CHAPTER 15 Bidding Strategy and Auction Design

TEACHING SUGGESTIONS

As was the case for Chapter 14 on voting, many students will be familiar with a number of different types of auctions. You can start a discussion related to their own experiences and bring in the appropriate terminology as appropriate. In this way, you should be able to address the differences between common-value and private-value auctions and between sealed bid and open outcry; students may also be familiar with first- and second-price as well as ascending and descending auction mechanisms.

For the winner's curse, you can play the auction a penny-jar game, described below, or think of a different example in which the low bid wins. House painters and other service providers often participate in such low-bid auctions. A painter will come up with her estimate of the time necessary to do the job, add in the cost of the predicted necessary materials, and submit a bid. In these situations, the painter (and the purchaser of the service) can verify the quantity of materials used, so she can ask that these costs be covered in full even if they exceed (by some limited amount) the amount stated in the original bid. But the painter cannot ask her client to cover the costs of additional time under most such bids, because of the moral hazard problems involved. Suppose the true time needed to sand and paint a house is 14 person days of time (assuming every painter has the same abilities). Then each painter makes an estimate of the time it will take her, and each estimate will come with some error (you might be within 2 or 3 days on either side of 14, for example). If lots of painters make bids and these are all pooled, their arithmetic average would be an unbiased and probably pretty accurate indicator of the true time needed to do the job. But each painter has only her own estimate, and the lowest of these will be biased on the short side (2 or 3 days short of 14). Thus, if a homeowner chooses the lowest bid, the winning painter is likely to have bid too little and will find that she has to use more time than anticipated in completing the job. Here the painter wins a job only when her rivals make estimates higher than hers. You could ask your students about ways to combat this problem. For instance, what if all house painters looked ahead to the winner's curse and adjusted their bids upward to compensate for the bias?

The revenue equivalence outcome is another example that can be used successfully in class. The text notes that expected outcomes under all four primary types of auctions are identical when bidders are risk neutral and their valuations are independent but goes on to note that more-advanced mathematical techniques are needed to prove this result. Exercise 5 at the end of the chapter can be used to establish the equivalence result in this very simple situation. That question, however, is labeled "difficult" and may not be appropriate for all classes.

For classes in which Exercise 5 cannot be used, a simple numeric example (while not a proof) can also be used to illustrate the equivalence result, and (at least) perhaps lead students to accept that it is reasonable.

Consider the following example: There are two risk-neutral bidders, each of whom has a private value for the auctioned good that is a (independent) draw from a uniform distribution between 0 and 1. In other words, each person's value for the good is equally likely to lie anywhere between 0 and 1; the two values are independent.

In an English auction, the person who places the higher value on the item, call this value V_A, will win it. Given that V_A is the higher valuation, the other bidder's valuation V_B is equally likely to be anywhere between 0 and V_A. On average, therefore, the lower valuation $V_B = V_A/2$. Since the lower bidder drops out of the auction at $V_B = V_A/2$, the bidder with the higher valuation wins the item, pays (on average) $V_A/2$, and receives an expected profit equal to $V_A - V_A/2 = V_A/2$.

Now, suppose the same people are bidding on the item using a first-price sealed-bid auction. Obviously, the bidders will shade their bids below their true values. The question is: by how much? Since the bidders are risk neutral, each will act in a way that maximizes expected profit = (person's probability of winning) × (person's value – person's bid). Without deriving the optimal bids from scratch, it can be shown that a Nash equilibrium bidding outcome entails each bidder's submitting a bid equal to half his or her valuation.

