STANDARD 12 — PROBABILITY AND STATISTICS

K-12 Overview

All students will develop an understanding of statistics and probability and will use them to describe sets of data, model situations, and support appropriate inferences and arguments.

Descriptive Statement

Probability and statistics are the mathematics used to understand chance and to collect, organize, describe, and analyze numerical data. From weather reports to sophisticated studies of genetics, from election results to product preference surveys, probability and statistical language and concepts are increasingly present in the media and in everyday conversations. Students need this mathematics to help them judge the correctness of an argument supported by seemingly persuasive data.

Meaning and Importance

Probability is the study of random events. It is used in analyzing games of chance, genetics, weather prediction, and a myriad of other everyday events. Statistics is the mathematics we use to collect, organize, and interpret numerical data. It is used to describe and analyze sets of test scores, election results, and shoppers' preferences for particular products. Probability and statistics are closely linked because statistical data are frequently analyzed to see whether conclusions can be drawn legitimately about a particular phenomenon and also to make predictions about future events. For instance, early election results are analyzed to see if they conform to predictions from pre-election polls and also to predict the final outcome of the election.

Understanding probability and statistics is essential in the modern world, where the print and electronic media are full of statistical information and interpretation. The goal of mathematical instruction in this area should be to make students sensible, critical users of probability and statistics, able to apply their processes and principles to real-world problems. Students should not think that those people who did not win the lottery yesterday have a greater chance of winning today! They should not believe an argument merely because various statistics are offered. Rather, they should be able to judge whether the statistics are meaningful and are being used appropriately.

K-12 Development and Emphases

Statistics and probability naturally lend themselves to plenty of fun, hands-on cooperative learning and group activities. Activities with spinners, dice, and coin tossing can be used to investigate chance events. Students should discuss the theoretical probabilities of different events such as the possible sums of a pair of dice, and

check them experimentally. They can choose topics to investigate, such as how much milk and juice the cafeteria should order each day, gather statistics on current orders and student preferences, and make predictions on future use. Connections between these topics and everyday experiences provide motivation and a sense of relevance to students.

In the area of probability, young children start out simply learning to use **probability terms** correctly. Words like *possibly*, *probably*, and *certainly* have definite meanings, referring to the increasing likelihood of an event happening, and it takes children some time to begin to use them correctly. Beyond that, though, elementary age children are certainly able to understand the **probability of an event**. Starting with phrases like *once in six tosses*, children progress to more sophisticated probability language like *chances are one out of six*, and finally to standard fractional, decimal, and percent notation for the expression of a probability. To motivate and foster that maturation, students should be regularly engaged in **predicting and determining probabilities**.

Experiments leading to discussions about the difference between experimental and theoretical probability should be done by older elementary and middle school students**. The theoretical probability** is the probability based on a mathematical analysis of the physical properties and behavior of the objects involved in the event. For instance, when a fair die is rolled each face is equally likely to wind up on top, and so the probability of any particular face showing is one-sixth. **Experimental probabilities** are determined by data gathered through experiments. For example, students may be able to compare the experimental probabilities of rolling a sum of seven vs. a sum of four with two dice long before they can explain why the first is twice as likely from a theoretical point of view.

Older students should understand the difference between **simple and compound events,** like rolling one die vs. rolling two dice, and the difference between **independent and dependent events,** like picking five marbles out of a bag of blue and green marbles one at a time with replacement vs. without replacement. Again, the best way to approach this content is with open-ended investigations that allow the students to arrive at their own conclusions through experimentation and discussion. Eventually, students should feel comfortable representing real-life events using **probability models**.

In statistics, young children can start out as early as kindergarten with **data collection, organization, and graphing**. The focus on those skills, with obviously increasing sophistication, should last throughout their schooling. Students must be able to understand the tables, charts, and graphs used to present data, and they must be able to organize their own data into formats which make them easier to understand. While young students can do exhaustive surveys about some interesting question for *all* of the members of the class, older students should focus some time and energy on the questions involved with **sampling**, where information is obtained from only *some* of the members of a group. Identifying and obtaining data from a well-defined sample of the population is one of the most challenging tasks of a professional pollster.

As students progress through the elementary grades, an increased focus on **central tendency** and later, on **variance and correlation,** are appropriate. Students should be able to use the *average* or *mean*, the *median*, and the *mode* and understand the differences in their uses. Measures of the variance from the center of a set of data, or dispersion, also provide useful insights into sets of numbers. These can be introduced early with the *range* for the early grades, *box-and-whisker plots* showing quartiles of a data distribution for upper elementary school students, and progress to measures like *standard deviation* for older students.

The reason statistics grew as a branch of mathematics, however, was to provide tools that are helpful in **analysis and inference** in situations of uncertainty, and that focus should permeate everything students do in

this area. Whenever they look at data, they should be trying to answer a question, support a position, or discover a pattern. Students at all grade levels should have many opportunities to look for patterns, draw conclusions, and make predictions about the outcomes of future experiments, polls, surveys, and so on. They should examine data to see whether they are consistent with some hypotheses that a classmate may already have made, and learn to judge whether the data are reliable or whether the hypothesis might need revision.

IN SUMMARY, probability and statistics hold the key for enabling our students to better understand, process, and interpret the vast amounts of quantitative data that exist all around them, and to have a probabilistic sense in situations of uncertainty. To be able to judge the validity of a data-supported argument presented to them, to discern the believability of a persuasive advertisement that talks about the results of a survey of all of the users of a particular product, or to be knowledgeable consumers of the data-intensive government and electoral statistics that are ever-present, students need the skills that they can learn in a well-conceived probability and statistics curriculum strand.

NOTE: *Although each content standard is discussed in a separate chapter, it is not the intention that each be treated separately in the classroom. Indeed, as noted in the Introduction to this Framework, an effective curriculum is one that successfully integrates these areas to present students with rich and meaningful cross-strand experiences.*

Standard 12 — Probability and Statistics — Grades K-2

Overview

Students can develop a strong understanding of probability and statistics from consistent experiences in classroom activities where a variety of manipulatives and technology are used. The key components of this understanding in probability for early elementary students, as identified in the K-12 Overview, are **probability terms**, **the concept of the probability of an event,** and **predicting and determining probabilities.** In statistics they key components for early elementary students are **data collection, organization, and representation.**

The understanding of probability and statistics begins with their introduction and use at the earliest levels of schooling. Children are natural investigators and explorers — curious about the world around them, as well as about the opinions and the habits of their classmates, teachers, neighbors and families. Thus, a fertile setting already exists in children for the development of statistics and probability skills and concepts. As with most of the curriculum at these grade levels, the dominant emphasis should be experiential with numerous opportunities to use the concepts in situations which are real to the students. Statistics and probability can and should provide rich experiences to develop other mathematical content and relate mathematics to other disciplines.

Kindergarten students can **gather data** and **make simple graphs** to organize their findings. These experiences should provide opportunities to look for patterns in the data, to answer questions related to the data, and to generate new questions to explore. By playing games or conducting experiments related to chance, children begin to develop an understanding of probability terms.

First- and second-grade children should continue to **collect and organize data**. These activities should provide opportunities for students to have some beginning discussions on **sampling**, and to **represent their data** in charts, tables, or graphs which help them **draw conclusions,** such as *most children like pizza* or *everyone in the class has between 0 and 4 sisters and brothers,* and raise new questions suggested by the data. As they move through this level, they should be encouraged to design data collection activities to answer new questions. They should be encouraged to see how frequently statistical claims appear in their life by collecting and discussing appropriate items from advertising, newspapers, and television reports.

Students in these grades should experience probability at a variety of levels. Numerous children's games are played with random chance devices such as spinners and dice. Students should have opportunities to play games using such devices. Games where students can make decisions based upon their understanding of probability help to raise their levels of consciousness about the significance of probability. Gathering data can lead to issues of probability as well. Students should experience **probability terms** such as *possibly*, *probably*, and *certainly* in a variety of contexts. Statements from newspapers, school bulletins, and their own experiences should highlight their relation to probability. In preparation for later work, students need to have experiences which involve systematic listing and counting of possibilities, such as all the possible outcomes when three coins are tossed (see Standard 14, Discrete Mathematics.)

