Can Infinity Collapse to Zero?
A Universal Infinite-Digit Representation System

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Abstract

We explore a universal representation for real numbers using infinite digits on both sides of the decimal point. This framework merges properties of real analysis [7] and b-adic number theory [5]. In this system, the symmetric maximal-digit expansion

$$\dots(b-1)(b-1)(b-1)\ .\ (b-1)(b-1)(b-1)\dots$$

evaluates to 0, leading to the provocative question of whether the "largest" representable number collapses to zero. We show how negative numbers can be represented without an explicit minus sign, demonstrate generalized zero patterns, and unify left-infinite (b-adic) and right-infinite (real-analytic) interpretations.

1 Introduction

Traditional real numbers allow infinite expansion only to the right of the decimal point [4], while the left side remains finite. We propose a universal infinite-digit format:

$$\dots d_3 d_2 d_1 d_0\ .\ d'_0 d'_1 d'_2 \dots$$

This merges real decimal expansions with b-adic expansions [3]. We explore whether symmetric infinite expansions allow infinity and zero to coincide. A general numeration context is given in [2]. The broad philosophical motivation aligns with ideas in [1].

2 Standard Real Representations

Real numbers use finite left expansions and possibly infinite right expansions. Negative numbers require an explicit minus sign. Classical decimal structure is reviewed in [4].

3 Extending the Representation

We define the general infinite-digit representation and show how left-infinite digits behave analogously to b-adic integers while right-infinite digits behave like real limits.

4 Infinite Left-Digit Expansions

Consider the b-adic sequence:

$$(b^1-1),(b^2-1),(b^3-1),\dots$$

In the b-adic metric, $b^n \to 0$, so $b^n - 1 \to -1$. This follows directly from standard treatments of p-adic convergence [5]. Hence:

$$\dots(b-1)(b-1)(b-1) = -1.$$

Figure 1: Universal infinite-digit representation

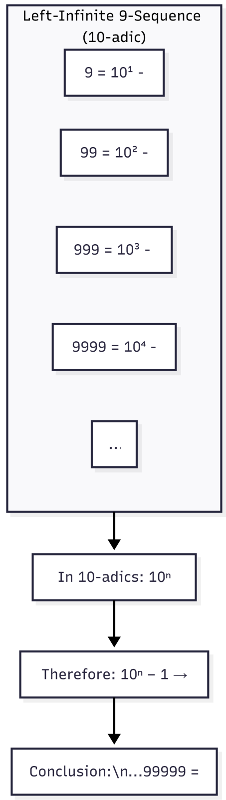


Figure 2: Limit-based interpretation of $\ldots$(b-1)(b-1)(b-1) as -1 in b-adic numbers

5 Why the Right Side Equals +1

Real analysis gives:

$$.(b{-}1)(b{-}1)(b{-}1)\ldots = \frac{b-1}{b} + \frac{b-1}{b^2} + \cdots = 1.$$

This is the classical limit of geometric series as treated in standard real-analysis texts [7]. The identity $0.999\ldots = 1$ is discussed in depth in [6].

6 Symmetric Combination: The Collapse to Zero

$$\ldots(b{-}1)(b{-}1)(b{-}1)\ .\ (b{-}1)(b{-}1)(b{-}1)\ldots \quad \Rightarrow \quad (-1)+1=0.$$

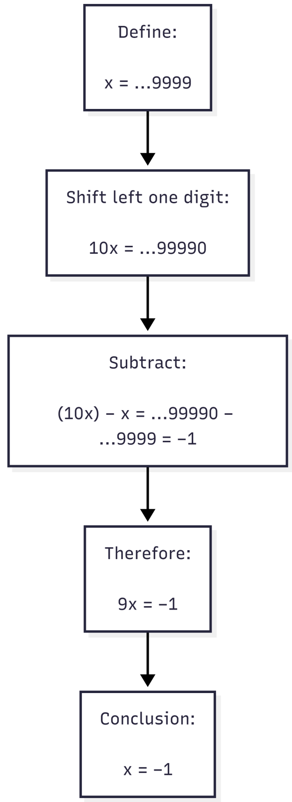


Figure 3: Algebraic shift proof that . . . (b-1)(b-1)(b-1) = -1

7 General Constant-Digit Zero Patterns

Any constant-digit symmetric expansion:

$$\dots dddd\,.\,dddd\dots$$

evaluates to zero. This follows from the scaling identity

$$d = \frac{d}{b-1}(b-1),$$

and the fact that the maximal-digit expansion equals zero. This connects with numeral system theory [2].

