# STAT603-Fall2023-FinalExam

December 11, 2023

## 1 Final Exam Fall 2023

Question 1.An airline runs a small commuter flight that has 10 seats. The probability that a passenger turns up for the flight is 0.92. What is the smallest number of seats the airline should sell to ensure that the probability the flight will be full is greater than 0.93?

[26]:
```
import math
from scipy.stats import norm
import numpy as np
from scipy.stats import expon
import matplotlib.pyplot as plt
import seaborn as sns
import pandas as pd
import warnings
warnings.filterwarnings('ignore')
```

[27]:
```
# Given probabilities
probability_turn_up = 0.92
desired_probability_full = 0.93

# Calculate the smallest number of seats
required_seats = math.ceil(math.log(desired_probability_full) / math.
 ↪log(probability_turn_up))

print(f"The smallest number of seats to ensure a probability greater than␣
 ↪{desired_probability_full} is: {required_seats}")
```

```
The smallest number of seats to ensure a probability greater than 0.93 is: 1
```

Question 2.The typical computer random number generator yields numbers in a uniform distribution between 0 and 1. If 45 random numbers are generated, find approximately the probability that their mean is below 0.465, using the central limit theorem.

[28]:
```
# Given values
sample_size = 45
mean_uniform = 1/2
std_dev_uniform = 1/math.sqrt(12)

# Calculate standard error of the mean
```

```
se = std_dev_uniform / math.sqrt(sample_size)

# Value for which we want to find the probability
X = 0.465

# Calculate Z-score
Z = (X - mean_uniform) / se

# Find the probability using the cumulative distribution function (CDF) of the␣
↪standard normal distribution
probability_below_X = norm.cdf(Z)

print(f"The probability that the mean is below {X} is approximately:␣
↪{probability_below_X:.4f}")
```

```
The probability that the mean is below 0.465 is approximately: 0.2080
```

Question 3.A farmer is interested in knowing the mean weight of his chickens when they leave the farm. Suppose that the standard deviation of the chicken's weight is 500 grams.

(a) What is the minimum number of chickens needed to ensure that the standard deviation of the sample mean is no more than 90 grams?

(b) Suppose the farm has three coops. The mean weights in each coop are 1.75, 1.85and 2.1 kg, and standard deviations are 450, 520, and 380 grams, respectively. Calculate the probability that a random sample of 30 chickens from the first coop will have a mean weight larger than 1.925 kg. Calculate the same probability for the second and third coops.

(c) Suppose the proportion of the three coops are 0.60, 0.25, 0.15. Given that a random sample of 30 chickens from some coop has a mean weight larger than 1.925 kg, find the posterior probability the sample is from the (i) first coop, (ii) second coop, (iii) third coop. Which coop did the sample of chickens most likely have come from?

[29]:
```
#Part(a)

# Given values
population_std_dev = 500
desired_SE = 90

# Calculate the minimum sample size
min_sample_size = math.ceil((population_std_dev / desired_SE)**2)

print(f"The minimum number of chickens needed is: {min_sample_size}")
```

```
The minimum number of chickens needed is: 31
```

[30]:
```
#Part(b)

# Given values for the first coop
```

```
mean_coop1 = 1.75
std_dev_coop1 = 450
sample_size_coop1 = 30
mean_weight_threshold = 1.925

# Calculate the z-score
z_score_coop1 = (mean_weight_threshold - mean_coop1) / (std_dev_coop1 / math.
 ↪sqrt(sample_size_coop1))

# Calculate the probability using the complementary cumulative distribution␣
 ↪function (1 - CDF)
probability_above_threshold_coop1 = 1 - norm.cdf(z_score_coop1)

print(f"Probability for the first coop: {probability_above_threshold_coop1:.
 ↪4f}")
```

```
Probability for the first coop: 0.4992
```

