this. The exact return type will be a hash table that maps from `DocPair` to a double representing the similarity.

```
public class DocPair {
  public int doc1, doc2;

  public DocPair(int d1, int d2) {
    doc1 = d1;
    doc2 = d2;
  }

  @Override
  public boolean equals(Object o) {
    if (o instanceof DocPair) {
      DocPair p = (DocPair) o;
      return p.doc1 == doc1 && p.doc2 == doc2;
    }
    return false;
  }

  @Override
  public int hashCode() { return (doc1 * 31) ^ doc2; }
}
```

It will also be useful to have a class that represents the documents.

```
public class Document {
  private ArrayList<Integer> words;
  private int docId;

  public Document(int id, ArrayList<Integer> w) {
    docId = id;
    words = w;
  }

  public ArrayList<Integer> getWords() { return words; }
  public int getId() { return docId; }
```

```
    public int size() { return words == null ? 0 : words.size(); }
}
```

Strictly speaking, we don't need any of this. However, readability is important, and it's a lot easier to read `ArrayList<Document>` than `ArrayList<ArrayList<Integer>>`.

Doing this sort of thing not only shows good coding style, it also makes your life in an interview a lot easier. You have to write a lot less. (You probably would not define the entire `Document` class, unless you had extra time or your interviewer asked you to.)

```
1  HashMap<DocPair, Double> computeSimilarities(ArrayList<Document> documents) {
2    HashMap<DocPair, Double> similarities = new HashMap<DocPair, Double>();
3    for (int i = 0; i < documents.size(); i++) {
4      for (int j = i + 1; j < documents.size(); j++) {
5        Document doc1 = documents.get(i);
6        Document doc2 = documents.get(j);
7        double sim = computeSimilarity(doc1, doc2);
8        if (sim > 0) {
9          DocPair pair = new DocPair(doc1.getId(), doc2.getId());
10         similarities.put(pair, sim);
11       }
12     }
13   }
14   return similarities;
15 }
16
17 double computeSimilarity(Document doc1, Document doc2) {
18   int intersection = 0;
19   HashSet<Integer> set1 = new HashSet<Integer>();
20   set1.addAll(doc1.getWords());
21
22   for (int word : doc2.getWords()) {
23     if (set1.contains(word)) {
24       intersection++;
25     }
26   }
27
28   double union = doc1.size() + doc2.size() - intersection;
29   return intersection / union;
30 }
```

Observe what's happening on line 28. Why did we make `union` a `double`, when it's obviously an `integer`?

We did this to avoid an integer division bug. If we didn't do this, the division would "round" down to an integer. This would mean that the similarity would almost always return 0. Oops!

Slightly Better Brute Force (Alternate)

If the documents were sorted, you could compute the intersection between two documents by walking through them in sorted order, much like you would when doing a sorted merge of two arrays.

This would take O(A + B) time. This is the same time as our current algorithm, but less space. Doing this on D documents with W words each would take $O(D^2 W)$ time.

Since we don't know that the arrays are sorted, we could first sort them. This would take O(D * W log W) time. The full runtime then is $O(D * W \log W + D^2 W)$.

We cannot necessarily assume that the second part "dominates" the first one, because it doesn't necessarily. It depends on the relative size of D and `log W`. Therefore, we need to keep both terms in our runtime expression.

Optimized (Somewhat)

It is useful to create a larger example to really understand the problem.

```
13: {14, 15, 100, 9, 3}
16: {32, 1, 9, 3, 5}
19: {15, 29, 2, 6, 8, 7}
24: {7, 10, 3}
```

At first, we might try various techniques that allow us to more quickly eliminate potential comparisons. For example, could we compute the min and max values in each array? If we did that, then we'd know that arrays with no overlap in ranges don't need to be compared.

The problem is that this doesn't really fix our runtime issue. Our best runtime thus far is $O(D^2 W)$. With this change, we're still going to be comparing all $O(D^2)$ pairs, but the $O(W)$ part might go to $O(1)$ sometimes. That $O(D^2)$ part is going to be a really big problem when D gets large.

Therefore, let's focus on reducing that $O(D^2)$ factor. That is the "bottleneck" in our solution. Specifically, this means that, given a document docA, we want to find all documents with some similarity—and we want to do this without "talking" to each document.

What would make a document similar to docA? That is, what characteristics define the documents with similarity > 0?

Suppose docA is `{14, 15, 100, 9, 3}`. For a document to have similarity > 0, it needs to have a 14, a 15, a 100, a 9, or a 3. How can we quickly gather a list of all documents with one of those elements?

The slow (and, really, only way) is to read every single word from every single document to find the documents that contain a 14, a 15, a 100, a 9, or a 3. That will take $O(DW)$ time. Not good.

However, note that we're doing this repeatedly. We can reuse the work from one call to the next.

If we build a hash table that maps from a word to all documents that contain that word, we can very quickly know the documents that overlap with docA.

```
1 -> 16
2 -> 19
3 -> 13, 16, 24
5 -> 16
6 -> 19
7 -> 19, 24
8 -> 19
9 -> 13, 16
...
```

When we want to know all the documents that overlap with docA, we just look up each of docA's items in this hash table. We'll then get a list of all documents with some overlap. Now, all we have to do is compare docA to each of those documents.

If there are P pairs with similarity > 0, and each document has W words, then this will take $O(PW)$ time (plus $O(DW)$ time to create and read this hash table). Since we expect P to be much less than D^2, this is much better than before.

Optimized (Better)

Let's think about our previous algorithm. Is there any way we can make it more optimal?

If we consider the runtime—`O(PW + DW)`—we probably can't get rid of the `O(DW)` factor. We have to touch each word at least once, and there are `O(DW)` words. Therefore, if there's an optimization to be made, it's probably in the `O(PW)` term.

It would be difficult to eliminate the `P` part in `O(PW)` because we have to at least print all P pairs (which takes `O(P)` time). The best place to focus, then, is on the `W` part. Is there some way we can do less than `O(W)` work for each pair of similar documents?

One way to tackle this is to analyze what information the hash table gives us. Consider this list of documents:

```
12: {1, 5, 9}
13: {5, 3, 1, 8}
14: {4, 3, 2}
15: {1, 5, 9, 8}
17: {1, 6}
```

If we look up document 12's elements in a hash table for this document, we'll get:

```
1 -> {12, 13, 15, 17}
5 -> {12, 13, 15}
9 -> {12, 15}
```

This tells us that documents 13, 15, and 17 have some similarity. Under our current algorithm, we would now need to compare document 12 to documents 13, 15, and 17 to see the number of elements document 12 has in common with each (that is, the size of the intersection). The union can be computed from the document sizes and the intersection, as we did before.

Observe, though, that document 13 appeared twice in the hash table, document 15 appeared three times, and document 17 appeared once. We discarded that information. But can we use it instead? What does it indicate that some documents appeared multiple times and others didn't?

Document 13 appeared twice because it has two elements (1 and 5) in common. Document 17 appeared once because it has only one element (1) in common. Document 15 appeared three times because it has three elements (1, 5, and 9) in common. This information can actually directly give us the size of the intersection.

We could go through each document, look up the items in the hash table, and then count how many times each document appears in each item's lists. There's a more direct way to do it.

1. As before, build a hash table for a list of documents.
2. Create a new hash table that maps from a document pair to an integer (which will indicate the size of the intersection).
3. Read the first hash table by iterating through each list of documents.
4. For each list of documents, iterate through the pairs in that list. Increment the intersection count for each pair.

Comparing this runtime to the previous one is a bit tricky. One way we can look at it is to realize that before we were doing `O(W)` work for each similar pair. That's because once we noticed that two documents were similar, we touched every single word in each document. With this algorithm, we're only touching the words that actually overlap. The worst cases are still the same, but for many inputs this algorithm will be faster.

```
HashMap<DocPair, Double>
computeSimilarities(HashMap<Integer, Document> documents) {
```

```
  HashMapList<Integer, Integer> wordToDocs = groupWords(documents);
  HashMap<DocPair, Double> similarities = computeIntersections(wordToDocs);
  adjustToSimilarities(documents, similarities);
  return similarities;
}

/* Create hash table from each word to where it appears. */
HashMapList<Integer, Integer> groupWords(HashMap<Integer, Document> documents) {
  HashMapList<Integer, Integer> wordToDocs = new HashMapList<Integer, Integer>();

  for (Document doc : documents.values()) {
    ArrayList<Integer> words = doc.getWords();
    for (int word : words) {
      wordToDocs.put(word, doc.getId());
    }
  }

  return wordToDocs;
}

/* Compute intersections of documents. Iterate through each list of documents and
 * then each pair within that list, incrementing the intersection of each page. */
HashMap<DocPair, Double> computeIntersections(
    HashMapList<Integer, Integer> wordToDocs {
  HashMap<DocPair, Double> similarities = new HashMap<DocPair, Double>();
  Set<Integer> words = wordToDocs.keySet();
  for (int word : words) {
    ArrayList<Integer> docs = wordToDocs.get(word);
    Collections.sort(docs);
    for (int i = 0; i < docs.size(); i++) {
      for (int j = i + 1; j < docs.size(); j++) {
        increment(similarities, docs.get(i), docs.get(j));
      }
    }
  }

  return similarities;
}

/* Increment the intersection size of each document pair. */
void increment(HashMap<DocPair, Double> similarities, int doc1, int doc2) {
  DocPair pair = new DocPair(doc1, doc2);
  if (!similarities.containsKey(pair)) {
    similarities.put(pair, 1.0);
  } else {
    similarities.put(pair, similarities.get(pair) + 1);
  }
}

/* Adjust the intersection value to become the similarity. */
void adjustToSimilarities(HashMap<Integer, Document> documents,
                          HashMap<DocPair, Double> similarities) {
  for (Entry<DocPair, Double> entry : similarities.entrySet()) {
    DocPair pair = entry.getKey();
    Double intersection = entry.getValue();
    Document doc1 = documents.get(pair.doc1);
```

```
    Document doc2 = documents.get(pair.doc2);
    double union = (double) doc1.size() + doc2.size() - intersection;
    entry.setValue(intersection / union);
  }
}

/* HashMapList<Integer, Integer> is a HashMap that maps from Integer to
 * ArrayList<Integer>. See appendix for implementation. */
```

For a set of documents with sparse similarity, this will run much faster than the original naive algorithm, which compares all pairs of documents directly.

Optimized (Alternative)

There's an alternative algorithm that some candidates might come up with. It's slightly slower, but still quite good.

Recall our earlier algorithm that computed the similarity between two documents by sorting them. We can extend this approach to multiple documents.

Imagine we took all of the words, tagged them by their original document, and then sorted them. The prior list of documents would look like this:

1_{12}, 1_{13}, 1_{15}, 1_{16}, 2_{14}, 3_{13}, 3_{14}, 4_{14}, 5_{12}, 5_{13}, 5_{15}, 6_{16}, 8_{13}, 8_{15}, 9_{12}, 9_{15}

Now we have essentially the same approach as before. We iterate through this list of elements. For each sequence of identical elements, we increment the intersection counts for the corresponding pair of documents.

We will use an Element class to group together documents and words. When we sort the list, we will sort first on the word but break ties on the document ID.

```
class Element implements Comparable<Element> {
  public int word, document;
  public Element(int w, int d) {
    word = w;
    document = d;
  }

  /* When we sort the words, this function will be used to compare the words. */
  public int compareTo(Element e) {
    if (word == e.word) {
      return document - e.document;
    }
    return word - e.word;
  }
}

HashMap<DocPair, Double> computeSimilarities(
    HashMap<Integer, Document> documents) {
  ArrayList<Element> elements = sortWords(documents);
  HashMap<DocPair, Double> similarities = computeIntersections(elements);
  adjustToSimilarities(documents, similarities);
  return similarities;
}

/* Throw all words into one list, sorting by the word and then the document. */
ArrayList<Element> sortWords(HashMap<Integer, Document> docs) {
  ArrayList<Element> elements = new ArrayList<Element>();
```

```
  for (Document doc : docs.values()) {
    ArrayList<Integer> words = doc.getWords();
    for (int word : words) {
      elements.add(new Element(word, doc.getId()));
    }
  }
  Collections.sort(elements);
  return elements;
}

/* Increment the intersection size of each document pair. */
void increment(HashMap<DocPair, Double> similarities, int doc1, int doc2) {
  DocPair pair = new DocPair(doc1, doc2);
  if (!similarities.containsKey(pair)) {
    similarities.put(pair, 1.0);
  } else {
    similarities.put(pair, similarities.get(pair) + 1);
  }
}

/* Adjust the intersection value to become the similarity. */
HashMap<DocPair, Double> computeIntersections(ArrayList<Element> elements) {
  HashMap<DocPair, Double> similarities = new HashMap<DocPair, Double>();

  for (int i = 0; i < elements.size(); i++) {
    Element left = elements.get(i);
    for (int j = i + 1; j < elements.size(); j++) {
      Element right = elements.get(j);
      if (left.word != right.word) {
        break;
      }
      increment(similarities, left.document, right.document);
    }
  }
  return similarities;
}

/* Adjust the intersection value to become the similarity. *
void adjustToSimilarities(HashMap<Integer, Document> documents,
                          HashMap<DocPair, Double> similarities) {
  for (Entry<DocPair, Double> entry : similarities.entrySet()) {
    DocPair pair = entry.getKey();
    Double intersection = entry.getValue();
    Document doc1 = documents.get(pair.doc1);
    Document doc2 = documents.get(pair.doc2);
    double union = (double) doc1.size() + doc2.size() - intersection;
    entry.setValue(intersection / union);
  }
}
```

The first step of this algorithm is slower than that of the prior algorithm, since it has to sort rather than just add to a list. The second step is essentially equivalent.

Both will run much faster than the original naive algorithm.

Advanced Topics

XI

This section includes topics that are mostly beyond the scope of interviews but can come up on occasion. Interviewers shouldn't be surprised if you don't know these topics well. Feel free to dive into these topics if you want to. If you're pressed for time, they're low priority.

XI

Advanced Topics

When writing the 6th edition, I had a number of debates about what should and shouldn't be included. Red-black trees? Dijkstra's algorithm? Topological sort?

On one hand, I'd had a number of requests to include these topics. Some people insisted that these topics are asked "all the time" (in which case, they have a very different idea of what this phrase means!). There was clearly a desire—at least from some people—to include them. And learning more can't hurt, right?

On the other hand, I know these topics to be rarely asked. It happens, of course. Interviewers are individuals and might have their own ideas of what is "fair game" or "relevant" for an interview. But it's rare. When it does come up, if you don't know the topic, it's unlikely to be a big red flag.

> Admittedly, as an interviewer, I *have* asked candidates questions where the solution was essentially an application of one of these algorithms. On the rare occasions that a candidate already knew the algorithm, they did not benefit from this knowledge (nor were they hurt by it). I want to evaluate your ability to solve a problem you haven't seen before. So, I'll take into account whether you know the underlying algorithm in advance.

I believe in giving people a fair expectation of the interview, not scaring people into excess studying. I also have no interest in making the book more "advanced" so as to help book sales, at the expense of your time and energy. That's not fair or right to do to you.

(Additionally, I didn't want to give interviewers—who I know to be reading this—the impression that they can or should be covering these more advanced topics. Interviewers: If you ask about these topics, you're testing knowledge of algorithms. You're just going to wind up eliminating a lot of perfectly smart people.)

But there are many borderline "important" topics. They're not often asked, but sometimes they are.

Ultimately, I decided to leave the decision in your hands. After all, you know better than I do how thorough you want to be in your preparation. If you want to do an extra thorough job, read this. If you just love learning data structures and algorithms, read this. If you want to see new ways of approaching problems, read this.

But if you're pressed for time, this studying isn't a super high priority.

▸ Useful Math

Here's some math that can be useful in some questions. There are more formal proofs that you can look up online, but we'll focus here on giving you the intuition behind them. You can think of these as informal proofs.

Sum of Integers 1 through N

What is 1 + 2 + ... + n? Let's figure it out by pairing up low values with high values.

If n is even, we pair 1 with n, 2 with n - 1, and so on. We will have $\frac{n}{2}$ pairs each with sum n + 1.

If n is odd, we pair 0 with n, 1 with n - 1, and so on. We will have $\frac{n+1}{2}$ pairs with sum n.

n is even			
pair #	**a**	**b**	**a + b**
1	1	n	n + 1
2	2	n - 1	n + 1
3	3	n - 2	n + 1
4	4	n - 3	n + 1
...	...	...	...
$\frac{n}{2}$	$\frac{n}{2}$	$\frac{n}{2}+1$	n + 1
total:	$\frac{n}{2}*(n+1)$		

n is odd			
pair #	**a**	**b**	**a + b**
1	0	n	n
2	1	n - 1	n
3	2	n - 2	n
4	3	n - 3	n
...	...	...	...
$\frac{n+1}{2}$	$\frac{n-1}{2}$	$\frac{n+1}{2}$	n
total:	$\frac{n+1}{2} * n$		

In either case, the sum is $\frac{n(n+1)}{2}$.

This reasoning comes up a lot in nested loops. For example, consider the following code:

```
for (int i = 0; i < n; i++) {
   for (int j = i + 1; j < n; j++) {
      System.out.println(i + j);
   }
}
```

On the first iteration of the outer for loop, the inner for loop iterates n - 1 times. On the second iteration of the outer for loop, the inner for loop iterates n-2 times. Next, n - 3, then n - 4, and so on. There are $\frac{n(n-1)}{2}$ total iterations of the inner for loop. Therefore, this code takes $O(n^2)$ time.

Sum of Powers of 2

Consider this sequence: $2^0 + 2^1 + 2^2 + ... + 2^n$. What is its result?

A nice way to see this is by looking at these values in binary.

	Power	Binary	Decimal
	2^0	00001	1
	2^1	00010	2
	2^2	00100	4
	2^3	01000	8
	2^4	10000	16
sum:	2^5-1	11111	32 - 1 = 31

Therefore, the sum of $2^0 + 2^1 + 2^2 + ... + 2^n$ would, in base 2, be a sequence of (n + 1) 1s. This is $2^{n+1} - 1$.

Takeaway: The sum of a sequence of powers of two is roughly equal to the *next* value in the sequence.

Bases of Logs

Suppose we have something in $\log_2$ (log base 2). How do we convert that to $\log_{10}$? That is, what's the relationship between $\log_b k$ and $\log_x k$?

Let's do some math. Assume $c = \log_b k$ and $y = \log_x k$.

```
log_b k = c --> b^c = k          // This is the definition of log.
log_x(b^c) = log_x k             // Take log of both sides of b^c = k.
c log_x b = log_x k              // Rules of logs. You can move out the exponents.
c = log_b k = log_x k / log_x b  // Dividing above expression and substituting c.
```

Therefore, if we want to convert $\log_2 p$ to $\log_{10}$, we just do this:

$$\log_{10} p = \frac{\log_2 p}{\log_2 10}$$

Takeaway: Logs of different bases are only off by a constant factor. For this reason, we largely ignore what the base of a log within a big O expression. It doesn't matter since we drop constants anyway.

Permutations

How many ways are there of rearranging a string of n unique characters? Well, you have n options for what to put in the first characters, then n - 1 options for what to put in the second slot (one option is taken), then n - 2 options for what to put in the third slot, and so on. Therefore, the total number of strings is n!.

$$n! = \underline{n} * \underline{n-1} * \underline{n-2} * \underline{n-3} * \ldots * \underline{1}$$

What if you were forming a k-length string (with all unique characters) from n total unique characters? You can follow similar logic, but you'd just stop your selection/multiplication earlier.

$$\frac{n!}{(n-k)!} = \underline{n} * \underline{n-1} * \underline{n-2} * \underline{n-3} * \ldots * \underline{n-k+1}$$

Combinations

Suppose you have a set of n distinct characters. How many ways are there of selecting k characters into a new set (where order doesn't matter)? That is, how many k-sized subsets are there out of n distinct elements? This is what the expression n-choose-k means, which is often written $\binom{n}{k}$.

Imagine we made a list of all the sets by first writing all k-length substrings and then taking out the duplicates.

From the above *Permutations* section, we'd have $n!/(n-k)!$ k-length substrings.

Since each k-sized subset can be rearranged k! unique ways into a string, each subset will be duplicated k! times in this list of substrings. Therefore, we need to divide by k! to take out these duplicates.

$$\binom{n}{k} = \frac{1}{k!} * \frac{n!}{(n-k)!} = \frac{n!}{k!(n-k)!}$$

Proof by Induction

Induction is a way of proving something to be true. It is closely related to recursion. It takes the following form.

Task: Prove statement P(k) is true for all k >= b.

- Base Case: Prove the statement is true for P(b). This is usually just a matter of plugging in numbers.
- Assumption: Assume the statement is true for P(n).
- Inductive Step: Prove that *if* the statement is true for P(n), then it's true for P(n+1).