To show that this strategy is a Nash equilibrium, assume that Bidder 2 uses it, and therefore submits a bid equal to $B_2 = V_2/2$. We must then demonstrate that Bidder 1's optimal response is to adopt the same strategy. To show this, note that Bidder 1's probability of winning the auction equals $\text{Prob}(B_1 > B_2)$. Given Bidder 2's assumed strategy, this probability equals $\text{Prob}(B_1 > V_2/2)$, which can be rewritten as $\text{Prob}(V_2 < 2B_1)$. Since we've assumed that valuations are equally likely to be anywhere between 0 and 1, the probability that $V_2 < 2B_1$ (assuming that B_1 is less than or equal to 1/2) equals $2B_1$. Thus (given Bidder 2's assumed strategy), the probability that Bidder 1's bid of B_1 will be the higher bid (and will thus win the auction) equals $2B_1$. If Bidder 1 does submit the higher bid, her profit from so doing will be $(V_1 - B_1)$. We can therefore say that Bidder 1's expected profit = (probability of winning) × (value – bid) = $2B_1 \times (V_1 - B_1) = 2(B_1V_1 - B_1^2)$. The remaining step is for Bidder 1 to choose B_1 in a way that maximizes her expected winnings.

(One could employ a calculus-based approach, using a simple derivative, to show that Bidder 1's expected winnings are maximized when $V_1 - 2B_1 = 0$, or $B_1 = V_1/2$. Such an approach has a critical flaw, however, because the bidder's payoff function is not accurate at any outcome that would entail a bid over 0.5. The formula is based on the probability of winning the auction equaling $2V_1$; obviously, this formula is in error for $B_1 > 0.5$ although it turns out that there is never any reason for Bidder 1 to bid more than 0.5. The optimality of the $B = V/2$ strategy is not altered by this complication, but a correct derivation of the optimal bidding strategy is more complicated than the simple derivative would suggest. For a rigorous, albeit more abstract, derivation of this result, see this chapter's Exercise 5 and its answer.)

To avoid the difficulties inherent in applying calculus to the problem, a series of tables of possible bids can be used to show that the optimal bid is $B_1 = V_1/2$. The procedure is to assume that Bidder 2 uses the bidding strategy $B_2 = V_2/2$, assume a particular value for V_1, and determine Bidder 1's optimal bid. Suppose, for example, that $B_2 = V_2/2$, $V_1 = 0.4$, and Bidder 1 bids 0.1. Bidder 1 will win the auction with probability 0.2 (since $B_1 = 0.1 > B_2$ only if $V_2 < 0.2$), and will collect earnings of $0.4 - 0.1 = 0.3$ if she does. Her expected earnings using this bid are thus $0.2 \times 0.3 = 0.6$. The following table shows how Bidder 1 would do if she used some other possible bids.

Bid	Probability of win	Profit	Expected profit = (probability of win) x profit
0.10	0.2	0.20	0.060
0.15	0.3	0.25	0.075
0.20	0.4	0.20	0.080
0.25	0.5	0.15	0.075
0.30	0.6	0.10	0.060

Under these assumptions, it is clear that $B_1 = 0.2$ is the best bid; note that this bid follows the rule $B_1 = V_1/2$.

Now, consider other possible values for V_1; for example, $V_1 = 0.6$. In this case (again assuming that $B_2 = V_2/2$), a table like the above reads:

Bid	Probability of win	Profit	Expected profit = (probability of win) x profit
0.20	0.4	0.40	0.160
0.25	0.5	0.35	0.175
0.30	0.6	0.30	0.180
0.35	0.7	0.25	0.175
0.40	0.8	0.20	0.160

Again, the optimal bid follows the rule $B_1 = V_1/2$.
One last example, assuming that $V_1 = 0.9$:

Bid	Probability of win	Profit	Expected profit = (probability of win) x profit
0.35	0.7	0.55	0.385
0.40	0.8	0.50	0.400
0.45	0.9	0.45	0.405
0.50	1.0	0.40	0.400

Again, the optimal bid is $B_1 = V_1/2$. [Note that the above table only has four entries because (given Bidder 2's assumed strategy) there is never any reason for Bidder 1 to bid more than 0.5; that bid guarantees that her bid will be the higher.]

In all of these examples, we see that the Bidder 1's optimal strategy is to bid $B_1 = V_1/2$. The same conclusion holds for other possible values of V_1. We can thus conclude that an outcome in which each person bids an amount equal to half of her evaluation is a Nash equilibrium; when $B_2 = V_2/2$, Bidder 1 wants to set $B_1 = V_1/2$ (and Bidder 2's incentive is the same).

