Engineering Innovation

A Summer Program for High School Students

Uncertainty, Measurements and Error Analysis

JOHNS HOPKINS
WHITING SCHOOL of ENGINEERING

Objectives

Engineering Innovation
A Summer Program for High School Students

1. Statistics
2. Probability
3. Normal Distibutions
4. Mean and Standard Deviation
5. Measurements
6. Significant Figures
7. Accuracy and Precision
8. Error Analysis

JOHNS HOPKINS
WHITING SCHOOL
of ENGINEERING

Why Study Statistics?

Engineering Innovation

A Summer Program for High School Students

<u>Statistics</u>: A mathematical science concerned with data collection, presentation, analysis, and interpretation.

Statistics can tell us about…

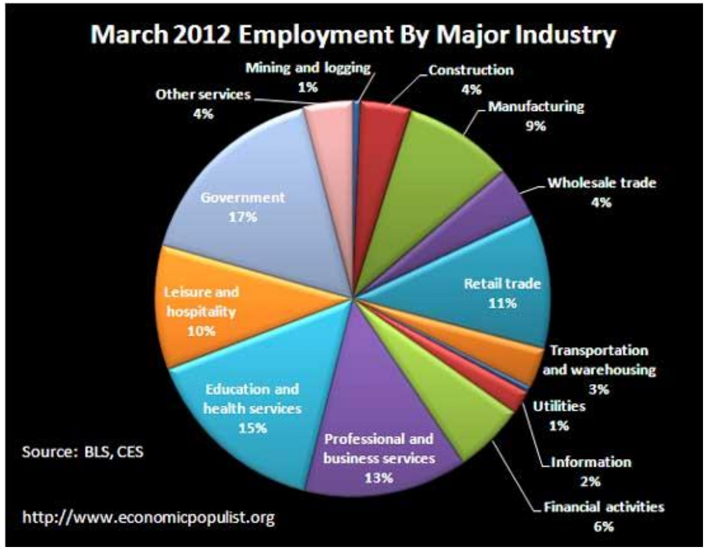
<u>Sports</u>

STATS

	GP	AB	R	H	2B	3B	HR	RBI
2014 Regular Season	159	644	88	181	30	2	29	96
2014 Postseason	7	27	6	6	0	0	1	3
Career	1105	4182	601	1169	204	24	169	577

http://espn.go.com/mlb/player/_/id/28513/adam-jones

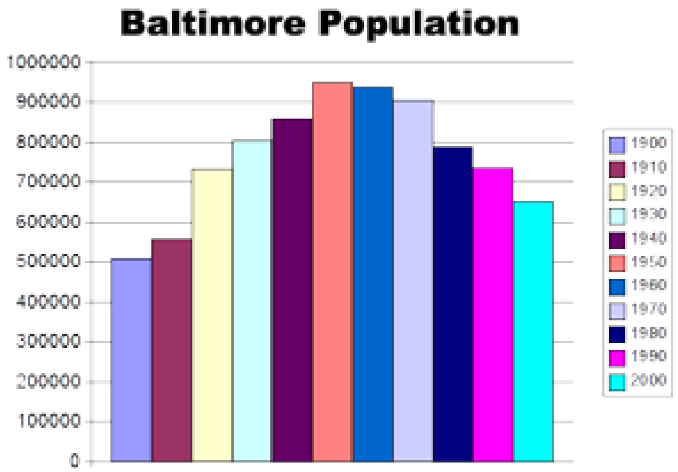
<u>Economy</u>



http://www.economicpopulist.org/content/peek-employment-report-establishment-survey

<u>Population</u>

http://baltimore-maryland.org/history/baltimore-population.png



JOHNS HOPKINS
WHITING SCHOOL of ENGINEERING

Why Study Statistics?

Engineering Innovation
A Summer Program for High School Students

Statistical analysis is also an integral part of scientific research!

Are your experimental results believable?

Example: Tensile Strength of Spaghetti

Data *suggests* a relationship between Type (size) and breaking strength

Pasta Type	Force (times g)
Angel Hair	21.5208
	25.431
Thin	33.7904
	41.454
Regular	62.622
	53.361
Thick	60.711
	5.9094

JOHNS HOPKINS
WHITING SCHOOL
of ENGINEERING

Why Study Statistics?

Engineering Innovation
A Summer Program for High School Students

Responses and measurements are variable!

Due to…

Systematic Error – same error value by using an instrument the same way

Random Error – may vary from observation to observation

Perhaps due to inability to perform measurements in exactly the same way every time.

Goal of statistics is to find the model that best describes a target population by taking sample data.

Represent randomness using <u>probability</u>.

JOHNS HOPKINS
WHITING SCHOOL
of ENGINEERING

Probability

Engineering Innovation
A Summer Program for High School Students

Experiment of chance: a phenomena whose outcome is uncertain.

Probabilities ⟷ Chances

Probability Model: Sample Space, Events, Probability of Events

Sample Space: Set of all possible outcomes

Event: A set of outcomes (a subset of the sample space). An event *E* occurs if any of its outcomes occurs. Rolling dice, measuring, performing an experiment, etc.

Probability: The likelihood that an event will produce a certain outcome.

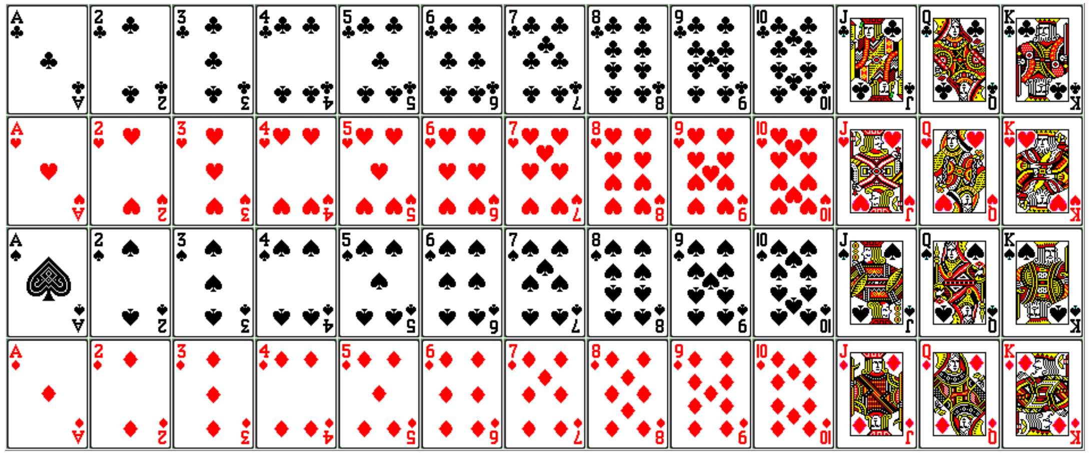
Independence: Events are independent if the occurrence of one does not affect the probability of the occurrence of another. Why important?

Johns Hopkins Whiting School of Engineering

Probability

Engineering Innovation
A Summer Program for High School Students

Consider a deck of playing cards…



Sample Space? Set of 52 cards

Event? *R*: The card is red.

F: The card is a face card.

H: The card is a heart.

3: The card is a 3.

Probability?

P(*R*) = 26/52

P(*F*) = 12/52

P(*H*) = 13/52

P(*3*) = 4/52

JOHNS HOPKINS
WHITING SCHOOL
of ENGINEERING

Events and variables

Engineering Innovation
A Summer Program for High School Students

Can be described as <u>random</u> or <u>deterministic</u>:

The outcome of a **random** event cannot be predicted:

The sum of two numbers on two rolled dice.

The time of emission of the i^{th} particle from radioactive material.

The outcome of a **deterministic** event can be predicted:

The measured length of a table to the nearest cm.

Motion of macroscopic objects (projectiles, planets, space craft) as predicted by classical mechanics.

Johns Hopkins
Whiting School of Engineering

Extent of randomness

Engineering Innovation
A Summer Program for High School Students

A variable can be more random or more deterministic depending on the degree to which you account for relevant parameters:

Mostly deterministic: Only a small fraction of the outcome cannot be accounted for.

Length of a table - only slightly dependent upon:

- Temperature/humidity variation
- Measurement resolution
- Instrument/observer error
- Quantum-level intrinsic uncertainty

Mostly Random: Most of the outcome cannot be accounted for.