8 Negative Numbers Without a Minus Sign

We represent negative numbers without writing a minus sign:

$$-1 = \dots 1110.1111\dots$$

$$-37 = \dots 666629.66666\dots$$

$$-37 = \dots 444407.444444\dots$$

This parallels complement-based constructions in numeral systems [2].

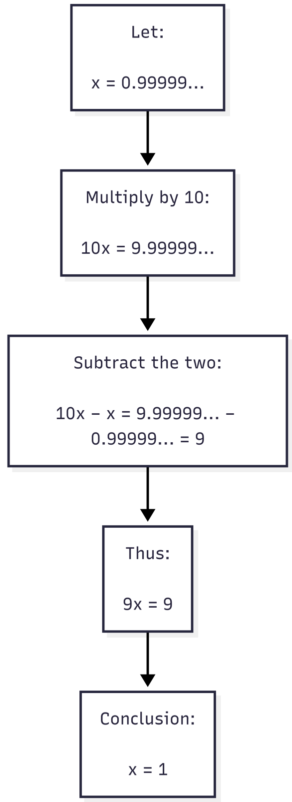


Figure 4: Algebraic proof that $0.999\ldots = 1$

9 Invalid Patterns: Why 9999999... Does Not Represent Anything

Patterns without a decimal anchor do not define a left/right structure. This is consistent with definitions of positional numeral systems [2].

10 Unified Architecture

11 Conclusion

We unify real and b-adic number systems into a single infinite-digit representation. The collapse of the maximal symmetric pattern to zero, and the ability to encode negative numbers without a minus sign, suggest the presence of a unified two-metric numerical structure combining real limits [7] and b-adic convergence [5].

References

[1] Can infinity collapse to zero? (video inspiration). `https://youtu.be/tRaq4aYPzCc?si=88m_NhE_9kM2Vz7C`. YouTube reference, accessed 2025.

[2] Numeral system. `https://en.wikipedia.org/wiki/Numeral_system`. Accessed 2025.

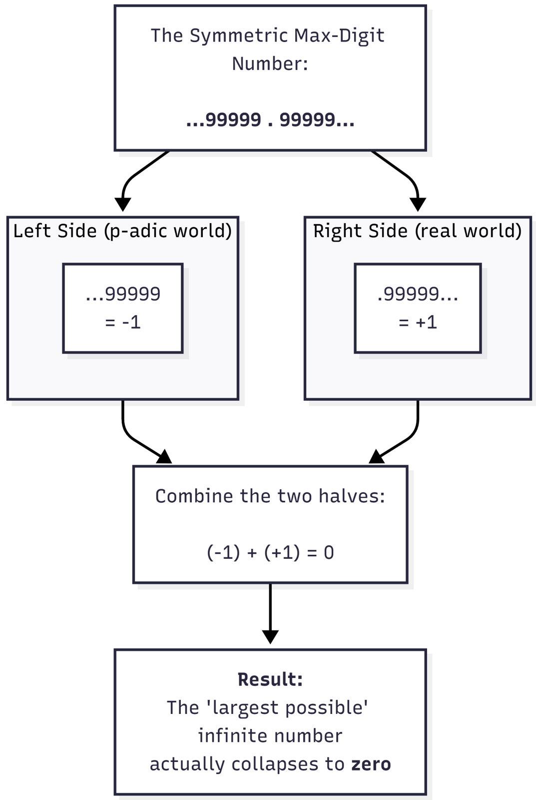


Figure 5: The largest symmetric expansion collapses to zero

[3] p-adic number. `https://en.wikipedia.org/wiki/P-adic_number`. Accessed 2025.

[4] Repeating decimal. `https://en.wikipedia.org/wiki/Repeating_decimal`. Accessed 2025.

[5] Fernando Q. Gouvêa. *p-adic Numbers: An Introduction*. Springer, 2nd edition, 1997.

[6] Konrad Knopp. On the representation 0.999... = 1. *Mathematische Zeitschrift*, 1928. Classic discussion of repeating decimal limits.

[7] Walter Rudin. *Principles of Mathematical Analysis*. McGraw-Hill, 3rd edition, 1976.

