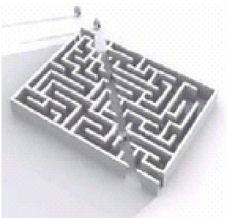
Quant Shortcuts for CAT 2007
by Total Gadha - Sunday, 11 November 2007, 04:23 PM



I have written on this theme before also. But before CAT 2007 happens, I would like to remind all the TGites about these methods once again. What is a shortcut? By shortcuts, I do not mean a sudden inspiration or am erratic method which can be applied to only one or two problems. By shortcut I mean a set method, a pattern which can be repeated again and again. These shortcuts can be learned and applied under pressure also. The reason to apply these shortcuts is one- getting to answers fast and in an easier way than by actually solving the problems. I have discussed two basic methods here which I predominantly use:

1. Examine the options
2. Start small

Examine the Options:

Enough stress cannot be laid on this procedure. Those who do not understand that even the options given in an MCQ are tools provided to help them, miss a big part of test-taking. The shortcoming of a multiple choice question is that it had to provide you with 4 or 5 options. And you can make use of it. Even if you know how to solve a particular question, examine the options; you might be able to solve it faster. Sometimes, you can easily obtain 2- 3 marks quickly if you work through the options when the original procedure to solve the problem might be lengthy or complex.

The sum of four consecutive two-digit odd numbers, when divided by 10, becomes a perfect square. Which of the following can possibly be one of these four numbers? **(CAT- 2006- 4 marks)**

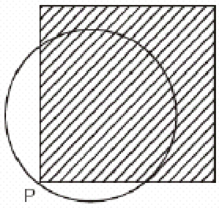
1. 21 2. 25 3. 41 4. 67 5. 73

If the sum of four consecutive odd numbers is ending in 0 (the sum is a multiple of 10) then the odd numbers must be ending in 7, 9, 1 and 3. Also, the sum would be nothing but 4 times the middle even number that is a multiple of 10. Looking at the options, the numbers can be

Option 1: 17, 19, 21 and 23 $\Rightarrow$ Sum = $4 \times 20 = 80 \Rightarrow 80/10 = 8$, not a perfect square.
Option 2: not possible
Option 3: 37, 39, 41 and 43 $\Rightarrow$ Sum = $160 \Rightarrow 160/10 = 16$, a perfect square.

3 is the answer.

A punching machine is used to punch a circular hole of diameter two units from a square sheet of aluminium of width 2 units, as shown below. The hole is punched such that the circular hole touches one corner P of the square sheet and the diameter of the hole originating at P is in line with a diagonal of the square. **(CAT 2006- 4 marks each)**



The proportion of the sheet area that remains after punching is:

1. $\frac{\pi+2}{8}$ 2. $\frac{6-\pi}{8}$ 3. $\frac{4-\pi}{4}$ 4. $\frac{\pi-2}{4}$ 5. $\frac{14-3\pi}{6}$

Find the area of the part of the circle (round punch) falling outside the square sheet.

1. $\frac{\pi}{4}$ 2. $\frac{\pi-1}{2}$ 3. $\frac{\pi-1}{4}$ 4. $\frac{\pi-2}{2}$ 5. $\frac{\pi-2}{4}$

These two questions were tailor-made to give 8 marks to any student in 1 minute. The first question asked for the fraction of the square left after the punch. Therefore, if you multiplied the correct answer of the question 1 with 4 (area of the square), you got the area left after the punch. Therefore, if you subtracted this area from the area of the square, you got the area of the punch. And if you added this area of the punch to the correct answer of question 2 you should get the area of the circle, i.e. π. Therefore, let the answer to question 1 be x and to question 2 be y.

$\Rightarrow 4 - 4x + y = \pi \Rightarrow y + 4 = \pi + 4x$.

$\pi + 4x = \frac{3\pi+2}{2}, \frac{\pi+6}{2}, 2\pi - 1, 3\pi - 4, 2\pi - 2$, respectively.

$y + 4 = \frac{\pi+16}{4}, \frac{\pi+7}{2}, \frac{\pi+15}{4}, \frac{\pi+6}{2}, \frac{\pi+14}{4}$, respectively.

Hence, option 2 and option 4 are the answers for questions 1 and 2, respectively.

Ram and Shyam run a race between points **A** and **B**, 5 km apart. Ram starts at 9 a.m. from **A** at a speed of 5 km/hr, reaches **B**, and returns to **A** at the same speed. Shyam starts at 9:45 a.m. from **A** at a speed of 10 km/hr, reaches **B** and comes back to **A** at the same speed. **(CAT 2005)**

At what time do Ram and Shyam first meet each other?