- Trajectory of a given molecule in a solution

JOHNS HOPKINS
WHITING SCHOOL
of ENGINEERING

Random variables

Engineering Innovation
A Summer Program for High School Students

Can be described as <u>discrete</u> or <u>continuous</u>:

- A discrete variable has a countable number of values.

  Number of customers who enter a store before one purchases a product.

- The values of a continuous variable can not be listed:

  Distance between two oxygen molecules in a room.

Consider data collected for undergraduate students:

Random Variable	Possible Values
Gender	Male, Female
Class	Fresh, Soph, Jr, Sr
Height (inches)	Integer in interval {30,90}
College	Arts, Education, Engineering, etc.
Shoe Size	3, 3.5 … 18

Is height a discrete or continuous variable?

How could you measure height and shoe size to make them continuous variables?

JOHNS HOPKINS
WHITING SCHOOL of ENGINEERING

Probability Distributions

If a random event is repeated many times, it will produce a distribution of outcomes (statistical regularity).

(Think about the sum of the dots on two rolled dice)

The distribution can be represented in two ways:

- **Frequency distribution function:** represents the distribution as the number of occurrences of each outcome
- **Probability distribution function:** represents the distribution as the percentage of occurrences of each outcome

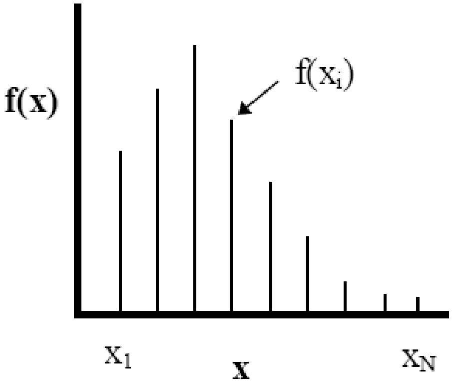
Discrete Probability Distributions

Engineering Innovation

A Summer Program for High School Students

Consider a <u>discrete</u> random variable, X:

$f(x_i)$ is the probability distribution function



Probability distribution function

What is the range of values of $f(x_i)$?

Therefore, $Pr(X=x_i) = f(x_i)$

JOHNS HOPKINS
WHITING SCHOOL of ENGINEERING

Discrete Probability Distributions

Engineering Innovation
A Summer Program for High School Students

Properties of discrete probabilities:

$$\Pr(X = x_i) = f(x_i) \geq 0$$ for all i

$$\sum_{i=1}^{k} \Pr(X = x_i) = \sum_{i=1}^{k} f(x_i) = 1$$ for k possible discrete outcomes

$$\Pr(a < X \leq b) = F(b) - F(a) = \sum_{a < x_i \leq b} f(x_i)$$

Where: $$F(x) = \Pr(X \leq x)$$

Johns Hopkins Whiting School of Engineering

Discrete Probability Distributions

Engineering Innovation

A Summer Program for High School Students

Example: Waiting for a success

Consider an experiment in which we toss a coin until heads turns up.

Outcomes, w = {H, TH, TTH, TTTH, TTTTH…}

Let X(w) be the number of tails before a heads turns up.

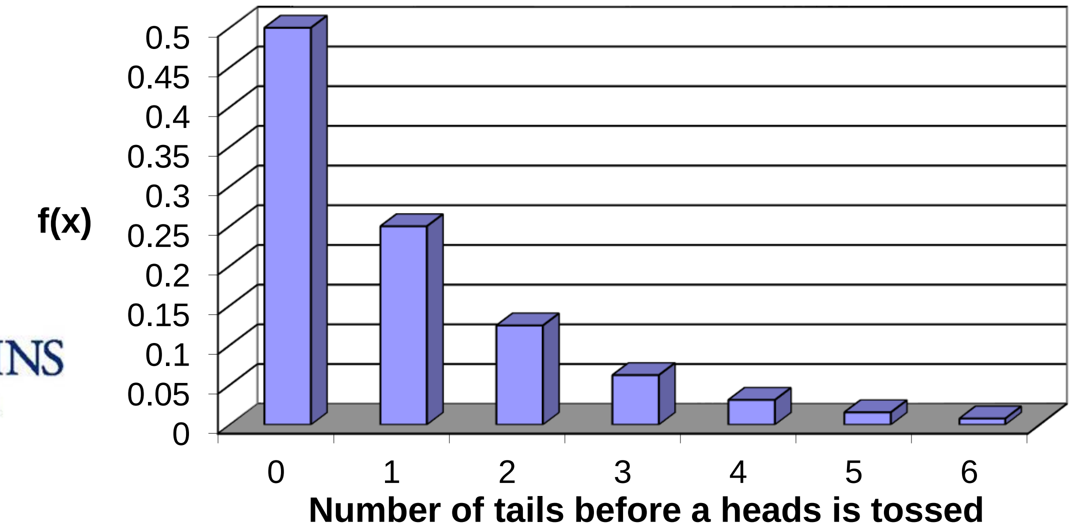
What is the Probability Distribution Function for the sample space?

$$f(x) = \frac{1}{2^{x+1}}$$

For x = 0, 1, 2….

$$\Pr(a < X \le b) = F(b) - F(a) = \sum_{a < x_i \le b} f(x_i)$$

$$\sum_{i=1}^{k} \Pr(X = x_i) = \sum_{i=1}^{k} f(x_i) = 1$$



JOHNS HOPKINS WHITING SCHOOL of ENGINEERING

Discrete Probability Distributions

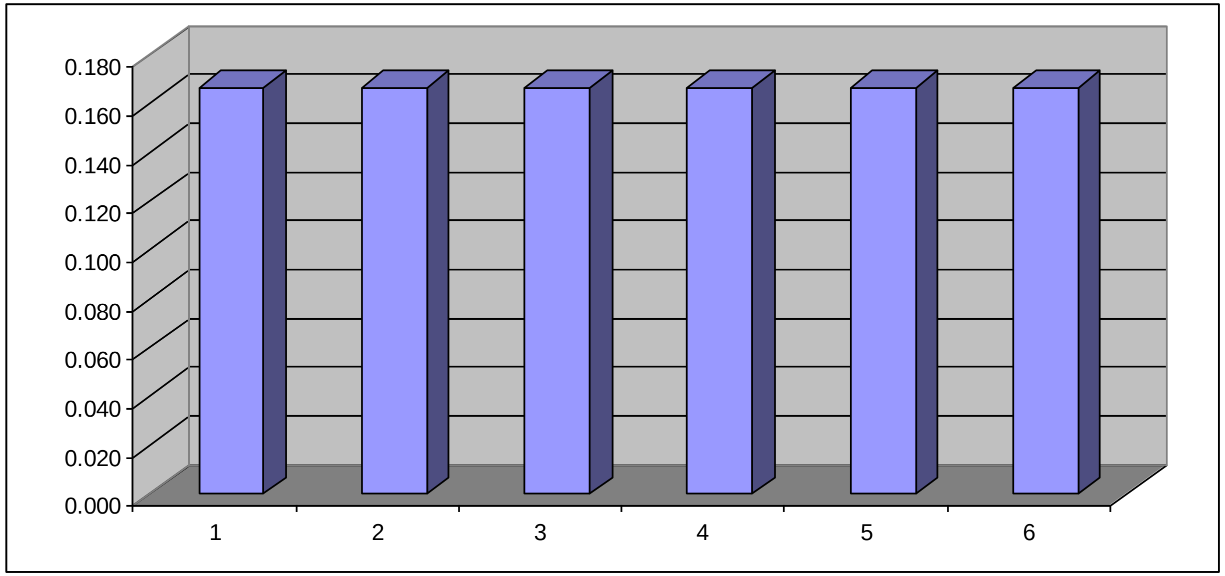
Engineering Innovation

A Summer Program for High School Students

Example: Distribution Function for Die/Dice

Distribution function for throwing a die:

Outcomes, w = {1, 2, 3, 4, 5, 6} → $f(x_i) = 1/6$ for $i = 1,6$



JOHNS HOPKINS WHITING SCHOOL of ENGINEERING

Discrete Probability Distributions

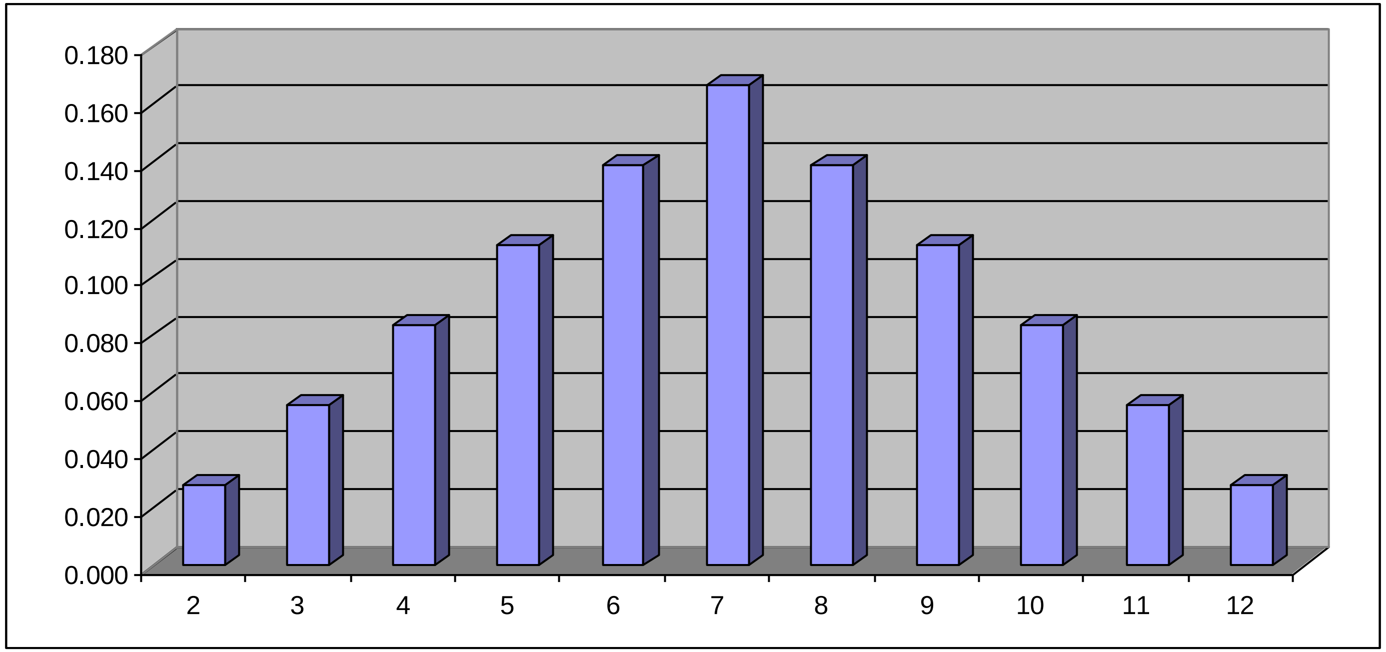
Example: Distribution Function for Die/Dice

Distribution function for the sum of two thrown dice:

$f(x_i) = 1/36$ for $x_1 = 2$

$2/36$ for $x_2 = 3$

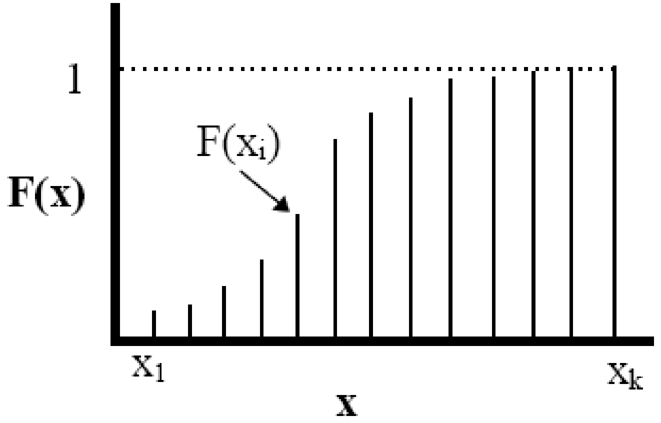
…



Cumulative Discrete Probability Distributions

$$\Pr(X < x') = F(x') = \sum_{i=1}^{j} f(x_i)$$

Where x_j is the largest discrete value of X less than or equal to x'



Cumulative distribution function

→ $$\Pr(X \leq x_k) = 1$$

<u>Continuous</u> Probability Distributions

Engineering Innovation
A Summer Program for High School Students

Properties of the cumulative distribution function:

$$F(-\infty) = 0$$

$$0 \leq F(x) \leq 1$$

$$F(x) = \Pr(X \leq x)$$

$$F(\infty) = 1$$

Properties of the probability density (distribution) function:

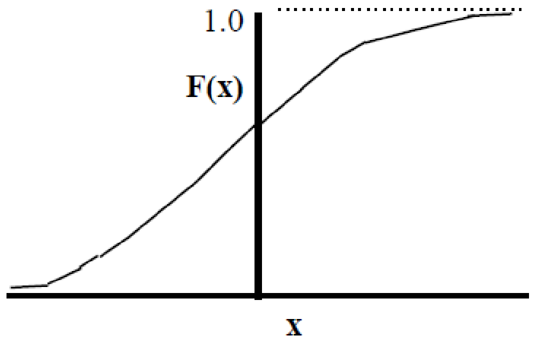
$$\Pr(a < X \leq b) = F(b) - F(a) = \int_a^b f(x)dx$$

JOHNS HOPKINS
WHITING SCHOOL of ENGINEERING

Continuous Probability Density Function

Cumulative Distribution Function (cdf): Gives the fraction of the total probability that lies at or to the left of each x

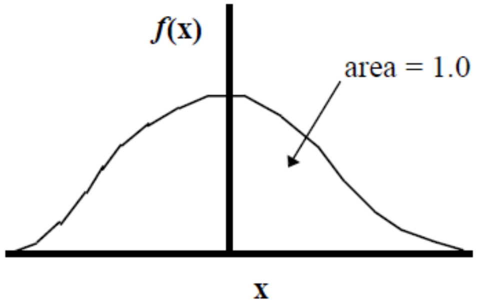
$$\Pr(X \leq x) = F(x) = \int_{-\infty}^{x} f(x)dx$$



Cumulative distribution function

Probability Density (Distribution) Function (pdf): Gives the density of concentration of probability at each point

$$f(x) = dF(x)/dx$$



Probability density function

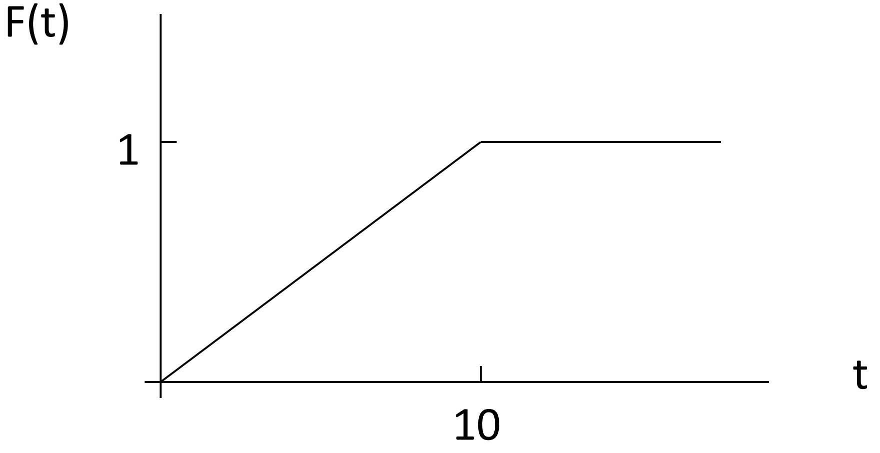
Continuous Probability Distributions

For continuous variables, the events of interest are intervals rather than isolated values.