[31]:

```
# Part(c)
#P(Coop Mean weight > 1.925 kg)=P(Mean weight > 1.925 kg)/P(Mean weight > 1.925␣
 ↪kg Coop)×P(Coop)


# Given values for the coops
mean_coop1 = 1.75
std_dev_coop1 = 450
sample_size_coop1 = 30
mean_weight_threshold = 1.925

# Calculate the z-score and probability for the first coop
z_score_coop1 = (mean_weight_threshold - mean_coop1) / (std_dev_coop1 / math.
 ↪sqrt(sample_size_coop1))
probability_above_threshold_coop1 = 1 - norm.cdf(z_score_coop1)

# Repeat for the second coop
mean_coop2 = 1.85
std_dev_coop2 = 520
sample_size_coop2 = 30

z_score_coop2 = (mean_weight_threshold - mean_coop2) / (std_dev_coop2 / math.
 ↪sqrt(sample_size_coop2))
probability_above_threshold_coop2 = 1 - norm.cdf(z_score_coop2)

# Repeat for the third coop
mean_coop3 = 2.1
std_dev_coop3 = 380
sample_size_coop3 = 30
```

```
z_score_coop3 = (mean_weight_threshold - mean_coop3) / (std_dev_coop3 / math.sqrt(sample_size_coop3))
probability_above_threshold_coop3 = 1 - norm.cdf(z_score_coop3)

print(f"Probability for the first coop: {probability_above_threshold_coop1:.4f}")
print(f"Probability for the second coop: {probability_above_threshold_coop2:.4f}")
print(f"Probability for the third coop: {probability_above_threshold_coop3:.4f}")
```

```
Probability for the first coop: 0.4992
Probability for the second coop: 0.4997
Probability for the third coop: 0.5010
```

Question 4.An engineering team has designed a lamp with two light bulbs. Let X be the lifetime for bulb 1 and Y be the lifetime for bulb 2, both in thousands of hours.Suppose that X and Y are independent and they follow an exponential distribution with mean = 2.

(a) What is the probability a bulb lasts more than 2000 hours?

(b) If the lamp works when at least one bulb is lit, what is the probability that the lamp works for more than 2000 hours?

(c) What is the probability that the lamp works no more than 1000 hours?

[32]:
```
# Given mean lifetime
beta = 2

# (a) Probability a bulb lasts more than 2000 hours
prob_a = 1 - expon.cdf(2, scale=beta)

# (b) Probability that the lamp works for more than 2000 hours
prob_b = 1 - expon.cdf(2, scale=beta)**2

# (c) Probability that the lamp works no more than 1000 hours
prob_c = expon.cdf(1, scale=beta)**2

print(f"(a) Probability a bulb lasts more than 2000 hours: {prob_a:.4f}")
print(f"(b) Probability that the lamp works for more than 2000 hours: {prob_b:.4f}")
print(f"(c) Probability that the lamp works no more than 1000 hours:{prob_c:.4f}")
```

```
(a) Probability a bulb lasts more than 2000 hours: 0.3679
(b) Probability that the lamp works for more than 2000 hours: 0.6004
(c) Probability that the lamp works no more than 1000 hours:0.1548
```

## 2 Health Insurance

[33]:
```
#Google Drive
from google.colab import drive
drive.mount('/content/drive',force_remount=True)
```

```
Mounted at /content/drive
```

[34]:
```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import seaborn as sns
import datetime as dt
from scipy import stats
from scipy.stats import norm
pd.options.display.float_format = '{:.0f}'.format
```

[35]:
```
# Reading the data from csv file
data=pd.read_csv('/content/drive/MyDrive/Colab Notebooks/CRA/insurance.csv')
data.head()
```

[35]:

| | age | sex | bmi | children | smoker | region | charges |
|---|---|---|---|---|---|---|---|
| 0 | 19 | female | 28 | 0 | yes | southwest | 16885 |
| 1 | 18 | male | 34 | 1 | no | southeast | 1726 |
| 2 | 28 | male | 33 | 3 | no | southeast | 4449 |
| 3 | 33 | male | 23 | 0 | no | northwest | 21984 |
| 4 | 32 | male | 29 | 0 | no | northwest | 3867 |

### 2.1 Dataset Description

This dataset can be helpful in a simple yet illuminating study in understanding the risk underwriting in Health Insurance, the interplay of various attributes of the insured and see how they affect the insurance premium. This dataset contains 1338 rows of insured data, where the Insurance charges are given against the following attributes of the insured: Age, Sex, BMI, Number of Children, Smoker and Region. There are no missing or undefined values in the dataset.

[36]:
```
# Get Data Summary
def GetSummary(text,df):
        print(f'{text} shape: {df.shape}')
        summ = pd.DataFrame(df.dtypes, columns=['dtypes'])
        summ['null'] = df.isnull().sum()
        summ['unique'] = df.nunique()
        summ['min'] = df.min()
        summ['median'] = df.median()
        summ['max'] = df.max()
        summ['mean'] = df.mean()
        summ['std'] = df.std()
        summ['duplicate'] = df.duplicated().sum()
```

```
        return summm
```

[37]:
```
# Get summary data
GetSummary('data',data)
```

```
data shape: (1338, 7)
```

[37]:
```
           dtypes  null  unique        min  median        max   mean    std  \
age         int64     0      47         18      39         64     39     14
sex        object     0       2     female     NaN       male    NaN    NaN
bmi       float64     0     548         16      30         53     31      6
children    int64     0       6          0       1          5      1      1
smoker     object     0       2         no     NaN        yes    NaN    NaN
region     object     0       4  northeast     NaN  southwest    NaN    NaN
charges   float64     0    1337       1122    9382      63770  13270  12110

          duplicate
age               1
sex               1
bmi               1
children          1
smoker            1
region            1
charges           1
```

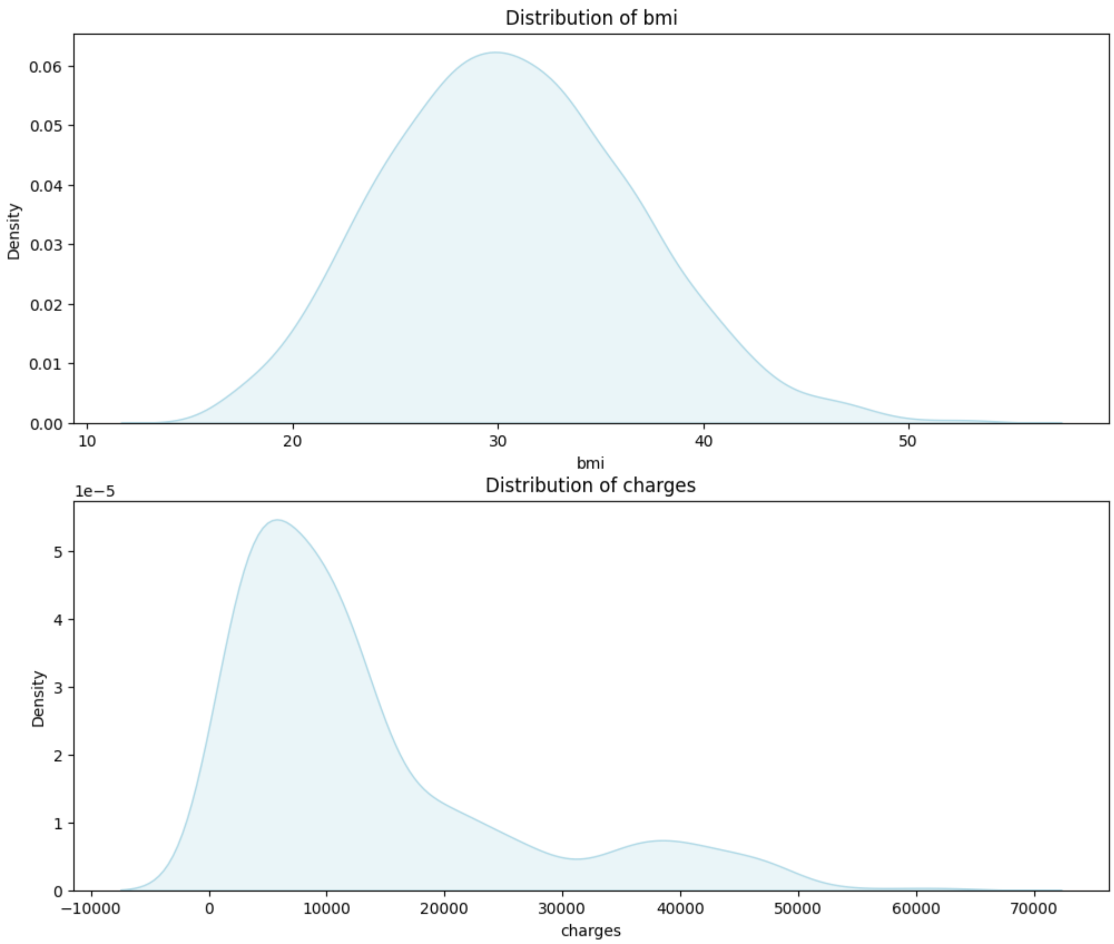
[38]:
```
# Drop duplicate data
data.drop_duplicates(inplace = True)
GetSummary('data',data)
```

```
data shape: (1337, 7)
```

