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Solution(1)

- Given Total number of documents: 12,000
- Documents retrieved by the system: 3,000
- Relevant documents among retrieved: 2,000
- Total relevant documents: 4,000

TP = Relevant documents that were retrieved = 2000

FP = Irrelevant documents that were retrieved. = 3000 – 2000 =1000

FN = Total relevant - Relevant retrieved = 4000 - 2000 = 2,000

TN = Total documents - (TP + FP + FN)= 12000 - (2000 + 1000 + 2000) = 7000

Confusion Matrix

	Relevant	Irrelevant
Retrieved	TP=2000	FP=1000
Not Retrieved	FN=2000	TN=7000

Recall (Sensitivity): TP/TP+FN = 2000/2000+2000 = 0.5

Precision: TP/TP+FP = 2000/3000=0.667

True Positive Rate (TPR) (same as Recall/Sensitivity): 0.5

False Positive Rate (FPR): FP/FP+TN = 1000/1000+7000 = 0.125

Sensitivity (Recall): 0.5

Specificity: TN/TN+FP = 7000/7000+1000 = 0.875

Solution(2)

Given

Sample#	Actual Class	Predicted probability of Yes
1	Yes	0.95
2	No	0.7
3	Yes	0.95
4	Yes	0.4

5	No	0.75
6	No	0.65
7	Yes	0.99
8	Yes	0.98
9	No	0.55
10	No	0.97

Sort the data in the descending order of Prediction probability

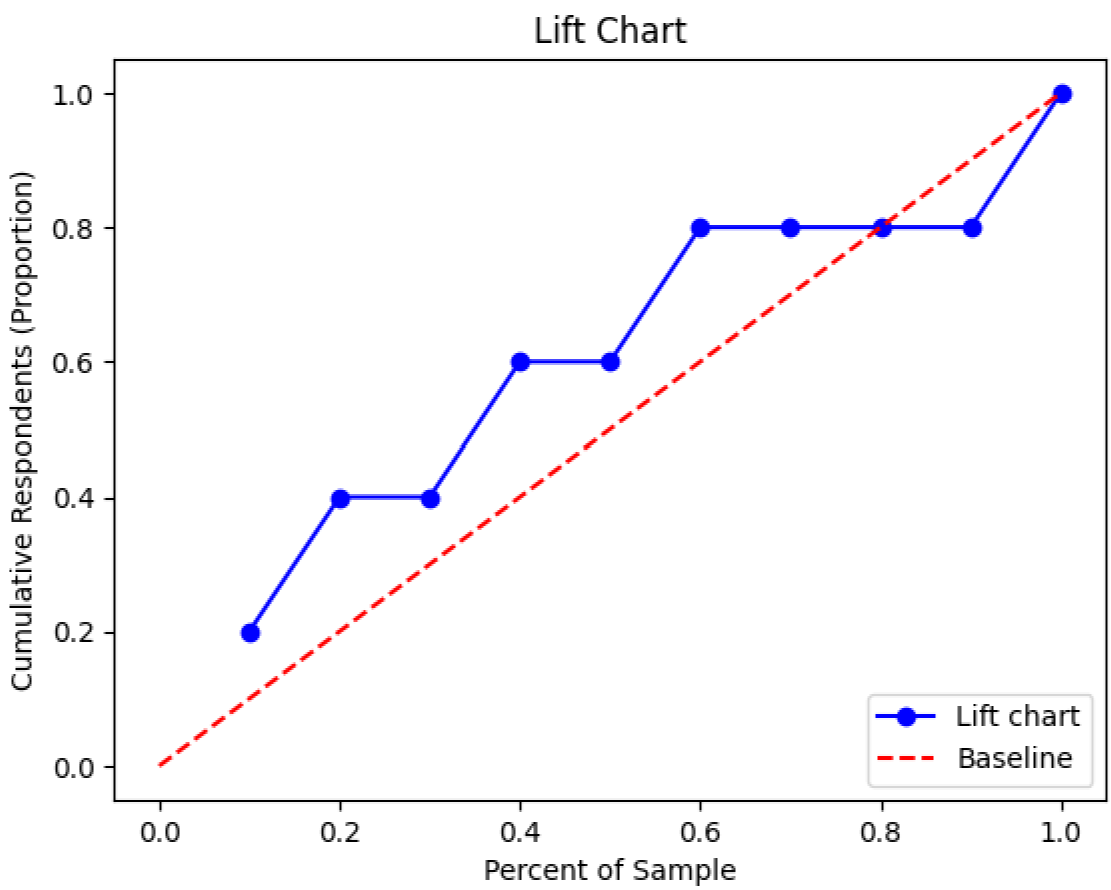
Sample#	Actual Class	Predicted probability of Yes
7	Yes	0.99
8	Yes	0.98
10	No	0.97
1	Yes	0.95
3	Yes	0.95
5	No	0.75
2	No	0.7
6	No	0.65
9	No	0.55
4	Yes	0.4

