Homework 5 Stat 603

3.43 You take a standard deck of playing cards, and remove one card at random. You then draw a single card. Write S for the event that the card you remove is a six. Write N for the event that the card you remove is not a six. Write R for the event that the card you remove is red. Write B for the event the card you remove is black.

(a) Write A for the event you draw a 6. What is P(A|S)?

Solution: P(A|S) = P (A S)/P(S), P(A|S) = Number of ways to draw a 6 from the remaining three 6s / Total number of remaining cards in the deck.

Hence 3 remaining 6s, and there are 51 remaining cards in the deck,

So, P(A|S) = (3/3) / (51/3) = 1/17.

(b) Write A for the event you draw a 6. What is P (A|N)?

Solution: P(A|N) = P (A N) /P(N), so P(N) = 48/52, P(A) = 4/51

P(A|N) = (4/51 * 48/52)/48/52 = 4/51

(c) Write A for the event you draw a 6. What is P(A)?

Solution: P(A) = Number of ways to draw a 6 from the entire deck / Total number of cards in the entire deck. P(A) = 4/52 = 1/13

(d) Write D for the event you draw a red six. Are D and an independent? why?

Solution: For events to be independent, P(D|A) = P(D) and P(A|D) = P(A).

P(D) = 2/52, P(A) = 1/13, Since P(D|A) = P(A|D) = 2/52 ≠ P(D) = 2/52 ≠ P(A) = 1/13, D and A are not independent.

(e) Write D for the event you draw a red six. What is P(D)?

Solution: P(D) = Number of ways to draw a red six / Total number of cards in the entire deck

P(D) = 2/51.

3.44 A student takes a multiple-choice test. Each question has N answers. If the student knows the answer to a question, the student gives the right answer, and otherwise guesses uniformly and at random. The student knows the answer to 70% of the questions. Write K for the event a student knows the answer to a question and R for the event the student answers the question correctly.
(a) What is P(K)?

Solution: P(K) = 70/100 = 0.7
(b) What is P(R|K)?

Solution: P(R|K) = P (R K) /P(K) = 1
(c) What is P(K|R), as a function of N?

Solution: Using Bayes' probability function,

P(K/R) = P (K and R)/P (R) = P(K) * P(R|K) / (P(K) * P(R|K) + P(K') P(R|K')

= 0.7*1 / (0.7* 1 + (1-0.7) * 1/N)

= 0.7/ (0.7 + 0.3/N)

(d) What values of N will ensure that P(K|R) > 99%?

Solution: 99/100 < 0.7/ (0.7 + 0.3/N)

= 0.99 < 0.7/(1/N)

= N> 42.86 == 43

3.Pollution of the rivers in the United States has been a problem for many years. Consider
the following events:

A: the river is polluted

B: a sample of water tested detects pollution

C: fishing is permitted

Assume that $P(A) = 0.3$, $P(B|A) = 0.75$, $P(B|A') = 0.20$, $P(C|(A \cap B)) = 0.20$
$P(C|(Ac \cap B)) = 0.15$, $P(C|(A \cap B')) = 0.80$, and $P(C|(Ac \cap B')) = 0.90$.

(a) Find $P(A \cap B \cap C)$.

Solution: $P(A \cap B \cap C) = P(C|A \text{ and } B) * P(A \text{ and } B)$

$= P(C|A \text{ and } B) * P(A)*(B|A)$

$= 0.2 * 0.75*0.3 = 0.045$

(b) Find $P(B' \cap C)$.

Solution: Since $B'\cap C = (A \cap B'\cap C) \cup (A'\cap B'\cap C)$

$P(B'\cap C) = P(A \cap B'\cap C) + P(A'\cap B'\cap C)$

But $P(A \cap B'\cap C) = P(C|A\cap B') * P(A\cap B')$

$=0.8*(1 – 0.75) * 0.3 = 0.06$

And $P(A'\cap B'\cap C) = 0.9 *(1 – 0.2) *(1 -0.3) = 0.504$

$P(B'\cap C) = 0.06+0.504 = 0.564$

(c) Find $P(C)$.

Solution: $C = (A \cap B' \cap C) \cup (A'\cap B'\cap C) \cup (A\cap B\cap C) \cup (A'\cap B\cap C)$

$P(C) = (A \cap B' \cap C) + (A' \cap B' \cap C) + (A \cap B \cap C) + (A' \cap B \cap C)$

But $P(A' \cap B \cap C) = P(C | A' \cap B) * P(A' \cap B)$

$= 0.15 * P(B | A') * P(A')$

$= 0.15 * 0.20 * 0.07 = 0.021$

Hence $P(C) = 0.06 + 0.504 + 0.045 + 0.021 = 0.63$

(d) Find the probability that the river is polluted given that fishing is permitted and the sample tested did not detect pollution

Solution: The required probability $= P(A | C \cap B') = P(A \cap B' \cap C)/P(C \cap B') = 0.06/0.546$

$= 0.106$