Learning probability and statistics provides an excellent opportunity for connections with the rest of the mathematics standards as well as with other disciplines. Probability provides a rich opportunity for children to begin to gain a sense of fractions. Geometry is frequently involved through use of student-made spinners

of varying-sized regions and random number generating devices such as dice cubes or octahedral (eight-sided) shapes. The ability to explain the results of data collection and attempts at verbal generalizations are the foundations of algebra. Making predictions in both probability and statistics provides students opportunities to use estimation skills. Measurement using non-standard units occurs in the development of histograms using pictures or objects and in discussions of how the frequency of occurrence for the various options are related. Even the two areas of this standard are related through such things as the use of statistical experiments to determine estimates of the probabilities of events as a means for solving problems such as how many blue and red marbles are in a bag.

The topics that should comprise the probability and statistics focus of the kindergarten through second grade mathematics program are:

collecting data
organizing and representing data with tables, charts and graphs
beginning analysis of data using concepts such as range and “most”
drawing conclusions based on data
using probability terms correctly
predicting and determining probability of events

Standard 12 — Probability and Statistics — Grades K-2

Indicators and Activities

The cumulative progress indicators for grade 4 appear below in boldface type. Each indicator is followed by activities which illustrate how it can be addressed in the classroom in kindergarten and grades 1 and 2.

Experiences will be such that all students in grades K-2:

1. **Formulate and solve problems that involve collecting, organizing, and analyzing data.**
    - Students collect objects such as buttons, books, blocks, counters, etc. which can be sorted by color, shape, or size. They classify the objects and color one square of a bar graph for each item using different colors for each category. Then they compare the categories and discuss the relationships among them.
    - As an assessment following activities such as the one described above, young students are given a sheet of picture stickers and a blank sheet of paper. They sort the stickers according to some classification scheme and then stick them onto the paper to form a pictograph showing the number in each category.
    - At the front of the room is a magnetic board and, for every child in the class, a magnet with that child's picture. At the start of each day, the teacher has a different question on the board and the children place their magnet in the appropriate area. It might be a bar graph tally for whether they prefer vanilla, chocolate or strawberry ice cream or a Venn diagram where students place their magnet in the appropriate area based on whether they have at least one brother, at least one sister, at least one of both, or neither.
    - Students survey their classmates to determine preferences for things such as food, flavors of ice cream, shoes, clothing, or toys. They analyze the data collected to develop a cafeteria menu or to decide how to stock a store.
    - Second graders record and graph the times of sunrise and sunset one day a week over the entire year. They calculate the time from sunrise to sunset, make a graph of the amount of daylight, and interpret these weekly results over the year.
    - A second grader, upset because she had wanted to watch a TV show the night before but had to go to bed instead, asks the teacher if the class can do a survey to find out when most children her age go to bed.

2. **Generate and analyze data obtained using chance devices such as spinners and dice.**
    - Students roll a die, spin a spinner, or reach blindly into a container to select a colored marble, with replacement, a dozen times. They then color the appropriate square in a bar graph for each pick. *Did some results happen more often or less often than others? Do you think some results are more likely to happen than others?* They repeat the experiment, this time without replacement, and compare the results.
    - Students spill out the contents of cups containing five two-colored counters and record the number of red sides and the number of yellow sides. They perform the experiment twenty

times, examine their data, and then discuss questions such as *Does getting four red sides happen more often than two red sides?* They explain their reasoning.

- Each student has a 4-section spinner. Working in pairs, the students spin their spinners simultaneously and together they record whether they have a match. After doing this several times, they predict how many times they would have a match in 20 spins. Then they compare their prediction with what happens when they actually spin the spinners 20 times. They repeat the activity with a different number of equal sections marked on their spinners. Students in the second grade combine the results of all the students in the class, and compare their predictions with the class total.

3. Make inferences and formulate hypotheses based on data.

- Students roll a pair of dice 100 times and make a frequency bar graph of the sums. They compare their results with those of their classmates. *Do your graphs look essentially alike? Which sum or sums came up the most? Does everyone have a 'winning' sum? Is it the same for everyone? Why do some sums come up less than others?*
- Children are regularly asked to think about their data. *Is there a pattern in the dice throws, bean growth, weather, temperature, or other data? What causes the patterns? Are the patterns in their data the same as those of their classmates?*

4. Understand and informally use the concepts of range, mean, mode, and median.

- When performing experiments, children are regularly asked to find the largest and smallest outcomes (range) for numerical data and the outcome that appeared most often (mode). They are asked to compare the mode they obtained for an experiment with the modes found by their classmates.

5. Construct, read, and interpret displays of data such as pictographs, bar graphs, circle graphs, tables, and lists.

- After collecting and sorting objects, children develop a pictograph or histogram showing the number of objects in each category.
- Students design and make tallies and bar graphs to display data on information such as their birth months.
- Students list all possible outcomes of probability experiments, such as tossing a penny, nickel, and dime together.
- Working in cooperative groups, students are given six sheets of paper each containing an outline of a circle which has been divided into eight equal sectors. The students color each whole circle a different color and then cut their circles into individual sectors so each group has 8 sectors in each of 6 colors. Then they roll a die eight times keeping a tally of the results using orange for rolls of 1, blue for rolls of 2, and so on. They use these eight colored sectors to record their results in a circle graph, which they put aside. They repeat this twice and get two other circle graphs. Finally, as a whole class activity, they gather the circle graphs from all the groups, and rearrange the sectors to make as many solid color circles as they can. They discuss the results.
- Students regularly read and interpret displays of data; they also read information from their classmates' graphs and discuss the differences in their results.

6. Determine the probability of a simple event, assuming equally likely outcomes.

- Children roll a die ten times and record the number of times each number comes up. They combine their tallies and discuss the class results.
- Children predict how often heads and tales come up when a coin is tossed. They toss a coin ten times and tally the number of heads and tails. *Are there the same number of heads and tails?* They combine their tallies and compare their class results with their predictions. (See *Making Sense of Data*, in the *Addenda Series*, by Mary Lindquist.)

7. Make predictions that are based on intuitive, experimental, and theoretical probabilities.

- Second graders are presented with a bag in which they are told are marbles of two different colors, twice as many of one color as the other. They are asked to guess the probability for drawing each color if a single marble is drawn. *Is this the same as flipping a coin? Will one color be picked more often than the other?* The experiment is performed repeatedly and tallies are recorded. The chosen marble is returned to the bag each time before a new marble is drawn. The children discuss whether their estimates of the probabilities made sense in light of the outcome.
- Students are told that a can contains ten beads, some red ones, some yellow ones, and some blue ones. They are asked to predict how many beads of each color are in the can. The students attempt to determine the answer by doing a statistical experiment. One at a time, each child in the class draws a bead, records the color with a class tally, and replaces it. At various times in the process, the teacher asks the children to return to their prediction to determine if they want to modify it.
- As an informal assessment of the students' understanding of these concepts, they are presented with a bag in which they are told there are 10 yellow marbles and 2 blue ones. They are asked to predict what color marble they will pick out of the bag if they pick without looking, and about how many students in the class will pick a blue marble.

8. Use concepts of certainty, fairness, and chance to discuss the probability of actual events.

- Students work through the *Elevens Alive!* lesson that is described in the Introduction to this *Framework*. They make number sentences adding up to 11 by dropping 11 chips which are yellow on one side and red on the other, and writing 11= 4+7 when four chips land yellow-side-up and seven chips land red-side-up. They notice that they are writing some number sentences more frequently than others, and these observations lead into a discussion of probability.
- Each child plants five seeds of a fast growing plant. They count the number of seeds which sprout and discuss how many seeds might sprout if they had each planted ten, or twenty, or a hundred seeds. They explain their reasoning. (The numbers can be adjusted for different grade levels.)
- Students predict how many M&Ms of each color are in a large unopened mystery bag. To help make these predictions, cooperative groups are given a handful of M&Ms from the bag; they tally the count of the colors, report their results, and prepare graphs of their results. Students refine their predictions by looking at the class totals. The mystery bag is then opened and the colors counted. Students discuss how their prediction matches the actual

count and how the experiment helped them make their prediction.