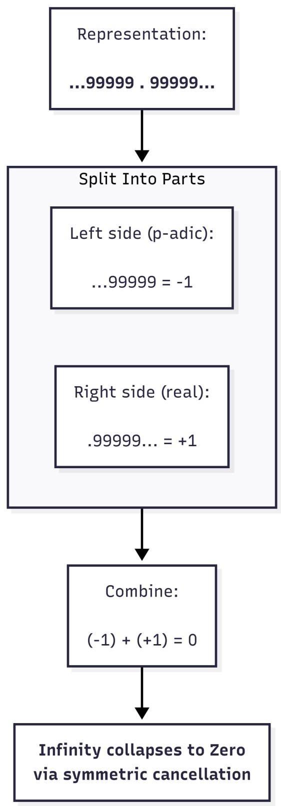


Figure 6: Cancellation of left-infinite and right-infinite contributions

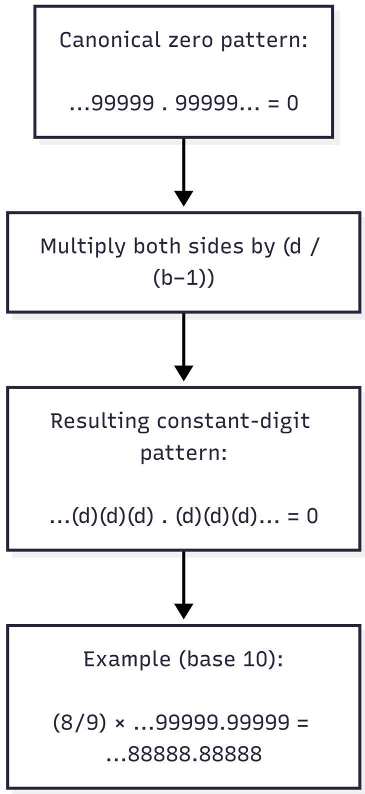


Figure 7: All constant-digit symmetric patterns represent zero

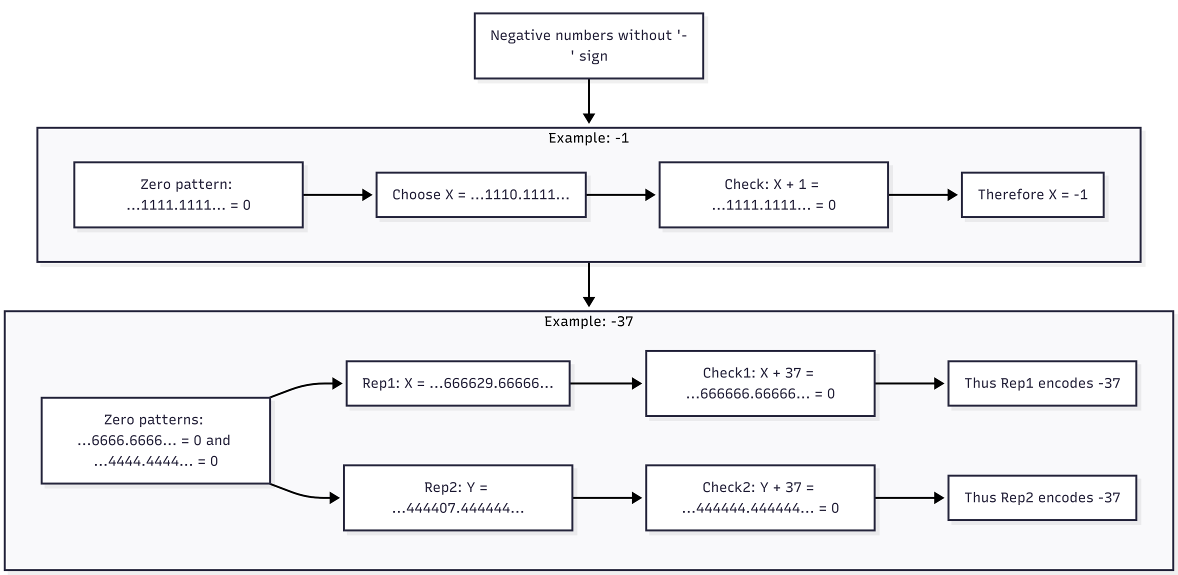


Figure 8: Representing negative numbers without using the minus sign

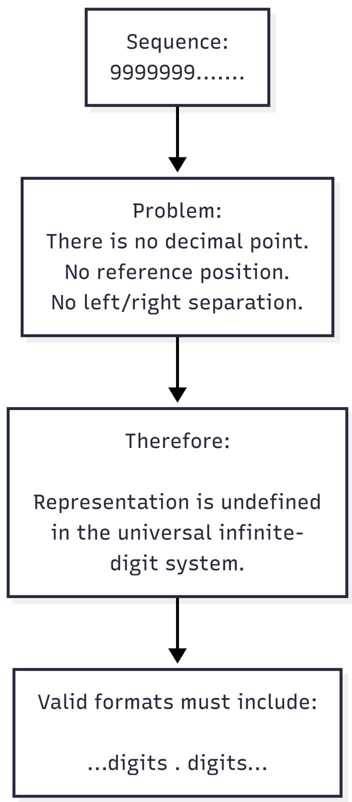
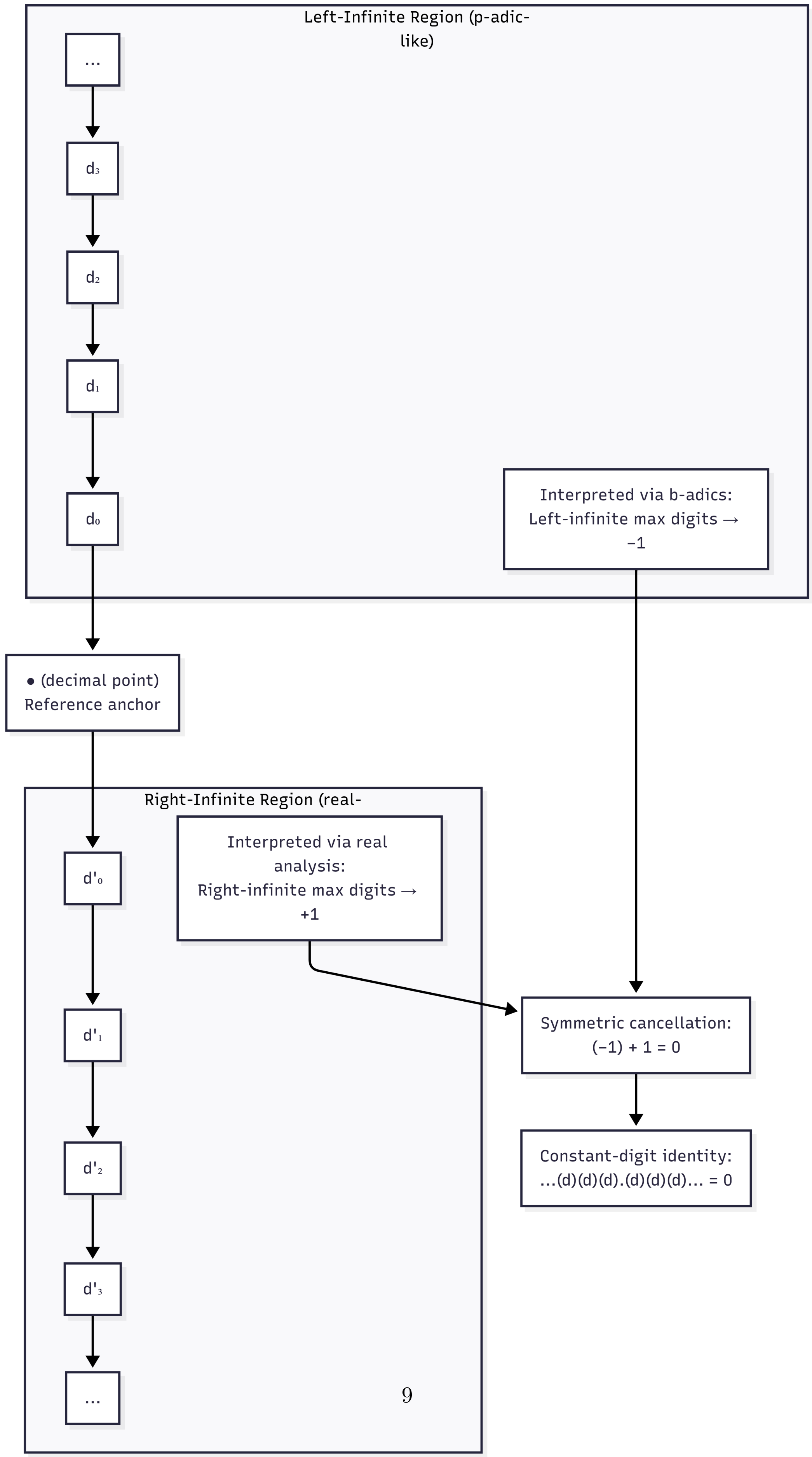


Figure 9: Why patterns without a decimal point are undefined



Left-Infinite Region (p-adic-like)
...
d₃
d₂
d₁
d₀
Interpreted via b-adics: Left-infinite max digits → −1
• (decimal point) Reference anchor
Right-Infinite Region (real-
Interpreted via real analysis: Right-infinite max digits → +1
d'₀
d'₁
d'₂
d'₃
...
Symmetric cancellation: (−1) + 1 = 0
Constant-digit identity: …(d)(d)(d).(d)(d)(d)… = 0