CHAPTER 4

JET PUMPS

SECTION 4.1
JET PUMP THEORY

RICHARD G. CUNNINGHAM

INTRODUCTION

The jet pump transfers energy from a liquid or gas primary fluid to a secondary fluid. The latter may be a liquid, a gas, a two-phase gas-in-liquid mixture, or solid particles transported in a gas or a liquid. Examples of all these combinations have been reported in the technical literature. Reference 1, the major bibliography in this field, contains over 400 abstracts. Although the terms “ejector” and “eductor” are also applied, the term “jet pump” will be used here. The jet pump offers significant advantages over mechanical pumps: no moving parts for improved reliability, adaptability to installation in remote or hazardous environments, simplicity, and low cost. The primary drawback is efficiency: both frictional losses and unavoidable mixing losses are incurred. Nevertheless, careful design can produce pumps with efficiencies on the order of 30–40%. The jet pump in Figure 1 is typical of liquid-jet pumps and low Mach-number gas-jet/gas pumps. Compressible-flow pumps, for example, steam-jet ejectors, employ converging-diverging nozzles for full expansion of the jet.

NOMENCLATURE

A = area, ft^2 (m^2)
A_w = throat wall area, ft^2 (m^2)
C = velocity of sound, ft/sec (m/s)
CR = cavitation resistance
D = diameter, ft (m)

E = energy rate, ft lb/sec (joule/s)
K = friction loss coefficient
°K = absolute temperature, Kelvin
LJL = liquid-jet liquid pump
LJG = liquid-jet gas compressor
LJGL = liquid-jet gas and liquid pump
M = liquid/liquid flow ratio, Q_2/Q_1
MN = Mach number
N = pressure ratio, LJL jet pump
NPSH = net positive suction head
P, P = pressure: static, total psia (kPa abs.)
P_v = vapor pressure, psia (kPa abs.)
Q = volumetric rate, ft^3/sec (m^3/s)
R = gas constant, ft lbs/slug °R (joules/kg °K)
°R = absolute temperature, Rankine
S = density ratio, ρ_2/ρ_1
T = temperature °R (°K)
V = velocity, ft/sec (m/sec)
W = work rate, ft lbs/sec (joules/s)
Z = jet dynamic pressure, psi (kPa)
a = diffuser area ratio, A_i/A_d
b = jet pump area ratio, A_n/A_t
c = $A_{2G0}/A_n = (A_t - A_n)/A_n = (1 - b)/b$
m = mass flow rate, slugs/s (kg/s)
s = seconds
psi = pounds per square inch
psia = pounds per square inch, absolute
r_v = gas/liquid vol. flow rate ratio Q_G/Q_2
r_{v0} = gas/liquid vol. flow rate ratio at 0: Q_{G0}/Q_2
r_m = gas/liquid mass flow-rate ratio
sp = nozzle-to-throat spacing, ft(m)
sp/D_{th} = spacing, throat diameters
η = efficiency
γ = gas density ratio at s, ρ_{Gs}/ρ_1
$\gamma\phi_s$ = m_{Gs}/m_1
ρ = density, slugs/ft^3 (kg/m^3)
τ = shear stress, psi (kPa)
ϕ = gas flow ratio Q_G/Q_1
ϕ_s = gas flow ratio Q_{Gs}/Q_1 at s

Subscripts

1 = liquid primary flow
2 = liquid secondary flow

G = gas secondary flow
$2G$ = bubbly secondary flow
$2G0$ = bubbly sec. flow at 0
mep = maximum efficiency point
op = operating point
L = limit for cavitating flow
3 = combined fluids 1, 2, and G
i, s, n = locations (see Fig. 1)
$0, t, d$ = locations (see Fig. 1)
$c0$ = flow-ratio cut off
f = friction loss
n = nozzle
en = throat entry
th = mixing throat
di = diffuser
td = throat and diffuser

Subscript Examples

Q_2 = secondary liquid vol. flow rate
Q_{G0} = secondary gas flow rate, at 0
Q_{2G0} = flow rate of bubbly mixture of gas in the secondary liquid, at 0

NOTE: The convention for pressure and stress in this section is to use psi (and psia) for pounds per square inch (and pounds per square inch absolute) instead of the conventional lb/in^2 used throughout the rest of the handbook.

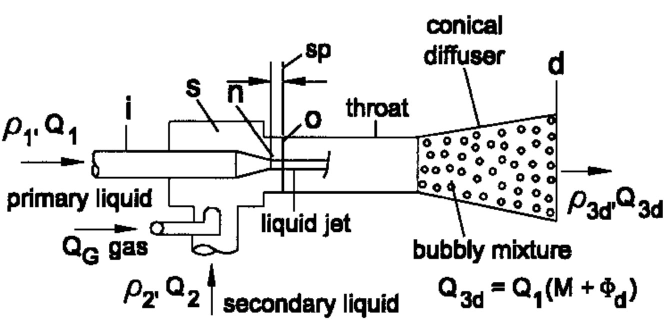


FIGURE 1 Liquid-jet gas and liquid pump and nomenclature

LIQUID-JET PUMP THEORY FOR THREE SECONDARY-FLOW TYPES

The liquid-jet pump model is based on conservation equations for energy, momentum, and mass. Real-fluid losses are accounted for by friction-loss coefficients (K). The primary or motive fluid is a liquid of density ρ_1. In the following derivation, the secondary/pumped fluid can be a second liquid of density ρ_1 or ρ_2, or a gas-in-liquid bubbly mixture, or a gas. These three jet pump flow regimes are referred to as liquid-jet liquid (LJL), liquid-jet gas liquid (LJGL), and liquid-jet gas (LJG). Equations (1), (3), (5), and (7) below apply to all three.

Assumptions:

a. The primary and secondary streams enter the mixing throat with uniform velocity distributions, and the mixed flows leave the throat and diffuser with a uniform velocity profile.

b. The gas phase—if present—undergoes isothermal compression in the throat and diffuser.

c. All two-phase flows at the throat entry and exit consist of homogeneous bubble mixtures of a gas in a continuous liquid.

d. Heat transfer from the gas to the liquid is negligible—the liquid temperature remains constant.

e. Change in solubility of the gas in the liquid from pressure P_s to P_d is negligible.

f. Vapor evolution from and condensation to the liquid are negligibly small.

