Homework 4

1. The probability that a student owns a microwave oven is 0.75, and that a student owns TV is 0.25. Probability that a student owns both a microwave and a TV is 0.16. Find the probability that a student owns either a microwave or a TV, but not both.

Solution: P(M) = 0.75, P(T) = 0.25, P (M intersection T) = 0.16

P(MUT) = P(M) + P(T) - P (M intersection T)

= 0.75 + 0.25 – 0.16

= 1.0 – 0.16

= 0.84

2. Five cards are drawn from a standard deck of cards without replacement. Find the probability of getting
a. All red cards
b. All diamonds
c. All aces

Solution:

P (All Red) =

26C5/52C5 = 65780/2598960= 0.0253

P (All diamonds) =

13C5/52C5= 1287/2598960 = 0.0049

P (All Aces) =

(4C4*48C1)/52C5= 1*48/2598960 = 0.000018

3. Suppose a person is randomly selected from a population of 1000 people with the distribution given below in the table.

Disease Status
Age None Mild Moderate Severe Totals
18 – 40 213 51 33 23 320
Over 40 430 121 98 31 680
Totals 643 172 131 54 1,000
Find the probabilities of the following events that the person is
a. P (Over 40)
b. P (Mild and Over 40)
c. P (Mild or Over 40)
d. P (not Mild)
e. P (Mild | Over 40)

Solution:

P (Over 40) = 680/1000 = 0.68

P (Mild and Over 40) = 121/1000 = 0.121

P (Mild or Over 40) = P(Mild) + P (Over 40) - P (Mild and Over 40)

= 0.172 + 0.68 – 0.121 = 0.731

P (not Mild) = 1 – P(Mild) = 1 – 0.172 = 0.828

P (Mild | Over 40) = P (Mild | Over 40 already happened)

= P (Mild ∩ Over40) / P(Over40)

= 0.121/0.68

= 0.1779

Events

3.8 At a particular University, 1/2 of the students drink alcohol and 1/3 of the students' smoke cigarettes.

(a) What is the largest possible fraction of students who do neither?

Solution: P(A) = ½ P(S) = 1/3

P(AUS) = P(A)+P(S) - P (A intersection S)

P(AUS)' = 1 -[P(A) + P(S) - P (A intersection S)]

= 1 - [1/2+1/3 - P (A intersection S)]

= 1 - 5/6 + P (A intersection S)]

Hence P (who do neither) >=1/6

(b) It turns out that, in fact, 1/3 of the students do neither. What fraction of the students does both?

Solution: If P(AUS)' = 1/3, then

1/3 = 1/6 + P (A intersection S)

P (A intersection S) = 1/3-1/6= 1/6

3.19 You shuffle a standard deck of cards, then draw four cards.

(a) What is the probability all four are the same suit?

Solution: Total cards = 52, each suit card = 13

Total cases = 52C4 = 270725

4 cards can be drawn from each suit = 4 * 13C4 = 2860

P (All 4 same suit) = 2860/270725 = 2707252860=0.0106

(b) What is the probability all four are red?

Solution: Sample space = 52C4

Red cards can be drawn as 26C4

P (All red) 26C4/52C4 = 0.0552

(c) What is the probability each has a different suit?

Solution: Sample space = 52C4.

All 4 cards are from different suits = 4*13C1

P (Different Suit) = 4*13C1/52C4 = 0.1055

Permutations and Combinations

3.26 You shuffle a standard deck of playing cards, and deal a hand of 10 cards. With what probability does this hand have five red cards?

Solution: Sample space = 52C10 = 15820024220.

5 red cards can be selected as 13+13C5 = 26C5 = 65780.

P (5 red cards) = 65780/15820024220 = 0.000004158.