...tions for questions 53 and 54:

...t us tabulate the number of tokens A and B have after each meeting.

Meetings	A (a tokens)	B (b tokens)
I	a – b	2b
II	a – 3b	4b
III	a – b	2b
IV	a – 3b	4b
V	a – 7b	8b
VI	a – 3b	4b
VII	a – 7b	8b
VIII	a – 15b	16b
IX	a – 7b	8b

From the above table, we can observe the number of tokens B has is of the form $2^n \cdot b$ and the number of tokens A has is of the form $a - (2^n - 1) \cdot b$.

$\therefore$ As B started with only one token and A finished with 70, the number of coins A has is

53. $a - (2^n - 1) = 70$

$a = 70 + (2^n - 1)$

$\therefore$ the number of tokens A has will be of the form $70 + 2^n - 1$. From the answer choices only $101 = 70 + 31 = 70 + (2^5 - 1)$ will satisfy. Choice (1)

54. From the above table, we can observe that the number of tokens with B in first meeting and third meeting are same and 4^{th} and 6^{th} meeting are same.

$\therefore$ the number of tokens in $3k^{th}$ meeting and $(3k - 2)^{th}$ meeting is same. Choice (2)

Solutions for questions 55 to 64:

55.

From 1 to 9	9.1 = 9 digits are written
From 10 to 99	90.2 = 180 digits are written
From 100 to 999	900.3 = 2,700 digits are written
From 1,000 to 9,999	9,000.4 = 36,000 digits are written
From 10,000 to 10,221	222 x 5 = 1,111 digits are written

Total 39,999 digits

Therefore, the $40,000^{th}$ digit we write is 1 (i.e., the first digit of 10,222). Choice (1)

56. Let the sum be P and the interest rate = r

Then after n years,

$A = P\left(1 + \frac{r}{100}\right)^n$

$\Rightarrow A = PR^n$ where $\left(R = \left(1 + \frac{r}{100}\right)\right)$

Now, $32,940 = PR^3$ - (1)

and $49,410 = PR^6$ - (2)

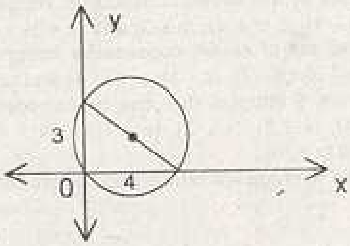
$\Rightarrow R^3 = \frac{49,410}{32,940}$ - (3)

and putting (3) in (1) again

$32,940 = P \times \frac{49,410}{32,940}$

=> P = Rs.21,960 Choice (3)

57. The vertices of the triangle are (0, 0), (0, 3) and (4, 0). It is a right angled triangle.



Diameter $= \sqrt{3^2 + 4^2} = 5$ units

Circumference $= \pi \times d$

$= \pi \times 5 = 5\pi$ units Choice (3)

58. Let the roots be α and β.

$\alpha + \beta = \frac{-q}{p}$ and $\alpha\beta = \frac{r}{p}$

Now, $q^3 + p^2r + pr^2 - 3pqr = 0$

Dividing both sides by p^3,

$\left(\frac{q}{p}\right)^3 + \frac{r}{p} + \left(\frac{r}{p}\right)^2 - 3\left(\frac{q}{p}\right)\left(\frac{r}{p}\right) = 0$

$\Rightarrow -(\alpha + \beta)^3 + \alpha\beta + \alpha^2\beta^2 + 3(\alpha + \beta)(\alpha\beta) = 0$

$\Rightarrow -\alpha^3 - \beta^3 - 3\alpha\beta(\alpha + \beta) + \alpha\beta + \alpha^2\beta^2 + 3\alpha\beta(\alpha + \beta) = 0$

$\Rightarrow \alpha^3 + \beta^3 - \alpha^2\beta^2 - \alpha\beta = 0 \Rightarrow \alpha(\alpha^2 - \beta) - \beta^2(\alpha^2 - \beta) = 0$

$\Rightarrow (\alpha - \beta^2)(\alpha^2 - \beta) = 0$

either $\alpha - \beta^2 = 0 \quad \Rightarrow \alpha = \beta^2$ or,

$\alpha^2 - \beta = 0 \quad \Rightarrow \beta = \alpha^2$

$\therefore$ One of the roots is the square of the other. Choice (1)

59. In the first cycle of 3 days, P, Q, R each worked for 2 days

$\therefore$ Portion of work completed in 3 days

$= 2\left[\frac{1}{20} + \frac{1}{40} + \frac{1}{50}\right]$

In $(5 \times 3 = 15 \text{ days}) \frac{5 \times 19}{100} = \frac{95}{100}$ portion of work is done.

