CAT 2025

MBA Fastrack

Lecture – 2 | **ALGEBRA**

Equations – 2

By– RAHUL BATHLA

TOPICS *to be covered*

1. Linear Equations - Advance — HARD — LAST CLASS ✓ focus ✓
2. Quadratic Equation - Discriminant Method — EASY
3. Theory of Roots — MED

Topic: Linear Equation Continued

If $a + 2b + c = 8$
$2a + b + 3c = 13,$
$2a - b + 6c = 18$

2

Eq 2 + Eq 3

5 $4a + 9c = 31$

$4a + 27 = 31$

$4a = 4$

$a = 1$

① & ③

$a + 2b + c = 8$

$4a - 2b + 12c = 36$

4 $5a + 13c = 44$

$20a + 52c = 176$

$20a + 45c = 155$

$7c = 21$

$c = 3$

$a = 1$

$2 - b + 18 = 18$

$b = 2$

QUESTION- 1

#Q. If 3a + 5b = 22 , Find a + b given that 'a' and 'b' are positive integer.

(A) 5

(B) 8

(C) 6

(D) Can not be determined.

22↓ 22↓

$3a + 5b = 22$ ✓

tve tve

2 VAs & 1 eqn

VAR → tve Integer

$a = 4$

$3a = 12$

$5b = 10$

$b = 2$

~~17 + 5 = 22~~

12 + 10 = 22

~~7 + 15 = 22~~

~~2 + 20 = 22~~

~~+ 25~~

QUESTION- 2

#Q. If 4a + 7b = 100, Find number of positive integral values of (a,b).

A. 3

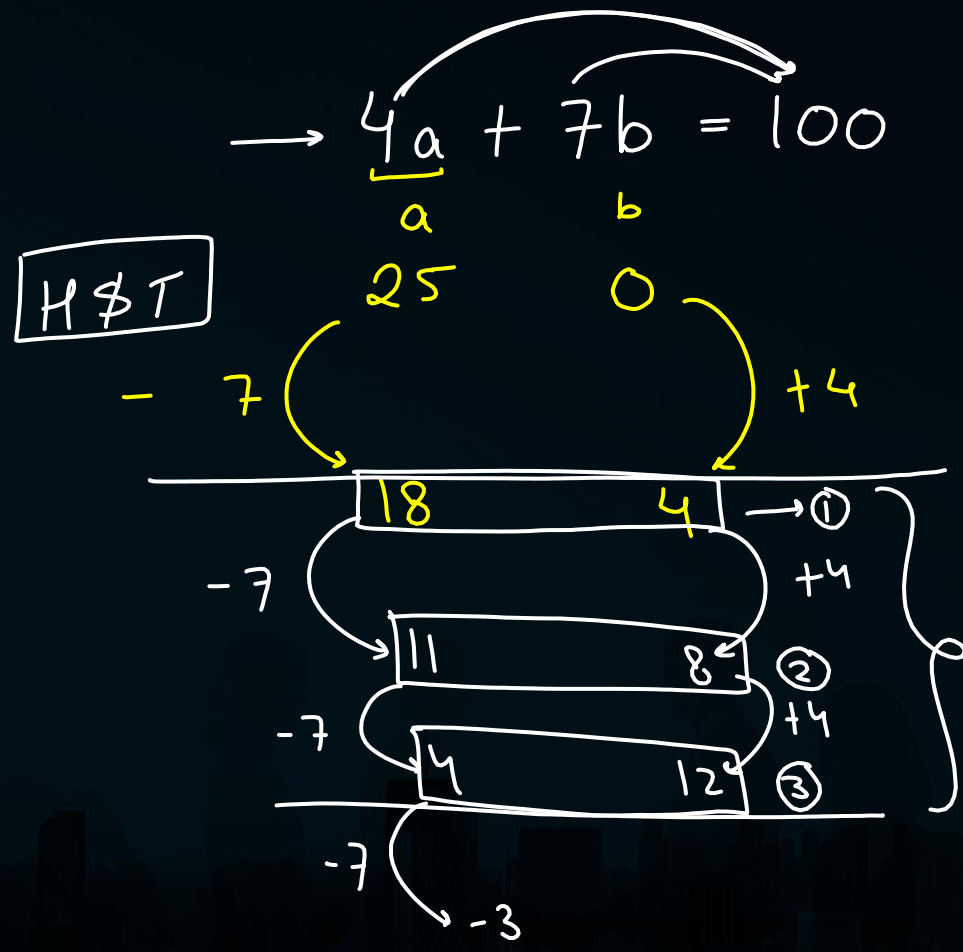
B. 4

C. 5

D. 6

$4(18)+7(4)$

$72+28=100$



$8a+14b=200$

$4a+7b=100$

QUESTION- 3 (CAT 2022)