- Students examine various types of raisin bran cereal. They experiment with scoops of cereal and determine the number of raisins that appear in each scoop. They make inferences about which brand might be the "raisiniest."

References

Lindquist, M., et al. *Making Sense of Data. Curriculum and Evaluation Standards for School Mathematics Addenda Series, Grades K-6.* Reston, VA: National Council of Teachers of Mathematics, 1992.

General References

Burton, G., et al. *First Grade Book. Curriculum and Evaluation Standards for School Mathematics Addenda Series, Grades K-6.* Reston, VA: National Council of Teachers of Mathematics, 1991.

Burton, G., et al. *Kindergarten Book. Curriculum and Evaluation Standards for School Mathematics Addenda Series, Grades K-6.* Reston, VA: National Council of Teachers of Mathematics, 1991.

On-Line Resources

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Standard 12 — Probability and Statistics — Grades 3-4

Overview

Students can develop a strong understanding of probability and statistics from consistent experiences in classroom activities where a variety of manipulatives and technology are used. The key components of this understanding in probability for elementary school students, as identified in the K-12 Overview, are: **probability terms, the concept of the probability of an event, predicting and determining probabilities,** and **the relationship between theoretical and experimental probabilities.** In statistics the key components for elementary school students are **data collection, organization, and representation, central tendency,** and **analysis and inference.**

Based on their earlier experiences with data, third- and fourth-graders should strengthen their ability to **collect, organize, and represent data**. They should build on their informal discussions of data by developing their ability to **analyze data, formulate hypotheses, and make inferences from the data.** As their numerical skills increase, they should begin to understand and to use the **mean** and **median**, as well as range and mode, as **measures of central tendency.** Frequent probability experiments should help students extend their ability to **make predictions** and understand probability as it relates to events around them, and should provide the intuition they will need in order to **determine probabilities** in simple situations.

As in the previous grade levels, probability and statistics understanding is best developed through frequent opportunities to perform experiments and gather and analyze data. Such activities are most valuable when students choose a topic to investigate based on a real problem or based on an attempt to answer a question of interest to them. Children should experience new activities, but they should have the opportunity to revisit problems introduced in grades K-2 when doing so would allow them to practice or develop new understandings.

Probability and statistics are closely related. Students should use known data to predict future outcomes and they should grapple with the concept of uncertainty using **probability terms** such as *likely,* not *likely, more likely,* and *less likely*. Developing an understanding of randomness in probability is crucial to acquiring a more thorough understanding of statistics.

Third and fourth grade is a wonderful time for students to see connections among subjects. Most science programs at this level involve collection and analysis of data as well as a focus on the likelihood of events. Social studies programs usually ask children to begin to develop ideas of the world around them. Discussions might focus on their school, neighborhood, and community. Such explorations can be enhanced through analysis and discussion of data such as population changes over the last century. Third- and fourth-graders are more attuned to their environment and are more sensitive to media information than early elementary school children. Discussions about such things as the claims in TV advertisements or commercials, or newspaper articles on global warming, help students develop the ability to use their understandings in real situations.

At all grade levels, probability and statistics provide students with rich experiences for practicing their skills in content areas such as number sense, numerical operations, geometry, estimation, algebra, patterns and functions, and discrete mathematics.

The topics that should comprise the probability and statistics focus of the mathematics program in grades three and four are:

collecting, organizing, and representing data
analyzing data using the concepts of range, mean, median, and mode
making inferences and formulating hypotheses from their analysis
determining the probability of a simple event assuming outcomes are equally likely
making valid predictions based on their understandings of probability

Standard 12 — Probability and Statistics — Grades 3-4

Indicators and Activities

The cumulative progress indicators for grade 4 appear below in boldface type. Each indicator is followed by activities which illustrate how it can be addressed in the classroom in grades 3 and 4.

Building upon knowledge and skills gained in the preceding grades, experiences in grades 3-4 will be such that all students:

1. **Formulate and solve problems that involve collecting, organizing, and analyzing data.**
    - Students wish to study the differences in temperature between their hometown and a school they have connected with in Sweden through the Internet. They exchange highs and lows for each Monday over the three-month period from January through March. They note whether the temperatures are given in degrees Celsius or Fahrenheit, and use a thermometer with both markings to change from one to the other if necessary. They organize and represent the data and develop questions about possible differences in lifestyle that are prompted by the temperature. They then exchange their questions with their sister school to learn more about their culture.
    - While studying about garbage and recycling, children notice the amount of waste generated in the cafeteria each day. A variety of questions begin to surface such as: *What types of waste are there? How much of each? Can we measure it? How? How often should we measure it to get an idea of the average amount of waste generated each day? How can we help make less waste?* The class considers how it can find answers to these questions, designs a way to obtain the data, and finds answers to their questions.
    - Students perform experiments such as rolling a toy car down a ramp and measuring the distance the car rolled beyond the bottom of the ramp. This experiment is repeated, holding the top of the ramp at various heights above the ground. Students discuss the patterns and relationships they see in the data and use their discoveries to predict the distances obtained for ramps of other heights.

2. **Generate and analyze data obtained using chance devices such as spinners and dice.**
    - Each child in the class rolls a die 20 times and records the outcomes in a frequency table. The class combines the results in a class frequency table. They discuss which outcome occurred most often and least often and then whether the class results differ from their individual results and why that might be.
    - Students make their own cubes from cardstock and label the sides 1, 2, 2, 3, 4, 5. They roll their cubes 20 times each, recording the results. After combining their results, the class discusses the experiment and the reasons the results differ from the results obtained when using a regular die.
    - As a question on a class test, students are told that Sarah rolled a die 20 times and she got twelve *1s*, two *2s*, three *3s*, and three *6s*. They are asked what they would conclude about Sarah's experiment and what might have accounted for her results.

3. **Make inferences and formulate hypotheses based on data.**

   - Students read *A Three Hat Day* by Laura Geringer. They use concrete objects (different colored beans, hats, or pattern blocks) to show different orders for wearing three different hats. They investigate how many different ways there are to wear four different hats.
   - After collecting, organizing, and analyzing data on the favorite sport of the fourth graders in their school, third graders are asked to interpret the findings. *Why do you suppose soccer was chosen as the favorite sport? How close were other sports? What if we collected data on the same question from fourth graders in another county or another state? Do you think first graders would answer similarly? Why?*
   - Students read *Mr. Archimedes' Bath* and *Who Sank the Boat* by Pamela Allen and discuss what happens to the water level in a container as things are added and why.
   - The fourth grade class is planning a walking tour of a local historic district in February. They want to take hot chocolate but don't know which type of cup to take so that it stays warm as long as possible after being poured. In the science unit on the cooling of liquids, the students discussed notions of variables and constants. They set up an experiment using cups of the same size but of different materials and measure the temperatures in each at equal intervals over a 30-minute period. They plot the data and use their graphs to discuss which cup would be best.

4. **Understand and informally use the concepts of range, mean, mode, and median.**

   - Before counting the number of raisins contained in each of 24 individual boxes of raisins, students are asked to estimate the number of raisins in each box. They count the raisins and compare the actual numbers to their estimates. Students discover that the boxes contain different numbers of raisins. They construct a frequency chart on the blackboard and use the concepts of range, mean, median, and mode to discuss the situation.
   - In a fourth grade assessment, students are asked to prepare an argument to convince their parents that they need a raise in their allowance. Students discuss what type of data would be needed to support their argument, gather the data, and use descriptive measures as a basis for their argument. In a cooperative effort, sixth grade students play the part of parents and listen to the arguments. The sixth graders provide feedback as to whether the students had enough information to convince them to raise the allowance and, if not, what more they might use.