Remaining portion of work $= \frac{5}{100} = \frac{1}{20}$

This work has to be done by P and Q who complete $\left(\frac{1}{20} + \frac{1}{40} = \frac{3}{40}\right)$ work in one day. So they will take 2/3 days to complete $1/20^{th}$ of work.

Total time taken $= 15^2/_3$ days. Choice (3)

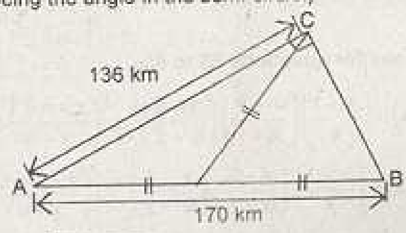
60. Let the number be xyxyxy i.e.,

xy xy xy $= 10^5x + 10^4y + 10^3x + 10^2y + 10x + y$

$= 10(10^4 + 10^2 + 1)x + (10^4 + 10^2 + 1)y$

$= 10101(10x + y)$ Choice (1)

61. From the given information, it is implied that the three cars make a right angled triangle (with their distances from the starting point being the radii of a circle and the distance between the first two cars the diameter. $\angle ACB = 90°$, being the angle in the semi-circle.)



Now, $CB = \sqrt{170^2 - 136^2} = 102$ km Choice (1)

62. The first set of the seven consecutive integers will be $(x-2)$, $(x-1)$, x, $(x+1)$, $(x+2)$ and $(x+3)$, $(x+4)$ and the second set of seven consecutive integers will be $(x-4)$, $(x-3)$, $(x-2)$, $(x-1)$, x, $(x+1)$ and $(x+2)$.
$\therefore$ There are 9 integers that can be considered. They are $(x-4)$, $(x-3)$, $(x-2)$, $(x-1)$, x, $(x+1)$ $(x+2)$, $(x+3)$ and $(x+4)$.
$\therefore$ Required average will be the middle number x.
Choice (1)

63. Area gazed by 4 buffaloes in 1 day = $4 \times \frac{1}{4} = 1$ unit.
Similarly, 10 sheep can graze a field of $\left(10 \times \frac{1}{5}\right) = 2$ units. 18 cows can graze an area = $18 \times \frac{1}{6} = 3$ units. Hence total area that can be grazed by the herd in one day = 1 + 2 + 3 = 6 units. Hence, the herd takes 1 day to completely graze the specified field.
Choice (3)

64. 173 actually equals 371 and 425 is 524
$\therefore$ 173 x 425 actually means 371 x 524 = 1,94,404 which in the number system of that country would mean 4,04,491.
Choice (1)

Solutions for questions 65 to 67:

We work out this problem backward, we get the following table.

Day	Sum at the start of day (before spending and donation)	Food expenses	Donation
Saturday	2 x 225 = 450	200 + 25 = 225	200
Friday	2 x 650 = 1,300	450 + 200 = 650	200
Thursday	2 x 1,500 = 3,000	1,300 + 200 = 1,500	200
Wednesday	2 x 3,200 = 6,400	3,000 + 200 = 3,200	200
Tuesday	2 x 6,600 = 13,200	6,400 + 200 = 6,600	200
Monday	2 x 13,400 = 26,800	13,200 + 200 = 13,400	200

65. Choice (4)

66. Choice (3)

67. Since Rs.26,800 is the amount in the beginning and Rs.25/- is the last, the amount is Rs.26,775/-.
Choice (2)

Solutions for questions 68 to 85:

68. $f(x) = \left(\frac{x+1}{x}\right)\left(\frac{x+2}{x+1}\right)\left(\frac{x+3}{x+2}\right)\cdots\cdots\left(\frac{x+n+1}{x+n}\right) = \frac{x+n+1}{x}$ and

$$g(x) = \left(\frac{x-1}{x}\right)\left(\frac{x-2}{x-1}\right)\left(\frac{x-3}{x-2}\right)\cdots\cdots\left(\frac{x-n-1}{x-n}\right) = \frac{x-n-1}{x}$$

$\therefore f(x) + g(x) = \frac{x+n+1+x-n-1}{x} = 2$ Choice (3)

69. $\frac{n(n+1)}{2} < 2{,}500 \Rightarrow n(n+1) \cong 5000$

$\Rightarrow$ n is close to (but less than) $\sqrt{5000}$ i.e. 70 or less.
Trying 70 first,
For, n = 70,
$\frac{n(n+1)}{2} = \frac{70 \times 71}{2} = 2{,}495$, hence n = 70.
$\therefore$ 15 was added as a result of those three numbers being added twice.
Now, x + (x + 1) + (x + 2) = 15 $\Rightarrow$ x = 4. Choice (2)

70. $p = (a+b)\left(\frac{a+b}{ab}\right) = \frac{(a+b)^2}{ab} = \frac{a^2+b^2}{ab} + 2$

Now, $\frac{a^2+b^2}{2} \geq \sqrt{a^2 b^2}$ (Since A.M. of a^2 and $b^2 \geq$ G.M. of a^2 and b^2)

$\Rightarrow a^2 + b^2 \geq 2ab$

$\therefore p \geq \frac{2ab}{ab} + 2$ $p \geq 4$

Alternatively
If a = 1, b = 1
$\Rightarrow$ P = (1 + 1) (1 + 1) = 4
Obviously, choices 1, 3 and 4 will be wrong. Choice (2)

71. We have to check by taking each and every answer choice. From answer choice (2), we get,
$x = 2 + 2^{2/3} + 2^{1/3}$ $\Rightarrow (x-2) = 2^{2/3} + 2^{1/3}$
Cubing both sides :
$x^3 - 6x^2 + 12x - 8 = 4 + 2 + 3.2^{2/3}.2^{1/3}(2^{2/3} + 2^{1/3})$
$\Rightarrow x^3 - 6x^2 + 12x - 8 = 6 + 6(x-2)$
$\Rightarrow x^3 - 6x^2 + 6x - 2 = 0$ Choice (2)

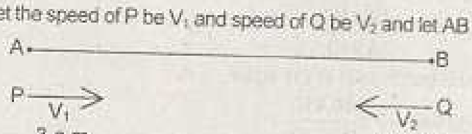
72. Since $(a^b)^c = a^{bc}$, $\left[x^{n^3}\right]^n = x^{n^3 \times n} = x^{n^4}$

On the other side we have $\left[x^{3^n}\right]^3 = x^{3^n \times 3} = x^{3n+1}$

$\Rightarrow n^4 = 3^{n+1}$
Which is possible only when n = 3.
$\therefore$ 2n + 1 = 7
$\Rightarrow \sqrt[7]{n^{21}} = n^{21/7} = n^3 = 3^3 = 27$. Choice (2)

73. Let the number of persons be 'n' and the sum of their ages be S.
S = 1 + 2 + 4 + 7 + ---------- + 67
S = 1 + 2 + 4 + --------- + t_{n-1} + 67
Subtracting we get,
{1 + 2 + 3 + --------- upto (n – 1) terms} – 66 = 0
$\Rightarrow \frac{(n-1)(n)}{2} = 66 \Rightarrow (n-1)(n) = 132$
$\Rightarrow n^2 - n - 132 = 0$ $\Rightarrow (n-12)(n+11) = 0$
$\Rightarrow$ n = 12(since n $\neq$ –11) Choice (1)

74. Let the speed of P be V_1 and speed of Q be V_2 and let AB = x



P starts at 3 a.m. and Q starts at 8 a.m. From 3 a.m. to 8 a.m., P covers $5V_1$. Also they meet at 12 noon. So, P will cover a further $4V_1$, while Q covers $4V_2$ by 12 noon.

