

Exercise - 1

OPERATION OF THE GENERATOR AND OSCILLOSCOPE. DETERMINATION OF AMPLITUDE AND PHASE CHARACTERISTICS ON THE EXAMPLE OF RC FILTER.

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1 Purpose of the exercise

- Getting to know the basic functions of the generator and oscilloscope.
- Consolidation of the method of determining the amplitude and phase characteristics using two methods.

2 Theoretical basics

2.1 Frequency characteristics

The basic method of determining the properties of linear systems is the determination of Bode characteristics, i.e. frequency characteristics. The frequency response describes the response of the system to a sinusoidal input with a frequency varying within a given range.

Let the input signal be determined as follows

$$u_{we}(t) = U_{we} \sin(2\pi ft - \varphi_{we}), \quad (1)$$

then the steady-state response of the linear system can be described as

$$u_{wy}(t) = U_{wy} \sin(2\pi ft - \varphi_{wy}). \quad (2)$$

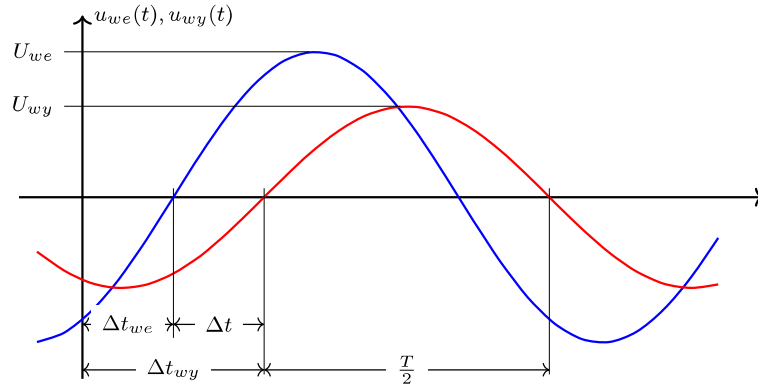


Figure 1: Input signal waveform and system response

The system response is a sinusoidal signal of the same frequency but generally different in amplitude and phase. The change in the amplitude and phase of the signal after passing through the system depends on the frequency f .

Amplitude characteristics $K_u = K_u(f)$ it is the ratio of the amplitude of the output signal to the amplitude of the input signal as a function of frequency:

$$K_u(f) = \frac{U_{wy}(f)}{U_{we}(f)} \quad (3)$$

In electronics, a logarithmic measure is often given $K_{u[dB]}$ amplitude ratio $K_u = \frac{U_{wy}}{U_{we}}$. We define the logarithmic measure, the unit of which is the decibel, as follows:

$$K_{u[dB]} = 20 \log_{10} K_u = 20 \log_{10} \frac{U_{wy}}{U_{we}} [dB].$$

Phase characteristics $\varphi = \varphi(f)$ it is the phase shift of the output signal with respect to the input signal as a function of frequency:

$$\varphi(f) = \varphi_{we}(f) - \varphi_{wy}(f), \quad (4)$$

where:

- input signal phase

$$\varphi_{we}(f) = \frac{\Delta t_{we}}{T} 360 = \Delta t_{we} f 360, \quad (5)$$

- the phase of the output signal

$$\varphi_{wy}(f) = \frac{\Delta t_{wy}}{T} 360 = \Delta t_{wy} f 360, \quad (6)$$

- phase shift of the output signal to the input signal

$$\varphi = \frac{\Delta t_{we} - \Delta t_{wy}}{T} 360 = \frac{-\Delta t}{T} 360 = -\Delta t f 360.$$

When the offset is negative, it means that the output signal lags behind the input signal.

The amplitude characteristic shows how the system amplifies or attenuates certain spectral components of the signal depending on their frequency. The phase characteristics show how the circuit delays the output signal with respect to the input signal for different frequencies of the input signal.

The frequency axis (horizontal) of the amplitude and phase characteristic plot may be scaled in Hertz or radians per second, both linearly and logarithmically. The amplitude axis (vertical) is unitless, but can be scaled in decibels. The phase shift (vertical) axis can be graduated in degrees or radians.

The following terms are used to describe the system and its amplitude characteristics:

- **3 dB frequency response** - the frequency range for which the system introduces attenuation not greater than $3dB$, i.e. the output signal does not decrease more $\sqrt{2}$ - times,
- **cutoff frequency** - the frequency limit at which the system's bandwidth ends, i.e. the frequency for which the attenuation is $3dB$, so $K_u = \frac{U_{wy}}{U_{we}} = \frac{1}{\sqrt{2}}$
- **slope of the characteristic** - it is the rate of fall or rise of the amplitude characteristic, measured in decibels per decade (i.e. gain change in decibels per 10-fold frequency change) or decibels per octave (i.e. gain change in decibels per 2-fold frequency change)

2.2 Experimental determination of the amplitude and phase characteristics

The experimental determination of the amplitude characteristics of the system consists in applying a sinusoidal signal of constant amplitude to its input, and measuring the amplitude of the signal observed at the output of the system. The quotient of the output signal amplitude to the input signal amplitude is the vertical coordinate of a single point on the amplitude characteristic plot, where the horizontal coordinate is the signal frequency.

