

Lecture - 1

**ALGEBRA** 

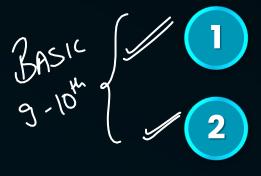
Linear Equations - 1

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# TIPICS

to be covered



Linear Equation

Quadratic Equation - Splitting the Middle term

System of Linear Equation



**Word Problems** 



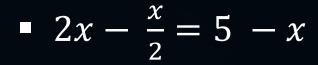


**Topic: LINEAR EQUATION** 

# LINEAR EQUATION (1 Variable)

$$-2(x-3) = 9 - x$$

$$2x-6=9-x$$





$$\frac{3x}{2} = 5 - x$$

$$3x = 10 - 2n$$

$$2x + y = 11$$

4x - y = 7

Substitution

$$y = 11-2x$$

$$4x - \begin{bmatrix} 11-2x \end{bmatrix} = 7$$

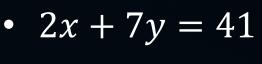
$$4x - 11 + 2x = 7$$

$$6x = 18$$

$$x = 3$$

$$4 = 11-2$$





$$2(3) + 7y = 41$$

$$7y = 35$$

$$4 = 5$$





$$\frac{9}{u} - \frac{6}{v} = 1$$

$$\frac{8}{y} - \frac{2}{y} = 2$$

Find 
$$u + v$$
.

$$\frac{9}{3} - \frac{6}{9} = 1$$

$$\frac{1}{0} = \frac{1}{3}$$

$$\sqrt{1} = 3$$

$$\frac{1}{0} = A = \frac{1}{3}$$

$$\frac{9}{0} - \frac{6}{9} = 1$$

$$9\left[\frac{1}{v}\right] - 6\left[\frac{1}{v}\right] = 1$$

$$\frac{8}{0} - \frac{2}{9} = 2$$

$$[84 - 58 = 5]3$$





Topic: QUADRATIC EQUATION (Splitting the Middle Term)

$$x^2 - 8x + 12 = 0$$



$$\chi^{2}-6\chi-2\chi+12=0$$

$$\chi(\chi-6)-2(\chi-6)=0$$

$$(\chi-2)(\chi-6)=0$$

$$\chi=2$$

$$\chi=6$$

$$\chi=6$$

$$1x^{2} - 8x + 12 = 0$$

$$1x^2 - 11x + 18 = 0$$



$$\chi = 9$$

9 + 2 = 1

$$7x^2 - 23x + 18 = 0$$

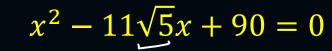
$$3x^2 - 29x + 18 = 0$$

$$27 + 2 = + 29$$

$$\lambda = 27 = 9$$

$$\chi = \frac{2}{3} = \frac{2}{3}$$

$$x^2 - 5\sqrt{3}x + 18 = 0$$





$$\chi = 2\sqrt{3}$$

$$\boxed{9} + \boxed{2} = 1$$





**Topic: System of Linear Equation** 

# Number of Solutions

# Unique Solution /

$$\begin{vmatrix} a_1x + b_1y + c_1 = 0 \\ a_2x + b_2y + c_2 = 0 \end{vmatrix}$$

$$\begin{vmatrix} a_1 \\ a_2 \neq b_1 \\ \vdots \\ \vdots \end{vmatrix}$$

$$2x + 3y = 7$$

$$4x - 3y = 10$$

$$7x + 7$$

$$7x + 7$$

$$7x + 7$$

$$7x + 7$$

# ... Infinite Solutions

$$a_{1}x + b_{1}y + c_{1} = 0$$

$$a_{2}x + b_{2}y + c_{2} = 0$$

$$\left(\frac{a_{1}}{a_{2}} = \frac{b_{1}}{b_{2}} = \frac{c_{1}}{c_{2}}\right)$$



# No Solutions

$$a_1 x + b_1 y + c_1 = 0$$

$$a_2 x + b_2 y + c_2 = 0$$

$$\left| \frac{a1}{a2} = \frac{b1}{b2} \neq \frac{c1}{c2} \right|$$

$$(2x + 3y = 10)x2$$

$$\frac{1}{1000} + 64 = 30$$

$$\frac{1}{1000} + 64 = 20$$

$$\frac{1}{1000} + 64 = 20$$

$$\frac{1}{2} = \frac{1}{2} + \frac{1}{3}$$



$$\int_{0}^{\infty} 2x + 3y = 10$$

$$\int_{0}^{\infty} 2x + 1.5y = 10$$

find k' such that pair of egn has represent overlapping line??

$$\frac{2}{1} = \frac{3}{1.5} = \frac{10}{k}$$

$$2 = \frac{10}{k}$$

$$k = \frac{10}{5} = 5$$

# **Geometrical Meaning**



# Unique Solution ~

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

$$\boxed{\frac{a1}{a2} \neq \frac{b1}{b2}}$$



intersecting lines

# **Infinite Solutions**

$$a_1 x + b_1 y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

$$\frac{a1}{a2} = \frac{b1}{b2} = \frac{c1}{c2}$$

Overlapping lines

# No Solutions

$$a_1 x + b_1 y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

$$\frac{a1}{a2} = \frac{b1}{b2} \neq \frac{c1}{c2}$$





# #Q: Find the value of 'p + q' if the pair of equations

$$2x + py = 8$$
$$qx - 6y = 24$$

$$\frac{a_1}{a_2} : \frac{b_1}{b_2} : \frac{c_1}{c_2}$$

has infinite solutions. parallel line overlapping.



$$\frac{2}{9} = \frac{P}{-6} = \frac{8}{24}$$

$$\frac{2}{9} \times \frac{1}{3}$$

$$\frac{1}{9} = \frac{1}{-6} \times \frac{1}{3}$$

$$\frac{1}{9} = \frac{6}{3} = -2$$

# **QUESTION - 4 (CAT 2020 SLOT 3)**



#Q: Let k be a constant. The equations kx + y = 3 and 4x + ky = 4 have a unique solution if and only if



$$k\neq\pm 2$$



$$K = \pm 2$$

$$\frac{k}{4} \neq \frac{1}{k}$$

$$k^{2} \neq 4$$

$$k^{3} \neq 4$$

# **QUESTION - 5 (CAT 2024 Slot - 3)**



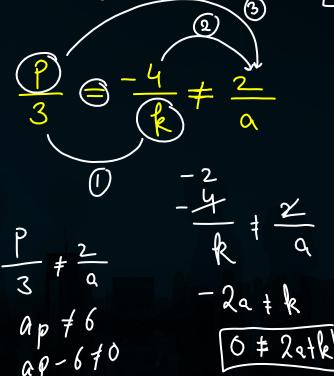
#Q. For some real numbers 'p', 'k' and 'a'. Consider the system of linear equations in x and y

$$\begin{cases} px - 4y = 2 \\ 3x + ky = a \end{cases}$$

A necessary condition for the system to have no solutions is

$$2a + k \neq 0$$

- **B** ap 6 = 0  $\chi$
- (c)  $ap + 6 = 0 \ x$
- $\bigcirc$  kp + 12  $\neq$  0



# **QUESTION - 6 (CAT 2023 Slot - 3)**



#Q. For some real numbers 'a' and 'b'. The equations x + y = 4 and  $(a + 5)x + (b^2 - 15)y = 8b$  has infinitely many solutions. Then the maximum possible value of ab is

- (A) 25
- **B** 15
- **C** 55
- **D** 33

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \qquad \begin{cases} (a+5)x + (b^2-15)y = 8b \\ 1x + 1y = 4 \end{cases}$$

$$b^{2}-15=2b$$
  
 $b^{2}-2b-15=0$ 

$$\frac{5}{5} + \frac{(-3)}{(-3)} = 2$$

$$5 \times \frac{(-3)}{(-3)} = -15$$

$$a+5 = b^{2}-15 = 2b$$

$$a+5 = 2b$$

$$a+5 = 2b$$

$$a+5 = 2b$$

$$a+5 = 2b$$

$$b=-3$$

$$a+5 = 10$$

$$a+5 = 10$$

$$a+5 = -6$$

$$a=-11$$

$$a+5 = 10$$

$$a+5 = -6$$

$$a=-11$$





# **Topic: Word Problems**

# **Words to Equation**

$$A = 6$$
  $B = 3$   $6 = 2(3)$ 



• A is twice of B Vs A is two more than B Vs A is two Times of B

• 5 more than A is twice of 2 more than B.

$$(A+5) = 2 \left[ B+2 \right]$$

- A father's age is 3 times his son.
- 3 Year Later,
- Father would be 20 Years older than the son

	F	5
PresentAge	3x	χ
3 YRo Poter	3x+3	x+3 3

# **Words to Equation**

- A Taxi cost structure comprises of two costs:
   Fixed Cost(x) and Per Km/Cost(y)
- Cost of 10 Kms is 58 Rs.

- A person agreed to work for a company in exchange of 45000 Rs and a Bike. Try to write the equation for monthly salary.
- After 7 Months he left and got 20000 and the Bike



# **Words to Equation**

Pw

- A person answered 10 questions. He got +3 for correct answer and -1 for wrong.
- He got 18 marks.

- A two digit number is such that....
- If 9 is added to the number, the digits are reversed.



#Q: In an exam a person is awarded +3 for correct answer and -1 for wrong answer. A person scored 17 marks for a certain response. If the total questions in the exam were 15. Find the total number of correctly answered questions given that the student attempted all questions.

A

5

B

6



3



$$\begin{array}{c}
15 \text{ Quu} \longrightarrow \text{2 (orrect)} \quad \text{y } \text{WRONG} \\
3 \times - \text{y} = 17 \\
2 \times + \text{y} = 15 \\
4 \times = 32 \\
2 \times = 8
\end{array}$$



#Q: Three years back, a father was 24 years older than his son. At present the father is 5 times as old as the son. How old will the son be three years from now?



12

B

6



3



	F	2	
3 42 BACK	5x-3	K-3	_
PRESSNI	5x=30	K	
THREEY MIAKR		9-	

$$(x-3)+24=(5x-3)$$
  
 $x+21=5x-3$   
 $24=4n$   
 $6=x$ 



#Q: A poultry farm has only chickens and pigs. When the manager of the poultry counted the heads of the stock in the farm, the number totalled up to 200. However, when the number of egs was counted, the number totalled up to 540. How many more chickens were there in the farm? Note: Each pig had 4 legs and each chicken had 2 legs.









$$2C + P = 200$$

$$2C + 4P = 540$$

$$2C + 2P = 400$$

$$2P = 140$$



#Q: An auto charges a fixed cost of  $x \$  for a ride and  $y \$  for each km travelled. If a person gave  $58 \$  for riding  $9 \$  Kms and another person paid  $43 \$  for travelling  $6 \$  Kms. Find the value of x - y.









$$\frac{1}{2}x + 9y = 58$$
 $\frac{1}{2}x + 6y = 43$ 
 $\frac{1}{3}y = 15$ 
 $\frac{1}{3}y = 15$ 



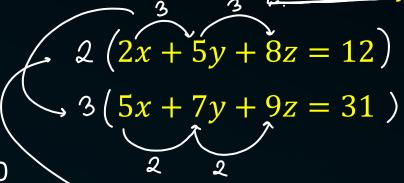


Linear Equation in 3 Variables



3VAR /2Ean

#Q. Find the value of 
$$11x + 11y + 11z$$



- **B** 69/
- **(c)** 38
- **D** 71



# #Q. Find the value of (x-23y).

$$2(2x - y + 3z = 12)$$
$$3(x + 7y + 2z = 3)$$

- **(A)** 12
- B 15
- **C** 18
- **D** 24

$$4x - 2y + 6z = 24$$
 $3x + 21y + 6z = 9$ 
 $3x - 23y = 15$ 



#Q. If 
$$(a + 2b + c = 30)^3$$
  
  $2a + b + 3c = 90$ 

Find 
$$\frac{a}{b}$$
.

$$3a+6b+3c=90$$
 $2a+b+3c=90$ 
 $a+5b=0$ 

a = - 5 b

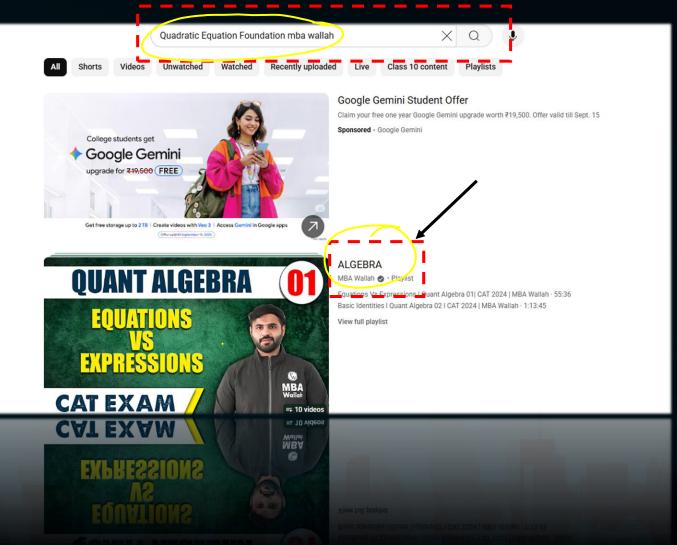
If 
$$a + 2b + c = 8$$
  
 $2a + b + 3c = 13$ ,  
 $2a - b + 6c = 18$ 

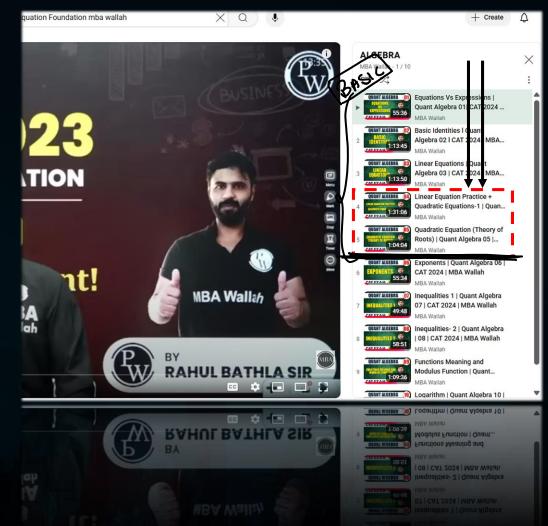






# ESSENTIALS EQ. MBAWALLAN





# **SUMMARY**