5. **Construct, read, and interpret displays of data such as pictographs, bar graphs, circle graphs, tables, and lists.**

   - Presented with a display of data from *USA TODAY*, students generate questions which can be answered from the display. Each child writes one question on a 3x5 card and gives it to the teacher. The cards are shuffled and redistributed to the students. Each student then answers the question he or she has been given and checks the answer with the originating student. Disagreements are presented to the class as a whole for discussion.
   - Following a survey of favorite TV shows of students in the entire third grade, groups of students develop their own pictographs using symbols of their choosing to represent multiple children.

6. **Determine the probability of a simple event assuming equally likely outcomes.**

   - Children toss a coin fifty times and record the results as a sequence of Hs and Ts. They tally the number of heads and tails. *Are there the same number of heads and tails?* The children discuss situations that often lead to misconceptions such as *If three tosses in a row come up heads, what is the chance that the next toss is a head? Is there a better chance than there would have been before the other tosses took place?* After what is a lively discussion, the children review their sequence of Hs and Ts to see what happened on the next toss each time that three consecutive heads appeared. This analysis should demonstrate that each result does not depend upon the previous ones.
   - Students discuss the probability that a particular number will come up when a die is thrown, and predict how many times that number will appear if the die is rolled 50 times. They then toss a die 50 times and compare the results with their predictions.

7. **Make predictions that are based on intuitive, experimental, and theoretical probabilities.**

   - Fourth-graders are presented with a bag in which there are marbles of three different colors, the same number of two of the colors, and twice as many of the third. They are asked what they would expect to happen if a marble were drawn twelve times and placed back in the bag after each time. The experiment is performed and the children discuss whether their estimates of the outcome made sense in light of the actual outcome.
   - During an ecology unit, students discuss the capture-recapture method of counting wildlife in a local refuge. A number of animals, say 30 deer, are captured, tagged, and released; later another group of deer is captured. If five of the twenty-five recaptured deer are tagged, then you might conclude that about one in five deer have been tagged, and therefore that the total number of deer in the refuge is about 5 x 30 or 150. The students perform a capture-recapture experiment using a large bag of lollipops to determine the number of lollipops in the bag.

8. **Use concepts of certainty, fairness, and chance to discuss the probability of actual events.**

   - Students discuss the probability of getting a zero or a seven on the roll of one die or picking a blue bead from a bag full of blue beads, and use this as an introduction to a discussion about the probability of certain events and impossible events.
   - Students discuss the relationship between events such as flipping a coin, a newborn baby being a girl, guessing on a true-false question, and other events which have an approximately equal chance of occurring.

References

Allen, Pamela. *Mr. Archimedes' Bath*. New York: Lothrop, Lee, and Shepard Books, 1980.

Allen, Pamela. *Who Sank the Boat.* New York: Putnam, 1990.

Lindquist, M., et al. *Making Sense of Data. Curriculum and Evaluation Standards for School Mathematics Addenda Series, Grades K-6.* Reston, VA: National Council of Teachers of Mathematics, 1992.

General References

Burton, G., et al. *Third Grade* Book. *Curriculum and Evaluation Standards for School Mathematics Addenda Series, Grades K-6*. Reston, VA: National Council of Teachers of Mathematics, 1992.

Geringer, Laura. *A Three Hat Day*. New York: Harper and Row, 1985

On-Line Resources

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Standard 12 — Probability and Statistics — Grades 5-6

Overview

Students can develop a strong understanding of probability and statistics from consistent experiences in classroom activities where a variety of manipulatives and technology are used. The key components of this understanding in probability for middle school students, as identified in the K-12 Overview, are: **probability terms, the concept of the probability of an event, predicting and determining probabilities, the relationship between theoretical and experimental probabilities,** and **compound events.** In statistics, the key components are: **data collection, organization, and representation, sampling, central tendency, variance and correlation,** and **analysis and inference.**

In grades K-4, students explored basic ideas of statistics by gathering data, organizing data, and representing data in charts and graphs, and then using this information to arrive at answers to questions and raise further questions. Students in grades 5 and 6 are keenly interested in movies, fashion, music, and sports. These areas provide a rich source of real problems for students at this age. The students should make the decision on how to **sample** and then **collect and organize data.** They should determine how best to **represent** the data and begin to develop a more formal understanding of summary statistics of **central tendency** such as the mean, the median, and the mode. They should recognize that for certain types of data, such as height, the mean is an appropriate measure, but it is inappropriate for other types of data, such as hair color. These activities should provide opportunities for students to **analyze data** and to **make inferences** regarding the data and to communicate their inferences in a convincing manner. They should further develop their understanding of statistics through the evaluation of arguments by others, whether they come from classmates, advertising, political rhetoric, or news sources.

While statistical investigations can be similar to those in earlier grades, fifth- and sixth-graders should have access to statistical software on computers or calculators which have statistical capability. This will allow them to carry out statistical work using real data without becoming mired in tedious calculations. The technology will be used to do the manipulation of the data and the students will focus on developing their skills in interpreting the data.

Students enter these grades having participated in a wide variety of activities designed to help them understand the nature of probability and chance. The emphasis in grades K-4 was primarily on simple events such as the roll of a die or the flip of one coin. Even when compound events such as the roll of two dice were considered, the outcomes were looked upon as a simple event. In grades 5 and 6, students begin to experiment with **compound events** such as flips of several coins and rolls of dice and to predict and evaluate their **theoretical and experimental probabilities.** As they develop their understanding of fractions, ratios, and percents, they should use them to represent probabilities in place of phrases such as “three out of four.” They begin to model probability situations and to use these models to **predict** events which are meaningful to them.

At all grade levels, probability and statistics provide students with rich experiences for practicing their skills in content areas such as number sense, numerical operations, geometry, estimation, algebra, patterns and functions, and discrete mathematics. Because most of the activities are hands-on and students are constantly dealing with numbers in a variety of ways, it assists the development of number sense as well.

The topics that should comprise the probability and statistics focus of the mathematics program in grades 5 and 6 are:

collecting, organizing, and representing data
analyzing data using range and measures of central tendency
making inferences and hypotheses from their analysis of data
evaluating arguments based upon data analysis
interpolating and/or extrapolating from data using a line of best fit
representing probability situations in a variety of ways
modeling probability situations
predicting events based on real-world data

Standard 12 — Probability and Statistics — Grades 5-6

Indicators and Activities

The cumulative progress indicators for grade 8 appear below in boldface type. Each indicator is followed by activities which illustrate how it can be addressed in the classroom in grades 5 and 6.

Building upon knowledge and skills gained in the preceding grades, experiences in grades 5-6 will be such that all students:

9. Generate, collect, organize, and analyze data and represent this data in tables, charts, and graphs.

- Students recognize that this is a time of growth for many of them. The class measures various lengths associated with a person, such as height, length of forearm, length of thigh, handspan, length of foot, and armspan. They enter the data into a spreadsheet and produce various graphs as well as statistical analyses of their measures. They update their data every month and discuss the change, both individually and as a class.
- Students survey another class to determine data of interest, such as the last movies seen, and then organize the data and produce reports discussing the interests of the grade level.
- Students work on problems like this one from the New Jersey State Department of Education's *Mathematics Instructional Guide (p. 7-95)*:

  A fair spinner with 4 congruent regions labeled A, B, C, and D is spun 20 times by each member of a class of 23 students. Assume that your class conducted the experiment and obtained the expected results. Make a bar graph illustrating the combined class results. Explain why you drew your bar graph the way you did . Explain why an individual student's results might be different from the class results.

10. Select and use appropriate graphical representations and measures of central tendency (mean, mode, and median) for sets of data.

