CHAPTER 8 The Prisoners' Dilemma Game

TEACHING SUGGESTIONS

The idea that tacit cooperation can be sustained in an ongoing relationship is very simple and students easily accept it. The formal analysis in a repeated game is much harder if your students do not have an economics or business background and are unused to compound interest, discounted present values, and summing infinite geometric sums. Be prepared to spend a lot of time on this, repeating the exposition and clarifying difficulties.

As usual, it helps to have a specific game with which they are familiar or which they have been required to play. (We describe a simple prisoners' dilemma game in Game 2 below that can be used to motivate the concept of leadership. A symmetric version of that game can be used as an example elsewhere in the presentation of material from this chapter.) Build up the intuitive nature of the cheat once option first, showing the gain from cheating and the loss that must be incurred one period later before cooperation can be resumed. (Of course, resumption of cooperation is possible only with an opponent who is playing TFT.) Those students who have had little experience with present value should be amenable to the argument that time is valuable and that money now is better than money later. Remind them about the possibility of placing money in the bank where it can earn interest, even over the span of a month or two. If your example has a gain from cheating that is exactly equal to the loss during the punishment phase, it is easier to see that cheating would be worthwhile in such a situation, at any positive rate of interest. Once students grasp this idea, you can move on to the cheat forever option and a more complicated present discounted value calculation. Don't forget to remind them that the conditions you derive relating to the interest rate at which cheating becomes worthwhile are example-specific; just because you need $r > 0.5$ in one game does not mean that $r > 0.5$ is a general condition that determines whether players will cheat.

By contrast, the solutions to the prisoners' dilemma based on penalties are much simpler and can largely be left for the students to read. You can use different illustrations of penalty systems depending on your audience. For example, to convey the idea that if the active players in the dilemma solve it and sustain cooperation, this can be bad for the rest of society, you can use the example of a cartel for economics students or of logrolling among a group of incumbent legislators for political science students. You can also introduce the resolution of the dilemma based on rewards for cooperating in which the credibility of promises to reward is questionable and can be achieved if some third party can hold a player's promise in an "escrow account." If you chose to have your students play the game of Zenda (in the Game Playing in Class section of this chapter) early in the term, they have already had an example of this approach. Now that they have seen a thorough analysis of the prisoners' dilemma, they may have additional insights into the game and their choices; this would be a good time to encourage discussion of their thoughts.

The possibility of solving the prisoners' dilemma through leadership is often less intuitive to students. Having them play an in-class game like Game 2 below can help them to appreciate the differing incentives faced by large and small players in a dilemma game. The game's framework can also be used to show how incentives change as the size disparity between players grows.

There are many, many examples of the prisoners' dilemma that you can point out to your students, from the stories provided at the end of the chapter or from your own experience. You may want to ask the students to try to find their own examples of dilemma situations; they can look for strategic situations that have the three identifying characteristics of a prisoner's dilemma. Students could suggest situations in class that might fit the criteria, and the class could debate whether the various situations qualify as prisoners' dilemmas.

Another good way to encourage discussion about the prisoners' dilemma is to get students thinking and talking about how the actual actions of players in such dilemmas often diverge from the predictions of the theory. This is easiest if they have been forced to play, either against each other, as suggested in Game 1 below, or against a computer. One web-based interactive version of the prisoners' dilemma is available at http://serendip.brynmawr.edu/~ann/pd.html. Each game played against the computer (Serendip) is of finite but unknown length (usually between 10 and 20 rounds), and the computer tracks the total as well as the average gain for each player during the game; there is a link to an explanation of the game that tells you the computer is playing tit-for-tat. Although most students don't read this first, virtually all of them figure it out. You can ask students to play against the computer before class and to keep track of their choices and their outcomes so that they can participate in a discussion during class. They will most certainly come up with a variety of different stories about how they tried to take advantage of the computer's forgiving play. You may also find that they play more to beat the computer than to maximize their own gains; this is your chance to suggest that sometimes predictions are wrong, not because the theory is wrong but because the theorist misunderstands the incentive structures or payoff functions of the players whose behavior she is predicting.

GAME PLAYING IN CLASS

GAME 1 Paired Prisoners' Dilemma

Students can be paired off and instructed to play several versions of a particular game with a prisoners' dilemma structure. Provide each pair with a sheet describing the payoff table for the game and with space available to indicate the choices made by each player in each round and in each version of the game.

