Suggested solution to HKCEE 2004 Additional Mathematics

1. Find

(*a*) $\int \cos(3x+1)dx$.

(*b*) $\int (2-x)^{2004}dx$.

(4 marks)

(*a*) $\int \cos(3x+1)dx = \frac{1}{3}\int \cos(3x+1)d(3x+1)$

$= \frac{1}{3}\sin(3x+1)+C$, where C is a constant.

(*b*) $\int (2-x)^{2004}dx = -\int (2-x)^{2004}d(2-x)$

$= -\frac{(2-x)^{2005}}{2005}$ + C, where C is a constant.

2. (*a*) Expand $(1 + 2x)^6$ in ascending powers of x up to the term x^3.

(*b*) Find the constant term in the expansion of $\left(1-\frac{1}{x}+\frac{1}{x^2}\right)(1+2x)^6$.

(4 marks)

(*a*) $(1 + 2x)^6 = 1 + 6(2x) + 15(2x)^2 + 20(2x)^3 + ...$

$= 1 + 12x + 60x^2 + 160x^3 + ...$

(*b*) $\left(1-\frac{1}{x}+\frac{1}{x^2}\right)(1+2x)^6=\left(1-\frac{1}{x}+\frac{1}{x^2}\right)(1+12x+60x^2+160x^3+\cdots)$

constant term = $1\times1 - 12 + 60 = 49$

3. The slope at any point (x, y) of a curve C is given by $\frac{dy}{dx}=3x^2+1$. If the x-intercept of C is 1, find the equation of C.

(4 marks)

$\frac{dy}{dx}=3x^2+1 \Rightarrow y=\int(3x^2+1)dx$

$y = x^3 + x + c$, where c is a constant

when $x = 1$, $y = 0 = 1 + 1 + c$, $c = -2$

$y = x^3 + x - 2$.

4.

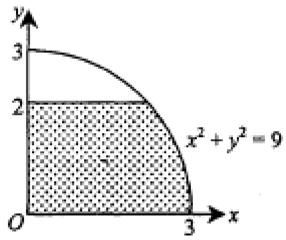

Figure 1

In Figure 1, the shaded region is bounded by the circle $x^2 + y^2 = 9$, the x-axis, the y-axis and the line $y = 2$. Find the volume of the solid generated by revolving the region about the y-axis.

(4 marks)

Volume V = $\int_0^2 \pi x^2 dy$

$=\pi\int_0^2(9-y^2)dy$

$=\pi\left(9y-\frac{1}{3}y^3\right)\Big|_0^2 = \frac{46\pi}{3}$ cubic units.

5. Find the general solution of the equation

$$\sin 3x + \sin x = \cos x.$$

(5 marks)

$2 \sin 2x \cos x = \cos x$

$\cos x = 0$ or $\sin 2x = \frac{1}{2}$

$x = 2n\pi \pm \frac{\pi}{2}$ or $x = \frac{n\pi}{2} + (-1)^n \frac{\pi}{12}$, where n is an integer.

6.

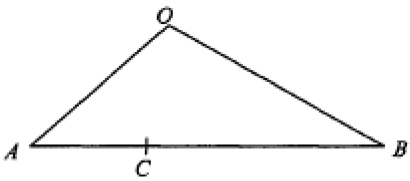

Figure 2

In Figure 2, OAB is a triangle. C is a point on AB such that $AC : CB = 1 : 2$. Let $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.

(*a*) Express $\overrightarrow{OC}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

(*b*) If $|\mathbf{a}| = 1$, $|\mathbf{b}| = 2$ and $\angle AOB = \frac{2\pi}{3}$, find $|\overrightarrow{OC}|$.

(5 marks)

(*a*) $\overrightarrow{OC} = \dfrac{2\vec{a} + \vec{b}}{3}$.