#Q. A donation box can receive only cheques of ₹100, ₹250, and ₹500. On one good day, the donation box was found to contain exactly 100 cheques amounting to a total sum of ₹15250. Then, the maximum possible number of cheques of ₹500 that the donation box may have contained, is

100 → x ✓
250 → y ✓
500 → z_{MAX} ✓

$x + y + z = 100$

$100x + 250y + 500z = 15250$

$2x + 5y + 10z = 305$

$2x + 2y + 2z = 200$

$3y + 8z = 105$

y	z
35	0
−8	+3
27	3
19	6
11	9
3	12

$z = 12$ ✓

QUESTION- 4

#Q. Aron bought some (x) pencils and (y) sharpeners. Spending the same amount of money as Aron, Aditya bought twice ($2x$) as many pencils and 10 less sharpeners ($y-10$). If the cost of one sharpener is 2 more than the cost of a pencil, then the minimum possible number of pencils bought by Aron and Aditya together is

A) 27

B) 33

C) 30

D) 36

x_{MIN}

$xP + yS = 2xP + (y-10)S$

$S = P + 2$

$yS = xP + yS - 10S$

$xP = 10S$

$xP = 10[P+2]$

$xP = 10P + 20$

$xP - 10P = 20$

$P(x-10) = 20$ (P: +ve, $(x-10)$: +ve, 20: +ve)

$x - 10 > 0$

$x > 10$

$x_{MIN} = 11$

ARON - x

ADITYA - $2x$

COMBINED - $3x = 3(11)$

$\hookrightarrow 33$

Topic: Quadratic Equation - Discriminant Method

❖ Quadratic Equation

Quadratic Equations is an equation which can be written in the form of

$$ax^2 + bx + c = 0$$

$$0x^2 + bx + c = 0$$

Where a, b and c are real numbers and a ≠ 0.

$ax^2 + bx + c = 0$

- Splitting The Middle Term
- Discriminant Method

❖ Discriminant Method

$1x^2 + 7x + 10 = 0$

a = 1, b = 7, c = 10

$(x+5)(x+2)$

$x = -5$

$x = -2$

Rem!

$D = b^2 - 4ac$

$= (7)^2 - 4(1)(10)$

$= 49 - 40$

$= 9$

$\sqrt{D} = \sqrt{9} = 3$

Rem!

$\alpha = \frac{-b+\sqrt{D}}{2a}$ $\quad \beta = \frac{-b-\sqrt{D}}{2a}$

$\alpha = \frac{-(7)+3}{2(1)}$ $\quad \beta = \frac{-7-3}{2(1)}$

$= \frac{-4}{2}$ $\quad = \frac{-10}{2}$

$= -2$ $\quad = -5$

❖ Conditions for D

$D = 0$

$x^2 + 7x + 10 = 0$

$D = 9$

$\alpha = -2 \qquad \beta = -5$

$D > 0$

Real & Unequal Roots

$x^2 - 6x + 9 = 0$

$D = (36) - 4(1)(9)$

$= 0$

$\alpha = \frac{-b + \sqrt{D}}{2a} = \frac{6}{2} = 3$ (D → 0)

$\beta = \frac{-b - \sqrt{D}}{2a} = \frac{-(-6)}{2} = 3$ (D → 0)

Real & Equal Roots

$x^2 + x + 1 = 0$

$D = 1 - 4(1)$

$= -3$

$\alpha = \frac{-b + \sqrt{-3}}{2a} \qquad \beta = \frac{-b - \sqrt{-3}}{2a}$

??

No Real Root.

$D < 0$

QUESTION- 5

#Q. Find the maximum integral value of k for which the equation has real solutions.

$x^2 + 3x + k = 0$

A. 2

B. 3

C. 0

D. *can't be determined*

$D > 0$

$9 - 4k > 0$

$9 > 4k$

$9 > 4 \quad k = 1$

$9 > 8 \quad k = 2$

$9 \not> 12 \quad k = 3$ ✗

QUESTION- 6

#Q. If a, b and c are positive integers, find the minimum value of $a + c$ for which the equation has real and equal solutions.

$$x^2 - 2bx + 7c + b^2 = 2a$$

(A) 7

(B) 9 ✓

(C) 5

(D) 3

$1\,x^2 - 2b\,x + (7c + b^2 - 2a) = 0$

$a = 1, \quad b = b, \quad c = 7c + b^2 - 2a$

$D = 0$

$4b^2 - 4(1)(7c + b^2 - 2a) = 0$

$4b^2 - 28c - 4b^2 + 8a = 0$

$8a = 28c$

$2a = 7c$

$\frac{a}{c} = \frac{7}{2}$

$a = 7k \rightarrow 7, 14, 21 \ldots$

$c = 2k \rightarrow 2, 4, 6 \ldots$

Important Concept - 1

$a, b, c \rightarrow$ RATIONAL

$x^2 - 2x - 7 = 0$

$D = 4 + 28 = 32$

$\alpha = \frac{2 + \sqrt{32}}{2}$ $\quad \beta = \frac{2 - \sqrt{32}}{2}$

IRRATIONAL

ROOT WAALE

$\sqrt{D} \rightarrow$ RATIONAL

$D \rightarrow$ perfect sq. $[1, 4, 9, 16, 25 \ldots]$

RATIONAL ROOTS

$D \rightarrow$ perfect sq. X

$D \rightarrow 32, 14 \ldots$

$\sqrt{D} \rightarrow$ IRRATIONAL

ROOTS $\rightarrow$ IRRATIONAL

$x^2 - 7x + 12 = 0$

$D = 49 - 48 = 1$

$\alpha = \frac{7 + 1}{2} = \frac{8}{2} = 4$ $\quad \beta = \frac{7 - 1}{2} = \frac{6}{2} = 3$

RATIONAL

If the quadratic equation $ax^2 + bx + c$ has rational roots then D is a perfect square given that a , b and c are rational numbers.

QUESTION- 7

#Q. $x^2 - 7x + p = 0$ has rational roots. How many positive integer values can 'p' take?

A. 3

B. 4

C. 5

D. 7

D → Perfect Sq.

$49 - 4p$

p=1	45	21	1
p=2	41	17	
	37	13	
	33	9	
	29		
	25	5	

Important Concept - 2

$x^2 + 4x + 2 = 0$

$D = 8$ → Not A perfect so.

$\sqrt{D} = \sqrt{8} = \sqrt{4 \times 2} = 2\sqrt{2}$

$\alpha = \frac{-4 + 2\sqrt{2}}{2} = -2 + \sqrt{2}$

$\beta = \frac{-4 - 2\sqrt{2}}{2} = -2 - \sqrt{2}$

Conjugate Pairs

$ax^2 + bx + c = 0$

$a, b, c \rightarrow$ Rational

$\alpha = \frac{3 + 2\sqrt{5}}{5}$

$\beta = \frac{3 - 2\sqrt{5}}{5}$

When a, b and c are rational and one of the root is $l + m\sqrt{n}$, then other is $l - m\sqrt{n}$

QUESTION- 8

#Q. If one root of the equation $ax^2 + bx + c = 0$ is $2 + \sqrt{3}$, and U represents the Sum of Roots and V represents the product of Roots. Find $U - V$.

Given that a, b and c are integers.

$2+\sqrt{3}$ $\quad$ $2-\sqrt{3}$

(A) 5

(B) 3 ✓

(C) 4

(D) 1

$U = 4$

$V = (2+\sqrt{3})(2-\sqrt{3})$

$= 4 - 3$

$= 1$

Topic: Theory Of Roots

Theory of ROOTS

Sum and Product of Roots

$2x^2 + 6x + 2 = 0$

SUM $\quad \alpha+\beta = \frac{-6}{2} = -3$

$\alpha\beta = \frac{2}{2} = 1$

$\alpha > \beta$

$\alpha - \beta = \frac{\sqrt{20}}{2}$

$= \frac{2\sqrt{5}}{2}$

$= \sqrt{5}$

$D = 36 - 4(2)(2)$

$= 36 - 16$

$= 20$

$ax^2 + bx + c = 0$

$\alpha > \beta$

$\alpha+\beta = \frac{-b}{a}$

$\alpha\beta = \frac{c}{a}$

$\alpha - \beta = \frac{\sqrt{D}}{a}$

} Rem!

$[ax^2 + bx + c = 0]$

$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$

$x^2 - \left[-\frac{b}{a}\right]x + \frac{c}{a} = 0$

$x^2 - [\text{SUM}]x + \text{Product} = 0$

QUESTION- 9

#Q. Find the value of '$b - c$' if one of the roots of the equation is $x^2 + bx + c = 0$ $4 + \sqrt{3}$ and b , c are rational.

(A) 5

(B) 8

(C) 21

(D) -21

$\alpha = 4 + \sqrt{3}$

$\beta = 4 - \sqrt{3}$

$\alpha + \beta = -b$

$8 = -b$

$b = -8$

$\alpha\beta = c$

$(4+\sqrt{3})(4-\sqrt{3}) = c$

$4^2 - (\sqrt{3})^2 = c$

$16 - 3 = c$

$13 = c$

Summary : Quadratic Equation

$ax^2 + bx + c = 0$

$\alpha = \frac{-b+\sqrt{D}}{2a}$

$\beta = \frac{-b-\sqrt{D}}{2a}$

$D = b^2 - 4ac$

Discriminant

$\alpha + \beta = -b/a$

$\alpha\beta = c/a$

$\alpha > \beta$

$\alpha - \beta = \frac{\sqrt{D}}{a}$

$D > 0$ Real & Unsq

$D = 0$ Real & Equal

$D < 0$ No Roots

❖ If roots of Quad. Eq. are rational and a, b *and* c are rational, then D is a perfect Square.

D → Perfect Sq. Nahi Aaya, a, b, c → R

$\ell + \sqrt{m}, \ell - \sqrt{m}$

EK IMPORTANT CATCH

$x^2 - 5x + 6 = 0$

$D = 1$

$\alpha = 2$

$\beta = 3$

$\alpha = \frac{+5+1}{2}$ $\quad \beta = \frac{5-1}{2}$

$2(x^2 - 5x + 6) = 0$

$2x^2 - 10x + 12 = 0$ ✓

$D = 100 - 4(2)(12)$

$= 100 - 96$

$= 4$

$\alpha = \frac{10+2}{4}$

$= \frac{12}{4} = 3$

$\beta = \frac{10-2}{4}$

$= \frac{8}{4}$

$= 2$

- Agar ek constant number se quadratic equation ko multiply karoge to roots change nahi honge.

Roots of the equation are 1 and 4

Find the quadratic equation

$x^2 - (\text{Sum of Roots})x + \text{Product of Roots} = 0$

$x^2 - [\quad 5 \quad]x + 4 = 0$

$x^2 - 5x + 4 = 0$

$2(x^2 - 5x + 4) = 0$

$3(x^2 - 5x + 4) = 0$

$4(x^2 - 5x + 4) = 0$

$4x^2 - 20x + 16 = 0$

$k(x^2 - 5x + 4) = 0$

$k \rightarrow$ Real no.

❖ If roots of Quadratic equation are given, then Equation : $k[x^2 - (\alpha + \beta)x + \alpha\beta] = 0$

QUESTION- 10 (CAT 2021)

#Q. Suppose one of the roots of the equation $ax^2 - bx + c = 0$ *is* $2 + \sqrt{3}$, . where a, b and c are rational numbers and a ≠ 0. If $b = c^3$ then "a" equals

(A) ± 2

(B) ± 3

(C) ± 4

(D) ± 1

$\alpha = 2 + \sqrt{3}$

$\beta = 2 - \sqrt{3}$

$\alpha + \beta = 4$

$-\frac{(-b)}{a} = 4$

$b = 4a$

$\alpha\beta = (2+\sqrt{3})(2-\sqrt{3})$

$= 2^2 - (\sqrt{3})^2$

$\frac{c}{a} = 1$

$c = a$

$b = c^3$

$4a = a^3$

$4 = a^2$

$a = \pm 2$

Quadratic Equation

Roots Type 2 Questions

$x^2 - 3x - 7 = 0$ → α, β

$\alpha + \beta = 3$ ✓

$\alpha\beta = -7$ ✓

$x^2 + px + q = 0$ → $\alpha+1$, $\beta+1$

$(\alpha+1)+(\beta+1) = -p$

$\alpha + \beta + 2 = -p$

$3 + 2 = -p$

$\boxed{-5 = p}$

$(\alpha+1)(\beta+1) = q$

$\alpha\beta + \alpha + \beta + 1 = q$

$-7 + 3 + 1 = q$

$\boxed{-3 = q}$

QUESTION- 11

#Q. If the roots of the Quadratic Equation $x^2 + px + q = 0$ are 2 more than the root of the equation $x^2 - 7x + 9 = 0$. Find $p + q$.

A. 7

B. 8

C. 4

D. 16 ✓

$x^2 - 7x + 9 = 0$ → α, β

$\alpha + \beta = 7$

$\alpha\beta = 9$

$x^2 + px + q = 0$ → $\alpha+2$, $\beta+2$

$\alpha + \beta + 4 = -p$

$7 + 4 = -p$

$11 = -p$

$\boxed{-11 = p}$

$(\alpha+2)(\beta+2) = q$

$\alpha\beta + 2\alpha + 2\beta + 4 = q$

$9 + 2(\alpha+\beta) + 4 = q$

$13 + 2(7) = q$

$27 = q$

$(\alpha+\beta)^2 = \alpha^2+\beta^2+2\alpha\beta$

$(\alpha-\beta)^2 = \alpha^2+\beta^2-2\alpha\beta$

$\alpha^2-\beta^2 = (\alpha-\beta)(\alpha+\beta)$

$\alpha^2+\beta^2 = (\alpha+\beta)^2-2\alpha\beta$

$= 9-2(-5)$

$= 19$

$\frac{\alpha}{\beta}+\frac{\beta}{\alpha} = \frac{\alpha^2+\beta^2}{\alpha\beta}$

$= -\frac{19}{5}$

$\alpha > \beta$

$\alpha^2-\beta^2 = (\alpha-\beta)(\alpha+\beta)$

$-3\sqrt{29}$ ✓

x^2-3x-5

$\alpha+\beta = -(-3) = 3$

$\alpha\beta = -5$

$|\alpha-\beta| = \sqrt{29}$

$\frac{1}{\alpha}+\frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta}$

$= -\frac{3}{5}$

QUESTION- 12

#Q. If the roots of the Quadratic Equation $x^2 - px + q = 0$ are square of the roots of the equation $x^2 - 7x + 9 = 0$. Find $q - p$.

A. 30

B. 50

C. 112

D. 52

$x^2 - 7x + 9 = 0$ → roots α, β

$\alpha + \beta = 7$

$\alpha\beta = 9$

$(\alpha+\beta)^2 = \alpha^2 + \beta^2 + 2\alpha\beta$

$49 = \alpha^2 + \beta^2 + 18$

$31 = \alpha^2 + \beta^2$

$x^2 - px + q = 0$ → roots α^2, β^2

$\alpha^2 + \beta^2 = p$

$31 = p$

$(\alpha^2\beta^2) = q$

$(\alpha\beta)^2 = q$

$81 = q$

HW

#Q. If the roots of the Quadratic Equation $3x^2 + px - 1 = 0$ are 'a' and 'b' such that $\frac{1}{a^2} + \frac{1}{b^2} = 15$ then the value of $(a^3 + b^3)^2$ *is*

