MBA Fastrack 2025 (CAT + OMETs)

QUANTITATIVE APTITUDE

DPP: 3

Exponents

Q1 Arrange the following in the ascending order of their magnitude.

$P = 4^{444}$; $Q = 444^4$; $R = 44^{44}$; $S = 4^{4^{4^4}}$

(A) QRPS (B) QPRS
(C) PQRS (D) PRSQ

Q2 If $4^{5x} - 3^{5x+2} = 295 \times 3^{5x-4} - \frac{4^{5x-2}}{121^{-\frac{1}{2}}}$, then find the value of x.

(A) $\frac{3}{2}$ (B) $\frac{5}{3}$
(C) $\frac{7}{5}$ (D) $\frac{9}{7}$

Q3 How many integer pairs (x, y) satisfy the following equation if

$(a - b + c)$: $(b - c + 2d) : (2a + c - d) = 3 : 4 : 7$

such that $\frac{5a-3d+5c}{a+d+c} = k$ and $\sqrt{x} + \sqrt{y} = k$

Q4 If $\frac{9^{2p-4}+13}{\frac{5}{2}\times 3^{2p-4}+1} = 4$, then find the sum of the possible roots of the equation.

Q5 Given that a and b are positive real numbers greater than 1, simplify the following expression and find the value of k if:

$$\left(\sqrt[3]{a^2 \cdot b}\right)^{\frac{3}{4}} \cdot \left(\sqrt[4]{a^3 \cdot b^2}\right)^{\frac{4}{3}} = a^k \cdot b^{\frac{11}{12}}$$

(A) $\frac{5}{2}$ (B) $\frac{3}{2}$
(C) $\frac{1}{2}$ (D) $\frac{7}{2}$

Q6 Simplify $\frac{9x^{-3}yz^{-4}}{3xy^{-4}z^3} \div \frac{6x^{-4}y^3z^{-2}}{2xy^{-3}z^4}$?

(A) $\frac{x}{yz}$
(B) $\frac{1}{3xyz}$
(C) $\frac{3y}{xz}$
(D) xyz

Q7 Find the value of x which satisfies the equation:-

$5 \times 2^{x+3} - 21 \times 2^{x-1} = 236$

Q8 If x = $(6561)^{5+2\sqrt{3}}$, then which of the following equals 81?

(A) $\frac{x^5}{x^{4\sqrt{3}}}$ (B) $\frac{x^{\frac{5}{26}}}{x^{2\sqrt{3}}}$
(C) $\frac{x^{\frac{5}{26}}}{x^{\frac{4}{\sqrt{3}}}}$ (D) $\frac{x^{\frac{5}{26}}}{x^{\frac{\sqrt{3}}{13}}}$

Q9 Let $2^a \cdot 3^b \cdot 5^c = \left(\frac{1}{2}\right)^2 (9)^3 (25)^4$. If $a^b b^c c^a = x^4$, then $\sqrt{x} = ?$

(A) -6 (B) 6
(C) 36 (D) 9

Q10 If $\frac{4^a}{5^b} = \left(\frac{32}{625}\right)^y$, then find $a + b$, where $y = \sqrt{2 + \sqrt{2 + \sqrt{2 + \ldots\infty}}}$.

(A) 10 (B) 11
(C) 12 (D) 13

Q11 If $(2^x - 7)^2 = 6\left(2^x - \frac{5}{2}\right)$, find the sum of the possible values of x.

(A) 5 (B) 6
(C) 7 (D) 8

Q12 If $7^m - 5^n = 117524$ and $7^{m-1} + 5^{n+1} = 17432$, then m+n equals:

(A) 10 (B) 9
(C) 8 (D) 7

Q13 Let $a^p = b^q = c^r$, where $a, b, c > 1$. Then, if $(abc)^{\frac{1}{p^{-1}+q^{-1}+r^{-1}}} = a^x$, find the value of x.

(A) p (B) q
(C) r (D) 1

Q14 If $x = \left[1 + \frac{1}{\sqrt{2}+1} + \frac{1}{\sqrt{3}+\sqrt{2}} + \frac{1}{\sqrt{4}+\sqrt{3}} + \ldots + \frac{1}{\sqrt{1024}+\sqrt{1023}}\right]$

.

Then, $\sqrt{x\sqrt{x\sqrt{x\ldots\infty}}} = a^b$, where a, b are integers, then find the minimum value of $(a+b)$.

(A) 5 (B) 6
(C) 7 (D) 33

Q15 The number of distinct integer solutions of the equation $(x^2-10)^{(x^3+x^2-14x-24)} = 1$ is:

(A) 5 (B) 4
(C) 3 (D) 2

Q16 If $22^x = 343^y = 154^z$ and x, y, z ≠ 0. Then find the value of $\left(\frac{yx-yz}{xz}\right)$.

(A) 3 (B) $\frac{1}{3}$
(C) 2 (D) $\frac{1}{2}$

Q17 If $\left(\sqrt[3]{\frac{3}{5}}\right)^{2x-y} = \frac{1375}{297}$ *and* $\left(\frac{3a}{2b}\right)^{5x-2y}$, where $= \left(\frac{5a}{3b}\right)^{2y-5x}$

a and b are non zero rational numbers, find (2x+3y+4).

