MBA

MBA Fastrack 2025 (CAT + OMETs)

QUANTITATIVE APTITUDE

DPP: 2

Equation - 2

Q1 Find the absolute difference between the roots of the following equation.

$x^2 - 11x + 18 = 0$

Q2 The smallest positive real number that is to be added to the smaller root of the quadratic equation $x^2 - 6x - 1 = 0$ to get a non-negative integer is:

(A) $-\sqrt{10} + 3$ (B) $-\sqrt{3} + 10$

(C) $\sqrt{10}$ (D) $\sqrt{10} - 3$

Q3 If the sum of the roots of the quadratic equation

$(m+1)x^2 + (m-1)x + (m^2+1) = 0$

is - 2, then find the product of its roots.

(A) - 5 (B) 0

(C) $\frac{5}{3}$ (D) 5

Q4 What is the absolute difference between the roots of the equation below

$\frac{1}{x} - \frac{1}{x+1} = \frac{1}{6}$?

(A) 0 (B) 2

(C) 1 (D) 5

Q5 In a quadratic equation of the form $ax^2 + bx + c = 0$, which of the following can be zero?

(A) Product of a and c

(B) Sum of a and c

(C) Difference between a and c

(D) All of the above

Q6 $2x^2 + bx + 8 = 0$ has non-real roots. How many integer values can 'b' take?

(A) 16 (B) 15

(C) 18 (D) 19

Q7 Construct a quadratic equation whose roots are 5 less than the roots of the equation $x^2 - 13x + 42 = 0$.

(A) x2– 5x + 2 = 0

(B) x2– 2x + 5 = 0

(C) x2– 5x +3 = 0

(D) x2– 3x +2 = 0

Q8 Find the quadratic equation whose roots are respectively two more than the roots of

$x^2 - ax + b = 0$

(A) $\mathrm{x}^2 - (\mathrm{a}+2)\mathrm{x} + 4b = 0$

(B) $\mathrm{x}^2 - (\mathrm{a}+4)\mathrm{x} + 4b = 0$

(C) $\mathrm{x}^2 - (\mathrm{a}+4)\mathrm{x} + (2\mathrm{a}+\mathrm{b}+4) = 0$

(D) $\mathrm{x}^2 - (4-a)\mathrm{x} + (\mathrm{b}-2\mathrm{a}+4) = 0$

Q9 Find the quadratic equation whose roots are reciprocals of the roots of the equation

$(p+q)x^2 - (p^2+q^2)x + 3pq = 0.$

(A) $(p^2+q^2)x^2 - (p+q)pqx + 3(p+q) = 0$

(B) $pqx^2 - pq(p^2+q^2)(p+q)x + 3 = 0$

(C) $(p+q)x^2 - pq(p^2+q^2)x + 3 = 0$

(D) $3pqx^2 - (p^2+q^2)x + (p+q) = 0$

Q10 If one of the roots of the equation $(a+1)x^2 + 2(a^2+1)x + 2a = 0$ is reciprocal of the other, then which of the following can be a root of the equation?

(A) - 2 (B) - 1

(C) 1 (D) 2

Q11 Let p and q be the roots of the quadratic equation $(x-3)(x-5) = a,\ a \neq 0$, then find the absolute difference between the sum and product of the roots of the quadratic equation $(x-p)(x-q) + a = 0$.

Q12 Find the least value of P such that the sum of the squares of the roots of the following quadratic equation is 0.

$Px^2 + (P-1)x + 2P = 0$

(A) $-\frac{1}{3}$ (B) - 1

(C) 1 (D) $\frac{1}{3}$

Q13 The maximum and the minimum possible values of the quadratic polynomial f(x) = $4x^2 + 9x - 5$, where x is a real number, respectively are:

(A) $-\frac{9}{8}, \; -\infty$

(B) $-\frac{9}{8}, \; -10$

(C) $\infty, \; -\frac{141}{8}$

(D) $\infty, \; -\frac{161}{16}$

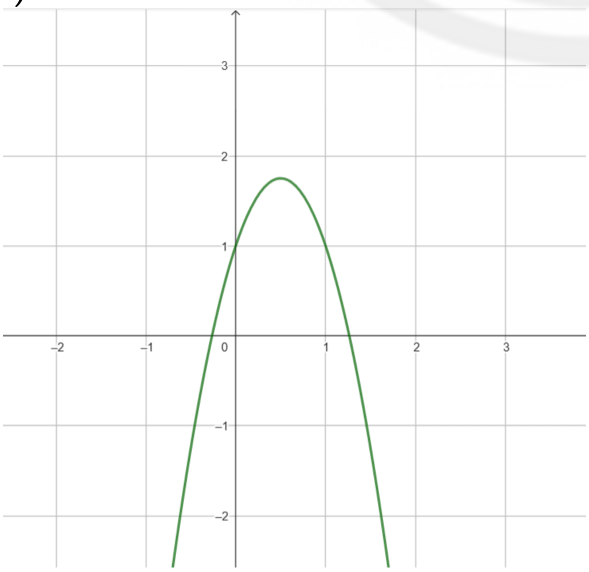
Q14 Let $(7-\sqrt{3})$ be one of the roots of the equation $px^2 + qx + r = 0$, where p, q, r all are rational numbers and s, t be the roots of the equation $qx^2 - 7rx + 22q = 0$, then what is the value of $|s-t|$?

(A) 19 (B) 21

(C) 23 (D) 27

Q15 Which of the following can be the correct graph of the quadratic polynomial $p(x) = -3x^2 + 3x - 1$?