Consider waiting time for a bus which is equally likely to arrive anytime in the next ten minutes:

Not interested in probability that the bus will arrive in 3.451233 minutes, but rather the probability that the bus will arrive in the subinterval (a,b) minutes:

$$P(a<T<b)=F(b)-F(a)=\frac{b-a}{10}$$

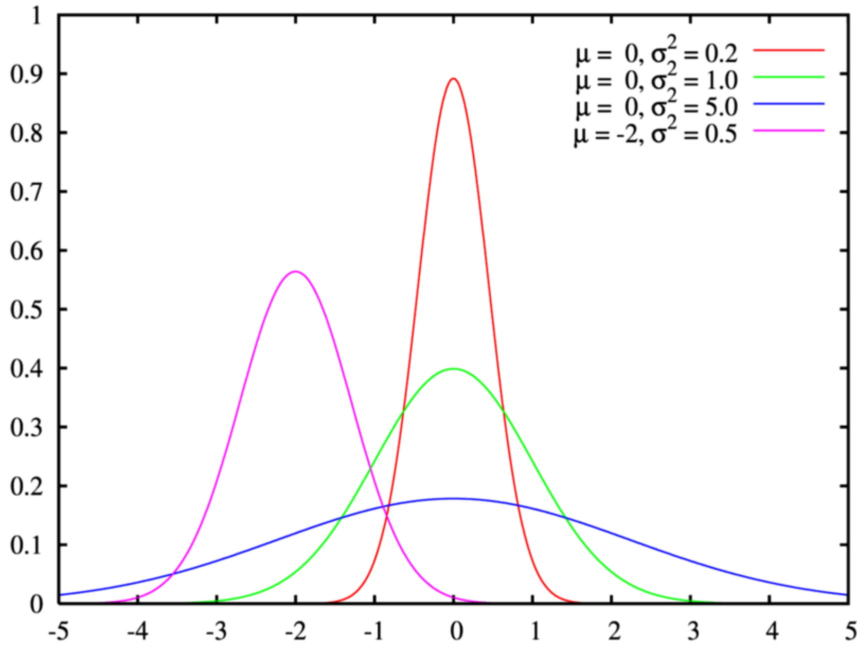


Continuous Probability Distributions

Engineering Innovation

A Summer Program for High School Students

Example: Gaussian (normal) distribution:



$Z = \dfrac{X-\mu}{\sigma}$	$X = \mu + \sigma Z$
$f(x) = \dfrac{1}{\sqrt{2\pi}\sigma} \exp\left[-\dfrac{(x-\mu)^2}{2\sigma^2}\right]$	

A normal distribution is described by the mean (μ) and variance (σ^2).

Standard normal curve: μ = 0, σ = 1.

JOHNS HOPKINS

WHITING SCHOOL *of* ENGINEERING

Mean, Standard Deviation, Variance

Engineering Innovation
A Summer Program for High School Students

Mean: The mean is the 1st moment about the origin; the average value of x

$$\mu = \nu_1 = \sum_{i=1}^{k} x_i f(x_i)$$

Standard Deviation: Standard deviation measures the spread of data about the mean.

$$\sigma = \frac{\Sigma(X-\mu)}{N}$$

Variance: Variance is second moment about the mean; the average squared distance of the d
from the mean.

$$\sigma^2 = \frac{\Sigma(X-\mu)^2}{N}$$

JOHNS HOPKINS
WHITING SCHOOL
of ENGINEERING

Practice!

Engineering Innovation
A Summer Program for High School Students

Two teams measure the height of a flagpole.

Height in cm

Team A		Team B
183		183.0
182		183.5
185		182.7
181		182.5
183		183.1
184		183.3
avg =		avg =
183		183.0
std dev =		std dev =
1.41		0.37

- Which team did the better job?
- Why do you think so?

JOHNS HOPKINS
WHITING SCHOOL
of ENGINEERING

Normal / Gaussian Distribution

Engineering Innovation
A Summer Program for High School Students

Normal (Gaussian) Distribution:

Can be used to approximately describe any variable that tends to cluster around the mean.

Central Limit Theorem:

The sum of a (sufficiently) large number of independent random variables will be approximately normally distributed.

Importance:

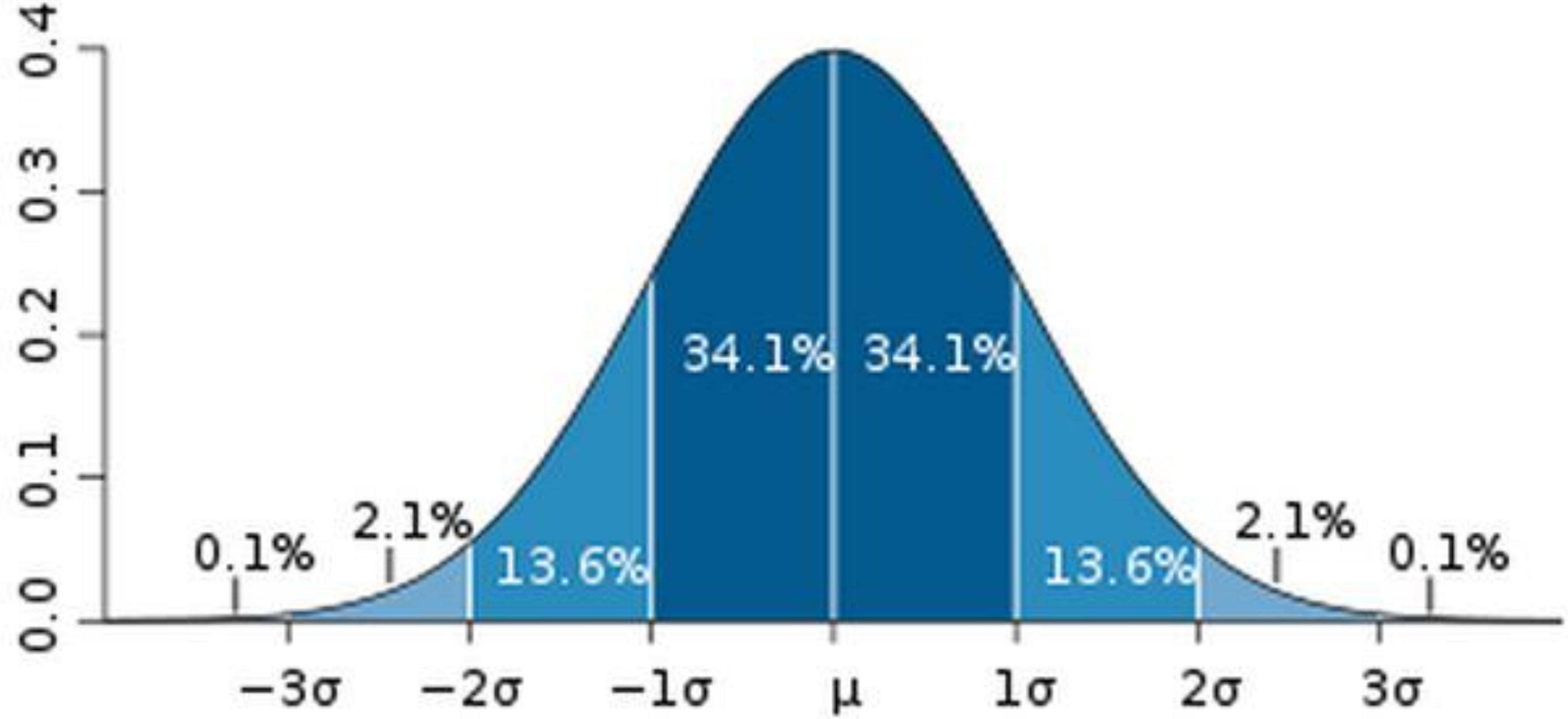
Used as a simple model for complex phenomena – statistics, natural science, social science [e.g., Observational error assumed to follow normal distribution]

JOHNS HOPKINS
WHITING SCHOOL of ENGINEERING

Examples of experiments/measurements that will produce Gaussian distribution?

Engineering Innovation

A Summer Program for High School Students



http://en.wikipedia.org/wiki/Standard_deviation

JOHNS HOPKINS
WHITING SCHOOL
of ENGINEERING

Standard Error

$$\frac{\sigma}{\sqrt{N}}$$

How do we reduce the size of our standard error?

1) Repeated Measurements
2) Different Measurement Strategy

Jacob Bernoulli (1731):
“For even the most stupid of men, by some instinct of nature, by himself and without any instruction (which is a remarkable thing), is convinced the more observations have been made, the less danger there is of wandering from one’s goal" (Stigler, 1986).