This is like dominoes. If the first domino falls, and one domino always knocks over the next one, then all the dominoes must fall.

Let's use this to prove that there are 2^n subsets of an n-element set.

- Definitions: let $S = \{a_1, a_2, a_3, \ldots, a_n\}$ be the n-element set.

- Base case: Prove there are 2^0 subsets of `{}`. This is true, since the only subset of `{}` is `{}`.
- Assume that there are 2^n subsets of $\{a_1, a_2, a_3, \ldots, a_n\}$.
- Prove that there are 2^{n+1} subsets of $\{a_1, a_2, a_3, \ldots, a_{n+1}\}$.

  Consider the subsets of $\{a_1, a_2, a_3, \ldots, a_{n+1}\}$. Exactly half will contain a_{n+1} and half will not.

  The subsets that do not contain a_{n+1} are just the subsets of $\{a_1, a_2, a_3, \ldots, a_n\}$. We assumed there are 2^n of those.

  Since we have the same number of subsets with x as without x, there are 2^n subsets with a_{n+1}.

  Therefore, we have $2^n + 2^n$ subsets, which is 2^{n+1}.

Many recursive algorithms can be proved valid with induction.

Topological Sort

A topological sort of a directed graph is a way of ordering the list of nodes such that if `(a, b)` is an edge in the graph then a will appear before b in the list. If a graph has cycles or is not directed, then there is no topological sort.

There are a number of applications for this. For example, suppose the graph represents parts on an assembly line. The edge (Handle, Door) indicates that you need to assemble the handle before the door. The topological sort would offer a valid ordering for the assembly line.

We can construct a topological sort with the following approach.

1. Identify all nodes with no incoming edges and add those nodes to our topological sort.
   - We know those nodes are safe to add first since they have nothing that needs to come before them. Might as well get them over with!
   - We know that such a node must exist if there's no cycle. After all, if we picked an arbitrary node we could just walk edges backwards arbitrarily. We'll either stop at some point (in which case we've found a node with no incoming edges) or we'll return to a prior node (in which case there is a cycle).
2. When we do the above, remove each node's outbound edges from the graph.
   - Those nodes have already been added to the topological sort, so they're basically irrelevant. We can't violate those edges anymore.
3. Repeat the above, adding nodes with no incoming edges and removing their outbound edges. When all the nodes have been added to the topological sort, then we are done.

More formally, the algorithm is this:

1. Create a queue `order`, which will eventually store the valid topological sort. It is currently empty.
2. Create a queue `processNext`. This queue will store the next nodes to process.
3. Count the number of incoming edges of each node and set a class variable `node.inbound`. Nodes typically only store their outgoing edges. However, you can count the inbound edges by walking through each node n and, for each of its outgoing edges `(n, x)`, incrementing `x.inbound`.
4. Walk through the nodes again and add to `processNext` any node where `x.inbound == 0`.
5. While `processNext` is not empty, do the following:
   - Remove first node n from `processNext`.

   - For each edge (n, x), decrement x.inbound. If x.inbound == 0, append x to processNext.
   - Append n to order.
6. If order contains all the nodes, then it has succeeded. Otherwise, the topological sort has failed due to a cycle.

This algorithm does sometimes come up in interview questions. Your interviewer probably wouldn't expect you to know it offhand. However, it would be reasonable to have you derive it even if you've never seen it before.

Dijkstra's Algorithm

In some graphs, we might want to have edges with weights. If the graph represented cities, each edge might represent a road and its weight might represent the travel time. In this case, we might want to ask, just as your GPS mapping system does, what's the shortest path from your current location to another point p? This is where Dijksta's algorithm comes in.

Dijkstra's algorithm is a way to find the shortest path between two points in a weighted directed graph (which might have cycles). All edges must have positive values.

Rather than just stating what Dijkstra's algorithm is, let's try to derive it. Consider the earlier described graph. We could find the shortest path from s to t by literally taking all possible routes using actual time. (Oh, and we'll need a machine to clone ourselves.)

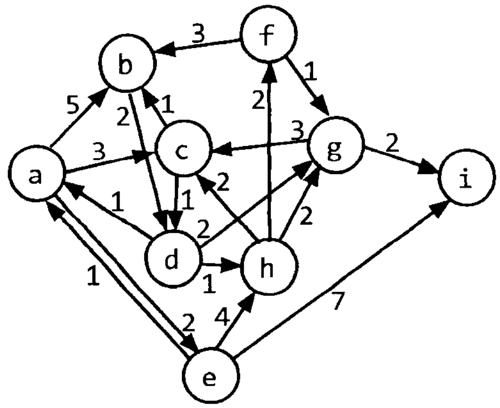
1. Start off at s.
2. For each of s's outbound edges, clone ourselves and start walking. If the edge (s, x) has weight 5, we should actually take 5 minutes to get there.
3. Each time we get to a node, check if anyone's been there before. If so, then just stop. We're automatically not as fast as another path since someone beat us here from s. If no one has been here before, then clone ourselves and head out in all possible directions.
4. The first one to get to t wins.

This works just fine. But, of course, in the real algorithm we don't want to literally use a timer to find the shortest path.

Imagine that each clone could jump immediately from one node to its adjacent nodes (regardless of the edge weight), but it kept a time_so_far log of how long its path would have taken if it did walk at the "true" speed. Additionally, only one person moves at a time, and it's always the one with the lowest time_so_far. This is sort of how Dijkstra's algorithm works.

Dijkstra's algorithm finds the minimum weight path from a start node s to *every* node on the graph.

Consider the following graph.



Assume we are trying to find the shortest path from a to i. We'll use Dijkstra's algorithm to find the shortest path from a to all other nodes, from which we will clearly have the shortest path from a to i.

We first initialize several variables:

- path_weight[node]: maps from each node to the total weight of the shortest path. All values are initialized to infinity, except for path_weight[a] which is initialized to 0.
- previous[node]: maps from each node to the previous node in the (current) shortest path.
- remaining: a priority queue of all nodes in the graph, where each node's priority is defined by its path_weight.

Once we've initialized these values, we can start adjusting the values of path_weight.

> A (min) **priority queue** is an abstract data type that—at least in this case—supports insertion of an object and key, removing the object with the minimum key, and decreasing a key. (Think of it like a typical queue, except that, instead of removing the oldest item, it removes the item with the lowest or highest priority.) It is an abstract data type because it is defined by its behavior (its operations). Its underlying implementation can vary. You could implement a priority queue with an array or a min (or max) heap (or many other data structures).

We iterate through the nodes in remaining (until remaining is empty), doing the following:

1. Select the node in remaining with the lowest value in path_weight. Call this node n.
2. For each adjacent node, compare path_weight[x] (which is the weight of the current shortest path from a to x) to path_weight[n] + edge_weight[(n, x)]. That is, could we get a path from a to x with lower weight by going through n instead of our current path? If so, update path_weight and previous.
3. Remove n from remaining.

When remaining is empty, then path_weight stores the weight of the current shortest path from a to each node. We can reconstruct this path by tracing through previous.

Let's walk through this on the above graph.

1. The first value of n is a. We look at its adjacent nodes (b, c, and e), update the values of path_weight (to 5, 3, and 2) and previous (to a) and then remove a from remaining.
2. Then, we go to the next smallest node, which is e. We previously updated path_weight[e] to be 2. Its adjacent nodes are h and i, so we update path_weight (to 6 and 9) and previous for both of those.

Observe that 6 is `path_weight[e]` (which is 2) + the weight of the edge `(e, h)` (which is 4).

3. The next smallest node is `c`, which has `path_weight` 3. Its adjacent nodes are b and d. The value of `path_weight[d]` is infinity, so we update it to 4 (which is `path_weight[c] + weight(edge c, d)`). The value of `path_weight[b]` has been previously set to 5. However, since `path_weight[c] + weight(edge c, b)` (which is 3 + 1 = 4) is less than 5, we update `path_weight[b]` to 4 and `previous` to `c`. This indicates that we would improve the path from `a` to `b` by going through `c`.

We continue doing this until `remaining` is empty. The following diagram shows the changes to the `path_weight` (left) and `previous` (right) at each step. The topmost row shows the current value for n (the node we are removing from `remaining`). We black out a row after it has been removed from `remaining`.

	INITIAL		n=a		n=e		n=c		n=b		n=d		n=h		n=g		n=f		FINAL	
	wt	pr	wt	pr	wt	pr	wt	pr	wt	pr	wt	pr	wt	pr	wt	pr	wt	pr	wt	pr
a	0	-	removed																0	-
b	∞	-	5	a			4	c	removed										4	c
c	∞	-	3	a			removed												3	a
d	∞	-					4	c			removed								4	c
e	∞	-	2	a	removed														2	a
f	∞	-											7	h			removed		7	h
g	∞	-									6	d			removed				6	d
h	∞	-			6	e					5	d	removed						5	d
i	∞	-	∞	-	9	e									8	g			8	g

Once we're done, we can follow this chart backwards, starting at `i` to find the actual path. In this case, the smallest weight path has weight 8 and is `a -> c -> d -> g -> i`.

Priority Queue and Runtime

As mentioned earlier, our algorithm used a priority queue, but this data structure can be implemented in different ways.

The runtime of this algorithm depends heavily on the implementation of the priority queue. Assume you have v vertices and e nodes.

- If you implemented the priority queue with an array, then you would call `remove_min` up to v times. Each operation would take $O(v)$ time, so you'd spend $O(v^2)$ time in the `remove_min` calls. Additionally, you would update the values of `path_weight` and `previous` at most once per edge, so that's $O(e)$ time doing those updates. Observe that e must be less than of equal to v^2 since you can't have more edges than there are pairs of vertices. Therefore, the total runtime is $O(v^2)$.
- If you implemented the priority queue with a min heap, then the `remove_min` calls will each take $O(\log v)$ time (as will inserting and updating a key). We will do one `remove_min` call for each vertex, so that's $O(v \log v)$ (v vertices at $O(\log v)$ time each). Additionally, on each edge, we might call one update key or insert operation, so that's $O(e \log v)$. The total runtime is $O((v + e) \log v)$.

Which one is better? Well, that depends. If the graph has a lot of edges, then v^2 will be close to e. In this case, you might be better off with the array implementation, as $O(v^2)$ is better than $O((v + v^2) \log v)$. However, if the graph is sparse, then e is much less than v^2. In this case, the min heap implementation may be better.

Hash Table Collision Resolution

Essentially any hash table can have collisions. There are a number of ways of handling this.

Chaining with Linked Lists

With this approach (which is the most common), the hash table's array maps to a linked list of items. We just add items to this linked list. As long as the number of collisions is fairly small, this will be quite efficient.

In the worst case, lookup is `O(n)`, where `n` is the number of elements in the hash table. This would only happen with either some very strange data or a very poor hash function (or both).

Chaining with Binary Search Trees

Rather than storing collisions in a linked list, we could store collisions in a binary search tree. This will bring the worst-case runtime to `O(log n)`.

In practice, we would rarely take this approach unless we expected an extremely nonuniform distribution.

Open Addressing with Linear Probing

In this approach, when a collision occurs (there is already an item stored at the designated index), we just move on to the next index in the array until we find an open spot. (Or, sometimes, some other fixed distance, like the `index + 5`.)

If the number of collisions is low, this is a very fast and space-efficient solution.

One obvious drawback of this is that the total number of entries in the hash table is limited by the size of the array. This is not the case with chaining.

There's another issue here. Consider a hash table with an underlying array of size 100 where indexes 20 through 29 are filled (and nothing else). What are the odds of the next insertion going to index 30? The odds are 10% because an item mapped to any index between 20 and 30 will wind up at index 30. This causes an issue called *clustering*.

Quadratic Probing and Double Hashing

The distance between probes does not need to be linear. You could, for example, increase the probe distance quadratically. Or, you could use a second hash function to determine the probe distance.

Rabin-Karp Substring Search

The brute force way to search for a substring S in a larger string B takes `O(s(b-s))` time, where `s` is the length of S and `b` is the length of B. We do this by searching through the first `b - s + 1` characters in B and, for each, checking if the next `s` characters match S.

The Rabin-Karp algorithm optimizes this with a little trick: if two strings are the same, they must have the same hash value. (The converse, however, is not true. Two different strings can have the same hash value.)

Therefore, if we efficiently precompute a hash value for each sequence of `s` characters within B, we can find the locations of S in `O(b)` time. We then just need to validate that those locations really do match S.

For example, imagine our hash function was simply the sum of each character (where space = 0, a = 1, b = 2, and so on). If S is `ear` and B = `doe are hearing me`, we'd then just be looking for sequences where the sum is 24 (e + a + r). This happens three times. For each of those locations, we'd check if the string really is `ear`.

char:	d	o	e		a	r	e		h	e	a	r	i	n	g		m	e
code:	4	15	5	0	1	18	5	0	8	5	1	18	9	14	7	0	13	5
sum of next 3:	24	20	6	19	24	23	13	13	14	24	28	41	30	21	20	18		

If we computed these sums by doing `hash('doe')`, then `hash('oe ')`, then `hash('e a')`, and so on, we would still be at `O(s(b-s))` time.

Instead, we compute the hash values by recognizing that `hash('oe ') = hash('doe') - code('d') + code(' ')`. This takes `O(b)` time to compute all the hashes.

You might argue that, still, in the worst case this will take `O(s(b-s))` time since many of the hash values could match. That's absolutely true—for this hash function.

In practice, we would use a better *rolling hash function*, such as the Rabin fingerprint. This essentially treats a string like doe as a base 128 (or however many characters are in our alphabet) number.

```
hash('doe') = code('d') * 128² + code('o') * 128¹ + code('e') * 128⁰
```

This hash function will allow us to remove the d, shift the o and e, and then add in the space.

```
hash('oe ') = (hash('doe') - code('d') * 128²) * 128 + code(' ')
```

This will considerably cut down on the number of false matches. Using a good hash function like this will give us expected time complexity of `O(s + b)`, although the worst case is `O(sb)`.

Usage of this algorithm comes up fairly frequently in interviews, so it's useful to know that you can identify substrings in linear time.

AVL Trees

An AVL tree is one of two common ways to implement tree balancing. We will only discuss insertions here, but you can look up deletions separately if you're interested.

Properties

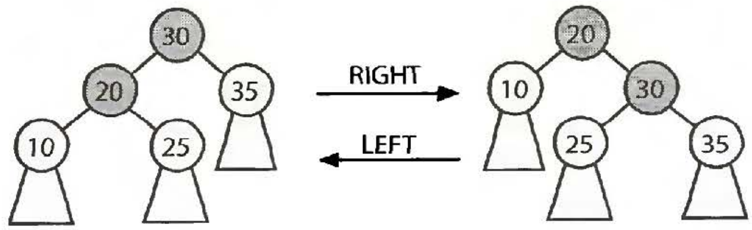
An AVL tree stores in each node the height of the subtrees rooted at this node. Then, for any node, we can check if it is height balanced: that the height of the left subtree and the height of the right subtree differ by no more than one. This prevents situations where the tree gets too lopsided.

```
balance(n) = n.left.height - n.right.height
-1 <= balance(n) <= 1
```

Inserts

When you insert a node, the balance of some nodes might change to -2 or 2. Therefore, when we "unwind" the recursive stack, we check and fix the balance at each node. We do this through a series of rotations.

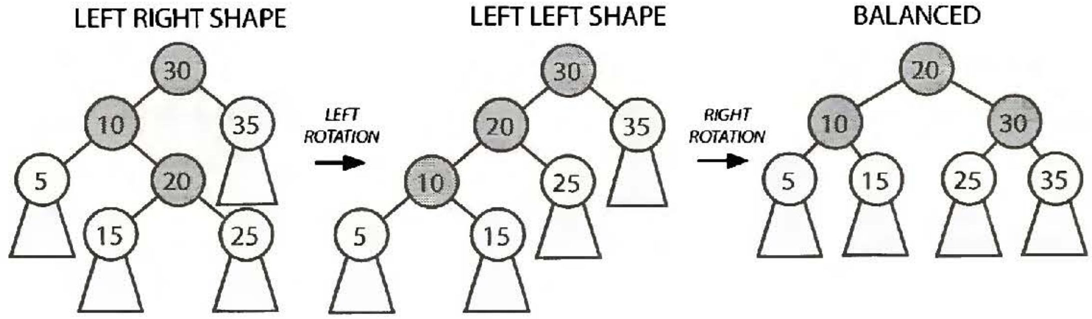
Rotations can be either left or right rotations. The right rotation is an inverse of the left rotation.



Depending on the balance and where the imbalance occurs, we fix it in a different way.

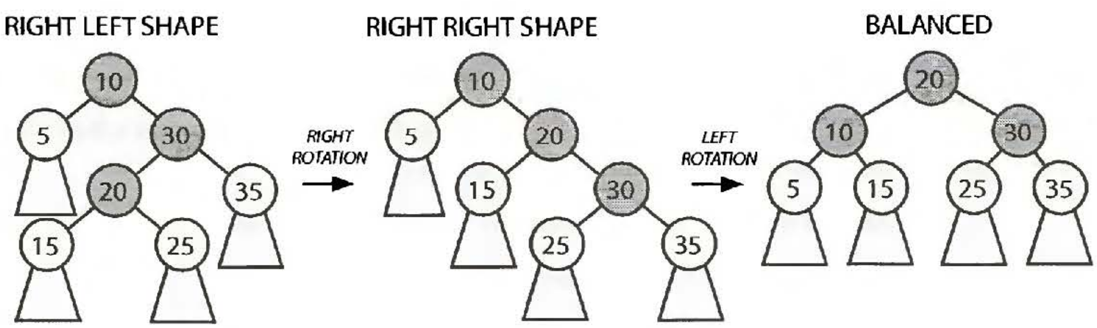
- *Case 1: Balance is 2.*

  In this case, the left's height is two bigger than the right's height. If the left side is larger, the left subtree's extra nodes must be hanging to the left (as in LEFT LEFT SHAPE) or hanging to the right (as in LEFT RIGHT SHAPE). If it looks like the LEFT RIGHT SHAPE, transform it with the rotations below into the LEFT LEFT SHAPE then into BALANCED. If it looks like the LEFT LEFT SHAPE already, just transform it into BALANCED.



- *Case 2: Balance is -2.*

  This case is the mirror image of the prior case. The tree will look like either the RIGHT LEFT SHAPE or the RIGHT RIGHT SHAPE. Perform the rotations below to transform it into BALANCED.



In both cases, "balanced" just means that the `balance` of the tree is between -1 and 1. It does not mean that the `balance` is 0.

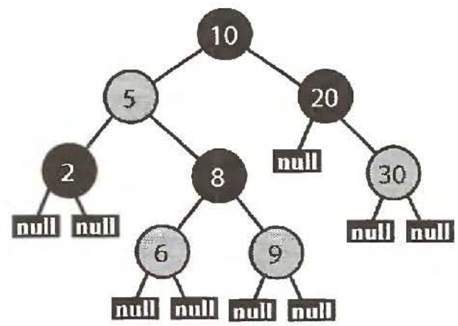
We recurse up the tree, fixing any imbalances. If we ever achieve a balance of 0 on a subtree, then we know that we have completed all the balances. This portion of the tree will not cause another, higher subtree to have a balance of -2 or 2. If we were doing this non-recursively, then we could break from the loop.

▸ Red-Black Trees

Red-black trees (a type of self-balancing binary search tree) do not ensure quite as strict balancing, but the balancing is still good enough to ensure O(log N) insertions, deletions, and retrievals. They require a bit less memory and can rebalance faster (which means faster insertions and removals), so they are often used in situations where the tree will be modified frequently.

Red-black trees operate by enforcing a quasi-alternating red and black coloring (under certain rules, described below) and then requiring every path from a node to its leaves to have the same number of black nodes. Doing so leads to a reasonably balanced tree.

The tree below is a red-black tree (where the red nodes are indicated with gray):



Properties

1. Every node is either red or black.
2. The root is black.
3. The leaves, which are NULL nodes, are considered black.
4. Every red node must have two black children. That is, a red node cannot have red children (although a black node can have black children).
5. Every path from a node to its leaves must have the same number of black children.

Why It Balances

Property #4 means that two red nodes cannot be adjacent in a path (e.g., parent and child). Therefore, no more than half the nodes in a path can be red.

Consider two paths from a node (say, the root) to its leaves. The paths must have the same number of black nodes (property #5), so let's assume that their red node counts are as different as possible: one path contains the minimum number of red nodes and the other one contains the maximum number.

- Path 1 (Min Red): The minimum number of red nodes is zero. Therefore, path 1 has b nodes total.
- Path 2 (Max Red): The maximum number of red nodes is b, since red nodes must have black children and there are b black nodes. Therefore, path 2 has 2b nodes total.