(*b*) $$|\overrightarrow{OC}|^2 = \overrightarrow{OC} \cdot \overrightarrow{OC} = \frac{2\vec{a} + \vec{b}}{3} \cdot \frac{2\vec{a} + \vec{b}}{3}$$
$$= \frac{1}{9}\left(4|\vec{a}|^2 + 4\vec{a} \cdot \vec{b} + |\vec{b}|^2\right)$$
$$= \frac{1}{9}\left(4 + 4 \times 1 \times 2 \times \cos\frac{2\pi}{3} + 2^2\right)$$
$$= \frac{4}{9}$$

7. Prove that $9^n - 1$ is divisible by 8 for all positive integers n.

(5 marks)

By induction on n. $n = 1$, $9 - 1 = 8$ which is divisible by 8. It is true for $n = 1$.
Suppose $9^k - 1 = 8m$, where k is a positive integer and m is an integer.
$9^{k+1} - 1 = 9(9^k) - 1 = 9(8m + 1) - 1$
$= 72m + 8 = 8(9m + 1)$, which is a multiple of 8.
Therefore, $9^{k+1} - 1$ is also divisible by 8 if $9^k - 1$ is divisible by 8 and k is a positive integer.
By the principle of mathematical induction, $9^n - 1$ is divisible by 8 for all positive integers n.

8. Solve the following equations:

(*a*) $|x - 3| = 1$.

(*b*) $|x - 1| = |x^2 - 4x + 3|$.

(6 marks)

(*a*) $x - 3 = 1$ or $x - 3 = -1$
$x = 4$ or 2

(*b*) $x - 1 = x^2 - 4x + 3$ or $-x + 1 = x^2 - 4x + 3$
$x^2 - 5x + 4 = 0$ or $x^2 - 3x + 2 = 0$
$(x - 1)(x - 4) = 0$ or $(x - 1)(x - 2) = 0$
$x = 1, 2$ or 4.

9.

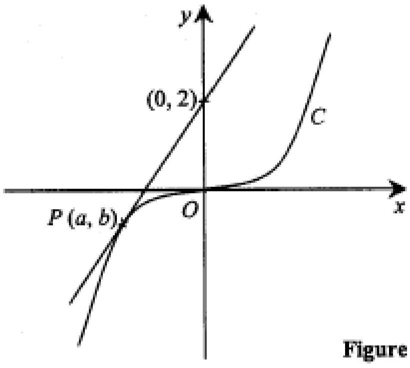

Figure 3

In Figure 3, $P(a, b)$ is a point on the curve C: $y = x^3$. The tangent to C at P passes through the point (0, 2).

(*a*) Show that $b = 3a^3 + 2$.

(*b*) Find the values of a and b.

(6 marks)

(*a*) Differentiate C with respect to x: $\frac{dy}{dx} = 3x^2$ = slope of tangent at P.

$3a^2 = \frac{b-2}{a-0} \Rightarrow 3a^3 = b - 2$

$b = 3a^3 + 2$

(*b*) $\because P(a, b)$ lies on the curve, $b = a^3$

$a^3 = 3a^3 + 2$

$a^3 = -1 \Rightarrow a = -1$

$b = -1$

10. Let O be the origin and A be the point (3, 4). P is a variable point such that the area of ΔOPA is always equal to 2.

Show that the locus of P is a pair of parallel lines.

Find the distance between these two lines. (6 marks)

Let $P(x, y)$, $\frac{1}{2}\left\|\begin{matrix} 0 & 0 \\ x & y \\ 3 & 4 \\ 0 & 0 \end{matrix}\right\| = 2$

$|4x - 3y| = 4$

$4x - 3y - 4 = 0$ or $4x - 3y + 4 = 0$

So the locus is a pair of parallel lines.

The distance is: $\left|\frac{4-(-4)}{\sqrt{4^2+(-3)^2}}\right| = \frac{8}{5}$.

11.

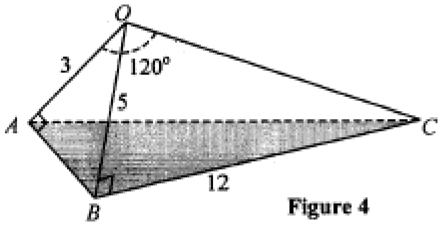

Figure 4

In Figure 4, $OABC$ is a pyramid such that $OA = 3$, $OB = 5$, $BC = 12$, $\angle AOC = 120°$ and $\angle OAB = \angle OBC = 90°$.