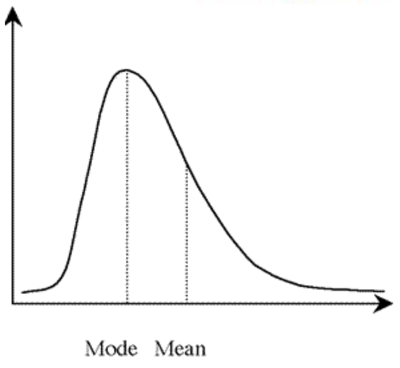
Moments

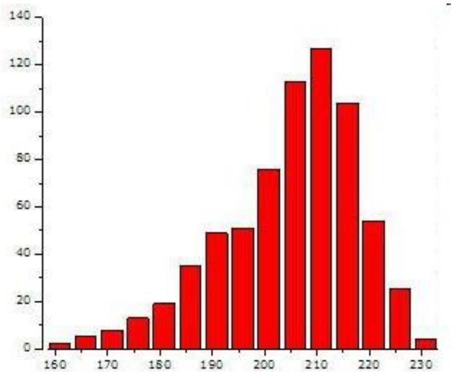
Engineering Innovation

A Summer Program for High School Students

Other values in terms of the moments:

Skewness: $\dfrac{\mu^3}{\left[\sigma^2\right]^{3/2}}$





→ 'lopsidedness' of the distribution

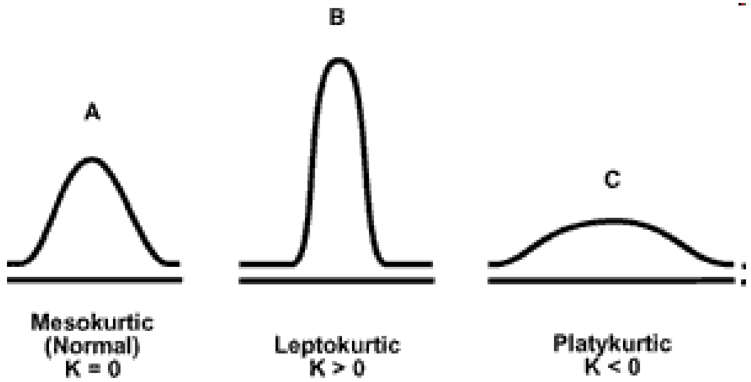
→ a symmetric distribution will have a skewness = 0

→ negative skewness, distribution shifted to the left

→ positive skewness, distribution shifted to the right

Kurtosis:

→ Describes the shape of the distribution with respect to the height and width of the curve ('peakedness')



JOHNS HOPKINS WHITING SCHOOL of ENGINEERING

http://hsc.csu.edu.au/maths/images/mc_images/DA5Q10AA.gif
http://www.connectseward.org/shs/da1/review07/Left%20Skewed.JPG
http://www.pqsystems.com/eline/2001/02/kurtosis.gif

Estimation of Random Variables

Engineering Innovation
A Summer Program for High School Students

Random Variable: a variable whose value is subject to variations due to chance

Assumptions/Procedures

→There exists a stable underlying probability density function for the random variable

→ Investigating the characteristics of the random variable consists of obtaining sample outcomes and making inferences about the underlying distribution

JOHNS HOPKINS
WHITING SCHOOL of ENGINEERING

Estimation of Random Variables

Engineering Innovation
A Summer Program for High School Students

Sample statistics on a random variable X

→The i^{th} sample outcome of the variable X is denoted X_i

→ The sample mean : $\bar{X} = \frac{1}{N}\sum_{i=1}^{N} X_i$ where N = sample size

→The sample variance: $s^2 = \frac{1}{N-1}\sum_{i=1}^{N}(X_i - \bar{X})^2$

$\bar{X}$ and s^2 are estimates of the mean, μ, and variance, σ^2, of the underlying p.d.f.

→ $\bar{X}$ and s^2 are estimates for the sample

→ μ and σ^2 are characteristics of the population from which the sample was taken

→$\bar{X}$ and s^2 are random variables

JOHNS HOPKINS
WHITING SCHOOL
of ENGINEERING

Expected Value, E(X)

The value that one would obtain if a very large number of samples were averaged together.

→$E[\bar{X}] = \mu$

→The expected value of the sample mean is the population mean

→$E[s^2] = \sigma^2$

→ The expected value of the sample variance is the population variance

Expected values allow us to use sample statistics to infer population statistics

Properties of Expected Values

Engineering Innovation
A Summer Program for High School Students

- $E[aX + bY] = aE[X] + bE[Y]$ where a and b are constants

- If Z = *g(X), then* E $[Z]$= *E[g(X)]* = $\sum_{all\ values\ x\ of\ X} g(x)Pr(X = x)$

→Example: Throw a die. If the die shows a “6” you win $5; else, you lose a $1.
What’s the expected value,E[Z], of this game?

Pr(X=1) = 1/6	g(1) = -1
Pr(X=2) = 1/6	g(2) = -1
Pr(X=3) = 1/6	g(3) = -1
Pr(X=4) = 1/6	g(4) = -1
Pr(X=5) = 1/6	g(5) = -1
Pr(X=6) = 1/6	g(6) = 5

E[Z] = ([(-1)*5*1]/6)+([5*1]/6) = 0, you would not expect to win or lose

JOHNS HOPKINS
WHITING SCHOOL
of ENGINEERING

Properties of Expected Values

- $E[XY] = E[X]E[Y]$ where X and Y are independent (samples of X cannot be used to predict anything about sample Y and Y cannot be used to predict X)

→Example: Find the area of a picture with height, X, and width, Y.

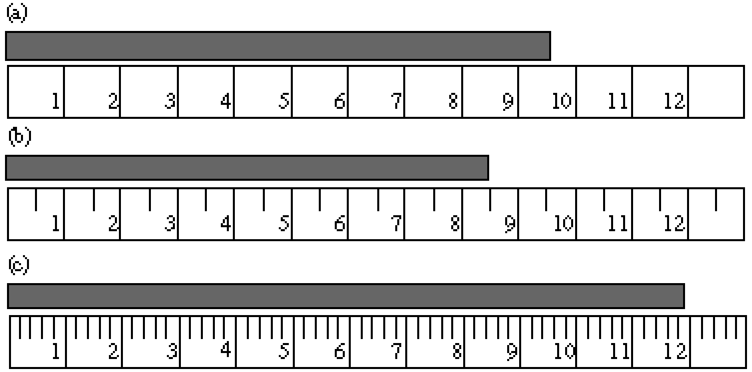
Measurements

All measurements have errors

Engineering Innovation
A Summer Program for High School Students

What are some sources of measurement errors?

- Instrument uncertainty (caliper vs. ruler)
  - Use half the smallest division (unless manufacturer provides precision information).



L = 9 ± 0.5 cm

L = 8.5 ± 0.3 cm

L = 11.8 ± 0.1 cm

- Measurement error (using an instrument incorrectly)
  - Measure your height - not hold ruler level.
- Variations in the size of the object (spaghetti is bumpy)
  - Statistical uncertainty