Calculate the %sample size and total respondents

Sample#	Actual Class	Predicted probability of Yes	%Sample	#Total Respondents
7	Yes	0.99	0.1	1
8	Yes	0.98	0.2	2
10	No	0.97	0.3	2
1	Yes	0.95	0.4	3
3	Yes	0.95	0.5	3
5	No	0.75	0.6	4
2	No	0.7	0.7	4
6	No	0.65	0.8	4
9	No	0.55	0.9	4
4	Yes	0.4	1	5

Plot Lift Curve

```python
import matplotlib.pyplot as plt

# Given data
percent_sample = [0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0]
cumulative_respondents = [1, 2, 2, 3, 3, 4, 4, 4, 4, 5]

# Calculate total respondents
total_respondents_at_end = cumulative_respondents[-1]

# Calculate cumulative percentage
cumulative_percentage = [x / total_respondents_at_end for x in cumulative_respondents]

# Plotting the lift chart
plt.figure()
plt.plot(percent_sample, cumulative_percentage, marker='o', linestyle='-', color='b', label='Lift chart')

# Adding reference line for baseline (random model)
plt.plot([0, 1], [0, 1], linestyle='--', color='r', label='Baseline')

# Adding labels and title
plt.xlabel('Percent of Sample')
plt.ylabel('Cumulative Respondents (Proportion)')
plt.title('Lift Chart')
plt.legend(loc='lower right')

# Show plot
plt.show()
```