A 4

B 1

C 16

D 9

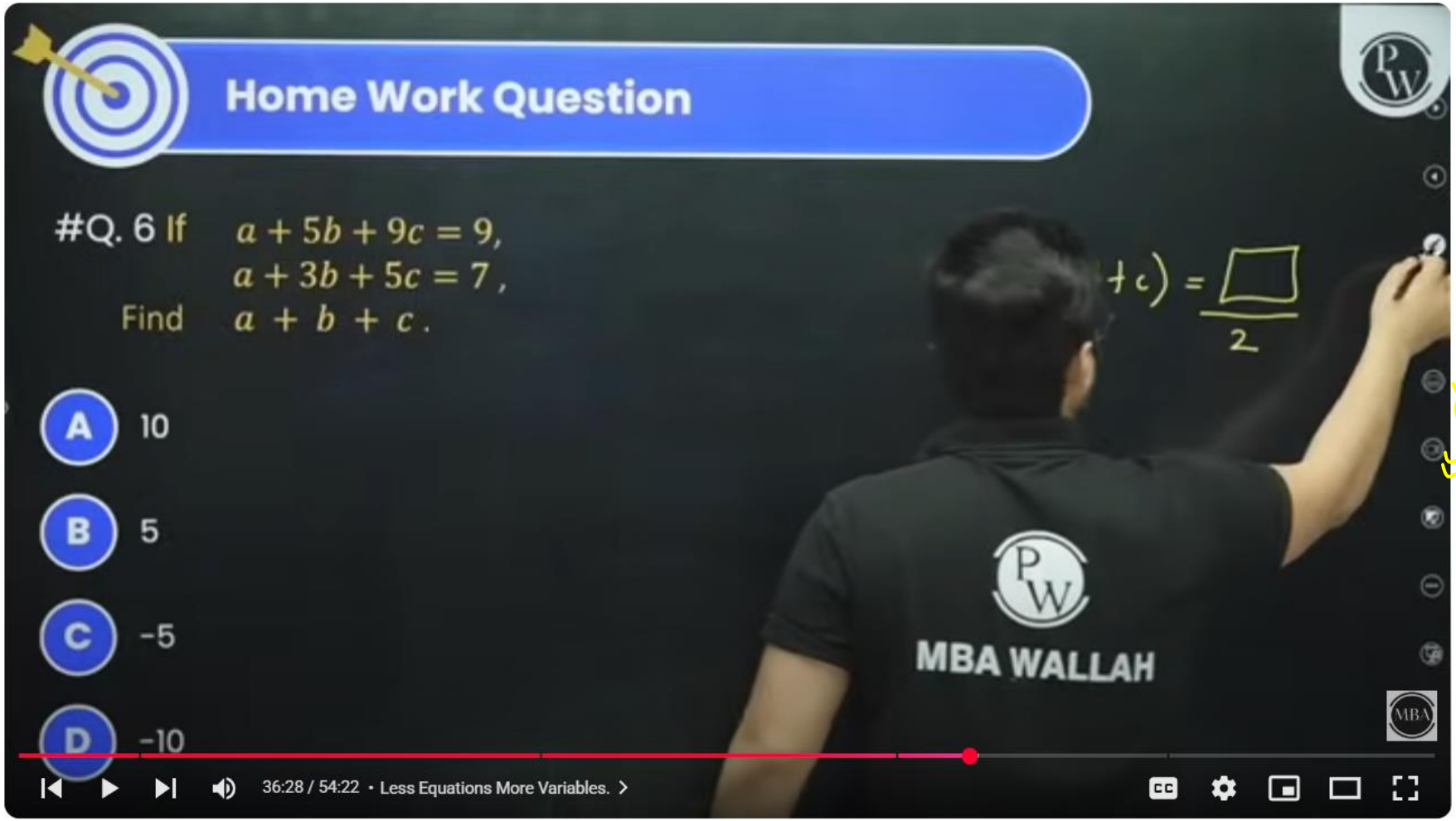
HW

HW

YouTube IN

mba wallah equations essential

Create



Quants Essentials CAT 2024

MBA Wallah - 3 / 12

1. Percentage Essentials | CAT 2024 | Quant Arithmetic |... — MBA Wallah — 1:20:49
2. Ratio Essentials | CAT 2024 | Lecture 2 | CAT 2024... — MBA Wallah — 1:33:05
3. CAT 2024 Essentials | L - 3 Equations Approaches and... — MBA Wallah — 54:23
4. Exponents Properties and Approaches | 04 | CAT 2024... — MBA Wallah — 1:04:44
5. CAT 2024 Essentials | 05 | Profit, Loss and Interest... — MBA Wallah — 1:00:22
6. Mixture & Allegation 06 | CAT 2024 Essentials | Quant — MBA Wallah — 1:06:24
7. Functions 07 | CAT 2024 Essentials | Quants — MBA Wallah — 45:23
8. Number System 08 | CAT 2024 Essentials | Quant — 1:18:53


THANK
You
52%
64%
85%
26,465.54
28,161.92
26,465.54
198.45
37,492.43
201.55
1,097.95
0.00
26,717.43
7,534.44
12,512.54
28,875.56