Finally, consider a bidder's expected winnings in this Nash equilibrium. Since both bidders are following the same strategy, the bidder who places the highest valuation on the good will win it. Furthermore, since the winning bid is equal to half the person's valuation, the person with the higher valuation will always collect winnings equal to exactly half his or her valuation.

Remember our conclusion from the English auction case analyzed above: The bidder (A) with the higher valuation (V_A) wins the item, pays (on average) $V_A/2$, and receives an expected amount of profit equal to $V_A - V_A/2 = V_A/2$. In the simple case we have analyzed, therefore, we can see that (in equilibrium) the English auction and the first-price sealed-bid auction produce identical outcomes in terms of expected value. The person who values the object the most always wins it and pays, either on average (in the English auction) or with certainty (in the first-price sealed-bid auction), a price equal to half of her valuation of the object.

We already know that with risk-neutral bidders and independent valuations (in equilibrium), the first-price, sealed-bid auction produces an outcome identical to that of the Dutch auction, while the English auction produces an outcome identical to that of a sealed-bid second-price auction. The conclusion that the first-price sealed-bid auction and the English auction produce identical outcomes (in this case) thus establishes the equivalence of all four auction designs.

Auctions are also now becoming popular on the Internet; on-line auction sites such as eBay (see http://www.ebay.com) are extremely well used. The bidding program used at eBay is described to users of the site as a way to simulate an English (eBay doesn't use that term) auction between interested bidders. The eBay procedure can (not surprisingly) also be viewed as a way to conduct a second-price sealed-bid auction (with an increment added to the second-highest bid). Students could be asked to write (for hypothetical posting on the eBay site) a new description that explains the eBay bidding procedure in terms of a second-price sealed-bid auction. For listings and information about other auction sites on the Internet, see http://www.auctioninsider.com or http://www.internetauctionlist.com.

For those who want to consider other auction-related issues, another possibility is to pursue in greater detail the Clark-Groves public goods valuation revelation scheme to supplement or buttress the discussion of Vickrey and second-price auctions. This connection is mentioned briefly in the text, but the details of the Clark-Groves scheme are found in many intermediate microeconomics texts; for example, see H. Varian, Intermediate Microeconomics, 5th ed. (New York: Norton, 1999), pages 634–637. We also provide a number of different in-class games that can be used to address a variety of topics from Vickrey's truth serum to the derivation of the optimal bid in an all-pay auction.

GAME PLAYING IN CLASS

GAME 1 Auctioning a Penny Jar (Winner's Curse)

This game was discussed as a possible game for the first or second day of class (see Chapters 1 and 2). Show a jar of pennies; pass it around so each student can have a closer look and form an estimate of the contents. Show the students a stack of 100 penniesto give them a better idea of what the jar might contain. While the jar is going around, explain the rules. Everyone plays and each submits a sealed bid; hand out blank cards and ask the students to write their names and bids and return the cards. The winner will pay his bid and get money (paper and silver, not pennies) equal to that in the jar. Ties for a positive top bid split both prize and payment equally. When you explain the rules, emphasize that the winner must pay his or her bid on the spot in cash.

After you have collected and sorted the cards, write the whole distribution of bids on the board. Our experience is that if the jar contains approximately $5, the bids average to $3.50 (including a few zeros). Thus the estimates are on the average conservative. But the winner usually bids about $6. Hold a brief discussion with the goal of getting across the idea of the winner's curse.

GAME 2 All-Pay Auction of $10

This game was also discussed as a possible game for the first or second day of class. Everyone plays. Show the students a $10 bill, and announce that it is the prize; the known value of the prize guarantees that there is no winner's curse. Hand out cards. Ask each student to write his name and a bid (in whole quarters). Collect the cards. The highest positive bid wins $10; if two or more tie with the highest positive bids, they share the $10 equally. *All* players pay the instructor what they bid, win or lose.

Be sure to emphasize before bids are submitted that "This is for real money; you must pay your bid in cash on the spot. You can make sure of not losing money by writing $0.00. But of course if almost everyone does that, then someone can win with $0.25 and walk away with a tidy profit of $9.75."