(*a*) Find AC.

(*b*) A student says that the angle between the planes OBC and ABC can be represented by $\angle OBA$. Determine whether the student is correct or not. (6 marks)

(a) In ΔOBC, $OC = \sqrt{5^2 + 12^2} = 13$
In ΔOAC, $AC^2 = 3^2 + 13^2 - 2(3)(13)\cos 120° = 217$
$AC = \sqrt{217}$

(b) In ΔOAB, $AB = \sqrt{5^2 - 3^2} = 4$
In ΔABC, $AB^2 + BC^2 = 4^2 + 12^2 = 160 < 217 = AC^2$
$\therefore \angle ABC \neq 90°$
The angle between the planes OBC and ABC is not $\angle OBA$, the student is incorrect.

12.

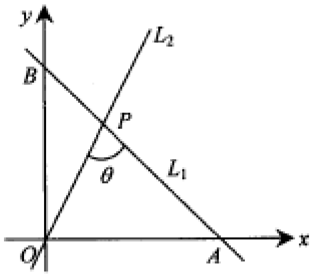

Figure 5

Figure 5 shows two lines L_1: $y = -x + c$ and L_2: $y = 2x$, where $c > 0$. The two lines intersect at point P.

(a) Let θ be the acute angle between L_1 and L_2. Find tan θ.

(b) L_1 intersects the x- and y-axes at the points A and B respectively. Find $AP : PB$.

(7 marks)

(a) $\tan\theta = \left|\dfrac{2-(-1)}{1+2(-1)}\right| = 3$

(b) $A = (c, 0)$, $B = (0, c)$.

Solving L_1: $y = -x + c$ and L_2: $y = 2x$; $y = -x + c = 2x$; $x = \dfrac{c}{3}$

Let $AP : PB = r : 1$, then by the section formula, $\dfrac{r\times 0 + 1\times c}{1+r} = \dfrac{c}{3}$

$\Rightarrow r = 2$; $AP : PB = 2 : 1$

13.

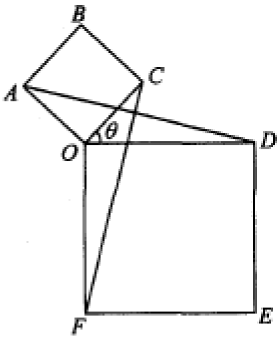

Figure 6

In Figure 6, $OABC$ and $ODEF$ are two squares such that $OA = 1$, $OF = 2$ and $\angle COD = \theta$, where $0° < \theta < 90°$. Let $\overrightarrow{OD} = 2\mathbf{i}$ and $\overrightarrow{OF} = -2\mathbf{j}$, where $\mathbf{i}$ and $\mathbf{j}$ are two perpendicular unit vectors.

(a) (i) Express $\overrightarrow{OC}$ and $\overrightarrow{OA}$ in terms of θ, $\mathbf{i}$ and $\mathbf{j}$.

(ii) Show that $\overrightarrow{AD} = (2 + \sin\theta)\,\mathbf{i} - \cos\theta\,\mathbf{j}$.

(4 marks)

(b) Show that $\overrightarrow{AD}$ is always perpendicular to $\overrightarrow{FC}$.

(4 marks)

(c) Find the value(s) of θ such that points B, C and E are collinear. Give your answer(s) correct to the nearest degree.