(A)



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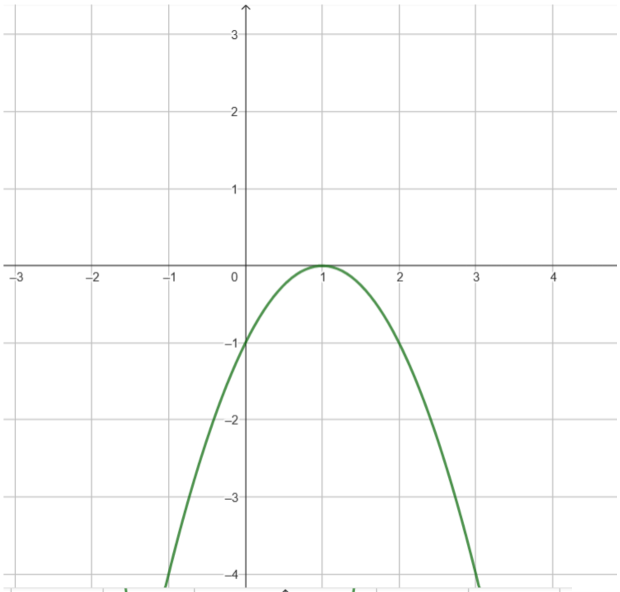
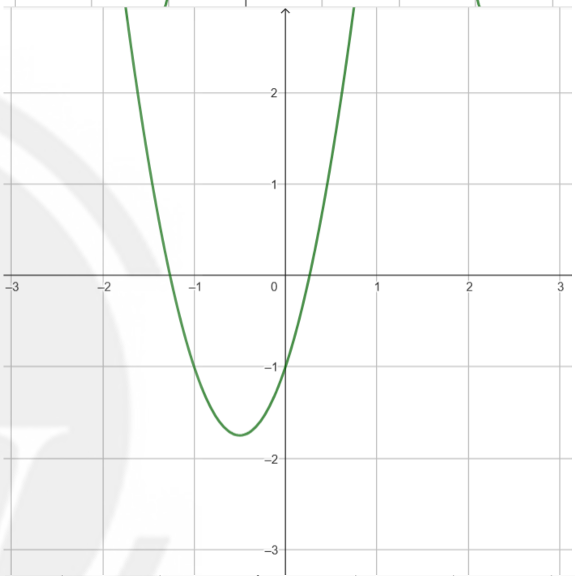
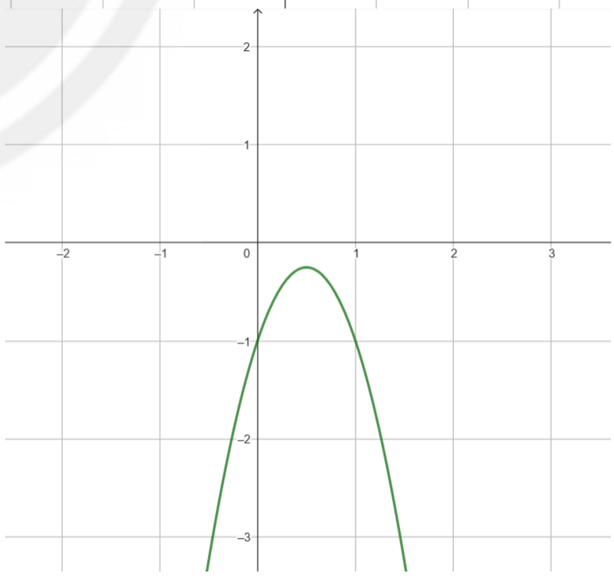


(C)



(D)



Q16 The two roots of a quadratic equation $2x^2 + ax + b = 0$ are 6 and 3. Find the minimum value of the quadratic polynomial (in 'x') whose zeroes are 3 more than the roots of

$2x^2+ax+b=0$ such that the co-efficient of x^2 in the new quadratic polynomial is 1.

(A) - 110.25 (B) - 1.5

(C) - 2.25 (D) - 7.5

Q17 If $\mathrm{l},\ \mathrm{m},\ \mathrm{n}$ are non-negative and real such that $4\mathrm{x}^2-4\mathrm{x}(\mathrm{l}+\mathrm{m}+\mathrm{n})+(\mathrm{lm}+\mathrm{mn}+\mathrm{nl})=0$ and $\mathrm{lmn}\neq 0$, then which of the following options is true?

(A) The equation has two distinct real roots.

(B) The equation has a pair of imaginary roots.

(C) The equation has repeated real roots.

(D) Nature of the roots cannot be determined.

Q18 If the equations $x^2+px+7=0$, $x^2+qx+38=0$ and $x^2+(q-p)x+15=0$ have a common negative root, then the value of (4p + 6q) is:

Q19 In the quadratic equation $px^2+2qx+2=0$, the values of $p,\ q$ lie in $\{2,\ 3,\ 4\}$. Then, for how many set of values of (p, q), does the equation have real roots?

Q20 If 'a' and 'b' are two distinct integers such that $-2x^2+ax-18\leq 0$ and $x^2-bx+4>0$ for all real values of 'x', then the maximum possible value of (a^2+b^2) is:

Q21 Let $(5+\sqrt{3})$ be one of the roots of the quadratic equation $ax^2-bx+c=0$, where a, b and c are all rational numbers. Then, find the ratio, a : b : c.

(A) 1 : 5 : 11 (B) 1 : 7 : 13

(C) 1 : 10 : 22 (D) 1 : 14 : 26

Q22 If p is a constant such that $x^2+6x-13=p\ \text{ and }\ x^2+6x-13=-p$ together have exactly three distinct real roots, then what is the product of all these three roots of the equation?

Q23 The maximum value of $mx^2 + 14x + n$ can be obtained at x = 7. If the product of the roots of $mx^2 + 14x + n = 0$ is - 2, then what can be the quadratic equation with the roots $\frac{1}{m+n}$ and $\frac{1}{m-n}$?

