Newton's Academy
MATHEMATICS AND STATISTICS

Time: 3 Hrs. **Max. Marks: 80**

General instructions:
The question paper is divided into ***FOUR*** *sections.*

(1) ***Section A:*** *Q. 1 contains* ***Eight*** *multiple choice type of questions, each carrying* ***Two*** *marks.*
Q. 2 contains ***Four*** *very short answer type questions, each carrying* ***one*** *mark.*

(2) ***Section B:*** *Q. 3 to Q. 14 contain* ***Twelve*** *short answer type questions, each carrying* ***Two*** *marks. (Attempt any* ***Eight****)*

(3) ***Section C:*** *Q. 15 to Q. 26 contain* ***Twelve*** *short answer type questions, each carrying* ***Three*** *marks. (Attempt any* ***Eight****)*

(4) ***Section D:*** *Q. 27 to Q. 34 contain* ***Eight*** *long answer type questions, each carrying* ***Four*** *marks. (Attempt any* ***Five****)*

(5) Use of log table is allowed. Use of calculator is not allowed.

(6) Figures to the right indicate full marks.

(7) Use of graph paper is not necessary. Only rough sketch of graph is expected.

(8) For each multiple choice type of question, it is mandatory to write the correct answer along with its alphabet, e.g. (a)……. / (b)……. /(c)……. /(d)……., etc. No marks shall be given, if ONLY the correct answer or the alphabet of correct answer is written. Only the first attempt will be considered for evaluation.

(9) Start answer to each section on a new page.

SECTION – A

Q.1. Select and write the correct answer for the following multiple choice type of questions: **[16]**

i. If $p \wedge q$ is F, $p \to q$ is F then the truth values of p and q are _______ respectively.
(a) T, T (b) T, F (c) F, T (d) F, F (2)

ii. In ΔABC, if $c^2 + a^2 - b^2 = ac$, then $\angle B =$ _______.
(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{6}$ (2)

iii. The area of the triangle with vertices (1, 2, 0), (1, 0, 2) and (0, 3, 1) in sq. unit is _______.
(a) $\sqrt{5}$ (b) $\sqrt{7}$ (c) $\sqrt{6}$ (d) $\sqrt{3}$ (2)

iv. If the corner points of the feasible solution are (0, 10), (2, 2) and (4, 0) then the point of minimum $z = 3x + 2y$ is _______.
(a) (2, 2) (b) (0, 10) (c) (4, 0) (d) (3, 4) (2)

v. If y is a function of x and $\log(x + y) = 2xy$, then the value of $y'(0) =$ _______.
(a) 2 (b) 0 (c) -1 (d) 1 (2)

vi. $\int \cos^3 x \, dx =$ _______.
(a) $\frac{1}{12}\sin 3x + \frac{3}{4}\sin x + c$ (b) $\frac{1}{12}\sin 3x + \frac{1}{4}\sin x + c$
(c) $\frac{1}{12}\sin 3x - \frac{3}{4}\sin x + c$ (d) $\frac{1}{12}\sin 3x - \frac{1}{4}\sin x + c$ (2)

vii. The solution of the differential equation $\frac{dx}{dt} = \frac{x\log x}{t}$ is _______
(a) $x = e^{ct}$ (b) $x + e^{ct} = 0$
(c) $x = e^t + t$ (d) $xe^{ct} = 0$ (2)

viii. Let the probability mass function (p.m.f.) of a random variable X be $P(X = x) = {}^4C_x \left(\frac{5}{9}\right)^x \times \left(\frac{4}{9}\right)^{4-x}$, for $x = 0, 1, 2, 3, 4$ then E(X) is equal to ________

(a) $\frac{20}{9}$ (b) $\frac{9}{20}$ (c) $\frac{12}{9}$ (d) $\frac{9}{25}$ (2)

Q.2. Answer the following questions: **[4]**

i. Write the joint equation of co-ordinate axes. (1)

ii. Find the values of c which satisfy $|c\bar{u}| = 3$ where $\bar{u} = \hat{i} + 2\hat{j} + 3\hat{k}$. (1)

iii. Write $\int \cot x \, dx$. (1)

iv. Write the degree of the differential equation $e^{\frac{dy}{dx}} + \frac{dy}{dx} = x$ (1)

SECTION – B

Attempt any EIGHT of the following questions: **[16]**

Q.3. Write inverse and contrapositive of the following statement:
If $x < y$ then $x^2 < y^2$ (2)

Q.4. If $A = \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$ is a non singular matrix, then find A^{-1} by elementary row transformations.

Hence write the inverse of $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ (2)

Q.5. Find the cartesian co-ordinates of the point whose polar co-ordinates are $\left(\sqrt{2}, \frac{\pi}{4}\right)$. (2)

Q.6. If $ax^2 + 2hxy + by^2 = 0$ represents a pair of lines and $h^2 = ab \neq 0$ then find the ratio of their slopes. (2)

Q.7. If $\bar{a}, \bar{b}, \bar{c}$ are the position vectors of the points A, B, C respectively and $5\bar{a} + 3\bar{b} - 8\bar{c} = \bar{0}$ then find the ratio in which the point C divides the line segment AB. (2)

Q.8. Solve the following inequations graphically and write the corner points of the feasible region:
$2x + 3y \leq 6$, $x + y \geq 2$, $x \geq 0$, $y \geq 0$ (2)

Q.9. Show that the function $f(x) = x^3 + 10x + 7$, $x \in R$ is strictly increasing. (2)

Q.10. Evaluate: $\int_0^{\frac{\pi}{2}} \sqrt{1 - \cos 4x} \, dx$ (2)

Q.11. Find the area of the region bounded by the curve $y^2 = 4x$, the X-axis and the lines $x = 1$, $x = 4$ for $y \geq 0$. (2)

Q.12. Solve the differential equation
$\cos x \cos y \, dy - \sin x \sin y \, dx = 0$ (2)

Q.13. Find the mean of number randomly selected from 1 to 15. (2)

Q.14. Find the area of the region bounded by the curve $y = x^2$ and the line $y = 4$. (2)

SECTION – C

Attempt any EIGHT of the following questions: **[24]**

Q.15. Find the general solution of $\sin\theta + \sin 3\theta + \sin 5\theta = 0$ (3)

Q.16. If $-1 \le x \le 1$, the prove that $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$ (3)

Q.17. If θ is the acute angle between the lines represented by $ax^2 + 2hxy + by^2 = 0$ then prove that $\tan\theta = \left|\frac{2\sqrt{h^2 - ab}}{a+b}\right|$ (3)

Q.18. Find the direction ratios of a vector perpendicular to the two lines whose direction ratios are $-2, 1, -1$ and $-3, -4, 1$. (3)

Q.19. Find the shortest distance between lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$. (3)

Q.20. Lines $\bar{r} = \left(\hat{i}+\hat{j}-\hat{k}\right)+\lambda\left(2\hat{i}-2\hat{j}+\hat{k}\right)$ and $\bar{r} = \left(4\hat{i}-3\hat{j}+2\hat{k}\right)+\mu\left(\hat{i}-2\hat{j}+2\hat{k}\right)$ are coplanar. Find the equation of the plane determined by them. (3)

Q.21. If $y = \sqrt{\tan x + \sqrt{\tan x + \sqrt{\tan x + ...+\infty}}}$, then

show that $\frac{dy}{dx} = \frac{\sec^2 x}{2y-1}$.

Find $\frac{dy}{dx}$ at $x = 0$. (3)

Q.22. Find the approximate value of $\sin(30°30')$.
Give that $1° = 0.0175^c$ and $\cos 30° = 0.866$ (3)

Q.23. Evaluate $\int x \tan^{-1} x \, dx$ (3)

Q.24. Find the particular solution of the differential equation $\frac{dy}{dx} = e^{2y}\cos x$, when $x = \frac{\pi}{6}$, $y = 0$ (3)

Q.25. For the following probability density function of a random variable X, find (a) $P(X < 1)$ and (b) $P(|X| < 1)$.

$f(x) = \frac{x+2}{18}$; for $-2 < x < 4$

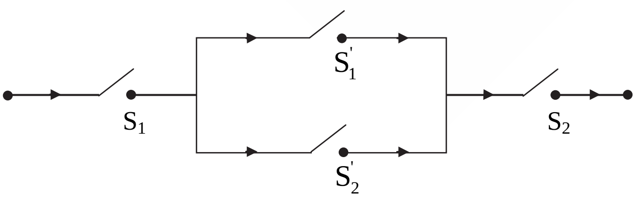
$= 0$, otherwise (3)

Q.26. A die is thrown 6 times. If 'getting an odd number' is a success, find the probability of at least 5 successes. (3)

SECTION – D

Attempt any FIVE of the following questions: **[20]**

Q.27. Simplify the given circuit by writing its logical expression. Also write your conclusion.



(4)

Q.28. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ verify that $A(\text{adj}A) = (\text{adj}A)A = |A|I$ (4)

Q.29. Prove that the volume of a tetrahedron with coterminus edges $\bar{a}$, $\bar{b}$ and $\bar{c}$ is $\frac{1}{6}\left[\bar{a}\,\bar{b}\,\bar{c}\right]$.

Hence, find the volume of tetrahedron whose coterminus edges are $\bar{a} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\bar{b} = -\hat{i} + \hat{j} + 2\hat{k}$ and $\bar{c} = 2\hat{i} + \hat{j} + 4\hat{k}$. (4)

Q.30. Find the length of the perpendicular drawn from the point P(3, 2, 1) to the line $\bar{r} = \left(7\hat{i} + 7\hat{j} + 6\hat{k}\right) + \lambda\left(-2\hat{i} + 2\hat{j} + 3\hat{k}\right)$ (4)

Q.31. If $y = \cos(m \cos^{-1} x)$ then show that $(1 - x^2)\frac{d^2 y}{dx^2} - x\frac{dy}{dx} + m^2 y = 0$ (4)

Q.32. Verify Lagrange's mean value theorem for the function $f(x) = \sqrt{x+4}$ on the interval [0, 5]. (4)

Q.33. Evaluate: $\int \frac{2x^2 - 3}{\left(x^2 - 5\right)\left(x^2 + 4\right)} dx$ (4)

Q.34. Prove that: $\int_0^{2a} f(x)\,dx = \int_0^{a} f(x)\,dx + \int_0^{a} f(2a - x)\,dx$ (4)