(4 marks)

(a) (i) $\overrightarrow{OC} = \cos\theta\,\vec{i} + \sin\theta\,\vec{j}$

$$\overrightarrow{OA} = \cos(90° + \theta)\vec{i} + \sin(90° + \theta)\vec{j}$$
$$= -\sin\theta\vec{i} + \cos\theta\vec{j}$$

(ii) $\overrightarrow{AD} = \overrightarrow{OD} - \overrightarrow{OA}$
$$= 2\vec{i} - (-\sin\theta\vec{i} + \cos\theta\vec{j})$$
$$= (2 + \sin\theta)\vec{i} - \cos\theta\vec{j}.$$

(b) $\overrightarrow{FC} = \overrightarrow{OC} - \overrightarrow{OF} = \cos\theta\vec{i} + \sin\theta\vec{j} - (-2\vec{j}) = \cos\theta\vec{i} + (\sin\theta + 2)\vec{j}$

$\overrightarrow{AD}\cdot\overrightarrow{FC} = [(2 + \sin\theta)\vec{i} - \cos\theta\ \vec{j}]\cdot[\cos\theta\vec{i} + (\sin\theta + 2)\vec{j}]$

$= (2 + \sin\theta)\cos\theta - \cos\theta(\sin\theta + 2) = 0$

$\overrightarrow{AD} \perp \overrightarrow{FC}$

(c) $\overrightarrow{CB} = \overrightarrow{OA} = -\sin\theta\vec{i} + \cos\theta\vec{j}$

$\overrightarrow{CE} = \overrightarrow{OC} - \overrightarrow{OE} = \cos\theta\vec{i} + \sin\theta\vec{j} - (2\vec{i} - 2\vec{j})$

$= (\cos\theta - 2)\vec{i} + (\sin\theta + 2)\vec{j}$

If B, C and E are collinear, then $\overrightarrow{CE} = k\overrightarrow{CB}$

$(\cos\theta - 2)\vec{i} + (\sin\theta + 2)\vec{j} = k(-\sin\theta\vec{i} + \cos\theta\vec{j})$

$\cos\theta - 2 = -k\sin\theta$ ..... (1)

$\sin\theta + 2 = k\cos\theta$ .... (2)

(1)÷(2) $\dfrac{\cos\theta - 2}{\sin\theta + 2} = -\dfrac{\sin\theta}{\cos\theta}$

$\cos^2\theta - 2\cos\theta = -\sin^2\theta - 2\sin\theta$

$1 = 2\cos\theta - 2\sin\theta$

$1 = 2\sqrt{2}\left(\cos\theta\cos 45^{\circ} - \sin\theta\sin 45^{\circ}\right)$

$\cos(\theta + 45°) = \dfrac{1}{2\sqrt{2}}$

$\theta + 45° = 69°$

$\theta = 24°$

14. C_1 and C_2 are the circles $x^2 + y^2 = 36$ and $x^2 + y^2 - 10x + 16 = 0$ respectively.

(a) (i) Show that, for all values of θ, the variable point $P(6\cos\theta, 6\sin\theta)$ always lies on C_1.

(ii) Find, in terms of θ, the equation of the tangent to C_1 at $P(6\cos\theta, 6\sin\theta)$.(3marks)

(b)

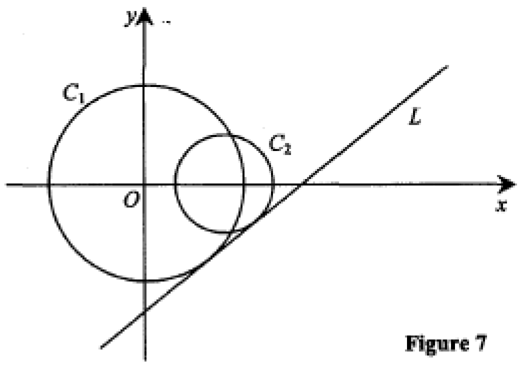

Figure 7

Let L be the common tangent to C_1 and C_2 with a positive slope (see Figure 7).

(i) Using (a), or otherwise, find the equation of L.

(ii) It is known that C_1 and C_2 intersect at two distinct points Q and R. A circle C_3, passing through Q and R, is bisected by L. Find the equation of C_3. (9 marks)

(a) (i) sub $P(6\cos\theta, 6\sin\theta)$ into C_1.

LHS $= (6\cos\theta)^2 + (6\sin\theta)^2 = 36 =$ RHS

so the point always lies on C_1.