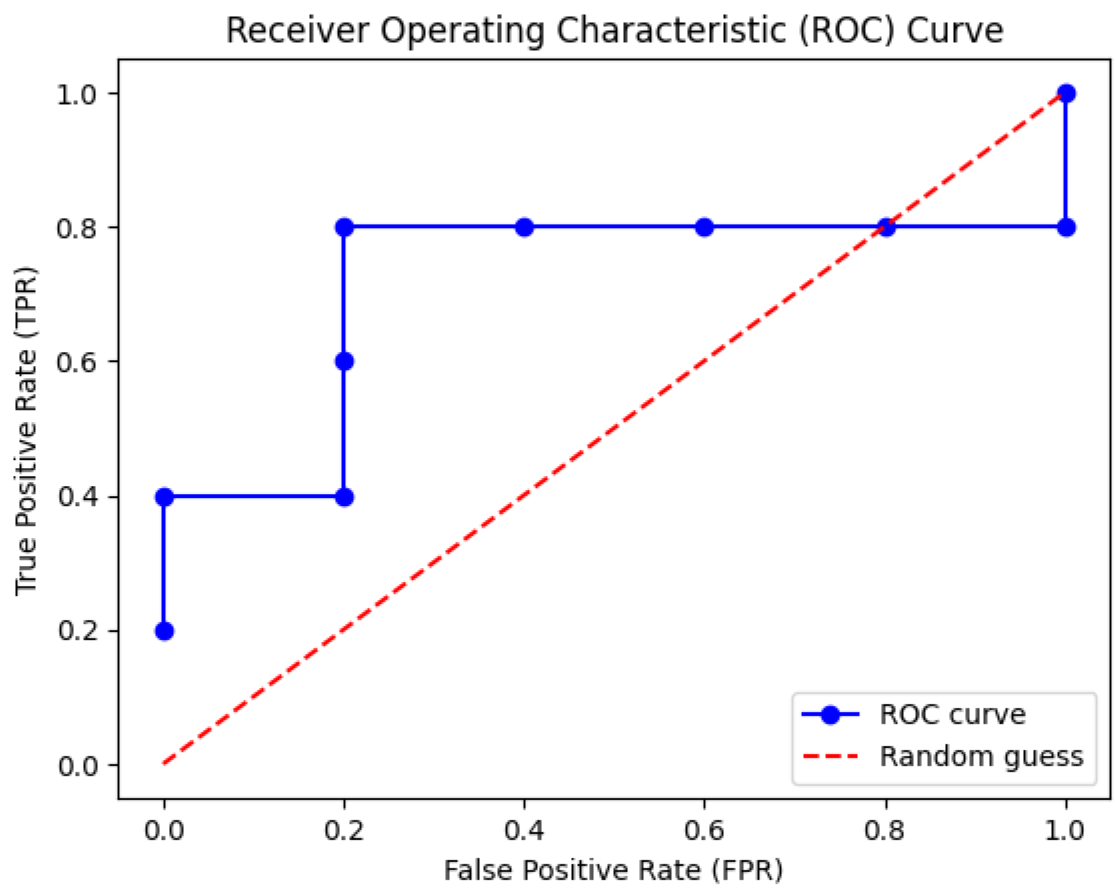


Calculate the TPR and FPR for ROC Curve

Sample#	Actual Class	Predicted probability of Yes	TP	FP	TPR	FPR	%FP	%TP
7	Yes	0.99	1	0	0.2	0	0	0
8	Yes	0.98	2	0	0.4	0	0	20
10	No	0.97	2	1	0.4	0.2	0	40
1	Yes	0.95	3	1	0.6	0.2	20	40
3	Yes	0.95	3	1	0.8	0.2	20	60
5	No	0.75	4	2	0.8	0.4	40	80
2	No	0.7	4	3	0.8	0.6	60	80
6	No	0.65	4	4	0.8	0.8	80	80
9	No	0.55	4	5	0.8	1	100	80
4	Yes	0.4	5	5	1	1	100	100

Plot ROC Curve

```python
import matplotlib.pyplot as plt

# Given data
tpr = [0.2, 0.4, 0.4, 0.6, 0.8, 0.8, 0.8, 0.8, 0.8, 1]
fpr = [0, 0, 0.2, 0.2, 0.2, 0.4, 0.6, 0.8, 1, 1]

# Plotting the ROC curve
plt.figure()
plt.plot(fpr, tpr, marker='o', linestyle='-', color='b', label='ROC curve')
plt.plot([0, 1], [0, 1], linestyle='--', color='r', label='Random guess')

# Adding labels and title
plt.xlabel('False Positive Rate (FPR)')
plt.ylabel('True Positive Rate (TPR)')
plt.title('Receiver Operating Characteristic (ROC) Curve')
plt.legend(loc='lower right')

# Show plot
plt.show()
```



Solution(3)

Student 1: Actual grade = F

Predicted probabilities: A: 0.1, B: 0.1, C: 0.2, D: 0.4, F: 0.2

y=[0,0,0,0,1]

Quadratic Loss:

(0−0.1)2+(0−0.1)2+(0−0.2)2+(0−0.4)2+(1−0.2)2=0.01+0.01+0.04+0.16+0.64=0.86

Information Loss (Log Loss):−log_2(0.2)= 2.32193

Student 2: Actual grade = C

Predicted probabilities: A: 0.2, B: 0.1, C: 0.5, D: 0.15, F: 0.05

y=[0,0,1,0,0]

Quadratic Loss:

(0−0.2)^2+(0−0.1) ^2+(1−0.5) ^2+(0−0.15) ^2+(0−0.05) ^2=0.04+0.01+0.25+0.0225+0.0025=0.325

Information Loss (Log Loss): −log_2(0.5)= 1

Student 3: Actual grade = B

Predicted probabilities: A: 0.3, B: 0.6, C: 0.15, D: 0.03, F: 0.02

y=[0,1,0,0,0]

Quadratic Loss:

(0−0.3) ^2+(1−0.6) ^2+(0−0.15) ^2+(0−0.03) ^2+(0−0.02) ^2=0.09+0.16+0.0225+0.0009+0.0004=0.2738

Information Loss (Log Loss):−log_2(0.6)= 0.7369

Student 4: Actual grade = A

Predicted probabilities: A: 0.7, B: 0.2, C: 0.05, D: 0.03, F: 0.02

y=[1,0,0,0,0]

Quadratic Loss:

(1−0.7) ^2+(0−0.2) ^2+(0−0.05) ^2+(0−0.03) ^2+(0−0.02) ^2=0.09+0.04+0.0025+0.0009+0.0004=0.1338

Information Loss (Log Loss):−log_2(0.7)= 0.5145

Student 5: Actual grade = D

Predicted probabilities: A: 0.1, B: 0.2, C: 0.1, D: 0.5, F: 0.1

y=[0,0,0,1,0]

Quadratic Loss:

(0−0.1) ^2+(0−0.2) ^2+(0−0.1) ^2+(1−0.5) ^2+(0−0.1) ^2=0.01+0.04+0.01+0.25+0.01=0.32

Information Loss (Log Loss):−log_2(0.5)= 1

Solution(4)

Given

Instance#	Actual Salary	Predicted Salary
1	75	85
2	95	70
3	105	100

4	65	55
5	85	100
6	75	75
7	80	60
8	95	100
9	90	75
10	70	85

RMSE=1/n∑(Actuali−Predictedi)2 from i=1 to n

Sum of squared errors = (85 – 75)^2+ (70 -95)^2+(100 – 105)^2+(55 – 65)^2+(100 -85)^2+(75 – 75)^2+(60 -80)^2+(100 – 95)^2 +(75 -90)^2+(85 – 70)^2= 100+625+25+100+225+0+400+25+225+225=1950

Mean Squared Error = 1950/10 = 195

Root mean squared Error =sqrt(195)=13.96

MAE=1/n∑|Actuali−Predictedi|, $i = 1$ to n

Sum of absolute error = (85 – 75)+ (70 -95)+(100 – 105)+(55 – 65)+(100 -85)+(75 – 75)+(60 -80)+(100 – 95)+(75 -90)+(85 – 70)=120

Mean Absolute Error = 120/10 = 12

Compute Actual Mean =Abar= 835/10 = 83.5

Compute Predicted Mean = Pbar=805/10 = 80.5

Compute Deviation from mean

Instance#	Actual Salary	Predicted Salary	Ai-Abar	Pi-Pbar
1	75	85	-8.5	4.5
2	95	70	11.5	-10.5
3	105	100	21.5	19.5
4	65	55	-18.5	-25.5
5	85	100	1.5	19.5
6	75	75	-8.5	-5.5
7	80	60	-3.5	-20.5
8	95	100	11.5	19.5
9	90	75	6.5	-5.5
10	70	85	-13.5	4.5

Compute the products of the deviations

(Ai -Abar)*(Pi -Pbar)

Instance#	Actual Salary	Predicted Salary	Ai-Abar	Pi-Pbar	(Ai-Abar)*(Pi-Pbar)
1	75	85	-8.5	4.5	-38.25
2	95	70	11.5	-10.5	-120.75
3	105	100	21.5	19.5	419.25
4	65	55	-18.5	-25.5	471.75
5	85	100	1.5	19.5	29.25
6	75	75	-8.5	-5.5	46.75
7	80	60	-3.5	-20.5	71.75
8	95	100	11.5	19.5	224.25
9	90	75	6.5	-5.5	-35.75
10	70	85	-13.5	4.5	-60.75

Sum the products of the deviations =
−38.25−120.75+419.25+471.75+29.25+46.75+71.75+224.25−35.75−60.75= 1007.5

SPA=1007.5/9=111.95

Compute the sum of squares of the deviations

Instance	Ai-Abar	(Ai - Abar)^2
1	-8.5	72.25
2	11.5	132.25
3	21.5	462.25
4	-18.5	342.25
5	1.5	2.25
6	-8.5	72.25
7	-3.5	12.25
8	11.5	132.25
9	6.5	42.25
10	-13.5	182.25

SA = 72.25+132.25+462.25+342.25+2.25+72.25+12.25+132.25+42.25+182.25=1452.5/n-1 = 161.39

Instance	(Pi- Pbar)	(Pi- Pbar)^2
1	4.5	20.25
2	-10.5	110.25
3	19.5	380.25
4	-25.5	650.25

5	19.5	380.25
6	-5.5	30.25
7	-20.5	420.25
8	19.5	380.25
9	-5.5	30.25
10	4.5	20.25

SP = 20.25+110.25+380.25+650.25+380.25+30.25+420.25+380.25+30.25+20.25=2422.5/n-1 = 269.17

R = SPA/sqrt(SP)*(SA) = 111.95/sqrt(161.39*269.17)= 0.5371