SECTION III

Number of Questions: 50

DIRECTIONS for Questions 101 to 113: Answer the questions independently of each other.

101. When the curves $y = \log_{10} x$ and $y = x^{-1}$ are drawn in the x-y plane, how many times do they intersect for values $x \geq 1$?

1. Never 2. Once 3. Twice 4. More than twice.

102. The sum of 3rd and 15th elements of an arithmetic progression is equal to the sum of 6th, 11th and 13th elements of the same progression. Then which element of the series should necessarily be equal to zero?

1. 1st 2. 9th 3. 12th 4. None of the above.

103. A test has 50 questions. A student scores 1 mark for a correct answer, $-1/3$ for a wrong answer, and $-1/6$ for not attempting a question. If the net score of a student is 32, the number of questions answered wrongly by that student cannot be less than

1. 6 2. 12 3. 3 4. 9

104. Let A and B be two solid spheres such that the surface area of B is 300% higher than the surface area of A. The volume of A is found to be k% lower than the volume of B. The value of k must be

1. 85.5 2. 92.5 3. 90.5 4. 87.5

105. Twenty-seven persons attend a party. Which one of the following statements can never be true?

1. There is a person in the party who is acquainted with all the twenty-six others.
2. Each person in the party has a different number of acquaintances.
3. There is a person in the party who has an odd number of acquaintances.
4. In the party, there is no set of three mutual acquaintances.

106. How many even integers n, where $100 \leq n \leq 200$, are divisible neither by seven nor by nine?

1. 40 2. 37 3. 39 4. 38

107. Which one of the following conditions must p, q and r satisfy so that the following system of linear simultaneous equations has at least one solution, such that $p+q+r \neq 0$?

$$\begin{aligned} x + 2y - 3z &= p \\ 2x + 6y - 11z &= q \\ x - 2y + 7z &= r \end{aligned}$$

1. $5p - 2q - r = 0$ 2. $5p + 2q + r = 0$ 3. $5p + 2q - r = 0$ 4. $5p - 2q + r = 0$

108. The function $f(x) = |x - 2| + |2.5 - x| + |3.6 - x|$, where x is a real number, attains a minimum at

1. $x = 2.3$ 2. $x = 2.5$ 3. $x = 2.7$ 4. None of the above.

109. Let $g(x) = \max(5 - x, x+2)$. The smallest possible value of $g(x)$ is

1. 4.0 2. 4.5 3. 1.5 4. None of the above.

110. A positive whole number M less than 100 is represented in base 2 notation, base 3 notation, and base 5 notation. It is found that in all three cases the last digit is 1, while in exactly two out of the three cases the leading digit is 1. Then M equals

1. 31 2. 63 3. 75 4. 91

111. A leather factory produces two kinds of bags, standard and deluxe. The profit margin is Rs. 20 on a standard bag and Rs. 30 on a deluxe bag. Every bag must be processed on machine A and on machine B. The processing times per bag on the two machines are as follows:

	Time required (Hours / bag)	
	Machine A	Machine B
Standard Bag	4	6
Deluxe Bag	5	10

The total time available on machine A is 700 hours and on machine B is 1250 hours. Among the following production plans, which one meets the machine availability constraints and maximizes the profit?

1. Standard 75 bags, Deluxe 80 bags
2. Standard 100 bags, Deluxe 60 bags
3. Standard 50 bags, Deluxe 100 bags
4. Standard 60 bags, Deluxe 90 bags

112. In a 4000 meter race around a circular stadium having a circumference of 1000 meters, the fastest runner and the slowest runner reach the same point at the end of the 5^{th} minute, for the first time after the start of the race. All the runners have the same starting point and each runner maintains a uniform speed throughout the race. If the fastest runner runs at twice the speed of the slowest runner, what is the time taken by the fastest runner to finish the race?

1. 20 min 2. 15 min 3. 10 min 4. 5 min

113. At the end of year 1998, Shepard bought nine dozen goats. Henceforth, every year he added p% of the goats at the beginning of the year and sold q% of the goats at the end of the year where p>0 and q >0. If Shepard had nine dozen goats at the end of year 2002, after making the sales for that year, which of the following is true?

1. $p = q$
2. $p < q$
3. $p > q$
4. $p = q/2$

DIRECTIONS for Questions 114 and 115: Answer the questions on the basis of the information given below.

New Age Consultants have three consultants Gyani, Medha and Buddhi. The sum of the number of projects handled by Gyani and Buddhi individually is equal to the number of projects in which Medha is involved. All three consultants are involved together in 6 projects. Gyani works with Medha in 14 projects. Buddhi has 2 projects with Medha but without Gyani, and 3 projects with Gyani but without Medha. The total number of projects for New Age Consultants is one less than twice the number of projects in which more than one consultant is involved.

