Newton's Academy
MATHEMATICS AND STATISTICS

Time: 3 Hrs. **Max. Marks: 80**

General instructions:

*The question paper is divided into **FOUR** sections.*

(1) ***Section A:*** *Q. 1 contains **Eight** multiple choice type of questions, each carrying **Two** marks.*
*Q. 2 contains **Four** very short answer type questions, each carrying **one** mark.*

(2) ***Section B:*** *Q. 3 to Q. 14 contain **Twelve** short answer type questions, each carrying **Two** marks. (Attempt any **Eight**)*

(3) ***Section C:*** *Q. 15 to Q. 26 contain **Twelve** short answer type questions, each carrying **Three** marks. (Attempt any **Eight**)*

(4) ***Section D:*** *Q. 27 to Q. 34 contain **Eight** long answer type questions, each carrying **Four** marks. (Attempt any **Five**)*

(5) *Use of log table is allowed. Use of calculator is not allowed.*

(6) *Figures to the right indicate full marks.*

(7) *Use of graph paper is not necessary. Only rough sketch of graph is expected.*

(8) *For each multiple choice type of question, it is mandatory to write the correct answer along with its alphabet, e.g., (a)……. / (b)…….. /(c)…….. /(d)……., etc. No marks shall be given, if ONLY the correct answer or the alphabet of correct answer is written. Only the first attempt will be considered for evaluation.*

(9) *Start answer to each section on a new page.*

SECTION – A

Q.1. Select and write the correct answer for the following multiple choice type of questions: **[16]**

i. The dual of statement $p \wedge \sim q$ is equivalent to ___________.

(a) $\sim p \wedge q$ (b) $p \leftrightarrow q$ (c) $\sim p \vee q$ (d) $\sim p \rightarrow \sim q$ (2)

ii. If $|\bar{a}| = 3$, $|\bar{b}| = 4$, then a value of λ for which $\bar{a} + \lambda\bar{b}$ is perpendicular to $\bar{a} - \lambda\bar{b}$ is _______.

(a) $\frac{9}{16}$ (b) $\frac{3}{4}$ (c) $\frac{3}{2}$ (d) $\frac{4}{3}$ (2)

iii. The acute angle between the lines $\frac{x-1}{1} = \frac{y-2}{-1} = \frac{z-3}{2}$ and $\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-3}{1}$ is _______.

(a) $60°$ (b) $30°$ (c) $45°$ (d) $90°$ (2)

iv. The vector equation of the plane passing through point $A(\bar{a})$ and parallel to $\bar{b}$ and $\bar{c}$ is _______.

(a) $(\bar{r} - \bar{a}) \times (\bar{b} \times \bar{c}) = \bar{0}$
(b) $(\bar{r} - \bar{a}) \cdot (\bar{b} + \bar{c}) = 0$
(c) $(\bar{r} - \bar{a}) \cdot (\bar{b} \times \bar{c}) = 0$
(d) $(\bar{r} - \bar{a}) \times (\bar{b} - \bar{c}) = \bar{0}$ (2)

v. If $x = at^4$, $y = 2at^2$, then $\frac{dy}{dx} =$ _______.

(a) $\frac{1}{t^2}$ (b) t^2 (c) $2t^2$ (d) $-\frac{1}{t^2}$ (2)

vi. The area bounded by the curve $y = 2x$, the Y-axis, the X-axis and $x = 3$ is _______.

(a) 3 sq. units (b) 6 sq. units (c) 9 sq. units (d) 12 sq. units (2)

vii. The differential equation $y\frac{dy}{dx}+x=0$ represents family of _______.

(a) circle (b) parabola (c) ellipse (d) hyperbola (2)

viii. If $f(x) = kx^2(1-x)$, for $0 < x < 1$
$= 0$, otherwise, is the probability distribution function of a random variable X, then the value of k is _______.

(a) 12 (b) 10 (c) –9 (d) –12 (2)

Q.2. Answer the following questions: **[4]**

i. Write the domain of inverse secant function. (1)

ii. Find the value of k, if lines represented by $kx^2 + 4xy - 4y^2 = 0$ are perpendicular to each other. (1)

iii. Evaluate: $\int \frac{1}{x\sqrt{\log x}}dx$ (1)

iv. Obtain the differential equation by eliminating the arbitrary constant from the equation $y^2 = 4ax$. (1)

SECTION – B

Attempt any EIGHT of the following questions: **[16]**

Q.3. Write the following compound statements symbolically:

(i) Nagpur is in Maharashtra and Chennai is in Tamilnadu.

