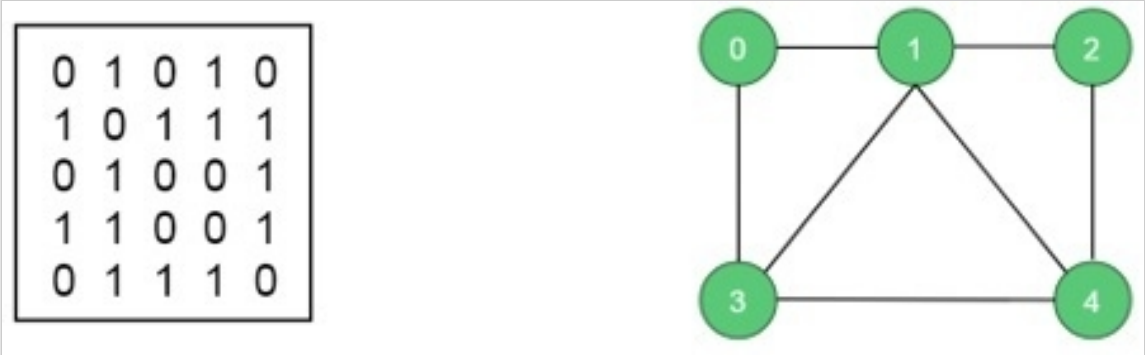
Hamiltonian Cycle

In an undirected graph, the Hamiltonian path is a path, that visits each vertex exactly once, and the Hamiltonian cycle or circuit is a Hamiltonian path, that there is an edge from the last vertex to the first vertex.

In this problem, we will try to determine whether a graph contains a Hamiltonian cycle or not. And when a Hamiltonian cycle is present, also print the cycle.

Input and Output

```
Input:
The adjacency matrix of a graph G(V, E).
```




```
Output:
The algorithm finds the Hamiltonian path of the given graph. For this case it is (0, 1,
If one graph has no Hamiltonian path, the algorithm should return false.
```

Algorithm

isValid(v, k)

Input – Vertex v and position k.

Output – Checks whether placing v in the position k is valid or not.

```
Begin
   if there is no edge between node(k-1) to v, then
      return false
   if v is already taken, then
      return false
   return true; //otherwise it is valid
End
```

cycleFound(node k)

Input – node of the graph.

Output – True when there is a Hamiltonian Cycle, otherwise false.

```
Begin
   if all nodes are included, then
      if there is an edge between nodes k and 0, then
         return true
      else
         return false;

   for all vertex v except starting point, do
      if isValid(v, k), then //when v is a valid edge
         add v into the path
         if cycleFound(k+1) is true, then
            return true
         otherwise remove v from the path
   done
   return false
End
```

Example

```
#include<iostream>
#define NODE 5
using namespace std;

int graph[NODE][NODE] = {
   {0, 1, 0, 1, 0},
   {1, 0, 1, 1, 1},
   {0, 1, 0, 0, 1},
   {1, 1, 0, 0, 1},
   {0, 1, 1, 1, 0},
};

/* int graph[NODE][NODE] = {
   {0, 1, 0, 1, 0},
   {1, 0, 1, 1, 1},
   {0, 1, 0, 0, 1},
   {1, 1, 0, 0, 0},
   {0, 1, 1, 0, 0},
}; */

int path[NODE];

void displayCycle() {
   cout<<"Cycle: ";

   for (int i = 0; i < NODE; i++)
```

```
      cout << path[i] << " ";
   cout << path[0] << endl;      //print the first vertex again
}

bool isValid(int v, int k) {
   if (graph [path[k-1]][v] == 0)   //if there is no edge
      return false;

   for (int i = 0; i < k; i++)   //if vertex is already taken, skip that
      if (path[i] == v)
         return false;
   return true;
}

bool cycleFound(int k) {
   if (k == NODE) {             //when all vertices are in the path
      if (graph[path[k-1]][ path[0] ] == 1 )
         return true;
      else
         return false;
   }

   for (int v = 1; v < NODE; v++) {       //for all vertices except starting point
      if (isValid(v,k)) {                 //if possible to add v in the path
         path[k] = v;
         if (cycleFound (k+1) == true)
            return true;
         path[k] = -1;               //when k vertex will not in the solution
      }
   }
   return false;
}

bool hamiltonianCycle() {
   for (int i = 0; i < NODE; i++)
      path[i] = -1;
   path[0] = 0; //first vertex as 0

   if ( cycleFound(1) == false ) {
      cout << "Solution does not exist"<<endl;
      return false;
   }

   displayCycle();
   return true;
}
```

```
int main() {
   hamiltonianCycle();
}
```

Output

```
Cycle: 0 1 2 4 3 0
```