114. What is the number of projects in which Gyani alone is involved?

1. Uniquely equal to zero.
2. Uniquely equal to 1.
3. Uniquely equal to 4.
4. Cannot be determined uniquely.

115. What is the number of projects in which Medha alone is involved?

1. Uniquely equal to zero.
2. Uniquely equal to 1.
3. Uniquely equal to 4.
4. Cannot be determined uniquely.

DIRECTIONS for Questions 116 to 133: Answer the questions independently of each other.

116. Each side of a given polygon is parallel to either the X or the Y axis. A corner of such a polygon is said to be convex if the internal angle is 90° or concave if the internal angle is 270°. If the number of convex corners in such a polygon is 25, the number of concave corners must be

1. 20
2. 0
3. 21
4. 22

117. Let p and q be the roots of the quadratic equation $x^2 - (\alpha - 2)x - \alpha - 1 = 0$. What is the minimum possible value of $p^2 + q^2$?

1. 0
2. 3
3. 4
4. 5

118. The 288[th] term of the series a,b,b,c,c,c,d,d,d,d,e,e,e,e,e,f,f,f,f,f,f... is

1. u
2. v
3. w
4. x

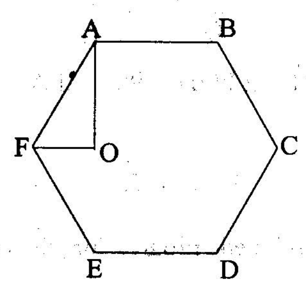
119. There are two concentric circles such that the area of the outer circle is four times the area of the inner circle. Let A, B and C be three distinct points on the perimeter of the outer circle such that AB and AC are tangents to the inner circle. If the area of the outer circle is 12 square centimeters then the area (in square centimeters) of the triangle ABC would be

1. $\pi\sqrt{12}$
2. $\frac{9}{\pi}$
3. $\frac{9\sqrt{3}}{\pi}$
4. $\frac{6\sqrt{3}}{\pi}$

120. Let a, b, c, d be four integers such that a+b+c+d = 4m+1 where m is a positive integer. Given m, which one of the following is necessarily true?

1. The minimum possible value of $a^2 + b^2 + c^2 + d^2$ is $4m^2-2m+1$
2. The minimum possible value of $a^2 + b^2 + c^2 + d^2$ is $4m^2+2m+1$
3. The maximum possible value of $a^2 + b^2 + c^2 + d^2$ is $4m^2-2m+1$
4. The maximum possible value of $a^2 + b^2 + c^2 + d^2$ is $4m^2+2m+1$

121. In the figure below, ABCDEF is a regular hexagon and $\angle AOF = 90^0$. FO is parallel to ED. What is the ratio of the area of the triangle AOF to that of the hexagon ABCDEF?

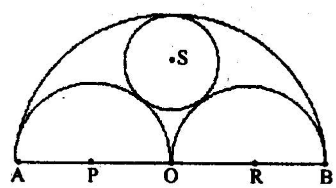


1. $\frac{1}{12}$
2. $\frac{1}{6}$
3. $\frac{1}{24}$
4. $\frac{1}{18}$

122. The number of non-negative real roots of $2^x - x - 1 = 0$ equals

1. 0 2. 1 3. 2 4. 3

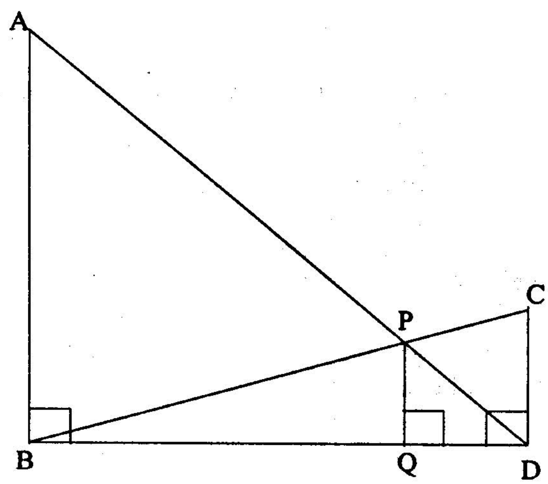
123. Three horses are grazing within a semi-circular field. In the diagram given below, AB is the diameter of the semi-circular field with centre at O. Horses are tied up at P, R and S such that PO and RO are the radii of semi-circles with centres at P and R respectively, and S is the centre of the circle touching the two semi-circles with diameters AO and OB. The horses tied at P and R can graze within the respective semi-circles and the horse tied at S can graze within the circle centred at S. The percentage of the area of the semi-circle with diameter AB that cannot be grazed by the horses is nearest to



1. 20 2. 28 3. 36 4. 40

124. In a triangle ABC, AB = 6, BC = 8 and AC = 10. A perpendicular dropped from B, meets the side AC at D. A circle of radius BD (with centre B) is drawn. If the circle cuts AB and BC at P and Q respectively, then AP:QC is equal to

1. 1:1 2. 3:2 3. 4:1 4. 3:8



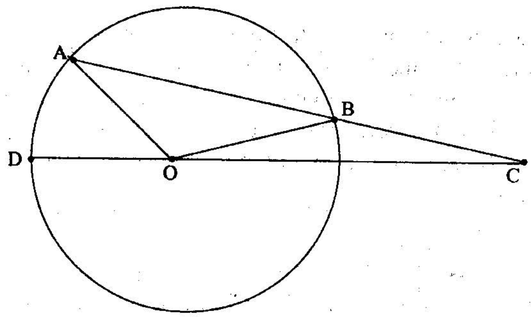
125. In the diagram given below, $\angle ABD = \angle CDB = \angle PQD = 90^0$. If AB:CD = 3:1, the ratio of CD: PQ is

1. 1 : 0.69 2. 1 : 0.75 3. 1 : 0.72 4. None of the above.

126. If $\log_3 2$, $\log_3 (2^x - 5)$, $\log_3 (2^x - 7/2)$ are in arithmetic progression, then the value of x is equal to

1. 5 2. 4 3. 2 4. 3

127. In the figure given below, AB is the chord of a circle with centre O. AB is extended to C such that BC = OB. The straight line CO is produced to meet the circle at D. If $\angle ACD = y$ degrees and $\angle AOD = x$ degrees such that $x = ky$, then the value of k is



1. 3 2. 2 3. 1 4. None of the above.

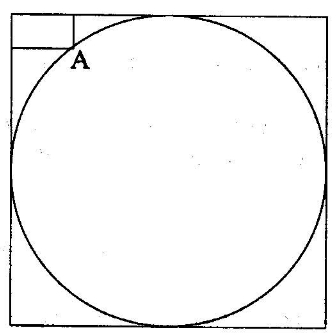
128. How many three digit positive integers, with digits x, y and z in the hundred's, ten's and unit's place respectively, exist such that $x < y$, $z < y$ and $x \neq 0$?

1. 245 2. 285 3. 240 4. 320

129. A vertical tower OP stands at the centre O of a square ABCD. Let h and b denote the length OP and AB respectively. Suppose $\angle APB = 60°$ then the relationship between h and b can be expressed as

1. $2b^2 = h^2$ 2. $2h^2 = b^2$ 3. $3b^2 = 2h^2$ 4. $3h^2 = 2b^2$

130. In the figure below, the rectangle at the corner measures 10 cm x 20 cm. The corner A of the rectangle is also a point on the circumference of the circle. What is the radius of the circle in cm?



1. 10 cm 2. 40 cm 3. 50 cm 4. None of the above.

131. There are 8436 steel balls, each with a radius of 1 centimeter, stacked in a pile, with 1 ball on top, 3 balls in the second layer, 6 in the third layer, 10 in the fourth, and so on. The number of horizontal layers in the pile is

1. 34 2. 38 3. 36 4. 32

132. If the product of n positive real numbers is unity, then their sum is necessarily

1. a multiple of n
2. equal to $n + \frac{1}{n}$
3. never less than n
4. a positive integer

133. Given that $-1 \le v \le 1$, $-2 \le u \le -0.5$ and $-2 \le z \le -0.5$ and $w = vz/u$, then which of the following is necessarily true?

1. $-0.5 \le w \le 2$
2. $-4 \le w \le 4$
3. $-4 \le w \le 2$
4. $-2 \le w \le -0.5$

DIRECTIONS for Questions 134 to 136: Answer the questions on the basis of the information given below.

A city has two perfectly circular and concentric ring roads, the outer ring road (OR) being twice as long as the inner ring road (IR). There are also four (straight line) chord roads from E1, the east end point of OR to N2, the north end point of IR; from N1, the north end point of OR to W2, the west end point of IR; from W1, the west end point of OR, to S2, the south end point of IR; and from S1, the south end point of OR to E2, the east end point of IR. Traffic moves at a constant speed of 30π km/hr on the OR road, 20π km/hr on the IR road, and $15\sqrt{5}$ km/hr on all the chord roads.

134. The ratio of the sum of the lengths of all chord roads to the length of the outer ring road is

1. $\sqrt{5}:2$
2. $\sqrt{5}:2\pi$
3. $\sqrt{5}:\pi$
4. None of the above.

135. Amit wants to reach N2 from S1. It would take him 90 minutes if he goes on minor arc S1 – E1 on OR, and then on the chord road E1 – N2. What is the radius of the outer ring road in kms?

1. 60
2. 40
3. 30
4. 20

136. Amit wants to reach E2 from N1 using first the chord N1 – W2 and then the inner ring road. What will be his travel time in minutes on the basis of information given in the above question?

1. 60
2. 45
3. 90
4. 105

DIRECTIONS for Questions 137 to 143: Answer the questions independently of each other.

137. There are 6 boxes numbered 1,2 , ... 6. Each box is to be filled up either with a red or a green ball in such a way that at least 1 box contains a green ball and the boxes containing green balls are consecutively numbered. The total number of ways in which this can be done is

1. 5
2. 21
3. 33
4. 60

138. A graph may be defined as a set of points connected by lines called edges. Every edge connects a pair of points. Thus, a triangle is a graph with 3 edges and 3 points. The degree of a point is the number of edges connected to it. For example, a triangle is a graph with three points of degree 2 each. Consider a graph with 12 points. It is possible to reach any point from any other point through a sequence of edges. The number of edges, e, in the graph must satisfy the condition

1. $11 \le e \le 66$
2. $10 \le e \le 66$
3. $11 \le e \le 65$
4. $0 \le e \le 11$

139. Let T be the set of integers {3,11,19,27,... 451,459,467} and S be a subset of T such that the sum of no two elements of S is 470. The maximum possible number of elements in S is

1. 32
2. 28
3. 29
4. 30

140. Consider the following two curves in the x-y plane:

$$y = x^3 + x^2 + 5$$
$$y = x^2 + x + 5$$

Which of the following statements is true for $-2 \le x \le 2$?

1. The two curves intersect once.
2. The two curves intersect twice.
3. The two curves do not intersect.
4. The two curves intersect thrice.

141. In a certain examination paper, there are n questions. For $j = 1,2 \ldots n$, there are 2^{n-j} students who answered j or more questions wrongly. If the total number of wrong answers is 4095, then the value of n is

1. 12
2. 11
3. 10
4. 9

142. If x,y,z are distinct positive real numbers then

$$\frac{x^2(y+z) + y^2(x+z) + z^2(x+y)}{xyz} \text{ would be}$$

1. greater than 4.
2. greater than 5.
3. greater than 6.
4. None of the above.

143. The number of positive integers n in the range $12 \le n \le 40$ such that the product $(n-1)(n-2)\ldots3.2.1$ is not divisible by n is

1. 5
2. 7
3. 13
4. 14

DIRECTIONS for Questions 144 to 148: Each question is followed by two statements, A and B. Answer each question using the following instructions.

Choose 1 if the question can be answered by one of the statements alone but not by the other.
Choose 2 if the question can be answered by using either statement alone.
Choose 3 if the question can be answered by using both the statements together, but cannot be answered by using either statement alone.
Choose 4 if the question cannot be answered even by using both the statements together.

144. Is $a^{44} < b^{11}$, given that a = 2 and b is an integer?

A. b is even
B. b is greater than 16

145. What are the unique values of b and c in the equation $4x^2 + bx + c = 0$ if one of the roots of the equation is (–1/2) ?

A. The second root is 1/2.
B. The ratio of c and b is 1.

146. AB is a chord of a circle. AB = 5 cm. A tangent parallel to AB touches the minor arc AB at E. What is the radius of the circle?

A. AB is not a diameter of the circle.
B. The distance between AB and the tangent at E is 5 cm.

147. Is $\left(\frac{1}{a^2}+\frac{1}{a^4}+\frac{1}{a^6}+\ldots\right)>\left(\frac{1}{a}+\frac{1}{a^3}+\frac{1}{a^5}+\ldots\right)$?

A. $-3 \leq a \leq 3$
B. One of the roots of the equation $4x^2-4x+1 = 0$ is a.

148. D, E, F are the mid points of the sides AB, BC and CA of triangle ABC respectively. What is the area of DEF in square centimeters?

A. AD = 1 cm, DF = 1 cm and perimeter of DEF = 3 cm
B. Perimeter of ABC = 6 cm, AB = 2 cm, and AC = 2 cm.

DIRECTIONS for Questions 149 and 150: Answer the questions on the basis of the information given below.

A certain perfume is available at a duty-free shop at the Bangkok international airport. It is priced in the Thai currency Baht but other currencies are also acceptable. In particular, the shop accepts Euro and US Dollar at the following rates of exchange:

US Dollar 1 = 41 Bahts
Euro 1 = 46 Bahts

The perfume is priced at 520 Bahts per bottle. After one bottle is purchased, subsequent bottles are available at a discount of 30%. Three friends S, R and M together purchase three bottles of the perfume, agreeing to share the cost equally. R pays 2 Euros. M pays 4 Euros and 27 Thai Bahts and S pays the remaining amount in US Dollars.

149. How much does R owe to S in Thai Baht?

1. 428   2. 416   3. 334   4. 324

150. How much does M owe to S in US Dollars?

1. 3   2. 4   3. 5   4. 6