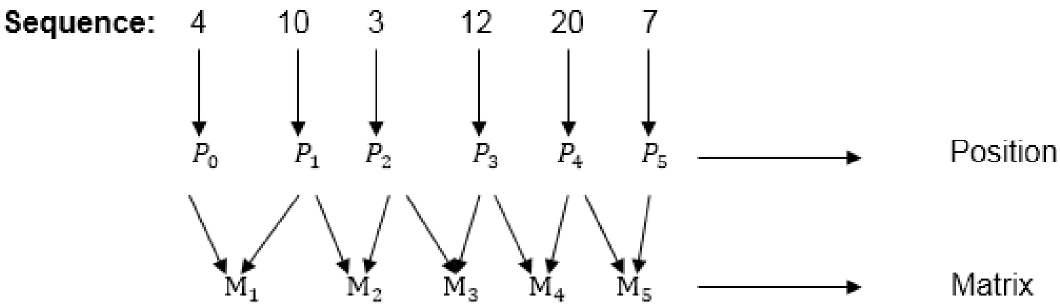
Example of Matrix Chain Multiplication

Example: We are given the sequence {4, 10, 3, 12, 20, and 7}. The matrices have size 4 x 10, 10 x 3, 3 x 12, 12 x 20, 20 x 7. We need to compute M [i,j], 0 ≤ i, j≤ 5. We know M [i, i] = 0 for all i.

1	2	3	4	5	
0					1
	0				2
		0			3
			0		4
				0	5

Let us proceed with working away from the diagonal. We compute the optimal solution for the product of 2 matrices.



Here P_0 to P_5 are Position and M_1 to M_5 are matrix of size (p_i to p_{i-1})

On the basis of sequence, we make a formula

For M_i ⟶ p [i] as column

p [i-1] as row

In Dynamic Programming, initialization of every method done by '0'.So we initialize it by '0'.It will sort out diagonally.

We have to sort out all the combination but the minimum output combination is taken into consideration.

Calculation of Product of 2 matrices:

1. m (1,2) = m_1 x m_2
   = 4 x 10 x 10 x 3
   = 4 x 10 x 3 = 120

2. m (2, 3) = m_2 x m_3
   = 10 x 3 x 3 x 12
   = 10 x 3 x 12 = 360

3. m (3, 4) = m_3 x m_4
   = 3 x 12 x 12 x 20
   = 3 x 12 x 20 = 720

4. m (4,5) = m_4 x m_5
   = 12 x 20 x 20 x 7
   = 12 x 20 x 7 = 1680

1	2	3	4	5	
0	120				1
	0	360			2
		0	720		3
			0	1680	4
				0	5

- We initialize the diagonal element with equal i,j value with '0'.
- After that second diagonal is sorted out and we get all the values corresponded to it

Now the third diagonal will be solved out in the same way.

Now product of 3 matrices:

```
M [1, 3] = M1 M2 M3
```

1. There are two cases by which we can solve this multiplication: (M_1 x M_2) + M_3, M_1+ (M_2x M_3)
2. After solving both cases we choose the case in which minimum output is there.

$$M\,[1,3] = \min \begin{Bmatrix} M\,[1,2] + M\,[3,3] + p_0\,p_2 p_3 = 120 + 0 + 4.3.12 \ = \ 264 \\ M\,[1,1] + M\,[2,3] + p_0\,p_1 p_3 = 0 + 360 + 4.10.12 \ = \ 840 \end{Bmatrix}$$

M [1, 3] =264

As Comparing both output **264** is minimum in both cases so we insert **264** in table and (M_1 x M_2) + M_3 this combination is chosen for the output making.

```
M [2, 4] = M2 M3 M4
```

1. There are two cases by which we can solve this multiplication: (M_2x M_3)+M_4, M_2+(M_3 x M_4)
2. After solving both cases we choose the case in which minimum output is there.

$$M\,[2,4] = \min \begin{Bmatrix} M[2,3] + M[4,4] + \ p_1 p_3 p_4 = 360 + 0 + 10.12.20 = 2760 \\ M[2,2] + \ M[3,4] + \ p_1 p_2 p_4 = 0 + 720 + 10.3.20 = \ 1320 \end{Bmatrix}$$