- Students demonstrate understanding of measures of central tendency by writing a letter to a fictional classmate explaining how the mean, the median, and the mode each help to describe data. They then extend their discussion by presenting a picture of an "average student" in their grade. The picture discusses height, color of hair, preference in movies, etc. In creating the picture, the students must choose the appropriate measures of central tendency based upon the type of data and justify their choice. (For example, the mean is not appropriate in discussing hair color.) They will likely want to present pictures of both an "average boy" and an "average girl" in the class.
- During a social studies unit, students determine a method to ascertain the value of the homes in their community. They determine the mean, the median, and the mode for the data and decide which provides the most accurate picture of the community. They include in their study homes from different sections of the town.
- Students perform an experiment where one group is given 10 words in a jumbled order while another group is given them in a sequence which facilitates remembering them. After giving

each group one minute to study the words, the students are asked to turn their papers over and write as many of the words as they remember. The papers are graded by fellow students and the scores reported. After considering various graphing methods, the students determine that a *box-and-whiskers* plot would be the best way to illustrate the results and compare the two groups.

11. Make inferences and formulate and evaluate arguments based on data analysis and data displays.

- Students are asked to develop a generalization about their classmates. They are allowed to make any hypothesis which is appropriate. For example, some boys might suggest that boys are stronger than girls or others might say that girls are taller than boys. They should determine how they would determine the validity of their hypotheses by designing a data collection activity related to it.
- The teacher in one fifth-grade class is especially alert for generalizations that students make about any topic. She writes them on slips of paper, and keeps them in a box. As an assessment of the students' ability to develop statistical activities to validate hypotheses, groups of students pull slips from the box, develop data collection activities, collect the data, analyze it, and make reports to the class about the validity of the generalizations originally made.
- Students are shown a newspaper article which states that 25% of fifth graders have smoked a cigarette. They discuss their reaction by indicating whether they believe the figure to be correct, too high, or too low. They then design a survey which they use to poll their fellow fifth graders in an effort to check the validity of the claim for the population of their school. They also send a letter to the newspaper requesting the sources of data for the article and compare the data in the article with their data.

12. Use lines of best fit to interpolate and predict from data.

- Given a jar with straight sides and half filled with water, students drop marbles in five at a time. After each group of five, they measure the height of the water and record in a table the number of marbles in the jar and the height of the water. The students then represent their data in a scatterplot on an x-y plane and find that the points lie almost exactly in a straight line. They draw a line through the data and use it to determine answers to questions like: *How high will the water be after 25 marbles have been added?* and *How many marbles will it take to have the water reach the top?* Activities like this one form the foundation for understanding graphs in algebra.

13. Determine the probability of a compound event.

- Students create a table to show all possible results of rolling two dice. At the left of the rows are the possible rolls of the first die and at the top of the columns are the possible rolls of the second die. They complete the table by putting in each cell the appropriate sum of the number in the top row and the left column. Counting the number of times each sum appears in the table, they determine the probability of getting each possible sum. They then roll two dice 100 times and compare the sums they get with the sums predicted from the table.
- Students make a list of all possible outcomes when four coins are tossed and determine the theoretical probability of having exactly two heads and two tails.

14. Model situations involving probability, such as genetics, using both simulations and theoretical models.

- Students examine the probability of a family with four children having two boys and two girls by simulating the situation using four coins. They first choose which side of the coin will represent males and which will represent females. They toss the set of coins 50 times and record their results as the number of boys and the number of girls in each "family." They compare the results of their experiment with the prediction based on probability. They also survey a large sample of students in the school and record the family composition of all families with four children. All of these are used to discuss the likelihood of an evenly-matched family.
- A 25¢ "prize" machine in the grocery store contains an equal number of each of six plastic containers with Power Ranger tattoos. Students are asked to determine how many containers they need to buy to have a good chance of getting all six. They simulate this situation with a bag containing an equal number of six different colored marbles. They draw out, record, and replace one marble at a time until they have drawn marbles of all six colors, recording the number of times that took. They repeat the simulation three times. The class results are gathered and discussed. One issue discussed is whether the model is a good one for the situation or whether it should be modified in some way to better represent reality.
- Students read *Caps for Sale* by Esphyr Slobodkina. The peddler in the story sells caps and wears his entire inventory on his head: a checked cap and four each of identical blue, gray, and brown hats. Students use concrete objects to model some of the different orders in which the hats can be worn. They come to realize that there are many ways and try to discover the total number of different ways. They search for an efficient way to determine the number of permutations.
- Students work through the *Two-Toned Towers* and *Pizza Possibilities* lessons that are described in the First Four Standards of the *Framework*. They make a systematic list of all the towers built out of four red and blue cubes (or of all the pizza combinations) and calculate the probability that a tower has three or four blue cubes.

15. Use models of probability to predict events based on actual data.

- Students examine weather data for their community from previous years, and then use their analysis of the data to predict the weather for the upcoming month. They compare the actual results with their predictions after the month has passed and then use the comparison to determine ways to improve their predictions.
- Using data from previous years, students determine the number of times their favorite professional football team scored a number of points in each of six ranges of scores (0-5, 6-10, 11-15, 16-20, 21-25, and 26 or more). They determine the fraction or percentage of games the score was in each of those ranges and make a spinner whose areas are divided the same way. Each Friday during football season, they spin their spinners to predict how many points the team will score and who will win the game. Toward the end of the season, they discuss the success or failure of their efforts and the probable causes.

16. Interpret probabilities as ratios and percents.

- The students are introduced to the game *Pass The Pigs* (Milton Bradley) where two small

hard-rubber pigs are rolled. Each pig can land on a side where there is a dot showing, a side where the dot does not show, on its hooves, on its back, leaning forward balancing itself on its snout, and balancing itself on its left foreleg, snout, and left ear. The students determine the fairness of the distribution of points on the sides of the pig by rolling the pig numerous times, recording the results, and using the ratios of successes for each, divided by the total number of rolls, to represent the probability of obtaining each situation.

- Students examine uses of probability expressed as percentages in such situations as weather forecasting, risks in medical operations, and reporting the confidence interval of surveys.

References

New Jersey State Department of Education, *Mathematics Instructional Guide: Linking Classroom Experiences to Current Statewide Assessments.* D. Varygiannes, Coord. Trenton, N.J., 1996.

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Zawojewski, Judith, et. al. *Dealing with Data and Chance.* A component of the *Curriculum and Evaluation Standards for School Mathematics Addenda Series, Grades 5-8*. Reston, VA: National Council of Teachers of Mathematics, 1991.

On-Line Resources

http://dimacs.rutgers.edu/nj_math_coalition/framework.html/

The *Framework* will be available at this site during Spring 1997. In time, we hope to post additional resources relating to this standard, such as grade-specific activities submitted by New Jersey teachers, and to provide a forum to discuss the *Mathematics Standards.*

Standard 12 — Probability and Statistics — Grades 7-8

Overview

Students can develop a strong understanding of probability and statistics from consistent experiences in classroom activities where a variety of manipulatives and technology are used. The key components of this understanding in probability for middle school students, as identified in the K-12 Overview, are: **probability terms, the concept of the probability of an event, predicting and determining probabilities, the relationship between theoretical and experimental probabilities,** and **compound events.** In statistics, the key components are: **data collection, organization, and representation, sampling, central tendency, variance and correlation,** and **analysis and inference.**

Students should enter the seventh grade with a strong intuitive understanding of probability and statistics as a result of their activities in grades K-6, and should have a basic understanding of the more formal methods which were introduced in grades 5-6. They will build on this foundation in grades 7 and 8.

Students in grades 7 and 8 present unique challenges. They are turning to their peer group for leadership and support and, at the same time, placing a strain on the relationships between themselves and significant adults in their lives. Some students begin to experiment with things they associate with being an adult: smoking, alcohol, drugs, and sex. The quantity of statistics in all of these areas provides an ideal opportunity to weave together statistical activities which dovetail with information provided by the the health and physical education department.

