CHAPTER 5 Simultaneous-Move Games with Mixed Strategies

TEACHING SUGGESTIONS

This chapter introduces mixed strategies and the methods used to solve for mixed strategy equilibria. Students are likely to accept the idea of randomization more readily if they think of it themselves. Here is one way to lead them to the idea: Draw or project the game table for the tennis game in Chapter 5 (Figure 5.1 or Figure 4.14), and ask one student to put herself in the shoes of Seles and think whether she would choose cross-court or down the line. Usually the answer will be one or the other, with some technical justification. Then ask another student how she would react if she were Hingis. Proceeding this way, get them to realize the importance of *keeping the other guessing*. Build this into the formal idea that randomization improves your maximin or minimax payoff. Similarly, to get across the idea of an *equilibrium* in mixed strategies, a good way to start is by getting students to trace out the succession of best responses that goes full circle in the tennis table or equivalent.

Once you have provided some intuition or motivation for the use (and usefulness) of mixed strategies, you can approach the problem of solving for the equilibrium mixtures. Students will quickly grasp the ideas that mixed strategies are a special case of continuous strategies in which players choose a value for p—the probability of using one of the two possible actions—from the interval [0, 1] and that the value of p at 0 or 1 coincide with the use of a pure strategy. Given this, we have been most successful starting with zero-sum games and the minimax analogy—arguing that the row for the p-mix and the column for the q-mix contain payoffs for all possible values of p against the rival's pure strategies. These payoffs are represented as functions of p or q and, as such, can be graphed. Then the maximin/minimax process implies that the row player's maximin occurs at that value of p which maximizes the entire set of minimum payoffs; graphically, this is the p value at the tip of the inverted V of minimum payoffs. Similarly for the column players. Students who have already learned the minimax method accept the logic of this process readily.

This method, which we call the prevent-exploitation method, works only for games in which a rival's best choice for herself is the worst choice for you, so it works only in zero-sum games. However, once you have derived the equilibrium mixtures, you can show that these have an additional feature; they serve to keep the opponent indifferent between her two pure strategies. This is the logic that we employ when searching for mixed-strategy equilibria in non-zero-sum games, in our keep-the-opponent-indifferent method; you can show the result arises in the zero-sum case and argue that equilibrium mixes must, in general, achieve this indifference property. Then it is much easier to show how the non-zero-sum analysis proceeds.

For games with multiple equilibria in pure strategies, as well as a mixed-strategy equilibrium, it is useful to show the best-response curve diagram for p and q. This diagram allows students to see *all* of the Nash equilibria of such games in one place; pure-strategy equilibria occur when the best-response curves meet at one or more of the corners, and mixed-strategy equilibria occur when the curves intersect at values of p and q in between 0 and 1. In games in which players' interests are opposed, like chicken, the pure-strategy equilibria occur at the top left, (0, 1), and bottom right, (1, 0), of the diagram, while in games in which the players' interests are aligned, like assurance games, the pure-strategy equilibria occur at the origin, (0, 0), and at the upper right corner, (1, 1).

One example that we have used successfully in class when covering basic 2-by-2 mixing is based on the Sherlock Holmes–Moriarty story that is told in John von Neumann and Oskar Morganstern's, *Theory of Games and Economic Behavior* (Princeton, N.J.: Princeton University Press, 1944), pages 176–178. The gist of the story is that Holmes is trying to get from London via Canterbury to Dover by train (and hence to the Continent) in order to escape from Moriarty. Moriarty has seen Holmes enter the train and can presumably get another himself. Each must then decide whether to go all the way to Dover or to get out at the single intermediate station at Canterbury; Holmes would prefer to make the opposite decision from Moriarty while Moriarty would prefer to make the same decision as Holmes.

Many students will not have read the story. For that matter, many may not be entirely familiar with the geography of England and France. [One of us (Dixit) has used a schematic drawing of the southeast corner of England showing the cities in question, the Channel, and the northern coast of France to make a joke about the modern student's ignorance of geography.] It may be worthwhile to add a bit of background related to Conan Doyle and the Holmes stories as well as Moriarty, who was not only the "godfather" of crime in London but a professor of mathematics as well.

