Discrete Math Final

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Start Time: 3:00
End Time: 3:27

Question 1

We assume, upon the information given in the problem, that since $a|bc$, there exists some k such that $k \in \mathbb{Z}$ and $bc = k \cdot a$
We first multiply $ax + by = 1$ by c. Doing do produces the following:

$$axc + byc = c$$

Using our assumptions from the beginning, we can substitute to generate the following:

$$axc + kay = c$$

Factoring a out gives us:

$$a \cdot (cx + ky) = c$$

Assuming that c, x, k, and y are all included in $\mathbb{Z}$, we can define some arbitrary value n such that $n = cx + ky$. Using this substitution, we can derive the following:

$$a \cdot n = c$$

This equation is the definition of $a|c$, so our statement is proven.

Question 2

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> ### Part A
>
> **How many pea plants had at least 1 characteristic?**
> The problem statement says that there were 50 plants in total, with 4 showing no characteristics at all. Using this information, we can conclude that 46 plants have at least one characteristic.

> ### Part B

Using the information that we derive from the venn diagram above, we can tell that the number of plants that are tall and have smooth peas will be $b + a$. Thus, we find that this number is equal to 17.

Question 3

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Question 4

Part A

$$n(A) = 100, P(A) = 0.04$$

$$A_{(\text{defective})} = 0.04 \cdot 100$$

$$A_{(\text{defective})} = 4$$

$$n(B) = 80, P(B) = 0.05$$

$$B_{(\text{defective})} = 0.05 \cdot 80$$

$$B_{(\text{defective})} = 4$$

$$A_{(\text{defective})} + B_{(\text{defective})} = 8$$

$$P(\text{Defective}) = \frac{8}{180}$$

Part B

From a sample of 180 items, 100 were from A, thus making the following statement true:

$$P(\frac{S}{A}) = \frac{100}{180} = \frac{5}{9}$$

This continues on with those from B, showing that

$$P(\frac{S}{B}) = \frac{80}{180} = \frac{4}{9}$$

We know that the following stands true, so we can substitute our known values:

$$P(\frac{A}{B}) = \frac{P(A) \cdot P(\frac{S}{A})}{P(A) \cdot P(\frac{S}{A}) + P(B) \cdot P(\frac{S}{B})}$$

Substituting our known values gives us:

$$\frac{0.04 \cdot \frac{5}{9}}{0.04 \cdot \frac{5}{9} + 0.05 \cdot \frac{4}{9}} = \frac{\frac{0.2}{9}}{\frac{0.2}{9} + \frac{0.2}{9}}$$

Simplifying, we see the solution must be the following:

$$\frac{0.0222}{0.0444} = \frac{1}{2} = 0.5$$

Thus, the probability given in the problem statement is 0.5 or

$$50\%$$

Question 5

Using induction and the information given to us in the problem, we assume that the following statement is true where $y \in \mathbb{Z}$:

$$5^k - 1 = 4 \cdot y$$

Using the above statement, we can derive the following:

$$5^k = 4y + 1$$

Incrementing k by 1 gives us:

$$5^{k+1} - 1 = 5 \cdot 5^k - 1$$

Using our equivalences from above, we can substitute to derive the following:

$$5 \cdot (4y + 1) - 1 = 20 \cdot y + 4$$

Factoring again gives us:

$$4 \cdot (5y + 1)$$

We know that $5y + 1 \in \mathbb{Z}$, so we can conclude that $5^{k+1} - 1$ is divisible by 4, proving the problem statement.