(A) 175 (B) 164
(C) 152 (D) 128

Q18 Find the sum of the series

$$\frac{2}{\sqrt{4}+\sqrt{8}} + \frac{2}{\sqrt{8}+\sqrt{12}} + \frac{2}{\sqrt{12}+\sqrt{16}} + \ldots + \frac{2}{\sqrt{192}+\sqrt{196}}$$

Q19 Let a and b be positive integers (a < b) such that there are exactly 35 integers between 9^a *and* 27^b, excluding both that can be expressed as powers of 3. Determine the minimum possible value of (a×b).

Q20 x is the largest positive integer that can divide all the values of $(3^a + 6^a + 9^a)$ and y is the largest positive integer that can divide all the values of $7^a + 5(7)^a + 7^{a+2}$. If a is a positive integer, find the value of (y-x).

Q21 If $y^{552} \times \left[\left(\sqrt{x^{\frac{5}{4}} \times y^{-\frac{6}{5}}}\right)^8 \times (xy)^{\frac{1}{5}}\right]^{5!}$, $= x^m y^n$

then find the absolute difference of the roots of the equation $x^2 - 35x - m = 0$.

(A) 60 (B) 61
(C) 62 (D) 63

Q22 If $6^a = 8^b = 12^c$, then which of the following will be 0 given that none of a, b and c is 0?

(A) $\frac{3}{a} - \frac{2}{b} + \frac{3}{c}$ (B) $\frac{3}{a} + \frac{1}{b} - \frac{3}{c}$
(C) $\frac{2}{a} + \frac{1}{b} + \frac{3}{c}$ (D) $\frac{2}{b} + \frac{3}{c} - \frac{2}{a}$

Q23 Let p,q,x, and y be natural numbers with p>1 and q>1. If $p^x \times q^y = 980^{101}$, find the maximum possible value of (y−x).

Q24 Determine which of the following options are equal to 64 if its given that x=$(256)^{3+2\sqrt{7}}$

(A) $\sqrt[76]{\frac{x^{8\sqrt{7}}}{x^{12}}}$ (B) $\sqrt[76]{\frac{x^{6\sqrt{7}}}{x^{9}}}$
(C) $\sqrt[16]{\frac{x^{6\sqrt{7}}}{x^{9}}}$ (D) $\sqrt[16]{\frac{x^{8\sqrt{7}}}{x^{12}}}$

Q25 $\frac{1}{\sqrt{23+\sqrt{140}-\sqrt{220}-\sqrt{308}}} = ?$

(A) $\frac{-\sqrt{5}-\sqrt{7}-\sqrt{11}}{1+2\sqrt{35}}$ (B) $\frac{\sqrt{5}+\sqrt{7}-\sqrt{11}}{1+2\sqrt{35}}$
(C) $\frac{-\sqrt{5}+\sqrt{7}+\sqrt{11}}{1+2\sqrt{35}}$ (D) $\frac{\sqrt{5}+\sqrt{7}+\sqrt{11}}{1+2\sqrt{35}}$

Q26 Find the maximum value of xy^2z^3 if x+y+z=12 and x, y , z are positive real numbers.

Q27 $If\ (33.33)^{x} = (0.3333)^{y} = 100$, then find the value of $2^{\frac{xy}{y-x}}$.

(A) $\frac{1}{4}$
(B) $\frac{1}{2}$
(C) 2
(D) 4

Q28 Given that

$p = \sqrt{3} + 2\sqrt{2},\ q = \sqrt{5} + \sqrt{6},\ r = 2\sqrt{11} - \sqrt{5}\ and\ s = 4\sqrt{3} - \sqrt{7}$

, which of the following lists arranges these values in descending order?

(A) q>p>r>s
(B) p>q>r>s
(C) r>q>s>p
(D) q>p>s>r

Q29 Find the value of 100(a-24b) if

$$\sqrt[3]{25^{4}} \times \sqrt[3]{5^{a}} \times \sqrt[6]{20} \times \sqrt[3]{8^{b}} = \frac{\left(\sqrt{50}\right)^{b} \times 64^{a} \times \sqrt[4]{125^{5}} \times \left(\sqrt[3]{64}\right)^{7}}{\sqrt[16]{16}}$$

(A) 3475
(B) 3865
(C) 4225
(D) 4685

Q30 If $81^{a} = 6^{b} = 108^{c}\ and\ c = \frac{4ab}{ax+b}$ such that a, b and c are non-zero, then find the value of $x^{x^{\frac{1}{3}}}$.

(A) 256
(B) 64
(C) 8
(D) 1

Answer Key

Q1	A
Q2	C
Q3	4
Q4	5
Q5	B
Q6	A
Q7	3
Q8	D
Q9	B
Q10	D
Q11	B
Q12	B
Q13	A
Q14	C
Q15	B
Q16	B
Q17	A
Q18	6
Q19	42
Q20	367
Q21	B
Q22	B
Q23	201
Q24	B
Q25	D
Q26	6912
Q27	C
Q28	A
Q29	B
Q30	B

Hints & Solutions

Note: scan the QR code to watch video solution

Q1 Text Solution:

$P = 4^{444}$; $Q = 444^4$

444 *lies between* 4^4 *and* 4^5

So, Q *lies between* 4^{16} *and* 4^{20}

$R = 44^{44}$

44 *lies between* 4^2 *and* 4^3

44^{44} *lies between* 4^{88} *and* 4^{132}

$S = 4^{4^{4^4}}$ *and it is greater than* 4^{444}

So, the ascending Order is $QRPS$.

Video Solution:

Q2 Text Solution:

The given equation is:

$$4^{5x} - 3^{5x+2} = 295 \times 3^{5x-4} - \frac{4^{5x-2}}{121^{-\frac{1}{2}}}$$

$$4^{5x} - 3^{5x+2} = 295 \times 3^{5x-4} - 11 \times 4^{5x-2}$$

$$4^{5x} + 11 \times 4^{5x-2} = 3^{5x+2} + 295 \times 3^{5x-4}$$

$$\frac{4^{5x} \times 16 + 11 \times 4^{5x}}{16} = \frac{3^{5x} \times 729 + 295 \times 3^{5x}}{81}$$

$$4^{5x} \times \frac{27}{16} = 3^{5x} \times \frac{1024}{81}$$

$$4^{5x} \times 2187 = 3^{5x} \times 16384$$

$$4^{5x} \times 3^7 = 3^{5x} \times 4^7$$

$$4^{5x-7} = 3^{5x-7}$$

$$5x - 7 = 0$$

$$x = \frac{7}{5}$$

Video Solution:

Q3 Text Solution:

Given that,

$$(a - b + c) : (b - c + 2d) : (2a + c - d)$$
$$= 3 : 4 : 7$$

We can notice that, 3 + 4 = 7

So,

$$(a - b + c) + (b - c + 2d) = 2a + c - d$$

$$a + 2d = 2a + c - d$$

$$a + c = 3d \ \ldots (i)$$

Now,

$$\frac{5a - 3d + 5c}{a + d + c} = k$$

$$\frac{5(a + c) - 3d}{a + c + d} = k$$

$$\frac{5 \times 3d - 3d}{3d + d} = k$$

$$\frac{12d}{4d} = k$$

$$k = 3$$

Therefore, our given equation becomes

$$\sqrt{x} + \sqrt{y} = 3$$

So, the possible integer pairs (x, y) satisfying the equation are:

(9, 0), (4, 1), (1, 4), (0, 9).

i.e., only 4 pairs are possible.

Video Solution:

Q4 Text Solution:

Given that,

$$\frac{9^{2p-4}+13}{\frac{5}{2}\times 3^{2p-4}+1}=4$$

$$\frac{9^{2p-4}+13}{5\times 3^{2p-4}+2}=2$$

$$9^{2p-4}+13=10\times 3^{2p-4}+4$$

$$9^{2p-4}+9=10\times 3^{2p-4}$$

$$\left(3^{2p-4}\right)^2-10\times 3^{2p-4}+9=0$$

$$\left[3^{2p-4}-1\right]\left[3^{2p-4}-9\right]=0$$

$$3^{2p-4}=1 \ \ or \ \ 3^{2p-4}=9$$

$$2p-4=0 \ \ or \ \ 2p-4=2$$

$$p=2 \ \ or \ \ p=3$$

So, the sum of the possible roots of the equation is (3 + 2) = 5.

Video Solution:

Q5 Text Solution:

Given :

$$\left(\sqrt[3]{a^2\cdot b}\right)^{\frac{3}{4}}\cdot\left(\sqrt[4]{a^3\cdot b^2}\right)^{\frac{4}{3}}=a^k\cdot b^{\frac{11}{12}}$$

Simplifying,

$$\Rightarrow \left(a^2.b\right)^{\frac{1}{3}\times\frac{3}{4}}.\ \left(a^3.b^2\right)^{\frac{1}{4}\times\frac{4}{3}}=a^k.b^{\frac{11}{12}}$$

$$\Rightarrow \left(a^2.b\right)^{\frac{1}{4}}.\ \left(a^3.b^2\right)^{\frac{1}{3}}=a^k.b^{\frac{11}{12}}$$

$$\Rightarrow a^{\frac{1}{2}}.b^{\frac{1}{4}}.a^1.b^{\frac{2}{3}}=a^k.b^{\frac{11}{12}}$$

$$\Rightarrow a^{\frac{1}{2}+1}.b^{\frac{1}{4}+\frac{2}{3}}=a^k.b^{\frac{11}{12}}$$

$$\Rightarrow a^{\frac{3}{2}}.b^{\frac{11}{12}}=a^k.b^{\frac{11}{12}}$$

$$\Rightarrow a^{\frac{3}{2}}=a^k$$

$$\Rightarrow k=\frac{3}{2}$$

Video Solution:

Q6 Text Solution:

Topic - Algebra

Sub - topic - Indices

$$\frac{9x^{-3}yz^{-4}}{3xy^{-4}z^3}\div\frac{6x^{-4}y^3z^{-2}}{2xy^{-3}z^4}$$

$$=\frac{9x^{-3}yz^{-4}}{3xy^{-4}z^3}\times\frac{2xy^{-3}z^4}{6x^{-4}y^3z^{-2}}$$

$$=\frac{3y^5}{x^4z^7}\times\frac{1x^5z^6}{3y^6}$$

$$=\frac{y^5}{y^6}\times\frac{x^5}{x^4}\times\frac{z^6}{z^7}$$

$$=\frac{x}{yz}$$

Hence, option (1) is the correct answer.

