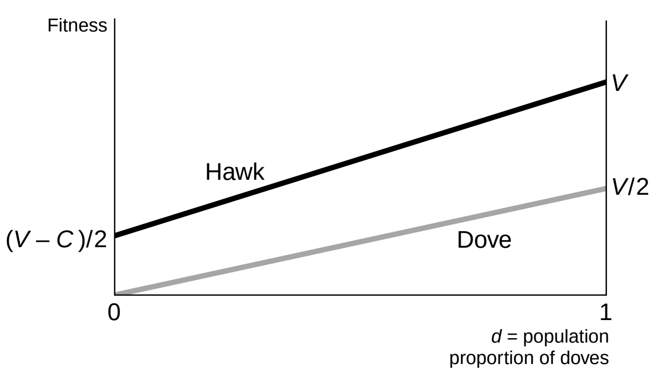
CHAPTER 10 Evolutionary Games

TEACHING SUGGESTIONS

How much time you want to spend on this material will depend on the focus of your course. For many social science courses, a general exposure to the ideas, based on a quick run through the special examples of the prisoners' dilemma and chicken games will suffice. If your class is comfortable with a bit of math, you can do the two together using the algebraic formulation of the hawk-dove game in Section 6.

We have found this last method to be a satisfying way to approach the topic of evolutionary games when you have time to devote only a single lecture to the topic. (If you want to use this type of presentation but avoid the algebra, you can substitute numbers for V and C in the following analysis.) You can set up the framework of the analysis first, comparing it to rational game-theoretic ideas. (Players here are phenotypes with hardwired strategies; fitness represents payoff to the players; equilibrium can occur as a monomorphism—pure ESS—or polymorphism—mixed ESS.) After you present the general hawk-dove payoffs, it is a straightforward task to show that when $V > C$, there is a prisoners' dilemma structure to the payoffs and that when $V > C$, there is a chicken structure.

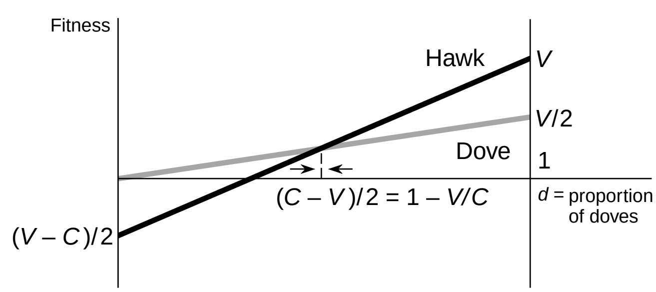
For the prisoners' dilemma case, the fitness graphs look as follows:



The evolutionary stable proportion of doves in the population is 0 when the game resembles a prisoners' dilemma. Hawk is the unique ESS. (You can note that Hawk is the dominant strategy for both players in the hawk-dove game if it is analyzed using standard, rational game-theoretic analysis.)

You may also want to draw a link between the repeated prisoners' dilemma game analyzed in this chapter and Axelrod's prisoners' dilemma tournaments discussed in Chapter 8. One point that could be made is that, since we now know that evolutionary success sometimes depends on starting conditions, Tit-for-tat may have failed to win Axelrod's tournaments if the starting population of strategies had been sufficiently different.

For the chicken case, rational game-theoretic analysis found two pure-strategy equilibria as well as a mixed-strategy equilibrium. For the hawk-dove game in the text, this type of analysis finds the equilibrium mixture to put probability V/C on Hawk and probability $1 – V/C = (C – V)/2$ on Dove. To make use of evolutionary analysis in this game, however, we need to look at the fitness graphs again:



This time, $(V – C)/2 < 0$, so the fitness lines cross each other. For low values of d, Dove fitness is higher than Hawk fitness while for high values of d, Hawk fitness is higher than Dove fitness; the value of d at the intersection is $1 – V/C$.

