CHAPTER 14 Strategy and Voting

TEACHING SUGGESTIONS

Most students will be familiar with at least a few of the various aggregation methods used to determine winners in elections. You may want to poll the class and come up with a list of different methods that they know about or have used. Even a simple discussion of this type will lay the ground work for coverage of the many methods available. There are additional examples beyond those found in the text; some of these are explained in the *Journal of Economic Perspectives* vol. 9, no. 1 (winter 1995), in the symposium on "Economics of Voting."

The best way to get students to appreciate that different voting methods can lead to very different final outcomes is to have them work through an example. The John Allen Paulos example found in Exercise 14.7 is an excellent one; you can change the characters to fit your own state, local, or university situation to make the idea more interesting to the students. Another classic example of how the choice of procedure can affect the outcome is based on a Roman legal case, extracted from the letters of Pliny the Younger [see Robin Farquharson, *Theory of Voting* (New Haven, Conn.: Yale University Press, 1969), pp. 6–7, or Peter Ordeshook, *Game Theory and Political Theory*, pp. 54–55]. Pliny, as a member of the Roman senate, had a distinct preference for the particular aggregation rule that would lead to his most-preferred outcome.

Many other examples of voting paradoxes can be found in the literature. Those who want to emphasize political science applications may want to refer to William Riker's *Liberalism Against Populism*, some of the papers of Peter Fishburn [including "Paradoxes of Voting," *American Political Science Review* vol. 68 (1974)], and the Farquharson and Ordeshook books mentioned above.

For a more detailed discussion of the Condorcet paradox, you can pick up on the idea mentioned briefly in the text that in the City Council example, the paradox arose "because the Council completely disagreed not only about which alternative was best but also about which was worst." This argument can be spelled out it more detail; doing so will help to clarify the conditions under which an agenda setter will be able to determine the outcome.

In a three-by-three case (in which votes are cast "truthfully"), it is clear that a Condorcet paradox can exist only when all three members rank a different alternative best. If two members agreed on their favorite, that alternative would beat both others. As the text implies, however, disagreement about the best alternative is not sufficient for the Condorcet paradox to exist. In addition, the three voters must also disagree about which alternative is worst.

Suppose that this isn't true, so that the three voters (who each prefer a different alternative) agree that one alternative, say A, is not the worst. In this case, Alternative A is ranked first by one voter, and second by the other two. Consider the voting outcome when Alternative A is matched against (for example) B: A gets two votes, one from the person who likes A the best, and one from the person who likes C the best but prefers A over B. Similarly, A gets two votes when matched against C, the second coming from the person who likes B best but ranks A over C. When all three voters agree that A is not the worst outcome, therefore, a Condorcet paradox cannot exist. We conclude that in the three-by-three case, a Condorcet paradox exists only when there is complete disagreement among the voters; they must disagree not only about which alternative is best but also about which is worst.

It is only when disagreement is so complete that the outcome can be determined by the preferences of the agenda setter. This is intuitive; if there is "enough" agreement among the voters, the voters' preferences fully determine the outcome. The power to set the agenda is decisive only when voters' preferences are quite diverse. One can note also that the above description is, of course, just another way to describe the well-known double-peaked-preferences condition. In addition, it may be worth pointing out to students that complete disagreement on rankings may be more likely if the three alternatives being voted on are three completely different options—like spending money on a park, a library, or a clinic—rather than a range of spending on a single program.

Another example that can be used in class comes from the story, mentioned briefly in the text, concerning how opponents of a federal school-construction-funding bill were able to defeat the bill by strategic voting. They voted in favor of an amendment, which they would have opposed on its own merits, anticipating that the amended bill was more likely to fail than the original bill was.

Bear in mind, however, that attempts to engage in such clever schemes sometimes fail. The first federal law that banned discrimination on the basis of gender passed only as a result of a failed strategic plan. The original bill banned racial discrimination (in certain contexts); opponents supported adding the gender-discrimination language expecting it to sink the whole bill. To their surprise, it passed.

When covering the median voter theorem, you may find that students balk at the idea that candidates all move toward the position of the median voter since this does not seem to be observed in reality. In actual political campaigns, candidates will often take positions in order to distinguish themselves from each other. It is worthwhile to explain to students that this observation does not destroy the essential conclusions of the median voter theorem.