First, tell students that the game will end after a known number of rounds, say 10. Have them play through 10 rounds, keeping track of their choices. Then have them play another 10-round game, and a third if there is time. As with the dime games, you should see convergence to the rollback equilibrium of always cheating within a few plays of the finite game; students who try to cooperate originally will lose out to cheating rivals at the end of the game and will cheat earlier and earlier in each successive play of the game.

Second, try a version in which the students play a round at a time without knowing when the game will end; you call out an end to the game after some number of minutes. This version is similar to the Serendip version available on the web (see URL above) except that the rival's strategy is not as predictable. In addition, there is uncertainty about how long the game will last since it could be very short (unlike the web version) or relatively long. You can do this several times and ask the students to describe how their behavior changes in future plays of the game.

Each of these games gives the students the opportunity to experience the actual play of a prisoners' dilemma. The discussion that follows can be used to consider discrepancies between predicted and actual behavior, changes in behavior in later plays of the same game, how players update information about a rival's play, and other issues related to players' willingness to cooperate (including cultural norms, friendships, rivalries, etc.)

GAME 2 Leadership in an Oil Production Game

Students can be broken into pairs to play this game once, with each student's representing one country; then each should switch partners and play as if she were the other country. Students could also play this game individually, indicating which action they choose when playing as each country; you can pair sets of answers to show outcomes. This game is based on the Rational-Pigs game in John McMillan's *Games, Strategies, and Managers* (London/New York: Oxford University Press, 1996), page 13; he uses the Saudi Arabia analogy on page 14.

Ask students to consider a very simplified version of the situation facing members of OPEC in the 1970s. In this simplified story, we assume that OPEC is made up of only two countries—Saudi Arabia and Kuwait. Suppose that each country can produce a unit of oil (maybe a million barrels) at a cost of $1. Saudi Arabia is a big country and can produce either 4 or 5 units of oil; Kuwait is a small country and can produce either 1 or 2 units of oil.

Given output and consumption in the rest of the world, the following formula tells us the price at which oil is sold: Price = $10 - (Q_S + Q_K)$, where Q_S is the number of units produced by Saudi Arabia (SA), and Q_K is the number of units produced by Kuwait (K). Then profits are calculated as follows. If SA produces 4 units and K produces 1 unit, the price of a unit of oil equals 5 (from the above formula). Because the cost of producing a unit of oil equals $1, a country earns a profit of $4 for each unit it makes. Because SA produces 4 units, its total profit equals $16; because K produces 1 unit, its total profit equals $4. Profit per unit obviously depends on total production, as the following table illustrates:

SA production + K production	Profit per unit
5	4
6	3
7	2

Joint profit (SA's profit plus K's profit, or OPEC profit) is maximized if total production equals 5 units.

Finally, tell students to put themselves in this situation, tell them to worry only about their own profits, and ask them: (1) How many units would you produce if you were Saudi Arabia? (2) How many units would you produce if you were Kuwait?

When discussing this game, you can first collect student input on their choices, writing on the board the number of students who chose 4 as Saudi Arabia and 2 as Kuwait. Then you can show how the game can be analyzed more formally. It is nice to show that the symmetric version of the game (in which Saudi Arabia and Kuwait each choose either 2 or 3 units) is a standard prisoners' dilemma; the smaller output level is the cooperative strategy. (You might even want to use this as your example of a standard prisoners' dilemma if you play the game before you start the material from Chapter 8.) Once you change the choices available to the players to be consistent with the description of the game and change the payoffs accordingly, you will be able to illustrate how the analysis changes. Encourage students to provide hypotheses for why Saudi Arabia's incentives have changed and build on their ideas to show that large players often incur a much higher cost of cheating than do small players. Ask students to think of other examples of dilemma situations with leadership or provide some of your own.

COMPUTER GAME Zenda

This game was invented by James Andreoni and Hal Varian; see their article, "Pre-Play Contracting in the Prisoners' Dilemma" (October 1993, unpublished, available from the author's web site, http://www.sims.berkeley.edu/~hal/Papers/Pre-Play.pdf). The paper also contains some code in C. Zenda is a prisoners' dilemma, but this is concealed behind a facade of playing cards and Pull and Push mouse clicks in such a way that students do not easily figure this out. (They could, from the name and word association, but few are sufficiently widely read or addicts of the right kind of movies.) Nevertheless, the game is best played during an early week of the semester, before you have treated the dilemma in class.

