CHAPTER 6 Combining Simultaneous and Sequential Moves

TEACHING SUGGESTIONS

The material in this chapter covers a variety of issues that require some knowledge of the analysis of both sequential-move and simultaneous-move games. Only one section deals with mixed strategies, so it is possible to present most of the material here after having introduced simultaneous-move games but before introducing mixing. We have used this approach in our own teaching.

To address rule changes in games, you will want to draw on examples used in previous class periods. We focus on the different outcomes that can arise when rules are changed. When a game is changed from simultaneous move to sequential move, for example, the change can create a first- or second-mover advantage. Games like the battle of the two cultures or chicken have first-mover advantages in their sequential-move versions; the tennis-point example has a second-mover advantage in its sequential-move version. Other games show no change in equilibrium as a result of the change in rules; games like the prisoners' dilemma, in which both players have dominant strategies, fall into this category.

Similarly, changing a game from sequential play to simultaneous play can mean that new equilibria arise—either multiple equilibria or equilibria in mixed strategies. Use the sequential-game examples you used to convey the material from Chapter 3 to show that there might be additional equilibria in the simultaneous-move versions of the game. This works for the tennis-point game if you teach it as a sequential game or for the three-person voting example from Ordeshook (see Chapter 3 discussion).

The most interesting component of the analysis in Chapter 6 is the representation of sequential-move games in strategic form and the solution of such games from that form. The second (and third) mover's strategies are more complex in sequential games, and the payoff table must have adequate rows (or columns or pages) to accommodate all of the possible contingent strategies available to players. Again, the tennis-point or voting examples can be used to illustrate this idea. One nice exercise is to assert the existence of a new number of Nash equilibria in the strategic form (like the eight found in Figure 6.11) and to show how one or two of these qualify as Nash equilibria; then use successive elimination of dominated strategies on the game to arrive at one cell of the table as the single reasonable equilibrium of the game. This helps motivate the idea of subgame perfection. Once you have shown that there may be multiple equilibria but that you can reduce the set of possible equilibria to one by eliminating (weakly) dominated strategies, you can show that the strategies associated with that one equilibrium coincide with the strategies found using rollback on the extensive form of the game. Students often have difficulty grasping the idea that the eliminated equilibria are unreasonable because of the strategies *associated* with them rather than because of the (often) lower *payoffs* going to the players, so you will want to reinforce this idea as often as possible.

In addition to the examples used in previous chapters, you might want to make use of the game from Exercise 9.4 in the text. This game can be played using either simultaneous or sequential moves, and there are several ways in which the sequential-move game can be set up; thus you have an opportunity to discuss rule changes as well as order changes. Also, the Boeing–Airbus example from Exercise 4 in Chapter 3 of the text can be used to show how multiple equilibria can arise when sequential-play games are represented in strategic form. This is another way to show, with a smaller payoff table, that all subgame-perfect equilibria are

Nash equilibria but that not all Nash equilibria are subgame perfect. This example is also useful for explaining why Boeing's threat to fight if Airbus enters is not credible. This concept will be developed in more detail and used extensively in Chapter 9, so it is useful to introduce it before.

GAME PLAYING IN CLASS

GAME 1 Color Coordination (with Delay)

This game should be played twice, once without the delay tactic and once with it, to show the difference between outcomes in the simultaneous and sequential versions. As usual, the game can be played by pairs of students, although it can also be played by all students simultaneously with the left-hand side of the room's playing against the right-hand side. Tell the students not to write down anything (except their names) until they hear all of the instructions. As with the tacit coordination game described in Chapter 4, you might want to provide some inducement for coordination here; chocolate usually works well.