As the text notes, part of the reason why candidates may distinguish themselves is that the median voter theorem does not hold when the political spectrum has two or more dimensions. There are also institutional (and other) features that explain some differentiation between candidates. A brief list includes:

- A candidate may need to win a primary election, by appealing to a left- or right-leaning subset of the total population, before running in the general election campaign.
- A candidate may attempt to increase the turnout of voters by appealing to her base of supporters.
- Prior public statements by a candidate may tie her to a particular position.
- A candidate's own belief system may lead some candidates to accept a reduced chance of winning an election in order to promote what she believes are correct stands.

In spite of these factors, however, the basic conclusion of the median voter theorem—a desire of candidates to move toward the middle of the political spectrum—still holds, even if the candidates do not meet at the exact midpoint.

For example, candidates who must take relatively extreme positions in order to win a primary often begin to moderate their positions as they move into the general election campaign. Consider also a candidate who has represented a congressional district that leans more to the left or the right than the candidate's whole state. If this person runs for a statewide office, she may modify some previous statements in order to sound more mainstream. Candidates may also alter or hide their own personal views.

In all of these cases, a candidate may not be able to move into the exact middle of the voter distribution because doing so would require the candidate to reverse some of her previous positions. The loss of credibility that might result could be very damaging to the candidate. Still, it is worth emphasizing that a candidate's incentive to moderate her positions is quite strong. Many candidates will behave in such a manner, up to the point at which the resulting loss of credibility becomes too costly.

ANSWERS TO EXERCISES FOR CHAPTER 5

1. Under truthful voting, A should match Geology and Anthropology in the first vote, with the winner (Geology) to face Philosophy in the second round. Under strategic voting, A should match Philosophy and Anthropology in the first round. In order to prevent a Geology outcome, B casts a strategic vote for Philosophy; Philosophy goes on to beat Geology.
2. Under truthful voting, B should match Philosophy and Geology in the first vote, with the winner (Philosophy) to face Anthropology in the second round. Under strategic voting, B should match Anthropology and Geology in the first round. In order to prevent a Philosophy outcome, C casts a strategic vote for Anthropology; Anthropology goes on to beat Philosophy.
3. Allocate points in the following way: 5 for a first-place vote, 2 for a second-place vote, and 1 for a third-place vote. A then receives 5 + 5 + 1 + 1 = 12 points, B gets 2 + 2 + 5 + 2 = 11, and C gets 1 + 1 + 2 + 5 = 9.
4. Allocate points in the following way: 3 for a first-place vote, 2 for a second-place vote, and 1 for a third-place vote. B then receives 2 + 2 + 3 + 2 = 9 points, A gets 3 + 3 + 1 + 1 = 8, and C gets 1 + 1 + 2 + 3 = 7.
5. (a) Under a plurality voting system (with truthful voting), Proposal 1 gets 20 votes and Proposals 2 and 3 get 15 votes each.
   (b) Under a 3-2-1 Borda count with truthful voting, Proposal 1 gets 60 + 15 + 30 = 105 points, Proposal 2 gets 40 + 45 + 15 = 100 points, and Proposal 3 gets 20 + 30 + 45 = 95 points.
   (c) For both the second and third type of voter to gain, Proposal 3 must win rather than Proposal 1. This

can be achieved if types 2 and/or 3 change their votes in such a way that Proposal 1 gets fewer points and/or Proposal 3 gets more. One example would be if types 2 and 3 vote as though their rankings were Proposal 3, Proposal 2, Proposal 1. With this voting pattern, Proposal 1 gets 60 + 15 + 15 = 90 points, Proposal 2 gets 40 + 30 + 30 = 100, and Proposal 3 gets 20 + 45 + 45 = 110.

Note two things. First, other voting schemes will also result in Proposal 3's winning. For example, if only the type 2s vote with a 3-2-1 ranking, Proposal 3 still wins. Second, a single member of types 2 and 3 can't alter the outcome by changing only her vote; coordinated action is required.

6. The preferences create a standard intransitive ordering, so none of the three alternatives would win a majority vote with truthful voting.