[38]:
```
           dtypes  null  unique        min  median        max   mean    std  \
age         int64     0      47         18      39         64     39     14
sex        object     0       2     female     NaN       male    NaN    NaN
bmi       float64     0     548         16      30         53     31      6
children    int64     0       6          0       1          5      1      1
smoker     object     0       2         no     NaN        yes    NaN    NaN
region     object     0       4  northeast     NaN  southwest    NaN    NaN
charges   float64     0    1337       1122    9386      63770  13279  12110

          duplicate
age               0
sex               0
bmi               0
children          0
smoker            0
region            0
```

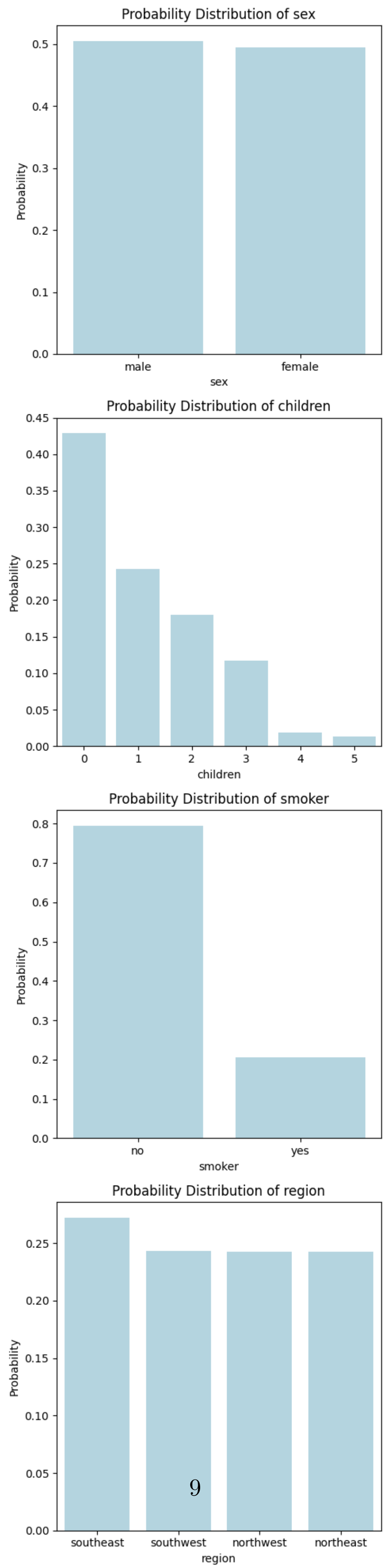
```
charges          0
```

[39]:
```
# Define variable type
# (a).Plot the histograms of primary (important) continuous variables and␣
↪probability distributions of categorical variables.

continuous= ['bmi','charges']
# Plot histograms for continuous variables
fig, axes = plt.subplots(nrows=len(continuous), ncols=1, figsize=(12, 10))
for i, variable in enumerate(continuous):
    sns.kdeplot(x=variable, data=data, ax=axes[i],color='lightblue',fill=True)
    axes[i].set_title(f'Distribution of {variable}')
plt.show()
```

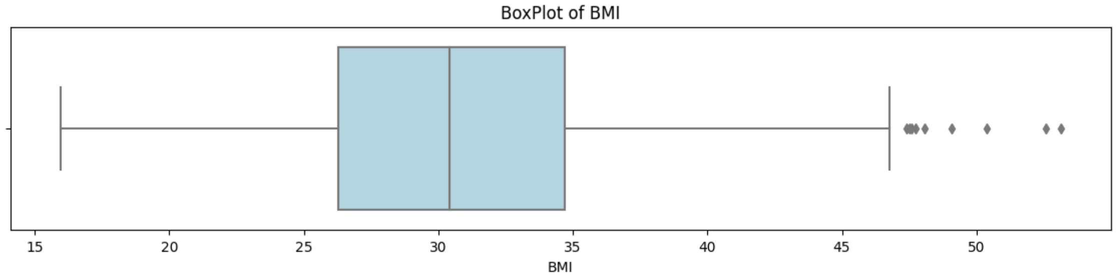


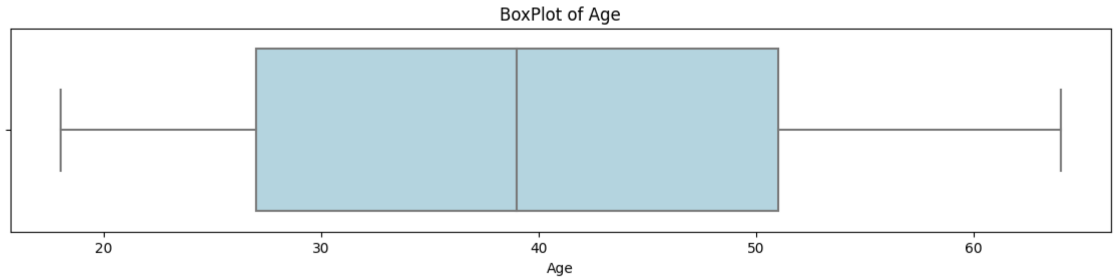

[40]:
```
# Plot probability distributions for categorical variables
categorical  = ['sex','children','smoker','region']
```