Make sure you have an even number of students. The program matches them randomly in pairs. Each student sees two cards for herself in the bottom half of her screen, and two cards for the player with whom she is matched in the top half of her screen. For each student, there is a low card called her pull card, and a high card called her push card. She can use her mouse to click on one of these. If she clicks on the low (pull) card, she gets from a central kitty a number of coins (points) equal to the value of that card. If she clicks on the high (push) card, *her opponent* gets from the same central kitty a number of coins (points) equal to the value of that card. The objective is to get as many coins for yourself as possible. The two matched in a pair make their choices simultaneously. They do not see each other's choice until both have clicked, when the actual transfer of coins takes place. Then new random pairings are formed, and the procedure is repeated. Depending on the time available, you can typically play up to 10 rounds of this. (Usually most students figure out after 2 or 3 rounds that pull is their dominant strategy.)

The values of the low and high cards a player has over her 10 rounds should be alternated in such a way as to allow each to get the same aggregate payoff if they play the correct strategies. This evenness is important if the exercise counts toward the course grade.

Then a second phase of the game begins. Here each player has the opportunity to bribe the other into playing Push; it shows how the prisoners' dilemma can be overcome if there is some mechanism by which the players can make credible promises. Again randomly matched pairs are formed. Again in each pair each player sees her and her opponent's cards. First each chooses how many coins she promises to pay her opponent if (and only if) the opponent plays Push. These bribes come from the player's own kitty (winnings from the first phase) and not from the central kitty. The bribes are put in an escrow box. Once both have set the bribes, each can see the bribe offered by the other. Then they play the actual game of clicking on the cards. When both have clicked, each gets the points from the central kitty depending on the push or pull choices as before. If your opponent plays Push, she gets the bribe you offered from your escrow box; if your opponent plays Pull, your bribe is returned to you from your escrow box. (The fact that the program resolves this disposition of the bribes makes the promise credible.) The bribe game is also played a number of times (typically 10 rounds) with fresh random matching of pairs for each round. Students quite quickly find the optimal bribing strategy.

You can try different variants (treatments) of the game: allow players to talk to one another or forbid talking, keep one pairing for several rounds to see if tacit cooperation develops, and so on. We append for your information the instructions given at the time of playing the game, and a report and analysis circulated later.

Instructions Given to Zenda Players

PHASE 1

In Zenda, you will play a simple card game with another player. Players are assigned to each other randomly, so that you will normally play a different person each time you play Zenda.

There are two phases to Zenda. We will first play Phase 1, which is called Push-Pull. When you play Push-Pull, you will see on your screen two cards for you, two cards for the other player, and a central pot with a pile of chips between the two of you.

Your cards are labeled Push and Pull. You can choose to play a card by clicking on it or the button under it with the mouse. When you choose a card it will become highlighted. You can only choose to play one card in a given round.

Your choice will not be final until you click on the Set Choice button!

At the same time you are deciding which card to play, the other player is making this decision too. Your payoff depends on the choice you make and the choice the other player makes.

If you choose the pull card, then you will pull the number of chips on that card from the central pot to your winnings. If you choose the push card, then you will push the number of chips on that card from the pot to the other player.

We will play 10 rounds of Phase 1; generally with different matchings in the different rounds. The objective is to maximize your own winnings; your score for this phase will equal the total number of chips you have at the end.

PHASE 2

Now you are in Phase 2 of Zenda. In this phase you have a new option called Pay to Push.

On your screen you will see a new button labeled Set Bribe and a slider. Each round you can offer to pay the other player a side payment if they choose to play their push card.

You set the amount of this bribe by moving the slider with the mouse. When you are satisfied with your decision about how much you are willing to pay the other player, you click on your Set Bribe button.

Your choice will not be final until you have clicked Set Bribe.

No player will see how much his or her opponent is willing to pay until both players have clicked on their Set Bribe button.

Your payoff will be as before, but if the other player chooses to Push, the bribe you offered will be subtracted from your winnings (and added to those of the other player). If the other player chooses to Pull, then you keep your side payment.

The other player makes the same sort of decision. The chips that you see in the box immediately in front of your cards are the amount the other player is willing to pay you if you choose to Push. The chips that you see in the box right above this are the payment you are willing to make to an opponent should she Push.

We will play 10 rounds of Phase 2, again with generally different pairings every time. Your final score for Zenda will be the total number of chips you have in your box at the end of the game.

REPORT AND ANALYSIS OF ZENDA (SENT TO STUDENTS)

In its basic structure, Zenda is a prisoners' dilemma. Many of you figured this out after a couple of rounds of play. To any who are literate or old movie buffs and figured it out from the name, special congratulations.