7. (a) Under the plurality method, Tsongas gets 18 votes, Clinton 12, Brown 10, Kerry 9, and Harkin 6. Under the runoff method, the above vote totals show that Tsongas and Clinton go into the second round, and Clinton wins that round 37 to 18. Under the elimination method, the above figures show that Harkin is eliminated in the first round. Second-round voting produces Tsongas 18, Clinton 16, Brown 12, and Kerry 9; Kerry is eliminated. Third-round voting produces Brown 21, Tsongas 18, and Clinton 16; Clinton is eliminated. Fourth-round voting produces Brown 37 and Tsongas 18. Under a 5-4-3-2-1 Borda count, point totals are Kerry 72 + 36 + 20 + 45 + 12 + 6 = 191, Harkin 54 + 48 + 30 + 27 + 20 + 10 = 189, Brown 36 + 24 + 50 + 36 + 8 + 8 = 162, Clinton 18 + 60 + 40 + 18 + 16 + 4 = 156, and Tsongas 90 + 12 + 10 + 9 + 4 + 2 = 127. Finally, under the Condorcet method Harkin beats Tsongas 37–18, Clinton 33–22, Brown 36–19, and Kerry 28–27.

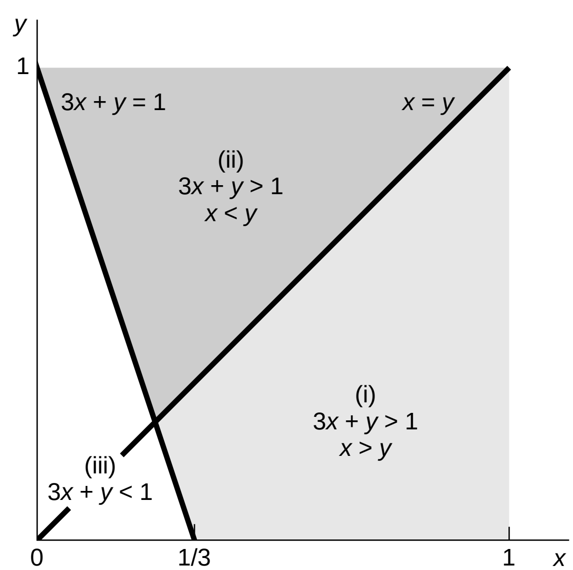
   (b) Realizing that their preferred candidates can't win (they are only playing the role of spoilers), supporters of Brown, Kerry, and Harkin can strategically vote for Clinton, since they each prefer Clinton over Tsongas.

   (c) Here are some examples of strategic voting (others can also be constructed); assume that all groups not described vote truthfully. (i) Under the runoff system, members of Group I could vote for Kerry instead of Tsongas in the first round; in a second-round vote between Kerry and Clinton, Kerry wins 29–26. Group I thus gets its 2nd choice instead of its 5th choice.
   (ii) Under the elimination method, members of Group I could vote as though Harkin were their first choice. This gives Harkin a solid 24 votes, which gets him into the final round. Harkin beats Brown in that round, so that Group I gets its third choice rather than its fourth.
   (iii) Under the Borda count, if members of Group V drop Kerry into the fifth spot on their ballots, his point total will fall below that of Harkin, so that Group V gets its 1st choice rather than its third.

8. (a) First, note that if you locate just to the left of the left-most candidate, your vote total equals x. If you locate just to the right of the right-most candidate, your vote total equals y. If you locate between the other two candidates, your vote total equals $(1 - y - x)/2$. The conditions given in the question determine which of x, y, and $(1 - y - x)/2$ is biggest and thus which of these locations gives you the most votes.

   (i) $x > y$ is given. $3x + y > 1$ can be rewritten as $x > (1 - y - x)/2$. Thus, locating just left of x gives the most votes.
   (ii) $y > x$ is given. $x + 3y > 1$ can be rewritten $y > (1 - y - x)/2$. Thus, locating just right of $(1 - y)$ gives the most votes.
   (iii) $3x + y < 1$ can be rewritten $(1 - y - x)/2 > y$. $x + 3y < 1$ can be rewritten: $(1 - y - x)/2 > y$. Thus, locating between the other two candidates gives the most votes.

   (b) Graph below:



   (c) Given the goal of candidates (as stated in the question), a Nash equilibrium in locations can exist only if each candidate is in the location that maximizes his or her share of the vote given the locations of the other candidates. There will always exist a candidate who can increase his vote share by moving. For example, any candidate who doesn't have another candidate located at a just slightly more moderate position can always gain votes by moving toward the center of the line. Thus, there is no Nash equilibrium in locations in the three-candidate game.

ADDITIONAL EXERCISES WITH ANSWERS

1. On November 17, 1998, the results of the voting for the 1998 National League (NL) Cy Young Award (which goes to the best pitcher in the league) were announced. Thirty-two people (two baseball writers from each of the 16 cities that have a NL team) voted for this award, and points are given using a Borda county system, in which a first-place vote is worth 5 points, a second-place vote 3 points, and a third-place vote 1 point. The following table shows how the votes were cast.

Pitcher	1st	2nd	3rd	Points
Tom Glavine	11	13	5	99
Trevor Hoffman	13	5	8	88
Kevin Brown	8	8	12	76

(Note: Four other pitchers received a total of 25 points. As reported at the time, this vote was the first time in the history of the Cy Young Award that the award winner did not receive the most first-place votes.)

(a) Assume that Voter X's true, personal ranking of the top pitchers in the league in '98 is Glavine, Hoffman, Brown. The only thing that Voter X is concerned about, however, is that Glavine win the award. How might Voter X (strategically) cast his votes in order to increase Glavine's chances of winning?

(b) Can you see any reason in the vote totals to suspect that some of the 32 voters might have cast their votes strategically? Explain.

ANSWER (a) Voter X would not put Hoffman in his list of three pitchers. (b) Out of the 32 voters, 3 did not put Glavine on their lists and 6 did not put Hoffman.

2. Consider a (public) vote in a (three-member) senate on whether to raise senators' salaries. Assume that each senator prefers a raise over no raise; however, each also knows that voting for a pay raise will cost him or her the support of some constituents. Thus, the best possible outcome for an individual senator is to vote against a pay raise that still wins majority approval, and the worst possible outcome is to vote for a pay proposal that fails. Not all senators agree on how to rank the other two outcomes. For Senators C and D, the second-best outcome is to have a raise proposal pass with their vote's being in favor. For Senator B, however, the second-best outcome is for a raise proposal to fail, with his vote's being against. The payoff ranking below (where large numbers represent more favorable outcomes) summarizes this situation. (You should assume that each senator knows the rankings of all three senators.)

   Assume that the senators vote simultaneously. The tables at the top of the next page can be used to show each senator's payoff in each of the eight possible voting outcomes. Note that Senator D's payoff has already been filled in.

Outcomes	Senator B's ranking	Senator C's and D's rankings
Best, 3 points	Raise passes, vote against	Raise passes, vote against
Second best, 2 points	Raise fails, vote against	Raise passes, vote for
Third best, 1 point	Raise passes, vote for	Raise fails, vote against
Worst, 0 points	Raise fails, vote for	Raise fails, vote for

Senator D votes:

For

		SENATOR C VOTES:	
		For	*Against*
SENATOR B VOTES:	For	, **,** 2	, **,** 2
	Against	, **,** 2	, **,** 0

Against

		SENATOR C VOTES:	
		For	*Against*
SENATOR B VOTES:	For	, **,** 3	, **,** 1
	Against	, **,** 1	, **,** 1

(a) Fill in Senator B's and Senator C's payoffs in the above tables (enter the payoffs in the following order: B's payoff, C's payoff, D's payoff).
(b) Find the Nash equilibrium (or all the Nash equilibria) of this simultaneous voting game.
(c) Explain (in words) why your answer to part b takes the form it does.

ANSWER (a)

Senator D votes:

For

		SENATOR C VOTES:	
		For	*Against*
SENATOR B VOTES:	For	1, ***2,*** 2	1, ***3,*** 2
	Against	3, ***2,*** 2	2, ***1,*** 0

Against

		SENATOR C VOTES:	
		For	*Against*
SENATOR B VOTES:	For	1, ***2,*** 3	0, ***1,*** 1
	Against	2, ***0,*** 1	2, ***1,*** 1

(b) Two Nash equilibria—(Against, For, For) and (Against, Against, Against). (c) Senator B has a dominant strategy—vote Against. Senators C and D are each willing to vote For as long as the other does also (so that the pay raise passes) but are not willing to cast the only For vote.

3. Suppose that nine people will vote on the amount of money to spend on a certain activity. Each person's preferences are as follows:

Number of voters	Most-preferred spending level
4	$10
2	$15
1	$30
2	$40

He or she has one spending level that is most preferred; furthermore, any spending closer to the most-preferred level is preferred over any level further from it. The accompanying table shows the number of people who hold each most-preferred spending level.

If the voting outcome is consistent with that described by the median voter theorem, then the spending level that will be selected by a vote among these people is _____.

(a) $10
(b) $15
(c) $20
(d) $30

ANSWER (b)