Therefore, even in the most extreme case, the lengths of paths cannot differ by more than a factor of two. That's good enough to ensure an O(log N) find and insert runtime.

If we can maintain these properties, we'll have a (sufficiently) balanced tree—good enough to ensure O(log N) insert and find, anyway. The question then is how to maintain these properties efficiently. We'll only discuss insertion here, but you can look up deletion on your own.

Insertion

Inserting a new node into a red-black tree starts off with a typical binary search tree insertion.

- New nodes are inserted at a leaf, which means that they replace a black node.
- New nodes are always colored red and are given two black leaf (NULL) nodes.

Once we've done that, we fix any resulting red-black property violations. We have two possible violations:

- Red violations: A red node has a red child (or the root is red).
- Black violations: One path has more blacks than another path.

The node inserted is red. We didn't change the number of black nodes on any path to a leaf, so we know that we won't have a black violation. However, we might have a red violation.

In the special case that where the root is red, we can always just turn it black to satisfy property 2, without violating the other constraints.

Otherwise, if there's a red violation, then this means that we have a red node under another red node. Oops!

Let's call N the current node. P is N's parent. G is N's grandparent. U is N's uncle and P's sibling. We know that:

- N is red and P is red, since we have a red violation.
- G is definitely black, since we didn't *previously* have a red violation.

The unknown parts are:

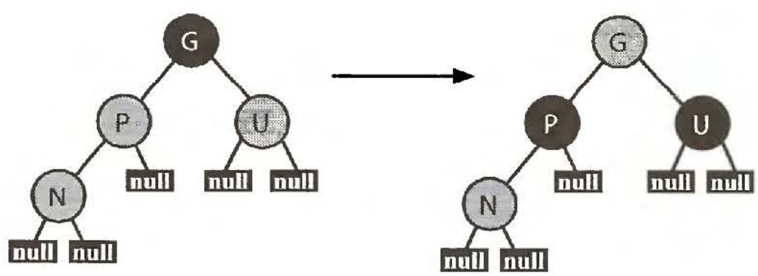
- U could be either red or black.
- U could be either a left or right child.
- N could be either a left or right child.

By simple combinatorics, that's eight cases to consider. Fortunately some of these cases will be equivalent.

- **Case 1: U is red.**

  It doesn't matter whether U is a left or right child, nor whether P is a left or right child. We can merge four of our eight cases into one.

  If U is red, we can just toggle the colors of P, U, and G. Flip G from black to red. Flip P and U from red to black. We haven't changed the number of black nodes in any path.



  However, by making G red, we might have created a red violation with G's parent. If so, we recursively apply the full logic to handle a red violation, where this G becomes the new N.

  Note that in the general recursive case, N, P, and U may also have subtrees in place of each black NULL (the leaves shown). In Case 1, these subtrees stay attached to the same parents, as the tree structure remains unchanged.

- **Case 2: U is black.**

  We'll need to consider the configurations (left vs. right child) of N and U. In each case, our goal is to fix up the red violation (red on top of red) without:

  » Messing up the ordering of the binary search tree.

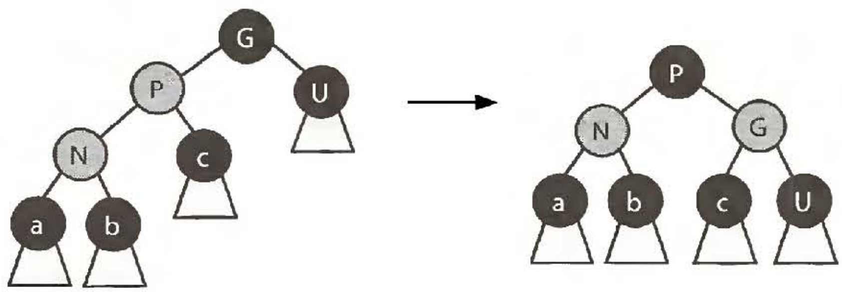
  » Introducing a black violation (more black nodes on one path than another).

  If we can do this, we're good. In each of the cases below, the red violation is fixed with rotations that maintain the node ordering.

  Further, the below rotations maintain the exact number of black nodes in each path through the affected portion of the tree that were in place beforehand. The children of the rotating section are either NULL leaves or subtrees that remain internally unchanged.

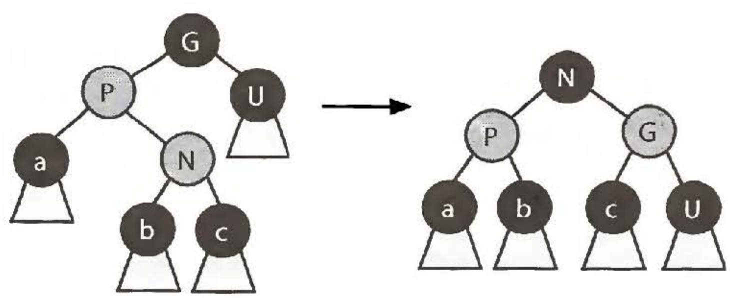
  Case A: N and P are both left children.

  We resolve the red violation with the rotation of N, P, and G and the associated recoloring shown below. If you picture the in-order traversal, you can see the rotation maintains the node ordering (`a <= N <= b <= P <= c <= G <= U`). The tree maintains the same, equal number of black nodes in the path down to each subtree a, b, c, and U (which may all be NULL).



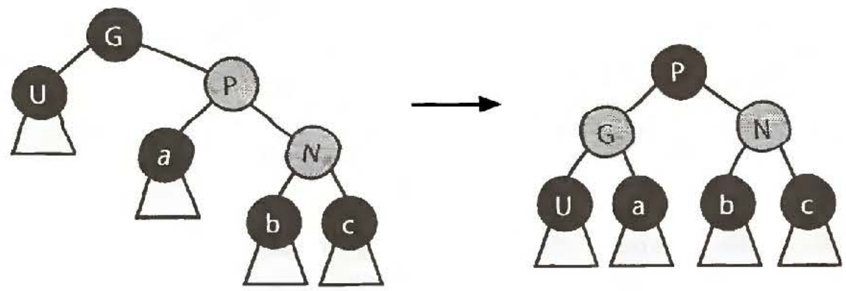
  Case B: P is a left child, and N is a right child.

  The rotations in Case B resolve the red violation and maintain the in-order property: `a <= P <= b <= N <= c <= G <= U`. Again, the count of the black nodes remains constant in each path down to the leaves (or subtrees).



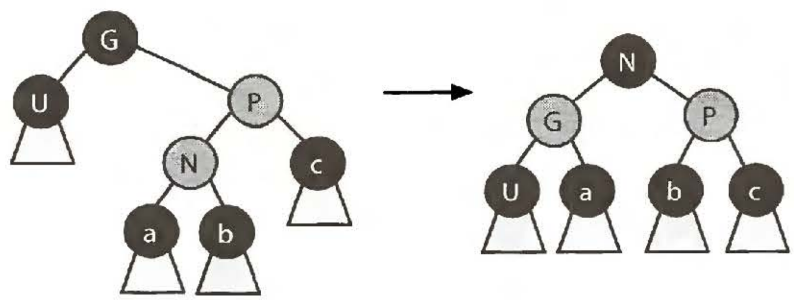
Case C: N and P are both right children.

This is a mirror image of case A.



Case D: N is a left child, and P is a right child.

This is a mirror image of case B.



In each of Case 2's subcases, the middle element by value of N, P, and G is rotated to become the root of what was G's subtree, and that element and G swap colors.

That said, do not try to just memorize these cases. Rather, study why they work. How does each one ensure no red violations, no black violations, and no violations of the binary search tree property?

▸ MapReduce

MapReduce is used widely in system design to process large amounts of data. As its name suggests, a MapReduce program requires you to write a Map step and a Reduce step. The rest is handled by the system.

1. The system splits up the data across different machines.
2. Each machine starts running the user-provided Map program.
3. The Map program takes some data and emits a `<key, value>` pair.
4. The system-provided `Shuffle` process reorganizes the data so that all <key, value> pairs associated with a given key go to the same machine, to be processed by `Reduce`.
5. The user-provided `Reduce` program takes a key and a set of associated values and "reduces" them in some way, emitting a new key and value. The results of this might be fed back into the `Reduce` program for more reducing.

The typical example of using MapReduce—basically the "Hello World" of MapReduce—is counting the frequency of words within a set of documents.

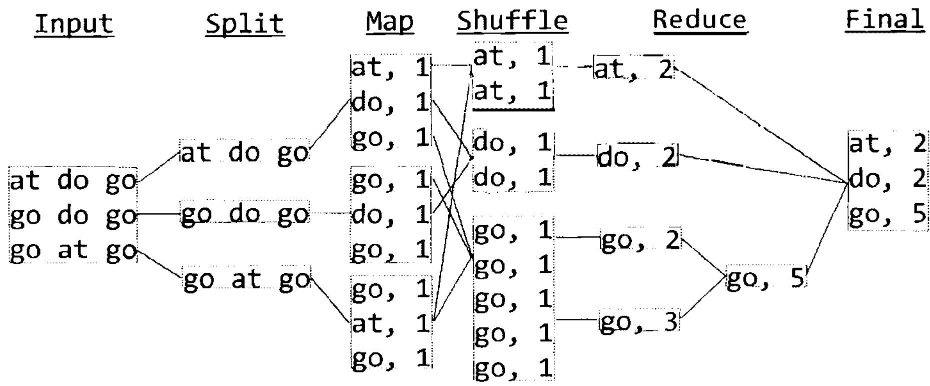
Of course, you could write this as a single function that reads in all the data, counts the number of times each word appears via a hash table, and then outputs the result.

MapReduce allows you to process the document in parallel. The Map function reads in a document and emits just each individual word and the count (which is always 1). The Reduce function reads in keys (words) and associated values (counts). It emits the sum of the counts. This sum could possibly wind up as input for another call to Reduce on the same key (as shown in the diagram).

```
void map(String name, String document):
  for each word w in document:
    emit(w, 1)

void reduce(String word, Iterator partialCounts):
  int sum = 0
  for each count in partialCounts:
    sum += count
  emit(word, sum)
```

The diagram below shows how this might work on this example.



Here's another example: You have a list of data in the form {City, Temperature, Date}. Calculate the average temperature in each city every year. For example {(2012, Philadelphia, 58.2), (2011, Philadelphia, 56.6), (2012, Seattle, 45.1)}.

- **Map**: The Map step outputs a key value pair where the key is `City_Year` and the value is `(Temperature, 1)`. The '1' reflects that this is the average temperature out of one data point. This will be important for the Reduce step.
- **Reduce**: The Reduce step will be given a list of temperatures that correspond with a particular city and year. It must use these to compute the average temperature for this input. You cannot simply add up the temperatures and divide by the number of values.

  To see this, imagine we have five data points for a particular city and year: 25, 100, 75, 85, 50. The Reduce step might only get some of this data at once. If you averaged {75, 85} you would get 80. This might end up being input for another Reduce step with 50, and it would be a mistake to just naively average 80 and 50. The 80 has more weight.

  Therefore, our Reduce step instead takes in {(80, 2), (50, 1)}, then sums the *weighted* temperatures. So it does 80 * 2 + 50 * 1 and then divides by (2 + 1) to get an average temperature of 70. It then emits (70, 3).

  Another Reduce step might reduce {(25, 1), (100, 1)} to get (62.5, 2). If we reduce this with (70, 3) we get the final answer: (67, 5). In other words, the average temperature in this city for this year was 67 degrees.

We could do this in other ways, too. We could have just the city as the key, and the value be (Year, Temperature, Count). The Reduce step would do essentially the same thing, but would have to group by Year itself.

In many cases, it's useful to think about what the Reduce step should do first, and then design the Map step around that. What data does Reduce need to have to do its job?

Additional Studying

So, you've mastered this material and you want to learn even more? Okay. Here are some topics to get you started:

- **Bellman-Ford Algorithm**: Finds the shortest paths from a single node in a weighted directed graph with positive and negative edges.
- **Floyd-Warshall Algorithm**: Finds the shortest paths in a weighted graph with positive or negative weight edges (but no negative weight cycles).
- **Minimum Spanning Trees**: In a weighted, connected, undirected graph, a spanning tree is a tree that connects all the vertices. The minimum spanning tree is the spanning tree with minimum weight. There are various algorithms to do this.
- **B-Trees:** A self-balancing search tree (not a binary search tree) that is commonly used on disks or other storage devices. It is similar to a red-black tree, but uses fewer I/O operations.
- **A***: Find the least-cost path between a source node and a goal node (or one of several goal nodes). It extends Dijkstra's algorithm and achieves better performance by using heuristics.
- **Interval Trees**: An extension of a balanced binary search tree, but storing intervals (low -> high ranges) instead of simple values. A hotel could use this to store a list of all reservations and then efficiently detect who is staying at the hotel at a particular time.
- **Graph coloring**: A way of coloring the nodes in a graph such that no two adjacent vertices have the same color. There are various algorithms to do things like determine if a graph can be colored with only K colors.
- **P, NP, and NP-Complete**: P, NP, and NP-Complete refer to classes of problems. P problems are problems that can be quickly solved (where "quickly" means polynomial time). NP problems are those where, given a solution, the solution can be quickly verified. NP-Complete problems are a subset of NP problems that can all be reduced to each other (that is, if you found a solution to one problem, you could tweak the solution to solve other problems in the set in polynomial time).

  It is an open (and very famous) question whether P = NP, but the answer is generally believed to be no.
- **Combinatorics and Probability**: There are various things you can learn about here, such as random variables, expected value, and n-choose-k.
- **Bipartite Graph**: A bipartite graph is a graph where you can divide its nodes into two sets such that every edge stretches across the two sets (that is, there is never an edge between two nodes in the same set). There is an algorithm to check if a graph is a bipartite graph. Note that a bipartite graph is equivalent to a graph that can be colored with two colors.
- **Regular Expressions**: You should know that regular expressions exist and what they can be used for (roughly). You can also learn about how an algorithm to match regular expressions would work. Some of the basic syntax behind regular expressions could be useful as well.

There is of course a great deal more to data structures and algorithms. If you're interested in exploring these topics more deeply, I recommend picking up the hefty *Introduction to Algorithms* ("CLRS" by Cormen, Leiserson, Rivest and Stein) or *The Algorithm Design Manual* (by Steven Skiena).

XII Code Library

Certain patterns came up while implementing the code for this book. We've tried to generally include the full code for a solution with the solution, but in some cases it got quite redundant.

This appendix provides the code for a few of the most useful chunks of code.

The complete compilable solutions can be downloaded from CrackingTheCodingInterview.com.

XI

Code Library

Certain patterns came up while implementing the code for this book. We've tried to generally include the full code for a solution with the solution, but in some cases it got quite redundant.

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▸ HashMapList<T, E>

The `HashMapList` class is essentially shorthand for `HashMap<T, ArrayList<E>>`. It allows us to map from an item of type of T to an `ArrayList` of type E.

For example, we might want a data structure that maps from an integer to a list of strings. Ordinarily, we'd have to write something like this:

```
HashMap<Integer, ArrayList<String>> maplist =
  new HashMap<Integer, ArrayList<String>>();
for (String s : strings) {
  int key = computeValue(s);
  if (!maplist.containsKey(key)) {
    maplist.put(key, new ArrayList<String>());
  }
  maplist.get(key).add(s);
}
```

Now, we can just write this:

```
HashMapList<Integer, String> maplist = new HashMapList<Integer, String>();
for (String s : strings) {
  int key = computeValue(s);
  maplist.put(key, s);
}
```

It's not a big change, but it makes our code a bit simpler.

```
public class HashMapList<T, E> {
  private HashMap<T, ArrayList<E>> map = new HashMap<T, ArrayList<E>>();

  /* Insert item into list at key. */
  public void put(T key, E item) {
    if (!map.containsKey(key)) {
      map.put(key, new ArrayList<E>());
    }
    map.get(key).add(item);
```

```
  }

  /* Insert list of items at key. */
  public void put(T key, ArrayList<E> items) {
    map.put(key, items);
  }

  /* Get list of items at key. */
  public ArrayList<E> get(T key) {
    return map.get(key);
  }

  /* Check if hashmaplist contains key. */
  public boolean containsKey(T key) {
    return map.containsKey(key);
  }

  /* Check if list at key contains value. */
  public boolean containsKeyValue(T key, E value) {
    ArrayList<E> list = get(key);
    if (list == null) return false;
    return list.contains(value);
  }

  /* Get the list of keys. */
  public Set<T> keySet() {
    return map.keySet();
  }

  @Override
  public String toString() {
    return map.toString();
  }
}
```

▸ TreeNode (Binary Search Tree)

While it's perfectly fine—even good—to use the built-in binary tree class when possible, it's not always possible. In many questions, we needed access to the internals of the node or tree class (or needed to tweak these) and thus couldn't use the built-in libraries.

The TreeNode class supports a variety of functionality, much of which we wouldn't necessarily want for every question/solution. For example, the TreeNode class tracks the parent of the node, even though we often don't use it (or specifically ban using it).

For simplicity, we'd implemented this tree as storing integers for data.

```
public class TreeNode {
  public int data;
  public TreeNode left, right, parent;
  private int size = 0;

  public TreeNode(int d) {
    data = d;
    size = 1;
  }
```

```
  public void insertInOrder(int d) {
    if (d <= data) {
      if (left == null) {
        setLeftChild(new TreeNode(d));
      } else {
        left.insertInOrder(d);
      }
    } else {
      if (right == null) {
        setRightChild(new TreeNode(d));
      } else {
        right.insertInOrder(d);
      }
    }
    size++;
  }

  public int size() {
    return size;
  }

  public TreeNode find(int d) {
    if (d == data) {
      return this;
    } else if (d <= data) {
      return left != null ? left.find(d) : null;
    } else if (d > data) {
      return right != null ? right.find(d) : null;
    }
    return null;
  }

  public void setLeftChild(TreeNode left) {
    this.left = left;
    if (left != null) {
      left.parent = this;
    }
  }

  public void setRightChild(TreeNode right) {
    this.right = right;
    if (right != null) {
      right.parent = this;
    }
  }

}
```

This tree is implemented to be a binary search tree. However, you can use it for other purposes. You would just need to use the `setLeftChild`/`setRightChild` methods, or the `left` and `right` child variables. For this reason, we have kept these methods and variables `public`. We need this sort of access for many problems.

▸ LinkedListNode (Linked List)

Like the TreeNode class, we often needed access to the internals of a linked list in a way that the built-in linked list class wouldn't support. For this reason, we implemented our own class and used it for many problems.

```
public class LinkedListNode {
  public LinkedListNode next, prev, last;
  public int data;
  public LinkedListNode(int d, LinkedListNode n, LinkedListNode p){
    data = d;
    setNext(n);
    setPrevious(p);
  }

  public LinkedListNode(int d) {
    data = d;
  }

  public LinkedListNode() { }

  public void setNext(LinkedListNode n) {
    next = n;
    if (this == last) {
      last = n;
    }
    if (n != null && n.prev != this) {
      n.setPrevious(this);
    }
  }

  public void setPrevious(LinkedListNode p) {
    prev = p;
    if (p != null && p.next != this) {
      p.setNext(this);
    }
  }

  public LinkedListNode clone() {
    LinkedListNode next2 = null;
    if (next != null) {
      next2 = next.clone();
    }
    LinkedListNode head2 = new LinkedListNode(data, next2, null);
    return head2;
  }
}
```

Again, we've kept the methods and variables public because we often needed this access. This would allow the user to "destroy" the linked list, but we actually needed this sort of functionality for our purposes.

