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# Uncertainty Quantification of Hydrologic Model

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## Abstract

Generalized Likelihood Uncertainty Estimation (GLUE), a simplified Bayesian method, was adopted to determine the parametric uncertainty in hydrological modeling. A preliminary analysis of the summer flows of the Kootenay Watershed, Canada, was modeled to portray a typical uncertainty analysis procedure. SLURP, a robust hydrologic model was chosen for this procedure. The results demonstrated the viability of applying the GLUE method in conjunction with the SLURP hydrological model, following which the posterior probability distributions of the parameters was analyzed. The performance of this technique was verified by examining the flows' prediction intervals for a period of 2 years, enabling valid future hydrological forecasting for the watershed.

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## 1. Introduction

Modeling is indispensable in many fields including hydrology. Given the devastating nature of floods on human lives and economy, the use of modeling in predicting river flow is essential. This critical aspect of hydrologic modeling seeks to quantify effects of flows resulting from different patterns of precipitation in conjunction with other physical processes. Furthermore, modeling in hydrology has diversified from the primary task of predicting runoff into a wide range of applications. This places a further need to analyze the nature of the model outputs using different techniques. The predictive efficiencies of the hydrological models can be improved for a specific case generally by using optimization techniques. However, due to the

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diversification in the applications of hydrological models, general analysis of model outputs gains pre-eminence over improving accuracy for a particular case study. This analysis of model outputs to determine their uncertainty is performed by using different techniques. Divided broadly into Bayesian based and non-Bayesian based techniques, the uncertainty in most hydrologic models can be analyzed. In this study, a robust uncertainty analysis technique, i.e. Generalized Likelihood Uncertainty Estimation (GLUE) method, is used.

## 2. Methodology and Study Case

The hydrological model employed in this study is the Simple Lumped Reservoir Parametric (SLURP) model developed by Kite [1]. The SLURP model has been applied for different snow watersheds and has been determined robust [2], [3]. The technical details of the model can be referred to Kite [1]. To enable a thorough study of the parametric uncertainty of the SLURP model, the Kootenay Watershed was modeled. The temperature data and relative humidity data for this particular case study were obtained from CFSR website. Other flow data and meteorological data at the different stations were obtained from the Hydat database and the Canada Daily Climate Data. The Kootenay Watershed is divided into three different ASAs and their characteristics could be found in Kite [1]. The streamflows from the Crossing ASA flow into Canal ASA and then onto Skookum ASA, which makes Skookum the outlet for the watershed. The calibration period (1979-1988) was performed using 3653 continuous daily flow data followed by a verification period (1989-1990) for 760 continuous daily outflows at Skookum. The Kootenay Basin is a remote mountainous watershed and has not seen any significant human settlements during the calibration-verification period. Hence it is assumed that the land-cover percentages are constant throughout the entire period. No visible major water diversionary structures were found in the period of study, removing any need to alter the output of the model to reflect this.

The uncertainty analysis method is based on GLUE. It is an Equifinality based method presented by Beven and Binley [4] using likelihood measures. It has been proven to be robust for uncertainty analysis of different case studies [5]. This method is a Bayesian-type uncertainty analysis technique which makes use Maximum Likelihood Estimators to evaluate the performance of different parameter ensembles. The method considers every parameter ensemble to have equal likelihood to yield sufficiently accurate outputs i.e. flows. The significant advantage of the GLUE method is its ability to determine parameter uncertainty from all sources: parameter, model structure and input [6]. Another advantage of the GLUE method is its flexibility and simplicity which guarantees a wide range of application as evidenced in numerous examples in literatures [6, 7, 8, 9]. Therefore, this methodology which combines GLUE with SLURP, examines the parametric uncertainty using likelihood estimators, proving to be a novel application of this method. The SLURP manual by Kite [1] describes the ten most sensitive parameters and their bounds.