1. 10 a.m. 2. 10:10 a.m. 3. 10:20 a.m. 4. 10:30 a.m.

Do you need to solve this question? Work with options and logic. At 10:00 am Ram would be at B and Shyam would be Midway. Now Ram will return and Shyam would be moving towards B. Since Shyam reaches B at 10:15, they would meet between 10:00 am and 10:15 am. Answer B. No need to calculate.

At what time does Shyam overtake Ram?
1. 10:20 a.m. 2. 10:30 a.m. 3. 10:40 a.m. 4. 10:50 a.m.

Again, work with options. When Shyam overtakes Ram, both would have traveled the same distance. Shyam will reach B at 10:15 and return. By 10:30 he will be halfway. Ram will reach B at 10:00 and return. By 10:30 he would also be halfway. Curtains.

If $a_1 = 1$ and $a_{n+1} - 3a_n + 2 = 4n$ for every positive integer n, then a_{100} equals **(CAT 2005- 2 marks)**
1. $3^{99} - 200$ 2. $3^{99} + 200$ 3. $3^{100} - 200$ 4. $3^{100} + 200$

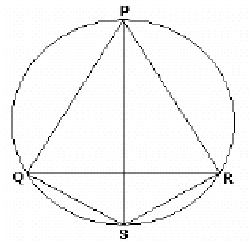
This would take some smart thinking. If you see the options, a small formula for a_{100} is given. If formula for a_{100} is given the formula for a_n would be there and the formula for a_n according to the options will be

Option 1: $a_n = 3^{n-1} - 2n$ **Option 2:** $a_n = 3^{n-1} + 2n$ **Option 3:** $a_n = 3^n - 2n$ **Option 2:** $a_n = 3^n + 2n$

Now $a_1 = 1$. Now we just keep n = 1 in all the formulas and check. Option 3 is correct.

P, Q, S, and **R** are points on the circumference of a circle of radius *r*, such that **PQR** is an equilateral triangle and **PS** is a diameter of the circle. What is the perimeter of the quadrilateral **PQSR**? **(CAT 2005- 2 marks)**
1. $2r(1+\sqrt{3})$ 2. $2r(2+\sqrt{3})$ 3. $r(1+\sqrt{5})$ 4. $2r+\sqrt{3}$

This question is so easy with the options that I was tempted to start this article with this question. Here is the figure:



You would agree that the perimeter of PQSR would be less than the perimeter of the circle? In other words, the perimeter would be less than $2\pi r = 2 \times 3.14r = 6.28r$. Option is out. Also, $PQ + QS > PS \Rightarrow PQ + QS > 2r$. Similarly, $PR + RS > 2r \Rightarrow PQ + QS + RS + PR > 4r$. Therefore, the perimeter is greater than 4r. Only option 1 works.

Let $x = \sqrt{4+\sqrt{4-\sqrt{4+\sqrt{4-\ldots\text{to infinity}}}}}$. Then x equals **(CAT 2005- 2 marks)**

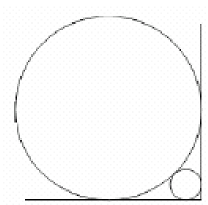
1. 3 2. $\frac{\sqrt{13}-1}{2}$ 3. $\frac{\sqrt{13}+1}{2}$ 4. $\sqrt{13}$

$x = \sqrt{4+\text{something...}} \Rightarrow$ x will be greater than $\sqrt{4} = 2$.
$x = \sqrt{4+\sqrt{4-\text{something..}}} \Rightarrow$ x will be less than $\sqrt{4+\sqrt{4}} = \sqrt{6} \approx 2.4$
By taking $\sqrt{13} \approx 3.6$, only option C satisfies the criteria.

Let $y = \cfrac{1}{2+\cfrac{1}{3+\cfrac{1}{2+\cfrac{1}{3+..}}}}$. What is the value of y? **(CAT 2004)**

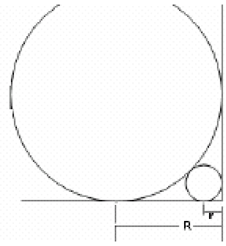
1. $\frac{3+\sqrt{13}}{2}$ 2. $\frac{\sqrt{13}-2}{2}$ 3. $\frac{3+\sqrt{15}}{2}$ 4. $\frac{\sqrt{15}-3}{2}$

Easiest of questions!
$y = \frac{1}{2+\text{something..}} \Rightarrow$ y will be less than $\frac{1}{2} = 0.5$.
Also, $y = \cfrac{1}{2+\cfrac{1}{3+\text{something..}}} \Rightarrow$ y will be greater than $\cfrac{1}{2+\cfrac{1}{3}} = 3/7 = 0.42$.
By scanning the options, options A and C can be counted out without solving them as they are greater than 1.
Now option B ≈ 0.6/2 ≈ 0.3 and option D = 0.436. (Taking $\sqrt{15} = 3.87$)
Hence, option D is the correct answer.