The payoff table we have used for this game is:

		HOLMES		
		Dover	*Canterbury*	
MORIARTY	Dover	75	0	Min = –0
	Canterbury	–50	100	Min = –50
		Max = 75	Max = 100	

The payoffs (which differ slightly from those found in von Neumann and Morganstern) are to Moriarty, who is most truly defeated if he exits at Canterbury to find that Holmes has continued safely on to Dover. The maximin of 0 does not equal the minimax of 75 here; the game has no equilibrium in pure strategies.

Moving on to consider mixed strategies, you can add a row for Moriarty's p-mix (see new payoff table below) and show how this new strategy allows Moriarty a way of improving his maximin payoff.

		HOLMES		
		Dover	*Canterbury*	
MORIARTY	Dover	75	0	Min = 0
	Canterbury	–50	100	Min = –50
	p-mix: $pD + (1 - p)C$	$75p - 50(1 - p)$	$100(1 - p)$	Min = ?

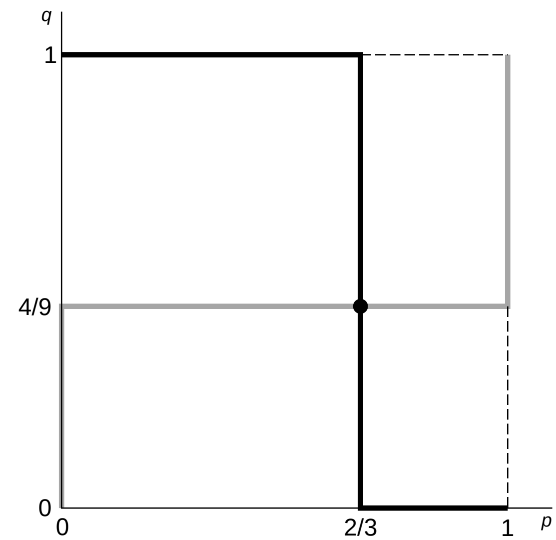
Graphing Moriarty's payoffs against Holmes playing either Dover (–50 payoff at $p = 0$ and 75 payoff at $p = 1$) or Canterbury (100 payoff at $p = 0$ and 0 payoff at $p = 1$) allows you to show Moriarty's minimum payoff in the p-mix row as an inverted V on the graph. The maximin occurs at the intersection of the two lines on the graph or at the value of p that solves $75p - 50(1 - p) = 100(1 - p)$; this value of p is 2/3. Moriarty's maximin payoff is $100(1 - p)$ (or $-50(1 - p) + 75p$) for $p = 2/3$ or 100/3, or $33\frac{1}{3}$.

Similar calculations can be done for Holmes in determining his q-mix. You can show his mixture as a third column in the payoff table:

		HOLMES		
		Dover	*Canterbury*	q-mix: $qD + (1 - q)C$
MORIARTY	Dover	75	0	$75q$
	Canterbury	–50	100	$-50q + 100(1 - q)$
		Max = 75	Max = 100	Max = ?

Graphing payoffs for Holmes against Moriarty playing his pure strategies as in Figure 5.5—using Moriarty's payoffs on the vertical axis (since Holmes gets the opposite) and q on the horizontal axis—allows you to show Holmes's maximum payoffs as a V-shaped curve on the graph. The minimax value of q occurs at the intersection of the line representing Moriarty's choice of Dover (intercept 0 at $q = 0$ and 75 at $q = 1$) and the line representing Moriarty's choice of Canterbury (intercept 100 at $q = 0$ and –50 at $q = 1$); this value of q solves $75q = -50q + 100(1 - q)$. The minimax q in this case is 4/9; Holmes's minimax payoff, given this q, is 100/3, or $33\frac{1}{3}$.