NOZZLE EQUATION With reference to Figure 1

$$P_i + \rho_1 \frac{V_i^2}{2} = P_o + \rho_1 \frac{V_n^2}{2} + K_n \rho_1 \frac{V_n^2}{2} \tag{1}$$

For

$$P_i = \overline{P_i}$$

the nozzle equation is

$$P_i - P_O = Z(1 + K_n)$$

THROAT-ENTRY EQUATION The two-phase secondary flow is described by

$$\frac{dP}{\rho} = VdV + d\left(K_{en}\frac{V^2}{2}\right) = 0 \tag{2}$$

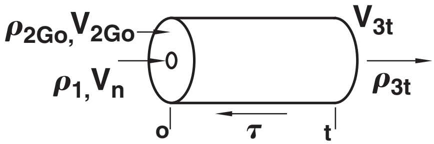
Density of the secondary fluid as a function of static pressure and flow ratios M and ϕ is

$$\rho_{2G} = \frac{m_2 + m_G}{Q_2 + Q_G} = \frac{m_1\left[\dfrac{m_2}{m_1} + \dfrac{m_G}{m_1}\right]}{Q_1(M + \phi)} = \rho_1\left[\frac{SM + \gamma\phi_s}{m + \phi}\right]$$

Integration of Eq. (2) using this density relation and continuity results in the throat-entry equation:

$$M(P_s - P_o) + P_s\phi_s \ln\frac{P_s}{P_o} = Z\,\frac{SM + \gamma\phi_s}{c^2}(1 + K_{en})(M + \phi_o)^2 \tag{3}$$

MIXING THROAT MOMENTUM EQUATION Equating control volume forces and fluid momentum changes:



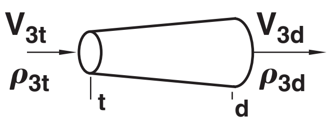
$$(P_0 - P_t)A_{th} - \tau A_w = (m_1 + m_2 + m_G)\, V_{3t} m_1 V_n - (m_2 + m_G)\, V_{2G0} \tag{4}$$

where
$$\frac{\tau A_w}{A_{th}} = \frac{K_{th}\rho_{3t}V_{3t}^2}{2}$$

Substituting the density and continuity relations in Eq. (4) and diving A_{th} produces a quadratic equation in P_t. The throat momentum equation becomes

$$P_t^2 - Z\Bigg[2b - b^2(2 + K_{th})(1 + SM + \gamma\phi_s)(1 + M) + 2(SM + \gamma\phi_s)(M + \phi_0)\frac{b^2}{(1 - b)} + \frac{P_0}{Z}\Bigg]P_t + Z[b^2(2 + K_{th})(1 + SM + \gamma\phi_s)P_s\phi_s] = 0 \tag{5}$$

DIFFUSER EQUATION The mixed primary and secondary fluids flow from t to d:



The t-d flow is described by Eq. (6):

$$\int_t^d \frac{dP}{\rho} + \int_t^d V dV + \int_t^d \frac{\Delta P_f}{\rho_{3t}} = 0 \tag{6}$$

Integrating Eq. (6) and substituting the density and continuity relations, the diffuser equation becomes

$$(P_d - P_t) + \frac{P_s\phi_s}{1 + M}\ln\frac{P_d}{P_t} = Zb^2\left[\frac{1 + SM + \gamma\phi_s}{1 + M}\right] \times [(1 + M + \phi_t)^2 - a^2(1 + M + \phi_d)^2 - K_{di}(1 + M + \phi_t)(1 + M)] \tag{7}$$

LIQUID-JET PUMP EQUATIONS The liquid jet pump is described in terms of the four flow processes by Equations (1), (3), (5), and (7). The term $\gamma\phi_s = \rho_{Gs}\, Q_{Gs}/\rho_1 Q_1$. For isothermal flow, $\gamma\phi_s = 144P_s\phi_s/\mathbf{R}T_s\rho_1$. For air, with $\mathbf{R}$ = 1716 ft lb/slug °R, and water, ρ_1 = 1.94 slugs/ft^3 and $\gamma\phi_s = .0432P_s\phi_s/T_s$. In SI, $\mathbf{R}$ = 286.92 mN/kg °K, ρ_1 = 1000 kg/m^3, and $\gamma\phi_s = .00348P_s\phi_s/T_s$, with P_s in kN/m2, and T_s in °K. The $\mathbf{R}$ and/or ρ_1 values must of course be replaced for fluids other than water and air.

In the LJGL pump, the secondary flow is a bubbly mixture of a gas in a liquid. Compressible flow phenomena must be considered because the velocity of sound is quite low in bubbly fluids. For example, the velocity of sound in a 50/50 uniform mixture of air in water is about 70 ft/s (21.3 m/s), far below sonic velocities in air (1100 ft/s or 335 m/s) or water (5000 ft/s or 1524 m/s).

The Mach number of a bubbly secondary flow at the throat entrance is (see Reference 5):

$$\mathbf{MN}_{2G0} = \frac{V_{2G0}}{C_{2G0}} = \frac{\phi_0}{c}\sqrt{\frac{2Z}{P_s\phi_s}(SM + \gamma\phi_s)} \tag{8}$$

When this Mach number is 1.0, the LJGL pump has reached limiting flow; that is, a reduction in back pressure P_d no longer causes an increase in the bubbly secondary-flow rate.

Pump Efficiencies The LJGL pump produces two useful work results:

- Static-pressure increase of the liquid component of the secondary flow stream.
- If a gas is entrained in this liquid stream, isothermal compression of the gas component.

With W as the work rate, ft-lb/s, (power) $W_L = Q_2\,(P_d - P_s)$ is the work rate on the liquid component, and $W_G = \rho_{Gs}Q_{Gs}\mathbf{R}Tln(P_d - P_s)$ is the work rate on the gas component. The energy rate input is $E_{\text{in}} = Q_1(P_i - P_s)$. The LJGL pump mechanical efficiency is the total work rate divided by the energy rate in

$$\eta = \frac{M(P_d - P_s) + P_s\phi_s \ln \dfrac{P_d}{P_s}}{\mathrm{P_i - P_d}} = \eta_L + \eta_G \tag{9}$$

The Jet Loss Jet pumps in practical applications have nozzle-to-throat spacings sp/*Dth* of one or more mixing-throat diameters. The power jet traverses from a static pressure at or near P_s down to P_o, with no useful work recognized in the one-dimensional theory. Thus a "jet loss" occurs, which is in addition to the frictional and mixing losses (see Reference 9).

In the LJL and LJGL (but not the LJG) pumps, throat-inlet pressure drops—and hence jet losses—are significant (Ref. 6).

Pump Efficiency, Incorporating Jet Loss In Eq. (9), $(P_i - P_d)$ is expanded: $(P_i - P_d) = (P_i - P_s) - (P_d - P_s)$, and $(P_i - P_s) = (P_i - P_o) - (P_s - P_o) = Z(1 + K_n) - \mathrm{j}(P_s - P_o)$, where $j = 1$ for a fully inserted nozzle, *no jet loss:* and $j = 0$ for the usual case of retracted nozzle, which produces *full jet loss.* Eq. 9 now becomes

$$\eta = \frac{M(P_d - P_s) + P_s\phi_s \ln \dfrac{P_d}{P_s}}{Z(1 + K_n) - j(P_s - P_o) - (P_d - P_s)} = \eta_L + \eta_G \tag{10}$$

Eq. (10) is recommended for predicting liquid-jet pump efficiencies as follows: Use $j = 0$ for pumps with normally-retracted nozzles (full jet loss); use $j = 1$ for no-jet-loss pumps (thin-walled nozzle tip fully inserted so sp = 0). The pressure in Eq. (10) should be calculated from the one-dimensional theory using Eqs (1), (3), (5), and (7). (See below for the LJL jet pump.)

Computer Programs for LJGL and LJG Models Solutions for the compressible flow cases are generated using computer spreadsheet or Fortran programs. Values for Z, b, P_s, Ts, $\mathbf{R}$, ρ_1, S and the four K coefficients are fixed/assumed for each pump and operating conditions. Eqs. (3), (5), and (7) are then solved for each step increase in flow-ratio M, with ϕ_s held constant. Alternatively, M may be held constant and the equations solved for step increase in ϕ_s. Eqs. (3), (5), and (7) are interdependent: solution of Eq. (5) requires P_o values from Eq. (3) and solution of Eq. (7) requires P_t values from Eq. (5). The program outputs at each flow-ratio step are static pressure P_o, P_tP_d, and the three pump efficiencies defined by Eq. 10.