The experimental determination of the phase characteristic consists in measuring the shift between the input and output signal. The phase shift can be measured using two methods: the classical method and the Lissajous figure method. The classical method is based on measuring the phase shift as the angle corresponding to the delay time interval between the output and input signal. The Lissajous figure method consists in determining the angle on the basis of an ellipse obtained for a given frequency.

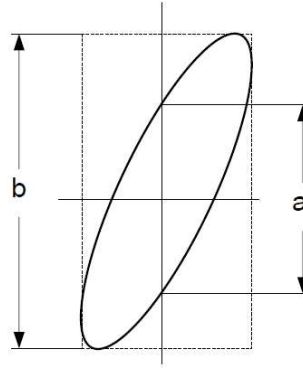


Figure 2: Displacement measurement with a Lissajous ellipse

Knowing the parameters a and b Lissajous ellipses, the shift is defined as

$$\varphi = \arcsin \frac{a}{b} \quad (7)$$

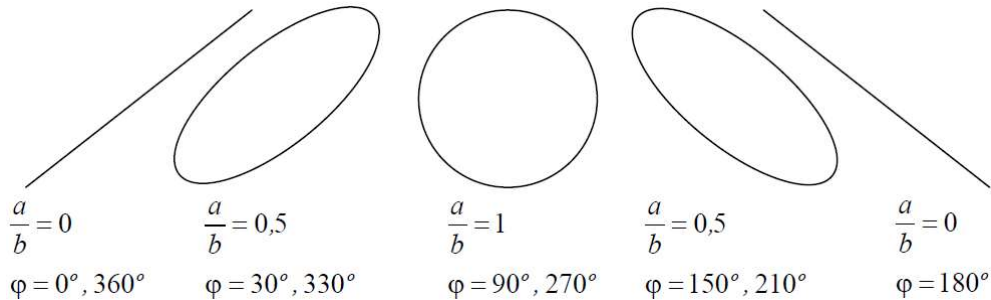


Figure 3: Examples of Lissajous ellipses

The precise drawing of the characteristics requires the repeated repetition of such a measurement for a wide range of frequencies and connecting the points with a continuous line.

2.3 1st order RC low-pass filter

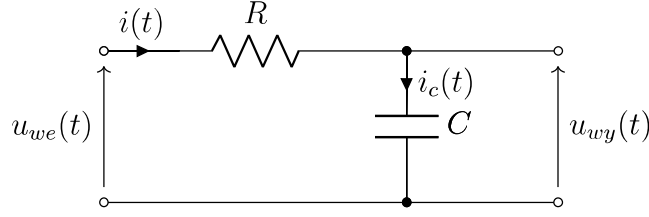


Figure 4: 1st order RC low-pass filter

For the above system, the following equations and the resulting differential equation can be written:

$$\left. \begin{aligned} u_{we}(t) &= u_{wy}(t) + Ri(t) \\ i(t) &= i_c(t) = C \frac{du_{wy}(t)}{dt} \end{aligned} \right\} \Rightarrow u_{we}(t) = u_{wy}(t) + RC \frac{du_{wy}(t)}{dt}$$

Applying the Laplace transform to the above equation we get **operator transmittance of the first order RC filter** :

$$H(s) = \frac{U_{wy}(s)}{U_{we}(s)} = \frac{1}{RCs+1}.$$

Substituting $s = j\omega$ we get relationship describing the amplitude characteristic:

$$K_u(\omega) = |H(s = j\omega)| = \frac{1}{\sqrt{1 + (RC\omega)^2}}. \quad (8)$$

The relationship describing the phase characteristics:

$$\varphi(\omega) = \arg(H(s = j\omega)) = \arctan(-RC\omega). \quad (9)$$

Knowing the dependence that describes the amplitude characteristics, the cutoff frequency ω_g the first order RC low-pass filter can be determined from the following equation:

$$-3dB = 20 \log_{10} K_u(\omega_g). \quad (10)$$

Taking into account the properties of logarithm, dependence 8 and $\omega_g = 2\pi f_g$ z 10 we get the formula for **cut-off frequency of the 1st order RC filter** :

$$f_g = \frac{1}{2\pi RC}.$$

3 The course of the exercise

3.1 Getting to know the operation of the oscilloscope and generator

Tasks to be completed:

- oscilloscope, generator - overview - connection of the oscilloscope to the generator,
- channel on and off, timeline and amplitude positioning, timeline and amplitude gain, trigger, AC and DC coupling, XT mode, XY mode
- measurements of signal parameters, cursors, measurement of displacements, saving waveforms (pendrive 1GB)

3.2 Determination of the amplitude and phase characteristics

Tasks to be completed:

- performing measurements using two methods:
 - classic method - oscilloscope in XT mode,
 - Lissajous figure method - oscilloscope in XY mode.
- making calculations and plotting the amplitude and phase characteristics

