CHAPTER 16 Bargaining

TEACHING SUGGESTIONS

Newspapers often print articles of advice on bargaining for buyers of houses, autos, and so on. We found a particularly good one that we mention in the text: Andrée Brooks, "Honing Haggling Skills" (*New York Times*, December 5, 1993). It mentions (not in technical language) several points that relate to the concepts in this chapter: Best Alternative to a Negotiated Agreement (BATNA), patience, signaling, screening, using mandated agents for credible commitment, and how multiple dimensions of bargaining facilitate agreement by exploiting differences in the parties' relative valuations of the dimensions. You can circulate such an article to your class a day in advance and start with a discussion that elicits these concepts and builds them into the game-theoretic framework of the chapter.

You can also look for other popular press publications or websites and interpret their content using the bargaining terminology found in this chapter. Consider the book *Getting to Yes* by R. Fisher and W. Ury (Baltimore: Penguin, 1981) or an Internet site that addresses bargaining in the process of purchasing a car: http://www.edmunds.com/edweb/burkleon/contents.html.

When you cover the alternating-offer models of bargaining, you may find it easier for your students to follow the progression of the game if they have a simple table showing the offers and amounts going to each player in each round. If there is gradual decay in the total value available, we say that the total value drops by x_1 after the first offer is rejected and further rejections lead to drops of x_2, x_3, . . . (all drops are the same size). For a game where \$1 is being split and the value drops by 10¢ after each rejected offer, the last possible offer would occur when only x_{10} was left. This bargaining situation can be illustrated with the following table:

Bid	Probability of win	Profit	Expected profit = (probability of win) x profit
0.20	0.4	0.40	0.160
0.25	0.5	0.35	0.175
0.30	0.6	0.30	0.180
0.35	0.7	0.25	0.175
0.40	0.8	0.20	0.160

The final row of the table shows the division that can be anticipated as the Nash equilibrium of the bargaining game.

For a bargaining game with impatience, let A regard \$1 immediately as equivalent to $\$(1 + r) > \$1$ one offer later and B regard \$1 immediately as equivalent to $\$(1 + s) > \$1$ one offer later. The values r and s measure impatience. Let $a = 1/(1 + r)$ and $b = 1/(1 + s)$. Then in a game of dividing \$1 that could go, say, 100 rounds, the table of offers and payoffs is as follows:

Round	Offer by	A gets	B gets
100	B	0	1
99	A	$1 - b$	b
98	B	$a(1 - b)$	$1 - a + ab$
97	A	$1 - b + ab - ab^2$	$b(1 - a + ab)$
96	B	$a(1 - b + ab - ab^2)$	$1 - a + ab - a^2b + a^2b^2$

Whenever A makes an offer, she will ask for

$$\begin{aligned} x &= 1 - b + ab - ab^2 + a^2b^2 - a^2b^3 + \cdots \\ &= (1 - b)(1 + ab + a^2b^2 + a^3b^3 + \cdots \end{aligned}$$

The limit, with an indefinite repetition of offers, yield

$$\begin{aligned} x &= (1 - b)/(1 - ab) \\ &= (s + rs)/(r + s + rs) \end{aligned}$$

Similarly, whenever B makes an offer, she will ask for

$$y = (r + rs)/(r + s + rs).$$

GAME PLAYING IN CLASS

You have probably already played some simple bargaining games in the very first session or two of your class—the take-it-or-leave-it-offer game, the centipede game, and so on. Now you can simply remind the class or play some of the games again, compare their behavior after learning game theory with their behavior before, and discuss this as a matter of interest in its own right.

GAME 1 Bargaining over $10

This is a collection of short bargaining situations that students can be asked to consider for homework or in class. Variants on these situations are easily created. Instructions: The minimum offer in all of these situations is 50¢. Each of you will be paired against the instructor when determining points. The instructor will respond to your offers using the Nash equilibrium strategy. If your offer is declined in any of the games, it will be assumed that the Nash equilibrium outcome is obtained at the next offer.

Situation 1

You and a partner are given $10 to split between you. You make the first move and offer your partner part of the $10. If she accepts, the bargain is complete and you each get an amount determined by your offer. If she declines, she gets a turn and makes you an offer. Again, the bargain is complete if you accept and the split depends on her offer. This process continues, infinitely if necessary, until one of you makes an offer that the other accepts.

If you are making the first offer in this bargaining game, what do you offer? ________________

Situation 2

You and a partner are given $10 to split between you. You will make the first move and offer your partner a portion of the $10. If she accepts, the bargain is complete and you each get an amount determined by your offer. If she declines, the $10 shrinks to $5 and she then gets a turn to make you an offer. Again, the bargain is complete if you accept and the division is made according to the terms of her offer. If you decline her offer, the $5 shrinks to zero, the game ends, and both of you receive nothing.

If you are making the first offer in this bargaining game, what do you offer? ________________

Situation 3

You and a partner are given $10 to split between you. You will make the first move and offer your partner a portion of the $10. If she accepts, the bargain is complete and you each get an amount determined by your offer. If she declines, the $10 shrinks to $6.67 and she then gets a turn to make you an offer. Again, the bargain is complete if you accept, and the division is made according to the terms of her offer. If you decline her offer, the sum shrinks to $3.33 and you get to make an offer. If your partner accepts, the division is made according to your offer, but if she declines, the sum available shrinks to zero and you each get nothing.

If you are making the first offer in this bargaining game, what do you offer? ________________

Situation 4

You are given $10 and told that you will be allowed to make one offer to split the sum with a partner who will either accept or reject. Regardless of her answer, the game ends after her move. If she accepts your offer, the $10 is split accordingly. If she rejects it, you both get nothing.

What amount do you offer your partner in this game? ______________________

Situation 5

You are given $10 and told that you will be allowed to make only one offer to split the sum with a partner. Your partner must either accept or reject. Regardless of her answer, the game ends after her move. If she accepts your offer, the $10 is split accordingly. If she rejects your offer, you get the full $10.

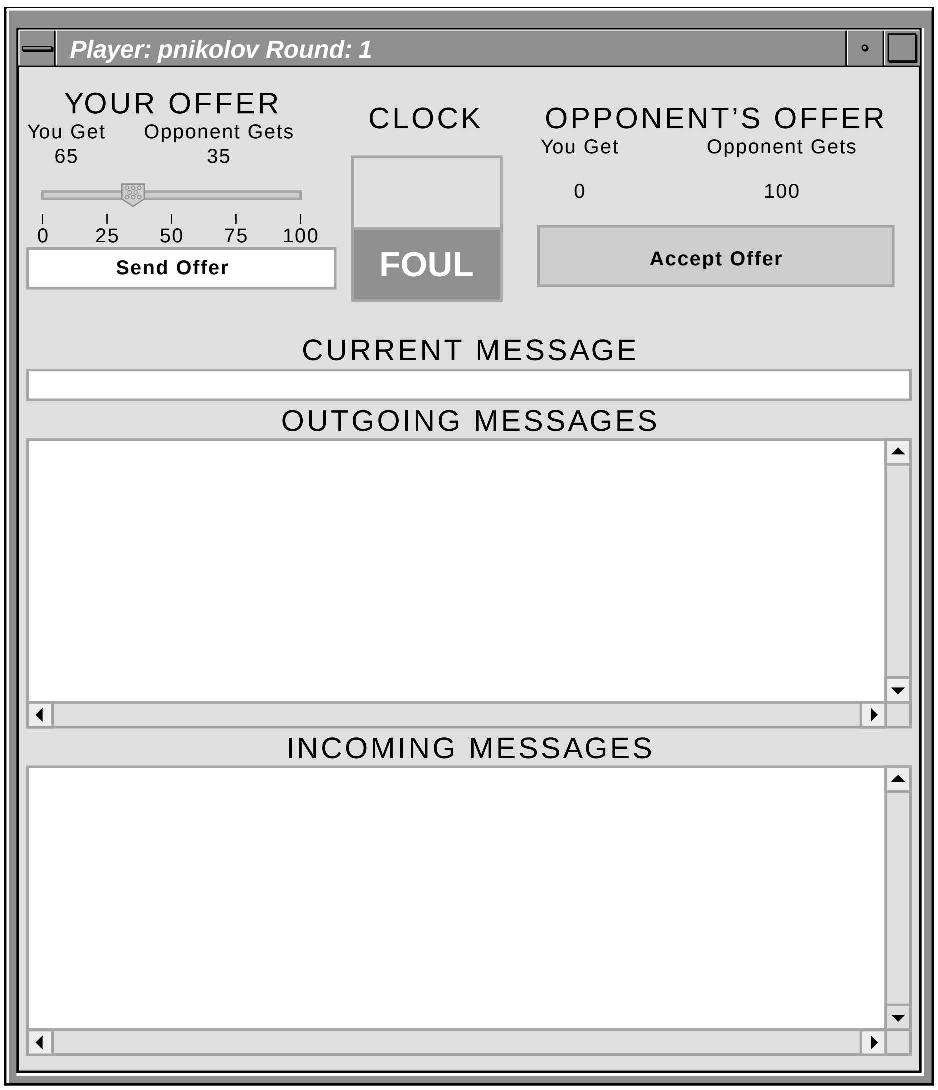
What amount do you offer your partner in this game? ______________________

The results from these bargaining games can be tabulated and given to students for discussion in class. The first three situations allow discussion of the need to look ahead to later rounds to consider the best possible first offer. (Variants with impatience can make these more interesting.) In the last two games, the Nash equilibrium first offer is the same, although many students do not offer the minimum (here 50¢) in Situation 4 because of what they identify as fear of rejection. This provides an additional opportunity to consider the validity of the Nash equilibrium concept.

COMPUTER GAME: Bargaining in Pairs

In this game the students are matched in pairs and attempt to agree to a division of the total of 100 points that are on the table between them; see the instructions below for details. This game is best played just *before* you do Chapter 16 in class. With the direct experience of making offers and counteroffers, the cost of delay, and so on, the students can better appreciate the theory.

Again you can try variants: (1) End the game at a precise known time instead of the random one we used. (2) Restrict the kind of communication that is allowed. (3) Fix the pairing for several rounds to see if more cooperation emerges.



INSTRUCTIONS FOR GAME 1

The game will be played in scheduled discussion sections. Be on time; once the server starts the game we cannot accommodate latecomers. To play the game you will have to log in using your e-mail account.

Connecting to Play the Game

1. In the cluster, choose a terminal. You should see a box where you are to enter your computer user ID. If you don't, press Return and it should appear. Type in your user ID and press Return. A new box for your password will appear; type this in carefully (it will not show on the screen as you type) and press Return.
2. If you have any system messages, choose q from the menu offered (ynq).
3. You need a window labeled "Terminal" with a prompt such as "beam.princeton.edu%."

   If any other windows appear, close them by double-clicking on the drawer symbol in the top left corner. Now observe a strip with several icons at the bottom of the screen. Just above the scratchpad-and-pencil icon there is an arrow. Click on the arrow. A window with several choices will appear. Click on the choice "Terminal." This will get you the terminal window.

Playing the Game

4. When all players have connected, the game window will appear; a sample image is shown at the end of these instructions. A message box will also appear, telling you that the game will start in 10 seconds. The instructions for playing the game are given below; read them carefully in advance and bring this with you.
5. The game consists of a number of rounds. We will play one round for practice and then several (usually 10) rounds for real.
6. Keep the game window in the foreground. Do not minimize it. Do not open any other windows. Do not check your e-mail, and do not use the terminal for any other purpose during idle moments. With so many simultaneous users, the system is sensitive and may crash. If you cause a crash, you will get zero points.
7. When the game is over, you must log out properly or someone else might use your account. When the server ends the game, the game window will disappear and the strip of icons at the bottom of the screen will reappear. To log out, click on the button marked "Exit." This will produce a window of options. Click on the button marked "Continue Logout."

INSTRUCTIONS FOR GAME 2

1. We will play several rounds. In each round, different randomly matched pairs of players will bargain over a total of 100 points. Your score for the game will be the average of the points you get in all of your rounds.
2. Each round will last no less than 3 minutes and no more than 5 minutes; the duration is determined randomly at the beginning of each round and not known to any player. A warning screen will pop up for 2 seconds after the time of the round has passed 3 minutes.
3. If a pair reaches an agreement within the duration of their round, their 100 points are divided between them as they agreed. If time runs out before a pair reaches agreement, both get zero for that round. If one player secures an agreement that gives him or her 51 or more points, then he or she will get an extra 10 points (from a central kitty, not from the other player).
4. You can make your opponent an offer by dragging the scale bar in the window labeled YOUR OFFER, located in the upper left-hand corner of the screen in the box, with the left mouse button. The number on the scale bar shows how many points you are offering to the *other* players; the numbers in the window above the slider bar show how much you will get and how much another will get if he or she accepts your offer. After you change the offer by moving the slider, the "Send Offer" button will turn yellow, signaling to you that the offer has not yet been sent. To send your offer to the other, click on the "Send Offer" button with the left mouse button.

   Important: If you do not click the "Send Offer" button, your offer will change on your screen but will not change on your opponent's screen or in the game server's records. In such a situation your opponent can still accept your previous offer if he or she so desires. So be sure to click the "Send Offer" button as soon as you have make a first firm offer and whenever you make a new offer. When you have clicked the "Send Offer" button, it will turn green.
5. In the upper right-hand corner of your screen is a box labeled "Opponent's Offer." This is what your opponent is currently offering you. You can accept this current offer and only this current offer by clicking the left mouse button on the "Accept Offer" button. This ends the round for your pair. When your opponent makes an offer and sends it to you, your "Accept Offer" button will turn yellow.

   Important: Both when sending an offer and when deciding whether to accept an offer, be careful about the content of the offer—how many points you get and how many your opponent gets. People have lost points by confusing the two numbers.
6. In the top center is a clock, showing the elapsed time for this round. There is also a red button labeled "Foul" whose purpose will be explained soon.
7. You can communicate with your opponent. To send a message, move the cursor to the window in the lower middle portion of the screen labeled CURRENT

MESSAGE and click the left mouse button. Then type your message. Messages are sent line by line; there is no word wrapping. As you complete each line of your message, press the "Return" key to send that line. Your opponent sees it on his or her INCOMING MESSAGES display, and you see it on your OUTGOING MESSAGES display. You can reread long messages by scrolling these windows.

8. The messages have no direct effect on the offers or the scores. An agreement reached in the message window is not recorded by the game server. Only offers that are sent and accepted as described in points 4 and 5 above are official.
9. The purpose of communication is to convey any ancillary information that you think may cause your opponent to accept your offer. An example may be: "I am taking this course pass-fail and don't really care about my score. So I will stick out for 95. You had better accept your 5; else you will get 0."
10. The matchings are random and differ from one round to the next; you do not know who your current opponent is. You can of course identify yourself to your opponent by revealing your name or your location in the cluster in one of your messages. But we advise you not to do so: as long as you are anonymous, your opponent cannot threaten you.
11. Even when you know who your opponent is, threats against his or her person or property are not allowed. Thus, "I will refuse to help you with problem sets in the future" is a permissible threat; "I will trash your room" is not permitted.
12. If you receive a threat against your person or property (but not merely because you are upset by something the opponent says), you can call a foul by clicking on the "Foul" button immediately below the clock. This is an option for rare and extreme situations, not to be invoked lightly. To give you time to cool down, a dialog box will pop up asking you if you really want to call the foul. If you click yes, then this will immediately end the round for your pair, and the identities of the players and the offending message will be stored on a file and read later by the game referee. If your foul call is judged valid, you will get 100 points and the person who sent the impermissible threat will get zero. But if it is judged that you called a foul without good reason, you will get 0 and the opponent will get 100. The referee's decision will be final.
13. We will play one practice round, to let you become familiar with the various windows and their operation. The results of this round will not count in your score. After the practice round is complete, we will play several (usually 10) rounds for real.

ANSWERS TO EXERCISES FOR CHAPTER 16

1. The businessman's BATNA is probably quite low; holding on to the domain name may be worth very little to him. Owning the domain name, which makes it easier for people to find the Alta Vista site, is important to Compaq, so its BATNA is also, apparently, millions below the value of a negotiated agreement. Compaq is probably quite impatient to make a deal. There are many firms in the portal market, but many of them may not survive. Winners and losers may be determined quickly, perhaps within a year or two. If some people have trouble finding the Alta Vista site during that time, the firm may be at a disadvantage during a crucial period of its industry. (Of course, if Alta Vista collapses, the domain name would cease to be worth much, so the businessman may also be impatient.) We now know that Compaq was trying to sell Alta Vista to Time Warner. After those talks collapsed, Compaq decided to establish Alta Vista as an independent, partly publicly held company. Compaq may well have wanted to acquire the altavista.com domain name before trying to sell Alta Vista. If so, this would increase Compaq's impatience. The businessman's bargaining strength may have been greater than Compaq's due to his rights over the domain name and his relative level of patience.

   Part of any cooperative outcome is that all possible mutual gains should be exploited (and exploited as quickly as possible). In this situation, a mutual gain is created by getting the www.altavista.com domain name into Compaq's hands as soon as possible. The name is certainly worth more when it is in Compaq's hands than when it is in the businessman's hands. Since at least some time passed before the domain name was sold, it is apparent that the players did not achieve the cooperative outcome. It would seem that disputes over how to divide the surplus between them prevented the two parties from reaching immediate agreement.
2. We start by assuming that the total value v exactly vanishes after n rounds, so $nc = v$. At the start of round n (after $n - 1$ refusals), the remaining surplus is $v - (n - 1)c$. Given $nc = v$, this implies that $v - (n - 1)c = c$.

   Assume that A makes the first offer; who makes the last offer depends on whether n is an odd or even number.

   When n is even, B makes the offer in the last round. B must then propose a division in which A gets his BATNA of a, which leaves B with $c - a$. Assume (for now) that doing this is worthwhile for B, so $c - a > b$. (We consider below the case in which this is not true.) In round $n - 1$, the value remaining is $v - (n - 2)c = 2c$. A proposes and must leave B the larger of $c - a$ or b. We've already assumed that $c - a > b$. So A then keeps $v - (n-2)c - (c-a) = 2c - (c-a) = c + a$. In round $n - 2$, the value remaining is $v - (n-3)c = 3c$. B proposes and must leave A the value $c + a$. B keeps $3c - (c + a) = 2c - a$. In round $n - 3$, the value remaining is $v - (n-4)c = 4c$. A proposes and must leave B the value $2c - a$. A keeps $4c - (2c - a) = 2c + a$. Result (for $nc = v$ and n even): A gets $(n/2)c + a = (v/2) + a$ and B gets $(n/2)c - a = (v/2) - a$.

Now assume n is an odd number, so that A makes the offer in the last round (where the remaining value again equals c). A must propose a division in which B gets his BATNA of b. Doing so leaves A with $c - b$ (assume that $c - b > a$). In round $n - 1$, B offers A the value $c - b$ and keeps $v - (n-2)c - (c-b) = 2c - (c - b) = c + b$. In round $n - 2$, A offers B $c + b$ and keeps $3c - (c + b) = 2c - b$. Result (for $nc = v$ and n odd): A gets $[(n+1)/2]c - b = [(n+1)/n](v/2) - b$ and B gets $[(n-1)/2]c + b = [(n-1)/n](v/2) + b$.

Now, return to assuming n is even, but assume that $2c - a > b > c - a$. In (the last) round n, B would get to make the offer and would let A get a, which would leave B with $c - a$. By assumption, however, $b > c - a$, so at this point of the game, both A and B receive their BATNAs. In round $n - 1$, A makes the offer and leaves B with b; this means A keeps $2c - b$ (by assumption, $2c - b > a$). In round $n - 2$, B makes the offer, leaves A with $2c - b$, and keeps $3c - (2c - b) = c + b$. In round $n - 3$, A makes the offer and leaves B with $c + b$; this means A keeps $4c - (c + b) = 3c - b$. Result (for $nc = v$, n even, and $2c - a > b > c - a$): A gets $[(n/2) + 1]c - b = [(1/2) + (1/n)]v - b$ and B gets $[(n/2) - 1]c + b = [(1/2) - (1/n)]v + b$.

Compare the results when $nc = v$ and n is even, but the size of b varies. When $b < c - a$, A gets $v/2 + a$ and B gets $v/2 - a$. When $2c - a > b > c - a$, A gets $[(1/2) + (1/n)v - b$ and B gets $[(1/2) - (1/n)]v + b$. The condition $b > c - a$ can be rewritten as $a > c - b$. Remembering that $nc = v$, this is equivalent to $a > v/n - b$. B's payoff when b is small is $v/2 - a$; when b is big, it is $[(1/2) - (1/n)]v + b$. The second expression is bigger than the first when $a > v/n - b = c - b$. As we expect, therefore, B does better when b is larger.

3. The general formulas tell us that $x = a + h(v - a - b)$ and $y = b + k(v - a - b)$. Letting Donna's payoff equal x and Pierce's equal y, we have $a = 132$, $b = 70$, and $v = 216$. If Donna's has 2.5 times as much bargaining power as Pierce's, $h = 2.5k$. Since $h + k = 1$, it must be that $k = 0.286$ and $h = 0.714$. Thus Donna's must get $x = 132 + 0.714(14) = 142$, and Pierce's gets $y = 70 + 0.286(14) = 74$. The natural distribution of profits in the (High, High) outcome is 156 to Donna's and 60 to Pierce's. To achieve the bargaining outcome, Pierce's agrees to charge High and Donna's agrees to transfer 14 to Pierce's. Both stores gain relative to the Nash outcome, Donna's by 10 and Pierce's by 4.
4. The government of Euphoria's BATNA is 0; the government of Militia's BATNA is 100. In any negotiated settlement, therefore, Euphoria will get just a bit above 0, while Militia will get just a bit below 100. It doesn't matter which country moves first because the outcome is determined by bargaining strength alone. The countries could reach agreement immediately or could wait until October.
5. In the previous question, both countries knew the other's bargaining strength, so that there was nothing to be gained by delaying an agreement. In negotiations, there is some degree of imperfect information about bargaining strengths. As a result, both parties will often attempt to signal that they have bargaining strength (a high BATNA, for example, or a great deal of patience) in order to obtain a more favorable settlement. The only credible way to signal bargaining strength may be to delay reaching an agreement.
6. If B is twice as impatient as is A, then s will be twice as large as r, or $s = 2r$. In this case, $x = s/(r + s) = 2r/(r + 2r) = 2/3 = 0.667$ and $y = r/(r + s) = r/(r + 2r) = 1/3 = 0.333$. When $r = 0.01$ and $s = 0.02$, the approximation formulas give $x = 0.667$ and $y = 0.333$. The exact formulas show that when A makes the first offer, A gets $x = (0.02 + 0.0002)/(0.01 + 0.02 + 0.0002) = 0.0202/0.0302 = 0.669$ and B gets $(1 - 0.669) = 0.331$. When B makes the first offer, B gets $y = (0.01 + 0.0002)/(0.01 + 0.02 + 0.0002) = 0.0102/0.0302 = 0.338$ and A gets $(1 - 0.338) = 0.662$. The approximation formulas are off by no more than 0.005. Only the exact formulas, however, show that being able to make the first offer is worth 0.007 to the player who does so.

ADDITIONAL EXERCISE WITH ANSWER

1. Consider the following description of a bargaining situation. Person A has a backstop payoff (or BATNA) equal to a and a bargaining strength equal to h. Person B's backstop payoff is b and bargaining strength is k. The value that A and B can split if they reach an agreement is v. The Nash bargaining solution indicates that, in this situation, person A receives _____ and Person B receives _____.

   (a) $a + h(v - a - b)$; $b + k(v - a - b)$
   (b) $a + hv$; $b + kv$
   (c) $h(v - b)$; $k(v - a)$
   (d) hv; kv

   ANSWER (a)