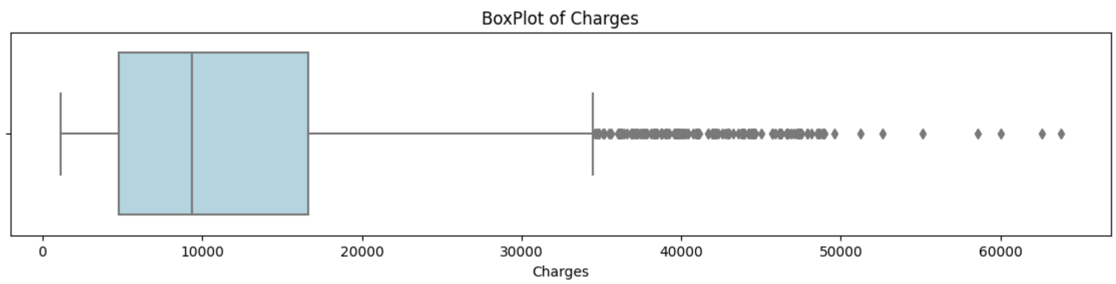
```
fig, axes = plt.subplots(nrows=len(categorical), ncols=1, figsize=(5, 20))

for i, variable in enumerate(categorical):
    # Calculate the relative frequencies (probabilities) for each category
    counts = data[variable].value_counts(normalize=True)

    # Plot the bar plot
    sns.barplot(x=counts.index, y=counts.values, ax=axes[i],color='lightblue')
    axes[i].set_title(f'Probability Distribution of {variable}')
    axes[i].set_xlabel(variable)
    axes[i].set_ylabel('Probability')

plt.tight_layout()
plt.show()
```


Probability Distribution of sex
Probability
0.0
0.1
0.2
0.3
0.4
0.5
male
female
sex
Probability Distribution of children
Probability
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
0.45
0
1
2
3
4
5
children
Probability Distribution of smoker
Probability
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
no
yes
smoker
Probability Distribution of region
Probability
0.00
0.05
0.10
0.15
0.20
0.25
southeast
southwest
northwest
northeast
region

[41]:
```
# (b).Plot the box plots of all the primary variables in the data. Identify and␣
↪delete a couple of extreme outliers from the data if there are any.
plt.figure(figsize= (12,10))
plt.subplot(3,1,1)
sns.boxplot(x= data.bmi, color='lightblue')
plt.title('BoxPlot of BMI')
plt.xlabel('BMI')

plt.subplot(3,1,2)
sns.boxplot(x= data.age, color='lightblue')
plt.title('BoxPlot of Age')
plt.xlabel('Age')

plt.subplot(3,1,3)
sns.boxplot(x= data.charges, color='lightblue')
plt.title('BoxPlot of Charges')
plt.xlabel('Charges')
plt.tight_layout(pad=3.5)
plt.show()
```




[42]:
```
# Detect outlier based on quantiles
def detect_outliers_iqr(column, threshold=1.5):
    q1 = column.quantile(0.25)
    q3 = column.quantile(0.75)
    iqr = q3 - q1
    lower_bound = q1 - threshold * iqr
    upper_bound = q3 + threshold * iqr
    outliers = (column < lower_bound) | (column > upper_bound)
    return outliers

# Select the specific columns you want to test for outliers
columns_to_test = ['age', 'bmi', 'charges']
outliers = data[columns_to_test].apply(detect_outliers_iqr)

print("Number of outliers in each selected column:")
print(outliers.sum())
```

```
Number of outliers in each selected column:
age          0
```

```
bmi           9
charges     139
dtype: int64
```

