

Solution(1)

Given:

- $P(M=T)=0.2$
- $P(B=T|M=T)=0.2, P(B=T|M=F)=0.05$
- $P(S=T|M=T)=0.8, P(S=T|M=F)=0.2$
- $P(C=T|B,S)$:
 - $P(C=T|B=T,S=T)=0.8$
 - $P(C=T|B=T,S=F)=0.8$
 - $P(C=T|B=F,S=T)=0.8$
 - $P(C=T|B=F,S=F)=0.05$
- $P(H=T|B=T)=0.8, P(H=T|B=F)=0.6$

Step 1: Calculate the joint probability for the given evidence

We need to calculate the joint probabilities for both cases $M=T$ and $M=F$ given the evidence.

When $M=T$: We need the joint probability for the patient having metastatic cancer ($M=T$), severe headache ($H=T$), brain tumor ($B=T$), not in coma ($C=F$), and no increased serum calcium ($S=F$): $P(H=T, B=T, C=F, S=F|M=T) \cdot P(M=T)$

$$P(H=T, B=T, C=F, S=F|M=T) \cdot P(M=T)$$

- $P(B=T|M=T)=0.2$
- $P(S=F|M=T)=1-0.8=0.2$
- $P(C=F|B=T, S=F)=1-0.8=0.2$
- $P(H=T|B=T)=0.8$

$$P(H=T, B=T, C=F, S=F|M=T) = P(H=T|B=T) \cdot P(C=F|B=T, S=F) \cdot P(S=F|M=T) \cdot P(B=T|M=T) = 0.8 \cdot 0.2 \cdot 0.2 \cdot 0.2 = 0.0064$$

$$P(H=T, B=T, C=F, S=F, M=T) = 0.0064 \cdot 0.2 = 0.00128$$

Step 2:

When $M=F$: Compute the probability of not having metastatic cancer and the symptom

We calculate the same joint probability assuming the patient does not have metastatic cancer ($M=F$): $P(H=T, B=T, C=F, S=F|M=F) \cdot P(M=F)$, Using the given probabilities:
 $= 0.8 \cdot 0.2 \cdot 0.8 \cdot 0.05 = 0.0064$

$$P(H=T, B=T, C=F, S=F, M=F) = 0.0064 * 0.8 = 0.00512$$

Step 3: Use Bayes' Theorem to find $P(M=T|H=T, B=T, C=F, S=F)$

$$P(H=T, B=T, C=F, S=F) = P(H=T, B=T, C=F, S=F, M=T) + P(H=T, B=T, C=F, S=F, M=F)$$

$$= 0.00128 + 0.00512$$

$$P(M=T | H=T, B=T, C=F, S=F) = P(H=T, B=T, C=F, S=F | M=T) * P(M=T) / P(H=T, B=T, C=F, S=F)$$

$$0.00128 / 0.0064 = 0.2.$$

The probability that the patient has metastatic cancer given the observed symptoms (severe headache, brain tumor, not in coma, and no increased serum calcium) is **20%**

Solution(2)

Given weather data from book.

SNo	Outlook	Temperature	Humidity	Windy	Play
1	sunny	hot	high	FALSE	no
2	sunny	hot	high	TRUE	no
3	overcast	hot	high	FALSE	yes
4	rainy	mild	high	FALSE	yes
5	rainy	cool	normal	FALSE	yes
6	rainy	cool	normal	TRUE	no
7	overcast	cool	normal	TRUE	yes
8	sunny	mild	high	FALSE	no
9	sunny	cool	normal	FALSE	yes
10	rainy	mild	normal	FALSE	yes
11	sunny	mild	normal	TRUE	yes
12	overcast	mild	high	TRUE	yes
13	overcast	hot	normal	FALSE	yes
14	rainy	mild	high	TRUE	no

Original Rule:

Rule: Outlook=sunny and temp=cool and humidity=normal and windy=false → Play = yes

- Instances satisfying this condition: Instance 9
- Support: 1 (instance 9)

- Accuracy: 100% (1 out of 1 are yes)

Pruning Steps:

Rule: Outlook=sunny and humidity=normal → Play = yes

Instances satisfying this condition:

- Instance 9: sunny, cool, normal, false → yes
- Instance 11: sunny, mild, normal, true → yes

Support: 2 (instances 9 and 11) **Accuracy: 100%** (2 out of 2 are yes)

Support is less than 3. We need to relax the rule further.

Rule: Outlook=sunny → Play = yes

Instances satisfying this condition:

- Instance 1: sunny, hot, high, false → no
- Instance 2: sunny, hot, high, true → no
- Instance 8: sunny, mild, high, false → no
- Instance 9: sunny, cool, normal, false → yes
- Instance 11: sunny, mild, normal, true → yes

Support: 5 (instances 1, 2, 8, 9, 11) **Accuracy: 40%** (2 out of 5 are yes)

Accuracy is less than 50%. We need to try a different combination.

Rule: humidity=normal → Play = yes

Instances satisfying this condition:

- Instance 5: rainy, cool, normal, false → yes
- Instance 6: rainy, cool, normal, true → no
- Instance 9: sunny, cool, normal, false → yes
- Instance 10: rainy, mild, normal, false → yes
- Instance 11: sunny, mild, normal, true → yes
- Instance 13: overcast, hot, normal, false → yes

Support: 6 (instances 5, 6, 9, 10, 11, 13) **Accuracy: 83.33%** (5 out of 6 are yes)

Support is 6 and accuracy is 83.33%, meeting the criteria.

Final Pruned Rule

humidity=normal → Play = yes

Other possible combinations

Rule: Outlook=sunny

Instances satisfying this condition:

- Instance 1: sunny, hot, high, false → no
- Instance 2: sunny, hot, high, true → no
- Instance 8: sunny, mild, high, false → no
- Instance 9: sunny, cool, normal, false → yes
- Instance 11: sunny, mild, normal, true → yes

Support: 5 Accuracy: 40% (2/5 are yes)

Rule: Outlook=overcast

Instances satisfying this condition:

- Instance 3: overcast, hot, high, false → yes
- Instance 7: overcast, cool, normal, true → yes
- Instance 12: overcast, mild, high, true → yes
- Instance 13: overcast, hot, normal, false → yes

Support: 4 Accuracy: 100% (4/4 are yes)

Rule: Outlook=rainy

Instances satisfying this condition:

- Instance 4: rainy, mild, high, false → yes
- Instance 5: rainy, cool, normal, false → yes
- Instance 6: rainy, cool, normal, true → no
- Instance 10: rainy, mild, normal, false → yes
- Instance 14: rainy, mild, high, true → no

Support: 5 Accuracy: 60% (3/5 are yes)

Rule: Temperature=hot

Instances satisfying this condition:

- Instance 1: sunny, hot, high, false → no
- Instance 2: sunny, hot, high, true → no
- Instance 3: overcast, hot, high, false → yes

- Instance 13: overcast, hot, normal, false → yes

Support: 4 Accuracy: 50% (2/4 are yes)

Rule: Temperature=mild

Instances satisfying this condition:

- Instance 4: rainy, mild, high, false → yes
- Instance 8: sunny, mild, high, false → no
- Instance 10: rainy, mild, normal, false → yes
- Instance 11: sunny, mild, normal, true → yes
- Instance 12: overcast, mild, high, true → yes
- Instance 14: rainy, mild, high, true → no

Support: 6 Accuracy: 66.67% (4/6 are yes)

Rule: Temperature=cool

Instances satisfying this condition:

- Instance 5: rainy, cool, normal, false → yes
- Instance 6: rainy, cool, normal, true → no
- Instance 7: overcast, cool, normal, true → yes
- Instance 9: sunny, cool, normal, false → yes

Support: 4 Accuracy: 75% (3/4 are yes)

Rule: Humidity=high

Instances satisfying this condition:

- Instance 1: sunny, hot, high, false → no
- Instance 2: sunny, hot, high, true → no
- Instance 3: overcast, hot, high, false → yes
- Instance 4: rainy, mild, high, false → yes
- Instance 8: sunny, mild, high, false → no
- Instance 12: overcast, mild, high, true → yes
- Instance 14: rainy, mild, high, true → no

- Support: 7
- Accuracy: 42.86% (3/7 are yes)

Rule: Windy=true

Instances satisfying this condition:

- Instance 2: sunny, hot, high, true → no

- Instance 6: rainy, cool, normal, true → no
- Instance 7: overcast, cool, normal, true → yes
- Instance 11: sunny, mild, normal, true → yes
- Instance 12: overcast, mild, high, true → yes
- Instance 14: rainy, mild, high, true → no

Support: 6 Accuracy: 50% (3/6 are yes)

Rule: Windy=false

Instances satisfying this condition:

- Instance 1: sunny, hot, high, false → no
- Instance 3: overcast, hot, high, false → yes
- Instance 4: rainy, mild, high, false → yes
- Instance 5: rainy, cool, normal, false → yes
- Instance 8: sunny, mild, high, false → no
- Instance 9: sunny, cool, normal, false → yes
- Instance 10: rainy, mild, normal, false → yes
- Instance 13: overcast, hot, normal, false → yes

Support: 8 Accuracy: 62.5% (5/8 are yes)

Additional Rules Meeting Criteria

Rule: Outlook=overcast → Play = yes

- Support: 4
- Accuracy: 100%

Rule: Outlook=rainy → Play = yes

- Support: 5
- Accuracy: 60%

Rule: Temperature=mild → Play = yes

- Support: 6
- Accuracy: 66.67%

Rule: Temperature=cool → Play = yes

- Support: 4
- Accuracy: 75%

Rule: Windy=false → Play = yes

- Support: 8
- Accuracy: 62.5%

Summary of Possible Rules

- humidity=normal → Play = yes (Support: 6, Accuracy: 83.33%)
- Outlook=overcast → Play = yes (Support: 4, Accuracy: 100%)
- Outlook=rainy → Play = yes (Support: 5, Accuracy: 60%)
- Temperature=mild → Play = yes (Support: 6, Accuracy: 66.67%)
- Temperature=cool → Play = yes (Support: 4, Accuracy: 75%)
- Windy=false → Play = yes (Support: 8, Accuracy: 62.5%)