(A) $2x^2 - 3x - 1 = 0$

(B) $3x^2 + 2x + 1 = 0$

(C) $2x^2 + 3x + 1 = 0$

(D) $3x^2 - 2x - 1 = 0$

Q24 If the equations m^2 + 7nm + 10 = 0 and m^2 + 3pm - 85 = 0 have a common root such that n and p are integral values. Find the number of possible values of p?

(A) 0 (B) 2

(C) 4 (D) 6

Q25 For some real numbers p and q, the system of equations x + y = 3 and (p + 7)x +(q^2 - 28)y = 9q has infinitely many solutions for x and y. Then, the maximum possible value of $532\left(\frac{1}{p}-\frac{1}{q}\right)$ is:

Q26 If a and b are roots of equation $x^2-3x-4k=0$ and b and c are roots of equation $x^2-5x-2k=0$, what will be the positive integral value of $a\times b\times c$?

(A) 0 (B) 56

(C) 84 (D) Both a and b

Q27 If $x^2+\frac{1}{x^2}=3$ where x is a positive real number, find the value of $x^{27}+\frac{1}{x^{27}}$.

(A) $5777\ \times\ 34\sqrt{5}$

(B) $6777\ \times\ 36\sqrt{5}$

(C) $8777\ \times\ 38\sqrt{5}$

(D) $9777\ \times\ 40\sqrt{5}$

Q28 If the roots of the quadratic equation $x^2+\left(\sqrt{m^2+13m+18}\right)x+3m+3=0$ are in the ratio 3:1, what is the sum of all the potential values of m?

Q29 A bamboo shoot is cut into two unequal pieces such that the ratio of the length of the longer piece to the length of the shorter piece is the same as the ratio of the length of the original bamboo shoot to the length of the longer piece. If the length of the shorter piece is 6 meters, find the length of the original bamboo shoot.

(A) $9+3\sqrt{5}\,m$ (B) $9-3\sqrt{5}\,m$

(C) 9 m (D) Either a or b

Q30 $\frac{M^2}{x+1}+\frac{N^2}{x-1}=1$

If M and N are real numbers such that $(NM)^2>2$, how many real values of 'x' exist for the given equation?

Answer Key

Q1	7
Q2	D
Q3	A
Q4	D
Q5	D
Q6	B
Q7	D
Q8	C
Q9	D
Q10	B
Q11	7
Q12	B
Q13	D
Q14	B
Q15	D
Q16	C
Q17	A
Q18	104
Q19	7
Q20	153
Q21	C
Q22	105
Q23	D
Q24	B
Q25	105
Q26	B
Q27	A
Q28	3
Q29	A
Q30	2

Hints & Solutions

Note: scan the QR code to watch video solution

Q1 Text Solution:

We have $x^2 - 11x + 18 = 0$

Sum of roots = 11 and Product of roots = 18

Now, we just need to think of two numbers whose sum is 11 and Product is 18

So, the numbers will be 9 and 2

Their absolute difference is |9 - 2| = 7

Alternatively:

The given quadratic equation is:

$x^2 - 11x + 18 = 0$ ...(i)

To find the absolute difference between the roots of the equation, first, we need to solve the equation, i.e., we have to find the values of x.

To find the values of x from (i), we have to write it in the form (x - a)(x - b) = 0, where a, and b are the roots of the equation.

Now, to write equation (i) into the factorized form, we will use the Middle-term splitting method.

According to this method, first, we have to split the middle term - 11 so that when the obtained numbers are multiplied gives the product of the coefficient of x^2 and the constant term, i.e., gives 18.

Therefore, we only can write - 11 as - 9 + (- 2) and again, (- 9)(- 2) = 1 × 18 = 18

So, we can rewrite (i) as:

$x^2 + (-9 - 2)x + 18 = 0$

$x^2 - 9x - 2x + 18 = 0$

$x(x - 9) - 2(x - 9) = 0$

$\left[\text{Grouping and taking out the common factors from each group}\right]$

$(\mathrm{x} - 2)(\mathrm{x} - 9) = 0$

$\left[\text{Further factorizing}\right]$

So, the roots of the equation (i) are: x = 2, 9

Thus, the absolute difference between the roots = |9 - 2| = 7

Video Solution:

Q2 Text Solution:

The given quadratic equation is:

$x^2 - 6x - 1 = 0$ ...(i)

Clearly, here we cannot use the Middle Term Splitting method because - 6 cannot be spit in such a way that the product of the obtained numbers becomes equal to - 1.

So, we need to use the Sridharacharya Method to solve this Quadratic Equation.

According to Sridharacharya Method, the solution to a quadratic equation of the form $ax^2 + bx + c = 0$ is $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

So, here,

$x = \frac{6 \pm \sqrt{36 + 4}}{2}$

$= \frac{6 \pm \sqrt{40}}{2}$

$= 3 \pm \sqrt{10}$

$= 3 + \sqrt{10},\ 3 - \sqrt{10}$

So, the smallest root of the given quadratic equation is $3 - \sqrt{10}$.

Therefore, the smallest positive real number that is to be added to the smallest root of the given equation to get a non-negative integer is $\sqrt{10} - 3$.

Video Solution:

Q3 Text Solution:

The given quadratic equation can be written as:

$$\left(m+1\right)x^2+\left(m-1\right)x+\left(m^2+1\right)=0$$

$$x^2-\frac{(1-m)}{(1+m)}x+\frac{m^2+1}{1+m}=0$$

So, the sum of the roots of the quadratic equation $=\frac{1-m}{1+m}=-2\ \left(\text{given}\right)$

$\Rightarrow 1-m=-2-2m$

$\Rightarrow m=-3$

So, the product of the roots

$=\frac{m^2+1}{1+m}=\frac{9+1}{1-3}=\frac{10}{-2}=-5.$

Video Solution:

Q4 Text Solution:

$$\frac{1}{x}-\frac{1}{x+1}=\frac{1}{6}$$

$$\frac{x+1-x}{x^2+x}=\frac{1}{6}$$

$$6=x^2+x$$

$$x^2+x-6=0$$

$$(x-2)(x+3)=0$$

$$x=2 \text{ or } x=(-3)$$

So, the difference is $2-(-3)=5$ or $-3-2=-5$.

Required difference will be 5 or -5 ,thus the absolute difference will be 5

Video Solution:

Q5 Text Solution:

From the theory of quadratic equations, we know that 'a' cannot be zero. But 'c' can be zero. So, obviously, the product of 'a' and 'c' can be zero.

Now, both 'a' and 'c' can be equal. So, their difference can be zero.

In , a – c = 0

Again, a + c can also be zero. In , a + c = 0

Hence, all the options can be zero.

Hence, option (D) is the correct answer.

Video Solution:

Q6 Text Solution:

For non-real roots, discriminant D < 0.

b^2 - 4ac < 0

b^2 - 4 × 2 × 8 < 0

b^2 < 64

-8 < b < 8

Then, the possible values of b are –7, –6, –5, –4, –3, –2, –1, 0, 1, 2, 3, 4, 5, 6, and 7.

Hence, option B is the correct answer.

Video Solution:

Q7 Text Solution:

The given equation is x^2 - 13x + 42 = 0.

Then, x^2 - (6 + 7)x + 42 = 0

=> x^2 - 6x - 7x + 42 = 0

=> x(x - 6) - 7(x - 6) = 0

=> (x - 7)(x - 6) = 0

=> x = 7, 6

Then, the roots of the quadratic equation whose roots are 5 less than the roots of the

equation x^2 - 13x + 42 = 0, are 2, 1.
So, the required quadratic equation is
=> $x^2 - (2 + 1)x + 2 = 0$
=> $x^2 - 3x + 2 = 0$.
Alternatively, we can directly substitute 'x+5' in place of 'x' in the original quadratic equation to form the new quadratic equation.

Video Solution:

Q8 Text Solution:
The given quadratic equation is:
$x^2 - ax + b = 0$ ...(i)
Then, the sum of the roots of (i) is 'a' and the product of the roots is 'b'.
i.e., if the roots of the equation are p and q, then
$p + q = a \ \ and \ \ pq = b$
Therefore, the required quadratic equation whose each of the roots is 2 more than each of the roots of (i), is:
$$x^2 - (p + 2 + q + 2)x + (p + 2)(q + 2) = 0$$
$$x^2 - (p + q + 4)x + pq + 2(p + q) + 4 = 0$$
$$x^2 - (a + 4)x + b + 2a + 4 = 0$$
$$[\text{Substituting the values of } (p + q) \text{ and } pq]$$
$$x^2 - (a + 4)x + (2a + b + 4) = 0$$
Alternative Solution:
We can directly replace 'x-2' in place of 'x' in the given quadratic equation to form a new quadratic equation whose roots are 2 more than the roots of the given quadratic equation.

Video Solution:

Q9 Text Solution:
The given quadratic equation is:
$$(p + q)x^2 - (p^2 + q^2)x + 3pq = 0$$
We know that the quadratic equation whose roots are the reciprocal to the roots of the equation $ax^2 + bx + c = 0$ is of the form:
$cx^2 + bx + a = 0$.
Therefore, the required quadratic equation becomes:
$$3pqx^2 - (p^2 + q^2)x + (p + q) = 0.$$

- **Prerequisite:**

[***Statement:*** The quadratic equation whose roots are the reciprocals of the roots of the equation $ax^2 + bx + c = 0$ is of the form:
$cx^2 + bx + a = 0$.
Proof: Let m and n be the roots of the equation $ax^2 + bx + c = 0$.
Then, $m + n = -\frac{b}{a} \ \ and \ mn = \frac{c}{a}$
Now, $\frac{1}{m} + \frac{1}{n} = \frac{m+n}{mn} = \frac{-\frac{b}{a}}{\frac{c}{a}} = -\frac{b}{c}$ and
$\frac{1}{m} \times \frac{1}{n} = \frac{1}{mn} = \frac{1}{\frac{c}{a}} = \frac{a}{c}$
So, the required quadratic equation becomes:
$x^2 - \left(-\frac{b}{c}\right)x + \frac{a}{c} = 0, \ \text{i.e.,} \ cx^2 + bx + a = 0$
]

Video Solution:

Q10 Text Solution:

The given quadratic equation can be written as:

$$(a+1)x^2+2(a^2+1)x+2a=0$$

$$x^2+\frac{2(a^2+1)}{(a+1)}x+\frac{2a}{a+1}=0$$

Let the roots of the quadratic equation be 'm' and '$\frac{1}{m}$'. Then, we have

$$m+\frac{1}{m}=-\frac{2(a^2+1)}{(a+1)} \text{ ...(i) and}$$

$$m\times\frac{1}{m}=\frac{2a}{a+1}\ldots(ii)$$

$$\frac{2a}{a+1}=1$$

$$2a=a+1$$

$$a=1$$

So, from (i), we get

$$m+\frac{1}{m}=-\frac{2\times 2}{2}=-2$$

$$m^2+1=-2m$$

$$m^2+2m+1=0$$

$$(m+1)^2=0$$

$$m=-1,\ -1$$

Video Solution:

Q11 Text Solution:

The first given equation is:

$$(x-3)(x-5)=a,\ a\neq 0$$

$$x^2-8x+(15-a)=0$$

Therefore, we have

p + q = 8 ...(i) and

pq = 15 - a ...(ii)

Now, the second equation is given as:

$$(x-p)(x-q)+a=0$$

$$x^2-(p+q)x+pq+a=0$$

$$x^2-8x+15-a+a=0$$

$$[\text{Using (i) and (ii)}]$$

$$\mathrm{x}^2-8\mathrm{x}+15=0$$

So, the required absolute difference = |8 - 15| = 7

Video Solution:

Q12 Text Solution:

The given quadratic equation can be written as:

$$Px^2+(P-1)x+2P=0$$

$$x^2+\frac{P-1}{P}x+2=0$$

Let the roots of the equation be 'a' and 'b'. Then, we have

The sum of the roots = a + b = $-\frac{P-1}{P}$ ...(i)

The product of the roots = ab = 2 ...(ii)

Now, given that, 0

$$\Rightarrow (a+b)^2-2ab=0$$

$$\Rightarrow \frac{(P-1)^2}{P^2}-2(2)=0$$

$$\Rightarrow (P-1)^2-4P^2=0$$

$$\Rightarrow (P-1)^2-(2P)^2=0$$

$$\Rightarrow (P-1+2P)(P-1-2P)=0$$

$$\Rightarrow (3P-1)(P+1)=0$$

$$\Rightarrow P=\frac{1}{3},\ -1$$

So, the least value of P = -1.

Video Solution:

Q13 Text Solution:

The given quadratic polynomial is:

f(x) = $4x^2 + 9x - 5$ ...(i)

Here, in equation (i), the coefficient of x^2 is +ve, so the graph of the function (i.e., the parabola) is open upwards.

So, it must have the **maximum value** as $+\infty$.

But, for the minimum value, we know that the minimum value of a quadratic polynomial of the form $ax^2 + bx + c, \ (a > 0)$ can be found at $x = -\frac{b}{2a}$ and the minimum value will be = $-\frac{Discriminant}{4a}$.

So, for eq. (i), the minimum value will be obtained at $x = -\frac{9}{2(4)} = -\frac{9}{8}$

The **minimum value** of the polynomial =

$-\frac{9^2 - 4(4)(-5)}{4 \times 4} = -\frac{81+80}{16} = -\frac{161}{16}$

Video Solution:

Q14 Text Solution:

Since, $(7 - \sqrt{3})$ is one of the roots of the quadratic equation $px^2 + qx + r = 0$, so the other root must be $(7 + \sqrt{3})$, as the irrational roots of a quadratic equation with rational coefficients appear in conjugate pair.

Therefore, the sum of the roots,

$7 - \sqrt{3} + 7 + \sqrt{3} = -\frac{q}{p}$

$\Rightarrow q = -14p$ ... (i)

Also, the product of the roots,

$(7 + \sqrt{3})(7 - \sqrt{3}) = \frac{r}{p}$

$\Rightarrow r = (49 - 3)p$

$\Rightarrow r = 46p \ldots (ii)$

Now, by dividing (ii) by (i), we get

$\frac{r}{q} = -\frac{46p}{14p}$

$i.e., \ \frac{r}{q} = -\frac{23}{7} \ldots \left(iii\right)$

Now, Since, s, t are the roots of the equation $qx^2 - 7rx + 22q = 0$, so

$s + t = \frac{7r}{q} = -7 \times \frac{23}{7} = -23$ [Using (iii)]

Also, $st = 22$

Therefore, we have

$(s - t)^2 = (s + t)^2 - 4st = (-23)^2 - 4 \times 22 = 529 - 88 = 441$

$|s - t| = 21$

Video Solution:

Q15 Text Solution:

The given polynomial is:

$p(x) = -3x^2 + 3x - 1$

Since, the coefficient of x^2 is negative, so the curve of the polynomial (parabola) is open downwards.

Now, let's check the discriminant of the quadratic equation $-3x^2 + 3x - 1 = 0$.

Here, the discriminant,

$(3)^2 - 4 \times (-3) \times (-1) = -3 < 0$

So, the quadratic equation has no real roots.

Thus, the curve of the polynomial must not touch or cut the x-axis.

Also, it cuts the y-axis at 'c' i.e. -1

Thus, option D represents the correct graph.

Video Solution:

Q16 Text Solution:

Given that, the roots of the quadratic equation $2x^2 + ax + b = 0$ are 6 and 3.

So, the zeros of the new quadratic polynomial are (6 + 3) and (3 + 3) i.e., 9 and 6.

Since, the co-efficient of x^2 in the new quadratic polynomial is 1, so the quadratic polynomial becomes

p(x) = (x - 6)(x - 9) = x^2 - 15x + 54

Since the co-efficient of x^2 is +ve, the curve of the parabola faces upwards

Now, the minimum value of p(x) will be obtained at $\frac{15}{2} = 7.5$.

Thus, the minimum value of p(x) = (7.5 - 6)(7.5 - 9) = - (1.5)(1.5) = - 2.25

Alternatively,

The minimum value of p(x) will be obtained at x = $-\left(\frac{-15}{2\times1}\right) = 7.5$

Thus, the required minimum value of p(x) is

$$p\left(7.5\right) = (7.5)^2 - 15\left(7.5\right) + 54 = -2.25$$

Video Solution:

Q17 Text Solution:

The given quadratic equation is:

$$4x^2 - 4x\left(l + m + n\right) + \left(lm + mn + nl\right) = 0$$

...(i)

Now, to find the nature of the roots of the given equation, we need to find the 'sign' of the discriminant.

$$D = 16(l + m + n)^2 - 4 \times 4 \times (lm + mn + nl)$$
$$= 16\left(l^2 + m^2 + n^2 + 2lm + 2mn + 2nl\right) - 16\left(lm + mn + nl\right)$$
$$= 8\left[\left(l^2 + 2lm + m^2\right) + \left(m^2 + 2mn + n^2\right) + \left(n^2 + 2nl + l^2\right)\right]$$
$$= 8\left[(l + m)^2 + (m + n)^2 + (n + l)^2\right]$$

Since, $l,\ m,\ n$ are non-negative reals and $lmn \neq 0$, so D > 0.