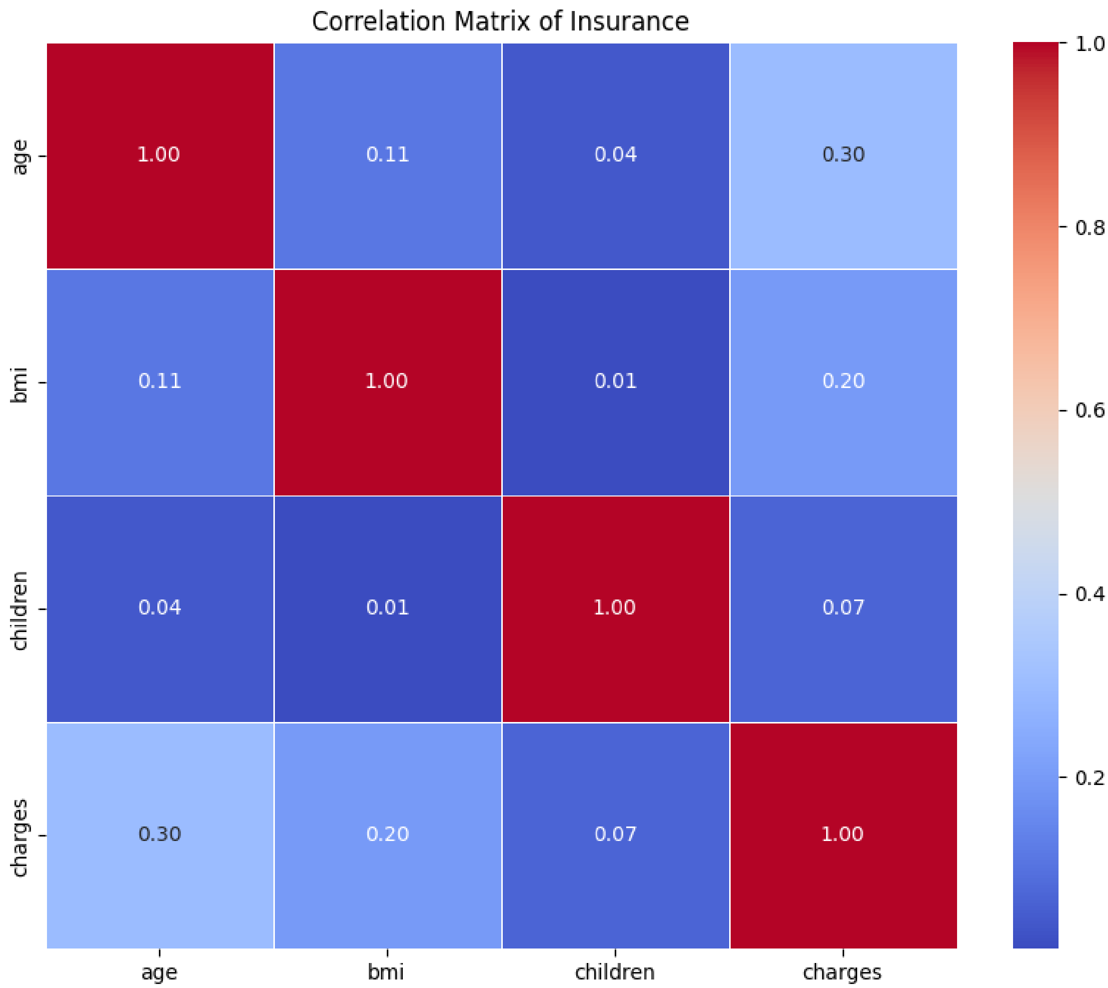
[43]:

```
# Delete the data from dataset after removing outliers
df_cleaned = data[~outliers.any(axis=1)]
print("Original DataFrame shape:", data.shape)
print("Cleaned DataFrame shape:", df_cleaned.shape)
```

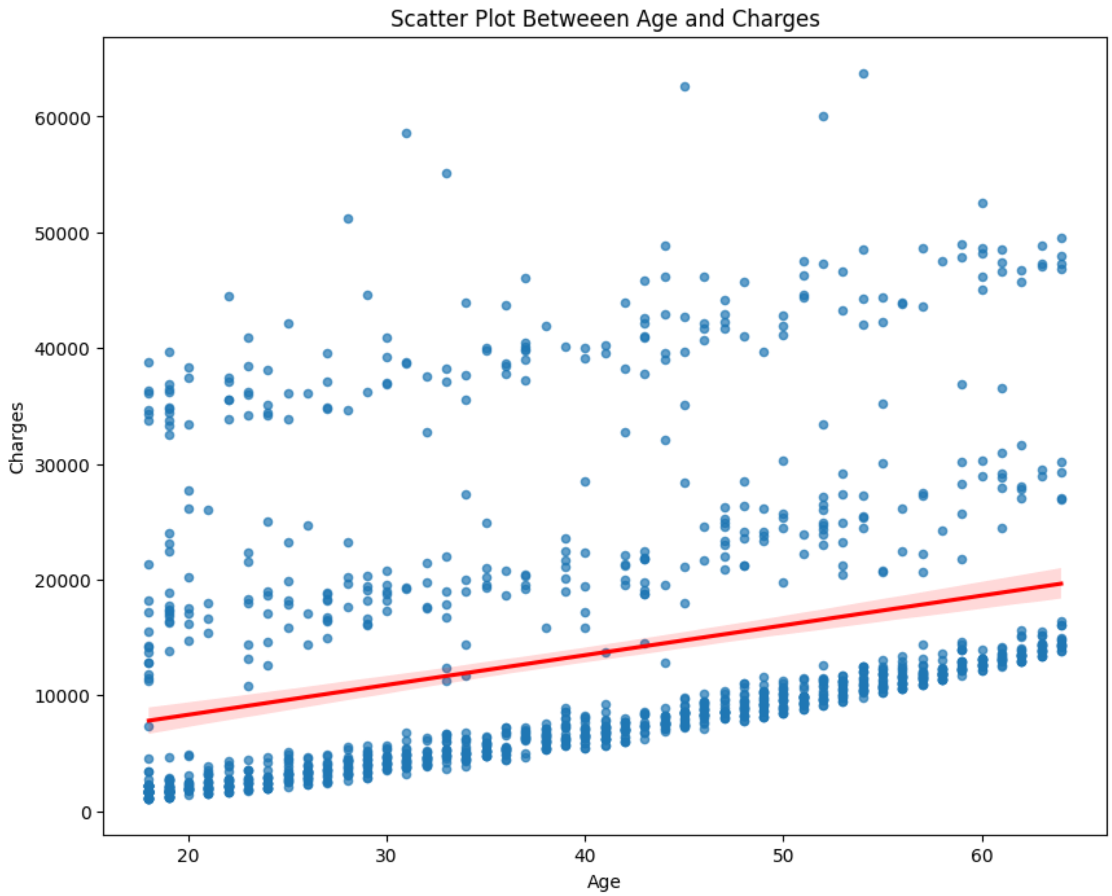
```
Original DataFrame shape: (1337, 7)
Cleaned DataFrame shape: (1192, 7)
```

[44]:

```
# (c).Compute the matrix of sample correlations between every pair of variables.
numeric_df = data.select_dtypes(include=['number'])
# Compute the matrix of sample correlations
correlation_matrix = data.corr()
# Plot a heatmap of the correlation matrix
plt.figure(figsize=(10, 8))
sns.heatmap(correlation_matrix, annot=True, cmap='coolwarm', fmt='.2f',␣
 ↪linewidths=0.5)
plt.title('Correlation Matrix of Insurance')
plt.show()
```