▸ Trie & TrieNode

The trie data structure is used in a few problems to make it easier to look up if a word is a prefix of any other words in a dictionary (or list of valid words). This is often used when we're recursively building words so that we can short circuit when the word is not valid.

```
public class Trie {
  // The root of this trie.
  private TrieNode root;

  /* Takes a list of strings as an argument, and constructs a trie that stores
   * these strings. */
  public Trie(ArrayList<String> list) {
    root = new TrieNode();
    for (String word : list) {
      root.addWord(word);
    }
  }


  /* Takes a list of strings as an argument, and constructs a trie that stores
   * these strings. */
  public Trie(String[] list) {
    root = new TrieNode();
    for (String word : list) {
      root.addWord(word);
    }
  }

  /* Checks whether this trie contains a string with the prefix passed in as
   * argument. */
  public boolean contains(String prefix, boolean exact) {
    TrieNode lastNode = root;
    int i = 0;
    for (i = 0; i < prefix.length(); i++) {
      lastNode = lastNode.getChild(prefix.charAt(i));
      if (lastNode == null) {
        return false;
      }
    }
    return !exact || lastNode.terminates();
  }

  public boolean contains(String prefix) {
    return contains(prefix, false);
  }

  public TrieNode getRoot() {
    return root;
  }
}
```

The Trie class uses the TrieNode class, which is implemented below.

```
public class TrieNode {
  /* The children of this node in the trie.*/
  private HashMap<Character, TrieNode> children;
  private boolean terminates = false;

  /* The character stored in this node as data.*/
  private char character;

  /* Constructs an empty trie node and initializes the list of its children to an
   * empty hash map. Used only to construct the root node of the trie. */
```

```
    public TrieNode() {
      children = new HashMap<Character, TrieNode>();
    }

    /* Constructs a trie node and stores this character as the node's value.
     * Initializes the list of child nodes of this node to an empty hash map. */
    public TrieNode(char character) {
      this();
      this.character = character;
    }

    /* Returns the character data stored in this node. */
    public char getChar() {
      return character;
    }

    /* Add this word to the trie, and recursively create the child
     * nodes. */
    public void addWord(String word) {
      if (word == null || word.isEmpty()) {
        return;
      }

      char firstChar = word.charAt(0);

      TrieNode child = getChild(firstChar);
      if (child == null) {
        child = new TrieNode(firstChar);
        children.put(firstChar, child);
      }

      if (word.length() > 1) {
        child.addWord(word.substring(1));
      } else {
        child.setTerminates(true);
      }
    }

    /* Find a child node of this node that has the char argument as its data. Return
     * null if no such child node is present in the trie. */
    public TrieNode getChild(char c) {
      return children.get(c);
    }

    /* Returns whether this node represents the end of a complete word. */
    public boolean terminates() {
      return terminates;
    }

    /* Set whether this node is the end of a complete word.*/
    public void setTerminates(boolean t) {
      terminates = t;
    }
}
```

Hints

XIII

Interviewers usually don't just hand you a question and expect you to solve it. Rather, they will typically offer guidance when you're stuck, especially on the harder questions. It's impossible to totally simulate the interview experience in a book, but these hints are designed to get you closer.

Try to solve the questions independently when possible. But it's okay to look for some help when you are really struggling. Again, struggling is a normal part of the process.

I've organized the hints somewhat randomly here, such that all the hints for a problem aren't adjacent. This way you won't accidentally see the second hint when you're reading the first hint.

I

Hints for Data Structures

#1.	1.2	Describe what it means for two strings to be permutations of each other. Now, look at that definition you provided. Can you check the strings against that definition?
#2.	3.1	A stack is simply a data structure in which the most recently added elements are removed first. Can you simulate a single stack using an array? Remember that there are many possible solutions, and there are tradeoffs of each.
#3.	2.4	There are many solutions to this problem, most of which are equally optimal in runtime. Some have shorter, cleaner code than others. Can you brainstorm different solutions?
#4.	4.10	If T2 is a subtree of T1, how will its in-order traversal compare to T1's? What about its pre-order and post-order traversal?
#5.	2.6	A palindrome is something which is the same when written forwards and backwards. What if you reversed the linked list?
#6.	4.12	Try simplifying the problem. What if the path had to start at the root?
#7.	2.5	Of course, you could convert the linked lists to integers, compute the sum, and then convert it back to a new linked list. If you did this in an interview, your interviewer would likely accept the answer, and then see if you could do this without converting it to a number and back.
#8.	2.2	What if you knew the linked list size? What is the difference between finding the Kth-to-last element and finding the Xth element?
#9.	2.1	Have you tried a hash table? You should be able to do this in a single pass of the linked list.
#10.	4.8	If each node has a link to its parent, we could leverage the approach from question 2.7 on page 95. However, our interviewer might not let us make this assumption.
#11.	4.10	The in-order traversals won't tell us much. After all, every binary search tree with the same values (regardless of structure) will have the same in-order traversal. This is what in-order traversal means: contents are in-order. (And if it won't work in the specific case of a binary search tree, then it certainly won't work for a general binary tree.) The pre-order traversal, however, is much more indicative.
#12.	3.1	We could simulate three stacks in an array by just allocating the first third of the array to the first stack, the second third to the second stack, and the final third to the third stack. One might actually be much bigger than the others, though. Can we be more flexible with the divisions?

#13. 2.6 Try using a stack.

#14. 4.12 Don't forget that paths could overlap. For example, if you're looking for the sum 6, the paths `1->3->2` and `1->3->2->4->-6->2` are both valid.

#15. 3.5 One way of sorting an array is to iterate through the array and insert each element into a new array in sorted order. Can you do this with a stack?

#16. 4.8 The first common ancestor is the deepest node such that p and q are both descendants. Think about how you might identify this node.

#17. 1.8 If you just cleared the rows and columns as you found 0s, you'd likely wind up clearing the whole matrix. Try finding the cells with zeros first before making any changes to the matrix.

#18. 4.10 You may have concluded that if `T2.preorderTraversal()` is a substring of `T1.preorderTraversal()`, then `T2` is a subtree of `T1`. This is almost true, except that the trees could have duplicate values. Suppose `T1` and `T2` have all duplicate values but different structures. The pre-order traversals will look the same even though `T2` is not a subtree of `T1`. How can you handle situations like this?

#19. 4.2 A minimal binary tree has about the same number of nodes on the left of each node as on the right. Let's focus on just the root for now. How would you ensure that about the same number of nodes are on the left of the root as on the right?

#20. 2.7 You can do this in `O(A+B)` time and `O(1)` additional space. That is, you do not need a hash table (although you could do it with one).

#21. 4.4 Think about the definition of a balanced tree. Can you check that condition for a single node? Can you check it for every node?

#22. 3.6 We could consider keeping a single linked list for dogs and cats, and then iterating through it to find the first dog (or cat). What is the impact of doing this?

#23. 1.5 Start with the easy thing. Can you check each of the conditions separately?

#24. 2.4 Consider that the elements don't have to stay in the same relative order. We only need to ensure that elements less than the pivot must be before elements greater than the pivot. Does that help you come up with more solutions?

#25. 2.2 If you don't know the linked list size, can you compute it? How does this impact the runtime?

#26. 4.7 Build a directed graph representing the dependencies. Each node is a project and an edge exists from A to B if B depends on A (A must be built before B). You can also build it the other way if it's easier for you.

#27. 3.2 Observe that the minimum element doesn't change very often. It only changes when a smaller element is added, or when the smallest element is popped.

#28. 4.8 How would you figure out if p is a descendent of a node n?

#29. 2.6 Assume you have the length of the linked list. Can you implement this recursively?

#30. 2.5 Try recursion. Suppose you have two lists, `A = 1->5->9` (representing 951) and `B = 2->3->6->7` (representing 7632), and a function that operates on the remainder of the lists (`5->9` and `3->6->7`). Could you use this to create the `sum` method? What is the relationship between `sum(1->5->9, 2->3->6->7)` and `sum(5->9, 3->6->7)`?

#31. 4.10 Although the problem seems like it stems from duplicate values, it's really deeper than that. The issue is that the pre-order traversal is the same only because there are null nodes that we skipped over (because they're null). Consider inserting a placeholder value into the pre-order traversal string whenever you reach a null node. Register the null node as a "real" node so that you can distinguish between the different structures.

#32. 3.5 Imagine your secondary stack is sorted. Can you insert elements into it in sorted order? You might need some extra storage. What could you use for extra storage?

#33. 4.4 If you've developed a brute force solution, be careful about its runtime. If you are computing the height of the subtrees for each node, you could have a pretty inefficient algorithm.

#34. 1.9 If a string is a rotation of another, then it's a rotation at a particular point. For example, a rotation of `waterbottle` at character 3 means cutting `waterbottle` at character 3 and putting the right half (`erbottle`) before the left half (`wat`).

#35. 4.5 If you traversed the tree using an in-order traversal and the elements were truly in the right order, does this indicate that the tree is actually in order? What happens for duplicate elements? If duplicate elements are allowed, they must be on a specific side (usually the left).

#36. 4.8 Start with the root. Can you identify if root is the first common ancestor? If it is not, can you identify which side of root the first common ancestor is on?

#37. 4.10 Alternatively, we can handle this problem recursively. Given a specific node within `T1`, can we check to see if its subtree matches `T2`?

#38. 3.1 If you want to allow for flexible divisions, you can shift stacks around. Can you ensure that all available capacity is used?

#39. 4.9 What is the very first value that must be in each array?

#40. 2.1 Without extra space, you'll need $O(N^2)$ time. Try using two pointers, where the second one searches ahead of the first one.

#41. 2.2 Try implementing it recursively. If you could find the (`K-1`)th to last element, can you find the Kth element?

#42. 4.11 Be very careful in this problem to ensure that each node is equally likely and that your solution doesn't slow down the speed of standard binary search tree algorithms (like `insert`, `find`, and `delete`). Also, remember that even if you assume that it's a balanced binary search tree, this doesn't mean that the tree is full/complete/perfect.

#43. 3.5 Keep the secondary stack in sorted order, with the biggest elements on the top. Use the primary stack for additional storage.

#44. 1.1 Try a hash table.

#45. 2.7 Examples will help you. Draw a picture of intersecting linked lists and two equivalent linked lists (by value) that do not intersect.

#46. 4.8 Try a recursive approach. Check if `p` and `q` are descendants of the left subtree and the right subtree. If they are descendants of different subtrees, then the current node is the first common ancestor. If they are descendants of the same subtree, then that subtree holds the first common ancestor. Now, how do you implement this efficiently?

#47. 4.7 Look at this graph. Is there any node you can identify that will definitely be okay to build first?

#48. 4.9 The root is the very first value that must be in every array. What can you say about the order of the values in the left subtree as compared to the values in the right subtree? Do the left subtree values need to be inserted before the right subtree?

#49. 4.4 What if you could modify the binary tree node class to allow a node to store the height of its subtree?

#50. 2.8 There are really two parts to this problem. First, detect if the linked list has a loop. Second, figure out where the loop starts.

#51. 1.7 Try thinking about it layer by layer. Can you rotate a specific layer?

#52. 4.12 If each path had to start at the root, we could traverse all possible paths starting from the root. We can track the sum as we go, incrementing `totalPaths` each time we find a path with our target sum. Now, how do we extend this to paths that can start anywhere? Remember: Just get a brute-force algorithm done. You can optimize later.

#53. 1.3 It's often easiest to modify strings by going from the end of the string to the beginning.

#54. 4.11 This is your own binary search tree class, so you can maintain any information about the tree structure or nodes that you'd like (provided it doesn't have other negative implications, like making `insert` much slower). In fact, there's probably a reason the interview question specified that it was your own class. You probably need to store some additional information in order to implement this efficiently.

#55. 2.7 Focus first on just identifying if there's an intersection.

#56. 3.6 Let's suppose we kept separate lists for dogs and cats. How would we find the oldest animal of any type? Be creative!

#57. 4.5 To be a binary search tree, it's not sufficient that the `left.value <= current.value < right.value` for each node. Every node on the left must be less than the current node, which must be less than all the nodes on the right.

#58. 3.1 Try thinking about the array as circular, such that the end of the array "wraps around" to the start of the array.

#59. 3.2 What if we kept track of extra data at each stack node? What sort of data might make it easier to solve the problem?

#60. 4.7 If you identify a node without any incoming edges, then it can definitely be built. Find this node (there could be multiple) and add it to the build order. Then, what does this mean for its outgoing edges?

#61. 2.6 In the recursive approach (we have the length of the list), the middle is the base case: `isPermutation(middle)` is true. The node `x` to the immediate left of the middle: What can that node do to check if `x->middle->y` forms a palindrome? Now suppose that checks out. What about the previous node `a`? If `x->middle->y` is a palindrome, how can it check that `a->x->middle->y->b` is a palindrome?

#62. 4.11 As a naive "brute force" algorithm, can you use a tree traversal algorithm to implement this algorithm? What is the runtime of this?

#63. 3.6 Think about how you'd do it in real life. You have a list of dogs in chronological order and a list of cats in chronological order. What data would you need to find the oldest animal? How would you maintain this data?

#64. 3.3 You will need to keep track of the size of each substack. When one stack is full, you may need to create a new stack.

#65. 2.7 Observe that two intersecting linked lists will always have the same last node. Once they intersect, all the nodes after that will be equal.

#66. 4.9 The relationship between the left subtree values and the right subtree values is, essentially, anything. The left subtree values could be inserted before the right subtree, or the reverse (right values before left), or any other ordering.

#67. 2.2 You might find it useful to return multiple values. Some languages don't directly support this, but there are workarounds in essentially any language. What are some of those workarounds?

#68. 4.12 To extend this to paths that start anywhere, we can just repeat this process for all nodes.

#69. 2.8 To identify if there's a cycle, try the "runner" approach described on page 93. Have one pointer move faster than the other.

#70. 4.8 In the more naive algorithm, we had one method that indicated if `x` is a descendent of n, and another method that would recurse to find the first common ancestor. This is repeatedly searching the same elements in a subtree. We should merge this into one `firstCommonAncestor` function. What return values would give us the information we need?

#71. 2.5 Make sure you have considered linked lists that are not the same length.

#72. 2.3 Picture the list `1->5->9->12`. Removing 9 would make it look like `1->5->12`. You only have access to the 9 node. Can you make it look like the correct answer?

#73. 4.2 You could implement this by finding the "ideal" next element to add and repeatedly calling `insertValue`. This will be a bit inefficient, as you would have to repeatedly traverse the tree. Try recursion instead. Can you divide this problem into subproblems?

#74. 1.8 Can you use $O(N)$ additional space instead of $O(N^2)$? What information do you really need from the list of cells that are zero?

#75. 4.11 Alternatively, you could pick a random depth to traverse to and then randomly traverse, stopping when you get to that depth. Think this through, though. Does this work?

#76. 2.7 You can determine if two linked lists intersect by traversing to the end of each and comparing their tails.

#77. 4.12 If you've designed the algorithm as described thus far, you'll have an $O(N \log N)$ algorithm in a balanced tree. This is because there are N nodes, each of which is at depth $O(\log N)$ at worst. A node is touched once for each node above it. Therefore, the N nodes will be touched $O(\log N)$ time. There is an optimization that will give us an $O(N)$ algorithm.

#78. 3.2 Consider having each node know the minimum of its "substack" (all the elements beneath it, including itself).

#79. 4.6 Think about how an in-order traversal works and try to "reverse engineer" it.

#80. 4.8 The `firstCommonAncestor` function could return the first common ancestor (if p and q are both contained in the tree), p if p is in the tree and not q, q if q is in the tree and not p, and `null` otherwise.

#81. 3.3 Popping an element at a specific substack will mean that some stacks aren't at full capacity. Is this an issue? There's no right answer, but you should think about how to handle this.

#82. 4.9 Break this down into subproblems. Use recursion. If you had all possible sequences for the left subtree and the right subtree, how could you create all possible sequences for the entire tree?

#83. 2.8 You can use two pointers, one moving twice as fast as the other. If there is a cycle, the two pointers will collide. They will land at the same location at the same time. Where do they land? Why there?

#84. 1.2 There is one solution that is `O(N log N)` time. Another solution uses some space, but is `O(N)` time.

#85. 4.7 Once you decide to build a node, its outgoing edge can be deleted. After you've done this, can you find other nodes that are free and clear to build?

#86. 4.5 If every node on the left must be less than or equal to the current node, then this is really the same thing as saying that the biggest node on the left must be less than or equal to the current node.

#87. 4.12 What work is duplicated in the current brute-force algorithm?

#88. 1.9 We are essentially asking if there's a way of splitting the first string into two parts, x and y, such that the first string is xy and the second string is yx. For example, `x = wat` and `y = erbottle`. The first string is `xy = waterbottle`. The second string is `yx = erbottlewat`.

#89. 4.11 Picking a random depth won't help us much. First, there's more nodes at lower depths than higher depths. Second, even if we re-balanced these probabilities, we could hit a "dead end" where we meant to pick a node at depth 5 but hit a leaf at depth 3. Re-balancing the probabilities is an interesting , though.

#90. 2.8 If you haven't identified the pattern of where the two pointers start, try this: Use the linked list 1->2->3->4->5->6->7->8->9->?, where the ? links to another node. Try making the ? the first node (that is, the 9 points to the 1 such that the entire linked list is a loop). Then make the ? the node 2. Then the node 3. Then the node 4. What is the pattern? Can you explain why this happens?

#91. 4.6 Here's one step of the logic: The successor of a specific node is the leftmost node of the right subtree. What if there is no right subtree, though?

#92. 1.6 Do the easy thing first. Compress the string, then compare the lengths.

#93. 2.7 Now, you need to find where the linked lists intersect. Suppose the linked lists were the same length. How could you do this?

#94. 4.12 Consider each path that starts from the root (there are N such paths) as an array. What our brute-force algorithm is really doing is taking each array and finding all contiguous subsequences that have a particular sum. We're doing this by computing all subarrays and their sums. It might be useful to just focus on this little subproblem. Given an array, how would you find all contiguous subsequences with a particular sum? Again, think about the duplicated work in the brute-force algorithm.

#95. 2.5 Does your algorithm work on linked lists like 9->7->8 and 6->8->5? Double check that.

#96. 4.8 Careful! Does your algorithm handle the case where only one node exists? What will happen? You might need to tweak the return values a bit.

#97. 1.5 What is the relationship between the "insert character" option and the "remove character" option? Do these need to be two separate checks?

#98. 3.4 The major difference between a queue and a stack is the order of elements. A queue removes the oldest item and a stack removes the newest item. How could you remove the oldest item from a stack if you only had access to the newest item?

#99. 4.11 A naive approach that many people come up with is to pick a random number between 1 and 3. If it's 1, return the current node. If it's 2, branch left. If it's 3, branch right. This solution doesn't work. Why not? Is there a way you can adjust it to make it work?

#100. 1.7 Rotating a specific layer would just mean swapping the values in four arrays. If you were asked to swap the values in two arrays, could you do this? Can you then extend it to four arrays?

#101. 2.6 Go back to the previous hint. Remember: There are ways to return multiple values. You can do this with a new class.

#102. 1.8 You probably need some data storage to maintain a list of the rows and columns that need to be zeroed. Can you reduce the additional space usage to `O(1)` by using the matrix itself for data storage?

#103. 4.12 We are looking for subarrays with sum `targetSum`. Observe that we can track in constant time the value of $\texttt{runningSum}_i$, where this is the sum from element 0 through element `i`. For a subarray of element `i` through element `j` to have sum `targetSum`, $\texttt{runningSum}_{i-1}$ + `targetSum` must equal $\texttt{runningSum}_j$ (try drawing a picture of an array or a number line). Given that we can track the `runningSum` as we go, how can we quickly look up the number of indices `i` where the previous equation is true?

#104. 1.9 Think about the earlier hint. Then think about what happens when you concatenate `erbottlewat` to itself. You get `erbottlewaterbottlewat`.

#105. 4.4 You don't need to modify the binary tree class to store the height of the subtree. Can your recursive function compute the height of each subtree while also checking if a node is balanced? Try having the function return multiple values.

#106. 1.4 You do not have to—and should not—generate all permutations. This would be very inefficient.

#107. 4.3 Try modifying a graph search algorithm to track the depth from the root.

#108. 4.12 Try using a hash table that maps from a `runningSum` value to the number of elements with this `runningSum`.

#109.	2.5	For the follow-up question: The issue is that when the linked lists aren't the same length, the head of one linked list might represent the 1000's place while the other represents the 10's place. What if you made them the same length? Is there a way to modify the linked list to do that, without changing the value it represents?
#110.	1.6	Be careful that you aren't repeatedly concatenating strings together. This can be very inefficient.
#111.	2.7	If the two linked lists were the same length, you could traverse forward in each until you found an element in common. Now, how do you adjust this for lists of different lengths?
#112.	4.11	The reason that the earlier solution (picking a random number between 1 and 3) doesn't work is that the probabilities for the nodes won't be equal. For example, the root will be returned with probability $\frac{1}{3}$, even if there are 50+ nodes in the tree. Clearly, not all the nodes have probability $\frac{1}{3}$, so these nodes won't have equal probability. We can resolve this one issue by picking a random number between 1 and `size_of_tree` instead. This only resolves the issue for the root, though. What about the rest of the nodes?
#113.	4.5	Rather than validating the current node's value against `leftTree.max` and `rightTree.min`, can we flip around the logic? Validate the left tree's nodes to ensure that they are smaller than `current.value`.
#114.	3.4	We can remove the oldest item from a stack by repeatedly removing the newest item (inserting those into the temporary stack) until we get down to one element. Then, after we've retrieved the newest item, putting all the elements back. The issue with this is that doing several pops in a row will require `O(N)` work each time. Can we optimize for scenarios where we might do several pops in a row?
#115.	4.12	Once you've solidified the algorithm to find all contiguous subarrays in an array with a given sum, try to apply this to a tree. Remember that as you're traversing and modifying the hash table, you may need to "reverse the damage" to the hash table as you traverse back up.
#116.	4.2	Imagine we had a `createMinimalTree` method that returns a minimal tree for a given array (but for some strange reason doesn't operate on the root of the tree). Could you use this to operate on the root of the tree? Could you write the base case for the function? Great! Then that's basically the entire function.
#117.	1.1	Could a bit vector be useful?
#118.	1.3	You might find you need to know the number of spaces. Can you just count them?
#119.	4.11	The issue with the earlier solution is that there could be more nodes on one side of a node than the other. So, we need to weight the probability of going left and right based on the number of nodes on each side. How does this work, exactly? How can we know the number of nodes?
#120.	2.7	Try using the difference between the lengths of the two linked lists.
#121.	1.4	What characteristics would a string that is a permutation of a palindrome have?
#122.	1.2	Could a hash table be useful?
#123.	4.3	A hash table or array that maps from level number to nodes at that level might also be useful.