So, the roots of the quadratic equation are real and distinct.

Video Solution:

Q18 Text Solution:

The given quadratic equations are:

$$x^2 + px + 7 = 0 \ldots (i)$$
$$x^2 + qx + 38 = 0 \ldots (ii)$$
$$x^2 + \left(q - p\right)x + 15 = 0 \ldots (iii)$$

Now, add equations (i) and (iii) and then subtract (ii) from the result. So, we get

$$x^2 + px + 7 + x^2 + \left(q - p\right)x + 15 - x^2 - qx - 38 = 0$$
$$x^2 + px + 7 + x^2 + qx - px + 15 - x^2 - qx - 38 = 0$$
$$x^2 - 16 = 0$$
$$x = \pm 4$$

Since, all these equations have a common negative root, so the common root is - 4.

Since, the common root must satisfy each equation, so substituting x = - 4 in equation (i), we get

16 - 4p + 7 = 0

i.e., p = $\frac{23}{4}$

Similarly, by substituting x = - 4 in equation (ii), we get

16 - 4q + 38 = 0

i.e., 4q = 54

i.e., q = $\frac{54}{4} = \frac{27}{2}$

Thus, 4p + 6q = 23 + 81 = 104

Video Solution:

Q19 Text Solution:

We know that a quadratic equation has real roots if the Discriminant is greater than or equals zero.

i.e., here for the equation $px^2 + 2qx + 2 = 0$, we must have

$4q^2 - 8p \geq 0, \;\; i.e., \; q^2 - 2p \geq 0$ to get real roots.

Let's analyze all the possible cases:

q-values	p-values
2	2
3	2
3	3
3	4
4	2
4	3
4	4

So, the total number of sets of possible values of (p, q) is 7.

Video Solution:

Q20 Text Solution:

The given inequations can be expressed as:

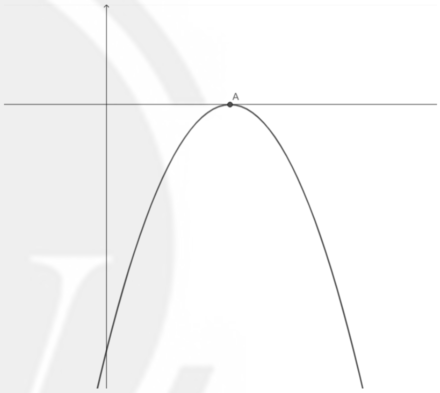
$f(x) = -2x^2 + ax - 18 \leq 0$ ...(i) and

$g(x) = x^2 - bx + 4 > 0$ ...(ii)

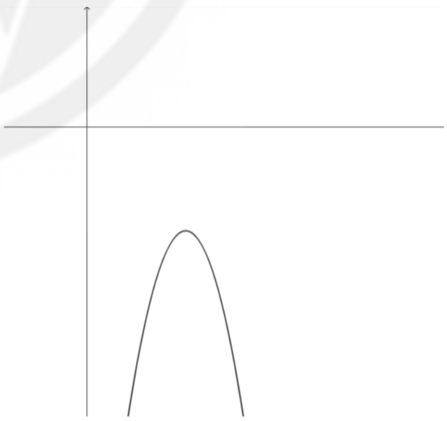
Now, the graph of both functions is parabolic.

For the first equation, the coefficient of x^2 is -ve, and so the graph of the parabola is open downwards. Also, $f(x) \leq 0$ which means, the graph of the parabola is located below the X-axis and its vertex touches the X-axis at one point. (However, if somehow, the graph cuts the X-axis twice, then some portion of the graph will be above the X-axis and that will make f(x) positive, which is a contradiction. Thus, it cannot have two distinct real roots.)

Therefore, the roots of the quadratic equation $-2x^2 + ax - 18 = 0$ ...(iii) are either imaginary or repeated reals. For more clarity, look at the below diagram.



or,



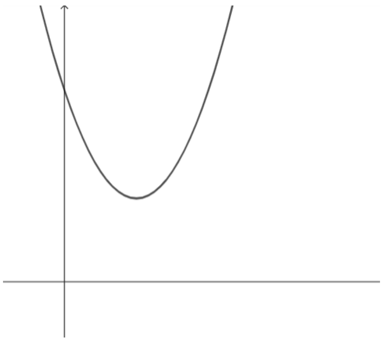
Thus, the discriminant for equation (iii) will be:

$a^2 - 144 \leq 0$

i.e., $a^2 \leq 144$ ...(iv)

Now, for the second equation, the coefficient of x^2 is +ve, and so the graph of the parabola is open upwards. Also, g(x) > 0. So, the graph of the parabola is located above the X-axis and it will neither touch nor cut the X-axis. So, the roots of the quadratic equation

$x^2 - bx + 4 = 0$ will always be imaginary.



Therefore, the discriminant for this equation will be: $b^2 - 16 < 0$

i.e., $b^2 < 16$ ...(v)

So, the maximum possible integer value of $a^2 + b^2$ = 144 + 9 = 153 [Since, b is an integer, the maximum value of b^2 can only be 9.]