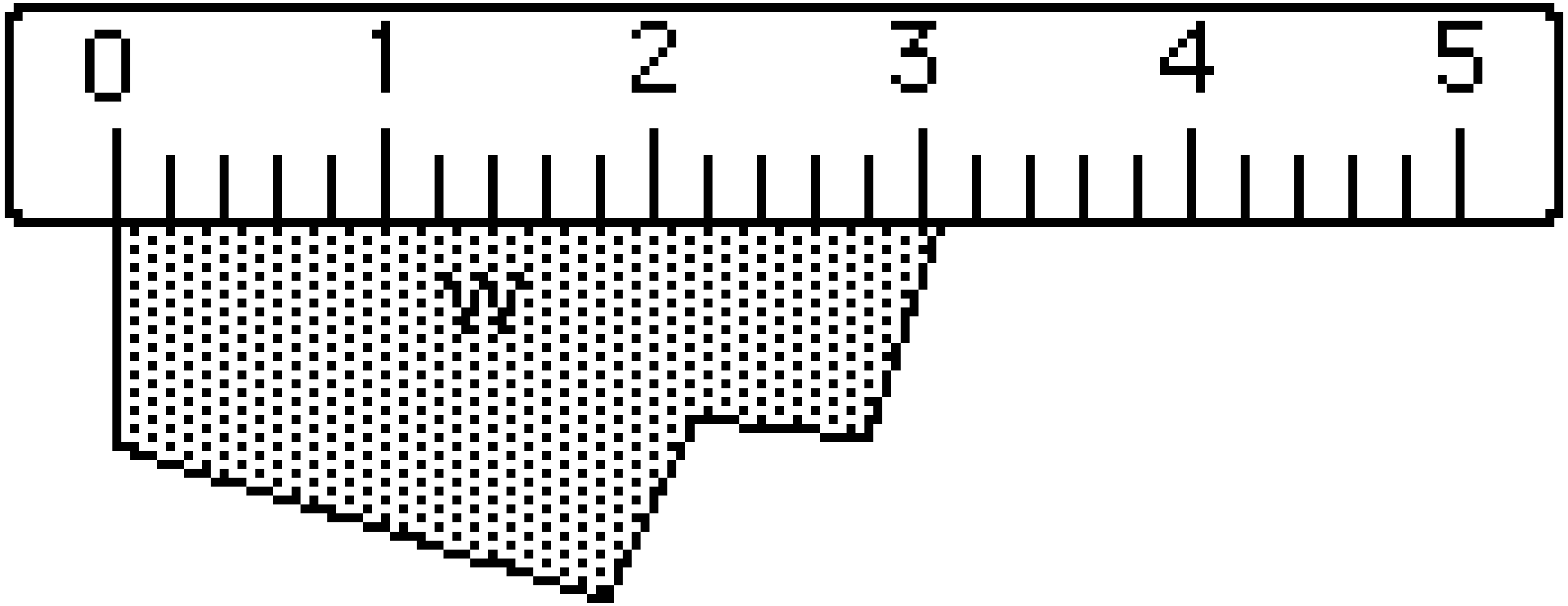
JOHNS HOPKINS
WHITING SCHOOL of ENGINEERING

Estimating and Accuracy

Engineering Innovation
A Summer Program for High School Students

- Measurements often don't fit the gradations of scales
- Two options:
  - Estimate with a single reading (take ½ the smallest division)
  - Independently measure several times and take an average – try to make each trial independent of previous measurement (different ruler, different observer)

3.1 ±0.1 cm



JOHNS HOPKINS
WHITING SCHOOL
of ENGINEERING

http://scidiv.bellevuecollege.edu/physics/measure&sigfigs/C-Uncert-Estimate.html

Accuracy vs. Precision

Engineering Innovation
A Summer Program for High School Students

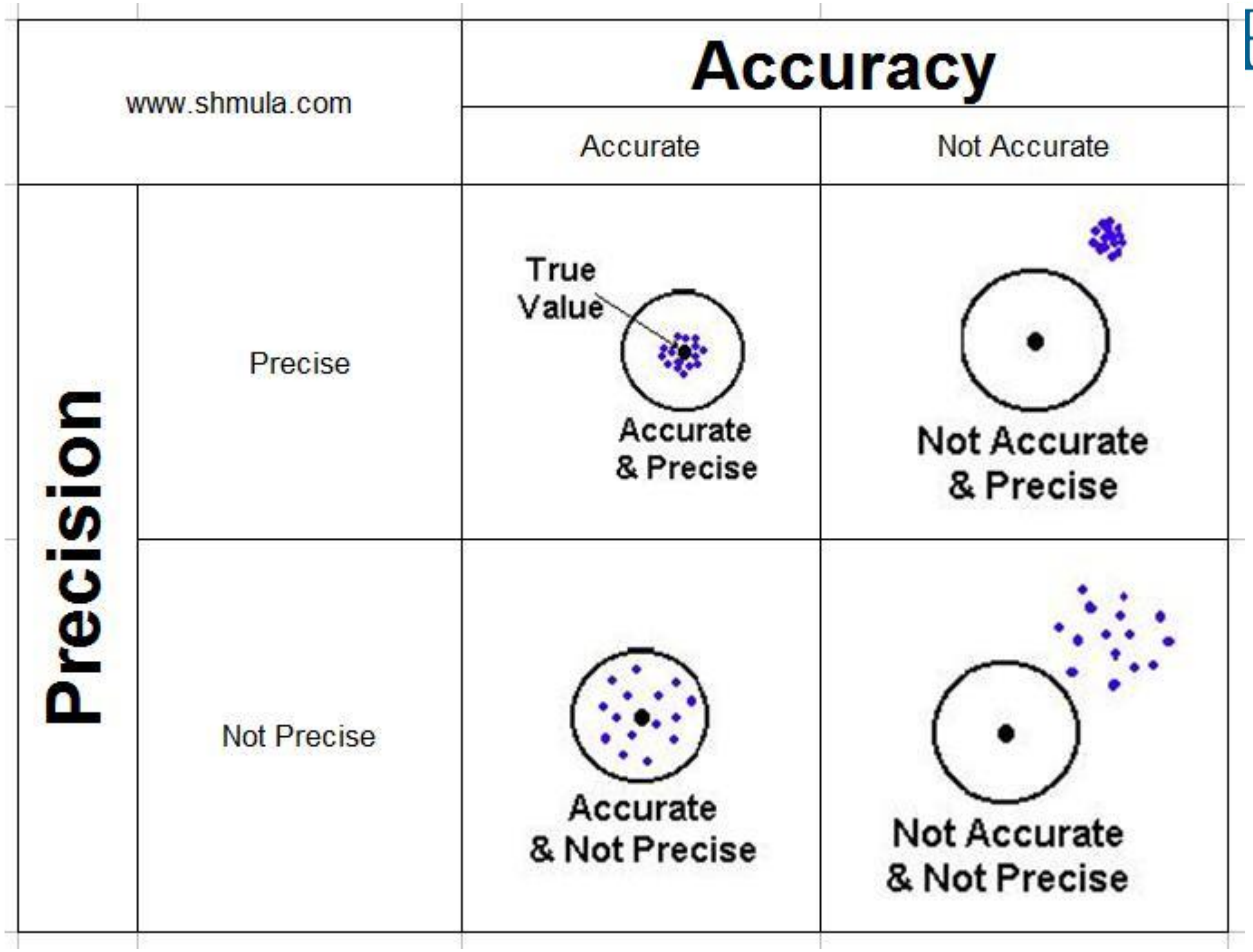
- Accuracy refers to the agreement between a measurement and the true or accepted value
  - Cannot be discussed meaningfully unless the true value is known or knowable
  - The *true* value is not usually known or may never be known)
  - We generally have an estimate of the true value

- Precision refers to the repeatability of measurement
  - Does not require us to know the true value

JOHNS HOPKINS
WHITING SCHOOL
of ENGINEERING

Engineering Innovation

A Summer Program for High School Students



http://www.shmula.com/2092/precision-accuracy-measurement-system

JOHNS HOPKINS
WHITING SCHOOL of ENGINEERING

Significant Figures

Engineering Innovation
A Summer Program for High School Students

- Digits that are:
  - Actual Measured Values
  - Defined Numbers:
    - Unit conversions, e.g. 2.54 cm in one inch
    - Pi
    - *e*, base of natural logarithms
    - Integers, e.g. counting, what calendar year
    - Rational fractions, e.g. 2/5
- EXACT NUMBERS HAVE INFINITE NUMBER OF SIGNIFICANT FIGURES

Significant Figures

Engineering Innovation
A Summer Program for High School Students

How many significant digits in each measurement taken with a meter stick?