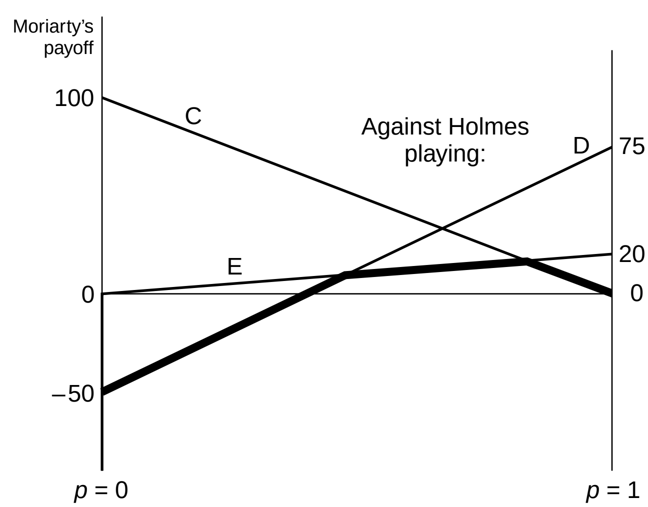
In this example we see that the maximin payoff for Moriarty is equal to the minimax payoff for Holmes so that the mixture values we calculated above—$p = 2/3$ and $q = 4/9$—constitute a mixed-strategy equilibrium of the game. The best-response curves for the two players in this game are illustrated below; as shown, there is only one equilibrium of the game and that is the equilibrium in mixed strategies for $p = 2/3$ and $q = 4/9$.



You can extend this example to one in which Holmes has three pure strategies (although this goes beyond the scope of the analysis in von Neumann and Morganstern). Here Holmes has a third strategy, Emergency, which is to pull the train's emergency stop cord somewhere between Canterbury and Dover and to exit the train in this way. This strategy also forces Moriarty's trailing train to stop when it discovers the track ahead blocked. What are the payoffs from this new strategy? If Moriarty has gotten off at Canterbury, then for Holmes Emergency is obviously better than Canterbury but worse than Dover because it still leaves him in the interior and not immediately on the boat to France; hence Emergency gets Holmes 0 if Moriarty gets off at Canterbury. But if Moriarty goes on to Dover, then his train will be held up due to the train emergency, and Holmes will find Emergency better than Dover but worse than Canterbury; we give Holmes a –20 (and Moriarty a 20) for this combination. Then we get the following payoff matrix, with a row added for Moriarty's p-mix:

		HOLMES		
		Dover	*Canterbury*	*Emergency*
MORIARTY	Dover	75	0	20
	Canterbury	– 50	100	0
	p-mix: pD + (1 – p)C	$75p - 50(1 - p)$	$100(1 - p)$	$20p$

Moriarty's minimum payoffs here no longer created an inverted-V shape as p varies from 0 to 1 because Holmes has three strategies. Rather we see Moriarty's set of minimum payoffs as the thick line in the diagram below:



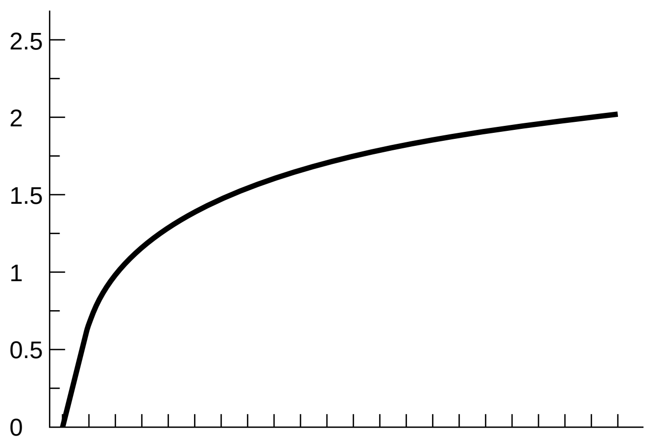
The maximin value of p chosen by Moriarty is the one that yields the highest point on the thick line in the diagram, or the p at the intersection of Holmes's C(anterbury) and E(mergency) lines. That p solves $20p = 100(1 – p)$; $p = 5/6 = 0.83$. Moriarty's maximin payoff is 100/6 = 16.67.

Note that Moriarty's mix would be better against Holmes's use of the pure strategy Dover but Holmes does not use Dover in his mix. Holmes chooses C and E with probabilities z and $(1 – z)$. The equilibrium value of z solves $0z + 20(1 – z) = 100z + 0(1 – z)$ which yields $z = 20/120 = 1/6 = 0.17$. Holmes's minimax payoff is 100/6 = 16.67, which is the same as Moriarty's maximin.

TEACHING SUGGESTIONS FOR THE APPENDIX ON PROBABILITY AND EXPECTED UTILITY

Most students know the elementary combinatorial rules for probability algebra and need only a refresher with some examples. We have used card examples; you can easily construct similar ones with coins or dice.

The concept of risk aversion is simple at an intuitive level, but its treatment using expected utility can be difficult to get across. We have found it useful to involve the students. Take a particular utility function, say the logarithmic, and calculate the sure prospect that gives the same utility as the expected utility of a particular lottery. The logarithmic utility function is shown in the diagram below, with payoffs on the vertical axis representing the log of the dollar amount on the horizontal axis:



In this case, $U(10) = 1$ and $U(100) = 2$. One possible lottery to consider might be that in which there is a 50–50 chance of getting 10 or 100 (55 on average). With risk aversion, $U(0.5 \times 10 + 0.5 \times 100) = 1.74 > 0.5 \times U(10) + 0.5 \times U(100)$. Rather, $0.5 \times U(10) + 0.5 \times \mathrm{U}(100) = 1.5 = \mathrm{U}(31.6)$. Thus, \$31.60 gives the same amount of utility as the 50-50 lottery between \$10 and \$100 under this utility function. Now ask for a vote on how many students would accept the sure prospect (\$31.60) and how many the lottery (50% chance of \$10 and 50% chance of \$100). If a majority would accept the sure prospect, say, "Most of you seem more risk-averse than this. Let us try a more concave function, say $U(x) = –1/x$" and repeat the experiment. You can use this process to try to find the risk aversion of the median student.

A few students get sufficiently intrigued by this to want more. If your class gets interested, and if you have time, you can talk about the history of the subject (St. Petersburg paradox and all that) or about the recent work in psychology and economic theory on non-expected-utility approaches. For a discussion of the St. Petersburg paradox, see http://plato.stanford.edu/archives/spr1999/entries/paradox-stpetersburg/; or consider using the following simple example of the Allais paradox that can help students see that they do not always make choices consistent with maximizing their expected utility.

Describe first a choice between two lotteries: Lottery A pays \$3,000 with probability 1 and Lottery B pays \$0 with probability 0.2 and \$4,000 with probability 0.8. Ask students to choose which lottery they would prefer to enter at a price of zero (and ask them to make note of their choices). Most choose A over B. Then describe a choice between two different lotteries: Lottery C pays \$0 with probability 0.8 and \$4,000 with probability 0.2; Lottery D pays \$0 with probability 0.75 and \$3,000 with probability 0.25. Again ask students to pick. Most choose C over D.

Now consider how the paired choices fit with the idea that people maximize expected utility. Set $U(0) = 0$. For those who chose A and C, this implies that $EU(A) > EU(B)$ or that $1U(3{,}000) > 0.8U(4{,}000)$; but choosing C implies that $EU(C) > EU(D)$ or that $0.2U(4{,}000) > 0.25U(3{,}000)$. The latter is equivalent to $0.8U(4{,}000) > 1U(3{,}000)$. This is in direct contradiction to the implication made when choosing A over B. Similar calculations can be used to show that those who choose B and D also violate the expected utility hypothesis. The choices of both A and D, or both B and C are consistent with maximization of expected utility.

GAME PLAYING IN CLASS

GAME 1 Rock-Scissors-Paper

This game entails playing three different versions of the children's game rock-scissors-paper. In rock-scissors-paper, two people simultaneously choose either Rock, Scissors, or Paper, usually by putting their hands into the shape of one of the three choices. The game is scored as follows: A person choosing Scissors beats a person choosing Paper (because scissors cut paper). A person choosing Paper beats a person choosing Rock (because paper covers rock). A person choosing Rock beats a person choosing Scissors (because rock breaks scissors). If two players choose the same object, they tie. Technically, players end up mixing over three possible pure strategies, and the analysis for this is not presented until Chapter 7. You can modify this in-class game so that it can be used to teach about two-by-two games, use it as is but focus your discussion on the points relevant to two-by-two games, or play it immediately before covering the analysis of larger games found in Chapter 7.

Instructions for students: In this experiment, each individual game (and we will play multiple games) is worth 10 points. Because it is a zero-sum game, the winner gets 10 points and the loser gives up 10 points. The following matrix shows the possible outcomes in the game.

		PROFESSOR		
		Rock	*Scissors*	*Paper*
STUDENT	Rock	0	10	–10
	Scissors	–10	0	10
	Paper	10	–10	0

In all three of the games described below, you are the row player and your professor is the column player. For each situation, we will simulate 60 repetitions of a game. Because actually signaling with our hands 60 times would be too time-consuming, you are instead asked to describe your behavior by writing down how many times (on average) out of 60 games you would play Rock, how many times you would play Scissors, and how many times you would play Paper.

Your professor will choose a particular strategy to use in each of three situations; her strategy in Situation 1 differs from her strategy in Situation 2, and so on. In each situation, your professor uses the same strategy against every student. You will be told your professor's exact strategies for Situation 1 and Situation 3 when we analyze the outcomes of the games; your professor's exact strategy for Situation 2 is explained below. Your task in each of the three games is to choose the strategy that you think (or guess) is most likely to maximize your total payoff.

Situation 1

This is the regular version of the game, using the payoffs above. You are asked to write down how many times (on average) out of 60 you would play Rock, how many times you would play Scissors, and how many times you would play Paper. In a similar way, your professor has written down numbers that describe her behavior. You can assume that your professor plays her equilibrium strategy here.

Situation 2

In this version of the game, the payoffs remain as above, but your professor commits (for some reason) to picking Rock 24 times out of 60 (40%), picking Scissors 18 times out of 60 (30%), and picking Paper 18 times out of 60 (30%). (The order in which she will make these picks is unknown.) No matter what anybody writes down, your professor's behavior will follow this rule. Knowing this, again try to maximize your total payoff.

Situation 3

In this version of the game, the row player (you) has an advantage. If you pick Rock and your professor picks Scissors, you win 20 points from your professor. The payoff matrix is now:

		PROFESSOR		
		Rock	*Scissors*	*Paper*
STUDENT	Rock	0	20	–10
	Scissors	–10	0	10
	Paper	10	–10	0

This time, your professor will use a strategy that she thinks will be beneficial for her (but she does not know for certain that it will be beneficial). Again, try to maximize your total payoff.

When discussing the results of these games, you will be able to note that everyone's payoff in Situation 1 is identical, regardless of her choice. This is a direct result of the fact that equilibrium mixtures keep a rival indifferent; any pure strategy or combination of pure strategies yields the same payoff against someone playing her equilibrium mixture. In Situation 2, the professor does not play her equilibrium mix. Some students (but perhaps not many) will figure out that this is not the equilibrium mix and that it can therefore be exploited. You can show on the board during the discussion that the student, using pure Rock, can obtain an expected payoff of 0 per game; the student, using pure Scissors, can obtain an expected payoff of –1 per game (so –60 over 60 plays of the game); or the student, using pure Paper, can obtain an expected payoff of 1 per game and 60 over 60 plays of the game. Given the disequilibrium mixture, the student can take advantage of the professor's "error" by playing Rock 100% of the time.

Finally, in Situation 3, you can consider a number of different strategies. If you again use your equilibrium mix, then the payoff to you is 5/6 (on average) per game, for a total of 50 over the 60 games. We have also used a number of nonequilibrium mixes on occasion; the most interesting one to analyze is the one that avoids the use of Scissors entirely, putting equal weight on Rock and Paper. This helps to make the point about the counter-intuitive outcome—that the student wants to avoid Rock in the new equilibrium mix—even more stark. Against a professor mixing only Rock and Paper in equal proportions, the student does best to use all Paper.

This game can be used to motivate: (1) the calculation of equilibrium mixes for larger games; (2) that a rival's equilibrium mix keep you indifferent among your pure strategies or any mixture, so makes your choice of strategy irrelevant; (3) that exploitation is possible if a rival is using a nonequilibrium mix—one of your pure strategies will be dominant; and (4) the counterintuitive outcome that, in equilibrium, you decrease your use of a strategy for which one payoff has increased.

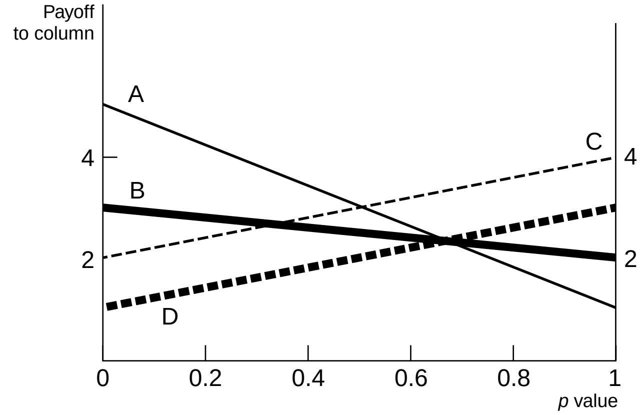
ANSWERS TO EXERCISES FOR CHAPTER 5

1. (a) For Row: p = probability of Up = 1/6 and $(1 – p)$ = probability of Down = 5/6.
   For Column: q = probability of Left = 1/2 and $(1 – q)$ = probability of Right = 1/2.
   (b) For Row: p = probability of Up = 3/4 and $(1 – p)$ = probability of Down = 1/4.
   For Column: q = probability of Left = 1/2 and $(1 – q)$ = probability of Right = 1/2.
   (c) For Row: p = probability of Up = 1/2 and $(1 – p)$ = probability of Down = 1/2.
   For Column: q = probability of Left = 1/2 and $(1 – q)$ = probability of Right = 1/2.
2. False. A player's equilibrium mixture is devised in order to keep her opponent indifferent among all of her (the opponent's) possible mixed strategies; thus, a player's equilibrium mixture yields the opponent the same expected payoff against each of the player's pure strategies. Note in part c that Row's p-mix yields Column the same expected payoff from playing Left ($0 + 1 \times 1/2 = 1/2$) as she gets from playing Right ($2 \times (1/2) + (–1) \times (1/2) = 1/2$). Similarly for Column's q-mix which yields Row an expected payoff of $4 \times (1/2) + (–1) \times (1/2) = 3/2$ from playing Up and an expected payoff of $1 \times (1/2) + 2 \times (1/2) = 3/2$ from playing Down. The statement is true for zero-sum games because, when your opponent is indifferent in such a game, it must also be true that you are indifferent as well.
3. Payoff matrix shown below. There are two pure strategy Nash equilibria: (Help, Not) and (Not, Help). There is also a mixed-strategy equilibrium.

		YOU		
		Help	*Not*	*q*-mix
I	Help	2, ***2***	2, ***3***	$2q + 2(1 – q)$, $\boldsymbol{2q + 3(1 – q)}$
	Not	3, ***2***	0, ***0***	$3q$, $\boldsymbol{2q}$
	p-mix	$2p + 3(1 – p)$, $\boldsymbol{2p + 2(1 – p)}$	$2p$, $\boldsymbol{3p}$	

The optimal p for I should keep you indifferent, so p must satisfy $2p + 2(1 – p) = 3p$, or $3p = 2$, so $p = 2/3$. Similarly (the game is symmetric), $q = 2/3$. The mixed-strategy equilibrium is that I Help 2/3 of the times and Not 1/3 of the times and You play the same strategy. The expected equilibrium payoff for I is $2(2/3)(2/3) + 2(2/3)(1/3) + 3(1/3)(2/3) + 0(1/3)(1/3) = 8/9 + 4/9 + 6/9 = 18/9 = 2$. For You, the expected payoff is also 2.

4. (a) There is no pure-strategy Nash equilibrium here, hence the search for an equilibrium in mixed strategies. Row's p-mix (probability p on Up) must keep Column indifferent so must satisfy $16p + 20(1 – p) = 6p + 40(1 – p)$; this yields $p = 2/3 = 0.67$ and $(1 – p) = 0.33$. Column's expected payoff = 17.33. Similarly, Column's q-mix (probability q on Left) must keep Row indifferent so must satisfy $q + 4(1 – q) = 2q + 3(1 – q)$; correct q here is 0.5. Row's expected payoff is 2.5
   (b) Joint payoffs are larger when Row plays Down, but the highest possible payoff to Row occurs when Row plays Up. Thus, in order to have a chance of getting 4, Row must play Up occasionally. If the players could reach an agreement to always play Down and Right, both would get higher expected payoffs than in the mixed-strategy equilibrium; this might be possible if the game were repeated or if guidelines for social conduct were such that players gravitated toward the outcome that maximized total payoff.
5. False. When a game has several Nash equilibria, these may be either all pure-strategy Nash equilibria or a combination of pure-strategy equilibria along with a mixed-strategy equilibrium. A mixed-strategy equilibrium does not, however, imply an outcome in which playing the game entails a random mixture between all the other equilibria. Rather, the outcome of such a game depends on whether any of the equilibria are focal, as in Exercise 3, in which case one might argue that the (Help, Help) pure-strategy equilibrium is focal.
6. (a) Two pure-strategy Nash equilibria: (2, A) yielding payoffs of (2, 5) and (1, C) yielding payoffs of (3, 4).
   (b) When Row mixes strategies 1 and 2 in proportions p and $(1 – p)$ respectively, we can show Column's payoffs on a graph as follows:



For $p < 0.5$, Column will play A, and for $p > 0.5$, Column will play C. When $p = 0.5$, Column is indifferent, so willing to mix A and C; Column's mixture will never involve B or D. To find Column's mixture probabilities, choose q = probability of playing A and $(1 - q)$ = probability of playing C to keep Row indifferent between strategies 1 and 2. The correct q satisfies $1q + 3(1 - q) = 2q + 1(1 - q)$; here, $q = 2/3 = 0.67$. The equilibrium mixtures entail Row mixing over 1 and 2 with probability 0.5 each and Column mixing over A and C only with probabilities 0.67 and 0.33 respectively.

7. (a) Row has a dominant strategy if C > A; Column has a dominant strategy if C > B.
   (b) If neither of the conditions in part a hold, then we have C < A and C < B. In this case there can be no Nash equilibrium in pure strategies.
   (c) The conditions for part b, A > C and B > C, lead to no Nash equilibrium in pure strategies, but there will be an equilibrium in mixed strategies.
   (d) The mixed-strategy equilibrium value of p (probability of choosing Up) for Row must satisfy $p\text{A} = (\text{B} - \text{C})(1 - p)$ or $p/(1 - p) = (\text{B} - \text{C})/\text{A}$. Then $p = (\text{B} - \text{C})/(\text{A} + \text{B} - \text{C})$.

ADDITIONAL EXERCISES WITH ANSWERS

1. Suppose that a person owes \$100 in taxes to the government. The person can either pay the \$100 or cheat and pay nothing. The government can either audit this person or not audit him; the decision about whether to audit is made without knowing what choice the person made.

   If the government audits a person who picked Cheat, the tax evasion is discovered and the person must pay \$150 to cover both his taxes and a fine imposed on detected cheaters.

   Auditing is costly; when the government audits a person, both the government and the person lose \$10 (this cost must be paid regardless of whether the person chose Cheat or Pay).

   The payoffs in this non-constant-sum game are measured so that (for example) if the person chooses Pay and the government chooses Not audit, the person's payoff equals 0 and the government's payoff equals 100.

   (a) Fill in the two missing numbers in the following payoff matrix.

		GOVERNMENT	
		Audit	*Not*
PERSON	Cheat	–60, ***140***	, ***0***
	Pay	–10,	0, ***100***

   (b) Determine whether there is an equilibrium in pure strategies in this game.
   (c) Suppose that the government chooses Audit with probability q (and, obviously, chooses Not with probability $1 - q$). Determine the value at which q must be set to ensure that the person gets the same expected payoff from Cheat as he gets from Pay (this is equivalent to asking you to find the value of q that holds in a mixed-strategy equilibrium in this non-constant-sum game).
   (d) Suppose that the person feels (somewhat) guilty when he chooses Cheat. We will represent this feeling of guilt by saying that the person's payoff when he chooses Cheat (whether he is audited or not) falls by 20. In this case, at which value must q be set to ensure that the person gets the same expected payoff from Cheat as he gets from Pay?
   (e) Compare the outcomes from parts c and d. Bearing in mind the probability with which the government chooses Audit, would you expect to see any difference between the person's equilibrium behavior in part c and his equilibrium behavior in part d? If not, is there any benefit to society from the person's guilty conscience in part d?

   ANSWER (a)

		GOVERNMENT	
		Audit	*Not*
PERSON	Cheat	–60, ***140***	100, ***0***
	Pay	–10, ***90***	0, ***100***

   (b) No pure-strategy equilibrium.
   (c) Solve $-60q + 100(1 - q) = -10q$ to find $q = 2/3$.
   (d) Solve $-80q + 80(1 - q) = -10q$ to find $q = 8/15$.
   (e) Guilty conscience means that (costly) auditing need occur with a probability of only 53% rather than of 67%.

2. Using this constant-sum, simultaneous game, in which the numbers in the matrix are payoffs to Player 1 in which Z is some number larger than 1, consider the following argument: "Since Action A offers Player 1 the possibility of an outcome that is more favorable than is any outcome that he can obtain from Action B, we can be certain that Player 1 will choose Action A more often than he will choose Action B." State whether this argument is correct, and briefly explain why or why not.

		PLAYER 2	
		Action X	*Action Y*
PLAYER 1	Action A	Z	–1
	Action B	–1	1

ANSWER Incorrect. A large Z leads Player 2 to choose Action X less often than Action Y. In equilibrium, Player 1 responds by picking Action A less often than Action B.

3. Consider the following game between two opposing baseball managers. A runner is currently on first base, and the manager of the team that is batting has to decide whether to choose to try to Steal second base or to choose No Steal. The manager of the pitching team has to decide whether to choose to call for a Pitchout or to choose No Pitchout.

   The four possible outcomes can be evaluated as follows. Choosing to pitchout when a steal is attempted is favorable for the pitching team, since the steal becomes relatively unlikely to succeed. Choosing not to pitchout when a steal is attempted is unfavorable for the pitching team, since the steal is more likely to succeed. Pitching out when no steal is called is slightly unfavorable to the pitching team, because the batter will be in better position on the next pitch. A No pitchout-No steal outcome is neutral.

   This is a constant-sum, simultaneous game. On the basis of the identity of the current baserunner and the current game situation, suppose that point values (where the points measure value to the pitching team) can be assigned to the various outcomes as shown.

		BATTING	
		Steal	*No Steal*
PITCHING	Pitchout	3	–1
	No Pitchout	–2	0

   Given these payoffs and assuming the managers act as they would in a mixed-strategy equilibrium, which is more likely: a pitchout or a steal attempt? (Show all your calculations.)

   ANSWER A pitchout is more likely; it occurs with probability 1/3, while a steal occurs with probability 1/6.

4. State whether the following is true or false and explain why. In a constant-sum game, when you are behaving just as you would in a mixed-strategy equilibrium, there is one particular thing you need to fear; namely, you are open to being exploited, in the sense that there is one strategy available to your opponent that would (in expected value) leave him better off and leave you worse off than would any other strategy available to him.

   ANSWER False. In a constant-sum game, a player chooses a mixed strategy to prevent being exploited.

5. Suppose that both players in a game (that will be repeated multiple times) are using mixed strategies. “Using mixed strategies” means that the players are _____.

   (a) mixing the order of play; sometimes one player moves first, and sometimes the other moves first
   (b) mixing the side of the game that each person takes; sometimes one person takes the part of Player A, and sometimes that same person takes the part of Player B
   (c) Both of the above are true.
   (d) None of the above are true.

   ANSWER (d)

6. The following matrix shows that payoffs for a constant-sum game (the players are dividing up a total payoff of 4) in which two players (simultaneously) call out either “heads” or “tails.” The numbers in the matrix show the payoffs received by Player 1. Note that Player 1 does well if both players call out the same word (and does especially well if they both call out “heads”); Player 2 does well when different words are called out.

		PLAYER 2	
		Heads	*Tails*
PLAYER 1	Heads	4	1
	Tails	1	3
	p-mix	*A*	*B*

   Suppose that we wish to find the mixed-strategy equilibrium of this game, in which Player 1 calls out “heads” with probability p. In one of the methods used to compute this equilibrium, which formulas should replace the A and B in the matrix?

   (a) $A + 4p + (1 - p)$, $B = p + 3(1 - p)$
   (b) $A + 4(1 - p) + p$, $B = (1 - p) + 3p$
   (c) $A + 4p + 4(1 - p)$, $B = p + (1 - p)$
   (d) $A + p + (1 - p)$, $B = 3p + 3(1 - p)$

   ANSWER (a)

7. Suppose that the game in Exercise 6 is repeated a number of times and that both players use the strategies that make up a mixed-strategy equilibrium. In this situation, we can compute that Player 1 calls out “heads” _____ than half the time and that Player 2 calls out “heads” _____ than half the time.

   (a) more; more
   (b) less; less
   (c) more; less
   (d) less; more

   ANSWER (b)