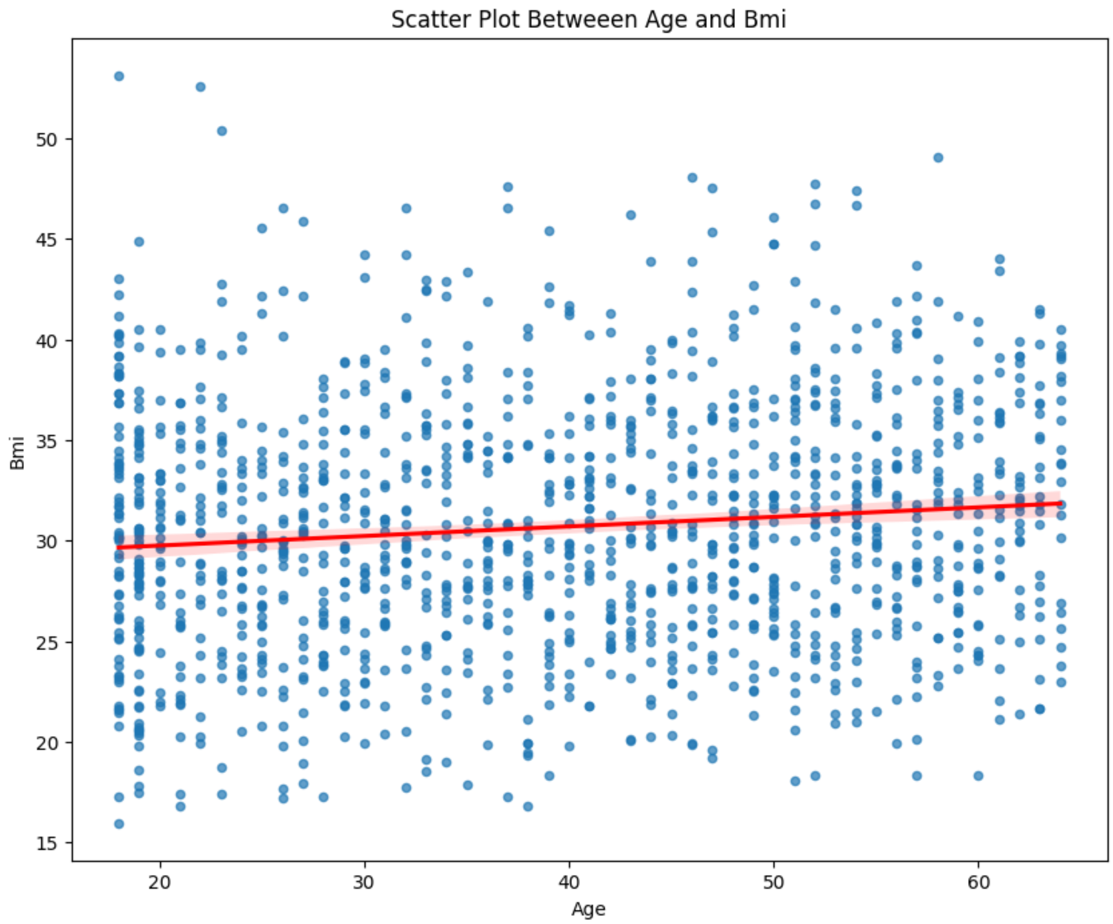



[45]:
```python
# (d).Choose two variables that are highly correlated (positively or␣
↪negatively) and plot their scatter plot and the best regression line
numeric_df = data.select_dtypes(include=['number'])
# Compute the matrix of sample correlations
correlation_matrix = numeric_df.corr()
# Create a scatter plot with the best-fitting regression line
plt.figure(figsize=(10, 8))
sns.regplot(x=data.age, y=data.charges, data=data, scatter_kws={'s': 20,␣
↪'alpha': 0.7}, line_kws={'color': 'red'})
plt.title(f'Scatter Plot Betweeen Age and Charges')
plt.xlabel('Age')
plt.ylabel('Charges')
plt.show()
```




[46]:

```
numeric_df = data.select_dtypes(include=['number'])
# Compute the matrix of sample correlations
correlation_matrix = numeric_df.corr()
# Create a scatter plot with the best-fitting regression line
plt.figure(figsize=(10, 8))
sns.regplot(x=data.age, y=data.bmi, data=data, scatter_kws={'s': 20, 'alpha': 0.
  ↪7}, line_kws={'color': 'red'})
plt.title(f'Scatter Plot Betweeen Age and Bmi')
plt.xlabel('Age')
plt.ylabel('Bmi')
plt.show()
```




(4). Write a brief description of your data and summarize your findings on the variables.

## 2.2 Observations

- Age shows uniform distribution.
- Bmi shows normal distribution.
- Charges shows normal distribution, but it skewed to right.
- BMI shows 9 row of outliers and charges show 139 rwo outliers.
- Many customers are no smoke.
- Sex distribution is relatively even.
- Male smokers are more than the female smoker.
- Female no smoker are more than male no smoker.
- Region distribution is fairly balanced, with the exception of the Southeast which has the highest number of customers.
- Many customers have one children and fewer customers have more than 4 childrens.
- As customer age increases, charges also tend to rise.
- Two customer groups exist: smokers and non-smokers
- Smoker customers incur higher charges compared to non-smokers.
- As customer age increases, charges also tend to rise/

- Typically, the highest charges are from male and female smoker customers.
- The lowest charges are from male and female no smoker customers.

[ ]:
```
%%capture
!pip install nbconvert
!sudo apt-get install texlive-xetex texlive-fonts-recommended␣
↪texlive-plain-generic
```

[ ]:
```
# https://saturncloud.io/blog/convert-google-colab-notebook-to-pdf-html/
!jupyter nbconvert  '/content/drive/MyDrive/Colab Notebooks/
↪STAT603-Fall2023-FinalExam.ipynb' --to pdf
```