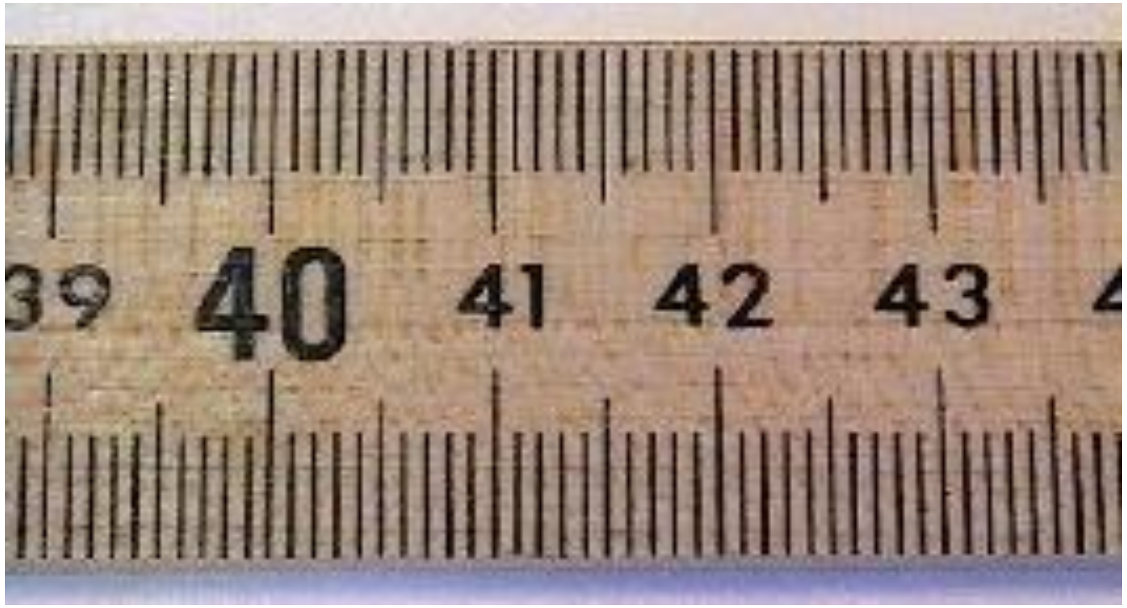
Three or four?:

40.05 cm

41.20 cm

42.43 cm

42.72 cm



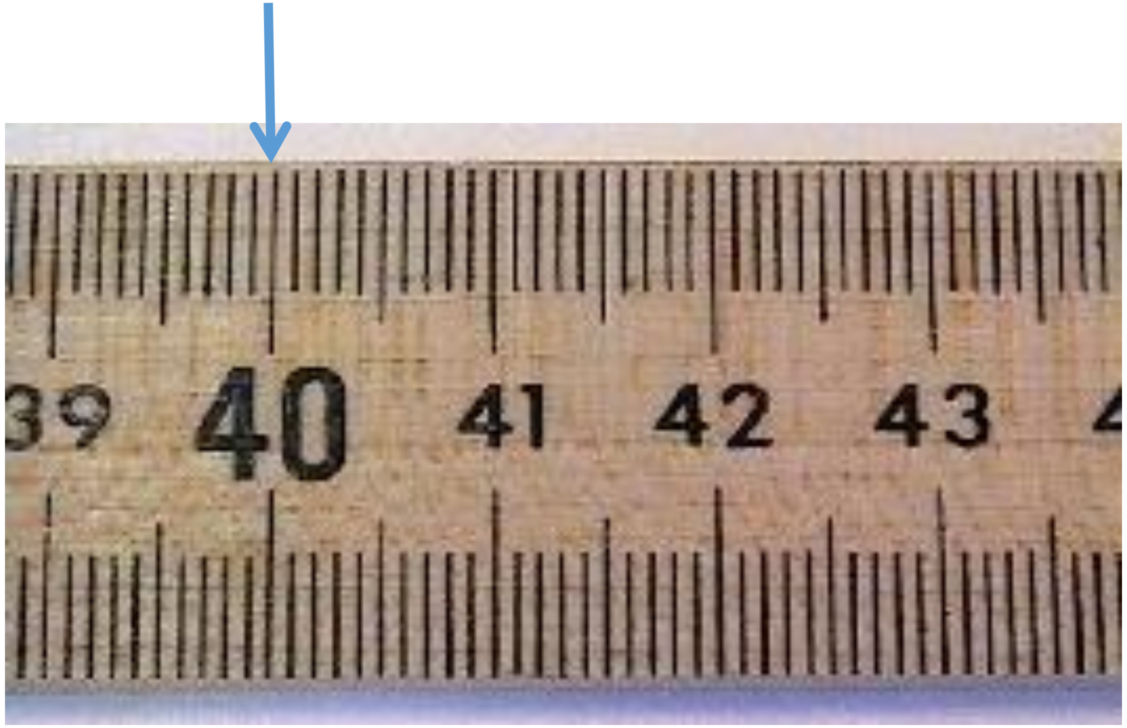
http://serc.carleton.edu/quantskills/methods/quantlit/DeepTime.html

JOHNS HOPKINS
WHITING SCHOOL of ENGINEERING

Significant Figures

Engineering Innovation
A Summer Program for High School Students

- Be clear in your communication
- Which is it?
  - 40 cm
  - 40.0 cm
  - 4×10^1 cm



http://serc.carleton.edu/quantskills/methods/quantlit/DeepTime.html

JOHNS HOPKINS
WHITING SCHOOL
of ENGINEERING

Significant Figures

Engineering Innovation
A Summer Program for High School Students

- State the number of significant figures:

Value	Number of significant figures
5280	3 or 4 (unclear)
0.35	2
0.00307	3
204100	4, 5 or 6 (unclear)
180.00	5

JOHNS HOPKINS
WHITING SCHOOL of ENGINEERING

Significant Figures

Engineering Innovation
A Summer Program for High School Students

- State the number of significant figures for the number described in each phrase below:

Statement	Number of significant figures
My mattress is 182 inches long	3
My car gets twenty miles per gallon	1 or 2
5280 feet per mile	Infinite – exact number by definition
There are ten cars in that train	Infinite – exact number (assuming you can count to ten accurately)
I am going to the Seven-Eleven	0

JOHNS HOPKINS
WHITING SCHOOL of ENGINEERING

Significant Figures

Engineering Innovation
A Summer Program for High School Students

Rounding:

If you do not round after a computation, you imply a greater accuracy than you actually measured

1. Determine how many digits you will keep
2. Look at the first rejected digit
3. If digit is less than 5, round down
4. If digit is more than 5, round up
5. If digit is 5, round up or down in order to leave an even number as your last significant figure

273.92 rounded to **4 digits** is **273.9**

1.97 rounded to **2 digits** is **2.0**

2.55 rounded to **2 digits** is **2.6**

4.45 rounded to **2 digits** is **4.4**

JOHNS HOPKINS
WHITING SCHOOL of ENGINEERING

Significant Figures

Engineering Innovation
A Summer Program for High School Students

Rounding after math operations:

- Multiplication or Division

  # of sig figs in result is equal to the # of sig figs in least accurate value used in the computation

  273.92 x 3.25 = 890.24; Result is rounded to 890

  1/3 x 5.20 = 1.73333; Result is rounded to 1.73

  1.97 x 2 = 3.94; Result is rounded to 4

  2.0 x Pi = 6.28318...; Result is rounded to 6.3

JOHNS HOPKINS
WHITING SCHOOL
of ENGINEERING

Significant Figures

Engineering Innovation

A Summer Program for High School Students

Rounding after math operations:

- Addition or Subtraction

  Place of last sig fig is important

$$\begin{array}{r} 235.68 \\ \underline{-235.12} \\ 0.56 \end{array} \qquad \begin{array}{l} \;\;0.1232 \\ \underline{+4\qquad\quad} \\ \;\;4 \end{array} \qquad \begin{array}{r} 200 \\ \underline{-\quad 5} \\ 195 \end{array} \qquad \begin{array}{r} 45.67 \\ \underline{+65.765} \\ 111.43 \end{array}$$

What’s the problem here?

Johns Hopkins

Whiting School of Engineering

Significant Figures

Engineering Innovation
A Summer Program for High School Students

Multiple Calculations

- The least error will come from combining all terms algebraically, then computing all at once.
- If you need to show intermediate steps to a reader, calculate sig figs at every step. Keep an extra sig fig until the last calculation.

Johns Hopkins
Whiting School of Engineering

Calculators and significant digits:

Engineering Innovation
A Summer Program for High School Students

Let the uncertain digit determine the precision to which you quote a result

Calculator:	12.6892
Estimated Error:	+/- 0.07
Quote:	12.69 +/- 0.07

Engineering Innovation
A Summer Program for High School Students

Error Analysis

JOHNS HOPKINS
WHITING SCHOOL
of ENGINEERING

What is an error?