LJGL FLOW CUT-OFF Compressible-flow choking of the secondary stream at the throat entrance will occur at $\mathbf{MN}_o = 1$. The flow ratio at which this will occur can be predicted from critical-flow theory. For further details, see Reference 5.

Performance of the gas compressor (LJG) can be calculated from Eqs. (1), (3), (5), and (7) by setting $M = 0$. Although simplified, the equations are still coupled as in the LJGL case. The one-dimensional theory predicts actual performance quite well (see References 6 and 7) provided the mixing is completed within the length of the mixing throat. Theory-experiment agreement fails and the gas compressor efficiency declines—mixing is allowed to extend into the diffuser.

THE LIQUID-JET LIQUID (LJL) PUMP

Equations for the LJL pump are much simpler than the corresponding LJGL and LJG equations because of the elimination of all ϕ terms. In this case, Eqs. (1), (3), (5), and (7) reduce to the following set:

Nozzle $\quad P_i - P_o = Z(1 + K_n) \qquad (1)$

Throat Entry $\quad P_s - P_o = ZS(1 + K_{en})M^2/c^2 \qquad (11)$

Throat $\quad P_t - Po = Z[2b + 2SM^2b^2/(1 - b) - b^2(2 + K_{th})(1 + SM)(1 + M)] \qquad (12)$

Diffuser $\quad P_d - P_t = Zb^2(1 + SM)(1 + M)(1 - K_{di} - a^2) \qquad (13)$

Pump efficiency η is defined as the ratio of useful work rate on the secondary fluid Q_2 to the energy extracted from the primary liquid:

$$\eta = Q_2(P_d - P_s)/Q_1(P_i - P_d) = MN \tag{14}$$

Two other definitions of efficiency are found in the literature, as follows:

$$\eta' = Q_2(P_d - P_s)/Q_1(P_i - P_s) = \eta/N + 1$$

$$\eta'' = (M + 1)(P_d - P_s)/(P_i - P_s)$$

These two other definitions assume that the primary/power stream pressure falls to P_s, not P_d, as shown in Figure 2. Efficiency conversions are possible only if all three pressures and two flow rates involved are given. Comparisons of efficiencies reported in the literature should be made with caution.

Combining Eqs. 1, 11–14, the theoretical pressure characteristic N for the LJL pump is

$$N = \frac{2b + \dfrac{2SM^2b^2}{1 - b} - b^2(1 + K_{td} + a^2)(1 + M)(1 + SM) - \left(\dfrac{SM^2}{c^2}\right)(1 + K_{en})}{1 + K_n - 2b - \dfrac{2SM^2b^2}{1 - b} + b^2(1 + K_{td} + a^2)(1 + M)(1 + SM) + (1 - j)\left(\dfrac{SM^2}{c^2}\right)(1 + K_{en})} \tag{15}$$

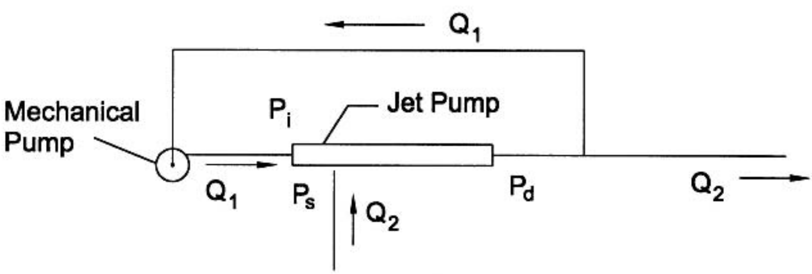


FIGURE 2 LJL jet pump and installation

where $K_{td} = K_{th} + K_{di}$. The term $j = 0$ for the normal (jet loss) case, and $J = 1$ for no jet loss. Jet pumps normally are designed with a finite nozzle-throat spacing, typically $sp/D_{th} = 1$; jet loss is experienced, and thus $j = 0$ for this (usual) case. Eq. (15) becomes

$$N = \frac{2b + \dfrac{2SM^2b^2}{1-b} - b^2(1 + K_{td} + a^2)(1 + M)(1 + SM) - \left(\dfrac{SM^2}{c^2}\right)(1 + K_{en})}{1 + K_n - \text{numerator}} \quad (16)$$

In terms of pressure,

$$N = (P_d - P_s)/(P_i - P_d)$$

Two simplifications of Eq. (16) are often appropriate: 1) The area ratio term $a^2 = 0$ for the usual 5°–8° included-angle diffuser ($a = A_t/A_d$ is small). 2) The density ratio $S = 1$ for similar primary and secondary liquids, for example, a water primary jet pumping water as the secondary fluid. With these simplifications, Eq. (16) for the normal (jet loss) case becomes

$$N = \frac{2b + \dfrac{2SM^2b^2}{1-b} - b^2(1 + K_{td})(1 + M)^2 - \left(\dfrac{M^2}{c^2}\right)(1 + K_{en})}{1 + K_n - \text{numerator}} \quad (17)$$

And pump efficiency is

$$\eta = \eta_L = MN \quad (18)$$

For the LJL jet pump, note also that

$$(P_d - P_s)/(P_i - P_s) = N/(N + 1) \quad (19)$$

Computer Programs for LJL Models It is convenient to use Eqs. (17) and (18) in spreadsheet form to generate tables of "$N(b, M, K_n, K_{td}, K_{en})$;" that is, the LJL jet pump pressure characteristic N as a function of the bracketed variables. Typically the Ks and the area ratio b are held constant and a table is generated using step increases in M to show resultant N values. Table 1 shows $N(M)$ for $b = .25$; and $K_n = .05$, $K_{td} = .2$, and $K_{en} = 0$. The performance of this pump is shown in Figure 3 as $N(M)$ and $\eta(M)$.

TABLE 1 Performance versus M, for $b = .25$, $K_n = .05$, $K_{td} = .2$, $K_{en} = 0$

M	$(P_d - P_s)/Z$	$(P_i - P_d)/Z$	N	$\eta\%$
0.000	0.4250	0.6250	0.6800	0.00
0.200	0.3942	0.6558	0.6012	12.02
0.400	0.3619	0.6881	0.5259	21.04
0.600	0.3280	0.7220	0.4543	27.26
0.800	0.2926	0.7574	0.3862	30.90
1.000	0.2556	0.7944	0.3217	32.17
1.200	0.2170	0.8330	0.2605	31.26
1.400	0.1769	0.8731	0.2026	28.36
1.600	0.1352	0.9148	0.1478	23.65
1.800	0.0920	0.9580	0.0960	17.29
2.000	0.0472	1.0028	0.0471	9.42
2.200	0.0009	1.0491	0.0008	0.19
2.204	0.0000	1.0500	0.0000	0.00
0.676	0.3147	0.7353	0.4280	28.93
1.014	0.2529	0.7971	0.3173	32.17