(ii) If ΔABC is right angled at B, then $m\angle A + m\angle C = 90°$ (2)

Q.4. Find the co-factors of the elements a_{11} and a_{21} of matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$. (2)

Q.5. Find the solution of $\cos\theta = \frac{1}{2}$, where $0 \le \theta < 2\pi$. (2)

Q.6. Find the joint equation of lines passing through the origin having slopes, 2 and 3. (2)

Q.7. If the vectors $-3\hat{i}+4\hat{j}-2\hat{k}$, $\hat{i}+2\hat{k}$ and $\hat{i}-p\hat{j}$ are coplanar, then find the value of p. (2)

Q.8. Find the direction cosines of the vector $2\hat{i}+2\hat{j}-\hat{k}$. (2)

Q.9. Find the equation of tangent to the curve $y = x^2 + 2e^x + 2$ at the point (0, 4). (2)

Q.10. Evaluate: $\int \frac{3^x - 4^x}{5^x}dx$ (2)

Q.11. Evaluate: $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \cos x \, dx$ (2)

Q.12. Find the area of the region bounded by the curve $y = x^2$, the X-axis and the lines $x = 1$, $x = 3$. (2)

Q.13. Find the integrating factor (I. F.) of the differential equation:

$\frac{dy}{dx} + y = e^{-x}$ (2)

Q.14. Given that X ~ B (n, p).

If p = 0.6 and E(X) = 6, find n and Var(x). (2)

SECTION – C

Attempt any EIGHT of the following questions: **[24]**

Q.15. Prove that $\tan^{-1} 1 + \tan^{-1} 2 + \tan^{-1} 3 = \pi$ (3)

Q.16. If one of the lines given by $ax^2 + 2hxy + by^2 = 0$ bisects an angle between the co-ordinate axes then show that $(a + b)^2 = 4h^2$ (3)

Q.17. Let $A(\bar{a})$ and $B(\bar{b})$ be any two points in the space and $R(\bar{r})$ be the third point on the line AB dividing the segment AB externally in the ratio m : n. Then prove that $\bar{r} = \dfrac{m\bar{b} - n\bar{a}}{m - n}$ (3)

Q.18. Find the position vector of point P such that OP is inclined to X – axis at 45° and to Y – axis at 60° and OP = 12 units. (3)

Q.19. Find the vector equation of the line passing through the point having position vector $2\hat{i} + \hat{j} - 3\hat{k}$ and perpendicular to vectors $\hat{i} + \hat{j} + \hat{k}$ and $\hat{i} + 2\hat{j} - \hat{k}$. (3)

Q.20. The foot of perpendicular drawn from the origin to a plane is M(1, 2, 0). Find the vector equation of the plane. (3)

Q.21. If $\log_{10}\left(\dfrac{x^3 - y^3}{x^3 + y^3}\right) = 2$, show that $\dfrac{dy}{dx} = \dfrac{-99x^2}{101y^2}$ (3)

Q.22. A wire of length 36 meters is bent in the form of rectangle. Find its dimensions if the area of the rectangle is maximum. (3)

Q.23. Evaluate: $\displaystyle\int \frac{e^x}{1 + e^{-x}}\, dx$ (3)

Q.24. Solve the differential equation:

$\dfrac{dy}{dx} = (4x + y + 1)^2$ (3)

Q.25. The probability distribution of X is as follows:

X	0	1	2	3	4
$P(X = x)$	0.1	K	2K	2K	K

Find: (i) K (ii) $P(X < 2)$ (iii) $P(X \geq 3)$ (3)

Q.26. Ten eggs are drawn successively with replacement from a lot containing 10% defective eggs. Find the probability that there is at least one defective egg. (3)

SECTION – D

Attempt any FIVE of the following questions: [20]

Q.27. Construct the truth table for the statement pattern $(p \to q) \wedge [(q \to r) \to (p \to r)]$ and interpret your result. (4)

Q.28. Solve the following equations by using method of inversion:

$x - y + z = 4,\ 2x + y - 3z = 0,\ x + y + z = 2$ (4)

Q.29. Prove that, in ΔABC,

$\tan\left(\dfrac{A - B}{2}\right) = \left(\dfrac{a - b}{a + b}\right) \cot\left(\dfrac{c}{2}\right)$ (4)

Q.30. Solve the linear programming problem by graphical method:

Maximize: $z = 3x + 5y$

Subject to: $x + 4y \leq 24$

$3x + y \leq 21$

$x + y \leq 9$ and $x \geq 0, y \geq 0$

Also find maximum value of z. (4)

Q.31. If $y = f(x)$ is a differentiable function of x on an interval 1 and y is one-one, onto and $\frac{dy}{dx} \neq 0$ on 1,

then prove that $\frac{dx}{dy} = \frac{1}{\frac{dy}{dx}}$.

where $\frac{dy}{dx} \neq 0$. Hence prove that $\frac{d}{dx}(\cot^{-1}x) = \frac{-1}{1+x^2}$ (4)

Q.32. The volume of the spherical ball is increasing at the rate of 4π cc/sec. Find the rate at which the radius and the surface area are changing when the volume is 288π cc. (4)

Q.33. Prove that: $\int \frac{1}{x^2 - a^2}\, dx = \frac{1}{2a} \log \left|\frac{x-a}{x+a}\right| + c$

Hence evaluate: $\int \frac{1}{x^2 - 3} dx$ (4)

Q.34. Evaluate: $\int_0^{\frac{\pi}{2}} \cos^3 x dx$ (4)