- No measurements – however carefully made- can be completely free of errors
- In data analysis, engineers use
  - error = uncertainty
  - error ≠ mistake.
- Mistakes in calculation and measurements should always be corrected before calculating experimental error.
- Measured value of $x = x_{best} \pm \delta x$
  - x_{best} = best estimate or measurement of x
  - δx = uncertainty or error in the measurements
- Experimental uncertainties should almost always be rounded to one sig. fig
- Uncertainty in any measured quantity has the same dimensions as the measured quantity itself

Error

- Error – difference between an observed/measured value and a true value.
  - We usually don't know the true value
  - We usually *do* have an estimate
- Systematic Errors
  - Faulty calibration, incorrect use of instrument
  - User bias
  - Change in conditions – e.g., temperature rise
- Random Errors
  - Statistical variation
  - Small errors of measurement
  - Mechanical vibrations in apparatus

Accuracy and Error

Engineering Innovation
A Summer Program for High School Students

- Which type of error, systematic or random?

Systematic

Low Error
Random

Random

JOHNS HOPKINS
WHITING SCHOOL
of ENGINEERING

Error

- Percent Error

$$\frac{error}{value\ (or\ mean)} \times 100\% = Percent\ Error$$

- Relative Error

$$\frac{error}{value\ (or\ mean)} = Relative\ Error$$

Engineering Innovation
A Summer Program for High School Students

How do you account for errors in calculations?

- The way you combine errors depends on the math function
  - added or subtracted –
    - The sum of two lengths is $L_{eq} = L_1 + L_2$. What is the error in L_{eq}?
  - multiplied or divide –
    - The area is of a room is $A = L \times W$. What is error in A?
  - other functions (trig functions, power relationships)
- A simple error calculation gives the largest probable error.

JOHNS HOPKINS
WHITING SCHOOL
of ENGINEERING

Sum or difference

Engineering Innovation
A Summer Program for High School Students

- What is the error if you add or subtract numbers?

$$x \pm \Delta x \qquad y \pm \Delta y \qquad z \pm \Delta z$$

$$w = x + y - z$$

- The absolute error is the sum of the absolute errors.

$$\Delta w \approx |\Delta x| + |\Delta y| + |\Delta z| \qquad \text{upperbound}$$

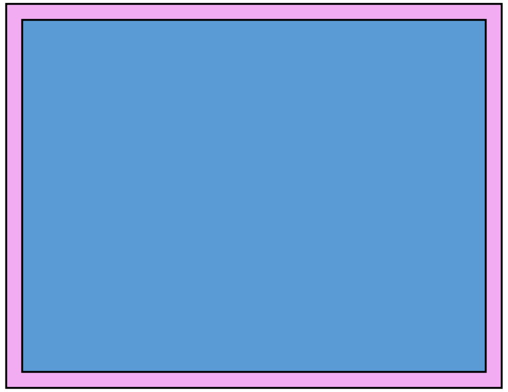
JOHNS HOPKINS
WHITING SCHOOL
of ENGINEERING

What is the error in length of molding to put around a room?

Engineering Innovation
A Summer Program for High School Students

- L_1 = 5.0cm $\pm$ 0.5cm and L_2 = 6.0cm $\pm$ 0.3cm.
- The perimeter is

$$L = L_1 + L_2 + L_1 + L_2$$
$$= 5.0\,\text{cm} + 6.0\,\text{cm} + 5.0\,\text{cm} + 6.0\,\text{cm}$$
$$= 22\,\text{cm}$$



- The error (upper bound) is:

$$\Delta L = \Delta L_1 + \Delta L_2 + \Delta L_1 + \Delta L_2$$
$$= 0.5\text{cm} + 0.3\text{cm} + 0.5\text{cm} + 0.3\text{cm}$$
$$= 1.6\text{cm}$$

JOHNS HOPKINS
WHITING SCHOOL of ENGINEERING

Errors can be large when you subtract similar values.

Engineering Innovation

A Summer Program for High School Students

- Weight of container = 30 ± 5 g
- Weight of container plus nuts = 35 ± 5 g
- Weight of nuts?

$$\text{Weight} = (35 - 30)g = 5g$$

$$\text{Error} = (5 + 5)g = 10g$$

$$\text{Result} = 5g \pm 10g \Rightarrow 200\%$$

JOHNS HOPKINS

WHITING SCHOOL of ENGINEERING

Product or quotient

Engineering Innovation
A Summer Program for High School Students

- What is error if you multiply or divide?

$$x \pm \Delta x \qquad y \pm \Delta y \qquad z \pm \Delta z$$

$$w = \frac{x \times y}{z}$$

$$\Delta w = \frac{(x + \Delta x) \times (y + \Delta y)}{z + \Delta z}$$

- The relative error is the sum of the relative errors.

$$\frac{\Delta w}{w} \approx \left|\frac{\Delta x}{x}\right| + \left|\frac{\Delta y}{y}\right| + \left|\frac{\Delta z}{z}\right| \qquad \text{upperbound}$$

JOHNS HOPKINS
WHITING SCHOOL
of ENGINEERING

What is the error in the area of a room?

Engineering Innovation
A Summer Program for High School Students

- L = 5.0cm $\pm$ 0.5cm and W = 6.0cm $\pm$ 0.3cm.

$$A = L \times W = 5.0\text{cm} \times 6.0\text{cm} = 30.0\text{cm}^2$$

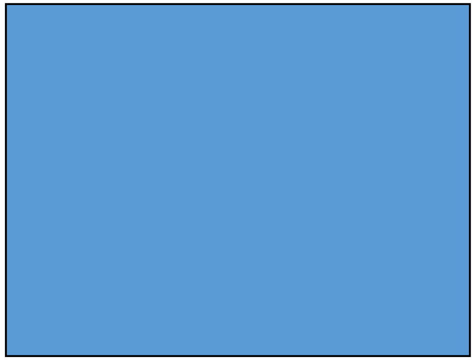
- What is the relative error?

$$\frac{\Delta A}{A} \approx \left|\frac{\Delta L}{L}\right| + \left|\frac{\Delta W}{W}\right|$$

$$= \left|\frac{0.5\text{cm}}{5.0\text{cm}}\right| + \left|\frac{0.3\text{cm}}{6.0\text{cm}}\right| = .15 \quad or\ 15\%$$

- What is the absolute error?

$$\Delta A = A \times 0.15 = 30.0\text{cm}^2 \times 0.15 = 4.5\text{cm}^2$$



JOHNS HOPKINS
WHITING SCHOOL
of ENGINEERING

Multiply by constant

- What if you multiply a variable x by a constant B?

$$w = Bx$$

- The error is the constant times the absolute error.

$$\Delta w = |B|\,|\Delta x|$$

What is the error in the circumference of a circle?

- $C = 2 \pi R$
  - For $R = 2.15 \pm 0.08$ cm

- $\Delta C = 2 \pi (0.08 \text{ cm})$

  $= 0.50$ cm

Powers and exponents

Engineering Innovation

A Summer Program for High School Students

- What if you square or cube a number?

$$w = x^n$$

- The relative error is the exponent times the relative error.

$$\frac{\Delta w}{w} = |n| \left| \frac{\Delta x}{x} \right|$$

JOHNS HOPKINS
WHITING SCHOOL
of ENGINEERING

What is the error in the volume of a sphere?

Engineering Innovation
A Summer Program for High School Students

- $V = 4/3\ \pi\ R^3$
  - For $R = 2.15 \pm 0.08$ cm
  - $V = 41.6\ cm^3$

- $\Delta V/V = 3 * (0.08\ cm/2.15\ cm)$

  $= 0.11$

- $\Delta V = 0.11 * 41.6\ cm^3$

  $= 4.6\ cm^3$

JOHNS HOPKINS
WHITING SCHOOL
of ENGINEERING

Trig Functions

Engineering Innovation
A Summer Program for High School Students

- What if you are using a trigonometric function?

$$w = \sin(x)$$

$$\Delta w = \sin(x + \Delta x) - \sin(x)$$

JOHNS HOPKINS
WHITING SCHOOL
of ENGINEERING

Engineering Innovation

A Summer Program for High School Students

Remote Measurement Lab
“Calculus of Errors” Explanation

JOHNS HOPKINS
WHITING SCHOOL
of ENGINEERING