#124. 4.4 Actually, you can just have a single `checkHeight` function that does both the height computation and the balance check. An integer return value can be used to indicate both.

#125. 4.7 As a *totally* different approach: Consider doing a depth-first search starting from an arbitrary node. What is the relationship between this depth-first search and a valid build order?

#126. 2.2 Can you do it iteratively? Imagine if you had two pointers pointing to adjacent nodes and they were moving at the same speed through the linked list. When one hits the end of the linked list, where will the other be?

#127. 4.1 Two well-known algorithms can do this. What are the tradeoffs between them?

#128. 4.5 Think about the `checkBST` function as a recursive function that ensures each node is within an allowable `(min, max)` range. At first, this range is infinite. When we traverse to the left, the min is negative infinity and the max is `root.value`. Can you implement this recursive function and properly adjust these ranges as you traverse the tree?

#129. 2.7 If you move a pointer in the longer linked list forward by the difference in lengths, you can then apply a similar approach to the scenario when the linked lists are equal.

#130. 1.5 Can you do all three checks in a single pass?

#131. 1.2 Two strings that are permutations should have the same characters, but in different orders. Can you make the orders the same?

#132. 1.1 Can you solve it in `O(N log N)` time? What might a solution like that look like?

#133. 4.7 Pick an arbitrary node and do a depth-first search on it. Once we get to the end of a path, we know that this node can be the last one built, since no nodes depend on it. What does this mean about the nodes right before it?

#134. 1.4 Have you tried a hash table? You should be able to get this down to `O(N)` time.

#135. 4.3 You should be able to come up with an algorithm involving both depth-first search and breadth-first search.

#136. 1.4 Can you reduce the space usage by using a bit vector?

II

Hints for Concepts and Algorithms

#137.	5.1	Break this into parts. Focus first on clearing the appropriate bits.
#138.	8.9	Try the Base Case and Build approach.
#139.	6.9	Given a specific door x, on which rounds will it be toggled (open or closed)?
#140.	11.5	What does the interviewer mean by a pen? There are a lot of different types of pens. Make a list of potential questions you would want to ask.
#141.	7.11	This is not as complicated as it sounds. Start by making a list of the key objects in the system, then think about how they interact.
#142.	9.6	First, start with making some assumptions. What do and don't you have to build?
#143.	5.2	To wrap your head around the problem, try thinking about how you'd do it for integers.
#144.	8.6	Try the Base Case and Build approach.
#145.	5.7	Swapping each pair means moving the even bits to the left and the odd bits to the right. Can you break this problem into parts?
#146.	6.10	Solution 1: Start with a simple approach. Can you just divide up the bottles into groups? Remember that you can't re-use a test strip once it is positive, but you can reuse it as long as it's negative.
#147.	5.4	Get Next: Start with a brute force solution for each.
#148.	8.14	Can we just try all possibilities? What would this look like?
#149.	6.5	Play around with the jugs of water, pouring water back and forth, and see if you can measure anything other than 3 quarts or 5 quarts. That's a start.
#150.	8.7	Approach 1: Suppose you had all permutations of `abc`. How can you use that to get all permutations of `abcd`?
#151.	5.5	Reverse engineer this, starting from the outermost layer to the innermost layer.
#152.	8.1	Approach this from the top down. What is the very last hop the child made?
#153.	7.1	Note that a "card deck" is very broad. You might want to think about a reasonable scope to the problem.
#154.	6.7	Observe that each family will have exactly one girl.
#155.	8.13	Will sorting the boxes help in any way?

#156.	6.8	This is really an algorithm problem, and you should approach it as such. Come up with a brute force, compute the worst-case number of drops, then try to optimize that.
#157.	6.4	In what cases will they not collide?
#158.	9.6	We've assumed that the rest of the eCommerce system is already handled, and we just need to deal with the analytics part of sales rank. We can get notified somehow when a purchase occurs.
#159.	5.3	Start with a brute force solution. Can you try all possibilities?
#160.	6.7	Think about writing each family as a sequence of Bs and Gs.
#161.	8.8	You could handle this by just checking to see if there are duplicates before printing them (or adding them to a list). You can do this with a hash table. In what case might this be okay? In what case might it not be a very good solution?
#162.	9.7	Will this application be write-heavy or read-heavy?
#163.	6.10	Solution 1: There is a relatively simple approach that works in 28 days, in the worst case. There are better approaches though.
#164.	11.5	Consider the scenario of a pen for children. What does this mean? What are the different use cases?
#165.	9.8	Scope the problem well. What will and won't you tackle as part of this system?
#166.	8.5	Think about multiplying 8 by 9 as counting the number of cells in a matrix with width 8 and height 9.
#167.	5.2	In a number like `.893` (in base 10), what does each digit signify? What then does each digit in `.10010` signify in base 2?
#168.	8.14	We can think about each possibility as each place where we can put parentheses. This means around each operator, such that the expression is split at the operator. What is the base case?
#169.	5.1	To clear the bits, create a "bit mask" that looks like a series of 1s, then 0s, then 1s.
#170.	8.3	Start with a brute force algorithm.
#171.	6.7	You can attempt this mathematically, although the math is pretty difficult. You might find it easier to estimate it up to families of, say, 6 children. This won't give you a good mathematical proof, but it might point you in the right direction of what the answer might be.
#172.	6.9	In which cases would a door be left open at the end of the process?
#173.	5.2	A number such as `.893` (in base 10) indicates $8 * 10^{-1} + 9 * 10^{-2} + 3 * 10^{-3}$. Translate this system into base 2.
#174.	8.9	Suppose we had all valid ways of writing two pairs of parentheses. How could we use this to get all valid ways of writing three pairs?
#175.	5.4	Get Next: Picture a binary number—something with a bunch of 1s and 0s spread out throughout the number. Suppose you flip a 1 to a 0 and a 0 to a 1. In what case will the number get bigger? In what case will it get smaller?

#176.	9.6	Think about what sort of expectations on freshness and accuracy of data is expected. Does the data always need to be 100% up to date? Is the accuracy of some products more important than others?
#177.	10.2	How do you check if two words are anagrams of each other? Think about what the definition of "anagram" is. Explain it in your own words.
#178.	8.1	If we knew the number of paths to each of the steps before step 100, could we compute the number of steps to 100?
#179.	7.8	Should white pieces and black pieces be the same class? What are the pros and cons of this?
#180.	9.7	Observe that there is a lot of data coming in, but people probably aren't reading the data very frequently.
#181.	6.2	Calculate the probability of winning the first game and winning the second game, then compare them.
#182.	10.2	Two words are anagrams if they contain the same characters but in different orders. How can you put characters in order?
#183.	6.10	Solution 2: Why do we have such a time lag between tests and results? There's a reason the question isn't phrased as just "minimize the number of rounds of testing." The time lag is there for a reason.
#184.	9.8	How evenly do you think traffic is distributed? Do all documents get roughly the same age of traffic? Or is it likely there are some very popular documents?
#185.	8.7	Approach 1: The permutations of `abc` represent all ways of ordering `abc`. Now, we want to create all orderings of `abcd`. Take a specific ordering of `abcd`, such as `bdca`. This `bdca` string represents an ordering of `abc`, too: Remove the `d` and you get `bca`. Given the string `bca`, can you create all the "related" orderings that include `d`, too?
#186.	6.1	You can only use the scale once. This means that all, or almost all, of the bottles must be used. They also must be handled in different ways or else you couldn't distinguish between them.
#187.	8.9	We could try generating the solution for three pairs by taking the list of two pairs of parentheses and adding a third pair. We'd have to add the third paren before, around, and after. That is: `()<SOLUTION>`, `(<SOLUTION>)`, `<SOLUTION>()`. Will this work?
#188.	6.7	Logic might be easier than math. Imagine we wrote every birth into a giant string of Bs and Gs. Note that the groupings of families are irrelevant for this problem. What is the probability of the next character added to the string being a B versus a G?
#189.	9.6	Purchases will occur very frequently. You probably want to limit database writes.
#190.	8.8	If you haven't solved 8.7 yet, do that one first.
#191.	6.10	Solution 2: Consider running multiple tests at once.
#192.	7.6	A common trick when solving a jigsaw puzzle is to separate edge and non-edge pieces. How will you represent this in an object-oriented manner?
#193.	10.9	Start with a naive solution. (But hopefully not too naive. You should be able to use the fact that the matrix is sorted.)

#194.	8.13	We can sort the boxes by any dimension in descending order. This will give us a partial order for the boxes, in that boxes later in the array must appear before boxes earlier in the array.
#195.	6.4	The only way they won't collide is if all three are walking in the same direction. What's the probability of all three walking clockwise?
#196.	10.11	Imagine the array were sorted in ascending order. Is there any way you could "fix it" to be sorted into alternating peaks and valleys?
#197.	8.14	The base case is when we have a single value, 1 or 0.
#198.	7.3	Scope the problem first and make a list of your assumptions. It's often okay to make reasonable assumptions, but you need to make them explicit.
#199.	9.7	The system will be write-heavy: Lots of data being imported, but it's rarely being read.
#200.	8.7	Approach 1: Given a string such as `bca`, you can create all permutations of `abcd` that have `{a, b, c}` in the order `bca` by inserting `d` into each possible location: `dbca`, `bdca`, `bcda`, `bcad`. Given all permutations of `abc`, can you then create all permutations of `abcd`?
#201.	6.7	Observe that biology hasn't changed; only the conditions under which a family stops having kids has changed. Each pregnancy has a 50% odds of being a boy and a 50% odds of being a girl.
#202.	5.5	What does it mean if `A & B == 0`?
#203.	8.5	If you wanted to count the cells in an 8x9 matrix, you could count the cells in a 4x9 matrix and then double it.
#204.	8.3	Your brute force algorithm probably ran in `O(N)` time. If you're trying to beat that runtime, what runtime do you think you will get to? What sorts of algorithms have that runtime?
#205.	6.10	Solution 2: Think about trying to figure out the bottle, digit by digit. How can you detect the first digit in the poisoned bottle? What about the second digit? The third digit?
#206.	9.8	How will you handle generating URLs?
#207.	10.6	Think about merge sort versus quick sort. Would one of them work well for this purpose?
#208.	9.6	You also want to limit joins because they can be very expensive.
#209.	8.9	The problem with the solution suggested by the earlier hint is that it might have duplicate values. We could eliminate this by using a hash table.
#210.	11.6	Be careful about your assumptions. Who are the users? Where are they using this? It might seem obvious, but the real answer might be different.
#211.	10.9	We can do a binary search in each row. How long will this take? How can we do better?
#212.	9.7	Think about things like how you're going to get the bank data (will it be pulled or pushed?), what features the system will support, etc.
#213.	7.7	As always, scope the problem. Are "friendships" mutual? Do status messages exist? Do you support group chat?
#214.	8.13	Try to break it down into subproblems.

#215. 5.1 It's easy to create a bit mask of 0s at the beginning or end. But how do you create a bit mask with a bunch of zeroes in the middle? Do it the easy way: Create a bit mask for the left side and then another one for the right side. Then you can merge those.

#216. 7.11 What is the relationship between files and directories?

#217. 8.1 We can compute the number of steps to 100 by the number of steps to 99, 98, and 97. This corresponds to the child hopping 1, 2, or 3 steps at the end. Do we add those or multiply them? That is: Is it `f(100) = f(99) + f(98) + f(97)` *or* `f(100) = f(99) * f(98) * f(97)`?

#218. 6.6 This is a logic problem, not a clever word problem. Use logic/math/algorithms to solve it.

#219. 10.11 Try walking through a sorted array. Can you just swap elements until you have fixed the array?

#220. 11.5 Have you considered both intended uses (writing, etc.) and unintended use? What about safety? You would not want a pen for children to be dangerous.

#221. 6.10 Solution 2: Be very careful about edge cases. What if the third digit in the bottle number matches the first or second digit?

#222. 8.8 Try getting the count of each character. For example, ABCAAC has 3 As, 2 Cs, and 1 B.

#223. 9.6 Don't forget that a product can be listed under multiple categories.

#224. 8.6 You can easily move the smallest disk from one tower to another. It's also pretty easy to move the smallest two disks from one tower to another. Can you move the smallest three disks?

#225. 11.6 In a real interview, you would also want to discuss what sorts of test tools we have available.

#226. 5.3 Flipping a 0 to a 1 can merge two sequences of 1s—but only if the two sequences are separated by only one 0.

#227. 8.5 Think about how you might handle this for odd numbers.

#228. 7.8 What class should maintain the score?

#229. 10.9 If you're considering a particular column, is there a way to quickly eliminate it (in some cases at least)?

#230. 6.10 Solution 2: You can run an additional day of testing to check digit 3 in a different way. But again, be very careful about edge cases here.

#231. 10.11 Note that if you ensure the peaks are in place, the valleys will be, too. Therefore, your iteration to fix the array can skip over every other element.

#232. 9.8 If you generate URLs randomly, do you need to worry about collisions (two documents with the same URL)? If so, how can you handle this?

#233. 6.8 As a first approach, you might try something like binary search. Drop it from the 50th floor, then the 75th, then the 88th, and so on. The problem is that if the first egg drops at the 50th floor, then you'll need to start dropping the second egg starting from the 1st floor and going up. This could take, at worst, 50 drops (the 50th floor drop, the 1st floor drop, the 2nd floor drop, and up through the 49th floor drop). Can you beat this?

#234. 8.5 If there's duplicated work across different recursive calls, can you cache it?

#235. 10.7 Would a bit vector help?

#236. 9.6 Where would it be appropriate to cache data or queue up tasks?

#237. 8.1 We multiply the values when it's "we do this then this." We add them when it's "we do this or this."

#238. 7.6 Think about how you might record the position of a piece when you find it. Should it be stored by row and location?

#239. 6.2 To calculate the probability of winning the second game, start with calculating the probability of making the first hoop, the second hoop, and not the third hoop.

#240. 8.3 Can you solve the problem in O(log N)?

#241. 6.10 Solution 3: Think about each test strip as being a binary indicator for poisoned vs. non-poisoned.

#242. 5.4 Get Next: If you flip a 1 to a 0 and a 0 to a 1, it will get bigger if the 0->1 bit is more significant than the 1->0 bit. How can you use this to create the next biggest number (with the same number of 1s)?

#243. 8.9 Alternatively, we could think about doing this by moving through the string and adding left and right parens at each step. Will this eliminate duplicates? How do we know if we can add a left or right paren?

#244. 9.6 Depending on what assumptions you made, you might even be able to do without a database at all. What would this mean? Would it be a good idea?

#245. 7.7 This is a good problem to think about the major system components or technologies that would be useful.

#246. 8.5 If you're doing `9*7` (both odd numbers), then you could do 4*7 and 5*7.

#247. 9.7 Try to reduce unnecessary database queries. If you don't need to permanently store the data in the database, you might not need it in the database at all.

#248. 5.7 Can you create a number that represents just the even bits? Then can you shift the even bits over by one?

#249. 6.10 Solution 3: If each test strip is a binary indicator, can we map, integer keys to a set of 10 binary indicators such that each key has a unique configuration (mapping)?

#250. 8.6 Think about moving the smallest disk from tower `X=0` to tower `Y=2` using tower `Z=1` as a temporary holding spot as having a solution for `f(1, X=0, Y=2, Z=1)`. Moving the smallest two disks is `f(2, X=0, Y=2, Z=1)`. Given that you have a solution for `f(1, X=0, Y=2, Z=1)` and `f(2, X=0, Y=2, Z=1)`, can you solve `f(3, X=0, Y=2, Z=1)`?

#251. 10.9 Since each column is sorted, you know that the value can't be in this column if it's smaller than the min value in this column. What else does this tell you?

#252. 6.1 What happens if you put one pill from each bottle on the scale? What if you put two pills from each bottle on the scale?

#253. 10.11 Do you necessarily need the arrays to be sorted? Can you do it with an unsorted array?

#254. 10.7 To do it with less memory, can you try multiple passes?

#255. 8.8 To get all permutations with 3 As, 2 Cs, and 1 B, you need to first pick a starting character: A, B, or C. If it's an A, then you need all permutations with 2 As, 2 Cs, and 1 B.

#256. 10.5 Try modifying binary search to handle this.

#257. 11.1 There are two mistakes in this code.

#258. 7.4 Does the parking lot have multiple levels? What "features" does it support? Is it paid? What types of vehicles?

#259. 9.5 You may need to make some assumptions (in part because you don't have an interviewer here). That's okay. Make those assumptions explicit.

#260. 8.13 Think about the first decision you have to make. The first decision is which box will be at the bottom.

#261. 5.5 If `A & B == 0`, then it means that A and B never have a 1 at the same spot. Apply this to the equation in the problem.

#262. 8.1 What is the runtime of this method? Think carefully. Can you optimize it?

#263. 10.2 Can you leverage a standard sorting algorithm?

#264. 6.9 Note: If an integer x is divisible by a, and `b = x / a`, then x is also divisible by b. Does this mean that all numbers have an even number of factors?

#265. 8.9 Adding a left or right paren at each step will eliminate duplicates. Each substring will be unique at each step. Therefore, the total string will be unique.

#266. 10.9 If the value x is smaller than the start of the column, then it also can't be in any columns to the right.

#267. 8.7 Approach 1: You can create all permutations of `abcd` by computing all permutations of `abc` and then inserting d into each possible location within those.

#268. 11.6 What are the different features and uses we would want to test?

#269. 5.2 How would you get the first digit in `.893`? If you multiplied by 10, you'd shift the values over to get `8.93`. What happens if you multiply by 2?

#270. 9.2 To find the connection between two nodes, would it be better to do a breadth-first search or depth-first search? Why?

#271. 7.7 How will you know if a user signs offline?

#272. 8.6 Observe that it doesn't really matter which tower is the source, destination, or buffer. You can do `f(3, X=0, Y=2, Z=1)` by first doing `f(2, X=0, Y=1, Z=2)` (moving two disks from tower 0 to tower 1, using tower 2 as a buffer), then moving disk 3 from tower 0 to tower 2, then doing `f(2, X=1, Y=2, Z=0)` (moving two disks from tower 1 to tower 2, using tower 0 as a buffer). How does this process repeat?

#273. 8.4 How can you build all subsets of `{a, b, c}` from the subsets of `{a, b}`?

#274. 9.5 Think about how you could design this for a single machine. Would you want a hash table? How would that work?

#275. 7.1 How, if at all, will you handle aces?

#276. 9.7 As much work as possible should be done asynchronously.

#277. 10.11 Suppose you had a sequence of three elements (`{0, 1, 2}`, in any order. Write out all possible sequences for those elements and how you can fix them to make `1` the peak.

#278. 8.7 Approach 2: If you had all permutations of two-character substrings, could you generate all permutations of three-character substrings?

#279. 10.9 Think about the previous hint in the context of rows.

#280. 8.5 Alternatively, if you're doing 9 * 7, you could do 4*7, double that, and then add 7.

#281. 10.7 Try using one pass to get it down to a range of values, and then a second pass to find a specific value.

#282. 6.6 Suppose there were exactly one blue-eyed person. What would that person see? When would they leave?

#283. 7.6 Which will be the easiest pieces to match first? Can you start with those? Which will be the next easiest, once you've nailed those down?

#284. 6.2 If two events are mutually exclusive (they can never occur simultaneously), you can add their probabilities together. Can you find a set of mutually exclusive events that represent making two out of three hoops?

#285. 9.2 A breadth-first search is probably better. A depth-first search can wind up going on a long path, even though the shortest path is actually very short. Is there a modification to a breadth-first search that might be even faster?

#286. 8.3 Binary search has a runtime of `O(log N)`. Can you apply a form of binary search to the problem?

#287. 7.12 In order to handle collisions, the hash table should be an array of linked lists.

#288. 10.9 What would happen if we tried to keep track of this using an array? What are the pros and cons of this?