Once you have collected the cards, write the distribution of bids on the board. Hold a brief discussion about the distribution and the value of the optimal bid. This game usually leads to gross overbidding; a profit of $50 in a class or section of 20 is not uncommon. If that happens, you will have to find ways of returning the profit to the class; we have done this by having a party if the sum is large enough or by bringing cookies for the next meeting if the sum is small. Of course, do not announce this plan in advance.

You can follow this game with a relatively in-depth discussion of the optimal bid in an all-pay auction. If your students are comfortable with the necessary mathematics, you could go on to derive the formula for the optimal bid.

GAME 3 Common-Value Asset Acution

Tell your students to imagine that you own an asset. The value to you of this asset is some number between 0 and 100 points. All values from 0 to 100 are equally likely to be the asset's value.

Whatever this asset is worth to the instructor, it is worth 1.5 times that amount to each student. Thus, the value to the student of this asset is some number between 0 and 150 points (each possible value is equally likely). Students are asked to make a bid for this asset. The instructor will sell the asset to a student if and only if the student's bid is larger than the asset's value to the instructor.

Students can receive points (or cash) from this game in two ways: The first outcome will involve luck; the second will be the average outcome. For the first outcome, the instructor will use a random-number table to choose a number between 0 and 100, which will be the value to her of the asset. Anybody who bids more than that value buys the asset and pays the number of points (or pennies) he or she bid; anybody who bids less than the amount found in the table does not buy the asset. If a student buys the asset, the gain or loss of points (or pennies) is computed as the asset's value to the student minus the amount paid to buy it. [Instructors can use the same asset value to compute each student's gain or loss of points (or pennies). A student's gain or loss depends on the amount bid and on the randomly selected value of the asset.] For the second outcome, students will receive the expected value of their bids, which is the average number of points per game that they would be expected to have if this game were repeated a number of times.

The results from this game can be used to lead a discussion about issues related to common-value auctions. (There are also ways in which this game can be construed as a type of coordination game if students are concerned not only about the outcome from their own bid but about how all other students fare in acquiring points from the game.)

GAME 4 Puppy Auction

The following is based on an auction experiment played at Northwestern University and available on the WWW at http://www.kellogg.nwu.edu/faculty/weber/ds-d52/auctsim.htm. Tell students:

> Suppose that you are one of two collectors involved in the sealed-bid auction of a classic 1997 "Spot" Beanie Baby puppy. This Spot is worth $*V* to you (you will determine *V* precisely below). Thus, you would be indifferent between losing Spot to your rival bidder and paying $*V* for Spot.
>
> Having sized up your opponent, you think Spot could be worth anything between $0 and $100 to her. (She has probably sized you up similarly.) The two valuations of the bidders in this auction are subjective—primarily matters of taste—and therefore we will assume that they are independent. This means that the value you assign to Spot will not affect your assessment of your rival's valuation. (For instance, if you have a valuation of $95, this does not change your opinion about your rival's being equally likely to value Spot at any value from $0 to $100.)
>
> The seller (the instructor) will unseal both bids, and will sell Spot to the high bidder. We will consider two different pricing rules: (1) Under the first set of rules, the seller will collect from the winner a price (in points or pennies) equal to that bidder's bid. (2) Under the second set of rules, the seller will collect from the winner a price (in points or pennies) equal to the lower, losing bid.
>
> To make it worthwhile to win the auction, all winning bidders will receive 10 points for each dollar of profit made on their purchase. *Profit* is defined as the difference between your *V* and the price you pay if you win the auction. In addition, you will all calculate a pair of values and will make two bids in each auction; this is to avoid discriminating against those with high valuations.
>
> You will be asked to use a piece of personal information to determine the actual value of Spot to you. You will then be asked to consider several questions pertaining to each type of auction.

HANDOUT FOR THIS GAME

Name: ____

Student ID number: ___________________________

Your value number 1 is found by taking the last two digits of your ID number.

Value #1: _______

Your value number 2 is found by subtracting value #1 from 99.

Value #2: _______

FIRST AUCTION

This is a standard sealed-bid auction, where the higher of the two submitted bids wins, and the winner pays the amount she bid. Assume the value of Spot to you is Value #1.

What will you bid for Spot? _________

How likely do you think it is that your bid will win? _______

(There are no points associated with the second question here.)

Now assume the value of Spot to you is Value #2.

What will you bid for Spot? _________

How likely do you think it is that your bid will win? _______

SECOND AUCTION

This is an auction in which the higher of the two submitted bids wins, but the winner only pays the amount of the (lower) losing bid.

Assume the value of Spot to you is Value #1.

What will you bid for Spot? _________

How likely do you think it is that your bid will win? _______

Now assume that the value of Spot to you is Value #2.

What will you bid for Spot? _________

How likely do you think it is that your bid will win? _______

Discussion of the results from these auction questions can be used to compare bidding results in different auction types. Students can also consider the amount of shading that occurs in different auctions and how bids may vary depending on the valuations of the bidders.

ANSWERS TO EXERCISES FOR CHAPTER 15

1. The painter can compare her estimated cost to a job's true cost only when she does the job. But, the painter only does a job when she agrees (through the bidding process) to do it for less than anybody else would charge. The fact that she submitted the lowest bid suggests that this is a job for which the painter has likely underestimated the real cost. This is therefore a winner's curse; the painter only wins contracts that tend to cost more than she expected.
2. If you turn out to be the lowest bidder and therefore fail to get the object, this must be because all of the others got a higher estimate of the value of the object than you did. Therefore you have reason to believe that you got an exceptionally low estimate—one with a large and negative error. This is a "loser's curse," just like the winner's curse that occurs when only one object is auctioned among many bidders, and you are the highest bidder only if you get an exceptionally high estimate. You will correct for this loser's curse by bidding somewhat more aggressively than would be justifiable on the basis of your own estimate alone. The precise calculation of course requires more information on the probability distribution of the errors, and so on. [See Wolfgang Pesendorfer and Jeroen Swinkels, "The Loser's Curse and Information Aggregation in Common Value Auctions," *Econometrica*, vol. 65, no. 6 (November 1997), pages. 1247–1281.]
3. Your vote makes a difference to the outcome only if the other 11 have all voted Guilty. Therefore your calculation of how to vote should focus on just this situation. Accordingly, your inferences about information should be as if all the others interpreted the evidence as pointing toward guilt. This is also like a loser's curse, as described in the answer to Exercise 2. You should vote Guilty more readily than you would on the basis only of your own interpretation of the evidence. Thus if jurors behaved in this way, too many innocent people would get convicted—quite the opposite of what the unanimity requirement in jury trials is supposed to achieve. In practice this problem may be ameliorated by the jurors' not being so strategically sophisticated and more importantly by explicit deliberation among the jurors, where they can share and compare their information [see Timothy Feddersen and Wolfgang Pesendorfer, "Convicting the Innocent: The Inferiority of Unanimous Jury Verdicts under Strategic Voting," *American Political Science Review*, vol. 92, no. 1 (March 1998), pages 23–35].
4. (a) If you offer \$3,000, the current owner sells only if the car's true value to him is less than \$3,000. Since all values between \$1,000 and \$3,000 are equally likely, a car that you'll buy has an expected value of \$2,000. On average, such a car will be worth $(4/3) \times \$2{,}000 = \$2{,}667$ to you. If you offer \$3,000, therefore, you can expect the transaction to cause you to lose \$333 in value.

   (b) Suppose that your offer equals B. The current owner will sell you his car if its value to him is between \$1,000 and B; the expected value of such a car is $1{,}000 + (B - 1{,}000)/2 = 500 + B/2$. The expected value to you of a car that is sold to you is thus $(4/3)(500 + B/2)$, and your net gain is $(4/3)(500 + B/2) - B$. If you are not going to lose money, this expression must equal 0, and this occurs at $B = 2{,}000$.
5. (a) First-Order Condition (FOC) is $v - B(x) - xB'(x) = 0$.

   (b) With $x = v$, FOC is $v - B(v) - xB'(v) = 0$.

   (c) When $B(v) = v/2$, so that $B'(v) = 1/2$, the FOC is $v - (v/2) - v(1/2) = 0$. This implies that $B(v) = v/2$ satisfies the FOC and thus is the equilibrium bid.

   In a sealed-bid auction, a bidder submits a bid that is shaded down from the value he actually places on the good. In this case, the shading entails submitting a bid that equals exactly half of the person's true value. The intuition is that the outcome produced by this strategy is, *in expected value*, equivalent to the outcome produced by an open-bid auction (which in turn is equivalent to the outcome produced by a second-price, sealed-bid auction).

When both bidders follow the $B(v) = v/2$ strategy, the person with the higher value always buys the good and pays half her value. Compare this with the expected outcome of an open-bid auction. One person places the higher value, call it H, on the good. Given that H is higher than the other person's true value and that private values are uniformly distributed over [0, 1], the other person's true value will, on average, be $H/2$. The loser will therefore, on average, drop out of the bidding at $H/2$, which implies that the high bidder, on average, will pay that amount to win the auction.

In equilibrium, in both the open- and sealed-bid, first-price auctions, therefore, the expected outcome is that the person who values the good most highly buys it at a price that equals half of her valuation; the payment is expected in the open-bid auction and certain in the (equilibrium) sealed-bid case.

6. A person who places value x on the item will have a higher value than any other bidder only if all the other $(n – 1)$ bidders value the item at less than x. Given the uniform distribution of private values, the probability that any one of the other bidders values the item at less than x is x; the probability that all $(n – 1)$ of the other bidders value the item at less than x is x^{n-1}.

   A person's expected payoff from bidding as though his value is x is thus $x^{n-1}[v – B(x)]$. The first-order condition is

$$(n-1)x^{n-2}[v - B(x)] - x^{n-1}B'(x) = 0$$

   Choosing $x = v$ produces

$$(n-1)v^{n-1}[v - B(v)] - v^{n-1}B'(v) = 0$$

   Substituting $B(v) = v(n – 1)/n$ produces

$$(n-1)v^{n-2}[v - v(n-1)/n] - v^{n-1}(n-1)/n)$$
$$= (n-1)v^{n-2}v/n - v^{n-1}(n-1)/n = 0$$

   which confirms that $B(v) = v(n – 1)/n$ is the equilibrium bidding strategy.

   Two intuitive stories can be told here. First, we expect the equilibrium outcome of the sealed-bid auction to match the expected outcome of an open-bid auction. As the number of bidders rises, the expected difference between the highest value and the second-highest value placed on the item shrinks. In an open-bid auction, the bidder who places the highest value on the good always wins it. As the number of bidders grows, however, her expected surplus (which is her value minus the second-highest value) shrinks. In a closed-bid auction, this shrinking surplus arises (with certainty) if an increase in the number of bidders leads every participant to raise her bid so that it is closer to her true valuation.

   Second, increasing the extent to which you shade your bid below your valuation may create either a benefit—if your bid remains highest, your surplus is larger—or a cost—there is a greater chance that your bid will not remain highest. The closer that somebody else's bid is to yours, the more likely it is that doing a little more shading will prevent you from winning the auction. As the number of other bidders rises (thus moving the expected value of the next-highest bid closer to yours), the possible cost of shading rises. Since the benefit of shading is unaffected, a bidder should respond to the increased cost of shading by doing less of it.

ADDITIONAL EXERCISES WITH ANSWERS

1. In February of 1999, James Surowiecki, who writes the "Moneybox" column for *Slate* magazine (www.slate.com), said:

   > As nearly everyone recognzes by now, most acquisitions are mistakes for the acquiring company, which tends to overpay in expectations of gains—either from synergy or from superior management—that never arrive.

   Explain why this conclusion is not surprising.

   ANSWER The acquiring companies may be falling victim to the winner's curse.

2. The auction that is probably seen most commonly on TV is an open-bid, ascending-price auction. Which of the following auctions is (under general conditions) most closely linked to the open-bid, ascending-price auction, in the sense that the two auction types are most likely to produce (approximately) the same outcome [the same winner and (roughly) the same price]?
   (a) A closed-bid, first-price auction
   (b) A closed-bid, second-price auction
   (c) An open-bid, descending-price auction
   (d) An open-bid, alternating-price auction

   ANSWER (b)