But what is the ESS in this case? Clearly a predominantly hawk population could be invaded by a mutant dove (since dove fitness is higher for small d) but a predominantly dove population can also be invaded by a mutant hawk (since hawk fitness is higher for large d). Thus neither $d = 0$ (all Hawk) nor $d = 1$ (all Dove) represents a stable equilibrium; neither Hawk nor Dove is an ESS. (Another way to think about this is that neither of the pure strategies is evolutionary stable.) The other two possibilities are (1) a stable polymorphic population (the population is mixed) and (2) a population in which each individual is hardwired to play a mixed strategy.

To test for a stable polymorphism, we look at the dynamics of the population proportion of doves in the fitness graph above. The analysis just above showed that, from either extreme, d converges to some central value; that value is at the intersection of the two fitness graphs, $d = 1 - V/C$. The graph indicates that when the population proportion of doves is $1 - V/C$—and the population proportion of hawks is V/C, there is a stable polymorphic ESS.

The final possibility is that there might be mixers—phenotypes hardwired to play hawk some proportion of the time and dove the rest of the time—in the population and that mixing could be an ESS. If you have already calculated the equilibrium mixed strategy from the rational analysis of this game, that is the mixture to suggest as a possible ESS for the mixing phenotype. If you haven't calculated it yet, you will want to do so now and then suggest it as the appropriate way to hardwire the mixing phenotype. The challenge then is to show that mixing could be an ESS when $V < C$. You will want to use the payoff table shown here that includes the third possibility for each phenotype:

		COLUMN		
		Hawk	*Dove*	*Mixer*
ROW	Hawk	$(V - C)/2$, ***(V – C)/2***	V, ***0***	$(1 - V/C)V/2$, ***V/C(V – C)/2***
	Dove	0, ***V***	$V/2$, ***V/2***	$(1 - V/C)V/2$, ***V/2(V/C + 1)***
	Mixer	$V/C(V - C)/2$, ***(1 – V/C)V/2***	$V/2(V/C + 1)$, ***(1 – V/C)V/2***	$(1 - V/C)V/2$, ***(1 – V/C)V/2***

To see if Mixer is an ESS, you need to see if any mutant doves or any mutant hawks could invade a predominantly mixer population. Here, you can appeal either to fitness graphs (as above) or to the algebra alone, since you will have shown several fitness graphs already. If there is a small proportion d of mutant doves in a predominantly mixer population, then dove fitness is

$$dV/2 + (1 - d)V/2(1 - V/C)$$

and mixer fitness is

$$dV/2(V/C + 1) + (1 - d)V/2(1 - V/C).$$

Note that the last parts of the two expressions are identical so doves can invade only if $V/2 > V/2(V/C + 1)$. But V/C + 1 > 1, so this cannot hold and doves cannot invade. Similarly, hawks cannot invade. Mixer is an ESS in this version of the game.

If you have time, you could consider a game in which phenotypes are more fit when more abundant; such a game would have a payoff structure similar to the assurance game. Then you will get two stable monomorphic equilibria at the extremes. There will also be a polymorphic equilibrium in such a game, but it will be unstable.

If you want to spend even more time on this material, you can make it come to life using some simulations of patterns that evolve according to specified rules of birth and death. Among the best known of such simulations is John Conway's game of life; it can be found at http://www.bitstorm.org/gameoflife. Also, the Santa Fe Institute is active in research on evolving cellular automata that can mimic the patterns of life cycles of some actual organisms. You can find this work at their website at http://www.santafe.edu/~evca .

For those who cover the population dynamics in the battle of the two cultures, you may find it easier to explain to a nonmathematical class if you first present it broken into two pieces. (This is best achieved with a series of overheads.) First draw the graph with a horizontal line at $y = 2/3$ and show x's decreasing above the line and increasing below it. Then draw a separate graph with a vertical line at $x = 2/3$ and show y's increasing to the left of it and decreasing to the right. These two diagrams can then be combined (stacked if you are using overheads) to get the one shown in the text. For the rare class that is sufficiently mathematically trained to cope with simple differential equations, the dynamics of Figures 10.10 and 10.13 can also be made more precise.

Thus for the battle-of-the-two-cultures game, suppose

$$dx/dt = 2/3 - y \quad \text{and} \quad dy/dt = 2/3 - x$$

Then

$$\begin{aligned} d[\,(2/3 - x)^2 - (2/3 - x)^2\,]/dt \\ = -2\,(2/3 - x)\,dx/dt + 2(2/3 - y)\,dy/dt = 0 \end{aligned}$$

Therefore the time paths are hyperbolas along which

$$(2/3 - x)^2 - (2/3 - x)^2 = \text{constant}$$

Similarly, in the rock-paper-scissors game, if you assume that

$$dq_1/dt = 1 - q_1 - 2q_2 \quad \text{and} \quad dq_2/dt = 2q_1 + q_2 - 1$$

then it is easy to verify that

$$(q_1 - 1/3)^2 + (q_2 - 1/3)^2 + (q_1 - 1/3)(q_2 - 1/3) = \text{constant}$$

and the time paths are ellipses. With a constant of proportionality $k \neq 1$ in one of the differential equations, you can get stable or unstable spirals.

ANSWERS TO EXERCISES FOR CHAPTER 10

1. Suppose the strategies are $S_1, S_2, \ldots, S_n$, and the payoff of a player genetically programmed to play i when matched against one genetically programmed to play j is A_{ij}. Suppose the proportions of the types are $p_1, p_2, \ldots, p_n$. Then the fitness of type i is

$$F_i = A_{il}p_1 + A_{i2}p_2 + \cdots + A_{in}p_n.$$

If strategy 1 is strictly dominated by strategy 2, then $A_{1j} < A_{2j}$ for all j. The proportions sum to 1, so at least one of them must be positive. Therefore $F_1 < F_2$. Then even a small initial proportion of 2-players will grow at the expense of that of 1-players and eventually drive the latter proportion to zero.

Next suppose strategy 1 is weakly dominated by strategy 2. The two can have equal fitnesses if the population consists of only those types against which the two have equal payoffs, but not if the population has positive proportions of all types. For example, suppose $A_{1j} = A_{2j}$ for $j = 1, 2, 3$ and $A_{1j} < A_{2j}$ for $j = 4, 5, \ldots, n$. Then type 1 can survive in a population that consists of types 1, 2, and 3 only but not in a population that has positive proportions of types $4, 5, \ldots, n$.

2. (a) Let D represent an always defector, and T a tit-for-tat player.

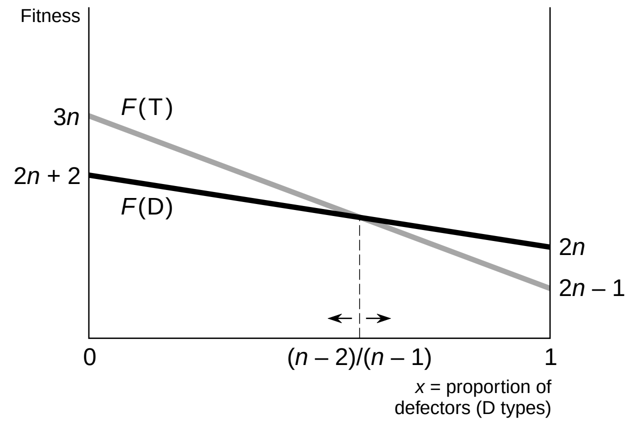
		PLAYER 2	
		D	*T*
PLAYER 1	D	$2n$, $\mathbf{2n}$	$2n + 2$, $\mathbf{2n - 1}$
	T	$2n - 1$, $\mathbf{2n + 2}$	$3n$, $\mathbf{3n}$

(b) and (c) $F(\text{D}) = p(2n) + (1 - p)(2n + 2) = 2(n + 1 - p)$; $F(\text{T}) = p(2n - 1) + (1 - p)3n = 3n - p(n + 1)$

(d) $F(\text{D}) > F(\text{T})$ when $2n + 2 - 2p > 3n - np - p$, or $2 - p > n - np$. This yields $np - p > n - 2$, and finally, $p(n - 1) > n - 2$, or $p > (n - 2)/(n - 1)$.

Similarly, $F(\text{T}) > F(\text{D})$ when $3n - np - p > 2n + 2 - 2p$, or $n - np > 2 - p$. This yields $n - 2 > np - p$, or $n - 2 > p(n - 1)$, and finally, $(n - 2)/(n - 1) > p$.

(e)



There are three possible equilibria. In one, $p = 0$; the entire population is tit-for-tat players. In another, $p = 1$; the entire population is defectors. In the third, $p = (n - 2)/(n - 1)$; the population is mixed: a proportion $(n - 2)/(n - 1)$ is always defectors, and $1 - (n - 2)/(n - 1)$ is tit-for-tat players. The first two equilibria are evolutionary stable; the polymorphic equilibrium is not.

(f) The population will move toward the all-tit-for-tat equilibrium (in which cooperation always occurs) whenever the original $p < (n - 2)/(n - 1)$. As n rises, $(n - 2)/(n - 1)$ also rises (approaching 1); this increases the likelihood that a given p will meet this condition.

Intuitively, the possible advantage of being hardwired to play tit-for-tat is that it gives a player the chance to establish long-lasting (and beneficial) cooperation if she meets another tit-for-tat player. The disadvantage is that a tit-for-tat player is left open for one-time exploitation when she meets an always defector. As the number of rounds rises, the possible benefits of (long-lasting) cooperation rise relative to the possible one-time cost. Thus, a larger n shifts the relative benefit-cost ratio in favor of tit-for-tat players.

3. (a)

		PLAYER 2			
		A	*T*	*N*	*S*
PLAYER 1	A	20, ***20***	11, 3***5***	2, ***50***	11, ***35***
	T	35, ***11***	6, ***6***	6, ***6***	28, ***4***
	N	50, ***2***	6, ***6***	6, ***6***	28, ***4***
	S	35, ***11***	4, ***28***	4, ***28***	13, ***13***

(b) Remember that payoffs are years in jail here, so smaller numbers represent better fitness. Then, no matter what mixture makes up the overall population, S is (strictly) fitter than N, so N will die out. In addition, S is (weakly) fitter than T, so T will also die out. When the population is reduced to only A and S types (no matter what their proportions in the population), A is fitter than S is, so S will also die out.

(c) Suppose N has died out. Then S strictly dominates N in a population consisting of A, T, and S, so N cannot reappear. Next suppose N and T have died out. In a population consisting of A and S alone, S strictly dominates N, and A strictly dominates T, so neither can reappear. Finally, suppose N, T, and S have all died out. In an all-A population, A gets a strictly better payoff than any of the other three, so none of them can reappear.

(d) The (only) ESS in this game is A. In the text example (which had no S strategy), T could be an ESS (provided that there were enough Ts in the population at the beginning of the game). In this problem, T cannot be an ESS because the S strategy is always fitter than T. Consider the outcomes when a player is matched with a T or an N player. During the first game of a two-game meeting, an S player gets the same benefit from cooperation as a T player does. An S player always, however, does better than a T player does in the second game. Thus T cannot be an ESS because an all-T population can be successfully invaded by a (fitter) S. Intuitively, an S player takes advantage of a T player by double-crossing her in the second game.

Note that if a fully rational player were matched against a player she knew to be a T type, the rational player would play the S strategy (which is essentially the defect-in-the-last-period-of-a-repeated-game strategy described in Chapter 7).

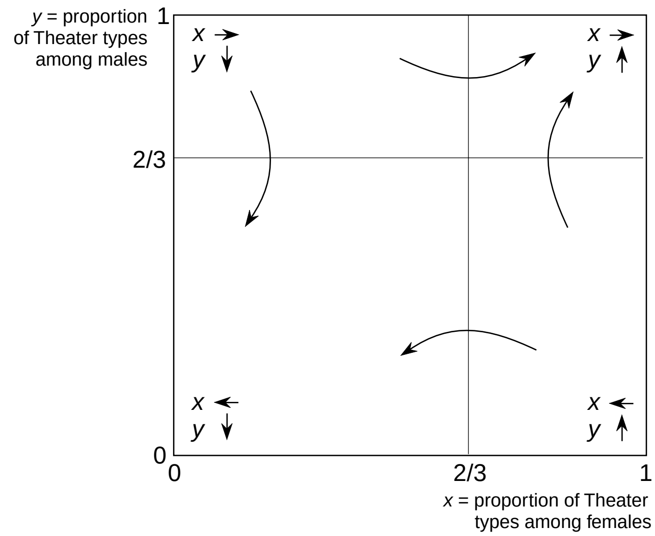
4. The payoff table is shown below:

		MALE	
		Theater (proportion y)	*Movie (proportion 1 – y)*
FEMALE	Theater (proportion *x*)	1, ***1***	0, ***0***
	Movie (proportion 1 – *x*)	0, ***0***	2, ***2***

Fitness of Female T individual = $1 \times y + 0 \times (1 - y) = y$.
Fitness of Female M individual = $0 \times y + 2 \times (1 - y) = 2(1 - y)$.

So, for females, Theater type is fitter than Movie type—and the population proportion x of Theater types, increases—if $y > 2(1 - y)$ or if $y > 2/3$. Similarly, for males, the Theater type is fitter than the Movie type—and the population proportion y of Theater types, increases—if $x > 2(1 - x)$ or if $x > 2/3$.

The diagram below shows the dynamics of the game. There are two ESS: (0, 0) and (1, 1).



ADDITIONAL EXERCISES WITH ANSWERS

1. The pattern of behavior displayed by the individuals ina multiplayer game can change over time, as one type of behavior is adopted by a greater share of the population (while another type of behavior becomes less commonly used).

   State whether the following is true or false and explain why. In such a situation, the original pattern of behavior (how many people are using each strategy) will have no effect on the final (equilibrium) pattern of behavior; the same final pattern will be reached no matter what original pattern existed.

   ANSWER False. In a game with multiple (stable) equilibria the original pattern of behavior can determine which equilibrium is reached.

2. Consider a population in which each of the members of the species may be either aggressive (and are called Hawks) or passive (and are called Doves). Every period, pairs of the species are (randomly) matched together, they interact, and each receives a payoff. A strategy is called evolutionary stable if, when the entire population uses that strategy, no other strategy can successfully “invade” the population. Use the following information to complete the sentence given below:

   When a dove meets a dove, the dove’s payoff is 4.
   When a dove meets a hawk, the dove’s payoff is 2.
   When a hawk meets a hawk, the hawk’s payoff is 1.
   When a hawk meets a dove, the hawk’s payoff is 6.

   Given these payoffs, _____.

(a) Hawk is an evolutionary stable strategy
(b) Dove is an evolutionary stable strategy
(c) both Hawk and Dove are evolutionary stable strategies
(d) neither Hawk nor Dove is an evolutionary stable strategy

ANSWER (d)

3. Consider a population in which individual members can display either Hawk or Dove behavior. Suppose that this population is now made up almost entirely of hawks, and that we wish to determine whether it is possible for doves to successfully invade the population. To do this, we should compare the payoff that a dove gets when it meets a hawk with the payoff that _____.

(a) a hawk gets when it meets a dove
(b) a hawk gets when it meets a hawk
(c) a dove gets when it meets a dove
(d) It is never possible for a dove to successfully invade such a population.

ANSWER (b)