So, by 12 noon, they cover $9V_1$ and $4V_2$ respectively.

$$A \xrightarrow{\quad 9V_1 \quad} X \xrightarrow{\quad 4V_2 \quad} B$$

12.00 noon

Now P, has to cover $4V_2$ to reach B and Q has to cover $9V_1$ to reach A.
Since they take the same time from 12 noon

$$\frac{9V_1}{V_2} = \frac{4V_2}{V_1}$$
$$9V_1^2 = 4V_2^2$$
$$\frac{V_1}{V_2} = \frac{2}{3}$$

$\therefore$ time after 12 noon = $\frac{9V_1}{V_2} = 9 \times \frac{2}{3} = 6$ hrs

They will reach at 6 p.m. Choice (2)

75. Let the radius of the first planet's orbit be R. Then the radius of the second planet will be 2R and the radii of the orbits of the seven planets will be R, 2R, 3R, 5R, 8R, 13R and 21R respectively.

Since the time taken for one orbit is proportional to the radius and if one monon is the time taken for the 1st planet, then, 1 duon = 2 monon, 1 trion = 3 monon and so on.

Then they align themselves, i.e. they meet once in every LCM of (Monon, Duon, Trion ......Septon)

= LCM of (1, 2, 3, 5, 8, 13, 21)
= 10,920 monons
In terms of hexons the time will be

$\frac{10{,}920}{13} = 840$ hexons. Choice (3)

76. The first, and the third statements imply that the Conductor's last name is not Sardesai. Also, from the sixth statement it is known that the Waiter's name is not Pandey. By the second statement, we know that the Conductor lives half-way between New Delhi and Mumbai. So, neither of the passengers who live in these locations is the passenger who lives nearest to the Conductor. Also one can conclude that the passenger Bedi does not live nearest to the Conductor and since he does not live in New Delhi either, he must live in Mumbai. So, he has the same last name as that of the Conductor.

So, the Conductor's last name is Bedi and the Waiter's last name is not Pandey, so it must be Sardesai. Then, the Engineer's last name is Pandey. Choice (2)

77. Let the common root be α.
$\alpha^2 + a\alpha + b = 0$ --------- (1)
$\alpha^2 + b\alpha + a = 0$ --------- (2)
(1) – (2) $\Rightarrow (a - b)\alpha + b - a = 0 \Rightarrow (\alpha - 1)(a - b) = 0$
Either $\alpha = 1$ or $a - b = 0$
If $\alpha = 1$; $a + b + 1 = 0$
and if $a - b = 0$ $\Rightarrow a = b$. Choice (4)

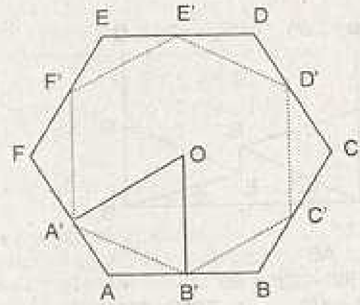
78. 6! = 720, which when divided by 13 gives a remainder 5. The remainder when given expression is divided by 13 is the remainder obtained when $(5)^{7! \times 13333}$ is divided by 13.

$(25)^{7! \times 13333/2}$

$= (26 - 1)^{\text{even number}} = 26K + 1$

So, remainder when $(25)^{(7! \times 13333)/2}$ is divided by 13 is 1. Choice (1)

79.



Each side of A'B'C'D'E'F' = A'B' = OB'

$= \frac{\sqrt{3}}{2} \times OB = \frac{\sqrt{3}}{2} \times AB = 5\sqrt{3}$ cm

Similarly each side of the next hexagon inside it

$= \frac{\sqrt{3}}{2} \times 5\sqrt{3} = \frac{15}{2}$ cm and so on.