Suppose the row player has the cards 2, 7, and the column player 4, 8. The payoff matrix:

		COLUMN	
		Pull	*Push*
ROW	Pull	2, ***4***	10, ***0***
	Push	0, ***11***	8, ***7***

Pull is the dominant strategy for both. But both do better if both play Push.

Phase 1

With a finite number of rounds, even against the same opponent, the game should unravel by backward induction, so both should play Pull every time. When opponents change from one round to the next and the opponent in any one round is unknown, building cooperation should be even harder.

But some cooperation does emerge. (1) Players experiment a little, and when both of a pair play Push, they are encouraged to try Push again. (2) Some people in some groups try to build up a "culture" of cooperation by exhorting everyone to Push. (Of course, some of these people are themselves persistent cheaters! They will probably become top political leaders in 20 years or so.) Generally, there is very little cooperation in the first couple of rounds. Then people realize its value, and the middle rounds have more cooperation. Sometimes as many as half the people in the group are playing Push at this point. Then cooperation declines, and in the last couple of rounds it collapses.

The minimum score for Phase 1 is zero (if you Push and your opponent Pulls every time); the maximum from a 10-round phase with alternating roles is when you Pull and your opponent Pushes every time: $5 \times 11 + 5 \times 10 = 105$. If you both play Push all the time, the score is $5 \times 8 + 5 \times 7 = 75$. No one came close to this. If everyone cheats all the time, everyone gets $5 \times 2 + 5 \times 4 = 30$. You could fall even lower if you pushed while your opponents were pulling.

Phase 2

Here each round has a little sequential structure: (1) Setting the bribe. (2) Choosing the card. Use backward induction. Begin with the choice of the card once the bribes are set.

Suppose you are offered a bribe x, your pull card is y, you offer your opponent the bribe z, and his or her push card is w. Your payoff matrix is:

		OTHER	
		Pull	*Push*
YOU	Pull	y	$y + w - z$
	Push	x	$x + w - z$

Therefore w and z are irrelevant to your decision. You have a dominant strategy; which one it is depends solely on whether $x > y$ or $x < y$.

If $x = y$, your choice makes no difference to you but may make a big difference to the other player. Whether you resolve your indifference in a way favorable to the other (play Push) or unfavorable (play Pull) depends on whether you are nice or mean, or on other concerns like relative payoffs (if the course is graded on a curve, more points for your opponent indirectly hurts you).

Now consider the stage of offering a bribe. It is in your interest that the other plays Push. But you don't want to waste more chips than are necessary to achieve this aim. Therefore you should set your bribe equal to the other's pull card plus 1. The plus 1 may be unnecessary if the other player is nice, but most players wouldn't take that chance. In fact in our class we found that some people played even safer and offered a bribe equal to the other's pull card plus *2*.

Remember that the column player's pull card is 4 and the row player's pull card is 2. So if the row player bribes, the bribe is 5; if the column player bribes, the bribe is 3. If your opponent offers you such a bribe, your dominant strategy will be to Push; if not, to Pull. If you plug in this calculation (this is the subgame perfection argument), the payoff matrix at the stage where each player is deciding whether to offer a bribe to the opponent is:

		COLUMN	
		Not Bribe	*Bribe*
ROW	Not Bribe	2, ***4***	3, ***4 + 7 – 3 = 8***
	Bribe	2 + 8 – 5 = 5, ***5***	3 + 8 – 5 = 6, ***5 + 7 – 3 = 9***

At this stage Bribe is the dominant strategy for each player. The maximum score for a 10-round Phase 2 with alternating roles is $5 \times 6 + 5 \times 9 = 75$. Many students and groups did figure this phase out after 1 or 2 rounds.

ANSWERS TO EXERCISES FOR CHAPTER 8

1. False. The players are not assured that they will reach the cooperative outcome. Rollback reasoning shows that the subgame-perfect equilibrium of a finitely played repeated prisoners' dilemma will entail constant cheating.
2. The demands for the two pizza restaurants are

$$Q_D = 12 - P_D + 0.5P_P$$
$$Q_P = 12 - P_P + 0.5P_D$$

To determine the best cheating price, use one firm's best-response rule:

$$P_P = 7.5 + 0.25P_D$$

Use Pierce as the potential cheater (all conclusions apply equally to Donna). If P_D = \$13.50, Pierce's profit-maximizing price is 10.88. In this outcome, Pierce sells 12 – 10.88 + (0.5)(13.50) = 7.87 thousand pizzas and earns a profit of (10.88 – 3)7.87 = \$62,016. When Pierce cheats, Donna sells 12 – 13.50 + (0.5)(10.88) = 3.94 thousand pizzas and earns a profit of (13.50 – 3)3.94 = \$41,370. When the firms collude and charge \$13.50, each sells 12 – 13.50 + (0.5)13.50 = 5.25 thousand pizzas and earns a profit of (13.50 – 3)5.25 = \$55,125. As shown in Chapter 4, in the Nash equilibrium outcome, each firm charges \$10, sells 7 thousand, and earns a profit of \$49,000.

To determine if collusion is sustainable, compare the stream of profits when collusion holds to the stream of profits earned by a cheater. It is reasonable to assume that the noncheating firm is using a tit-for-tat strategy (or, equivalently, a grim strategy in the cheat forever case). Then we look first at the cheat forever strategy:

Collusion yields 55,125, 55,125, 55,125, ...
Cheat forever yields 62,016, 49,000, 49,000, . . .
(Nash equilibrium in all periods after first).

Discounting the infinite stream of payoffs, Pierce finds cheating to offer a higher payoff if

$$62{,}016 + 49{,}000/r > 55{,}125 + 55{,}125/r$$
$$62{,}016 - 55{,}125 > (55{,}125 - 49{,}000)/r$$
$$6{,}891 > 6{,}125/r$$
$$r > 6{,}125/6{,}891 = 0.889$$

Considering the cheat forever strategy, therefore, Pierce will cheat only if the monthly interest rate $r > 0.889$. (The question asks for the discount factors, $d = 1/(1 + r)$, at which collusion is sustainable. Pierce will cheat only if the (monthly) discount factor $d < 0.529$.)

Now, consider the cheat once strategy. There are some ambiguities in this case that involve describing the outcome that holds when Pierce signals his intention to reestablish cooperation after his (one-time) cheat. Note first that this problem does not arise in the high price–low price version of the pricing game. When Pierce cheats in that version, the outcome is (High, Low), and when he signals that he is ready to again cooperate, the outcome is (Low, High). In the continuous-price version of the game, the outcome in the reestablish-cooperation period is less clear. One possibility is that Donna assumes that the game will go to a Nash equilibrium and thus charges 10, while Pierce demonstrates his intention to begin cooperating by charging 13.5. In this case, Pierce sells 12 – 13.50 + (0.5)(10) = 3.50 thousand pizzas and earns a profit of (13.50 – 3)3.50 = $36,750. Assume that in the period after this, Pierce and Donna return to the collusive outcome. Under these assumptions, the possibilities for Pierce are:

Collusion yields 55,125, 55,125, 55,125, ...
Cheat once yields 62,016, 36,750, 55,125, ...

Pierce finds cheating to offer a higher payoff if

$$62{,}016 + 36{,}750/(1 + r) > 55{,}125 + 55{,}125/(1 + r)$$
$$62{,}016 - 55{,}125 > (55{,}125 - 36{,}750)/(1 + r)$$
$$6{,}891 > 18{,}375/(1 + r)$$
$$1 + r > (18{,}375/6{,}891) = 2.667$$
$$r > 1.667$$

Considering the Cheat Once strategy, therefore, Pierce will cheat only if the monthly interest rate $r > 1.667$. In discount rate terms, Pierce cheats only if the (monthly) discount factor $d < 0.375$. Different assumptions about the pricing in the second period will, of course, produce different conclusions about the critical values of p and d.

3. The demands for the two food stores are

$$q_1 = 10 - p_1 - 0.5p_2$$
$$q_2 = 12 - p_2 - 0.5p_1$$

To determine the best cheating price, use the firms' best-response rules, which were derived in the answer to Exercise 4.12:

$$p_1 = 5.5 - 0.25p_2$$
$$p_2 = 7 - 0.25p_1$$

The Nash equilibrium prices are $p_1 = 4$ and $p_2 = 6$, sales are $q_1 = 3$ and $q_2 = 4$, and profits are La Boulangerie: (4 – 1)3 = 9 and La Fromagerie: (6 – 2)4 = 16. Joint profits are thus 25. In the collusive outcome (again from the answer to Exercise 4.12), joint-profit-maximizing prices are $p_1 = 3.17$ and $p_2 = 5.67$, sales are $q_1 = 10 - 3.17 - (0.5)5.67 = 4$ and $q_2 = 12 - 5.67 - (0.5)3.17 = 4.75$, and profits are $Y_1 = (3.17 - 1)4 = 8.67$ and $Y_2 = (5.67 - 2)4.75 = 17.42$. Joint profits are 26.09. Note that, while joint profit is higher in the collusive case than it is in the Nash case, La Boulangerie does (individually) worse in the collusive outcome than it does in the Nash equilibrium (8.67 to 9). Intuitively, this result occurs because potential profits are higher at La Fromagerie than at La Boulangerie (La Fromagerie has a demand curve that is 2 units higher but costs that are only 1 higher). In the joint-profit-maximizing outcome, therefore, La Boulangerie charges a relatively low price (and sacrifices its own profit) in order to boost La Fromagerie's sales and profit. The joint-profit-maximizing collusive outcome is thus impossible to maintain; La Boulangerie will always prefer to revert to the Nash equilibrium (and prefer even more to be the lone cheater) than to maintain collusion.

One way in which collusion may be maintained is through side payments from La Fromagerie to La Boulangerie. Of course, we can't predict the exact size of these side payments, but one possibility would be for the two stores to split approximately equally the 1.09 gain from collusion. If the firms collude and La Fromagerie transfers 0.87 to La Boulangerie, then La Boulangerie ends up with 8.67 + 0.87 = 9.54, and La Fromagerie with 17.42 – 0.87 = 16.55.

We can now consider whether this outcome will hold up against the cheat forever strategy. Look first at La Boulangerie's gain from cheating. Presumably, if La Boulangerie deviates from the collusive outcome, La Fromagerie will end all side payments. Then, if $p_2 = 5.67$, La Boulangerie's profit-maximizing price is found from its best-response rule: $p_1 = 5.5 - (0.25)5.67 = 4.08$, its sales are $q_1 = 10 - 4.08 - (0.5)5.67 = 3.09$, and its resulting profit is $Y_1 = (4.08 - 1)3.09 = 9.52$. This is less than the collusion-with-side-payment outcome just described, so we can expect La Boulangerie to never defect from the described outcome.

Now, consider La Fromagerie's gain from cheating. If $p_1 = 3.17$, La Fromagerie's profit-maximizing price is $p_2 = 7 - (0.25)3.17 = 6.21$, its sales are $q_2 = 12 - 6.21 - (0.5)3.17 = 4.20$, and its resulting profit is $Y_2 = (6.21 - 2)4.20 = 17.68$.

Collusion (after side payment) yields 16.55, 16.55, 16.55, ...
Cheat forever yields 17.68, 16, 16, . . .
(if Nash equilibrium holds in all periods after first).

Discounting the infinite stream of payoffs, La Fromagerie finds cheating to offer a higher payoff if

$$17.68 + 16/r > 16.55 + 16.55/r$$
$$17.68 - 16.55 > (16.55 - 16)/r$$
$$1.13 > 0.55/r$$
$$r > (0.55/1.13) = 0.487$$

Considering the cheat forever strategy, therefore, La Fromagerie will cheat only if the monthly interest rate $r > 0.487$. Converted to a discount factor, La Fromagerie will cheat only if the (monthly) discount factor $d < 0.672$.

Of course, the side payment we've analyzed is only one possibility; perhaps La Fromagerie's payment to La Boulangerie will be smaller. If so, it becomes possible that La Boulangerie will begin to gain from cheating at certain interest rates, while the critical r at which La Fromagerie becomes willing to cheat will rise. For any given side payment s, one can find the interest rate r at which cheating becomes optimal. Thus, one can find conditions on the pair (s, r) under which collusion can be maintained. If sustaining cooperation is expected to be a problem (i.e., if r is plausibly in the neighborhood of the values given above), one could maximize the likelihood that collusion would hold by finding the value of s that produced the largest value for r (for both firms).

Finally, note that a cheat once strategy could also be evaluated in a world with side payments. Doing so would, however, involve some of the same ambiguities that were described in the answer to Exercise 2.

4. The three-dimensional payoff table is:

C

Yes

		B	
		Yes	*No*
A	Yes	4, ***4,*** 4	4, ***10,*** 4
	No	10, ***4,*** 4	5, ***5,*** 0

No

		B	
		Yes	*No*
A	Yes	4, ***4,*** 10	0, ***5,*** 5
	No	5, ***0,*** 5	5, ***5,*** 5

A vote of No is a dominant strategy for each of the three council members; the Nash equilibrium is thus for all three to vote No.