Video Solution:

Q21 **Text Solution:**

Since, $\left(5 + \sqrt{3}\right)$ is a root of the quadratic equation $ax^2 - bx + c = 0$ and a, b, and c are all rationals, so the other root must be $\left(5 - \sqrt{3}\right)$, as irrational roots of a polynomial with rational coefficients always occur in conjugate pairs.

Therefore, the sum of the roots of the given equation,

$\frac{b}{a} = 5 + \sqrt{3} + 5 - \sqrt{3} = 10$

$b = 10a \ldots (i)$

Also, the product of the roots of the equation,

$\frac{c}{a} = \left(5 + \sqrt{3}\right)\left(5 - \sqrt{3}\right) = 5^2 - \left(\sqrt{3}\right)^2$

$= 25 - 3 = 22$

$c = 22a \ldots \left(ii\right)$

So, a : b : c = a : 10a : 22a = 1 : 10 : 22.

Video Solution:

Q22 **Text Solution:**

The given equations are:

$x^2 + 6x - 13 = p \ldots (i)$ and $x^2 + 6x - 13 = -p \ldots (ii)$

Now, according to the given problem, combining equations (i) and (ii), it has only 3 distinct real roots. We know that imaginary roots come in pairs. So, none of the two equations has an imaginary root.

So, **the only possibility is:** One of the equations has two distinct real roots and the other equation has repeated real roots. So, by combining them, we can get 3 distinct real roots.

Now, to get repeated roots from a quadratic equation, the equation must form a perfect square.

So, from equation (ii), we get

$x^2 + 2 \times x \times 3 + \left(p - 13\right) = 0$ ...(iii)

So, for the LHS of equation (iii) to be a perfect square, it is obvious that

$(p - 13) = 3^2$

$p = 9 + 13 = 22$

So, when p = 22, equation (ii) becomes

$\left(x + 3\right)^2 = 0$

$x = -3, \ -3$

When p = 22, equation (i) becomes

$x^2 + 6x - 13 = 22$

$x^2 + 6x - 35 = 0$

So, the product of all the three distinct roots (as stated in the problem) = (- 3)(- 35) = 105.

Video Solution:

Q23 Text Solution:

We know that, any quadratic expression $ax^2 + bx + c$ has its maximum/minimum value at x = $-\frac{b}{2a}$.

Therefore, the maximum value of $mx^2 + 14x + n$ can be obtained at x = $-\frac{14}{(2\times m)} = -\frac{7}{m}$

So, $-\frac{7}{m}$ = 7

m = - 1

Now, since the product of the roots for $mx^2 + 14x + n = 0$ is - 2, so

$\frac{n}{m}$ = - 2

=> n = - 2m = - 2(- 1) = 2

Thus, the required quadratic equation will be

$x^2 - (\frac{1}{m+n} + \frac{1}{m-n})x + \frac{1}{m^2-n^2} = 0$

$x^2 - (\frac{1}{-1+2} + \frac{1}{-1-2})x + \frac{1}{(-1)^2-2^2} = 0$

$x^2 - \frac{2x}{3} - \frac{1}{3} = 0$

$3x^2 - 2x - 1 = 0$

Video Solution:

Q24 Text Solution:

Given that the two equations m^2 + 7nm + 10 = 0 and m^2 + 3pm - 85 = 0 have exactly one root in common.

As the product of roots which is c/a of both the equations is different, the roots must be rational (Had they been irrational, it would have been in the form of a quadratic surd such as a + √b and other other root of both the equations would have been a - √b and therefore both the roots would be common)

Now as n and p are integral values, the sum of roots will be an integer. Further, the roots are rational and their product is also rational. Therefore, the roots of the above equation will also be integers. Now, the roots of m^2 + 7nm + 10 = 0 will have to be factors of 10 i.e. (± 1, ± 2, ± 5, ± 10)

And the roots of m^2 + 3pm - 85 = 0 will similarly be factors of 85 i.e. (± 1, ± 5, ± 17, ± 85)

Now, only if the roots of equation m^2 + 7nm + 10 = 0 are (2,5) or (-2, -5) will 'n' be an integral value (On the basis of sum of roots, this can be tested)

When the roots of the first equation are taken as (2,5), the common root will be 5.

Therefore, the other root of the second equation will be -17. Therefore, sum of roots which is -3p = 5-17 = -12 So p can be 4

When the roots of the first equation are taken as (-2,-5), the common root will be -5.

Therefore, the other root of the second equation will be 17. Therefore, sum of roots which is -3p = -5+17 = 12 So p can be -4

As both the values of p are integers, we will consider both of these. Hence, there are two possible values of p satisfying the above conditions.

Video Solution:

Q25 Text Solution:

The given system of linear equations is:

x + y = 3 ...(i)

$(p + 7)x + (q^2 - 28)y = 9q$ ...(ii)

Since, the system has infinitely many solutions, so we have

$\frac{1}{p+7}=\frac{1}{q^2-28}=\frac{3}{9q}$

$\Rightarrow 3p+21=9q$ and $3q^2-84=9q$

$\Rightarrow p+7=3q\ldots(iii)$ and q^2-28

$=3q\ldots(iv)$

Solving (iv), we get

$q^2-3q-28=0$

$q^2-7q+4q-28=0$

$q(q-7)+4(q-7)=0$

$(q+4)(q-7)=0$

$q=-4,\ 7$

Now, if q = - 4, then p = 3(- 4) - 7 = - 19 and thus, pq = (- 4)(- 19) = 76.