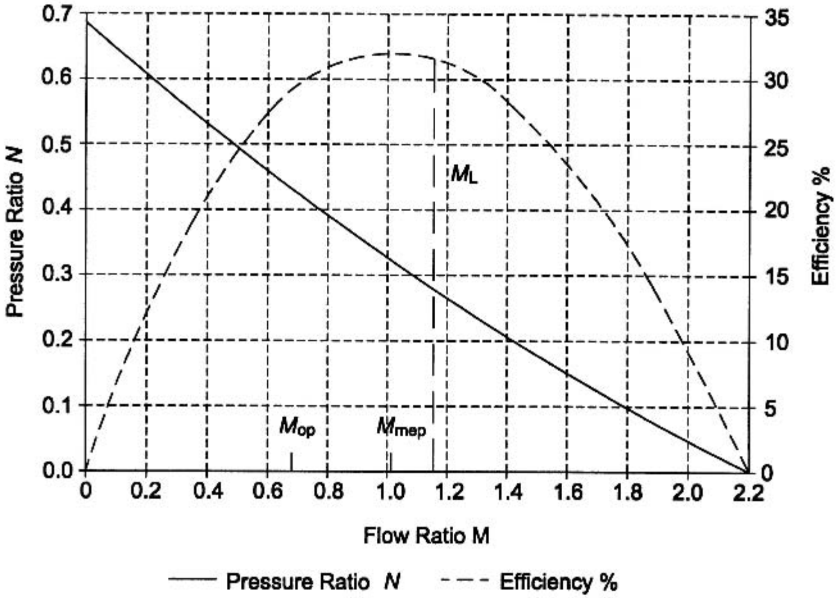


FIGURE 3 $K_n = .05, K_{td} = .20, K_{en} = 0$

It has been shown (see References 2, 3, 4) that one-dimensional analyses successfully predict actual LJL pump performance. But theory-experiment agreement obtains only if the test-pump flow conforms with the model assumptions. Two conditions that will cause departure of measured N and η data from the theoretical curves are

- Cavitation, which occurs in the mixing throat
- Extension of the throat mixing process out into the diffuser

Operating Flow Ratio M_{op} In Figure 3, the flow ratio M_{op} is indicated on the left slope of the "parabolic" efficiency curve, which peaks at $M = M_{mep}$. The recommended location of the design operating point is to set $M_{op} = \frac{2}{3} M_{mep}$. Higher operating M ratios would provide slightly higher efficiencies, but at a greater risk of cavitation. (Cavitation as part of the design process is included in the following examples given.) Finding M_{mep} can be readily accomplished from Table 1 type data using spreadsheet successive approximations. Alternatively, Eqs. (17)–(18) can be differentiated and set equal to zero to find the peak efficiency M_{mep} value.

Cavitation LJL pumps may encounter cavitation, which occurs in the mixing throat. With reference to Figure 3, the LJL pump normally responds to a reduction in back pressure P_d (Q_1 and P_s constant) by producing a larger Q_2 secondary flow, and hence a larger M. Measured pressure ratios (N) and efficiencies (η) track along these theory-based characteristic curves as shown in Figure 3. But after the throat-inlet pressure (P_o) is reduced to the vapor pressure (P_v) of the secondary liquid, any further drop in the back pressure has no effect on the flow ratio, which stabilizes at $M = M_L$, the *cavitation-limited flow ratio.* Note the vertical dashed line in Figure 3: measured N and η values fall on this vertical line, under M_L operating conditions. In this manner, cavitating-pump performance departs radically from predicted/normal behavior.

Published studies (see References 1 and 8) have shown that NPSH-type correlations adequately explain and predict cavitation-limited flow phenomena. **Comparing the predicted M_L with the intended M_{op} is an essential step in designing a jet pump**

installation. If $M_{op} < M_L$, the LJL pump can be expected to perform "on design;" that is, to follow the Eq. (17) $N(M)$ relation, with no cavitation. M_L can be predicted from the operating conditions as follows (see Reference 8):

$$M_L = c\sqrt{\frac{P_s - P_v}{\sigma Z}} \tag{20}$$

where σ is a cavitation coefficient.

Experiments, primarily with water and lubricating oils, have shown a σ range of 0.8–1.4 (see Reference 1). One investigator (see Reference 10) found that improving the nozzle exterior profile and related throat-inlet internal profile reduced the measured σ from 1.4 to 1.0, indicating a significant improvement in cavitation resistance. For design use, a conservative value of $\sigma = 1.35$ is recommended (see Reference 8).

Area Ratio b The nozzle-to-throat area ratio b is the only geometric parameter in these liquid-jet pump models, and including the LJL case, Eq. (17). Area ratio b is all-important, affecting pump efficiency, flow capacity, cavitation, and pressure characteristic $N(M)$. Comparisons of b-value series of pumps (see References 1–4) show that peak efficiency is highest for $b = .2$ to .3. This optimum occurs because efficiency of the jet pump reflects the two unavoidable losses: friction and the mixing loss. The latter (a maximum at zero M) *decreases* with rising M, whereas frictional losses *increase* with M. The sum of these losses is a minimum for pumps in the $b = .2 - .3$ range, each operating at or near the respective M_{mep} (see Reference 9). As indicated in the numerical Examples 1 and 2 further on, it is recommended that jet pump designs start—in the absence of other restraints—with $b = .25$. This initial area ratio can then later be adjusted, for example, to handle a cavitation problem. In specific cases the jet pump placement, or a desired flow ratio, may determine N or M, and hence b. Example 3 illustrates the situation.

Friction-Loss Coefficients Solutions of Eqs. (11)–(18) require use of appropriate K loss coefficients. The following sources are suggested:

1. Adopt published results from jet pump studies that have included measurements of K values (see References 3, 10, 11).
2. Use published loss coefficients for the (four) components of the LJL pump; that is, converging nozzle, tube entry, short tube, and diffuser.
3. Measure Ks directly by running flow test of jet pumps or component parts. Full evaluation of the four Ks requires measurement of two flow rates and five static pressures, P_i, P_s, P_o, P_t, and P_d. Most studies, however, have omitted P_o and P_t pressure taps, and measured only three of these pressures: P_i, P_s, and P_d. This omission limits K evaluations based on pump tests to K_n and K_{td}. Fortunately, the theory works quite well under this limitation, largely because the throat and diffuser Ks are additive ($K_{td} = K_{th} + K_{di}$) in the $N(M)$ equations.

In regard to Item 1 above, Table 2 shows the range and recommended values for the Ks which appear in the theoretical model equations.