A circle with radius 2 is placed against a right angle. Another smaller circle is also placed, as shown in the figure. What is the radius of the smaller circle? **(CAT 2004- 2 marks)**
1. $3 - 2\sqrt{2}$ 2. $4 - 2\sqrt{2}$ 3. $7 - 4\sqrt{2}$ 4. $6 - 4\sqrt{2}$

Looks technical isn't it? But it can be solved it by calculating the options and seeing the figure. The values of the options are 0.172, 1.172, 1.344, 0.344. By visual inspection of the figure we can see that r is less than half of R, or r is less than 1.



Hence, option B and C are ruled out. For options A and D, the values of the ratio R/r are 11.62 and 5.81 respectively. We can again see from the figure that R cannot be 11.62 times of r. Hence, D is the correct answer.

NOTE: The given figure may not be drawn to scale but as long as the two circles are drawn in this manner, the ratio R/r will be same for all figures.

If $\log_{10} x - \log_{10}\sqrt{x} = 2\log_x 10$, then a possible value of x is given by **(CAT 2003)**

1. 10   2. 1/100   3. 1/1000   4. None of these

Nothing to this. Just put the values given in the options and solve. Option B satisfies the equation.

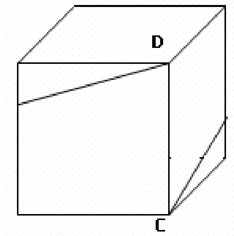
The infinite sum $1 + 4/7 + 9/7^2 + 16/7^3 + 25/7^4 + \ldots$ equals **(CAT 2003)**

1. 27/14   2. 21/13   3. 49/27   4. 256/147

A student does not need knowledge of arithmetico-geometric series to solve this problem.
4/7 = 0.571, 9/49 = 0.183, 16/343 = 0.046
➔ $1 + 4/7 + 9/7^2 + 16/7^3 = 1.800$
As further terms are negligible, the value will lie closer to 1.8.
Only 49/27 satisfies the given criterion as 49/27 = 1.81.

A string, when wound on the exterior four walls of a cube of side *n* cms, starting at point C and ending at point D, can give exactly one turn (see figure, not drawn to scale). The length of the string, in cms, is **(CAT 2003)**

1. √2 *n*   2. √17 *n*   3. *n*   4. √13 *n*



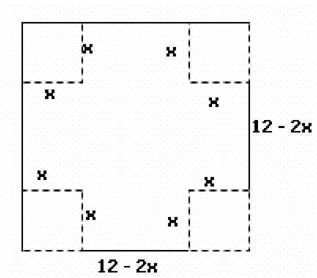
Answer: What's the mystery in this? The length of the string will be more than 4n as it is taking 4 turns around the cube.

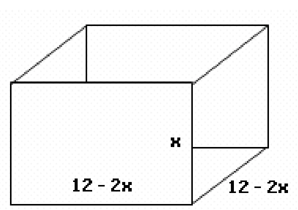
The only option satisfying the criterion is B.

A square tin sheet of side 12 inches is converted into a box with open top in the following steps: The sheet is placed horizontally. Then, equal sized squares, each of side x inches, are cut from the four corners of the sheet. Finally, the four resulting sides are bent vertically upwards in the shape of a box. If x is an integer, then what value of x maximizes the volume of the box? **(CAT 2003)**

1. 3   2. 4   3. 1   4. 2

This question takes more time to read than to solve! Can you make a mental picture of the construction of the box? Here it is.





Hence, the volume of the created box will be $(12 - 2x) \times (12 - 2x) \times x = x(12 - 2x)^2$.
Now all that is needed is to keep the values from the options and check. By keeping the values of x from options A, B, C and D, the volumes of the box will come out to be 108, 64, 100 and 128. Hence, D is the answer.

Instead of walking along two adjacent sides of a rectangular field, a boy took a short cut along the diagonal and a saved a distance equal to half the longer side. Then the ratio of the shorter side to the longer side is **(CAT 2002)**

1. 1/2   2. 2/3   3. 1/4   4. 3/4

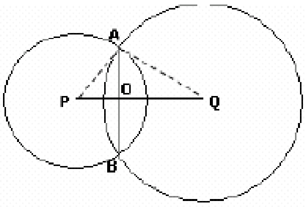
A student can take the sides of the rectangle equal to that given in the options and check. For example, in option A, the sides can be taken as 1 m and 2 m. The diagonal will become √5 m. By checking the answer this way, D will be found as the correct option.

NOTE: a simple knowledge of Pythagorean triplets will make a student check option D first, thereby saving time.

The length of the common chord of two circles of radii 15 cm and 20 cm, whose centers are 25 cm apart, is (cm) **(CAT 2002)**

1. 24 2. 25 3.15 4. 20

Work with the options again.



Let AB be the length of the common chord. Then AO = AB/2. We first pick up the options which give a whole number when divided by 2. From option A, AO = 12.
➔ PO = $(AP^2 - AO^2)^{1/2} = (15^2 - 12^2)^{1/2} = 9$. Similarly, OQ = 16.
➔ PO + OQ = 25.
Since, the distance between the centres comes out to be as given in the question, option A is correct.

Number S is obtained by squaring the sum of digits of a two digit number D. If difference between S and D is 27, then the two digit number D is **(CAT 2002)**

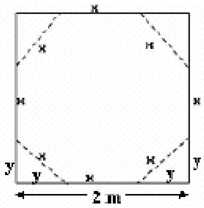
1. 24 2. 54 3. 34 4. 45

Options again. B is correct.

A square, whose side is 2 meters, has its corners cut away so as to form an octagon with all sides equal. Then the length of each side of the octagon, in meters is **(CAT 2001)**

1. $\frac{\sqrt{2}}{1+\sqrt{2}}$ 2. $\frac{2}{1+\sqrt{2}}$ 3. $\frac{2}{\sqrt{2}-1}$ 4. $\frac{\sqrt{2}}{\sqrt{2}-1}$

Do you need a good knowledge of geometry to solve this question? All that is needed is that you should be able to draw the picture.



As can be seen from the figure, x will be less than 2. By looking at the options, the last two can be eliminated easily as they are greater than 2. Taking $\sqrt{2} = 1.414$, the approximate values of the first two options are 0.58 and 0.93.

For x = 0.58 ≈ 0.6, the length y will be 0.7. The lengths of dotted lines should also be x (= y$\sqrt{2}$). The first option does not satisfy the criteria because 0.7$\sqrt{2}$ will be greater than 0.6.

Hence, option B is the correct answer, which we can also be checked by using the above logic.

Start Small:

Whenever in a series or an algebraic expression, you cannot handle the question because the magnitude of the problem is too big, start small. Work with smaller numbers. Working with smaller numbers helps you identify the pattern and solve the problem. But in order to do this, you must identify the pattern in the problem to operate the same pattern at a smaller level.

If $R = \frac{30^{65} - 29^{65}}{30^{64} + 29^{64}}$, then **(CAT- 2005)**

1. $0 < R \le 0.1$ 2. $0.1 < R \le 0.5$ 3. $0.5 < R \le 1.0$ 4. $R > 1.0$

Here, you cannot calculate 30^{65} or 29^{65}. Therefore, operate with smaller numbers.

$$\frac{30^2 - 29^2}{30 + 29} = \frac{(30-29)(30+29)}{30+29} = 1$$

$$\frac{30^3 - 29^3}{30^2 + 29^2} = \frac{(30-29)(30^2 + 29^2 + 30 \times 29)}{30^2 + 29^2} = 1 + \frac{30 \times 29}{30^2 + 29^2} \ge 1$$

You can now see the pattern that R is increasing. Therefore, option 4.

Let $n! = 1 \times 2 \times 3 \times \ldots \times n$ for integer $n \ge 1$. If $p = 1! + (2 \times 2!) + (3 \times 3!) + \ldots + (10 \times 10!)$, then $p+2$ when divided by 11! leaves a remainder of **(CAT- 2005- 2 marker)**

1. 10 2. 0 3. 7 4. 1

Let's start small.

Let $p = 1! = 1$, $p + 2 = 1 + 2 = 3$. $p + 2$ when divided by $2!$ leaves 1 as remainder.

Let $p = 1! + (2 \times 2!) = 5$, $p + 2 = 7$. $p + 2$ when divided by $3!$ gives remainder 1.

Let $p = 1! + (2 \times 2!) + (3 \times 3!) = 25$. $p + 2$ when divided by $4!$ gives remainder 1.

Now do you see the pattern and the answer? Yes, option 4 is correct.