(*ii*) Equation of tangent: $6\cos\theta\, x + 6\sin\theta\, y = 36$

$\Rightarrow x\cos\theta + y\sin\theta = 6$

(*b*) (*i*) C_2: $x^2 + y^2 - 10x + 16 = 0$, centre (5, 0), radius = $\sqrt{5^2 - 16} = 3$

since $x\cos\theta + y\sin\theta = 6$ is a common tangent to C_1, C_2.

so the distance from centre (5, 0) to the line = radius

$$\left|\frac{5\cos\theta - 6}{\sqrt{\cos^2\theta + \sin^2\theta}}\right| = 3$$

$5\cos\theta - 6 = 3$ or $5\cos\theta - 6 = -3$

$\cos\theta = \frac{9}{5}$ (rejected) or $\frac{3}{5}$

$\sin\theta = \frac{4}{5}$ or $-\frac{4}{5}$

when $\sin\theta = \frac{4}{5}$, the slope of $L = -\frac{\cos\theta}{\sin\theta} < 0$, contradict the positive slope.

when $\sin\theta = -\frac{4}{5}$, slope of $L = -\frac{\cos\theta}{\sin\theta} = -\frac{\frac{3}{5}}{-\frac{4}{5}} = \frac{3}{4}$

L: $\frac{3}{5}x - \frac{4}{5}y = 6$

$3x - 4y = 30$

(*ii*) First we find the radical axis of C_1 and C_2: $C_1 - C_2$

$10x - 16 = 36$

$5x = 26$

Next, we find the family of circles through the intersections Q and R.

$x^2 + y^2 - 36 + k(5x - 26) = 0$

$x^2 + y^2 + 5kx - (36 + 26k) = 0$

centre = $\left(-\frac{5k}{2}, 0\right)$

since L bisects C_3, so the centre lie on L.

$3\left(-\frac{5k}{2}\right) - 4(0) = 30$

$k = -4$

C_3: $x^2 + y^2 - 20x + 68 = 0$

15. Given two curves C_1: $y = f(x)$, where $f(x)$ is a quadratic function, and

$$C_2: y = -\frac{1}{5}x^2 - \left(\frac{h-20}{10}\right)x + h.$$

C_1 has the vertex (4, 9) and passes through the point (10, 0).

(*a*) Show that $f(x) = -\frac{1}{4}x^2 + 2x + 5$. (3 marks)

(*b*) (*i*) Show that C_2 also passes through the point (10, 0).

(*ii*) If C_1 and C_2 meet at two points, find, in terms of h, the x-coordinate of the point other than (10, 0). (5 marks)

(*c*) Figure 8 shows a fountain. A vertical water pipe OP of height 15 units is installed on the horizontal ground. Two streams of water are ejected continuously from two small holes D_1 and D_2 in the pipe, with D_2 above D_1. The two streams of water lie in the same vertical plane. A rectangular coordinate system is introduced in this plane, with O as the origin and OP on the

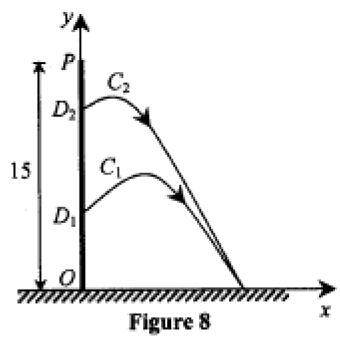

Figure 8

positive y-axis. The fountain is designed such that the stream of water ejected from D_1 lies on the curve C_1, and that ejected from D_2 lies on C_2.

(i) Find OD_1.

(ii) If the two streams of water do not cross each other in the air before meeting at the same point on the ground, find the range of possible values of OD_2.

(4 marks)

(a) $f(x) = a(x-4)^2 + 9$

$0 = a(10-4)^2 + 9$

$a = -\frac{1}{4}$

$f(x) = -\frac{1}{4}(x-4)^2 + 9$

$= -\frac{1}{4}x^2 + 2x + 5$.

(b) (i) Put (10, 0) into C_2: RHS $= -\frac{1}{5}(10)^2 - \left(\frac{h-20}{10}\right)(10) + h = -20 - h + 20 + h = 0 =$ LHS

so C_2 also passes through the point (10, 0).

(ii) $C_1 = C_2$: $y = -\frac{1}{4}x^2 + 2x + 5 = -\frac{1}{5}x^2 - \left(\frac{h-20}{10}\right)x + h$

$-5x^2 + 40x + 100 = -4x^2 - 2(h-20)x + 20h$

$x^2 - 2hx + 20(h-5) = 0$

$(x-10)(x-2h+10) = 0$

so the x-coordinate of the point other than (10, 0) is $2h - 10$.

(c) (i) C_1: $y = -\frac{1}{4}x^2 + 2x + 5$; when $x = 0$, $y = 5$

$OD_1 = 5$

(ii) they do not cross $\Rightarrow$ x-coordinate of the other intersection ≤ 0 or ≥ 10

$2h - 10 \leq 0$ or $2h - 10 \geq 10$

$h \leq 5$ or $h \geq 10$

C_2: $y = -\frac{1}{5}x^2 - \left(\frac{h-20}{10}\right)x + h$; when $x = 0$, $y = h$

$0 \leq OD_2 \leq 5$ or $15 \geq OD_2 \geq 10$

16.

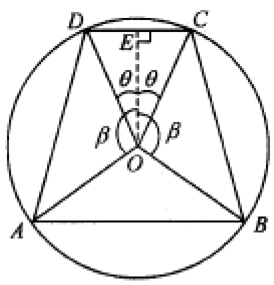

Figure 9

In Figure 9, $ABCD$ is a quadrilateral inscribed in a circle centred at O and with radius r, such that $AB//DC$ and O lies inside the quadrilateral. Let $\angle COD = 2\theta$ and reflex $\angle AOB = 2\beta$, where $0 < \theta < \frac{\pi}{2} < \beta < \pi$. Point E denotes the foot of perpendicular from O to DC. Let S be the area of $ABCD$.

(a) Show that $S = \frac{r^2}{2}[\sin 2\theta - \sin 2\beta + 2\sin(\beta - \theta)]$. (3 marks)

(b) Suppose β is fixed. Let S_β be the greatest value of S as θ varies.

Show that $S_\beta = 2r^2 \sin^3(\frac{2\beta}{3})$ and the corresponding value of θ is $\frac{\beta}{3}$.

[Hint: You may use the identity $\sin 3\alpha = 3\sin\alpha - 4\sin^3\alpha$.] (6 marks)

(*c*) A student says:

> Among all possible values of β, the quadrilateral *ABCD* becomes a square when S_β in (*b*) attains its greatest value.

Determine whether the student is correct or not. (3 marks)

(*a*) S = area of ΔOCD + 2 area of ΔOBC + area of ΔOAB

$$= \frac{r^2}{2}\sin 2\theta + 2[\frac{r^2}{2}\sin(\beta-\theta)] + \frac{r^2}{2}\sin(2\pi - 2\beta)$$

$$= \frac{r^2}{2}[\sin 2\theta - \sin 2\beta + 2\sin(\beta-\theta)]$$

(*b*) $$\frac{dS}{d\theta} = \frac{r^2}{2}[2\cos 2\theta - 2\cos(\beta-\theta)] = r^2[\cos 2\theta - \cos(\beta-\theta)]$$

Let $\frac{dS}{d\theta} = 0$; $\cos 2\theta - \cos(\beta-\theta) = 0$

$$-2\sin\frac{\theta+\beta}{2}\sin\frac{3\theta-\beta}{2} = 0$$

$\theta + \beta = 0$ (rejected) or $3\theta - \beta = 0$

$$\theta = \frac{\beta}{3}$$

$$\frac{d^2S}{d\theta^2} = r^2[-2\sin 2\theta + \sin(\beta-\theta)]$$

$$\left.\frac{d^2S}{d\theta^2}\right|_{\theta=\frac{\beta}{3}} = r^2\left[-2\sin\frac{2\beta}{3} + \sin\frac{2\beta}{3}\right] < 0$$

$\therefore$ when $\theta = \frac{\beta}{3}$, S is a maximum

$$\text{maximum } S = \frac{r^2}{2}\left[\sin\frac{2\beta}{3} - \sin 2\beta + 2\sin\frac{2\beta}{3}\right]$$

$$= \frac{r^2}{2}\left[3\sin\frac{2\beta}{3} - 3\sin\frac{2\beta}{3} + 4\sin^3\frac{2\beta}{3}\right]$$

$$= 2r^2\sin^3(\frac{2\beta}{3})$$

(*c*) $\because S_\beta = 2r^2\sin^3(\frac{2\beta}{3}) \le 2r^2 \times 1$

when $\sin^3(\frac{2\beta}{3}) = 1$, S_β is a maximum. $\frac{2\beta}{3} = \frac{\pi}{2}$

$$\beta = \frac{3\pi}{4}$$

when $\beta = \frac{3\pi}{4}$, $\angle AOB = 2\pi - 2\beta = \frac{\pi}{2}$; $\theta = \frac{\beta}{3} = \frac{\pi}{4}$; $\angle COD = 2\theta = \frac{\pi}{2}$.

Similarly, $\angle AOD = \angle BOC = \beta - \theta = \frac{\pi}{2}$, so *ABCD* is a square when S_β is a maximum.

The student is correct.

17. (*a*) Let $y = (x - \pi)\sin x + \cos x$.

(*i*) Show that $\dfrac{dy}{dx} = (x - \pi)\cos x$.

Hence find $\int (x-\pi)\cos x\,dx$.

(*ii*) Figure 10 shows the graph of $y = (x - \pi)\cos x$ for $0 \le x \le \dfrac{3\pi}{2}$.

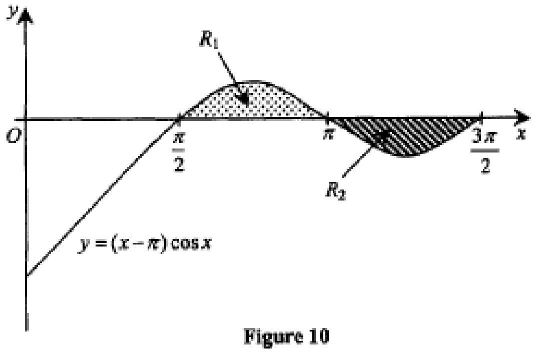

Figure 10

(1) Find the areas of the two shaded regions R_1 and R_2 as shown in Figure 10.

(2) Find $\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} (x-\pi)\cos x\,dx$.

(7 marks)

(*b*)

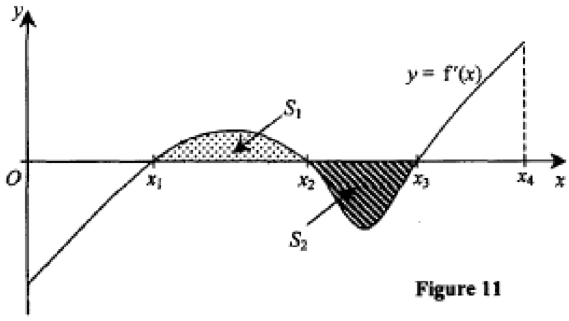

Figure 11

Let $f(x)$ be a continuous function. Figure 11 shows a sketch of the graph of $y = f\,'(x)$ for $0 \le x \le x_4$. It is known that the areas of the shaded regions S_1 and S_2 as shown in Figure 11 are equal.

(*i*) Show that $f(x_1) = f(x_3)$.

(*ii*) Furthermore, $f(0) = f(x_4) = 0$ and $f(x) \ne 0$ for $0 < x < x_4$. In Figure 12, draw a sketch of the graph of $y = f(x)$ for $0 \le x \le x_4$.

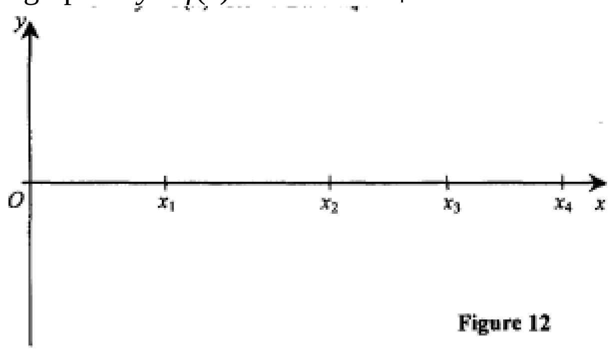

Figure 12

(5 marks)

(*a*) (*i*) $\dfrac{dy}{dx} = (x - \pi)\cos x + \sin x - \sin x = (x - \pi)\cos x$.

$\int (x-\pi)\cos x\,dx = \int dy = y + c = (x - \pi)\sin x + \cos x + c$, where c is a constant.

(*ii*) (1) $R_1 = \int_{\frac{\pi}{2}}^{\pi} (x-\pi)\cos x dx$

$= [(x-\pi)\sin x + \cos x]_{\frac{\pi}{2}}^{\pi}$

$= [(\pi - \pi)\sin\pi + \cos\pi] - [(\frac{\pi}{2} - \pi)\sin\frac{\pi}{2} + \cos\frac{\pi}{2}]$

$= -1 + \frac{\pi}{2}$

$R_2 = |\int_{\pi}^{\frac{3\pi}{2}} (x-\pi)\cos x dx|$

$= |[(x-\pi)\sin x + \cos x]_{\pi}^{\frac{3\pi}{2}}|$

$= |[(\frac{3\pi}{2} - \pi)\sin\frac{3\pi}{2} + \cos\frac{3\pi}{2}] - [(\pi - \pi)\sin\pi + \cos\pi]|$

$= |-\frac{\pi}{2} + 1| = -1 + \frac{\pi}{2}$

(2) $\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} (x-\pi)\cos x dx = [(x-\pi)\sin x + \cos x]_{\frac{\pi}{2}}^{\frac{3\pi}{2}}$

$= [(\frac{3\pi}{2} - \pi)\sin\frac{3\pi}{2} + \cos\frac{3\pi}{2}] - [(\frac{\pi}{2} - \pi)\sin\frac{\pi}{2} + \cos\frac{\pi}{2}]$

$= -\frac{\pi}{2} + \frac{\pi}{2} = 0$

(*b*) (*i*) $\int_{x_1}^{x_2} f'(x)dx = -\int_{x_2}^{x_3} f'(x)dx$

$f(x_2) - f(x_1) = -[f(x_3) - f(x_2)]$

$f(x_1) = f(x_3)$

(*ii*) $f'(x)$ changes from -ve to +ve at x_1, so $f(x_1)$ is a relative minimum

$f'(x)$ changes from +ve to -ve at x_2, so $f(x_2)$ is a relative maximum

$f'(x)$ changes from -ve to +ve at x_3, so $f(x_3)$ is a relative minimum

$f(0) = f(x_4) = 0$ and $f(x) \neq 0$ for $0 < x < x_4$; from the graph, $f'(x) < 0$ at $x = 0$

so $f(x)$ is decreasing at $x = 0 \Rightarrow f(x) < 0$ for all x: $0 < x < x_4$

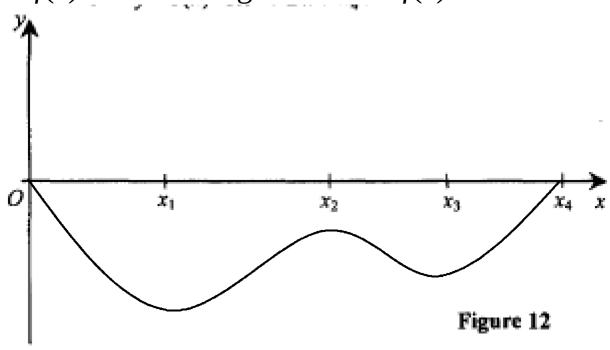

Figure 12