In this application of the GLUE method, the Nash Sutcliffe Efficiency (NSE) is utilized as the likelihood estimator [10]. The NSE determines the magnitude of residual variance when the simulations are compared to the observed flow. It has been the likelihood estimator for many GLUE studies using other models, including SWAT [6], Institute of Hydrology Distributed Model [4], ADM model [5], etc. In the absence of a prior distribution, the uniform distribution was adopted as recommended by previous researchers [6]. The random values were generated using Matlab software subject to the limits described in Kite [1]. To ensure a rigorous process of choosing the parameter ensembles, one million parameter ensembles were created. To further improve the sampling process, Latin Hypercube Sampling Scheme was employed. To account for the three different land-covers (i.e. Impervious, Forest and Crop/Grass) three different sets of the million ensembles were created (a total of three million ensembles) for analysis. In the GLUE methodology, the definition of threshold is subjective. The likelihood measure is accorded a value which each parameter ensemble must exceed to be considered a physically representative ensemble. Usually, when the NSE is employed as the likelihood measure, a threshold value of approximately 0.5 has been suggested [6]. However, given that a

larger number of parameter ensembles were created, the threshold level for the likelihood measure was set at 0.75.

### 3. Results and Discussion

#### 3.1. Parameter analysis

The model was run using the parameter ensembles, and the NSE values were obtained for each run. Based on the NSE values, 32221 distinct parameter ensembles were found to be successful. An analysis of the statistical properties of each parameter, for each land-cover is conducted. The Initial contents of snow storage parameter demonstrated the effect of land-cover in yielding more accurate outflows, by averaging differently in each land-cover. Similarly, for the Forest land-cover, Initial contents of snow storage exhibited a significantly higher skewness, indicative of higher asymmetry in the probability distributions. The modeled distribution for each parameter was based on the Kolmogorov-Smirnov Test. The modeled distributions for Initial contents of snow storage in Impervious land-cover is the Beta General distribution, Forest land-cover is the Weibull distribution and Crop land-cover is the Weibull distribution (Figure 1).

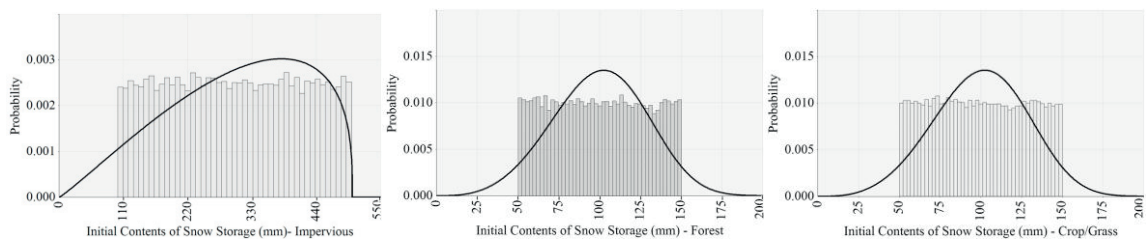


Fig. 1. Probability Distribution Function plots for Initial Contents of Snow Storage for different Land-Covers

The distribution breakdown for the thirty parameters indicated that the Uniform distribution was the most appropriate distribution fitting, followed by the Beta General distribution. Uniform distribution specifies that the prior and the posterior distributions remain unchanged for parameters such as Initial contents of slow storage (Impervious), Maximum infiltration rate (all land-covers), Retention capacity for slow storage (Imp. and Crop/grass), Maximum capacity for slow storage (Imp., Forest), Precipitation factor (Imp., Crop/grass) and P10 (Imp.). The variance exhibited by the successful parameter ensembles signifies the variability of the posterior parameter. The variance of P 10 in the Forest land-cover is of particular significance given the contrasting values obtained when compared to other land-covers. This may be attributed to the statistical accuracy of this parameter being centered within a narrow band, also simultaneously underscoring the variation in the averages of the parameter. Also important to note is the variation of skewness which may change signs for the same parameter in different land-covers (for Maximum capacity for slow storage). This may be ascribed to the nature of the parameter, which is used in computing slow storage in conjunction with parameters: Initial contents of slow storage and Retention capacity for slow storage. The strongly positive skewness for fast and slow storages, indicate that in the watershed, the runoff is more likely to be high from the forest land-cover. Another aspect of the GLUE method is the property of Equifinality. In the one million runs, many parameter ensembles generated similar likelihood values. For example, 36 different parameter ensembles yielded the NSE value of 0.69.