Students at these ages also become more aware of community issues. Integrating statistics activities with topics in the social studies curriculum can enhance their work in both areas as well as fit in with their growing interests. Hands-on science activities require good statistical methods and understanding in order to develop accurate and appropriate conclusions. At the same time, students need to understand how often statistics and probability statements are incomplete, misunderstood, or purposely used to mislead. Having students read books such as *How to Lie with Statistics* by Darryl Huff or *Innumeracy* by John Allen Paulos provides excellent opportunities to discuss how statistics and probability are misused.

In statistics, students continue to **collect, organize, and represent data** and to use various **measures of central tendency** to describe their data. But they should now become more focused on **sampling** techniques that justify making **inferences** about entire populations. Examples of this appear frequently in the news media. They also begin to explore **variance and correlation** as additional tools in describing sets of data.

Many of the probability experiments should continue to be related to games and other fun activities. Students in these grades should continue to develop their understanding **of compound events** and their related probabilities, and should continue to consider and compare **experimental and theoretical probabilities**. Furthermore, the connection between probability and statistics should help them understand issues such as **sampling** and **reliability.** Students need to develop a sense of the application of probability to the world around them as well. Everyday life is rich with "coincidences" which are actually likely to occur. For example, they should examine the probability that two people in their class, or any group of 25 or more people, have the same birthday. The results always stir up considerable interest and disbelief.

At all grade levels, probability and statistics provide students with rich experiences for practicing their skills

in content areas such as number sense, numerical operations, geometry, estimation, algebra, patterns and functions, and discrete mathematics. Because most of the activities are hands-on and students are constantly dealing with numbers in a variety of ways, it assists the development of number sense as well.

The topics that should comprise the probability and statistics focus of the mathematics program in grades 7 and 8 are:

collecting, organizing, and representing data
analyzing data using range and measures of central tendency
making inferences and hypotheses from their analysis of data
evaluating arguments based upon data analysis
interpolating and/or extrapolating from data using a line of best fit
representing probability situations in a variety of ways
modeling probability situations
analyzing probability situations theoretically
predicting events based on real-world data

Standard 12 — Probability and Statistics — Grades 7-8

Indicators and Activities

The cumulative progress indicators for grade 8 appear below in boldface type. Each indicator is followed by activities which illustrate how it can be addressed in the classroom in grades 7 and 8.

Building upon knowledge and skills gained in the preceding grades, experiences in grades 7-8 will be such that all students:

9. Generate, collect, organize, and analyze data and represent this data in tables, charts, and graphs.

- Most students in grades 7 and 8 have major physical growth activity. Students can continue to maintain the statistics related to their body that they began to collect in the fifth and sixth grades. They should continually update what the average person in the grade would look like in terms of this data.
- In the spring, the social studies teacher and the mathematics teacher plan a unit on the school board elections. Students are broken into groups to study questions such as *What percent of the registered voters can be expected to vote? Will the budget pass?* and *Who will be elected to the board of education?* Students plan their survey, how they will choose the sample, how best to gather the data, and how best to report the information to the class.

10. Select and use appropriate graphical representations and measures of central tendency (mean, mode, and median) for sets of data.

- Students study the sneakers worn by students in the school. They form into a human histogram based upon their brand of sneakers. The data is recorded and a discussion is encouraged about the distribution of sneakers throughout the school. Students discuss in their journals which of the measures of central tendency a sporting goods store would use in determining which brands to stock and in what proportion. The students gather prices for a variety of brands and styles and enter the data into a spreadsheet. They respond in their journal as to whether and why the mean, median, or mode would be most useful to discuss sneaker prices.
- Presented with a list of OPEC countries and their estimated crude oil production in a recent year, students determine how best to report the data. Some present their graphs as box plots, others use histograms, and others use circle graphs. They use the three measures of central tendency and discuss what each result would mean in this situation and which would be best to use in other situations.
- Students work on problems like this one from the New Jersey Department of Education's *Mathematics Instructional Guide (p. 7-99)*:

  A set of test scores in Mrs. Ditkof's class of 20 students is shown below.

  62 77 82 88 73 64 82 85 90 75
  74 81 85 89 96 69 74 98 91 85

Determine the mean, median, mode, and range for the data. Suppose each student completes an extra-credit assignment worth 5 points, which is then added to his/her score. What is the mean of the set of scores now if each student received the extra five points? Explain how you calculated your answer.

11. Make inferences and formulate and evaluate arguments based on data analysis and data displays.

- Students are presented with data from *The World Almanac* showing the number of cigarettes smoked per year per adult and the rate of coronary heart disease in 21 countries. They produce a scatterplot and recognize a relatively high correlation between the two factors. They write an essay on the possible causes of this relationship and their interpretation of it.
- Students are asked to predict how many drops of water will fit on a penny. They write their prediction on a post-it note along with an explanation of their reasoning. The predictions are collected and displayed on bar graphs or stem-and-leaf plots. Students perform the experiment and record their results on another post-it note. They compare their hypotheses with the conclusions. A science lesson on surface tension can easily be integrated with this lesson.
- Students are studying their community's recycling efforts in an integrated unit. In getting ready for discussion in this area, the mathematics teachers ask the students to predict how many pounds of junk mail comes in to their community in a month. The students collect all junk mail sent to their house over the course of a month. They weigh the junk mail weekly and record the results. At the end of the month, all the students bring in their data. The class determines the mean, median, and mode for the collected data, decides which of these measures would be the best to use, and agrees on a method to use to estimate the amount of junk mail for the entire community.

12. Use lines of best fit to interpolate and predict from data.

- Presented with the problem of determining how long it would take *the wave* to go around Giants Stadium, students design an experiment to gather data from various numbers of students. They produce a scatterplot and use it to determine a line of best fit. They pick two points on the line and determine the equation for that line. Last, they estimate the number of people around the stadium and answer the question.
- Given some of the winning times for the Men's and Women's Olympic 100 meter freestyle events during the past century, students plot the data and produce a line of best fit for each event. They use their equations to estimate the winning times in those years for which the information was not recorded, and they predict when the women's winning times will equal the men's current winning times.
- Students are presented with an article that states that police have discovered a human radius bone which is 25 centimeters long. Students perform measurements of the lengths of radius bones of various-sized people and their heights, produce a scatterplot, fit a line to the data, and determine their prediction of the height of the person whose bone was found. They write a letter to the chief of police, predicting the height of the person, with justifications for their conclusion.

13. Determine the probability of a compound event.

- Students watch the long-range weekend weather forecast and learn that the probability of rain is 40% on Saturday and 50% on Sunday. They determine that the probability that it will rain on both days is 20% by multiplying the two percentages together (.40 x .50 = .20 or 20%), and similarly then find that the probability that it will not rain on either day is 30%. Following the weekend, they discuss the success or failure of their prediction methods.
- Two teams are in a playoff for the division title. If the probability of the Eagles defeating the Falcons in an individual game is 40%, what is the probability that they will win a three game playoff? What about a five-game playoff?

14. Model situations involving probability, such as genetics, using both simulations and theoretical methods.

- During an integrated unit with their science and health classes, students discuss the various gender possibilities for children within a family. For each large family, that is, number of children, up to 6, they calculate the probability of each possible gender mix. Three groups of students conduct simulations — one with coins, one with dice (1, 2, or 3 on a die represent a female) and one with spinners. They also collect this data for all of the students in their school. They report their findings and compare the theoretical possibilities, the simulated probabilities, and the actual outcomes, and discuss the differences and similarities.
- Students study the chances of winning the New Jersey Pick 3 lottery. They model the problem by using spinners with 10 numbers and calculate the theoretical probability. They may also use a computer program to randomly generate a million 3-digit numbers and see how close to 1 out of 1000 times their favorite number comes up.

15. Use models of probability to predict events based on actual data.

- Students are presented with data collected by an ecologist tallying the number of deer of one species that died at ages from 1 to 8 years. Students use the data to discuss the probability of living to various given ages and what they would expect the life expectancy of this species to be.

