

Homework 5 Stat 603

3.43 You take a standard deck of playing cards, and remove one card at random. You then draw a single card. Write S for the event that the card you remove is a six. Write N for the event that the card you remove is not a six. Write R for the event that the card you remove is red. Write B for the event the card you remove is black.

(a) Write A for the event you draw a 6. What is $P(A|S)$?

Solution: $P(A|S) = P(A \cap S)/P(S)$, $P(S) = 1 - 48/52 = 4/52$, $P(A) = 4/51$

$$P(A|S) = (4/51 * 4/52) / (4/52) = 4/51$$

(b) Write A for the event you draw a 6. What is $P(A|N)$?

Solution: $P(A|N) = P(A \cap N) / P(N)$, so $P(N) = 48/52$, $P(A) = 4/51$

$$P(A|N) = (4/51 * 48/52) / 48/52 = 4/51$$

(c) Write A for the event you draw a 6. What is $P(A)$?

Solution: $P(A) = 4/51$

(d) Write D for the event you draw a red six. Are D and A independent? why?

Solution: For events to be independent, $P(A) = P(A|B) = 1/2$, $P(D) = P(D|A)$.

$P(D) = 2/51$, $P(A) = 4/51$, $P(D|A) = P(D \cap A) / P(A) = (2/51 * 4/51) / (4/51) = 2/51$
 $= P(D)$, Hence they are independent events.

(e) Write D for the event you draw a red six. What is $P(D)$?

Solution: $P(D) = 2/51$.

3.44 A student takes a multiple-choice test. Each question has N answers. If the student knows the answer to a question, the student gives the right answer, and otherwise guesses uniformly and at random. The student knows the answer to 70% of the questions. Write K for the event a student knows the answer to a question and R for the event the student answers the question correctly.

(a) What is $P(K)$?

Solution: $P(K) = 70/100 = 0.7$

(b) What is $P(R|K)$?

Solution: $P(R|K) = P(R \cap K) / P(K) = 1$

(c) What is $P(K|R)$, as a function of N ?

Solution: Using Bayes' probability function,

$$P(K|R) = P(K \text{ and } R) / P(R) = P(K) * P(R|K) / (P(K) * P(R|K) + P(K') * P(R|K'))$$

$$= 0.7 * 1 / (0.7 * 1 + (1-0.7) * 1/N)$$

$$= 0.7 / (0.7 + 0.3/N)$$

(d) What values of N will ensure that $P(K|R) > 99\%$?

Solution: $99/100 < 0.7 / (0.7 + 0.3/N)$

$$= 0.99 < 0.7 / (1/N)$$

$$= N > 42.86 \Rightarrow 43$$

3. Pollution of the rivers in the United States has been a problem for many years.

Consider

the following events:

A: the river is polluted

B: a sample of water tested detects pollution

C: fishing is permitted

Assume that $P(A) = 0.3$, $P(B|A) = 0.75$, $P(B|A') = 0.20$, $P(C|(A \cap B)) = 0.20$

$P(C|(A^c \cap B)) = 0.15$, $P(C|(A \cap B')) = 0.80$, and $P(C|(A^c \cap B')) = 0.90$.

(a) Find $P(A \cap B \cap C)$.

Solution: $P(A \cap B \cap C) = P(C|A \text{ and } B) * P(A \text{ and } B)$

$$= P(C|A \text{ and } B) * P(A) * P(B|A)$$

$$= 0.2 * 0.75 * 0.3 = 0.045$$

(b) Find $P(B' \cap C)$.

Solution: Since $B' \cap C = (A \cap B' \cap C) \cup (A' \cap B' \cap C)$

$$P(B' \cap C) = P(A \cap B' \cap C) + P(A' \cap B' \cap C)$$

$$\text{But } P(A \cap B' \cap C) = P(C|A \cap B') * P(A \cap B')$$

$$= 0.8 * (1 - 0.75) * 0.3 = 0.06$$

$$\text{And } P(A' \cap B' \cap C) = 0.9 * (1 - 0.2) * (1 - 0.3) = 0.504$$

$$P(B' \cap C) = 0.06 + 0.504 = 0.564$$

(c) Find $P(C)$.

Solution: $C = (A \cap B' \cap C) \cup (A' \cap B' \cap C) \cup (A \cap B \cap C) \cup (A' \cap B \cap C)$

$$P(C) = P(A \cap B' \cap C) + P(A' \cap B' \cap C) + P(A \cap B \cap C) + P(A' \cap B \cap C)$$

$$\text{But } P(A' \cap B \cap C) = P(C|A' \cap B) * P(A' \cap B)$$

$$= 0.15 * P(B|A') * P(A')$$

$$= 0.15 * 0.20 * 0.07 = 0.021$$

$$\text{Hence } P(C) = 0.06 + 0.504 + 0.045 + 0.021 = 0.63$$

(d) Find the probability that the river is polluted given that fishing is permitted and the sample tested did not detect pollution

Solution: The required probability = $P(A|C \cap B') = P(A \cap B' \cap C) / P(C \cap B') = 0.06 / 0.546$
 $= 0.106$