So,

$532\left(\frac{1}{p}-\frac{1}{q}\right)=532\times\frac{q-p}{pq}=532\times\frac{-4+19}{76}$

$=105$

if q = 7, then p = 3(7) - 7 = 14 and thus, pq = (14)(7) = 98

So,

$532\left(\frac{1}{p}-\frac{1}{q}\right)=532\times\frac{q-p}{pq}=532\times\frac{7-14}{98}$

$=-38$

Thus, the required maximum possible value is 105.

Video Solution:

Q26 Text Solution:

a+b = 3

and b+c = 5

Using b as the common root

$b^2-3b-4k=0$

and

$b^2-5b-2k=0$

Finding difference between the two equations

2b=2k

⇒b = k

So using k as b,

$k^2-3k-4k=0$

$\Rightarrow k(k-7)=0$

So

$k=0$ *or* $k=7$

So

If b = k = 0

a=3 and c = 5

Then

$a\times b\times c=0$

If b = k = 7

a = -4 and c = -2

Then

$a\times b\times c=-4\times 7\times -2=56$

Since the question is asking about the positive integral value of the product of a, b and c, it is going to be 56 since 0 is not a positive integer.

Video Solution:

Q27 Text Solution:

Given that,

$x^2+\frac{1}{x^2}=3$

$\left(x+\frac{1}{x}\right)^2-2\times x\times\frac{1}{x}=3$

$\left(x+\frac{1}{x}\right)^2=5$

$x+\frac{1}{x}=\sqrt{5}$ [Since, x is a positive real number.]

Now, $x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x}\right)^3 - 3 \times x$
$\times \frac{1}{x}\left(x + \frac{1}{x}\right)$
$= \left(\sqrt{5}\right)^3 - 3 \times \sqrt{5}$
$= 2\sqrt{5}$
Thus, $\left(x^9 + \frac{1}{x^9}\right) = \left(x^3\right)^3 + \left(\frac{1}{x^3}\right)^3$
$= \left(x^3 + \frac{1}{x^3}\right)^3 - 3\left(x^3 + \frac{1}{x^3}\right) = \left(2\sqrt{5}\right)^3$
$- 3 \times 2\sqrt{5} = 34\sqrt{5}$
Therefore,

$x^{27} + \frac{1}{x^{27}} = \left(x^9\right)^3 + \left(\frac{1}{x^9}\right)^3$

$= \left(x^9 + \frac{1}{x^9}\right)^3 - 3 \times x^9 \times \frac{1}{x^9}\left(x^9 + \frac{1}{x^9}\right)$

$= \left(x^9 + \frac{1}{x^9}\right)^3 - 3\left(x^9 + \frac{1}{x^9}\right)$

$= \left(34\sqrt{5}\right)^3 - 3 \times 34\sqrt{5}$

$= 34\sqrt{5}\left[\left(34\sqrt{5}\right)^2 - 3\right] = 34 \times 5777$

$\times \sqrt{5}$

Video Solution:

Q28 Text Solution:

If a quadratic equation $ax^2 + bx + c = 0$ has roots in the ratio p:q, then taking the roots as pk and qk

pk+qk= $\frac{-b}{a}$

$\Rightarrow k = \frac{-b}{a(p+q)}$

and $pqk^2 = \frac{c}{a}$

$\Rightarrow pq\left(\frac{-b}{a(p+q)}\right)^2 = \frac{c}{a}$

$\Rightarrow pqb^2 = ac(p+q)^2$
So using this equation for given quadratic equation

$\left(3\right)\left(1\right)\left(\sqrt{m^2 + 13m + 18}\right)^2$

$= \left(1\right)\left(3m + 3\right)(3 + 1)^2$

$\Rightarrow 3m^2 + 39m + 54 = 48m + 48$
$\Rightarrow 3m^2 - 9m + 6 = 0$
$\Rightarrow \left(3m - 6\right)\left(m - 1\right) = 0$
$\Rightarrow m = 1 \text{ or } 2$
So sum of all potential values of m = 2+1 = 3

Video Solution:

Q29 Text Solution:

Let the length of the bamboo shoot be x meters long

So

$\frac{x-6}{6} = \frac{x}{x-6}$

$\Rightarrow x^2 - 12x + 36 = 6x$
$\Rightarrow x^2 - 18x + 36 = 0$
$\Rightarrow x = \frac{18 \pm \sqrt{324-144}}{2}$

$\Rightarrow x = \frac{18 \pm 6\sqrt{5}}{2} = 9 \pm 3\sqrt{5}$

So x = 9+3$\sqrt{5}$ meters or x = 9-3$\sqrt{5}$ meters, but since x should be greater than 6 meters so x = $9 + 3\sqrt{5}$ m

Video Solution:

Q30 Text Solution:

$\frac{M^2}{x+1} + \frac{N^2}{x-1} = 1$

$\Rightarrow M^2\left(x-1\right) + N^2\left(x+1\right) = x^2 - 1$

$\Rightarrow M^2x - M^2 + N^2x + N^2 = x^2 - 1$

$\Rightarrow x^2 - \left(M^2 + N^2\right)x$

$+ \left(M^2 - N^2 - 1\right) = 0$

So

$D = \left(-(M^2 + N^2)\right)^2$

$- 4\left(1\right)\left(M^2 - N^2 - 1\right)$

$= N^4 + M^4 + 2N^2M^2 - 4M^2 + 4N^2$
$+ 4$

$= N^4 + 4N^2 + 4 + M^4 - 4M^2 + 4$
$+ 2N^2M^2 - 4$

$= \left(N^2 + 2\right)^2 + \left(M^2 - 2\right)^2$

$+ 2\left(N^2M^2 - 2\right)$

First term will be greater than 0, the second term will be greater than or equal to 0 and the third term will always be greater than 0, so D>0

and hence the equation will have 2 real and distinct roots

Video Solution: