Assignment-3

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PART 1

Section 7.1 Solution #4

a. Compute a 95% CI for μ when $n = 25$ and $\bar{x} = 58.3$.

```
# Constants
x_bar <- 58.3
sigma <- 3.0
n1 <- 25
alpha1 <- 0.05  # 1 - confidence level

# Z-score for 95% confidence level
z1 <- qnorm(1 - alpha1 / 2)

# Confidence Interval Calculation
ci1_lower <- x_bar - z1 * (sigma / sqrt(n1))
ci1_upper <- x_bar + z1 * (sigma / sqrt(n1))

cat("CI1_Lower:",ci1_lower , "\n")
```

```
## CI1_Lower: 57.12402
```

```
cat("CI1_Upper:",ci1_upper , "\n")
```

```
## CI1_Upper: 59.47598
```

b. Compute a 95% CI for μ when $n = 100$ and $\bar{x} = 58.3$.

```
# Constants
n2 <- 100

# Z-score for 95% confidence level
z2 <- qnorm(1 - alpha1 / 2)

# Confidence Interval Calculation
ci2_lower <- x_bar - z2 * (sigma / sqrt(n2))
```

```r
ci2_upper <- x_bar + z2 * (sigma / sqrt(n2))

cat("CI2_Lower:",ci2_lower , "\n")
```

```
## CI2_Lower: 57.71201
```

```r
cat("CI2_Upper:",ci2_upper , "\n")
```

```
## CI2_Upper: 58.88799
```

c. Compute a 99% CI for μ when $n = 100$ and $\bar{x} = 58.3$.

```r
# Constants
alpha3 <- 0.01  # 1 - confidence level
n3 <- 100

# Z-score for 99% confidence level
z3 <- qnorm(1 - alpha3 / 2)

# Confidence Interval Calculation
ci3_lower <- x_bar - z3 * (sigma / sqrt(n3))
ci3_upper <- x_bar + z3 * (sigma / sqrt(n3))

cat("CI3_Lower:",ci3_lower , "\n")
```

```
## CI3_Lower: 57.52725
```

```r
cat("CI3_Upper:",ci3_upper , "\n")
```

```
## CI3_Upper: 59.07275
```

d. Compute a 82% CI for μ when $n = 100$ and $\bar{x} = 58.3$.

```r
# Constants
alpha4 <- 0.18  # 1 - confidence level
n4<-100
# Z-score for 82% confidence level
z4 <- qnorm(1 - alpha4 / 2)

# Confidence Interval Calculation
ci4_lower <- x_bar - z4 * (sigma / sqrt(n4))
ci4_upper <- x_bar + z4 * (sigma / sqrt(n4))

cat("CI4_Lower:",ci4_lower , "\n")
```

```
## CI4_Lower: 57.89777
```

```r
cat("CI4_Upper:",ci4_upper , "\n")
```

```
## CI4_Upper: 58.70223
```

e.How large must be n if the width of the 99% interval of μ is to be 1.0.

```r
# Desired width
desired_width <- 1.0

# Z-score for 99% confidence level
z5 <- qnorm(1 - alpha3 / 2)

# Solve for n
n5 <- ((2 * z5 * sigma) / desired_width)^2

cat("N should be:",round(n5, digits = 0), "\n")
```

```
## N should be: 239
```

Section 7.1 Solution #6

Solution(a)

```r
# Given data for part (a)
sample_size_a <- 25
sample_mean_a <- 8439
z_CI <- 1.645
sigma_a <- 100

# Margin of error calculation for part (a)
margin_of_error_a <- z_CI * (sigma_a / sqrt(sample_size_a))

# Confidence interval calculation for part (a)
lower_limit_a <- sample_mean_a - margin_of_error_a
upper_limit_a <- sample_mean_a + margin_of_error_a

# Displaying the results for part (a)
cat(paste("90% Confidence Interval (Part a): [", lower_limit_a, ",", upper_limit_a, "]\n"))
```

```
## 90% Confidence Interval (Part a): [ 8406.1 , 8471.9 ]
```

Solution(b)

```r
# Given data for part (b)
confidence_level_b <- 0.92
z_CI<-1.751
# Margin of error calculation for part (b)
margin_of_error_b <- z_CI * (sigma_a / sqrt(sample_size_a))
```

```r
# Confidence interval calculation for part (b)
lower_limit_b <- sample_mean_a - margin_of_error_b
upper_limit_b <- sample_mean_a + margin_of_error_b

# Displaying the results for part (b)
cat("92% Confidence Interval (Part b): [", lower_limit_b, ",", upper_limit_b, "]\n")
```

```
## 92% Confidence Interval (Part b): [ 8403.98 , 8474.02 ]
```

PART 2

Solution (a)

To determine the sample size (n) for a 90% confidence interval estimate of the population mean, the formula is given by:

$$n = \frac{Z^2 \times \sigma^2}{E^2}$$

For a 90% confidence interval, the Z-score is approximately 1.645. Substituting the given values into the formula:

```r
# Given values for part (a)
Z_score_a <- 1.645
sigma_a <- 22.5
E_a <- 2

# Sample size calculation for part (a)
n_a <- ((Z_score_a * sigma_a) / E_a)^2
cat("N should be:",ceiling(n_a), "\n")
```

```
## N should be: 343
```

Solution (b)

The sample size is inversely proportional to the square of the margin of error (E) and directly

proportional to the square of the Z-score (Z). As the confidence level increases, the Z-score increases, which in

turn increases the sample size. Therefore, when the desired margin of error is fixed, increasing the confidence

level will lead to an increase in the sample size.