Table 2 recommendations assume that only P_i, P_s, and P_d are utilized; if this is the case, it is necessary to assume that $K_{en} = 0$, as indicated. A finite value of K_{en} in Eqs. (15)–(17)

TABLE 2 Recommended values for K friction-loss coefficients

	Range	Rec. Values	References
K_n	.04–1.0	.05	3, 10, 11
K_{en}	—	0	ditto
K_{td}	.17–.40	.20	ditto

TABLE 3 Values of $M_{op} = \frac{2}{3}M_{mep}$, and N_{op}, for Figure 4

b	M_{mep}	M_{op}	N_{op}
0.05	4.8520	3.2347	0.0739
0.10	2.7120	1.8080	0.1509
0.15	1.8294	1.2196	0.2344
0.20	1.3330	0.8887	0.3264
0.25	1.0142	0.6760	0.4280
0.30	0.7913	0.5275	0.5406
0.40	0.5020	0.3347	0.8039
0.50	0.3270	0.2180	1.1210
0.60	0.2117	0.1411	1.4913

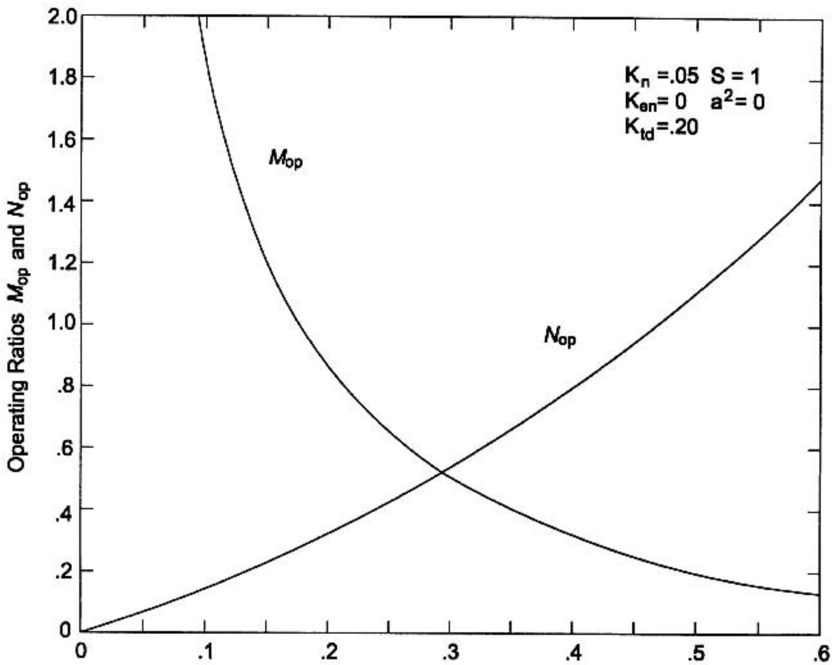


FIGURE 4 Response of LJL operating pressure ratio N_{op} and operating flow ratio M_{op} to jet pump design area ratio b

increases the slope of the theoretical $N(M)$ curve at low M values. At operating M values, however, this effect is negligible.

Finding b as a Function of N_{op} or M_{op} Figure 4 presents $N_{op}(b)$ and $M_{op}(b)$ for the recommended K friction coefficients of .05, 0, and .20. The Figure 4 curves are cross-plots of M_{op} values ($=\frac{2}{3} M_{mep}$) and associated N_{op} values. (To find M_{mep} value at b, Eqs. (17)–(18) spreadsheets of $\eta(M)$ and $N(M)$ were prepared for b values covering the range .05–.6; these numerical values are provided in Table 3.) Graphical representation of the LJL pump characteristics as shown in Figure 4 is useful in estimating b values when the jet pump application fixes N or M, and hence b. Example 3 illustrates the utility of Figure 4.

Straight-Line N(M) Approximation Early investigators (see Reference 2) noted a nearly linear behavior of N versus M. A "straight-line" approximation for $N(M)$ based on the axis intercepts N_o and M_o would, of course, be useful. A true parabolic efficiency curve would result so $M_{\text{mep}} = \frac{1}{2} M_o$, and M_o is easily found. Unfortunately, the degree of this linearity varies with area ratio b. At about $b = .12$, the N curve is in fact a straight line. But as b is increased, the $N(M)$ curve is increasingly concave down so the $\eta = MN$ "parabolic" curve is increasingly skewed to the left. The converse is true at very small b values. Accordingly, the straight-line approximation is not recommended for use in designing LJL jet pumps. Instead, the Eqs. (17) and (18) theoretical model should be solved and charted for each b-value pump under consideration. Again, the recommended operating flow ratio $M_{\text{op}} = \frac{2}{3} M_{\text{mep}}$.

Longitudinal Dimensions of the LJL Jet Pump One-dimensional theory includes dimensions perpendicular to flow, but provides no guidance for longitudinal shapes/profiles. Two hardware dimensions of key importance must be taken from the literature or determined experimentally: nozzle-to-throat spacing "sp" and mixing throat length "L." Both are expressed in terms of throat diameters; sp/D_{th} and L/D_{th}. Recommended values are given as follows.

NOZZLE-THROAT SPACING Many experimental searches for optimum sp have been reported. Sanger and Vogel both found (see References 10 and 12) that maximum efficiency (slightly above 40% for Reference 10) is obtained with sp/$D_{\text{th}} = 0$, using nozzles with an external concave tapering, leading to a thin lip at the outlet. This configuration matches the theoretical model: The jet discharges to pressure P_o at the throat inlet, and thus the normal jet loss was eliminated. But a zero spacing promotes cavitation, even with a thin-lipped nozzle tip, and certainly with the rounded-nose exterior profile used in Reference 4. Sanger found that retracting nozzles a distance of about one diameter provided good cavitation resistance and at only a small loss in efficiency. Other investigators have found that performance is insensitive in the range of sp/$D_{\text{th}} = 0.5$–2 diameters, and that small spacings do promote cavitation. (see Reference 1). *It is recommended that LJL jet pumps be designed with* sp/$D_{\text{th}} = 1$.

MIXING-THROAT LENGTH The parallel-walled throat should be long enough to allow complete mixing, but throat lengths should be as short as possible to minimize frictional losses. L/D_{th} values as well as length (a venturi shape) ranging 1.0 to 10 have been reported for LJL pumps. Several factors affect optimum throat length:

a. When sp/D_{th} is finite—true for most jet pumps—mixing takes place in the distance (sp plus part or all of L, the mixing-throat length); thus, sp and L are interrelated. Sanger (see Reference 10) found that optimum sp/D_{th} increased from 0 to 2.3, for pumps with $L/D_{\text{th}} = 7.5$ and 3.5 respectively.

b. The primary-flow nozzle affects required L: Long tapered nozzles promote boundary-layer build-up producing jets that delay mixing, increasing required L. Multi-hole nozzles and swirl-inducing nozzles promote mixing and reduce required length L, but pump efficiency suffers because of increased nozzle-flow losses. (See "Primary-Flow Nozzle Design.")

c. Pump area-ratio b can affect optimum throat length. Small-b pumps operate with high flow ratios and throat lengths of $L/D_{\text{th}} = 8$ were required. For pumps with larger b values, throat lengths of four diameters sufficed (see Reference 13).

d. The gas compressor jet pump (LJG) requires longer throats, as high as 10–30 throat diameters (see Reference 7).

$L/D_{\text{th}} = 6$ *is recommended for general LJL design use.* Efficiency of the proposed pump may subsequently be improved by optimizing L (experimentally) for the given pump and duty.

PRIMARY-FLOW NOZZLE DESIGN A short-entry internally-convex profile similar to the ASME metering flow nozzle, is recommended for the LJL pumps. Avoid long conical nozzles.

Liquid-jet flow from a sharp-edged orifice mixes readily and is recommended for the LJG gas compressor (see Reference 7). The annular-nozzle liquid-jet pump has been investigated. In this configuration the secondary flow is axial, surrounded by the primary flow at the throat entrance. This arrangement is advantageous in pumping sticky secondary fluids because it prevents wall contact of the sticky fluid at the throat entry (see Reference 1). An obvious disadvantage is the increased nozzle frictional loss caused by flow of the primary fluid over the comparatively large surface area of the annular nozzle.