With the given assumptions, two members of the council face no cost from voting Yes. The only gain from voting No is that such an action looks good to voters. When a council member doesn't have to face voters for over a year, however, the voters have forgotten about the vote and so will impose no punishment (nor will they provide a reward) based on the pay-raise vote. The voter's forgetfulness alters the payoffs given above, and the two council members whose reelection campaigns are delayed can safely vote Yes (and get the benefits of the pay raise). The one council member who will face the voters within a year and who thus still has the payoffs described in the question can vote No, while still receiving the pay raise that will pass by a 2 to 1 margin.

5. (a) High payoff from cheating (72) > cooperative payoff (64) > defect payoff (57) > low payoff from cooperating (20), so prisoners' dilemma. The Nash equilibrium strategies if play once are (Low, Low) and payoffs are (57, 57).
   (b) Total profits at the end of 4 years = $4 \times 57 = 228$. Firms know the game ends in 4 years, so they start at the end and use backward induction to find it's best to cheat in year 4 and in each preceding year. No cooperation is sustainable in the finite game.
   (c) One-time gain = $72 - 64 = 8$. The loss in *every* future period = $64 - 57 = 7$. Cheating is beneficial if the gain > the present discounted value of future losses, or $8 > 7/r$. Thus, $r > 7/8$ (or $d > 8/15$) makes cheating worthwhile, and $r < 7/8$ lets the grim strategy sustain cooperation between the firms in the infinite version of the game. If $r = 0.25$, cooperation can be sustained.
   (d) Total profits after 4 years = $4 \times 64 = 256$. With no known end of the world, can sustain cooperation if $r < 7/8$ (as in part c). This is different from part b because firms see no fixed end point of the game and can't use backward induction. Instead, they assume the game is infinite and use the grim strategy to sustain the cooperative outcome.
   (e) 10% probability of bankruptcy translates to 90% probability that the game continues, so $p = 0.9$. Then, for $r = 0.25$ ($d = 0.8$), $R = 39\%$. This would need to exceed 7/8 before cheating was worthwhile, so still get cooperation. For a 35% probability of bankruptcy, $p = 0.65$ and $R = 92\%$, so if bankruptcy becomes more certain, cheating becomes worthwhile.

6. See payoff table below:

		BAKER	
		10	*100*
CUTLER	50:50	5, ***5***	50, ***50***
	90:10	9, ***1***	90, ***10***

Both players have dominant strategies, 90:10 for Cutler and 100 for Baker. Therefore the equilibrium consists of these two strategies and the payoffs are 90 to Cutler and 10 to Baker, irrespective of whether the moves are simultaneous or sequential and, in the latter case, irrespective of the order of moves. But this is not a prisoners' dilemma; the outcome cannot be Pareto bettered. (Moral: Not every game with dominant strategies for both players is a prisoners' dilemma.)

7. In the $k < 1$ case, (Swerve, Swerve) maximizes the player's joint payoff. Maintaining this type of cooperation, however, is essentially impossible. This game differs from a prisoners' dilemma because a cheater in a prisoners' dilemma can rationally expect retaliation. When one player establishes a pattern of playing Defect, it is individually optimal for the other player also to play Defect. A potential cheater therefore must compare the immediate gain from cheating with the future loss from the breakdown of cooperation.

   In this chicken game, in contrast, if one driver succeeds in being the first to drive straight (and will continue to do so), it is *not* rational for the other driver to retaliate; if James is going straight, Dean's best response is to Swerve. James can thus lock in an outcome in which he achieves his most-preferred result. Any attempt to establish a pattern of (Swerve, Swerve) outcomes is thus likely to break down as each player tries to be the first to establish that he will choose Straight.

   In the $k > 1$ case, a pattern in which each player alternates between Swerve and Straight is, once established, almost certain to last. Both (Swerve, Straight) and (Straight, Swerve) are Nash equilibria in a single-play game. Thus, once the pattern of alternating actions has been established, neither driver can gain by deviating from it.

   One difficulty that is likely to arise in this situation is in determining who gets the k payoff (and who gets the –1 payoff) in the first round. With either discounting or an uncertain end to the game (or an odd number of rounds), the player who drives straight in the first round will have an advantage; if both players attempt to get this advantage, the alternating pattern may be hard to establish. Of course, either player would prefer to always drive straight, while having the other driver respond by choosing Swerve. This always straight strategy, however, is not optimal if you expect the other driver to alternate.