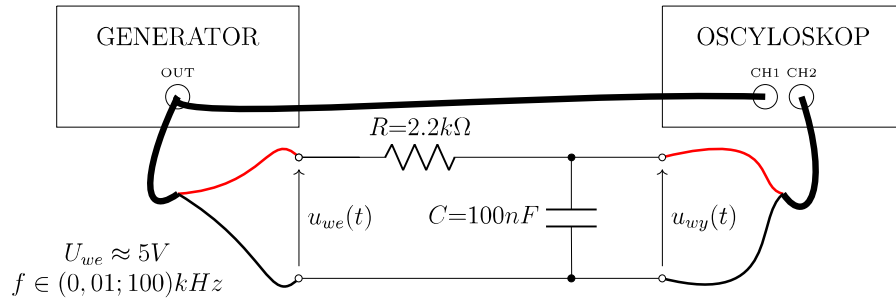


Figure 5: Scheme of the measuring system

Connect the circuit as shown in the picture 5. Set AC coupling on the first and second channels of the oscilloscope. Put a sinusoidal wave at the input of the system. Measure the peak-to-peak voltage U_{pp}^{we} at the input and U_{pp}^{wy} at the output. Measure the frequency of the input signal and the offset between the input signal and the output signal. Perform measurements for signals with a frequency ranging from $10Hz$ to $100kHz$. Make calculations K_u , $K_{u[dB]}$ i φ . Record the measurement results in the table 1. In the picture 7 and 8 plot the amplitude and phase characteristics.

3.3 Recording of waveforms from an oscilloscope

Observe the voltage waveforms at the input of the RC filter for different frequencies and for the following input signal shapes: sinusoidal, square and triangle. Then input a sinusoidal waveform and record Lissajous figures for different frequencies of the input signal.

4 Report

The report should contain:

- results of measurements and calculations
- sample calculations
- sample waveforms observed on the oscilloscope
- RC filter frequency characteristics obtained on the basis of measurements and comparing them with theoretical characteristics
- interpretation of characteristics taking into account the frequency response, cut-off frequency and steepness of the characteristic

5 Necessary equipment

- scientific calculator
- pendrive $1GB$ or a camera for recording waveforms from an oscilloscope
- protocol

Protocol

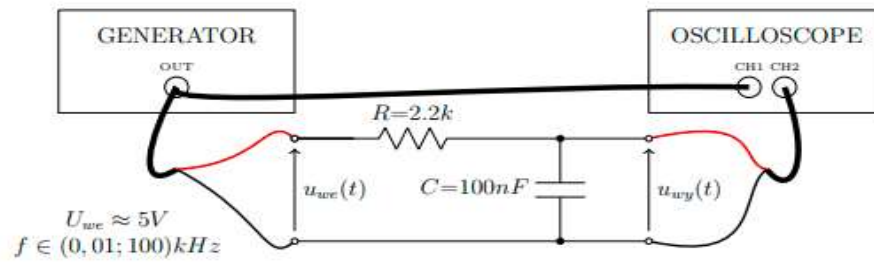


Figure 6: Scheme of the measuring system

Results of measurements and calculations

Measurements				Calculation results		
f[kHz]	Uwe V	u wy V	T	ku	kudb	shifted phase
0.049	9.84	9.68	0.32	0.9837398374	-0.1423948225	-5.6448
0.498	10	8.08	0.2	0.808	-1.851772785	-35.856
1	9.92	5.7	0.14	0.5745967742	-4.81273633	-50.4
2	10.2	3.48	0.1	0.3411764706	-9.340418556	-72
5	10.1	1.58	0.046	0.1564356436	-16.11328574	-82.8
10	10.2	0.816	0.023	0.08	-21.93820026	-82.8
15	10.2	0.556	0.016	0.05450980392	-25.2705076	-86.4
20	10.2	0.42	0.0116	0.04117647059	-27.70701763	-83.52
25	10.2	0.352	0.0096	0.03450980392	-29.24115017	-86.4
30	10.2	0.292	0.008	0.02862745098	-30.86434641	-86.4
50	10.1	0.189	0.0047	0.01871287129	-34.55719139	-84.6
60.5	10.1	0.156	0.004	0.01544554455	-36.22393551	-87.12
65	10.6	0.142	0.0035	0.01339622642	-37.46035042	-81.9
71.18	10.6	0.132	0.0033	0.01245283019	-38.09463868	-84.56184
77.47	10.6	0.122	0.0031	0.01150943396	-38.77892069	-86.45652
83.51	10.6	0.114	0.003	0.01075471698	-39.36802028	-90.1908
91.74	10.6	0.108	0.0029	0.01018867925	-39.8376422	-95.77656
93.8	10.6	0.102	0.0027	0.009622641509	-40.33411387	-91.1736
99.99	10.6	0.1	0.0026	0.009433962264	-40.50611731	-93.59064
113.7	10.6	0.088	0.0014	0.008301886792	-41.61646386	-57.3048