THROAT-INLET CONTOUR Many jet pumps reported in the early literature had long (small-convergence-angle) conical sections connecting the suction chamber to the throat, and usually including a sudden wall angle change at the throat entry. Later developments led to the short (large-convergence-angle) entry, well rounded at the throat. The long (small-angle) conical entry is wrong because its proximity to the nozzle exterior throttles the secondary flow and because it promotes cavitation. Secondly, a long approach section increases wall friction (reflected in a high K_{en} if it is measured). A short entry to the throat and a well-rounded profile connecting the suction chamber and throat is recommended.

Laboratory Flow Tests Performance testing of LJL pumps requires a facility with appropriate instrumentation, pumps, flow meters, and control valves. The flow rates Q_1 and Q_2, and at least three static pressures, P_i, P_s, and P_d, must be recorded at each test point. In addition, it is recommended that the throat section(s) contain static pressure taps for measurement of P_o and P_t. Jet pump test data will then permit measurement of K_{en}, K_{th} and K_{di}. For on-design operation, these *K*s vary little, if at all, with change in flow ratio M. Departures of $K(M)$ from nominal levels serve to reveal otherwise hidden problems. One example is that K_{th} will change if an increase in flow ratio causes mixing to extend/persist from the throat section into the diffuser (inadequate throat length). Another example is that a sudden rise in K_{di} may indicate diffuser-wall separation.

LIQUID-JET LIQUID (LJL) PUMP DESIGN EXAMPLES

Equations (1), (3), (5) and (7) will model jet pumps for LJL, LJG and LJGL configurations. Equations (1), (11–19) apply specifically to the LJL jet pump. The three numerical examples provided here are for this widely used pump only. LJG and LJGL liquid-jet pumps are less common: please see References 5, 6 and 7.

Example 1 Design a jet pump to handle 50 gpm (11.36 m^3/h) of water at 80°F (26.7°C) from a suction at 14.7 psia atmospheric pressure (101.325 kPa) to discharge at 40 psi (275.8kPa). Determine the required primary flow rate, jet nozzle pressure and dimensions of the jet pump.

Solution: For best efficiency, select $b = .25$. From Table 2, adopt $K_n = 0.5, K_{td} = .20$, and $K_{en} = 0$. Eq. (17) then produces $N(M)$. A computer spreadsheet table for Eqs. (17) and (18) showing output values based on increments of M is recommended, for example, see Table 1. As shown by the bottom line in Table 1, the $M_{mep} = 1.104$ (found by successive approximations using the spreadsheet program). The operating flow ratio is $M_{op} = \frac{2}{3}M_{mep} = .676$. The spreadsheet program at this M_{op} produces the N_{op} (.428) and η(28.934%) values shown in Table 1, second line from the bottom. From Eq. 19 $(P_d - P_s)/P_i - P_s) = N/(N + 1)$, so and $P_i - P_s = (1.428/.4828)40 = 1.3346$ psi (920.212 kPa). Thus $P_i = 1.3346 + 14.7 = 148.16$ psia (1021.56kPa). Liquid jet pumps—with very few exceptions—operate with the nozzle tip withdrawn from the throat entry by one throat-diameter or more. The nozzle tip experiences a discharge pressure close to P_s, not P_o, and "jet loss" thus occurs. For this jet loss condition, Eq. (1) changes to $(P_i - P_s) = Z(1 + K_n)$. The jet velocity head $Z = 1.3346/1.05 = 1.2710$ psi (876.35kPa).

CAVITATION-LIMITED FLOW RATIO M_L The flow ratio M_L is now evaluated using Eq. (20) to answer the question: will $M_{op} = .676$ avoid pump cavitation? The area ratio $c = (1 - .25)/$

.25 = 3. For the secondary fluid (water at 80°F, 26.7°C), P_v = .506 psia (3.49 kPa). The conservative value of σ = 1.35 is used in Eq. (20) as follows:

$$M_L = 3\sqrt{\frac{14.7 - .506}{1.35 \times 1.2710}} = .863$$

Because M_{op} (.676) < M_L (.863), this pump will not cavitate at the specified operating condition.

JET PUMP DIMENSIONS $Q_1 = Q_2/M_{op}$ = 50/.676 − 73.96 gpm (16.8 m^3/h) = 73.96/(7.481 × 60) = .165 ft^3/s. Longitudinal dimensions: sp/D_{th} = 1.0, thus sp = .937 in (23.81 mm). For L/D_{th} = 6, L = 6 × .937 = 5.62 in (142.75 mm). A diffuser with an included angle of 5° (conservative) and area ratio a = .224 would have length of approximately 12 in (304.8 mm). A shorter diffuser may be desirable, but kinetic-energy leaving losses will be higher.

EXAMPLE 1 USING OTHER AREA RATIOS An infinite number of different jet pumps can be designed to handle the Example 1 duty of pumping 50 gpm at 40 psi discharge pressure. The previous numerical example (b = .25) was repeated using b = .1, .4, and .6. Table 4 compares the results of these four b-ratio pumps. In each case the design is based on $M_{op} = \frac{2}{3}M_{mep}$; the assumed K values and calculation procedure are similar for the four pumps.

The expression CR% in Table 4 indicates the $(M_L - M_{op})/M_{op}$ % separation of operating flow ratio M_{op} and the limiting flow ratio M_L; the larger this number, the better. For this constant-duty example (Q_2 = 50 gpm and P_d = 40 psi), Table 4 shows that cavitation resistance can be improved by using a larger b ratio, i.e., the $(M_L - M_{op})/M_{op}$ % figure increases with b. Note that at the smallest b value (.1), the cavitation value (sixth column) is CR = −5.3%: this indicates that $M_{op} > M_L$, and this pump *would* encounter cavitation: b = .1 should not be used at these M_{op} and σ values. A redesign of the pump suction chamber (nozzle-external profile and throat-inlet profile) leading to an improvement in cavitation coefficient σ could render the b = .1 pump usable (the value σ = 1.35 used previously is conservative). For example, an improvement to σ = 1.0 would raise M_L to 2.0. With this change, a b = .1 pump could be used, cavitation-free.

Table 4 shows two other important facts: (1) The b = .25 pump provides the highest efficiency, even though all four are designed to operate at $\frac{2}{3}M_{mep}$, i.e., each at a similar position on the efficiency curve specific to that pump. (2) Small b-value pumps operate at large nozzle pressure-drops ($(P_i - P_s)$) and with small Q_1 primary-values. Conversely, high b-value pumps operate with small nozzle pressure-drops, but use a large flow rate Q_1. The pumps have in common similar expenditures of energy (Q_1 × pressure change, and allowing for efficiency differences) to handle the duty, which is the same in all four cases. In some design problems the nozzle-pressure—or possibly the primary-fluid flow volume—might outweigh the importance of mechanical efficiency. Table 4 shows how an adjustment in b might be used to achieve improved cavitation, albeit with efficiency sacrifices.

REDUCING THE FLOW RATIO M_{OP} TO COPE WITH CAVITATION For a given b-value LJL jet pump, a reduction in M_{op} offers a way to improve cavitation resistance. The b = .1 pump used

TABLE 4 Four b-value jet pumps for duty of 50 gpm (11.35 m^3/h) at P_d = 40 lb/in² (275 kPa)

b	M_{op}	N_{op}	η%	M_L	CR%	$(P_i - P_s)$, lb/in²	Q_1, gpm	D_n, in
0.10	1.8080	0.1509	27.28	1.7122	−5.30	305.08	27.65	0.233
0.25	0.6760	0.4280	28.93	0.8628	27.65	133.46	73.96	0.469
0.40	0.3347	0.8039	26.91	0.5261	57.19	89.76	149.39	0.736
0.60	0.1411	1.4913	21.04	0.2711	92.13	66.82	354.36	1.219

[Note: for SI conversion: in × 25.4 = mm]

TABLE 5 Reducing M_{op} improves CR% cavitation resistance ($b = .1$)

M_{op}	N_{op}	η%	M_L	CR%	$(P_i - P_s)$, lb/in²	Q_1, gpm	D_n, in
1.808	0.1509	27.28	1.7122	−5.30	305.08	27.65	0.233
1.500	0.1631	24.46	1.7704	18.01	285.08	33.33	0.260
1.000	0.1823	18.23	1.8566	85.67	259.45	50.00	0.326
0.500	0.2006	10.03	1.9326	286.56	239.36	100.00	0.471

[For SI conversions: lb/in² × 6.89 = kPa; gpm × 0.277 = m³/h; in × 25.4 = mm]

in this example is examined for the effects of M_{op} reduction, in Table 5. The conditions for all four pumps in Table 5 are $b = .1$, and again, $K_n = .05$, $K_{td} = .2$, $K_{en} = 0$. The Table 5 top line is $M_{op} = 1.808$, followed by three .5 decrements of M_{op}. The corresponding N_{op} and η_{op} values are found from successive approximations using the spreadsheet based on Eqs. (17) and (18). The M_L values are calculated from Eq. (20). The CR% cavitation merit figure (5th column) is the same as used in Table 4. Table 5 shows that while a $b = .1$ pump at $M_{op} = 1.808$ flow ratio will cavitate, the three lower M_{op} cases all exhibit increased CR% cavitation resistance. In Table 5, M_L (4th column) changes little with M_{op}; the cavitation improvements largely result from the decreases in M_{op}. The primary flow rate Q_1 increases markedly, and the nozzle pressure decreases somewhat as M_{op} is reduced. Compared with adjustment in b, cavitation avoidance by reducing M_{op} has obvious disadvantages: Efficiency is severely reduced, and the pump size (see D_n, 8th column) must be increased to handle the larger flow Q_1.

In summary, every jet pump design should be evaluated for cavitation resistance. The CR% index used here ($(M_L - M_{op})/M_{op}$ %) is suggested if several options are to be compared. Otherwise, the simpler test $M_{op} < M_L$, is adequate. Cavitation resistance for a given pump duty can be improved by increasing the area ratio b, or by reducing M_{op} of a particular pump, or by reducing the cavitation coefficient σ by improved internal design.

Example 2 Design a jet pump to lift 80°F (27.67°C) water to 20 ft (6.1m) to discharge at an atmospheric pressure of 14.7 psia or 34 ft of water (10.3 m). A mechanical pump provides the primary flow stream; fluid power available at the jet pump is 10 kW. Find the secondary flow Q_2, and pump dimensions.

Solution: Assume the same K values used in Example 1. Again select $b = .25$, for maximum efficiency. Eq. 17 (via spreadsheet program) produces $M_{mep} = 1.014$ and $M_{op} = \frac{2}{3}(1.014) = 6.76$, where $N_{op} = .428$ and $\eta = 28.93\%$. $P_s = (34\text{-}20) = 14$ ft water abs. (4.261 m abs.) at the suction chamber. And $P_s = 14 \times 1.934 \times 32.174/144 = 6.05$ psia (41.72 kPa). $(P_d - P_s) = (14.7 - 6.05) = 8.65$ psi (59.64 kPa). From Eq. 19, $(P_i - P_s) = (P_d - P_s)(N + 1)/N$, so that $(P_i - P_s) = (14.7 - 6.05)(1.428/.428) = 28.86$ psi (199 kPa). Pi = 28.86 + 6.05 = 34.91 psia (40.7 kPa). $Z = (P_i - P_s)/(1 + K_n) = 28.86/1.05 = 27.486$ psi (189.52 kPa).

Cavitation-Limited Flow Ratio M_L   Limiting flow ratio (Eq. 20)

$$M_L = 3\sqrt{\frac{6.05 - .506}{1.35 \times 27.486}} = 1.16$$

Because M_{op} (.676) < M_L (1.16), the proposed LJL jet pump will not cavitate.

JET PUMP DIMENSIONS Power in = $Q_1(P_i - P_d)$; Q_1 = power in/$(P_i - P_d)$, where $(P_i - P_d) = (34.91 - 14.7) = 20.21$ psi. With 10 kW treated here as the net "fluid" power available at the nozzle. (In Figure 2 the jet and mechanical pumps are at the same elevation)

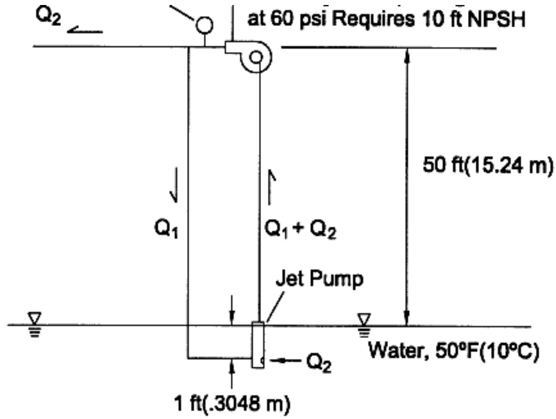


FIGURE 5 Jet and centrifugal pump system, for Example 3

$$Q_1 = \frac{10 \times 7.481 \times 60}{1.356 \times 10^{-3} \times 144 \times 20.21} = 1137.42 \text{ gal/min } (258.6 \text{ m}^3\text{/h})$$

(conversion factors 7.481 gal/ft³ and 1.356 watts/ft-lb/s were used previously)
Secondary flow $Q_2 = 1137.42 \times .676 = 768.9$ gpm (174.6 m³/h).

Nozzle area:

$$A_n = \frac{Q_1}{V_n}, \text{ where } V_n = \sqrt{2 \times 27.476 \times \frac{144}{1.934}} = 64 \text{ ft/s } (19.51 \text{ m/s})$$

$$A_n = (768.9 \times 144)/(64 \times 7.481 \times 60) = 3.85 \text{ in}^2 (24.84 \text{ cm}^2)$$

Nozzle diameter:

$$D_n = \sqrt{3.85 \times \frac{4}{3.1416}} = 2.22 \text{ in } (56.39 \text{ mm})$$

Example 3 As shown in Figure 5, a surface-mounted centrifugal pump requires a NPSH of 10 ft (3 m) of water, and the centrifugal-pump discharge pressure is 60 psi (414 kPa) at 100 gpm (22.7 m³/h). Water at 50°F (10°C) is to be lifted 50 ft (15.2 m). Design the jet pump including dimensions, and find the primary and secondary flow rates.

Solution: The jet pump's placement and centrifugal-pump discharge and inlet pressures determine all three jet-pump operating pressures, and hence the pressure characteristic N.

$$P_i = 60\, psi + (50 + 1) \times 1.94 \times 32.17/144 = 82.1\, psi\ (566.1\, kPa)$$

$$P_s = 1 \times 1.94 \times 32.17/144 = .433 \text{ psi } (299\text{kPa}).\ P_d = (50 + 10) \times$$

$$1.94 \times 32.17/144 = 26 \text{ psi } (179.27 \text{ kPa}).\ \ N_{op} = (P_d - P_s)/(P_i - P_d) = .456.$$

To estimate the b level which will operate at this N, enter Figure 4 with $N_{\text{op}} = .46$: the corresponding $b = .27$, and $M_{\text{op}} = .62$. Estimate b values should be confirmed—and adjusted if necessary—using a spreadsheet solution based on Eqs (17) and (18). In this case the spreadsheet results are $N_{op} = .461$, $b = .265$, $M_{op} = .626$, and $\eta = 28.8\%$. The cavitation check and calculation of dimensions follow next. *Note:* This solution is a first estimate because piping frictional losses have not been included in the calculations. Second and third—if necessary—calculations must include the pressure losses in the piping connecting the two pumps.

CAVITATION LIMITED FLOW RATIO M_L Jet velocity head $Z = (P_i - P_s)/(1 + K_n) = (82.1 - .433)/(1 + .05) = 77.78$ psi (536.29kPa).

For water at 50°F (10°C), $P_v = .178$ psia (1.23 kPa)

$$M_L = c\sqrt{\frac{P_s - P_v}{\sigma} \times Z} = 2.774\sqrt{\frac{15.133 - .178}{1.35} \times 77.78} = 1.047$$

Because M_{op} (.626) < M_L (1.047), the jet pump will not cavitate at the operating point.

JET PUMP DIMENSIONS $Q_1 + Q_2 = 100$ gpm (19.71 m³/h), and $Q_2/Q_1 = .626$, so $Q_1 = 61.5$ and $Q_2 = 38.5$ gpm (12.12 and 7.59 m³/h). $A_n = Q_1/V_n$.

Jet velocity:

$$V_n = \sqrt{\frac{2Z}{\rho_1}} = \sqrt{\frac{2 \times 77.8}{1.94}} = 107.46 \text{ ft/s } (32.75 \text{ m/s}).$$

$$Q_1 = 61.5/7.481 \times 60 = .137 \text{ ft}^3/\text{s}(13.97 \text{ m}^3/\text{h})$$

$$\textit{Nozzle area } A_n = (.137/107.46)144 = 1.87 \text{ in}^2(1.187 \text{ cm}^2).$$

Nozzle diameter:

$$D_n = \sqrt{\frac{4 \times A_n}{\pi}} = \sqrt{\frac{4(.184)}{3.1416}} = .483 \text{ in } (12.27 \text{ mm}).$$

Throat diameter:

$$D_{th} = \frac{D_n}{\sqrt{b}} = \frac{.483}{.515} = .938 \text{ in } (23.83 \text{ mm})$$

Nozzle-throat spacing recommended: $sp/D_{th} = 1$, hence sp $= .938$ in (23.83 mm)

Mixing-throat length recommended: $L/D_{th} = 6$, hence $L = 6\,(.938) = 5.63$ in (143 mm)

Centrifugal-Jet-Pump Interactions Oil- and water-well pumping are major applications of LJL pumps. Compact jet pumps, lowered down the bore hole near to or below the liquid surface, are powered by primary flow supplied from a mechanical pump located above, at the earth's surface. The jet pump pressurizes the surface pump's suction preventing cavitation. The design of a combination pump system normally requires designing for operation at the intersection of the two pumps' operating curves (Refs. 17, 18, and 19) and of course inclusion of frictional losses (neglected in Example 3) in the associated piping. One operating point (100 gpm at 60 psi) in Example 3 represented the centrifugal pump. The *60* psi discharge pressure of the centrifugal pump and the vertical placement fixed the jet pump's N_{op} operating point. This in turn determined the jet pump's b value. Consider now a change in the centrifugal pump to one providing 100 gpm at 100 psi (22.7 m³/h at 6.89 kPa): the jet pump's N_{op} (and b) would be reduced, and M_{op} raised. Details are omitted here, but the resultant area ratio would be about $b = .167$, which would provide $M_{op} = 1.09$, a considerable increase over $M_{op} = .626$, as found in Example 3.

SIGNIFICANCE OF ON-DESIGN OPERATING CONDITIONS

With reference to Figure 3, assume that the pump is operating on-design at $M_{op} = \frac{2}{3}M_{mep}$; that is, on the left branch of the efficiency "parabola." Primary flow rate Q_1 and suction pressure P_s are constant. If the back pressure P_d is then *reduced,* the secondary flow rate Q_2 (and hence M_{op}) will rise and pressure ratio N_{op} will fall. The new operating points will move to the right, along the characteristic curves for N and η. This increased-flow response to lowered P_d will end when cavitation-limited flow ratio M_L is reached, assuming the NPSH is low enough to allow cavitation. Any further reduction of P_d will cause the operating points to leave the N and η theoretical curves on paths vertically downward as shown by the dashed line in Fig. 3. Consider next the same initial condition (on design: $M_{op} = \frac{2}{3}M_{mep}$) followed by an *increase* in jet-pump discharge pressure P_d. In response, the secondary flow Q_2 (and hence M) will decrease, the N_{op} and η_{op} operating points moving left along the pump's N and η characteristic curves. Ultimately, increasing the back-pressure will result in reaching the $M = 0$ point, where $N = N_o$. (See Table 1, showing $N = N_o = .680$ at $M = 0$.)

Mechanical efficiency is important in most jet pump installations. Two low-efficiency situations that should obviously be avoided are

- Throttling the pump discharge pressure (to raise the P_d to the design point) wastes energy. Design P_d should match the pressure of the system into which the pump is to discharge.
- Operating the LJL pump with a discharge pressure P_d less than the design P_d (same Q_1 and P_s) will yield more Q_2 flow ($M > M_{op}$) and slightly higher efficiency. But such an increase in M may reach the cavitation point, that is, where $M = M_L$, causing a marked departure of actual/measured operating points downward from the N and η characteristic curves.

OTHER JET PUMP APPLICATIONS

Virtually any fluid can be pumped by the liquid-jet pump. Suspensions of solids in a liquid can be handled with the LJL analysis given by incorporating the "S" density-ratio term. In extreme cases, slurry pumping may also be affected by slip velocity and solids concentration (see References 1 and 15).

Steam jet pumps are found in the earliest literature (see Reference 1). Analyses have usually combined one-dimensional treatment with shock-wave compressibility phenomena. Theory has not contributed to the longitudinal dimensions problem, as is the case with liquid jet pumps. Advances in applying two- and three-dimensional flow theory have contributed to understanding low Mach-number mixing length requirements. Air/air jet pump theory and performance have been compared (see Reference 16). Analysis of high-velocity compressible-flow gas-jet pumps has been less successful.

Design of a two-pump combination system (illustrated by Examples 2 and 3) required system operation at an intersection of the mechanical- and jet-pump characteristic curves, selected so both pumps operate at or near their maximum efficiency points (see References 17, 18 and 19). And the system design must of course include consideration of the pressure losses that will occur in the piping connecting the two pumps.

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