8. The Nash equilibrium in the one-stage game is (Low, Low). Low is a dominant strategy for both players. The game is a prisoners' dilemma: for both players, the cheater payoff (10 or 11) exceeds the cooperative payoff (7 or 8), which exceeds the defect payoff (2 or 4), which exceeds the loser payoff (0).

   Consider the second stage of the two-stage game. Call the amount that Row puts in the escrow account (to reward Column for cooperative behavior) r, and the amount that Column puts in the escrow account (to reward Row for cooperative behavior) c. The second stage payoff table is thus:

		COLUMN	
		Low	*High*
ROW	Low	2, ***4***	$10 - r$, $\boldsymbol{0 + r}$
	High	$0 + c$, $\boldsymbol{11 - c}$	$8 - r + c$, $\boldsymbol{7 + r - c}$

For Row, High is a (weakly) dominant strategy when $c \geq 2$; for Column, High is a (weakly) dominant strategy when $r \geq 4$. For these values of c and r, therefore, (High, High) is a Nash equilibrium.

   To simplify the analysis, assume that a player always chooses High when both actions would give her the same payoff. Because of this assumption, we can say that the players will choose from between only two possible values for their payments into the escrow account. If Column wants to alter Row's incentive, it should set $c = 2$ (setting it higher hurts Column and has no impact on Row's choice); otherwise, Column should set $c = 0$. Similarly, Row should set r equal to either 4 or 0.

   The first stage of the game can now be analyzed. Column has two strategies in this first stage: set $c = 0$ or set $c = 2$; Row also has two strategies: set $r = 0$ or set $r = 4$.

		COLUMN	
		$c = 2$	$c = 0$
ROW	$r = 4$	6, ***9***	6, ***4***
	$r = 0$	2, ***9***	2, ***4***

The rollback equilibrium is then ($r = 4$, $c = 2$) in the first stage, and (High, High) in the second stage. The prisoners' dilemma is resolved in this game; players attain the joint-payoff- maximizing outcome of (High, High). The escrow accounts provide a way to credibly commit to sharing the benefits of cooperative behavior. In any prisoners' dilemma, Player A's decision to cooperate always reduces his own payoff but also increases the joint payoff received by the full group. In other words, Player B's gain exceeds Player A's loss. If Player B has a credible way to share that gain, it would be in her best interest to do so in order to induce Player A to cooperate. By providing a side payment of appropriate size, Player B can create an situation in which Player A's cooperation helps both players. The escrow system provides just such a credible way to share the gains from cooperation and thus to induce cooperative behavior.

In the actual play of the game, we have observed players choosing bribes of 5 and 3 rather than 4 and 2, hoping that the other player would break her indifference in their favor.

ADDITIONAL EXERCISE WITH ANSWER

1. In "The Permanent Insurrection: Bob Livingston's Unruly Inheritance," (*New Republic*, November 30, 1998), David Grann argues that changes both in the administrative rules of the U.S. Congress and in the personal plans of some representatives have made it more difficult for congressional leaders to exert control over the institution. Grann explains:

> The most notable of these changes is the most obscure: a new rule limiting committee chairmen to only three terms and the Speaker of the House to four. Slipped into a rules package in the first heady hours of the Republican revolution, it was supposed to contain lawmakers' ambitions; instead, it has unleashed them. [Bob] Livingston began campaigning for the speakership last summer only because his term as chairman of the powerful House Appropriations Committee was about to lapse. He had nowhere else to go but into retirement (which he also considered). Once he announced his candidacy, young members rushed to support his insurgency while launching their own: under the new system, they could climb the ladder more quickly by joining a coup than by stopping one. "The rule changes have created total chaos," explains [one] GOP Representative. . . . They're "the dumbest thing we've done," adds a . . . Committee Chairman.
>
> Even more destabilizing, though, are some members' self-imposed term limits. While Congress never passed term-limit legislation, several crusading lawmakers have voluntarily adopted them. Now they constitute a kind of roving band of mercenaries who wander the House floor, looking for somebody to topple. They have no incentive to work within the system—and no fear of reprisals from the Speaker. "The best thing about it is you get to really piss off some very powerful Republican every six months," a Congressman . . . told me earlier this year.

Explain how this passage relates to the chapter's discussion of the incentives in repeated strategic games.

ANSWER The adoption of term limits shortens the time horizon of the legislators; this reduces the gains from maintaining cooperation.