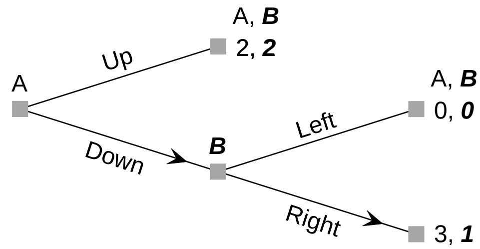
Ask the students to choose partners from the other side of the room or have them imagine that each is playing with one person who is sitting on the other side of the room. Each student will eventually be asked to write either *pink* or *purple*. If both students in the real or imaginary pair write *pink*, the person on the *right*-hand side of the room gets 50 points and the person on the *left*-hand side of the room gets 40 points. ("Right-hand" and "left-hand" are defined from the students' point of view.) If both write *purple*, the person on the *left*-hand side of the room gets 50 points and the person on the *right*-hand side of the room gets 40 points. If the answers don't match, neither player gets anything.

To play without the delay tactic, simply ask the students to choose a color and write the choice. Then play again, immediately, but explain that you will flip a coin first. If it comes up heads, those on the right-hand side of the room get to write their answers first; otherwise those on the left-hand side of the room write first.

Once you have collected answers from the students, you can discuss the implications of the game. Clearly, it is much more difficult to coordinate without the benefit of the delay tactic and there are two equilibria in pure strategies in the simultaneous-move game. The delay tactic makes the game sequential and creates a first-mover advantage; outcomes from this version often come much closer to complete coordination. As usual, you can ask students to try to come up with real-world situations that replicate some of the conditions of the game, or you can provide some examples.

ANSWERS TO EXERCISES FOR CHAPTER 6

1. (a) Tree is shown above. The subgame-perfect equilibrium is (Down, Right).



(b)

		PLAYER B	
		if Down, then Left	*if Down, then Right*
PLAYER A	Up	2, ***2***	~~2~~, ***2***
	Down	~~0~~, ~~***0***~~	3, ***1***

There are two Nash equilibria: (Up, Left) yields a payoff of (2, 2) and (Down, Right) yields a payoff of (3, 1). The (Down, Right) equilibrium is subgame perfect (see answer to part a); it does not rely on either player's taking an action that is not in her own interest at the time it is taken. The (Up, Left) equilibrium is not subgame perfect. This is a Nash equilibrium only if Player A expects Player B to choose Left, but in the sequential game, Player B would hurt herself (she would get a payoff of 0 rather than of 1) if she chose Left *after* Player A had already picked Down. Player B's threat to pick Left in this situation is not credible; it is just this sort of noncredible threat used to support an equilibrium that is eliminated when the requirement of subgame perfectness is imposed.

(c) One uses the basic tools for finding equilibrium in games illustrated in sequential form; that is, one looks for a dominant (or a dominated) strategy. In this case, Player B's strategy "if Down, then Right" weakly dominates "if Down, then Left." Assuming Player B uses her dominant strategy, it is obvious that Player A picks Down. This process thus finds the unique subgame-perfect (or rollback) equilibrium of (Down; if Down, then Right).

2. (a) For game in Exercise 3.2a:

		PLAYER B	
		If North, then Top	*If North, then Bottom*
PLAYER A	North	~~0~~, ***2***	2, ~~***1***~~
	South	1, ***0***	~~1~~, ***0***

The only Nash equilibrium is (South; if North, then Top) with payoffs of (1, 0).

(b) For game in Exercise 3.3c:

		PLAYER B			
		s if S	*n if S and s if S again*	*n if S and n if S again and s if S again*	*n if S and n if S again and n if S again*
PLAYER A	N	~~0~~, ***1***	~~0~~, ***1***	~~0~~, ***1***	~~0~~, ***1***
	S and N if n	5, ***4***	~~2~~, ~~***3***~~	~~2~~, ~~***3***~~	~~2~~, ~~***3***~~
	S and S if n and N if n again	5, ~~***4***~~	3, ~~***2***~~	4, ***5***	4, ***5***
	S and S if n and S if n again	5, ***4***	3, ***2***	~~2~~, ~~***2***~~	~~1~~, ~~***0***~~

There are four Nash equilibria: (S and N if n, s if S), (S and S if n and S if n again, s if S), (S and S if n and N if n again, n if S and n if S again and s if S again), and (S and S if n and N if n again, n if S and n if S again and n if S again). Only the third of these is subgame perfect and matches the equilibrium found using rollback in Chapter 3.

The remaining three equilibria are not subgame perfect. The first, (S and N if n, s if S), requires that A choose N at her second decision node even though that yields a 2, rather than the 3 or 4 she could get if she chose S; "S and N if n" is not credible for A. (Similarly, "s if S" is not credible for B since B could get 5 rather than 4 if she chose n over s at her first decision node.) The second equilibrium, (S and S if n and S if n again, s if S), has A choose S at her third decision node even though that gets her either 1 or 2 rather than the 4 she could get if she chose N; again, this strategy is not credible for A. (Again, "s if S" is also not credible for B.) Finally, the fourth equilibrium, (S and S if n and N if n again, n if S and n if S again and n if S again), requires that B choose s at her last decision node even though this action gets her a 0 instead of the 2 that she could get from choosing n; this strategy is not credible for player B. Note here that player A is using her equilibrium strategy.

3. (a) Airbus has two strategies: In and Out. Boeing has two strategies:" if In, then peace" and "if In then War."

(b)

		BOEING	
		If In, then Peace	*If In, then War*
AIRBUS	In	\$300m, ***\$300m***	–\$100m, ***–\$100m***
	Out	0, ***\$1b***	0, ***\$1b***

There are two Nash equilibria: (In; if In, then Peace) and (Out; if In, then War).

(c) Only the (In; if In, then Peace) equilibrium is subgame perfect.

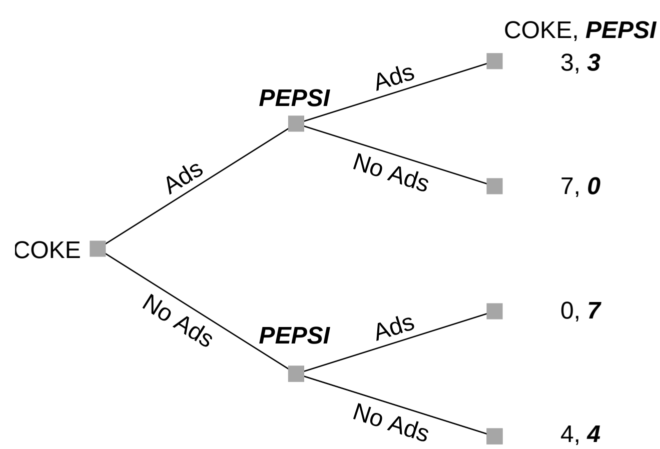
(d) The outcome (Out; if In, then War) is a Nash equilibrium but is not subgame perfect. This equilibrium hinges on Airbus's belief that Boeing will start a price war on Airbus's entry into the market. However, Boeing lowers its own payoff by starting such a price war. The threat to do so is therefore not credible.

4. (a)

		PEPSI	
		Ads	*No Ads*
COKE	Ads	3, ***3***	7, ~~***0***~~
	No Ads	~~0~~, ***7***	~~4~~, ~~***4***~~

This game has a dominant strategy equilibrium: (Ads, Ads).

(b)



The subgame-perfect equilibrium is again (Ads, Ads); when both players have a dominant strategy, the dominant-strategy equilibrium will hold in both the simultaneous and the sequential versions of the game.

(c) This game is a prisoners' dilemma. The (Ads, Ads) equilibrium is jointly inferior to the (No ads, No ads) equilibrium. The two firms would do better if they could somehow cooperate and both choose not to advertise. Whether such cooperation is likely to occur is the subject of later chapters.

5. (a)

Frieda's

Urban

		BIG GIANT	
		Urban	*Rural*
TITAN	Urban	11/3, ***11/3,*** 11/3	5, ***~~2~~,*** 5
	Rural	~~2~~, ***5,*** 5	~~4~~, ***~~4~~,*** ~~3~~

Rural

		BIG GIANT	
		Urban	*Rural*
TITAN	Urban	5, ***5,*** ~~2~~	~~3~~, ***~~4~~,*** ~~4~~
	Rural	~~4~~, ***~~3~~,*** ~~4~~	4, ***4,*** 4

There are two Nash equilibria: (Urban, Urban, Urban) and (Rural, Rural, Rural). Note that with the lottery, when all three stores choose Urban, each has a 2/3 probability of getting into Urban and receiving a payoff of 5 and each has a 1/3 probability of being left to Rural *after* the good mall locations are chosen and getting a payoff of 1. Thus, the expected payoff to each store in this case is (2/3)5 + (1/3)1 = 11/3.

(b) One Nash equilibrium (Rural, Rural, Rural) appears in both versions of the game. In both versions, a store that believes that the other two stores will pick Rural also wants to do so (to get a payoff of 4), rather than be the only store to pick Urban (to get a payoff of 3).

In the big name version of the game, if Frieda's believes that both other stores will pick Urban (so that Frieda's is shut out of the urban mall), it should pick Rural. Big Giant and Titan are happy in this outcome, since they achieve the highest payoff. Thus, the outcome in which Titan and Big Giant pick Urban while Frieda's picks Rural is an equilibrium.

In the equal chances version, if Frieda's believes that both other stores will pick Urban, it also wants to pick Urban (taking the chance of getting into the urban mall is more attractive than settling for the rural mall is). The other two stores feel the same. Thus, all three stores' picking Urban is an equilibrium.

It is hard to say which of (Urban, Urban, Urban) and (Rural, Rural, Rural) is focal. On one hand, the (Rural, Rural, Rural) equilibrium offers each store a higher payoff than (Urban, Urban, Urban) does. On the other hand, (Urban, Urban, Urban) offers each firm a 2/3 chance of receiving the highest payoff, which may attract the attention of the stores.

6. Cell-by-cell examination reveals the following eight Nash equilibria (where choices are given in the order Frieda's, Big Giant, Titan): (R, UU, UUUU), (R, UU, UUUR), (R, UU, URUU), (R, UU, URUR), (R, UR, UURR), (R, UR, URRR), (R, RR, URRR), and (R, RR, RRRR).

Note that the possible equilibria produce two possible outcomes. The first four equilibria produce an outcome in which Frieda's is alone in the rural mall (payoffs are (2, 5, 5)); the last four find all three stores in the rural mall (payoffs are (4, 4, 4)). The rollback procedure introduced in Chapter 3 found the equilibrium (R, UU, UUUR). This equilibrium is reasonable because each of the three stores makes the decision that is in its own best interest at every decision node that could possibly arise during the game (even those nodes that do not arise when the equilibrium is played).

In the other seven equilibria, one store (or more) uses a strategy in which, at some possible decision node, it makes a choice that lowers its own payoff. Of course, these self-defeating choices do not arise when all three stores use their equilibrium strategies. In other words, these seven equilibria are supported by beliefs about off-the-equilibrium-path behavior.

For example, consider the (R, UR, UURR) equilibrium, and use the labels provided in Figure 3.5. This equilibrium involves Titan's picking R at node *f*, which is harmful to itself. If, however, Big Giant believes that Titan will act in this way, Big Giant will choose Rural at node *c*, this moves the game along to node *g*. Given Big Giant's choice, Titan's strategy calls for the correct choice at node *g*.

In this equilibrium, as in the other seven unreasonable ones, each store makes the choice that is in its own best interest given what the other stores do (so the outcome does qualify as an equilibrium). However, some store takes the action it does only because it believes that another store will take an unreasonable action at some out-of-equilibrium node (which is why the equilibrium is unreasonable).

7. (a)

		VENDOR 2				
		A	*B*	*C*	*D*	*E*
VENDOR 1	A	85, ***85***	100, ***170***	125, ***195***	150, ***200***	160, ***160***
	B	170, ***100***	110, ***110***	150, ***170***	175, ***175***	200, ***150***
	C	195, ***125***	170, ***150***	120, ***120***	170, ***150***	195, ***125***
	D	200, ***150***	175, ***175***	150, ***170***	110, ***110***	170, ***100***
	E	160, ***160***	150, ***200***	125, ***195***	100, ***170***	85, ***85***

(b) Locations A and E are both dominated for both vendors.

(c) There are two pure-strategy equilibria: both involve one vendor's locating at B and the other's locating at D. To arrive at one of these equilibria requires coordination of some type. (If both players tried to locate at B or D, payoffs would be only 110 each instead of 175.)

(d) In sequential play of the game, the vendor that moves first will choose either B or D and the vendor that moves second will choose the other location (D or B). In the sequential game, the move order establishes the necessary coordination so that each vendor receives a payoff of 175 in equilibrium.

8. (a) Table payoffs for second pitch:

		PITCHER	
		Fast	*Curve*
BATTER	Swing	0.75	0
	Take	0	0.25

Batter's p-mix (probability p on Swing) must satisfy $0.75p = 0.25\ (1 - p)$, so we get $p = 0.25$. Pitcher's q-mix (probability q on Fast) must satisfy $0.75q = 0.25\ (1 - q)$, so we get $q = 0.25$. Batter's payoff in the mixed-strategy equilibrium on the second pitch is $0.75 \times 0.25 = 0.1875$.

We use the batter's expected payoff from the second pitch as the payoff to the batter from taking a curveball on the first pitch; thus, 3/16 is the payoff in the bottom right cell of the payoff table for the first pitch, as seen below.

		PITCHER	
		Fast	*Curve*
BATTER	Swing	0.75	0
	Take	0	0.1875

Batter's p-mix here satisfies $0.75p = 0.1875(1 - p)$, which yields $p = 0.2$. Thus, the batter should Take with probability $(1 - p) = 0.8$ on the first pitch and Take with probability 0.75 on the second pitch.

(b) The pitcher's equilibrium q-mix on the first pitch must satisfy $0.75q = 0.1875(1 - q)$, which yields $q = 0.2$. The pitcher's equilibrium strategy is: Throw a curve with probability 0.8 on the first pitch. If the game goes to a second pitch, then on that pitch throw a curve with probability 0.75.

(c) The batter's expected payoff in equilibrium here is the expected payoff calculated from the equilibrium mixtures at the first pitch $0.75 \times 0.2 = 0.15$. (The pitcher's equilibrium payoff is thus –0.15.)

(d) The batter's probability of swinging is so low because the pitcher is less likely to throw a fastball, which in turn is so because fastballs are expensive for the pitcher. If fastballs are thrown with greater probability, the batter will swing more often to take advantage and hit several home runs.

ADDITIONAL EXERCISE WITH ANSWER

1. This question is based on the football game described in Chapter 6, Section 3. The offense's probabilities of success are shown below.

		DEFENSE	
		10 yd	*20 yd*
OFFENSE	10 yd	1/2	1
	20 yd	1	1/4

(a) Determine the equilibrium (and the value) of the fourth-down-and-20-yards-to-go game.

(b) Determine the (mixed-strategy) equilibrium (and the value) of the fourth-down-and-10-yards-to-go game.

(c) Explain why the payoff matrix in the third-down situation is:

		DEFENSE	
		10 yd	*20 yd*
OFFENSE	10 yd	19/40	7/10
	20 yd	1	7/16

(d) Use the table in part c to determine the (mixed-strategy) equilibrium of the third-down game.

ANSWER (a) Offense uses 20-yard play; Defense defends against 20-yard play. Value = 1/4.

(b) Offense uses 10-yard play with probability 3/5; Defense defends against 10-yard play with probability 3/5. Value = (3/5)(1/2) + (2/5)1 = 7/10.

(c) (10 yd, 10 yd): probability of third-down success (1/2) × value of fourth-and-10 (7/10) + probability of third-down failure (1/2) × value of fourth-and-20 (1/4) = 19/40.

(10 yd, 20 yd): certain third-down success (1) × value of fourth-and-10 (7/10) = 7/10.

(20 yd, 20 yd): certain third-down success (1) × value of won game (1) = 1.

(20 yd, 20 yd): probability of third-down success (1/4) × value of won game (1) + probability of third-down failure (3/4) × value of fourth-and-20 (1/4) = 7/16.

(d) Offense uses the 10-yard play with probability 5/7; Defense defends against the 10-yard play with probability 1/3.