$\therefore$ Sum of the areas

$$= \frac{3\sqrt{3}}{2}\left(10^2 + \left(5\sqrt{3}\right)^2 + \left(\frac{15}{2}\right)^2 + \ldots\ldots\ldots\infty\right)$$

$$= \frac{3\sqrt{3}}{2}\left(\frac{100}{1 - \left(\frac{\sqrt{3}}{2}\right)^2}\right)$$

$$= \frac{3\sqrt{3}}{2} \times \frac{100 \times 4}{(4-3)} = 600\sqrt{3} \text{ sq.cm}$$ Choice (1)

80. Volume of 200 gm of silver = $\frac{200}{25} = 8$ cu.cm

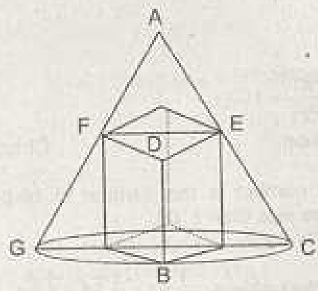
$\therefore$ Thickness of each leaf = $\frac{8}{40 \times 10{,}000}$ cm

Let the number of leaves required be n.

$$n \times \frac{8}{4{,}00{,}000} = \frac{1}{2}$$

$$\Rightarrow n = \frac{4{,}00{,}000}{8 \times 2} = 25{,}000$$ Choice (2)

81.



Let each side of the cube be a cm.
FE is the diagonal of one of the faces of cube, and hence, $FE = a\sqrt{2}$ cm.

Since $\Delta AFE \sim \Delta AGC$,

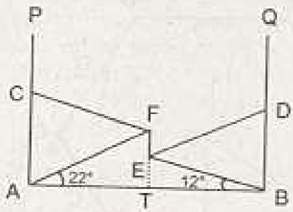
$$\frac{AD}{AB} = \frac{FE}{GC} \Rightarrow \frac{10 - a}{10} = \frac{a\sqrt{2}}{10}$$

$$\Rightarrow a\left(\sqrt{2} + 1\right) = 10 \Rightarrow a = \frac{10}{\sqrt{2} + 1} = 10\left(\sqrt{2} - 1\right)$$

$\therefore$ Total surface area = $6a^2 = 6 \times 100\left(\sqrt{2} - 1\right)^2$

$= 600\left(3 - 2\sqrt{2}\right)$ sq.cm Choice (1)

82.



Draw ET ⊥ AB
$\angle AFE = 90° - 22° = 68°$
$\therefore \angle FED = 68° \quad \Rightarrow \angle EDB = 68°$
But, $\angle DBE = 90° - 12° = 78°$
$\therefore \angle DEB = 180° - (68° + 78°) = 34°$ Choice (4)

83. The period for the remainder when 3^m is divided by 7 is 6. i.e. the remainder when 3^{6P} is divided by 7, it is the same as when 3^0 is divided by 7.

Now, we need to find the remainder when 5^n is divided by 6.

The period is 2 here and whenever n is even the remainder is 1 and when n is odd, the remainder is 5.

Hence, the remainder when 3^{5^n} divided by 7 will be

1) Same as that for $\frac{3^1}{7}$ when n is even, i.e. 3

2) Same as that for $\frac{3^5}{7}$ when n is odd, i.e. 5

$\therefore$When $(3^{5^n} + 1)$ is divided by seven,
R = 3 + 1= 4, when n is even.
R = 5 + 1 = 6, when n is odd. Choice (1)

84. Let, $2^{\log_3 4}$ be x
$\Rightarrow \log_3 4 = \log_2 x$ ---------- (1)
Also, let $4^{\log_3 2}$ be y
$\Rightarrow \log_3 2 = \log_4 y$
$\Rightarrow \frac{1}{2}\log_3 4 = \log_4 y$
$\Rightarrow \log_3 4 = 2\log_4 y$
$\log_3 4 = \log_2 y$ ---------- (2)
From (1) and (2)
$\therefore \log_2 x = \log_2 y$
=> x = y
i.e. $\frac{2^{\log_3 4}}{4^{\log_3 2}} = 1$
Similarly, $\frac{5^{\log_2 3}}{3^{\log_2 5}} = 1$
Given expression. Choice (2)