#### 3.2. Uncertainty in Simulations

In the peak of summer, the model was able to predict the peak flows with reasonable accuracy. This may be attributed to the model structure, which ensures that the precipitation incident accumulates as fast storage and combines with the snow melt producing peaks. Multiple peaks may account for different incidents of rainfall in the mountainous watershed over the summer period. The lag in the autumn of first year of the verification period can be attributed to the structure of the SLURP model. The preponderance of the flow to accumulate as slow storage, can lead to lags in the hydrographs. This is reflected by the parameters for the predominant land-cover – Forest. The related parameters exhibited higher values for slow storage when compared to fast storage within their bounds.

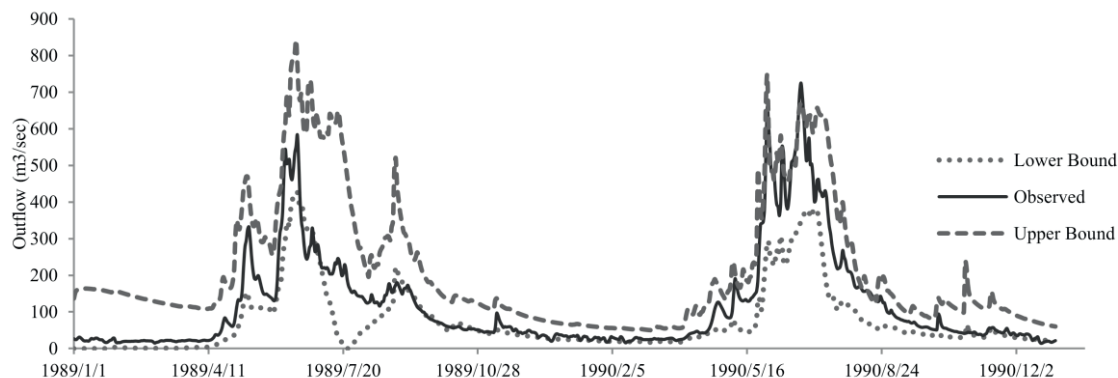


Fig. 2. Confidence interval at 95<sup>th</sup> percentile for the outflows in the watershed for the period 1989-1990

The SLURP model was able to predict the shapes of the hydrographs with accuracy. The contrast within the two year verification period manifested when the low and medium flow periods are tracked. The observed flows possessed similar magnitudes to the lower bound in the first year and were in comparable magnitudes to the upper bounds in the second year (Figure 2). There are certain anomalies in the low flow periods of the verification period, and this could arise as the model calibration is geared towards predicting large flows accurately in the watershed.

#### 4. Conclusion

The SLURP model was calibrated and verified for the Kootenay Watershed. After a rigorous calibration process, the output of the model was verified by comparing with observed flow for a period of two years. Using the GLUE method, the thirty parameters of the SLURP model were examined for their contribution to the uncertainty of the predictions. Some of the parameters were concluded to significantly contribute to the uncertainty, although variations across different land-covers were observed. The model predicted the outflows with reasonable accuracy and hence can be used for future modeling of similar watersheds. The upper and lower bounds were customarily in accordance to the overall flow regime of the watershed for the years 1989-1990 validating the use of this model for future studies in the basin. Although some concerns remain on the contribution to uncertainty by the model structure, analysis of the uncertainty by applying different hydrological models to the Kootenay Watershed would further the comparative analysis of the SLURP model with others.

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