M [2, 4] = 1320

As Comparing both output **1320** is minimum in both cases so we insert **1320** in table and M_2+(M_3 x M_4) this combination is chosen for the output making.

```
M [3, 5] = M3  M4  M5
```

1. There are two cases by which we can solve this multiplication: (M_3 x M_4) + M_5, M_3+ (M_4xM_5)
2. After solving both cases we choose the case in which minimum output is there.

$$M\,[3,5] = \min \begin{cases} M[3,4] + M[5,5] + p_2p_4p_5 = 720 + 0 + 3\,.20.7 = 1140 \\ M[3,3] + M\,[4,5] + p_2p_3p_5 = 0 + 1680 + 3.12.7 = 1932 \end{cases}$$

```
M [3, 5] = 1140
```

As Comparing both output **1140** is minimum in both cases so we insert **1140** in table and (M_3 x M_4) + M_5this combination is chosen for the output making.

1	2	3	4	5	
0	120				1
	0	360			2
		0	720		3
			0	1680	4
				0	5

→

1	2	3	4	5	
0	120	264			1
	0	360	1320		2
		0	720	1140	3
			0	1680	4
				0	5

Now Product of 4 matrices:

```
M [1, 4] = M1  M2 M3 M4
```

There are three cases by which we can solve this multiplication:

1. (M_1 x M_2 x M_3) M_4
2. M_1 x(M_2 x M_3 x M_4)
3. (M_1 xM_2) x (M_3 x M_4)

After solving these cases we choose the case in which minimum output is there

$$M[1,3] + M[4,4] + p_0p_3p_4 = 264 + 0 + 4.12.20 = 1224$$

M [1, 4] =1080

As comparing the output of different cases then '**1080**' is minimum output, so we insert 1080 in the table and (M_1 xM_2) x (M_3 x M_4) combination is taken out in output making,

```
M [2, 5] = M2 M3 M4 M5
```

There are three cases by which we can solve this multiplication:

1. (M_2 x M_3 x M_4)x M_5
2. M_2 x(M_3 x M_4 x M_5)
3. (M_2 x M_3)x (M_4 x M_5)

After solving these cases we choose the case in which minimum output is there

$$\left(\mathrm{M}[2,4] + \mathrm{M}[5,5] + \mathrm{p}_1\mathrm{p}_4\mathrm{p}_5 = 1320 + 0 + 10.20.7 = \quad 2720 \right)$$

```
M [2, 5] = 1350
```

As comparing the output of different cases then '**1350**' is minimum output, so we insert 1350 in the table and M_2 x(M_3 x M_4 xM_5)combination is taken out in output making.

1	2	3	4	5

1	2	3	4	5

Now Product of 5 matrices:

```
M [1, 5] = M1  M2 M3 M4 M5
```

There are five cases by which we can solve this multiplication:

1. $(M_1 \times M_2 \times M_3 \times M_4) \times M_5$
2. $M_1 \times (M_2 \times M_3 \times M_4 \times M_5)$
3. $(M_1 \times M_2 \times M_3) \times M_4 \times M_5$
4. $M_1 \times M_2 \times (M_3 \times M_4 \times M_5)$

After solving these cases we choose the case in which minimum output is there

```
M [1, 5] = 1344
```

As comparing the output of different cases then '**1344**' is minimum output, so we insert 1344 in the table and $M_1 \times M_2 \times (M_3 \times M_4 \times M_5)$combination is taken out in output making.

Final Output is:

1	2	3	4	5	
0	120	264	1080		1

1	2	3	4	5	
0	120	264	1080	1344	1

Step 3: Computing Optimal Costs: let us assume that matrix A_i has dimension $p_{i-1} \times p_i$ for i=1, 2, 3....n. The input is a sequence $(p_0, p_1, \ldots p_n)$ where length [p] = n+1. The procedure uses an auxiliary table m [1....n, 1.....n] for storing m [i, j] costs an auxiliary table s [1.....n, 1.....n] that record which index

of k achieved the optimal costs in computing m [i, j].

The algorithm first computes m [i, j] ← 0 for i=1, 2, 3.....n, the minimum costs for the chain of length 1.

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