16. Interpret probabilities as ratios and percents.

- Students examine uses of probability expressed as percentages in such things as weather forecasting, risks in medical operations, and reporting the confidence interval of surveys.
- Students work on problems like this one from the New Jersey State Department of Education's *Mathematics Instructional Guide* (p. 7-103):

  A dart board is composed of three concentric circles with radii 2 cm, 10 cm, and 20 cm [as indicated in an accompanying diagram]. A grand prize is earned if a dart is thrown in the 2 cm circle (bulls-eye). Given that a person is blindfolded and throws a dart somewhere on the board, find the probability that the grand prize will be won when the first dart is thrown. Explain the process you used to solve the problem.

References

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Zawojewski, Judith, et al. *Dealing with Data and Chance.* A Component of the *Curriculum and Evaluation Standards for School Mathematics Addenda Series, Grades 5-8*. Reston, VA: National Council of Teachers of Mathematics, 1991.

On-Line Resources

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Standard 12 — Probability and Statistics — Grades 9-12

Overview

Students can develop a strong understanding of probability and statistics from consistent experiences in classroom activities where a variety of manipulatives and technology are used. The key components of this understanding in probability for middle school students, as identified in the K-12 Overview, are: **probability terms, the concept of the probability of an event, predicting and determining probabilities, expected value, the relationship between theoretical and experimental probabilities,** and **compound events.** In statistics, the key components are: **data collection, organization, and representation, sampling, central tendency, variance and correlation,** and **analysis and inference.**

The field of statistics is relatively new. Beyond the work of scientists, Florence Nightingale was the great pioneer in gathering and analyzing statistical data for public health questions. During the great cholera epidemic of 1854 in London, England, statistics on the prevalence of cholera cases in various London neighborhoods were used to deduce that the cholera originated with a single well. In our own century statistics touches all of us through such diverse means as statistical quality control in industry, advertising claims, pre-election polls, television show ratings, and weather forecasts. To be successful members of present day society, high-school graduates need an understanding of statistics and probability which formerly was rare even among college graduates.

By the time students enter high school, they should have mastered basic descriptive statistical methods. On the basis of their varied experience, they should be able to set up a study, gather the data, and appropriately analyze and report their findings. Throughout grades 9 to 12, students should have numerous opportunities to continue to practice these skills in a variety of ways, and also to extend these skills, in connection with their growth in other mathematical areas. As students learn new algebraic functions, they might revisit a problem they had previously modeled linearly and apply a different model. For example, they may have linearly modeled the series of winning times of the men's Olympic marathon but now understand that there would probably be a limiting time and so attempt to fit a quadratic or logarithmic curve instead. Where appropriate, the content should be developed through a problem-centered approach. For example, if students are required to generate a report on two sets of data which have the same measures of **central tendency** only to find later they have very different **variance**, they should recognize the need for some way to identify that difference.

John Allen Paulos, in his book, *Innumeracy*, cites numerous problems associated with a lack of understanding of probability. If people are to make appropriate decisions, then they must understand the relationship of probability to real situations and be able to weigh the consequences against the odds. As with statistics, probability needs to be experienced, not memorized. Work done at this level should provide insight into the use of probability and probability distributions in a variety of real-world situations. The normal curve presents interesting opportunities to examine uses and abuses of mathematics.

Students should have access to appropriate technology for their work in probability and statistics, not only to simplify calculation and display charts and graphs, but also to generate appropriate data for activities and projects. They should make use of data taken from the Internet and CD-ROMs, and simulate experiments with Calculator Based Laboratories. Whenever possible, real data gathered from school, the community, or cooperating businesses should be used.

Probability and statistics offers a rich opportunity to integrate with other mathematics content and other disciplines. This content provides the opportunity to generate the numbers and situations which should be used in other areas such as geometry, algebra, functions, and discrete mathematics. The goal to have students become effective members of a democratic society requires them to practice and participate in decision-making experiences. The ability to make intelligent decisions rests on an understanding of statistics and probability, and students should regularly integrate this content with their experiences in social studies, science, and other disciplines.

The topics that should comprise the probability and statistics focus of the mathematics program in grades 9 through 12 are:

designing, conducting, and interpreting statistical work to solve problems
analyzing data using range, measures of central tendency, and dispersion
applying probability dispersions in real situations
evaluating arguments based upon their knowledge of sampling and data analysis
interpolating and/or extrapolating from data using curve fitting
using simulations to estimate probabilities
determining expected values
using the law of large numbers

Standard 12 — Probability and Statistics — Grades 9-12

Indicators and Activities

The cumulative progress indicators for grade 12 appear below in boldface type. Each indicator is followed by activities which illustrate how it can be addressed in the classroom in grades 9, 10, 11, and 12.

Building upon knowledge and skills gained in the preceding grades, experiences in grades 9-12 will be such that all students:

17. Estimate probabilities and predict outcomes from actual data.

- In a standard class test, students are asked to compute the probability that a given raffle ticket for a senior class raffle to raise money for the senior trip will win a prize. The class will be printing 500 tickets that they will sell for $1 each. First prize is a stereo worth $150. Second prize is a $100 shopping spree in the local Gap store. Third prize is a $50 gift certificate to The Golden Goose restaurant. There are ten fourth prizes of a commemorative T-shirt worth $8 each. Students also compute the expected value of each ticket.
- Students determine the area of an irregular closed figure drawn on a large sheet of paper using the Monte Carlo method: Each person in the group drops a handful of pennies over their shoulder (without looking) onto the paper containing the figure. They count the number of coins on the paper (total shots) and the number within the figure (hits). They thus produce the ratio of hits to total shots and multiply this fraction by this area of the paper to estimate the area of the figure.
- Students work through the *On the Boardwalk* lesson that is described in the Introduction to the *Framework*. In this lesson they explore the probability that a quarter thrown onto a rectangular grid will land entirely within one of the squares on the grid, and then discuss how changing the size of the squares will affect the probability.
- A point P inside a square is selected at random and is used to form a triangle with vertices A and B of the square. Students determine the probability that the triangle is acute using a simulation and a theoretical calculation.

18. Understand sampling and recognize its role in statistical claims.

- While studying United States history, students read about the prediction in the 1936 presidential race that Alfred Landon would defeat incumbent President Franklin Delanor Roosevelt. They raise questions as to why that prediction was so far off and research how TV stations can forecast winners of some elections with a very small percentage of the voting results reported. Students contact local radio and TV stations and newspapers to discover how they determine their population sample.

19. Evaluate bias, accuracy, and reasonableness of data in real-world contexts.

- After reading the chapter on sampling in the book *How to Lie With Statistics* by Darryl Huff, students bring in ads, graphs, charts, and articles from newspapers which all make statements or claims allegedly based on data. Students examine the articles for information

about the sample and identify those claims which may have little or no substantiation. They also discuss how the sample populations chosen could have influenced the outcomes.

- Students take statements such as "50% of the students failed the test," and "4 out of 5 dentists recommend" and discuss what data they would need to know in order to judge if the conclusions were reasonable. How many students took the test? How many dentists were queried? How were the students or dentists selected? What factors can be identified which would bias the results?

20. Understand and apply measures of dispersion and correlation.

- Students are presented with data gathered by an archaeologist at several sites. The data identifies the number of flintstones found at each site and the number of charred bones. The archaeologist claimed that the data showed that the flintstones were used to light the fires that charred the bones. Students produce a scatterplot, find the correlation between the two sets of figures, and use their work to support or criticize the claim.
- As an assessment activity using their journals, students respond to the claim that children with bigger feet spell better. They discuss whether they believe the claim is true, how statistics might have led to this claim, and whether it has any importance to a philosophy of language teaching.

21. Design a statistical experiment to study a problem, conduct the experiment, and interpret and communicate the outcomes.

- Based on a discussion among some members of the class, a question arises as to which are the most popular cars in the community. The students work in cooperative groups to design an experiment to gather the data, analyze the data, and design an appropriate report format for their results.
- Intrigued by the question *How long would it take dominoes set up one inch apart all the way across the room to fall?,* the class designs an experiment to gather data on smaller sets of dominoes and then extrapolates to estimate the answer.
- Students have just finished a unit in which they discussed the capture-recapture method for estimating the population of wildlife. Part of their assessment for the unit is a project where they work in groups to design and conduct a simulation of the capture-recapture method. One group uses the method to determine the number of lollipops in a large bag.

22. Make predictions using curve fitting and numerical procedures to interpolate and extrapolate from known data.

- Students are presented with this data comparing a student's test grade to the number of hours each studied.

Hours	1	2	3	4	4	6	8	9	10	10	12	12
Grade	60	55	65	65	77	80	83	80	75	90	72	80

  Earlier in the year they had produced a line of best fit for the data, but they had recognized that it was not a good model for the data. Now the students use calculators to help them fit a quadratic curve to the data and discuss the advantages it has over the straight line.

- Students conduct an experiment where they suspend a weight on a string from a hook in the doorway. They swing their homemade pendulum and time how long it takes for it to swing

10 times. They had performed this experiment in 8th grade and used the median-median line fit method to model the data. In this revisitation of the problem, the teacher insists they use very short lengths and very long lengths in addition to various ones in between. When the data is graphed, it becomes apparent that the data is not linear and would fit a quadratic curve better. (The median-median line, available on many calculators which have statistics capabilities, is found by dividing the data points on the x-y plane into three equal sets, grouped by x-value, finding a single point for each set whose coordinates are the medians of the respective coordinates of the points in the set, connecting the first and third points by a straight line, and shifting this line 1/3 of the way toward the second point. See *Contemporary Precalculus Through Applications*.)

- Students perform an Introductory Physical Science experiment where water is cooled by adding ice cubes and stirring. A Calculator Based Lab temperature probe is attached to a graphing calculator which is programmed to gather the data. Students (or each group of students) link their calculators to the original one to transfer the data to their calculators. They use the statistics functions to perform a quadratic fit, an exponential fit, and a logarithmic fit, and use the function graph capabilities to determine which is the best model.
- Students work through the *What's My Line* unit described in the *Keys to Success in the Classroom* chapter of the *Framework.* They use median–median and regression lines to estimate the height of a person whose thigh bone was found in a dig.

23. Use relative frequency and probability, as appropriate, to represent and solve problems involving uncertainty.

- After a unit where dependent and independent events were detailed, students are challenged by a problem containing this excerpt from The Miami Herald of May 5, 1983.

  An airline jet carrying 172 people between Miami and Nassau lost its engine oil, power, and 12,000 feet of altitude over the Atlantic Ocean before a safe recovery was made.

  When all three engines' low oil pressure warning lights all lit up at nearly the same time, the crew's initial reaction was that something was wrong with the indicator system, not the oil pressure.

  They considered the possibility of a malfunction in the indication system because it's such an unusual thing to see all three with low pressure indications. The odds are so great that you won't get three indications like this. The odds are way out of sight, so the first thing you would suspect is a problem with the indication system.

  Aviation records show that the probability of an engine failure in any particular hour is about 0.00004. If the failures of three engines were independent, what would the probability be of them failing within one hour? Discuss why the speaker in the article would refer to such a probability as "way out of sight." Discuss situations which might make the failures of three engines not independent events.

- Students keep a record of their trips through the town and whether or not they have to stop at each of the four traffic lights. After one month, the data is grouped and studied. They use their data to determine whether the timing of the lights is independent or not.
- While discussing the issue of mandatory drug testing in social studies, students examine the probability of misdiagnosing people as having AIDS with a test that would identify 99% of those who are true positives and misdiagnose 3% of those who don't have AIDS. They

examine situations where the prevalence of the disease is 50%, 10%, and 1% using 100,000 people as a base. They discuss the fact that, at the 1% level, 75% of the people identified as having AIDS would be false positives, the implications that fact has on mandatory testing, and potential ways to improve the predictive value of testing.

24. Use simulations to estimate probabilities.

- Students derive the theoretical probability of winning the New Jersey Pick 6 lottery and then write a computer program to simulate the lottery. The students enter the winning numbers and the computer generates sets of 6 numbers until it hits the winning combination. The computer prints out the number of sets generated including the winning one. Students run the program several times, attempting to verify experimentally the theoretical probability they derived.

25. Create and interpret discrete and continuous probability distributions, and understand their application to real-world situations.

- Students work on a project where they pick one form of insurance (life, car, home), and determine the variables which affect the premiums they would need to pay for this type of insurance and what it would cost for them to obtain it. Using their research, they write an essay summarizing how insurance companies use statistics and probabilities to determine their rates.
- An article in *Consumer Reports* indicates that 25% of 5-lb bags of sugar from a particular company are underweight. The class works with the local supermarket to develop and perform a consumer research project. Each group is given a commodity to study (e.g., potato chips, sugar). They design a method for randomly selecting and testing whether the product matches the claimed specifications or not. They use their data to determine the probability that a randomly selected bag would be underweight.
- Students repeatedly extracted five marbles from a bag containing 10 red and 10 blue marbles, and each time record the number of marbles of each color obtained. They combine the data for the entire class, tabulating the number of times there were 0, 1, 2, 3, 4, and 5 red marbles, and the percentages for each number. They compared their percentages to the theoretical percentages for this binomial distribution, and make the connection to the fifth row of Pascal's triangle.

26. Describe the normal curve in general terms, and use its properties to answer questions about sets of data that are assumed to be normally distributed.

- Students describe a typical student in the school. To do this, they first select a random sample of 30 students in their school. They then survey their sample for information they believe necessary to identify what would be "typical." Finally, they use appropriate displays and descriptive statistics to support their representation of a typical student.
- Students are introduced to the "central limit theorem" through this problem:

  A worker on the assembly line at Western Digital is involved in industrial sabotage by weakening a soldering joint that causes a hard drive to fail after 5 hours of use. At his station, he actually comes in contact with 30% of the drives produced. The other 70% will last 100 hours. If they are packed randomly in boxes of 36, what would be the average expected lifespan of the drives in the box?

Students prepare simulations of the problem by repeatedly extracting 36 cubes at random from a bag containing 30 yellow and 70 red cubes and calculating the average expected lifespan for each selection. They discover that the answers fall into a normal curve with a mean of approximately 70 hours.

- Students are given the administrator's summary of the school's standardized tests. Each group is given one area on which to focus. They prepare a presentation they would give to the Board of Education discussing the comparisons between local norms, national norms, suburban norms, urban norms, and independent norms using their understanding of normal distribution, percentile ranks, and graphical displays.

27. Understand and use the law of large numbers (that experimental results tend to approach theoretical probabilities after a large number of trials).

- Students are given two dice, each a different color and roll them repeatedly. For each roll, they record the result for each individual die as well as the total. After a large number of rolls they compare their relative frequencies to the expected outcomes. Then they combine the totals for the entire class and compare the experimental results with the theoretical predictions.
- Students are presented with a paper containing the following gambler's formula: *When playing roulette, bet red. If red does not win, double the bet on red. Continue in this manner*. They evaluate whether the formula makes sense, identify potential problems, and limitations, and discuss the fallacy that the odds improve for red to appear on the next roll every time red doesn't win.

References

Burrill, Gail, et al. *Data Analysis and Statistics Across the Curriculum*. A component of the *Curriculum and Evaluation Standards for School Mathematics Addenda Series, Grades 9-12*. Reston, VA: National Council of Teachers of Mathematics, 1992.

Huff, D. *How to Lie with Statistics*. New York: Norton, 1954.

Paulos, J. A. *Innumeracy: Mathematical Illiteracy and its Consequences*. New York: Hill and Wang, 1988.

The North Carolina School of Science and Mathematics. *Contemporary Precalculus Through Applications*. Providence, RI: Janson Publications, 1991.

On-Line Resources

http://dimacs.rutgers.edu/nj_math_coalition/framework.html/

The *Framework* will be available at this site during Spring 1997. In time, we hope to post additional resources relating to this standard, such as grade-specific activities submitted by New Jersey teachers, and to provide a forum to discuss the *Mathematics Standards.*