85. The required number is the number of co-primes of 2100 which are less than 2100.
$2100 = 2^2 \times 5^2 \times 3 \times 7$

Required number = $N\times\left(1-\frac{1}{2}\right)\left(1-\frac{1}{5}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{7}\right) = 480$
Choice (1)

Solutions for questions 86 and 87:

86. Let the salary be Rs.x.
Expenditure on food = x/3
Expenditure on rent = 1/4 (x – x/3) = x/6

Expenditure on clothing = $\frac{1}{5}\left(x - \left(\frac{x}{3} + \frac{x}{6}\right)\right) = \frac{x}{10}$

Expenditure on books = 1/6 [x – (x/3 + x/6 + x/10)] = x/15
Expenditure on medicines
= 2/5 [x – (x/3 + x/6 + x/10 + x/15)] = 2x/15

Now, $\frac{x}{3} + \frac{x}{6} + \frac{x}{10} + \frac{x}{15} + \frac{2x}{15} + 3{,}000 = x$

$\Rightarrow \frac{x}{5} = 3{,}000 \Rightarrow x = 15{,}000$

This question can also be done by back substitution.
Choice (3)

87. Expenditure on clothing = $\frac{15{,}000}{10}$ = Rs.1,500

Note: This problem can also be worked by back substitution by starting with the balance amount of Rs.3,000 and calculating each item. Choice (3)

Solutions for questions 88 to 94:

88. That the price consumption and expenditure be denoted by P, C and E. The assumed initial and the corresponding changed values are tabulated below.

	P	C	E
Assumed	10	10	100
Changed	12.5	x	110

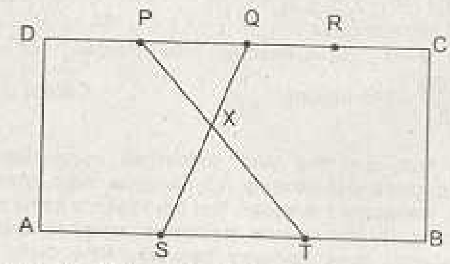
Here x is the final consumption (C)

$x = \frac{110}{12.5} = 8.8$

Percentage reduction in the consumption =

$\frac{1.2}{10} \times 100 = 12\%$ Choice (2)

89.

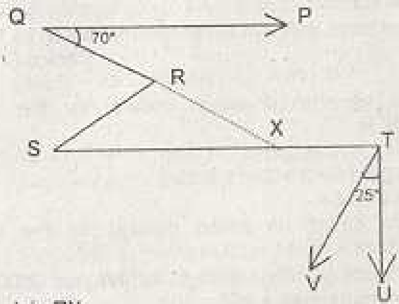


In triangle PQX and STX, ∠X is common angle
∠QPX = ∠STX (alternate angle are equal)
∠PQX = ∠TSX (alternate angel are equal)
$\therefore$ The two triangles PQX and STX are similar.
Ratio of the areas will be equal to the square of the ratio of the sides.
As P, Q and R divide CD into four equal parts
$PQ = \frac{1}{4} \times CD$

Similarly $ST = \frac{1}{3} \times AB = \frac{1}{3} \times CD$

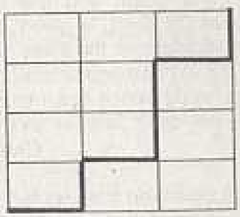
$\frac{\text{Area of PQX}}{\text{Area of STX}} = \frac{\left(\frac{1}{4}CD\right)^2}{\left(\frac{1}{3}CD\right)^2} = \frac{9}{16}$ Choice (2)

90. The first subset has one term, the second subset has two terms and so on. So, S_{50} will have 50 terms.
S_1 has 1 as the only term.
S_2 has 2 as the first term which is 1 + 1
S_3 has 4 as the first term which is 1 + 2 + 1
S_4 has 7 as the first term which is 1 + 2 + 3 + 1
So, S_{50} will have 1 + 2 + 3 .... + 49 + 1 as the first term = 1,226
Its last term = 1,275
Sum = (50/2) [1,226 + 1,275] = 62,525 Choice (4)



Join RX
$\angle RXS = \angle PQR = 70°$(Alternate angles)
Also, $\angle STV = \angle RSX$ (Alternate angels)
But $\angle STV = 90° - 25° = 65°$
$\therefore \angle RSX = 65°$
$\angle QRS = \angle RSX + \angle RXS = 70° + 65° = 135°$
Choice (3)

92.