Video Solution:

Q7 Text Solution:

Here we are given the equation

$5\times 2^{x+3}-21\times 2^{x-1}=236$ First we will rewrite the equation to get

$$5\times 2^4\times 2^{x-1}-21\times 2^{x-1}=236$$

Now taking 2^{x-1} common we get

$$2^{x-1}(80-21)=236$$

or, $2^{x-1}\times 59=236$

Taking 59 to RHS we get

$$2^{x-1}=\frac{236}{59}$$

or, $2^{x-1}=4$

or, $2^{x-1}=2^2$

Since the bases are same, we can equate the powers to get

$$x-1=2$$

or, $x = 3$
Hence the value of x is 3 .

Video Solution:

Q8 Text Solution:

x = $(6561)^{5+2\sqrt{3}}$

$x^{\frac{1}{(5+2\sqrt{3})}}$ = 6561

On rationalizing $\frac{1}{(5+2\sqrt{3})}$ we get

$\frac{1}{(5+2\sqrt{3})}$ =

$$\frac{(5-2\sqrt{3})}{(5+2\sqrt{3})(5-2\sqrt{3})} = \frac{(5-2\sqrt{3})}{25-12} = \frac{(5-2\sqrt{3})}{13}$$

So, $x^{\frac{(5-2\sqrt{3})}{13}}$ = 6561 = 81^2

=> $81 = x^{\frac{(5-2\sqrt{3})}{13\times2}}$

=> 81 = $\frac{x^{\frac{5}{26}}}{x^{\frac{\sqrt{3}}{13}}}$

Video Solution:

Q9 Text Solution:

Given that, $2^a \cdot 3^b \cdot 5^c = \left(\frac{1}{2}\right)^2 (9)^3 (25)^4$

$$2^a 3^b 5^c = 2^{-2} \times 3^6 \times 5^8$$

On comparing both sides we get,

$$\begin{aligned} \text{So, } a &= -2, \\ b &= 6, \\ c &= 8 \end{aligned}$$

Therefore,

$$\begin{aligned} a^b b^c c^a &= (-2)^6 (6)^8 (8)^{-2} \\ &= 2^6 \times 2^8 \times 3^8 \times 2^{-6} \\ &= 2^8 \times 3^8 = x^4 \text{ (Given)} \end{aligned}$$

So, $x = (2 \times 3)^{\frac{8}{4}} = 36$
So, $\sqrt{x} = 6$

Video Solution:

Q10 Text Solution:

$$y = \sqrt{2 + \sqrt{2 + \sqrt{2 + \ldots \infty}}}$$
$$\Rightarrow y^2 = 2 + y$$
$$\Rightarrow y^2 - y - 2 = 0$$
$$\Rightarrow y^2 - 2y + y - 2 = 0$$
$$\Rightarrow y = 2, -1$$
$$\Rightarrow y = 2 [\text{ Since, } y > 0, \text{ so } y \neq -1]$$

Therefore,

$$\frac{4^\alpha}{5^b} = \left(\frac{32}{625}\right)^y$$
$$\Rightarrow \frac{2^{2a}}{5^b} = \frac{2^{5y}}{5^{4y}} = \frac{2^{10}}{5^8}$$

So, $2^{2a} = 2^{10}$

$$\Rightarrow 2a = 10$$
$$\Rightarrow a = 5$$

Also, $5^b = 5^8$

$$\Rightarrow b = 8$$

So, $(a+b) = 5 + 8 = 13$

Video Solution:

Q11 Text Solution:

$$\left(2^x-7\right)^2=6\left(2^x-\frac{5}{2}\right)$$
$$\Rightarrow 2^{2x}-14\cdot 2^x+49=6\cdot 2^x-15$$
$$\Rightarrow 2^{2x}-20\cdot 2^x+64=0$$
$$\Rightarrow y^2-20y+64=0\,[\text{ Let } 2^x=y]$$
$$\Rightarrow (y-16)(y-4)=0$$
$$\Rightarrow y=4,16$$

Therefore, $2^x=16$, or $2^x=4$

$$\Rightarrow x=4, \text{ or } x=2.$$

Hence, the sum of the possible values of $x=4+2=6$

Video Solution:

Q12 Text Solution:

It is given that $7^m - 5^n = 117524$ and $7^{m-1} + 5^{n+1} = 17432$

Let $7^{m-1} = p$ and $5^n = q$

So, 7p - q = 117524 ---- (1)

p + 5q = 17432 ---- (2)

Substitute p = $\frac{117524+q}{7}$ into the equation (2).

$\frac{117524+q}{7}$ + 5q = 17432

$\frac{117524+36q}{7}$ = 17432

q = 125 = 5^3

Now, we know that, q = $5^n = 5^3$

So, n = 3.

Also, p = $\frac{117524+125}{7}$ = 16807 = 7^5

we know that, p = $7^{m-1} = 7^5$

m-1 = 5

=> m = 6

Thus, m + n = 6 + 3 = 9.

Video Solution:

Q13 Text Solution:

Let $a^p=b^q=c^r=k^{pqr}$

Then, $a=k^{qr}$

$$b=k^{pr}$$
$$c=k^{pq}$$

So, $abc=k^{pq+qr+pr}$

Therefore,

$$(abc)^{\frac{1}{p^{-1}+q^{-1}+r^{-1}}}=\left(k^{pq+qr+pr}\right)^{\frac{pqr}{pq+pr+qr}}$$
$$=k^{pqr}=a^p=a^x \text{ (Given)}$$

Hence, $x=p$

Video Solution:

Q14 Text Solution:

$$x = \left[1+\frac{1}{\sqrt{2}+1}+\frac{1}{\sqrt{3}+\sqrt{2}}+\frac{1}{\sqrt{4}+\sqrt{3}} + \ldots + \frac{1}{\sqrt{1024}+\sqrt{1023}}\right]$$
$$= 1+\sqrt{2}-1+\sqrt{3}-\sqrt{2}+\ldots + \sqrt{1024}-\sqrt{1023}$$
$$= 2^5$$

Also,

$$y = \sqrt{x\sqrt{x\sqrt{x\ldots\infty}}}$$
$$=> y^2 = xy$$
$$=> y = x = 2^5 [\text{ Since, } y \neq 0]$$

Therefore, $2^5 = a^b$

$$\Rightarrow a = 2, b = 5 \text{ or, } a = 32, b = 1$$

So, $(a+b)$ is having a minimum value of $(2+5) = 7$.

Video Solution:

Q15 Text Solution:

The given equation is

$$(x^2-10)^{(x^3+x^2-14x-24)} = 1$$

We know that, $a^b = 1$ implies 3 cases:

Case 1: When b = 0.

$x^3 + x^2 - 14x - 24 = 0$

$x^3 + 3x^2 - 2x^2 - 6x - 8x - 24 = 0$

=> $x^2(x+3) - 2x(x+3) - 8(x+3) = 0$

=> $(x+3)(x^2-2x-8)=0$

=> x = -3,

=> x = -3, -2, 4

Case 2: When a = 1.

$x^2 - 10 = 1$

$x^2 = 11$

=> x is not an integer.

Case 3: When a = -1 and b is even.

$x^2 - 10 = -1$

=> $x^2 = 9$

=> $x = \pm 3$

At x = 3,

$b = x^3 + x^2 - 14x - 24$

$= 3^3 + 3^2 - 14(3) - 24$

=27 + 9 - 42 - 24

=36 - 66

= - 30

At x = -3,

$b=x^3+x^2-14x-24=(-3)^3+(-3)^2-14(-3)-24$

= -27 + 9 + 42 – 24

= 51 – 51

= 0

Hence, the possible solutions are x = 3, -3, -2, 4.

So, the number of possible integer solutions is 4.

Video Solution:

Q16 Text Solution:

Let

$22^{x} = 343^{y} = 154^{z} = a$

So

$22^{x} = a$

$\Rightarrow a^{\frac{1}{x}} = 22$

Similarly

$a^{\frac{1}{y}} = 343$

and $a^{\frac{1}{z}} = 154$

So

$\frac{154}{22} = \frac{a^{\frac{1}{z}}}{a^{\frac{1}{x}}}$

$\Rightarrow 7 = a^{\frac{1}{z}-\frac{1}{x}}$

$\Rightarrow \sqrt[3]{343} = a^{\frac{1}{z}-\frac{1}{x}}$

$\Rightarrow 343^{\frac{1}{3}} = a^{\frac{1}{z}-\frac{1}{x}}$

$\Rightarrow a^{\frac{1}{3y}} = a^{\frac{1}{z}-\frac{1}{x}}$

$\Rightarrow \frac{1}{3y} = \frac{1}{z} - \frac{1}{x}$

So

$\frac{yx-yz}{xz}$

$= y\left(\frac{1}{z} - \frac{1}{x}\right)$

$= y \times \frac{1}{3y}$

$= \frac{1}{3}$

Video Solution:

Q17 Text Solution:

$\left(\sqrt[3]{\frac{3}{5}}\right)^{2x-y} = \frac{1375}{297}$

$\Rightarrow \left(\frac{3}{5}\right)^{\frac{2x-y}{3}} = \frac{125}{27}$

$\Rightarrow \left(\frac{3}{5}\right)^{\frac{2x-y}{3}} = \left(\frac{3}{5}\right)^{-3} \Rightarrow 2x - y = -9$

$\Rightarrow y = 2x + 9$

Also

$\left(\frac{3a}{2b}\right)^{5x-2y} = \left(\frac{5a}{3b}\right)^{2y-5x}$

$\Rightarrow \left(\frac{3a}{2b}\right)^{5x-2y} = \left(\frac{3b}{5a}\right)^{5x-2y}$

Since the powers are equal, either the base will be equal or the powers must be 0

Checking for the first case

$\frac{3a}{2b} = \frac{3b}{5a}$

$\Rightarrow \frac{a^2}{b^2} = \frac{2}{5}$

$\Rightarrow \frac{a}{b} = \sqrt{\frac{2}{5}}$

Since both a and b are rational so $\frac{a}{b}$ *must also be rational which is not the case here, so the powers must be* 0

So

$5x - 2y = 0$

$\Rightarrow 5x - 2\left(2x + 9\right) = 0$

$\Rightarrow 5x - 4x - 18 =$

$\Rightarrow x = 18$

and

$y = 45$

So

$2x + 3y + 4 = 175$

Video Solution:

Q18 Text Solution:

$$\frac{2}{\sqrt{4}+\sqrt{8}}+\frac{2}{\sqrt{8}+\sqrt{12}}+\frac{2}{\sqrt{12}+\sqrt{16}}+\ldots$$
$$+\frac{2}{\sqrt{192}+\sqrt{196}}$$
$$=\frac{2}{\sqrt{8}+\sqrt{4}}\times\frac{\sqrt{8}-\sqrt{4}}{\sqrt{8}-\sqrt{4}}+\frac{2}{\sqrt{12}+\sqrt{8}}\times\frac{\sqrt{12}-\sqrt{8}}{\sqrt{12}-\sqrt{8}}$$
$$+\frac{2}{\sqrt{16}+\sqrt{12}}\times\frac{\sqrt{16}-\sqrt{12}}{\sqrt{16}-\sqrt{12}}+\ldots+\frac{2}{\sqrt{196}+\sqrt{192}}$$
$$\times\frac{\sqrt{196}-\sqrt{192}}{\sqrt{196}-\sqrt{192}}$$
$$=2\left[\frac{\sqrt{8}-\sqrt{4}}{8-4}+\frac{\sqrt{12}-\sqrt{8}}{12-8}+\frac{\sqrt{16}-\sqrt{12}}{16-12}+\ldots\right.$$
$$\left.+\frac{\sqrt{196}-\sqrt{192}}{196-192}\right]$$
$$=\frac{2}{4}\left(\sqrt{8}-\sqrt{4}+\sqrt{12}-\sqrt{8}+\sqrt{16}\right.$$
$$\left.-\sqrt{12}+\ldots+\sqrt{196}-\sqrt{192}\right)$$
$$=\frac{1}{2}\left(\sqrt{196}-\sqrt{4}\right)$$
$$=\frac{1}{2}\times\left(14-2\right)$$
$$=6$$

Video Solution:

Q19 Text Solution:

$$9^a=3^{2a}$$
and
$$27^b=3^{3b}$$
So there are 35 integers between these two numbers in the form of powers of 3, let them be

$$3^{2a},\ 3^{2a+1},\ 3^{2a+2},\ldots,\ 3^{2a+35},\ 3^{2a+36}$$
So
$$3^{3b}=3^{2a+36}$$
$$\Rightarrow 3b=2a+36$$
$$\Rightarrow b=\frac{2a+36}{3}$$
So b will increase with increase in the value of a, we need to minimise a to minimise their product. But since a and b both are positive integers value of a should be such that the value of b is an integer. So minimum such value of a is 3, so

b=$\frac{6+36}{3}$=14

So minimum possible value of $(a\times b)$=
$$3\times 14=42$$

Video Solution:

Q20 Text Solution:

$$3^a+6^a+9^a$$
$$=3^a+(2\times 3)^a+(3\times 3)^a$$
$$=3^a\left(1^a+2^a+3^a\right)$$
$Since\ 1^a+2^a+3^a=odd+even+odd$ $=\ even,\ 1^a+2^a+3^a\ will\ be\ an\ even$ $number\ which\ will\ always\ be\ divisible$ $by\ 2.$

$Since\ minimum\ value\ of\ a\ is\ 1\ ,$

$at\ a=1$
$$1^a+2^a+3^a=6$$
$at\ a=2$
$$1^a+2^a+3^a=14$$
$So\ the\ largest\ positive\ integer\ that\ can$ $divide\ 3^a+6^a+9^a\ is,$

$2\times 3\ x\ 3\ =18,$

$x=18$
$$7^a+5(7)^a+7^{a+2}$$
$$=\left(1+5\right)\left(7^a\right)+7^{a+2}$$
$$=6\left(7^a\right)+7^{a+2}$$
$$=7^a\left(6+7^2\right)$$
$$=(55)(7^a)$$
$Since\ minimum\ value\ of\ a\ is\ 1,\ the$ $largest\ positive\ integer\ that\ can\ divide$ $all\ the\ values\ is$
$$=55\times 7^1$$
$$=385$$

So

$y - x = 385 - 18 = 367$

Video Solution:

Q21 Text Solution:

$$y^{552} \times \left[\left(\sqrt{x^{\frac{5}{4}} \times y^{-\frac{6}{5}}}\right)^{8} \times (xy)^{\frac{1}{5}}\right]^{5!}$$

$$= y^{552} \times \left(\sqrt{x^{\frac{5}{4}} \times y^{-\frac{6}{5}}}\right)^{8\times5!} \times (xy)^{4!}$$

$$= y^{552} \times \left(x^{\frac{5}{4}\times\frac{1}{2}} \times y^{-\frac{6}{5}\times\frac{1}{2}}\right)^{8\times5!} \times (xy)^{4!}$$

$$= y^{552} \times \left(x^{\frac{5}{8}} \times y^{-\frac{3}{5}}\right)^{8\times5!} \times (xy)^{4!}$$

$$= y^{552} \times \left(x^{\frac{5}{8}\times8\times5!} \times y^{-\frac{3}{5}\times8\times5!}\right) \times (xy)^{4!}$$

$$= x^{5\times5!+4!} \times y^{552-3\times8\times4!+4!}$$

$$= x^{624} \times y^{0}$$

So, m = 624, n = 0

Now, the given equation becomes

$x^2 - 35x - 624 = 0$

$x^2 - 48x + 13x - 624 = 0$

$x(x - 48) + 13(x - 48) = 0$

$(x + 13)(x - 48) = 0$

$x = -13,\ 48$

So, the absolute difference between the roots

= |48 + 13| = 61

Video Solution:

Q22 Text Solution:

Let

$6^a = 8^b = 12^c = k$

So

$6 = k^{\frac{1}{a}}$

$8 = k^{\frac{1}{b}}$

$12 = k^{\frac{1}{c}}$

So

$$\frac{k^{\frac{1}{c}}}{k^{\frac{1}{a}}} = \frac{12}{6}$$

$\Rightarrow k^{\frac{1}{c}-\frac{1}{a}} = 2$

$\Rightarrow k^{\frac{1}{c}-\frac{1}{a}} = (8)^{\frac{1}{3}}$

$\Rightarrow k^{\frac{1}{c}-\frac{1}{a}} = k^{\frac{1}{3b}}$

$\Rightarrow \frac{1}{c} - \frac{1}{a} = \frac{1}{3b}$

$\Rightarrow \frac{3}{c} - \frac{3}{a} = \frac{1}{b}$

$\Rightarrow \frac{3}{a} + \frac{1}{b} - \frac{3}{c} = 0$

Video Solution:

Q23 Text Solution:

Factorising

$980 = (2)^2 (5) (7)^2$

So

$$p^x \times q^y = \left(2^2 \times 5 \times 7^2\right)^{101} = 2^{202} \times 5^{101} \times 7^{202}$$

Since we have to obtain the maximum possible value of (y-x), we need to maximize y and minimze x, so

To maximize y, q=2, y=202 or q=7, y=202

and to minimze x, p=$\left(5^{101} \times 7^{202}\right)$, x=1 or p=$\left(5^{101} \times 2^{202}\right)$, x=1

i.e.,

$$p^x \times q^y = \left(5^{101} \times 7^{202}\right)^1 \times (2)^{202}\ or$$

$$\left(5^{101} \times 2^{202}\right)^1 \times (7)^{202}$$

So in both the cases

$(y-x)_{max}$=202-1 = 201

Video Solution:

Q24 Text Solution:

$$x=(256)^{3+2\sqrt{7}}$$
$$=\left(2^{8}\right)^{3+2\sqrt{7}}$$
$$=(2)^{24+16\sqrt{7}}$$
$$=\left(2^{6}\right)^{4+\frac{8\sqrt{7}}{3}}$$
$$=(64)^{\frac{12+8\sqrt{7}}{3}}$$
So
$$x=(64)^{\frac{12+8\sqrt{7}}{3}}$$
$$\Rightarrow 64=x^{\frac{3}{12+8\sqrt{7}}}=x^{\frac{3}{12+8\sqrt{7}}\times\frac{8\sqrt{7}-12}{8\sqrt{7}-12}}$$
$$=x^{\frac{24\sqrt{7}-36}{304}}=x^{\frac{6\sqrt{7}-9}{76}}$$
$$\Rightarrow 64=\sqrt[76]{\frac{x^{6\sqrt{7}}}{x^{9}}}$$

Video Solution:

Q25 Text Solution:

$$\frac{1}{\sqrt{23+\sqrt{140}-\sqrt{220}-\sqrt{308}}}$$
$$=\frac{1}{\sqrt{5+7+11+2\sqrt{35}-2\sqrt{55}-2\sqrt{77}}}$$
$$=\frac{1}{\sqrt{\left(\sqrt{5}+\sqrt{7}-\sqrt{11}\right)^{2}}}$$
$$=\frac{1}{\sqrt{5}+\sqrt{7}-\sqrt{11}}$$
$$=\frac{1}{\sqrt{5}+\sqrt{7}-\sqrt{11}}\times\frac{\sqrt{5}+\sqrt{7}+\sqrt{11}}{\sqrt{5}+\sqrt{7}+\sqrt{11}}$$
$$=\frac{\sqrt{5}+\sqrt{7}+\sqrt{11}}{5+\sqrt{35}+\sqrt{55}+\sqrt{35}+7+\sqrt{77}-\sqrt{55}-\sqrt{77}-11}$$
$$=\frac{\sqrt{5}+\sqrt{7}+\sqrt{11}}{1+2\sqrt{35}}$$

Video Solution:

Q26 Text Solution:

$$x+y+z=12$$
$$\Rightarrow x+\frac{y}{2}+\frac{y}{2}+\frac{z}{3}+\frac{z}{3}+\frac{z}{3}=12$$
The product of the 6 terms on the lhs will be maximum when all of them are equal, i.e.
$$x=\frac{y}{2}=\frac{y}{2}=\frac{z}{3}=\frac{z}{3}=\frac{z}{3}=\frac{12}{6}=2$$
So
$$\left[x\times\frac{y}{2}\times\frac{y}{2}\times\frac{z}{3}\times\frac{z}{3}\times\frac{z}{3}\right]_{max}=2^{6}$$
$$\Rightarrow\left[\frac{xy^{2}z^{3}}{2^{2}\times3^{3}}\right]_{max}=2^{6}$$
$$\Rightarrow\left[xy^{2}z^{3}\right]_{max}=2^{8}\times3^{3}=6912$$

Video Solution:

Q27 Text Solution:

Let

$(33.33)^x = (0.3333)^y = 100 = k$

So

$33.33 = k^{\frac{1}{x}}$

$0.3333 = k^{\frac{1}{y}}$

So

$0.3333 \times 100 = 33.33$

$\Rightarrow k^{\frac{1}{y}} \times k = k^{\frac{1}{x}}$

$\Rightarrow k^{\frac{1}{y}+1} = k^{\frac{1}{x}}$

$\Rightarrow \frac{1}{y} + 1 = \frac{1}{x}$

$\Rightarrow \frac{1}{x} - \frac{1}{y} = 1$

$\Rightarrow \frac{y-x}{xy} = 1$

$\Rightarrow \frac{xy}{y-x} = 1$

So

$2^{\frac{xy}{y-x}} = 2^1 = 2$

Video Solution:

Q28 Text Solution:

$p^2 = 3 + 8 + \sqrt{96} = 11 + 9.xx = 20.xx$

$q^2 = 5 + 6 + \sqrt{120} = 11 + 10.xx = 21.xx$

$r^2 = 44 + 5 - \sqrt{880} = 49 - 29.xx = 19.xx$

$s^2 = 48 + 7 - \sqrt{1344} = 55 - 36.xx = 18.xx$

So

$q^2 > p^2 > r^2 > s^2$

$\Rightarrow q > p > r > s$

$(\textit{Since all of them are positive numbers})$

Video Solution:

Q29 Text Solution:

$\sqrt[3]{25^4} \times \sqrt[3]{5^a} \times \sqrt[6]{20} \times \sqrt[3]{8^b}$

$= \frac{(\sqrt{50})^b \times 64^a \times \sqrt[4]{125^5} \times \left(\sqrt[3]{64}\right)^7}{\sqrt[16]{16}}$

$\Rightarrow 5^{\frac{8}{3}} \times 5^{\frac{a}{3}} \times 2^{\frac{1}{3}} \times 5^{\frac{1}{6}} \times 2^b = 2^{\frac{b}{2}} \times 5^b \times 2^{6a} \times 5^{\frac{15}{4}} \times 2^{14} \times 2^{-\frac{1}{4}}$

$\Rightarrow 2^{\left(\frac{1}{3}+b\right)} \times 5^{\left(\frac{8}{3}+\frac{a}{3}+\frac{1}{6}\right)} = 2^{\frac{b}{2}+6a+14-\frac{1}{4}} + 5^{b+\frac{15}{4}}$

So

$\frac{8}{3} + \frac{a}{3} + \frac{1}{6} = b + \frac{15}{4}$

$\Rightarrow \frac{a}{3} + \frac{17}{6} - \frac{15}{4} = b$

$\Rightarrow b = \frac{4a-11}{12}$

Similarly

$\frac{1}{3} + b = \frac{b}{2} + 6a + 14 - \frac{1}{4}$

$\Rightarrow \frac{1}{3} + b = \frac{b}{2} + 6a + \frac{55}{4}$

$\Rightarrow \frac{b}{2} = 6a + \frac{55}{4} - \frac{1}{3}$

$\Rightarrow \frac{4a-11}{24} = \frac{72a+165-4}{12}$

$\Rightarrow 4a - 11 = 144a + 322$

$\Rightarrow 140a = -333$

$\Rightarrow a = \frac{-333}{140}$

$b = \frac{4a-11}{12}$

So

$100\left(a - 24b\right) = 100\left[a - 24\left(\frac{4a-11}{12}\right)\right]$

$= -700a + 2200 = \frac{-700 \times -333}{140} + 2200$

$= 1665 + 2200 = 3865$

Video Solution:

Q30 Text Solution:

Given : $81^a = 6^b = 108^c$ = k (let)

Now, $81 = k^{\frac{1}{a}}$, $6 = k^{\frac{1}{b}}$, $108 = k^{\frac{1}{c}}$

We know, $81^{\frac{1}{4}} \times 6^2 = 108$

Substituting the values, we get

$\Rightarrow k^{\frac{1}{4a}} \times k^{\frac{2}{b}} = k^{\frac{1}{c}}$

$\Rightarrow \frac{1}{4a} + \frac{2}{b} = \frac{1}{c}$

$\Rightarrow \frac{8a+b}{4ab} = \frac{1}{c}$

$\Rightarrow c = \frac{4ab}{8a+b}$

But it is given that $c = \frac{4ab}{ax+b}$

Comparing both values of c, we get

$\frac{4ab}{ax+b} = \frac{4ab}{8a+b}$

$x = 8$

Therefore, $x^{x^{\frac{1}{3}}} = 8^{8^{\frac{1}{3}}} = 8^2 = 64$

Video Solution: