Hydrologic Science and Engineering
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Intensity-Duration-Frequency (IDF) Curves
Example

Intensity-Duration-Frequency (IDF) curves describe the relationship between rainfall intensity, rainfall duration, and return period (or its inverse, probability of exceedance). IDF curves are commonly used in the design of hydrologic, hydraulic, and water resource systems. IDF curves are obtained through frequency analysis of rainfall observations.

Procedure

Data. From rainfall measurements, for every year of record, determine the annual maximum rainfall intensity for specific durations (or the annual maximum rainfall depth over the specific durations). Common durations for design applications are: 5-min, 10-min, 15-min, 30-min, 1-hr, 2-hr, 6-hr, 12-hr, and 24-hr (see for example Table 1 below.)

As discussed in class, the development of IDF curves requires that a frequency analysis be performed for each set of annual maxima, one each associated with each rain duration. The basic objective of each frequency analysis is to determine the exceedance probability distribution function of rain intensity for each duration. In class, we discussed two options for this frequency analysis:

1) Use an empirical plotting position approach to estimate the exceedance probabilities based on the observations.
2) Fit a theoretical Extreme Value (EV) distribution (*e.g.*, Gumbel Type I) to the observations and then use the theoretical distribution to estimate the rainfall events associated with given exceedance probabilities.

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Empirical Plotting Position Approach

To illustrate the first approach, select for example the 30-min duration data from Table 1 and proceed as follows:

1) Rank the observations in descending order (Table 2, Column 1)
2) Compute the exceedance probability associated with each rainfall volume using he following expression (Table 2, Column 4):
$$p = \frac{1}{T} = \frac{rank}{m+1} \tag{1}$$
where m is the number of observations, p is the exceedance probability and T is the corresponding return period (Table 2, Column 5).
3) Transform the volume data into rainfall intensity by dividing volume by the corresponding duration (Table 2, Column 6).
4) Plot empirical distribution of rainfall intensity (Columns 5 and 6 in Figure 1).

As indicated above, this procedure is repeated for each of the desired durations.

Table 1. Maximum Annual Rainfall Intensity for the Shown Duration

	Mean Annual Rainfall Volume for the Shown Duration [mm]								
	5 min 0.08 hr	**10 min 0.17 hr**	**15 min 0.25 hr**	**30 min 0.50 hr**	**1 hr 1 hr**	**2 hr 2 hr**	**6 hr 6 hr**	**12 hr 12 hr**	**24 hr 24 hr**
1985	2.8	5.3	8.1	10.9	13.7	14.4	24.2	28	30.4
1986	2.5	3.9	4.4	5.9	8.6	14.6	36.8	56.3	84.7
1987	1.5	2.5	3.2	5.5	9.9	17.7	33.8	43.2	65.3
1988	2	3.2	4.2	5.3	6.8	11.1	27.7	45	51.8
1989	3	4.3	5.2	6.9	9.3	15.2	30	45.6	50.9
1990	2.4	2.9	3.5	6.2	10.5	17.7	41.4	52.1	78.6
1991	2.6	3.6	4.8	6.4	10.7	17.4	36	66.4	100.9
1992	1.7	2	3.1	5.3	9.1	15.3	26.1	43.9	54.4
1993	2.8	4	4.5	7.4	10.8	15.8	27.2	38.2	64.9
1994	1.8	2.7	3.6	5.8	10.1	15	30.9	40.1	60.6
1995	2.5	3.2	4.1	5.9	9.4	14.4	33.7	50.7	82.6
1996	4.4	6.9	9.9	15.9	21.2	24	46.7	50.3	60.9
1997	3.1	3.6	4.3	6.7	10.5	15.9	38.8	54.8	65.2
1998	1.9	2.3	2.9	5.3	8.8	14.4	33.5	44.3	48.5
1999	2	2.5	3.5	6	10.8	17.4	35.9	48	59.4
2000	2	3.5	4	5.9	8.7	15	30.1	45.2	47.6
2001	2.9	4	4.2	5.5	7.8	13.2	23.2	36.2	45.6
2002	4.4	4.8	4.8	5.7	9.3	14.5	30	38	64.9
2003	2.3	4.2	5.6	8.1	8.7	11.8	29.1	45.5	72.6
2004	3.9	6.3	7.6	9.2	10.2	15.2	27.7	33	41
2005	3.2	5	6.5	8.5	9.8	13.9	24.1	34.5	43.7
Mean	2.65	3.84	4.86	7.06	10.22	15.42	31.76	44.73	60.69
St. Dev.	0.82	1.28	1.80	2.50	2.87	2.61	6.04	8.79	16.69

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Table 2. 30-min rainfall – Frequency Analysis

1	2	3	4	5	6
		30 min			
Rank	Year	0.50 hr	p	T	Intensity [mm/hr]
1	1996	15.9	0.05	22.00	31.8
2	1985	10.9	0.09	11.00	21.8
3	2004	9.2	0.14	7.33	18.4
4	2005	8.5	0.18	5.50	17
5	2003	8.1	0.23	4.40	16.2
6	1993	7.4	0.27	3.67	14.8
7	1989	6.9	0.32	3.14	13.8
8	1997	6.7	0.36	2.75	13.4
9	1991	6.4	0.41	2.44	12.8
10	1990	6.2	0.45	2.20	12.4
11	1999	6	0.50	2.00	12
12	1986	5.9	0.55	1.83	11.8
13	1995	5.9	0.59	1.69	11.8
14	2000	5.9	0.64	1.57	11.8
15	1994	5.8	0.68	1.47	11.6
16	2002	5.7	0.73	1.38	11.4
17	1987	5.5	0.77	1.29	11
18	2001	5.5	0.82	1.22	11
19	1988	5.3	0.86	1.16	10.6
20	1992	5.3	0.91	1.10	10.6
21	1998	5.3	0.95	1.05	10.6

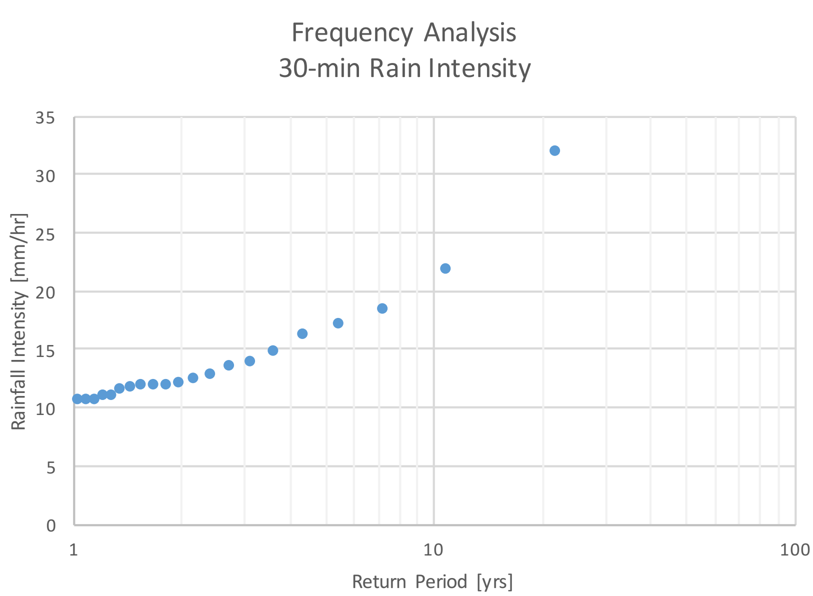

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Figure 1.

Theoretical Extreme Value (EV) Distribution Approach
To illustrate the second approach, let us select the Gumbel (Type I) distribution as our EV distribution. The Gumbel Type I distribution is,

$$G(x;\mu,\beta) = \frac{1}{\beta} e^{\frac{x-\mu}{\beta}} e^{-e^{\frac{x-\mu}{\beta}}} \tag{2}$$

where μ is the location parameter and β is the scale parameter.

It can be shown that the value of the random variable X_T associated with a given return period, T, may be obtained from the following expression,

$$X_T = \bar{X} + K_T S \tag{3}$$

where $\bar{X}$ is the mean of the observations (*e.g.*, arithmetic average of the observations), and S is the standard deviation of the observations. The frequency factor associated with return period T, K_T, is given by

$$K_T = -\frac{\sqrt{6}}{\pi}[0.5772 + \ln(\ln(\frac{T}{T-1}))] \tag{4}$$

Equations (1), (2) and (3) are applied to each set of annual maxima corresponding to each duration, as follows:

1. Compute the frequency factors associated with the desired return periods (*e.g.*, 2, 5, 10, 25, 50, 100, 1000) using equation (4).

Table 3. Frequency Factors

T	2	5	10	25	50	100	1000
K_T	-0.164272	0.7194574	1.3045632	2.0438459	2.5922880	3.1366806	4.9355236

2. For each duration (*e.g.*, 5-min, 10-min, …etc.), compute the sample mean and sample standard deviations of the series of annual maxima, ($x_1, \ldots\ldots, x_m$) (see Table 1).

$$\bar{X} = \frac{1}{m}\sum_{i=1}^{m} x_i \quad \text{and} \quad S = \frac{1}{m-1}\sum_{i=1}^{m}(x_i - \bar{X})^2$$

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3. Use equation (3) to compute the precipitation intensity associated with each return period.

	Return Period *T*						
	2	**5**	**10**	**25**	**50**	**100**	**1000**
Duration	30.213	38.904	44.658	51.928	57.322	62.676	80.366
5 min	30.213	38.904	44.658	51.928	57.322	62.676	80.366
10 min	21.795	28.585	33.080	38.759	42.973	47.155	60.976
15 min	18.248	24.600	28.806	34.121	38.063	41.976	54.907
30 min	13.303	17.719	20.642	24.336	27.076	29.797	38.785
1 hr	9.753	12.287	13.965	16.085	17.657	19.218	24.377
2 hr	7.497	8.651	9.415	10.380	11.096	11.807	14.155
6 hr	5.128	6.017	6.605	7.349	7.901	8.449	10.259
12 hr	3.607	4.254	4.683	5.225	5.626	6.025	7.343
24 hr	2.415	3.029	3.436	3.950	4.331	4.710	5.961

4. Plot the results (Figure 2).

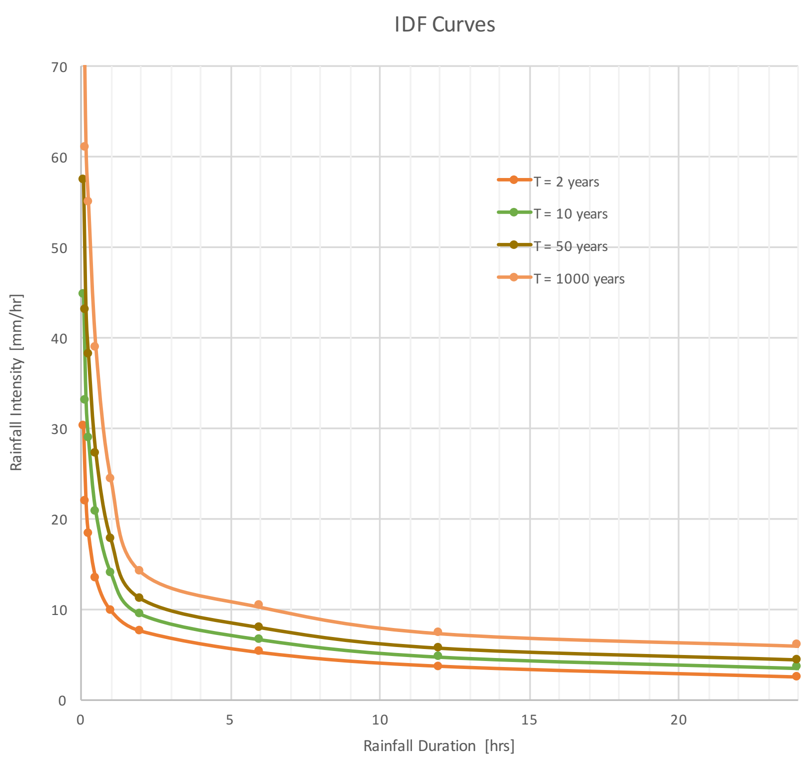

Figure 2.