The path is composed of 4 + 3 i.e. 7 segments of length equal to the length of the sides of the squares of the grid. Out of these 7 segments, 4 are horizontal and 3 vertical. We have to go 4 rows up and 3 column, to the right.
If we now consider the path as a long line divided into 7 segments, each different path in the grid is just a way to choose which of the segments are going to be horizontal and which ones vertical. (Note that once we have decided which are horizontal, the rest would be vertical) Thus, the number of paths equals the number of ways. 4 elements can be chosen from 7, which is ${}^7C_4 = 35$. Choice (2)

93. By selling a mixture of milk and water at the cost price of pure milk to make a profit of p%, we must add p% of water to pure milk.
In this case the required ratio of water and milk is
$16^2/_3 : 100 = 1 : 6$ Choice (2)

94. Let the father's age be f and the son's age be s.
$f + 6 = 2(s + 6) + 6$
$f - 2s = 12$ - (1)

$f + 18 = 2(s + 18) - 6$
$f - 2s = 12$ - (2)

Let, after x years father be twice the son's age,
$(f + x) = 2(s + n)$
$f - 2s = x$.
From (1) and (2) $x = 12$ Choice (3)

Solutions for questions 95 and 96:

Let the number of bags be x, compass boxes be y, lunch boxes be z
We have, $x + y + z = 134$ - (1)
$120x + 65y + 35z = 9{,}475$
i.e. $24x + 13y + 7z = 1{,}895$ - (2)

$24x + 13y + 7z = 1895$
$24x + 24y + 24z = 3216$ (Multiply (1) with 24)

$11y + 17z = 1321$ - (3)

i.e. $y = \dfrac{1{,}321 - 17z}{11}$

$= (120) + \left(\dfrac{1 - 17z}{11}\right)$

z can take the values 2, 13, 24, 35, 46, 57, 68. For the next possible value of z (i.e.79),y is negative.
$\therefore$ z can take 7 possible values i.e., the three items can be purchased in 7 different ways.

(2) We have $y = \dfrac{13{,}21 - 17z}{11}$ and z can take possible values 2, 13, 24, 35, 46, 57, 68.
Here, the value of y decreases successively by 17.

z	y		
2.	117		X
13.	100	even perfect square	✓
24.	83		X
35.	66		
46.	49	not an even perfect square	X
57.	32	not a perfect square	X
68.	15		X

$\therefore$ The only possibility is for z = 13.

95. Choice (3)

96. Choice (2)

Solutions for questions 97 to 100:

97. The given relations are
$J < T$ ---------------------- (1)
$J + T = A + D$ ------------- (2)
$A + T < D + J$ -------------- (3)
From (2), $T = A + D - J$
Substituting this value of T in equation (3), we get
$A + A + D - J < D + J \Rightarrow 2A < 2J \Rightarrow \boxed{J > A}$ ← (4)
$\therefore$ Amar < Jayesh < Tony [From (1) and (4)]
From (2) $J = A + D - T$
Substituting this value of J in (3), we get
$A + T < 2D + A - T$
$\Rightarrow 2D > 2T$
$\Rightarrow D > T$
$\Rightarrow$ David > Tony.
$\Rightarrow A < J < T < D$ Choice (2)

98. Pick up two marbles and test them against each other. There are now two possibilities.

1. The marbles do not balance. Thus, one (the heavier) is a glass marble and the other (lighter one) is a quartz marble. Now, put these two marbles together on one side of the balance scale. Pick up two more marbles and put them on the other side. If the two new marbles are heavier, they are glass. If the two new marbles are lighter, they are quartz. If the two new marbles balance, then one is quartz and one is glass. In any of these events, we can count how many glass and how many quartz marbles are there among the two new marbles. (note that we don't have to identify the marbles, just to count them). So, we set those two new marbles aside