#289. 10.8 Can you use a bit vector?

#290. 8.4 Anything that is a subset of `{a, b}` is also a subset of `{a, b, c}`. Which sets are subsets of `{a, b, c}` but not `{a, b}`?

#291. 10.9 Can we use the previous hints to move up, down, left, and right around the rows and columns?

#292. 10.11 Revisit the set of sequences for `{0, 1, 2}` that you just wrote out. Imagine there are elements before the leftmost element. Are you sure that the way you swap the elements won't invalidate the previous part of the array?

#293. 9.5 Can you combine a hash table and a linked list to get the best of both worlds?

#294. 6.8 It's actually better for the first drop to be a bit lower. For example, you could drop at the 10th floor, then the 20th floor, then the 30th floor, and so on. The worst case here will be 19 drops (10, 20, ..., 100, 91, 92, ..., 99). Can you beat that? Try not randomly guessing at different solutions. Rather, think deeper. How is the worst case defined? How does the number of drops of each egg factor into that?

#295. 8.9 We can ensure that this string is valid by counting the number of left and right parens. It is always valid to add a left paren, up until the total number of pairs of parens. We can add a right paren as long as `count(left parens) <= count(right parens)`.

#296. 6.4 You can think about this either as the probability(3 ants walking clockwise) + probability(3 ants walking counter-clockwise). Or, you can think about it as: The first ant picks a direction. What's the probability of the other ants picking the same direction?

#297. 5.2 Think about what happens for values that can't be represented accurately in binary.

#298. 10.3 Can you modify binary search for this purpose?

#299. 11.1 What will happen to the `unsigned int`?

#300. 8.11 Try breaking it down into subproblems. If you were making change, what is the first choice you would make?

#301. 10.10 The problem with using an array is that it will be slow to insert a number. What other data structures could we use?

#302. 5.5 If `(n & (n-1)) == 0`, then this means that n and `n - 1` never have a 1 in the same spot. Why would that happen?

#303. 10.9 Another way to think about this is that if you drew a rectangle around a cell extending to the bottom, right coordinate of the matrix, the cell would be bigger than all the items in this square.

#304. 9.2 Is there any way to search from both the source and destination? For what reason or in what case might this be faster?

#305. 8.14 If your code looks really lengthy, with a lot of if's (for each possible operator, "target" boolean result, and left/right side), think about the relationship between the different parts. Try to simplify your code. It should not need a ton of complicated if-statements. For example, consider expressions of the form <LEFT>OR<RIGHT> versus <LEFT>AND<RIGHT>. Both may need to know the number of ways that the <LEFT> evaluates to true. See what code you can reuse.

#306. 6.9 The number 3 has an even number of factors (1 and 3). The number 12 has an even number of factors (1, 2, 3, 4, 6, 12). What numbers do not? What does this tell you about the doors?

#307. 7.12 Think carefully about what information the linked list node needs to contain.

#308. 8.12 We know that each row must have a queen. Can you try all possibilities?

#309. 8.7 Approach 2: To generate a permutation of abcd, you need to pick an initial character. It can be a, b, c, or d. You can then permute the remaining characters. How can you use this approach to generate all permutations of the full string?

#310. 10.3 What is the runtime of your algorithm? What will happen if the array has duplicates?

#311. 9.5 How would you scale this to a larger system?

#312. 5.4 Get Next: Can you flip a 0 to a 1 to create the next biggest number?

#313. 11.4 Think about what load testing is designed to test. What are the factors in the load of a webpage? What criteria would be used to judge if a webpage performs satisfactorily under heavy load?

#314.	5.3	Each sequence can be lengthened by merging it with an adjacent sequence (if any) or just flipping the immediate neighboring zero. You just need to find the best choice.
#315.	10.8	Consider implementing your own bit vector class. It's a good exercise and an important part of this problem.
#316.	10.11	You should be able to design an O(n) algorithm.
#317.	10.9	A cell will be larger than all the items below it and to the right. It will be smaller than all cells above it and to the left. If we wanted to eliminate the most elements first, which element should we compare the value x to?
#318.	8.6	If you're having trouble with recursion, then try trusting the recursive process more. Once you've figured out how to move the top two disks from tower 0 to tower 2, trust that you have this working. When you need to move three disks, trust that you can move two disks from one tower to another. Now, two disks have been moved. What do you do about the third?
#319.	6.1	Imagine there were just three bottles and one had heavier pills. Suppose you put different numbers of pills from each bottle on the scale (for example, bottle 1 has 5 pills, bottle 2 has 2 pills, and bottle 3 has 9 pills). What would the scale show?
#320.	10.4	Think about how binary search works. What will be the issue with just implementing binary search?
#321.	9.2	Discuss how you might implement these algorithms and this system in the real world. What sort of optimizations might you make?
#322.	8.13	Once we pick the box on the bottom, we need to pick the second box. Then the third box.
#323.	6.2	The probability of making two out of three shots is probability(make shot 1, make shot 2, miss shot 3) + probability(make shot 1, miss shot 2, make shot 3) + probability(miss shot 1, make shot 2, make shot 3) + probability(make shot 1, make shot 2, make shot 3).
#324.	8.11	If you were making change, the first choice you might make is how many quarters you need to use.
#325.	11.2	Think about issues both within the program and outside of the program (the rest of the system).
#326.	9.4	Estimate how much space is needed for this.
#327.	8.14	Look at your recursion. Do you have repeated calls anywhere? Can you memoize it?
#328.	5.7	The value 1010 in binary is 10 in decimal or 0xA in hex. What will a sequence of 101010... be in hex? That is, how do you represent an alternating sequence of 1s and 0s with 1s in the odd places? How do you do this for the reverse (1s in the even spots)?
#329.	11.3	Consider both extreme cases and more general cases.
#330.	10.9	If we compare x to the center element in the matrix, we can eliminate roughly one quarter of the elements in the matrix.
#331.	8.2	For the robot to reach the last cell, it must find a path to the second-to-last cells. For it to find a path to the second-to-last cells, it must find a path to the third-to-last cells.
#332.	10.1	Try moving from the end of the array to the beginning.

#333. 6.8 If we drop Egg 1 at fixed intervals (e.g., every 10 floors), then the worst case is the worst case for Egg 1 + the worst case for Egg 2. The problem with our earlier solutions is that as Egg 1 does more work, Egg 2 doesn't do any less work. Ideally, we'd like to balance this a bit. As Egg 1 does more work (has survived more drops), Egg 2 should have less work to do. What might this mean?

#334. 9.3 Think about how infinite loops might occur.

#335. 8.7 Approach 2: To generate all permutations of abcd, pick each character (a, b, c, or d) as a starting character. Permute the remaining characters and prepend the starting character. How do you permute the remaining characters? With a recursive process that follows the same logic.

#336. 5.6 How would you figure out how many bits are different between two numbers?

#337. 10.4 Binary search requires comparing an element to the midpoint. Getting the midpoint requires knowing the length. We don't know the length. Can we find it?

#338. 8.4 Subsets that contain c will be subsets `{a, b, c}` but not `{a, b}`. Can you build these subsets from the subsets of `{a, b}`?

#339. 5.4 Get Next: Flipping a 0 to a 1 will create a bigger number. The farther right the index is the smaller the bigger number is. If we have a number like 1001, we want to flip the rightmost 0 (to create 1011). But if we have a number like 1010, we should not flip the rightmost 1.

#340. 8.3 Given a specific index and value, can you identify if the magic index would be before or after it?

#341. 6.6 Now suppose there were two blue-eyed people. What would they see? What would they know? When would they leave? Remember your answer from the prior hint. Assume they know the answer to the earlier hint.

#342. 10.2 Do you even need to truly "sort"? Or is just reorganizing the list sufficient?

#343. 8.11 Once you've decided to use two quarters to make change for 98 cents, you now need to figure out how many ways to make change for 48 cents using nickels, dimes, and pennies.

#344. 7.5 Think about all the different functionality a system to read books online would have to support. You don't have to do everything, but you should think about making your assumptions explicit.

#345. 11.4 Could you build your own? What might that look like?

#346. 5.5 What is the relationship between how n looks and how n - 1 looks? Walk through a binary subtraction.

#347. 9.4 Will you need multiple passes? Multiple machines?

#348. 10.4 We can find the length by using an exponential backoff. First check index 2, then 4, then 8, then 16, and so on. What will be the runtime of this algorithm?

#349. 11.6 What can we automate?

#350. 8.12 Each row must have a queen. Start with the last row. There are eight different columns on which you can put a queen. Can you try each of these?

#351.	7.10	Should number cells, blank cells, and bomb cells be separate classes?
#352.	5.3	Try to do it in linear time, a single pass, and `O(1)` space.
#353.	9.3	How would you detect the same page? What does this mean?
#354.	8.4	You can build the remaining subsets by adding c to all the subsets of `{a, b}`.
#355.	5.7	Try masks `0xaaaaaaaa` and `0x55555555` to select the even and odd bits. Then try shifting the even and odd bits around to create the right number.
#356.	8.7	Approach 2: You can implement this approach by having the recursive function pass back the list of the strings, and then you prepend the starting character to it. Or, you can push down a prefix to the recursive calls.
#357.	6.8	Try dropping Egg 1 at bigger intervals at the beginning and then at smaller and smaller intervals. The idea is to keep the sum of Egg 1 and Egg 2's drops as constant as possible. For each additional drop that Egg 1 takes, Egg 2 takes one fewer drop. What is the right interval?
#358.	5.4	Get Next: We should flip the rightmost non-trailing 0. The number 1010 would become 1110. Once we've done that, we need to flip a 1 to a 0 to make the number as small as possible, but bigger than the original number (1010). What do we do? How can we shrink the number?
#359.	8.1	Try memoization as a way to optimize an inefficient recursive program.
#360.	8.2	Simplify this problem a bit by first figuring out if there's a path. Then, modify your algorithm to track the path.
#361.	7.10	What is the algorithm to place the bombs around the board?
#362.	11.1	Look at the parameters for `printf`.
#363.	7.2	Before coding, make a list of the objects you need and walk through the common algorithms. Picture the code. Do you have everything you need?
#364.	8.10	Think about this as a graph.
#365.	9.3	How do you define if two pages are the same? Is it the URLs? Is it the content? Both of these can be flawed. Why?
#366.	5.8	First try the naive approach. Can you set a particular "pixel"?
#367.	6.3	Picture a domino laying down on the board. How many black squares does it cover? How many white squares?
#368.	8.13	Once you have a basic recursive algorithm implemented, think about if you can optimize it. Are there any repeated subproblems?
#369.	5.6	Think about what an XOR indicates. If you do a XOR b, where does the result have 1s? Where does it have 0s?
#370.	6.6	Build up from this. What if there were three blue-eyed people? What if there were four blue-eyed people?
#371.	8.12	Break this down into smaller subproblems. The queen at row 8 must be at column 1, 2, 3, 4, 5, 6, 7, or 8. Can you print all ways of placing eight queens where a queen is at row 8 and column 3? You then need to check all the ways of placing a queen on row 7.

#372.	5.5	When you do a binary subtraction, you flip the rightmost 0s to a 1, stopping when you get to a 1 (which is also flipped). Everything (all the 1s and 0s) on the left will stay put.
#373.	8.4	You can also do this by mapping each subset to a binary number. The `ith` bit could represent a "boolean" flag for whether an element is in the set.
#374.	6.8	Let X be the first drop of Egg 1. This means that Egg 2 would do X - 1 drops if Egg 1 broke. We want to try to keep the sum of Egg 1 and Egg 2's drops as constant as possible. If Egg 1 breaks on the second drop, then we want Egg 2 to do X - 2 drops. If Egg 1 breaks on the third drop, then we want Egg 2 to do X - 3 drops. This keeps the sum of Egg 1 and Egg 2 fairly constant. What is X?
#375.	5.4	Get Next: We can shrink the number by moving all the 1s to the right of the flipped bit as far right as possible (removing a 1 in the process).
#376.	10.10	Would it work well to use a binary search tree?
#377.	7.10	To place the bombs randomly on the board: Think about the algorithm to shuffle a deck of cards. Can you apply a similar technique?
#378.	8.13	Alternatively, we can think about the repeated choices as: Does the first box go on the stack? Does the second box go on the stack? And so on.
#379.	6.5	If you fill the 5-quart jug and then use it to fill the 3-quart jug, you'll have two quarts left in the 5-quart jug. You can either keep those two quarts where they are, or you can dump the contents of the smaller jug and pour the two quarts in there.
#380.	8.11	Analyze your algorithm. Is there any repeated work? Can you optimize this?
#381.	5.8	When you're drawing a long line, you'll have entire bytes that will become a sequence of 1s. Can you set this all at once?
#382.	8.10	You can implement this using depth-first search (or breadth-first search). Each adjacent pixel of the "right" color is a connected edge.
#383.	5.5	Picture n and n - 1. To subtract 1 from n, you flipped the rightmost 1 to a 0 and all the 0s on its right to 1s. If `n & n-1 == 0`, then there are no 1s to the left of the first 1. What does that mean about n?
#384.	5.8	What about the start and end of the line? Do you need to set those pixels individually, or can you set them all at once?
#385.	9.1	Think about this as a real-world application. What are the different factors you would need to consider?
#386.	7.10	How do you count the number of bombs neighboring a cell? Will you iterate through all cells?
#387.	6.1	You should be able to have an equation that tells you the heavy bottle based on the weight.
#388.	8.2	Think again about the efficiency of your algorithm. Can you optimize it?
#389.	7.9	The `rotate()` method should be able to run in `O(1)` time.
#390.	5.4	Get Previous: Once you've solved Get Next, try to invert the logic for Get Previous.
#391.	5.8	Does your code handle the case when `x1` and `x2` are in the same byte?
#392.	10.10	Consider a binary search tree where each node stores some additional data.

#393.	11.6	Have you thought about security and reliability?
#394.	8.11	Try using memoization.
#395.	6.8	I got 14 drops in the worst case. What did you get?
#396.	9.1	There's no one right answer here. Discuss several different technical implementations.
#397.	6.3	How many black squares are there on the board? How many white squares?
#398.	5.5	We know that n must have only one 1 if `n & (n-1) == 0`. What sorts of numbers have only one 1?
#399.	7.10	When you click on a blank cell, what is the algorithm to expand the neighboring cells?
#400.	6.5	Once you've developed a way to solve this problem, think about it more broadly. If you are given a jug of size X and another jug of size Y, can you always use it to measure Z?
#401.	11.3	Is it possible to test everything? How will you prioritize testing?

III

Hints for Knowledge-Based Questions

#402. 12.9 Focus on the concept firsts, then worry about the exact implementation. How should `SmartPointer` look?

#403. 15.2 A context switch is the time spent switching between two processes. This happens when you bring one process into execution and swap out the existing process.

#404. 13.1 Think about who can access private methods.

#405. 15.1 How do these differ in terms of memory?

#406. 12.11 Recall that a two dimensional array is essentially an array of arrays.

#407. 15.2 Ideally, we would like to record the timestamp when one process "stops" and the timestamp when another process "starts." But how do we know when this swapping will occur?

#408. 14.1 A GROUP BY clause might be useful.

#409. 13.2 When does a finally block get executed? Are there any cases where it won't get executed?

#410. 12.2 Can we do this in place?

#411. 14.2 It might be helpful to break the approach into two pieces. The first piece is to get each building ID and the number of open requests. Then, we can get the building names.

#412. 13.3 Consider that some of these might have different meanings depending on where they are applied.

#413. 12.10 Typically, `malloc` will just give us an arbitrary block of memory. If we can't override this behavior, can we work with it to do what we need?

#414. 15.7 First implement the single-threaded FizzBuzz problem.

#415. 15.2 Try setting up two processes and have them pass a small amount of data back and forth. This will encourage the system to stop one process and bring the other one in.

#416. 13.4 The purpose of these might be somewhat similar, but how does the implementation differ?

#417. 15.5 How can we ensure that `first()` has terminated before calling `second()`?

#418. 12.11 One approach is to call `malloc` for each array. How would we free the memory here?

#419. 15.3 A deadlock can happen when there's a "cycle" in the order of who is waiting for whom. How can we break or prevent this cycle?

#420. 13.5 Think about the underlying data structure.

#421. 12.7 Think about why we use virtual methods.

#422. 15.4 If every thread had to declare upfront what processes it might need, could we detect possible deadlocks in advance?

#423. 12.3 What is the underlying data structure behind each? What are the implications of this?

#424. 13.5 `HashMap` uses an array of linked lists. `TreeMap` uses a red-black tree. `LinkedHashMap` uses doubly-linked buckets. What is the implication of this?

#425. 13.4 Consider the usage of primitive types. How else might they differ in terms of how you can use the types?

#426. 12.11 Can we allocate this instead as a contiguous block of memory?

#427. 12.8 This data structure can be pictured as a binary tree, but it's not necessarily. What if there's a loop in the structure?

#428. 14.7 You probably need a list of students, their courses, and another table building a relationship between students and courses. Note that this is a many-to-many relationship.

#429. 15.6 The keyword `synchronized` ensures that two threads cannot execute synchronized methods on the same instance at the same time.

#430. 13.5 Consider how they might differ in terms of the order of iteration through the keys. Why might you want one option instead of the others?

#431. 14.3 First try to get a list of the IDs (just the IDs) of all the relevant apartments.

#432. 12.10 Imagine we have a sequential set of integers (3, 4, 5, ...). How big does this set need to be to ensure that one of the numbers is divisible by 16?

#433. 15.5 Why would using `boolean` flags to do this be a bad idea?

#434. 15.4 Think about the order of requests as a graph. What does a deadlock look like within this graph?

#435. 13.6 Object reflection allows you to get information about methods and fields in an object. Why might this be useful?

#436. 14.6 Be particularly careful about which relationships are one-to-one vs. one-to-many vs. many-to-many.

#437. 15.3 One idea is to just not let a philosopher hold onto a chopstick if he can't get the other one.

#438. 12.9 Think about tracking the number of references. What will this tell us?

#439. 15.7 Don't try to do anything fancy on the single-threaded problem. Just get something that is simple and easily readable.

#440. 12.10 How will we free the memory?

#441. 15.2 It's okay if your solution isn't totally perfect. That might not be possible. Discuss the tradeoffs of your approach.

#442. 14.7 Think carefully about how you handle ties when selecting the top 10%.

#443.	13.8	A naive approach is to pick a random subset size z and then iterate through the elements, putting it in the set with probability `z/list_size`. Why would this not work?
#444.	14.5	Denormalization means adding redundant data to a table. It's typically used in very large systems. Why might this be useful?
#445.	12.5	A shallow copy copies just the initial data structure. A deep copy does this, and also copies any underlying data. Given this, why might you use one versus the other?
#446.	15.5	Would semaphores be useful here?
#447.	15.7	Outline the structure for the threads without worrying about synchronizing anything.
#448.	13.7	Consider how you'd implement this first without lambda expressions.
#449.	12.1	If we already had the number of lines in the file, how would we do this?
#450.	13.8	Pick the list of all the subsets of an n-element set. For any given item x, half of the subsets contain x and half do not.
#451.	14.4	Describe INNER JOINs and OUTER JOINs. OUTER JOINs can have multiple types: left, right, and full.
#452.	12.2	Be careful about the null character.
#453.	12.9	What are all the different methods/operators we might want to override?
#454.	13.5	What would the runtime of the common operations be?
#455.	14.5	Think about the cost of joins on a large system.
#456.	12.6	The keyword `volatile` signals that a variable might be changed from outside of the program, such as by another process. Why might this be necessary?
#457.	13.8	Do not pick the length of the subset in advance. You don't need to. Instead, think about this as picking whether each element will be put into the set.
#458.	15.7	Once you get the structure of each thread done, think about what you need to synchronize.
#459.	12.1	Suppose we didn't have the number of lines in the file. Is there a way we could do this without first counting the number of lines?
#460.	12.7	What would happen if the destructor were not virtual?
#461.	13.7	Break this up into two parts: filtering the countries and then getting a sum.
#462.	12.8	Consider using a hash table.
#463.	12.4	You should discuss vtables here.
#464.	13.7	Can you do this without a `filter` operation?

IV

Hints for Additional Review Problems

#465. 16.3 Think about what you're going to design for.

#466. 16.12 Consider a recursive or tree-like approach.

#467. 17.1 Walk through binary addition by hand (slowly!) and try to really understand what is happening.

#468. 16.13 Draw a square and a bunch of lines that cut it in half. Where are those lines located?

#469. 17.24 Start with a brute force solution.

#470. 17.14 There are actually several approaches. Brainstorm these. It's okay to start off with a naive approach.

#471. 16.20 Consider recursion.

#472. 16.3 Will all lines intercept? What determines if two lines intercept?

#473. 16.7 Let k be 1 if a > b and 0 otherwise. If you were given k, could you return the max (without a comparison or if-else logic)?

#474. 16.22 The tricky bit is handling an infinite grid. What are your options?

#475. 17.15 Try simplifying this problem: What if you just needed to know the longest word made up of two other words in the list?

#476. 16.10 Solution 1: Can you count the number of people alive in each year?

#477. 17.25 Start by grouping the dictionary by the word lengths, since you know each column has to be the same length and each row has to be the same length.

#478. 17.7 Discuss the naive approach: merging names together when they are synonyms. How would you identify transitive relationships? A == B, A == C, and C == D implies A == D == B == C.

#479. 16.13 Any straight line that cuts a square in half goes through the center of the square. How then can you find a line that cuts two squares in half?

#480. 17.17 Start with a brute force solution. What is the runtime?

#481. 16.22 Option #1: Do you actually need an infinite grid? Read the problem again. Do you know the max size of the grid?

#482. 16.16 Would it help to know the longest sorted sequences at the beginning and end?

#483. 17.2 Try approaching this problem recursively.

#484. 17.26 Solution 1: Start with just a simple algorithm comparing all documents to all other documents. How would you compute the similarity of two documents as fast as possible?

#485. 17.5 It doesn't really matter which letter or number it is. You can simplify this problem to just having an array of As and Bs. You would then be looking for the longest subarray with an equal number of As and Bs.

#486. 17.11 Consider first the algorithm for finding the closest distance if you will run the algorithm only once. You should be able to do this in O(N) time, where N is the number of words in the document.

#487. 16.20 Can you recursively try all possibilities?

#488. 17.9 Be clear about what this problem is asking for. It's asking for the kth smallest number in the form $3^a * 5^b * 7^c$.

#489. 16.2 Think about what the best conceivable runtime is for this problem. If your solution matches the best conceivable runtime, then you probably can't do any better.

#490. 16.10 Solution 1: Try using a hash table, or an array that maps from a birth year to how many people are alive in that year.

#491. 16.14 Sometimes, a brute force is a pretty good solution. Can you try all possible lines?

#492. 16.1 Try picturing the two numbers, a and b, on a number line.

#493. 17.7 The core part of the problem is to group names into the various spellings. From there, figuring out the frequencies is relatively easy.

#494. 17.3 If you haven't already, solve 17.2 on page 186.

#495. 17.16 There are recursive and iterative solutions to this problem, but it's probably easier to start with the recursive solution.

#496. 17.13 Try a recursive approach.

#497. 16.3 Infinite lines will almost always intersect—unless they're parallel. Parallel lines might still "intercept"—if they're the same lines. What does this mean for line segments?

#498. 17.26 Solution 1: To compute the similarity of two documents, try reorganizing the data in some way. Sorting? Using another data structure?

#499. 17.15 If we wanted to know just the longest word made up of other words in the list, then we could iterate over all words, from longest to shortest, checking if each could be made up of other words. To check this, we split the string in all possible locations.

#500. 17.25 Can you find a word rectangle of a specific length and width? What if you just tried all options?

#501. 17.11 Adapt your algorithm for one execution of the algorithm for repeated executions. What is the slow part? Can you optimize it?

#502. 16.8 Try thinking about the number in terms of chunks of three digits.

#503. 17.19 Start with the first part: Finding the missing number if only one number is missing.

#504. 17.16 Recursive solution: You have two choices at each appointment (take the appointment or reject the appointment). As a brute force approach, you can recurse through all possibilities. Note, though, that if you take request `i`, your recursive algorithm should skip request `i + 1`.

#505. 16.23 Be very careful that your solution actually returns each value from 0 through 6 with equal probability.

#506. 17.22 Start with a brute force, recursive solution. Just create all words that are one edit away, check if they are in the dictionary, and then attempt that path.

#507. 16.10 Solution 2: What if you sorted the years? What would you sort by?

#508. 17.9 What does a brute force solution to get the kth smallest value for $3^a * 5^b * 7^c$ look like?

#509. 17.12 Try a recursive approach.

#510. 17.26 Solution 1: You should be able to get an `O(A+B)` algorithm to compute the similarity of two documents.

#511. 17.24 The brute force solution requires us to continuously compute the sums of each matrix. Can we optimize this?

#512. 17.7 One thing to try is maintaining a mapping of each name to its "true" spelling. You would also need to map from a true spelling to all the synonyms. Sometimes, you might need to merge two different groups of names. Play around with this algorithm to see if you can get it to work. Then see if you can simplify/optimize it.

#513. 16.7 If k were 1 when `a > b` and 0 otherwise, then you could return `a*k + b*(not k)`. But how do you create k?

#514. 16.10 Solution 2: Do you actually need to match the birth years and death years? Does it matter when a specific person died, or do you just need a list of the years of deaths?

#515. 17.5 Start with a brute force solution.

#516. 17.16 Recursive solution: You can optimize this approach through memoization. What is the runtime of this approach?

#517. 16.3 How can we find the intersection between two lines? If two line segments intercept, then this must be at the same point as their "infinite" extensions. Is this intersection point within both lines?

#518. 17.26 Solution 1: What is the relationship between the intersection and the union? Can you compute one from the other?

#519. 17.20 Recall that the median means the number for which half the numbers are larger and half the numbers are smaller.

#520. 16.14 You can't truly try all possible lines in the world—that's infinite. But you know that a "best" line must intersect at least two points. Can you connect each pair of points? Can you check if each line is indeed the best line?

#521. 16.26 Can we just process the expression from left to right? Why might this fail?

#522. 17.10 Start with a brute force solution. Can you just check each value to see if it's the majority element?

#523. 16.10 Solution 2: Observe that people are "fungible." It doesn't matter who was born and when they died. All you need is a list of birth years and death years. This might make the question of how you sort the list of people easier.

#524. 16.25 First scope the problem. What are the features you would want?

#525. 17.24 Can you do any sort of precomputation to make computing the sum of a submatrix `O(1)`?

#526. 17.16 Recursive solution: The runtime of your memoization approach should be `O(N)`, with `O(N)` space.

#527. 16.3 Think carefully about how to handle the case of line segments that have the same slope and y-intercept.

#528. 16.13 To cut two squares in half, a line must go through the middle of both squares.

#529. 16.14 You should be able to get to an $O(N^2)$ solution.

#530. 17.14 Consider thinking about reorganizing the data in some way or using additional data structures.

#531. 16.17 Picture the array as alternating sequences of positive and negative numbers. Observe that we would never include just part of a positive sequence or part of a negative sequence.

#532. 16.10 Solution 2: Try creating a sorted list of births and a sorted list of deaths. Can you iterate through both, tracking the number of people alive at any one time?

#533. 16.22 Option #2: Think about how an `ArrayList` works. Can you use an `ArrayList` for this?

#534. 17.26 Solution 1: To understand the relationship between the union and the intersection of two sets, consider a Venn diagram (a diagram where one circle overlaps another circle).

#535. 17.22 Once you have a brute force solution, try to find a faster way of getting all valid words that are one edit away. You don't want to create all strings that are one edit away when the vast majority of them are not valid dictionary words.

#536. 16.2 Can you use a hash table to optimize the repeated case?

#537. 17.7 An easier way of taking the above approach is to have each name map to a list of alternate spellings. What should happen when a name in one group is set equal to a name in another group?

#538. 17.11 You could build a lookup table that maps from a word to a list of the locations where each word appears. How then could you find the closest two locations?

#539. 17.24 What if you precomputed the sum of the submatrix starting at the top left corner and continuing to each cell? How long would it take you to compute this? If you did this, could you then get the sum of an arbitrary submatrix in `O(1)` time?

#540. 16.22 Option #2: It's not impossible to use an `ArrayList`, but it would be tedious. Perhaps it would be easier to build your own, but specialized for matrices.

#541. 16.10 Solution 3: Each birth adds one person and each death removes a person. Try writing an example of a list of people (with birth and death years) and then re-formatting this into a list of each year and a +1 for a birth and a -1 for a death.

#542. 17.16 Iterative solution: Take the recursive solution and investigate it more. Can you implement a similar strategy iteratively?

#543. 17.15 Extend the earlier idea to multiple words. Can we just break each word up in all possible ways?

#544. 17.1 You can think about binary addition as iterating through the number, bit by bit, adding two bits, and then carrying over the one if necessary. You could also think about it as grouping the operations. What if you first added each of the bits (without carrying any overflow)? After that, you can handle the overflow.

#545. 16.21 Do some math here or play around with some examples. What does this pair need to look like? What can you say about their values?

#546. 17.20 Note that you have to store all the elements you've seen. Even the smallest of the first 100 elements could become the median. You can't just toss very low or very high elements.

#547. 17.26 Solution 2: It's tempting to try to think of minor optimizations—for example, keeping track of the min and max elements in each array. You could then figure out quickly, in specific cases, if two arrays don't overlap. The problem with that (and other optimizations along these lines) is that you still need to compare all documents to all other documents. It doesn't leverage the fact that the similarity is sparse. Given that we have a lot of documents, we really need to not compare all documents to all other documents (even if that comparison is very fast). All such solutions will be $O(D^2)$, where D is the number of documents. We shouldn't compare all documents to all other documents.

#548. 16.24 Start with a brute force solution. What is the runtime? What is the best conceivable runtime for this problem?

#549. 16.10 Solution 3: What if you created an array of years and how the population changed in each year? Could you then find the year with the highest population?

#550. 17.9 In looking for the kth smallest value of $3^a * 5^b * 7^c$, we know that a, b, and c will be less than or equal to k. Can you generate all such numbers?

#551. 16.17 Observe that if you have a sequence of values which have a negative sum, those will never start or end a sequence. (They could be present in a sequence if they connected two other sequences.)

#552. 17.14 Can you sort the numbers?

#553. 16.16 We can think about the array as divided into three subarrays: `LEFT`, `MIDDLE`, `RIGHT`. `LEFT` and `RIGHT` are both sorted. The `MIDDLE` elements are in an arbitrary order. We need to expand `MIDDLE` until we could sort those elements and then have the entire array sorted.

#554. 17.16 Iterative solution: It's probably easiest to start with the end of the array and work back wards.

#555. 17.26 Solution 2: If we can't compare all documents to all other documents, then we need to dive down and start looking at things at the element level. Consider a naive solution and see if you can extend that to multiple documents.

#556. 17.22 To quickly get the valid words that are one edit away, try to group the words in the dictionary in a useful way. Observe that all words in the form `b_ll` (such as `bill`, `ball`, `bell`, and `bull`) will be one edit away. However, those aren't the only words that are one edit away from `bill`.

#557. 16.21 When you move a value a from array A to array B, then A's sum decreases by a and B's sum increases by a. What happens when you swap two values? What would be needed to swap two values and get the same sum?

#558. 17.11 If you had a list of the occurrences of each word, then you are really looking for a pair of values within two arrays (one value for each array) with the smallest difference. This could be a fairly similar algorithm to your initial algorithm.

#559. 16.22 Option #2: One approach is to just double the size of the array when the ant wanders to an edge. How will you handle the ant wandering into negative coordinates, though? Arrays can't have negative indices.

#560. 16.13 Given a line (slope and y-intercept), can you find where it intersects another line?

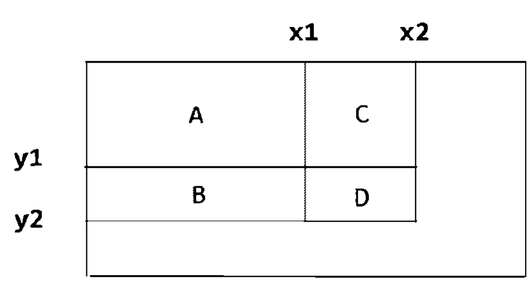
#561. 17.26 Solution 2: One way to think about this is that we need to be able to very quickly pull a list of all documents with some similarity to a specific document. (Again, we should not do this by saying "look at all documents and quickly eliminate the dissimilar documents." That will be at least $O(D^2)$.)

#562. 17.16 Iterative solution: Observe that you would never skip three appointments in a row. Why would you? You would always be able to take the middle booking.

#563. 16.14 Have you tried using a hash table?

#564. 16.21 If you swap two values, a and b, then the sum of A becomes `sumA - a + b` and the sum of B becomes `sumB - b + a`. These sums need to be equal.

#565. 17.24 If you can precompute the sum from the top left corner to each cell, you can use this to compute the sum of an arbitrary submatrix in $O(1)$ time. Picture a particular submatrix. The full, precomputed sum will include this submatrix, an array immediately above it (C), and array to the left (B), and an area to the top and left (A). How can you compute the sum of just D?



#566. 17.10 Consider the brute force solution. We pick an element and then validate if it's the majority element by counting the number of matching and non-matching elements. Suppose, for the first element, the first few checks reveal seven non-matching elements and three matching elements. Is it necessary to keep checking this element?

#567. 16.17 Start from the beginning of the array. As that subsequence gets larger, it stays as the best subsequence. Once it becomes negative, though, it's useless.

#568. 17.16 Iterative solution: If you take appointment `i`, you will never take appointment `i + 1`, but you will always take appointment `i + 2` or `i + 3`.

#569. 17.26 Solution 2: Building off the earlier hint, we can ask what defines the list of documents with some similarity to a document like {13, 16, 21, 3}. What attributes does that list have? How would we gather all documents like that?

#570. 16.22 Option #2: Observe that nothing in the problem stipulates that the label for the coordinates must remain the same. Can you move the ant and all cells into positive coordinates? In other words, what would happen if, whenever you needed to grow the array in a negative direction, you relabeled all the indices such that they were still positive?

#571. 16.21 You are looking for values a and b where `sumA - a + b = sumB - b + a`. Do the math to work out what this means for a and b's values.

#572. 16.9 Approach these one by one, starting with subtraction. Once you've completed one function, you can use it to implement the others.

#573. 17.6 Start with a brute force solution.

#574. 16.23 Start with a brute force solution. How many times does it call `rand5()` in the worst case?

#575. 17.20 Another way to think about this is: Can you maintain the bottom half of elements and the top half of elements?

#576. 16.10 Solution 3: Be careful with the little details in this problem. Does your algorithm/code handle a person who dies in the same year that they are born? This person should be counted as one person in the population count.

#577. 17.26 Solution 2: The list of documents similar to {13, 16, 21, 3} includes all documents with a 13, 16, 21, and 3. How can we efficiently find this list? Remember that we'll be doing this for many documents, so some precomputing can make sense.

#578. 17.16 Iterative solution: Use an example and work backwards. You can easily find the optimal solution for the subarrays $\{r_n\}$, $\{r_{n-1}, r_n\}$, $\{r_{n-2}, ..., r_n\}$. How would you use those to quickly find the optimal solution for $\{r_{n-3}, ..., r_n\}$?

#579. 17.2 Suppose you had a method `shuffle` that worked on decks up to `n - 1` elements. Could you use this method to implement a new `shuffle` method that works on decks up to n elements?

#580. 17.22 Create a mapping from a wildcard form (like `b_ll`) to all words in that form. Then, when you want to find all words that are one edit away from `bill`, you can look up `_ill`, `b_ll`, `bi_l`, and `bil_` in the mapping.

#581. 17.24 The sum of just D will be `sum(A&B&C&D) - sum(A&B) - sum(A&C) + sum(A)`.

#582. 17.17 Can you use a trie?

#583. 16.21 If we do the math, we are looking for a pair of values such that `a - b = (sumA - sumB) / 2`. The problem then reduces to looking for a pair of values with a particular difference.

#584. 17.26 Solution 2: Try building a hash table from each word to the documents that contain this word. This will allow us to easily find all documents with some similarity to {13, 16, 21, 3}.

#585. 16.5 How does a zero get into the result of `n!`? What does it mean?

#586. 17.7 If each name maps to a list of its alternate spellings, you might have to update a lot of lists when you set X and Y as synonyms. If X is a synonym of {A, B, C}, and Y is a synonym of {D, E, F} then you would need to add {Y, D, E, F} to A's synonym list, B's synonym list, C's synonym list, and X's synonym list. Ditto for {Y, D, E, F}. Can we make this faster?

#587. 17.16 Iterative solution: If you take an appointment, you can't take the next appointment, but you can take anything after that. Therefore, `optimal(`r_i`, ..., `r_n`) = max(`r_i` + optimal(`r_{i+2}`, ..., `r_n`), optimal(`r_{i+1}`, ..., `r_n`))`. You can solve this iteratively by working backwards.

#588. 16.8 Have you considered negative numbers? Does your solution work for values like 100,030,000?

#589. 17.15 When you get recursive algorithms that are very inefficient, try looking for repeated subproblems.

#590. 17.19 Part 1: If you have to find the missing number in O(1) space and O(N) time, then you can do a only constant number of passes through the array and can store only a few variables.

#591. 17.9 Look at the list of all values for $3^a * 5^b * 7^c$. Observe that each value in the list will be 3*(some previous value), 5*(some previous value), or 7*(some previous value).

#592. 16.21 A brute force solution is to just look through all pairs of values to find one with the right difference. This will probably look like an outer loop through A with an inner loop through B. For each value, compute the difference and compare it to what we're looking for. Can we be more specific here, though? Given a value in A and a target difference, do we know the exact value of the element within B we're looking for?

#593. 17.14 What about using a heap or tree of some sort?

#594. 16.17 If we tracked the running sum, we should reset it as soon as the subsequence becomes negative. We would never add a negative sequence to the beginning or end of another subsequence.

#595. 17.24 With precomputation, you should be able to get a runtime of $O(N^4)$. Can you make this even faster?

#596. 17.3 Try this recursively. Suppose you had an algorithm to get a subset of size m from n - 1 elements. Could you develop an algorithm to get a subset of size m from n elements?

#597. 16.24 Can we make this faster with a hash table?

#598. 17.22 Your previous algorithm probably resembles a depth-first search. Can you make this faster?

#599. 16.22 Option #3: Another thing to think about is whether you even need a grid to implement this. What information do you actually need in the problem?

#600. 16.9 Subtraction: Would a negate function (which converts a positive integer to negative) help? Can you implement this using the add operator?

#601. 17.1 Focus on just one of the steps above. If you "forgot" to carry the ones, what would the add operation look like?

#602. 16.21 What the brute force really does is look for a value within B which equals a - `target`. How can you more quickly find this element? What approaches help us quickly find out if an element exists within an array?

#603. 17.26 Solution 2: Once you have a way of easily finding the documents similar to a particular document, you can go through and just compute the similarity to those documents using a simple algorithm. Can you make this faster? Specifically, can you compute the similarity directly from the hash table?

#604. 17.10 The majority element will not necessarily look like the majority element at first. It is possible, for example, to have the majority element appear in the first element of the array and then not appear again for the next eight elements. However, in those cases, the majority element will appear later in the array (in fact, many times later on in the array). It's not necessarily critical to continue checking a specific instance of an element for majority status once it's already looking "unlikely."

#605. 17.7 Instead, X, A, B, and C should map to the same instance of the set `{X, A, B, C}`. Y, D, E, and F should map to the same instance of `{Y, D, E, F}`. When we set X and Y as synonyms, we can then just copy one of the sets into the other (e.g., add `{Y, D, E, F}` to `{X, A, B, C}`). How else do we change the hash table?

#606. 16.21 We can use a hash table here. We can also try sorting. Both help us locate elements more quickly.

#607. 17.16 Iterative solution: If you're careful about what data you really need, you should be able to solve this in `O(n)` time and `O(1)` additional space.

#608. 17.12 Think about it this way: If you had methods called `convertLeft` and `convertRight` (which would convert left and right subtrees to doubly linked lists), could you put those together to convert the whole tree to a doubly linked list?

#609. 17.19 Part 1: What if you added up all the values in the array? Could you then figure out the missing number?

#610. 17.4 How long would it take you to figure out the least significant bit of the missing number?

#611. 17.26 Solution 2: Imagine you are looking up the documents similar to {1, 4, 6} by using a hash table that maps from a word to documents. The same document ID appears multiple times when doing this lookup. What does that indicate?

#612. 17.6 Rather than counting the number of twos in each number, think about digit by digit. That is, count the number of twos in the first digit (for each number), then the number of twos in the second digit (for each number), then the number of twos in the third digit (for each number), and so on.

#613. 16.9 Multiply: it's easy enough to implement `multiply` using add. But how do you handle negative numbers?

#614. 16.17 You can solve this in `O(N)` time and `O(1)` space.

#615. 17.24 Suppose this was just a single array. How could we compute the subarray with the largest sum? See 16.17 for a solution to this.

#616. 16.22 Option #3: All you actually need is some way of looking up if a cell is white or black (and of course the position of the ant). Can you just keep a list of all the white cells?

#617. 17.17 One solution is to insert every suffix of the larger string into the trie. For example, if the word is dogs, the suffixes would be dogs, ogs, gs, and s. How would this help you solve the problem? What is the runtime here?

#618. 17.22 A breadth-first search will often be faster than a depth-first search—not necessarily in the worst case, but in many cases. Why? Can you do something even faster than this?

#619. 17.5 What if you just started from the beginning, counting the number of As and the number of Bs you've seen so far? (Try making a table of the array and the number of As and Bs thus far.)

#620. 17.10 Note also that the majority element must be the majority element for some subarray and that no subarray can have multiple majority elements.

#621. 17.24 Suppose I just wanted you to find the maximum submatrix starting at row r1 and ending at row r2, how could you most efficiently do this? (See the prior hint.) If I now wanted you find the maximum subarray from r1 to (r2+2), could you do this efficiently?

#622. 17.9 Since each number is 3, 5, or 7 times a previous value in the list, we could just check all possible values and pick the next one that hasn't been seen yet. This will result in a lot of duplicated work. How can we avoid this?

#623. 17.13 Can you just try all possibilities? What might that look like?

#624. 16.26 Multiplication and division are higher priority operations. In an expression like 3*4 + 5*9/2 + 3, the multiplication and division parts need to be grouped together.

#625. 17.14 If you picked an arbitrary element, how long would it take you to figure out the rank of this element (the number of elements bigger or smaller than it)?

#626. 17.19 Part 2: We're now looking for two missing numbers, which we will call a and b. The approach from part 1 will tell us the sum of a and b, but it won't actually tell us a and b. What other calculations could we do?

#627. 16.22 Option #3: You could consider keeping a hash set of all the white cells. How will you be able to print the whole grid, though?

#628. 17.1 The adding step alone would convert 1 + 1 -> 0, 1 + 0 -> 1, 0 + 1 -> 1, 0 + 0 -> 0. How do you do this without the + sign?

#629. 17.21 What role does the tallest bar in the histogram play?

#630. 16.25 What data structure would be most useful for the lookups? What data structure would be most useful to know and maintain the order of items?

#631. 16.18 Start with a brute force approach. Can you try all possibilities for a and b?

#632. 16.6 What if you sorted the arrays?

#633. 17.11 Can you just iterate through both arrays with two pointers? You should be able to do it in O(A+B) time, where A and B are the sizes of the two arrays.

#634. 17.2 You could build this algorithm recursively by swapping the nth element for any of the elements before it. What would this look like iteratively?

#635. 16.21 What if the sum of A is 11 and the sum of B is 8? Can there be a pair with the right difference? Check that your solution handles this situation appropriately.

#636. 17.26 Solution 3: There's an alternative solution. Consider taking all of the words from all of the documents, throwing them into one giant list, and sorting this list. Assume you could still know which document each word came from. How could you track the similar pairs?

#637. 16.23 Make a table indicating how each possible sequence of calls to `rand5()` would map to the result of `rand7()`. For example, if you were implementing `rand3()` with `(rand2() + rand2()) % 3`, then the table would look like the below. Analyze this table. What can it tell you?

```
1st   2nd   Result
0     0     0
0     1     1
1     0     1
1     1     2
```

#638. 17.8 This problem asks us to find the longest sequence of pairs you can build such that both sides of the pair are constantly increasing. What if you needed only one side of the pair to increase?

#639. 16.15 Try first creating an array with the frequency that each item occurs.

#640. 17.21 Picture the tallest bar, and then the next tallest bar on the left and the next tallest bar on the right. The water will fill the area between those. Can you calculate that area? What do you do about the rest?

#641. 17.6 Is there a faster way of calculating how many twos are in a particular digit across a range of numbers? Observe that roughly $\frac{1}{10}$ th of any digit should be a 2—but only roughly. How do you make that more exact?

#642. 17.1 You can do the add step with an XOR.

#643. 16.18 Observe that one of the substrings, either a or b, must start at the beginning of the string. That cuts down the number of possibilities.

#644. 16.24 What if the array were sorted?

#645. 17.18 Start with a brute force solution.

#646. 17.12 Once you have a basic idea for a recursive algorithm, you might get stuck on this: sometimes your recursive algorithm needs to return the start of the linked list, and sometimes it needs to return the end. There are multiple ways of solving this issue. Brainstorm some of them.

#647. 17.14 If you picked an arbitrary element, you would, on average, wind up with an element around the 50th percentile mark (half the elements above it and half the elements below). What if you did this repeatedly?

#648. 16.9 Divide: If you're trying to compute, where $x = \frac{a}{b}$, remember that a = bx. Can you find the closest value for x? Remember that this is integer division and x should be an integer.

#649. 17.19 Part 2: There are a lot of different calculations we could try. For example, we could multiply all the numbers, but that will only lead us to the product of a and b.

#650. 17.10 Try this: Given an element, start checking if this is the start of a subarray for which it's the majority element. Once it's become "unlikely" (appears less than half the time), start checking at the next element (the element after the subarray).

#651. 17.21 You can calculate the area between the tallest bar overall and the tallest bar on the left by just iterating through the histogram and subtracting out any bars in between. You can do the same thing with the right side. How do you handle the remainder of the graph?

#652. 17.18 One brute force solution is to take each starting position and move forward until you've found a subsequence which contains all the target characters.

#653. 16.18 Don't forget to handle the possibility that the first character in the `pattern` is `b`.

#654. 16.20 In the real world, we should know that some prefixes/substrings won't work. For example, consider the number 33835676368. Although 3383 does correspond to `fftf`, there are no words that start with `fftf`. Is there a way we can short-circuit in cases like this?

#655. 17.7 An alternative approach is to think of this as a graph. How would this work?

#656. 17.13 You can think about the choices the recursive algorithm makes in one of two ways: (1) At each character, should I put a space here? (2) Where should I put the next space? You can solve both of these recursively.

#657. 17.8 If you needed only one side of the pair to increase, then you would just sort all the values on that side. Your longest sequence would in fact be all of the pairs (other than any duplicates, since the longest sequence needs to strictly increase). What does this tell you about the original problem?

#658. 17.21 You can handle the remainder of the graph by just repeating this process: find the tallest bar and the second tallest bar, and subtract out the bars in between.

#659. 17.4 To find the least significant bit of the missing number, note that you know how many `0`s and `1`s to expect. For example, if you see three `0`s and three `1`s in the least significant bit, then the missing number's least significant bit must be a `1`. Think about it: in any sequence of `0`s and `1`s, you'd get a `0`, then a `1`, then a `0`, then a `1`, and so on.

#660. 17.9 Rather than checking all values in the list for the next value (by multiplying each by 3, 5, and 7), think about it this way: when you insert a value `x` into the list, you can "create" the values `3x`, `5x`, and `7x` to be used later.

#661. 17.14 Think about the previous hint some more, particularly in the context of quicksort.

#662. 17.21 How can you make the process of finding the next tallest bar on each side faster?

#663. 16.18 Be careful with how you analyze the runtime. If you iterate through $O(n^2)$ substrings and each one does an $O(n)$ string comparison, then the total runtime is $O(n^3)$.

#664. 17.1 Now focus on the carrying. In what cases will values carry? How do you apply the carry to the number?

#665. 16.26 Consider thinking about it as, when you get to a multiplication or division sign, jumping to a separate "process" to compute the result of this chunk.

#666. 17.8 If you sort the values based on height, then this will tell you the ordering of the final pairs. The longest sequence must be in this relative order (but not necessarily containing all of the pairs). You now just need to find the longest increasing subsequence on weight while keeping the items in the same relative order. This is essentially the same problem as having an array of integers and trying to find the longest sequence you can build (without reordering those items).

#667. 16.16 Consider the three subarrays: LEFT, MIDDLE, RIGHT. Focus on just this question: Can you sort middle such that the entire array becomes sorted? How would you check this?

#668. 16.23 Looking at this table again, note that the number of rows will be 5^k, where k is the max number of calls to `rand5()`. In order to make each value between 0 and 6 have equal probability, $\frac{1}{7}$ th of the rows must map to 0, $\frac{1}{7}$ th to 1, and so on. Is this possible?

#669. 17.18 Another way of thinking about the brute force is that we take each starting index and find the next instance of each element in the target string. The maximum of all these next instances marks the end of a subsequence which contains all the target characters. What is the runtime of this? How can we make it faster?

#670. 16.6 Think about how you would merge two sorted arrays.

#671. 17.5 When the above tables have equal values for the number of As and Bs, the entire subarray (starting from index 0) has an equal number of As and Bs. How could you use this table to find qualifying subarrays that don't start at index 0?

#672. 17.19 Part 2: Adding the numbers together will tell us the result of `a + b`. Multiplying the numbers together will tell us the result of `a * b`. How can we get the exact values for a and b?

#673. 16.24 If we sorted the array, we could do repeated binary searches for the complement of a number. What if, instead, the array is given to us sorted? Could we then solve the problem in `O(N)` time and `O(1)` space?

#674. 16.19 If you were given the row and column of a water cell, how can you find all connected spaces?

#675. 17.7 We can treat adding X, Y as synonyms as adding an edge between the X node and the Y node. How then do we figure out the groups of synonyms?

#676. 17.21 Can you do precomputation to compute the next tallest bar on each side?

#677. 17.13 Will the recursive algorithm hit the same subproblems repeatedly? Can you optimize with a hash table?

#678. 17.14 What if, when you picked an element, you swapped elements around (as you do in quicksort) so that the elements below it would be located before the elements above it? If you did this repeatedly, could you find the smallest one million numbers?

#679. 16.6 Imagine you had the two arrays sorted and you were walking through them. If the pointer in the first array points to 3 and the pointer in the second array points to 9, what effect will moving the second pointer have on the difference of the pair?

#680. 17.12 To handle whether your recursive algorithm should return the start or the end of the linked list, you could try to pass a parameter down that acts as a flag. This won't work very well, though. The problem is that when you call `convert(current.left)`, you want to get the end of `left`'s linked list. This way you can join the end of the linked list to `current`. But, if `current` is someone else's right subtree, `convert(current)` needs to pass back the start of the linked list (which is actually the start of `current.left`'s linked list). Really, you need both the start and end of the linked list.

#681. 17.18 Consider the previously explained brute force solution. A bottleneck is repeatedly asking for the next instance of a particular character. Is there a way you can optimize this? You should be able to do this in `O(1)` time.

#682. 17.8 Try a recursive approach that just evaluates all possibilities.

#683. 17.4 Once you've identified that the least significant bit is a 0 (or a 1), you can rule out all the numbers without 0 as the least significant bit. How is this problem different from the earlier part?

#684. 17.23 Start with a brute force solution. Can you try the biggest possible square first?

#685. 16.18 Suppose you decide on a specific value for the "a" part of a pattern. How many possibilities are there for b?

#686. 17.9 When you add x to the list of the first k values, you can add 3x, 5x, and 7x to some new list. How do you make this as optimal as possible? Would it make sense to keep multiple queues of values? Do you always need to insert 3x, 5x, and 7x? Or, perhaps sometimes you need to insert only 7x? You want to avoid seeing the same number twice.

#687. 16.19 Try recursion to count the number of water cells.

#688. 16.8 Consider dividing up a number into sequences of three digits.

#689. 17.19 Part 2: We could do both. If we know that `a + b = 87` and `a * b = 962`, then we can solve for a and b: `a = 13` and `b = 74`. But this will also result in having to multiply really large numbers. The product of all the numbers could be larger than 10^{157}. Is there a simpler calculation you can make?

#690. 16.11 Consider building a diving board. What are the choices you make?

#691. 17.18 Can you precompute the next instance of a particular character from each index? Try using a multi-dimensional array.

#692. 17.1 The carry will happen when you are doing 1 + 1. How do you apply the carry to the number?

#693. 17.21 As an alternative solution, think about it from the perspective of each bar. Each bar will have water on top of it. How much water will be on top of each bar?

#694. 16.25 Both a hash table and a doubly linked list would be useful. Can you combine the two?

#695. 17.23 The biggest possible square is NxN. So if you try that square first and it works, then you know that you've found the best square. Otherwise, you can try the next smallest square.

#696. 17.19 Part 2: Almost any "equation" we can come up with will work here (as long as it's not equivalent to a linear sum). It's just a matter of keeping this sum small.

#697. 16.23 It is not possible to divide 5^k evenly by 7. Does this mean that you can't implement `rand7()` with `rand5()`?

#698. 16.26 You can also maintain two stacks, one for the operators and one for the numbers. You push a number onto the stack every time you see it. What about the operators? When do you pop operators from the stack and apply them to the numbers?

#699. 17.8 Another way to think about the problem is this: if you had the longest sequence ending at each element `A[0]` through `A[n-1]`, could you use that to find the longest sequence ending at element `A[n-1]`?

#700. 16.11 Consider a recursive solution.

#701. 17.12 Many people get stuck at this point and aren't sure what to do. Sometimes they need the start of the linked list, and sometimes they need the end. A given node doesn't necessarily know what to return on its `convert` call. Sometimes the simple solution is easiest: always return both. What are some ways you could do this?

#702. 17.19 Part 2: Try a sum of squares of the values.

#703. 16.20 A trie might help us short-circuit. What if you stored the whole list of words in the trie?

#704. 17.7 Each connected subgraph represents a group of synonyms. To find each group, we can do repeated breadth-first (or depth-first) searches.

#705. 17.23 Describe the runtime of the brute force solution.

#706. 16.19 How can you make sure that you're not revisiting the same cells? Think about how breadth-first search or depth-first search on a graph works.

#707. 16.7 When `a > b`, then `a - b > 0`. Can you get the sign bit of `a - b`?

#708. 16.16 In order to be able to sort `MIDDLE` and have the whole array become sorted, you need `MAX(LEFT) <= MIN(MIDDLE and RIGHT)` and `MAX(LEFT and MIDDLE) <= MIN(RIGHT)`.

#709. 17.20 What if you used a heap? Or two heaps?

#710. 16.4 If you were calling `hasWon` multiple times, how might your solution change?

#711. 16.5 Each zero in `n!` corresponds to `n` being divisible by a factor of 10. What does that mean?

#712. 17.1 You can use an AND operation to compute the carry. What do you do with it?

#713. 17.5 Suppose, in this table, index `i` has `count(A, 0->i) = 3` and `count(B, 0->i) = 7`. This means that there are four more Bs than As. If you find a later spot `j` with the same difference (`count(B, 0->j) - count(A, 0->j)`), then this indicates a subarray with an equal number of As and Bs.

#714. 17.23 Can you do preprocessing to optimize this solution?

#715. 16.11 Once you have a recursive algorithm, think about the runtime. Can you make this faster? How?

#716. 16.1 Let `diff` be the difference between `a` and `b`. Can you use `diff` in some way? Then can you get rid of this temporary variable?

#717. 17.19 Part 2: You might need the quadratic formula. It's not a big deal if you don't remember it. Most people won't. Remember that there is such a thing as good enough.

#718. 16.18 Since the value of `a` determines the value of `b` (and vice versa) and either `a` or `b` must start at the beginning of the value, you should have only `O(n)` possibilities for how to split up the pattern.

#719. 17.12 You could return both the start and end of a linked list in multiple ways. You could return a two-element array. You could define a new data structure to hold the start and end. You could re-use the `BiNode` data structure. If you're working in a language that supports this (like Python), you could just return multiple values. You could solve the problem as a circular linked list, with the start's previous pointer pointing to the end (and then break the circular list in a wrapper method). Explore these solutions. Which one do you like most and why?

#720. 16.23 You can implement `rand7()` with `rand5()`, you just can't do it deterministically (such that you know it will definitely terminate after a certain number of calls). Given this, write a solution that works.

#721. 17.23 You should be able to do this in $O(N^3)$ time, where N is the length of one dimension of the square.

#722. 16.11 Consider memoization to optimize the runtime. Think carefully about what exactly you cache. What is the runtime? The runtime is closely related to the max size of the table.

#723. 16.19 You should have an algorithm that's $O(N^2)$ on an NxN matrix. If your algorithm isn't, consider if you've miscomputed the runtime or if your algorithm is suboptimal.

#724. 17.1 You might need to do the add/carry operation more than once. Adding `carry` to `sum` might cause new values to carry.

#725. 17.18 Once you have the precomputation solution figured out, think about how you can reduce the space complexity. You should be able to get it down to $O(SB)$ time and $O(B)$ space (where B is the size of the larger array and S is the size of the smaller array).

#726. 16.20 We're probably going to run this algorithm many times. If we did more preprocessing, is there a way we could optimize this?

#727. 16.18 You should be able to have an $O(n^2)$ algorithm.

#728. 16.7 Have you considered how to handle integer overflow in `a - b`?

#729. 16.5 Each factor of 10 in `n!` means `n!` is divisible by 5 and 2.

#730. 16.15 For ease and clarity in implementation, you might want to use other methods and classes.

#731. 17.18 Another way to think about it is this: Imagine you had a list of the indices where each item appeared. Could you find the first possible subsequence with all the elements? Could you find the second?

#732. 16.4 If you were designing this for an NxN board, how might your solution change?

#733. 16.5 Can you count the number of factors of 5 and 2? Do you need to count both?

#734. 17.21 Each bar will have water on top of it that matches the minimum of the tallest bar on the left and the tallest bar on the right. That is, `water_on_top[i] = min(tallest_bar(0->i), tallest_bar(i, n))`.

#735. 16.16 Can you expand the middle until the earlier condition is met?

#736. 17.23 When you're checking to see if a particular square is valid (all black borders), you check how many black pixels are above (or below) a coordinate and to the left (or right) of this coordinate. Can you precompute the number of black pixels above and to the left of a given cell?

#737. 16.1 You could also try using XOR.

#738. 17.22 What if you did a breadth-first search starting from both the source word and the destination word?

#739. 17.13 In real life, we would know that some paths will not lead to a word. For example, there are no words that start with `hellothisism`. Can we terminate early when going down a path that we know won't work?

#740. 16.11 There's an alternate, clever (and very fast) solution. You can actually do this in linear time without recursion. How?

#741. 17.18 Consider using a heap.

#742. 17.21 You should be able to solve this in O(N) time and O(N) space.

#743. 17.17 Alternatively, you could insert each of the smaller strings into the trie. How would this help you solve the problem? What is the runtime?

#744. 16.20 With preprocessing, we can actually get the lookup time down to O(1).

#745. 16.5 Have you considered that 25 actually accounts for two factors of 5?

#746. 16.16 You should be able to solve this in O(N) time.

#747. 16.11 Think about it this way. You are picking K planks and there are two different types. All choices with 10 of the first type and 4 of the second type will have the same sum. Can you just iterate through all possible choices?

#748. 17.25 Can you use a trie to terminate early when a rectangle looks invalid?

#749. 17.13 For early termination, try a trie.

XIV

About the Author

Gayle Laakmann McDowell has a strong background in software development with extensive experience on both sides of the hiring table.

She has worked for Microsoft, Apple, and Google as a software engineer. She spent three years at Google, where she was one of the top interviewers and served on the hiring committee. She interviewed hundreds of candidates in the U.S. and abroad, assessed thousands of candidate interview packets for the hiring committee, and reviewed many more resumes.

As a candidate, she interviewed with—and received offers from—twelve tech companies, including Microsoft, Google, Amazon, IBM, and Apple.

Gayle founded CareerCup to enable candidates to perform at their best during these challenging interviews. CareerCup.com offers a database of thousands of interview questions from major companies and a forum for interview advice.

In addition to ***Cracking the Coding Interview***, Gayle has written other two books:

- ***Cracking the Tech Career: Insider Advice on Landing a Job at Google, Microsoft, Apple, or Any Top Tech Company*** provides a broader look at the interview process for major tech companies. It offers insight into how anyone, from college freshmen to marketing professionals, can position themselves for a career at one of these companies.
- ***Cracking the PM Interview: How to Land a Product Manager Job in Technology*** focuses on product management roles at startups and big tech companies. It offers strategies to break into these roles and teaches job seekers how to prepare for PM interviews.

Through her role with CareerCup, she consults with tech companies on their hiring process, leads technical interview training workshops, and coaches engineers at startups for acquisition interviews.

She holds bachelor's degree and master's degrees in computer science from the University of Pennsylvania and an MBA from the Wharton School.

She lives in Palo Alto, California, with her husband, two sons, dog, and computer science books. She still codes daily.

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GAYLE LAAKMANN MCDOWELL

Gayle Laakmann McDowell is the founder and CEO of CareerCup and the author of Cracking the PM Interview and Cracking the Tech Career.

Cayle has a strong background in software development, having worked as a software engineer at Google, Microsoft, and Apple. At Google, she interviewed hundreds of software engineers and evaluated thousands of hiring packets as part of the hiring committee. She holds a B.S.E. and M.S.E. in computer science from the University of Pennsylvania and an MBA from the Wharton School.

She now consults with tech companies to improve their hiring process and with startups to prepare them for acquisition interviews.

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